Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.7-Miscellaneous/136-4.7.2-trig^m-a-trig+b-trig-^n

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [294]. This is test number [136].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (294)	0.00 (0)
Mathematica	100.00 (294)	0.00 (0)
Fricas	98.64 (290)	1.36 (4)
Mupad	98.64 (290)	1.36 (4)
Maple	98.30 (289)	1.70 (5)
Giac	95.58 (281)	4.42 (13)
Maxima	92.18 (271)	7.82 (23)
Sympy	22.79 (67)	77.21 (227)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

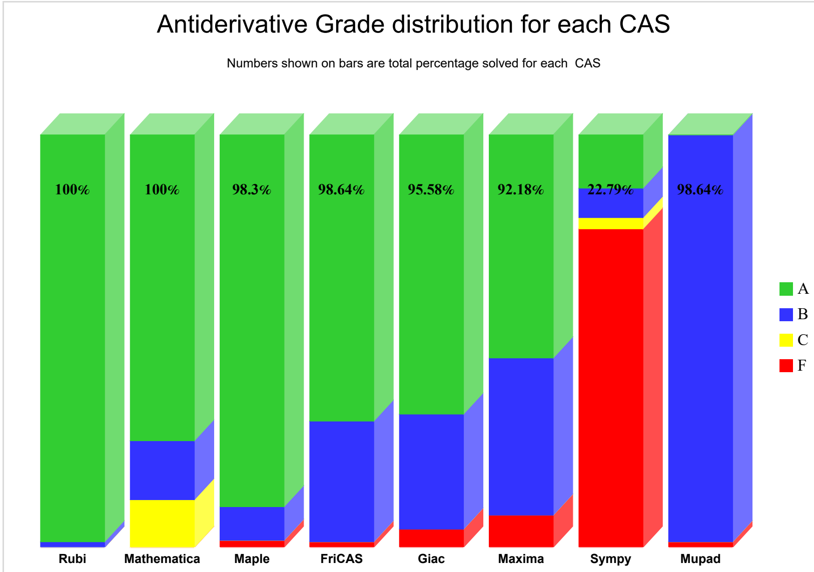
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

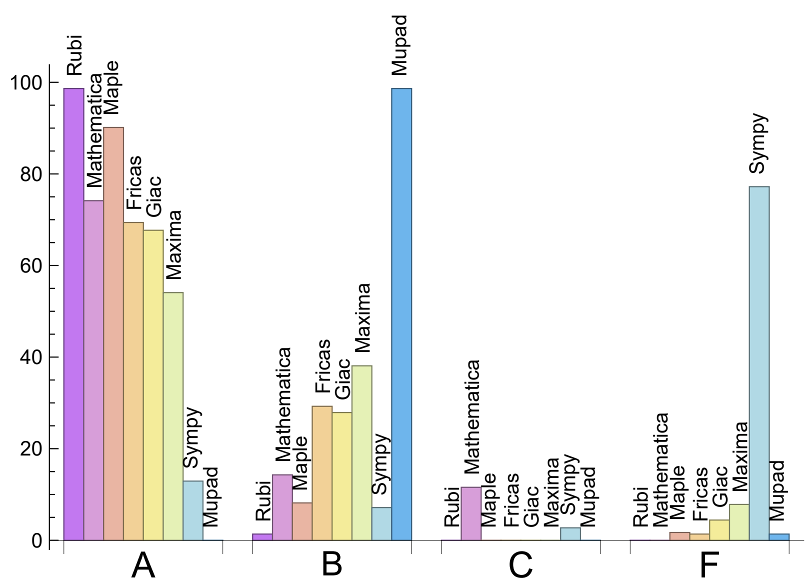
System	% A grade	% B grade	% C grade	% F grade
Rubi	98.64	1.36	0.00	0.00
Maple	90.14	8.16	0.00	1.70
Mathematica	74.15	14.29	11.56	0.00
Fricas	69.39	29.25	0.00	1.36
Giac	67.69	27.89	0.00	4.42
Maxima	54.08	38.10	0.00	7.82
Sympy	12.93	7.14	2.72	77.21
Mupad	N/A	98.64	0.00	1.36

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	5	100.00 %	0.00 %	0.00 %
Fricas	4	100.00 %	0.00 %	0.00 %
Giac	13	38.46 %	7.69 %	53.85 %
Maxima	23	17.39 %	0.00 %	82.61 %
Sympy	227	64.76 %	26.87 %	8.37 %
Mupad	4	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

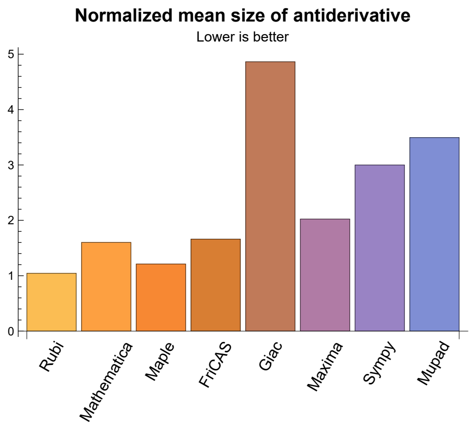
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

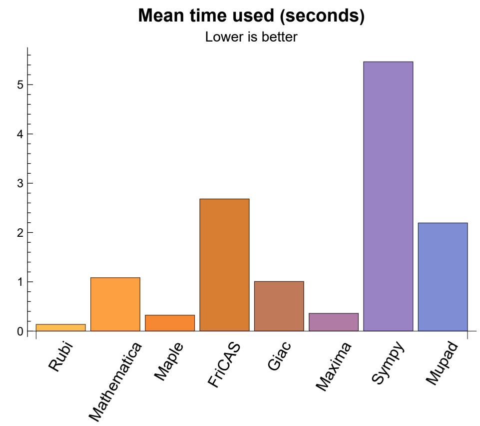
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.14	112.64	1.04	86.50	1.00
Mathematica	1.08	178.74	1.60	98.50	1.04
Maple	0.32	118.13	1.21	97.00	1.05
Maxima	0.36	177.42	2.02	118.00	1.50
Fricas	2.68	172.00	1.66	115.50	1.20
Sympy	5.46	277.63	3.00	175.00	1.87
Giac	1.01	414.11	4.86	118.00	1.47
Mupad	2.19	408.84	3.49	149.50	1.79

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
1
2
3
4
5
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
1
2
3
4
5
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{\}$

Mathematica $\{29, 272\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

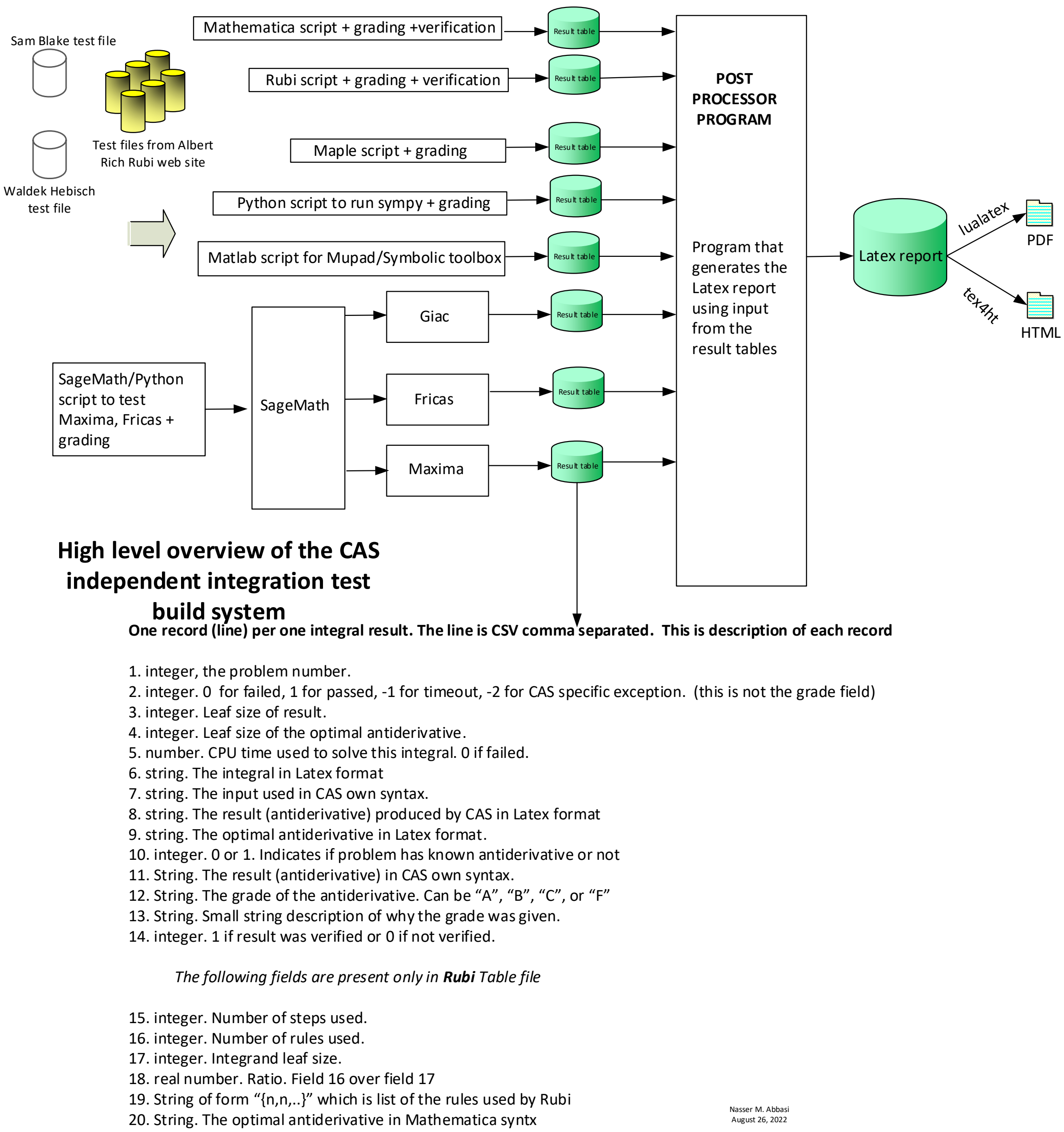
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

B grade: { 15, 23, 131, 142 }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 7, 9, 11, 12, 13, 14, 15, 17, 18, 19, 20, 23, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 64, 69, 71, 73, 74, 75, 76, 78, 79, 80, 82, 87, 89, 90, 91, 92, 93, 95, 96, 97, 101, 106, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 118, 120, 122, 123, 125, 126, 127, 128, 130, 133, 136, 137, 138, 140, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 170, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 186, 187, 189, 190, 194, 195, 196, 197, 198, 201, 202, 204, 206, 208, 210, 212, 213, 214, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 265, 266, 269, 270, 271, 273, 274, 275, 277, 279, 281, 283, 285, 287, 288, 289, 291, 293, 294 }

B grade: { 6, 21, 24, 63, 66, 67, 68, 70, 72, 81, 84, 85, 86, 88, 98, 99, 100, 103, 104, 105, 119, 121, 134, 143, 169, 171, 173, 179, 185, 188, 191, 192, 193, 199, 200, 203, 205, 207, 209, 211, 241, 242 }

C grade: { 8, 10, 16, 22, 25, 29, 50, 65, 77, 83, 94, 102, 112, 124, 129, 131, 132, 135, 139, 141, 142, 215, 264, 267, 268, 272, 276, 278, 280, 282, 284, 286, 290, 292 }

F grade: { }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 143, 145, 146, 147, 148, 149, 150, 152, 153, 154, 155, 156, 158, 160, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 179, 181, 182, 183, 185, 188, 189, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 204, 206, 207, 208, 210, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 273, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

B grade: { 23, 24, 25, 67, 85, 104, 133, 142, 144, 151, 157, 159, 161, 163, 173, 180, 184, 186, 190, 197, 203, 205, 209, 211 }

C grade: { }

F grade: { 29, 187, 271, 272, 274 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 9, 11, 16, 18, 20, 27, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 113, 115, 122, 124, 126, 128, 130, 143, 145, 147, 149, 155, 167, 168, 170, 172, 174, 178, 179, 180, 181, 193, 198, 200, 201, 202, 206, 208, 212, 214, 217, 218, 219, 224, 225, 227, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 271, 273, 275, 284, 294 }

B grade: { 8, 10, 12, 13, 14, 15, 17, 19, 21, 22, 23, 24, 25, 26, 28, 67, 85, 104, 110, 111, 112, 114, 116, 117, 118, 119, 120, 121, 123, 125, 127, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 146, 148, 156, 157, 158, 159, 160, 161, 162, 169, 171, 173, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 194, 195, 196, 197, 199, 203, 204, 205, 207, 209, 210, 211, 213, 215, 216, 220, 221, 222, 223, 226, 228, 264, 265, 266, 267, 268, 269, 270, 276, 277, 278, 279, 280, 281, 282, 283, 285, 286, 287, 288, 289, 290, 291, 292, 293 }

C grade: { }

F grade: { 29, 150, 151, 152, 153, 154, 163, 164, 165, 166, 175, 176, 177, 187, 257, 258, 259, 260, 261, 262, 263, 272, 274 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 7, 8, 10, 12, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 114, 116, 118, 119, 120, 121, 122, 126, 139, 150, 151, 152, 153, 154, 155, 156, 158, 160, 162, 163, 164, 165, 166, 167, 168, 169, 170, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 188, 189, 190, 193, 194, 195, 196, 197, 198, 200, 201, 202, 203, 204, 206, 207, 208, 209, 210, 212, 213, 214, 216, 217, 218, 219, 221, 222, 224, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 273, 276, 277, 278, 280, 281, 282, 283, 284, 286, 288, 290, 292 }

B grade: { 6, 9, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 37, 67, 85, 104, 113, 115, 117, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 157, 159, 161, 171, 172, 173, 184, 185, 186, 191, 192, 199, 205, 211, 215, 220, 223, 225, 226, 264, 265, 266, 267, 268, 269, 270, 271, 275, 279, 285, 287, 289, 291, 293, 294 }

C grade: { }

F grade: { 29, 187, 272, 274 }

2.1.6 Sympy

A grade: { 2, 3, 4, 5, 6, 7, 31, 33, 35, 43, 45, 47, 57, 58, 59, 60, 74, 76, 77, 78, 92, 93, 94, 95, 150, 152, 154, 155, 164, 166, 167, 168, 175, 177, 178, 180, 202, 232 }

B grade: { 1, 30, 32, 34, 44, 46, 48, 61, 62, 75, 79, 96, 97, 151, 153, 163, 165, 176, 179, 188, 195 }

C grade: { 9, 10, 16, 113, 114, 124, 275, 284 }

F grade: { 8, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 36, 37, 38, 39, 40, 41, 42, 49, 50, 51, 52, 53, 54, 55, 56, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 115, 116, 117, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 156, 157, 158, 159, 160, 161, 162, 169, 170, 171, 172, 173, 174, 181, 182, 183, 184, 185, 186, 187, 189, 190, 191, 192, 193, 194, 196, 197, 198, 199, 200, 201, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 276, 277, 278, 279, 280, 281, 282, 283, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 14, 15, 17, 18, 19, 20, 21, 24, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 38, 40, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 54, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 69, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 87, 89, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 106, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 124, 125, 126, 127, 128, 129, 130, 131, 134, 136, 137, 138, 139, 140, 143, 145, 147, 148, 149, 150, 151, 152, 153, 154, 155, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 174, 175, 176, 177, 178, 180, 181, 182, 183, 184, 185, 186, 189, 191, 192, 193, 194, 196, 198, 199, 200, 201, 202, 204, 206, 207, 208, 210, 212, 213, 214, 216, 217, 218, 219, 221, 223, 225, 228, 232, 241, 242, 253, 254, 256, 258, 259, 260, 263, 275, 276, 277, 278, 279, 280, 281, 282, 285, 286, 287, 288, 290, 291, 294 }

B grade: { 5, 6, 13, 16, 22, 23, 25, 37, 39, 41, 53, 55, 67, 68, 70, 72, 84, 85, 86, 88, 90, 97, 104, 105, 107, 109, 121, 123, 132, 133, 135, 141, 142, 144, 146, 156, 173, 179, 188, 190, 195, 197, 203, 205, 209, 211, 215, 220, 222, 224, 226, 227, 229, 230, 231, 233, 234, 235, 236, 237, 238, 239, 240, 250, 251, 252, 255, 257, 261, 262, 264, 265, 266, 267, 268, 269, 270, 283, 284, 289, 292, 293 }

C grade: { }

F grade: { 29, 187, 243, 244, 245, 246, 247, 248, 249, 271, 272, 273, 274 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 273, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

C grade: { }

F grade: { 29, 187, 272, 274 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **N.S.** in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	34	28	25	36	75	33	35
N.S.	1	1.00	0.94	0.78	0.69	1.00	2.08	0.92	0.97
time (sec)	N/A	0.034	0.010	0.129	0.272	2.602	0.143	0.395	0.526

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	20	20	27	27	25	32
N.S.	1	1.00	1.08	0.83	0.83	1.12	1.12	1.04	1.33
time (sec)	N/A	0.029	0.008	0.047	0.270	2.228	0.085	0.410	0.454

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	21	21	19	37	19	19
N.S.	1	1.00	1.00	0.84	0.84	0.76	1.48	0.76	0.76
time (sec)	N/A	0.018	0.008	0.039	0.350	3.040	0.055	0.401	0.412

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	10	10	8	10	10
N.S.	1	1.00	1.00	1.10	1.00	1.00	0.80	1.00	1.00
time (sec)	N/A	0.006	0.007	0.033	0.294	3.119	0.008	0.399	0.399

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	9	10	9	11	8	24	54
N.S.	1	1.00	1.00	1.11	1.00	1.22	0.89	2.67	6.00
time (sec)	N/A	0.014	0.008	0.065	0.286	3.264	0.664	0.434	0.540

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	25	19	24	33	24	33	24
N.S.	1	1.00	2.08	1.58	2.00	2.75	2.00	2.75	2.00
time (sec)	N/A	0.024	0.012	0.062	0.276	2.438	1.021	0.407	0.397

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	15	19	17	13	14
N.S.	1	1.00	1.00	0.93	1.00	1.27	1.13	0.87	0.93
time (sec)	N/A	0.032	0.014	0.066	0.280	1.507	2.184	0.412	0.409

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	94	97	209	93	0	148	2500
N.S.	1	1.00	1.03	1.07	2.30	1.02	0.00	1.63	27.47
time (sec)	N/A	0.082	0.205	0.118	0.523	2.706	0.000	0.401	7.501

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	62	84	106	144	706	94	94
N.S.	1	1.00	0.91	1.24	1.56	2.12	10.38	1.38	1.38
time (sec)	N/A	0.060	0.173	0.124	0.491	2.121	110.149	0.466	0.581

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	47	47	88	46	165	55	970
N.S.	1	1.00	1.34	1.34	2.51	1.31	4.71	1.57	27.71
time (sec)	N/A	0.041	0.063	0.066	0.477	1.571	0.348	0.419	2.141

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	35	61	96	0	61	31
N.S.	1	1.00	1.06	0.97	1.69	2.67	0.00	1.69	0.86
time (sec)	N/A	0.012	0.027	0.076	0.496	2.398	0.000	0.411	1.052

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	20	21	48	44	0	22	32
N.S.	1	1.00	0.87	0.91	2.09	1.91	0.00	0.96	1.39
time (sec)	N/A	0.049	0.052	0.097	0.273	1.965	0.000	0.394	0.575

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	67	81	107	133	0	108	170
N.S.	1	1.00	1.22	1.47	1.95	2.42	0.00	1.96	3.09
time (sec)	N/A	0.048	0.138	0.164	0.493	2.171	0.000	0.453	0.678

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	48	53	119	117	0	78	91
N.S.	1	1.00	0.87	0.96	2.16	2.13	0.00	1.42	1.65
time (sec)	N/A	0.086	0.172	0.119	0.277	1.665	0.000	0.411	0.571

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	283	107	140	253	240	0	186	224
N.S.	1	2.64	1.00	1.31	2.36	2.24	0.00	1.74	2.09
time (sec)	N/A	0.871	0.477	0.245	0.501	1.699	0.000	0.422	0.836

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	121	81	117	132	976	139	626
N.S.	1	1.00	1.89	1.27	1.83	2.06	15.25	2.17	9.78
time (sec)	N/A	0.090	0.285	0.128	0.475	1.945	0.660	0.408	7.064

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	62	97	128	164	0	103	86
N.S.	1	1.00	1.03	1.62	2.13	2.73	0.00	1.72	1.43
time (sec)	N/A	0.034	0.163	0.132	0.500	2.680	0.000	0.439	0.612

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	14	39	0	13	29
N.S.	1	1.00	1.00	0.82	0.82	2.29	0.00	0.76	1.71
time (sec)	N/A	0.010	0.025	0.085	0.271	2.192	0.000	0.400	0.431

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	72	85	128	220	0	109	492
N.S.	1	1.00	1.14	1.35	2.03	3.49	0.00	1.73	7.81
time (sec)	N/A	0.043	0.354	0.200	0.495	3.389	0.000	0.447	0.812

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	76	56	62	134	0	63	114
N.S.	1	1.00	1.55	1.14	1.27	2.73	0.00	1.29	2.33
time (sec)	N/A	0.054	0.220	0.145	0.283	3.224	0.000	0.402	0.621

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	270	158	242	345	0	215	511
N.S.	1	1.00	2.29	1.34	2.05	2.92	0.00	1.82	4.33
time (sec)	N/A	0.132	2.029	0.249	0.492	3.426	0.000	0.445	0.744

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	114	134	359	282	0	242	2500
N.S.	1	1.00	1.16	1.37	3.66	2.88	0.00	2.47	25.51
time (sec)	N/A	0.135	0.885	0.329	0.503	2.790	0.000	0.438	8.599

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	300	92	212	299	282	0	197	263
N.S.	1	3.26	1.00	2.30	3.25	3.07	0.00	2.14	2.86
time (sec)	N/A	0.513	0.448	0.411	0.494	2.557	0.000	0.456	0.797

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	19	47	29	84	116	0	20	48
N.S.	1	1.27	3.13	1.93	5.60	7.73	0.00	1.33	3.20
time (sec)	N/A	0.017	0.105	0.252	0.282	2.171	0.000	0.409	0.493

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	101	157	250	225	0	166	216
N.S.	1	1.00	1.38	2.15	3.42	3.08	0.00	2.27	2.96
time (sec)	N/A	0.026	0.185	0.283	0.495	2.920	0.000	0.452	0.755

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	96	73	172	220	0	77	131
N.S.	1	1.00	1.63	1.24	2.92	3.73	0.00	1.31	2.22
time (sec)	N/A	0.056	0.250	0.325	0.294	2.553	0.000	0.433	0.711

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	193	163	276	463	0	212	813
N.S.	1	1.00	1.05	0.89	1.50	2.52	0.00	1.15	4.42
time (sec)	N/A	0.156	0.833	0.497	0.510	2.584	0.000	0.453	1.005

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	208	128	308	385	0	146	253
N.S.	1	1.00	1.78	1.09	2.63	3.29	0.00	1.25	2.16
time (sec)	N/A	0.097	0.857	0.402	0.305	2.207	0.000	0.427	0.856

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	367	0	0	0	0	0	-1
N.S.	1	1.00	5.56	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	3.859	0.162	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	57	62	62	62	175	95	149
N.S.	1	1.00	0.66	0.71	0.71	0.71	2.01	1.09	1.71
time (sec)	N/A	0.068	0.123	0.221	0.295	2.117	0.440	0.455	4.205

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	63	46	49	51	87	85	67
N.S.	1	1.00	1.05	0.77	0.82	0.85	1.45	1.42	1.12
time (sec)	N/A	0.048	0.022	0.168	0.281	2.507	0.250	0.436	0.464

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	62	52	48	51	128	65	107
N.S.	1	1.00	0.95	0.80	0.74	0.78	1.97	1.00	1.65
time (sec)	N/A	0.054	0.107	0.153	0.276	3.119	0.181	0.427	4.106

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	36	35	38	63	55	47
N.S.	1	1.00	1.00	0.82	0.80	0.86	1.43	1.25	1.07
time (sec)	N/A	0.044	0.017	0.126	0.288	3.310	0.110	0.399	0.437

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	46	41	37	35	73	35	35
N.S.	1	1.00	1.07	0.95	0.86	0.81	1.70	0.81	0.81
time (sec)	N/A	0.029	0.061	0.086	0.274	4.573	0.079	0.408	0.428

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	46	25	24	23	31	24	38
N.S.	1	1.00	1.92	1.04	1.00	0.96	1.29	1.00	1.58
time (sec)	N/A	0.010	0.019	0.070	0.266	2.846	0.052	0.420	0.357

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	23	30	21	0	27	70
N.S.	1	1.00	1.00	1.35	1.76	1.24	0.00	1.59	4.12
time (sec)	N/A	0.018	0.017	0.138	0.285	3.054	0.000	0.422	0.572

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	32	40	54	0	54	38
N.S.	1	1.00	1.00	1.33	1.67	2.25	0.00	2.25	1.58
time (sec)	N/A	0.031	0.022	0.156	0.285	2.933	0.000	0.421	0.413

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	30	30	0	25	23
N.S.	1	1.00	1.00	0.89	1.07	1.07	0.00	0.89	0.82
time (sec)	N/A	0.042	0.020	0.192	0.282	2.938	0.000	0.436	0.399

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	50	61	74	0	99	105
N.S.	1	1.00	1.00	0.96	1.17	1.42	0.00	1.90	2.02
time (sec)	N/A	0.046	0.023	0.238	0.295	1.934	0.000	0.429	2.207

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	41	38	41	45	0	48	40
N.S.	1	1.00	0.93	0.86	0.93	1.02	0.00	1.09	0.91
time (sec)	N/A	0.044	0.105	0.230	0.273	2.807	0.000	0.445	0.523

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	68	63	86	88	0	141	175
N.S.	1	1.00	0.92	0.85	1.16	1.19	0.00	1.91	2.36
time (sec)	N/A	0.059	0.243	0.293	0.280	2.320	0.000	0.445	4.118

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	53	48	53	57	0	70	65
N.S.	1	1.00	0.88	0.80	0.88	0.95	0.00	1.17	1.08
time (sec)	N/A	0.049	0.193	0.277	0.297	2.458	0.000	0.449	0.674

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	154	108	98	94	187	155	176
N.S.	1	1.00	1.12	0.79	0.72	0.69	1.36	1.13	1.28
time (sec)	N/A	0.098	0.406	0.285	0.284	2.860	0.647	0.484	0.695

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	147	118	102	95	340	132	156
N.S.	1	1.00	0.84	0.68	0.59	0.55	1.95	0.76	0.90
time (sec)	N/A	0.124	0.270	0.236	0.286	2.443	0.454	0.464	0.610

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	116	88	77	74	138	114	115
N.S.	1	1.00	1.13	0.85	0.75	0.72	1.34	1.11	1.12
time (sec)	N/A	0.090	0.187	0.191	0.277	2.227	0.289	0.429	0.615

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	98	97	75	75	238	85	89
N.S.	1	1.00	0.78	0.77	0.60	0.60	1.89	0.67	0.71
time (sec)	N/A	0.098	0.255	0.170	0.277	2.238	0.199	0.443	0.584

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	64	52	52	53	85	73	77
N.S.	1	1.00	0.96	0.78	0.78	0.79	1.27	1.09	1.15
time (sec)	N/A	0.059	0.427	0.139	0.280	2.315	0.125	0.420	0.514

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	52	70	68	52	128	50	63
N.S.	1	1.00	0.95	1.27	1.24	0.95	2.33	0.91	1.15
time (sec)	N/A	0.013	0.118	0.125	0.273	2.172	0.093	0.400	0.482

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	84	53	60	62	0	89	66
N.S.	1	1.00	1.53	0.96	1.09	1.13	0.00	1.62	1.20
time (sec)	N/A	0.050	0.172	0.201	0.266	2.791	0.000	0.443	0.489

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	69	44	49	60	0	44	118
N.S.	1	1.00	1.77	1.13	1.26	1.54	0.00	1.13	3.03
time (sec)	N/A	0.041	0.148	0.217	0.494	2.657	0.000	0.442	0.679

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	83	89	96	0	122	106
N.S.	1	1.00	1.00	1.24	1.33	1.43	0.00	1.82	1.58
time (sec)	N/A	0.063	0.051	0.253	0.267	3.074	0.000	0.465	0.901

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	56	48	45	55	0	41	68
N.S.	1	1.00	1.87	1.60	1.50	1.83	0.00	1.37	2.27
time (sec)	N/A	0.035	0.744	0.261	0.268	3.288	0.000	0.463	0.491

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	120	118	129	120	0	249	216
N.S.	1	1.00	1.00	0.98	1.08	1.00	0.00	2.08	1.80
time (sec)	N/A	0.099	0.092	0.325	0.276	3.741	0.000	0.491	3.161

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	54	82	70	79	0	80	98
N.S.	1	1.00	0.64	0.96	0.82	0.93	0.00	0.94	1.15
time (sec)	N/A	0.053	0.217	0.313	0.272	3.137	0.000	0.467	0.621

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	104	149	180	142	0	343	328
N.S.	1	1.00	0.62	0.89	1.07	0.85	0.00	2.04	1.95
time (sec)	N/A	0.122	0.668	0.388	0.281	3.265	0.000	0.465	3.259

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	104	110	91	100	0	118	130
N.S.	1	1.00	0.83	0.88	0.73	0.80	0.00	0.94	1.04
time (sec)	N/A	0.073	0.755	0.247	0.281	2.745	0.000	0.491	0.827

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	235	175	163	150	508	218	523
N.S.	1	1.00	0.89	0.66	0.62	0.57	1.92	0.82	1.97
time (sec)	N/A	0.184	0.510	0.379	0.275	3.058	1.006	0.531	2.289

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	204	145	126	123	233	197	214
N.S.	1	1.00	1.17	0.83	0.72	0.70	1.33	1.13	1.22
time (sec)	N/A	0.129	0.438	0.280	0.280	3.166	0.659	0.513	0.784

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	171	155	131	128	400	157	407
N.S.	1	1.00	0.79	0.72	0.61	0.59	1.85	0.73	1.88
time (sec)	N/A	0.155	0.343	0.256	0.276	2.872	0.443	0.502	2.030

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	150	125	107	102	182	145	147
N.S.	1	1.00	1.07	0.89	0.76	0.73	1.30	1.04	1.05
time (sec)	N/A	0.114	0.317	0.212	0.275	3.958	0.275	0.478	0.698

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	94	114	91	100	272	104	281
N.S.	1	1.00	1.21	1.46	1.17	1.28	3.49	1.33	3.60
time (sec)	N/A	0.045	0.439	0.178	0.284	3.222	0.195	0.454	1.627

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	81	75	84	77	117	91	104
N.S.	1	1.00	1.40	1.29	1.45	1.33	2.02	1.57	1.79
time (sec)	N/A	0.017	0.388	0.158	0.277	3.148	0.134	0.425	0.572

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	401	98	91	79	0	93	156
N.S.	1	1.00	4.41	1.08	1.00	0.87	0.00	1.02	1.71
time (sec)	N/A	0.084	0.843	0.241	0.281	2.351	0.000	0.499	1.242

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	131	96	84	109	0	150	116
N.S.	1	1.00	1.52	1.12	0.98	1.27	0.00	1.74	1.35
time (sec)	N/A	0.079	1.115	0.277	0.288	2.228	0.000	0.473	1.051

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	79	70	85	88	0	71	183
N.S.	1	1.00	1.10	0.97	1.18	1.22	0.00	0.99	2.54
time (sec)	N/A	0.068	0.295	0.291	0.491	2.619	0.000	0.482	1.864

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	293	146	118	123	0	171	160
N.S.	1	1.00	2.84	1.42	1.15	1.19	0.00	1.66	1.55
time (sec)	N/A	0.088	1.697	0.341	0.326	1.866	0.000	0.476	2.378

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	79	72	87	78	0	57	88
N.S.	1	1.00	2.63	2.40	2.90	2.60	0.00	1.90	2.93
time (sec)	N/A	0.032	0.636	0.338	0.284	1.919	0.000	0.480	0.636

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	464	198	157	147	0	333	293
N.S.	1	1.00	2.94	1.25	0.99	0.93	0.00	2.11	1.85
time (sec)	N/A	0.133	1.399	0.418	0.268	3.101	0.000	0.521	4.278

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	54	127	122	105	0	112	123
N.S.	1	1.00	0.45	1.06	1.02	0.88	0.00	0.93	1.02
time (sec)	N/A	0.070	0.395	0.290	0.300	3.589	0.000	0.501	0.841

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	637	248	208	170	0	465	423
N.S.	1	1.00	3.03	1.18	0.99	0.81	0.00	2.21	2.01
time (sec)	N/A	0.156	2.127	0.359	0.293	3.937	0.000	0.498	4.282

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	115	173	154	128	0	166	156
N.S.	1	1.00	0.66	0.99	0.89	0.74	0.00	0.95	0.90
time (sec)	N/A	0.100	0.633	0.315	0.281	2.921	0.000	0.526	1.154

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	810	294	248	192	0	597	547
N.S.	1	1.00	3.13	1.14	0.96	0.74	0.00	2.31	2.11
time (sec)	N/A	0.187	4.190	0.447	0.286	4.222	0.000	0.526	4.557

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	177	219	184	150	0	220	189
N.S.	1	1.00	0.83	1.03	0.86	0.70	0.00	1.03	0.89
time (sec)	N/A	0.124	2.129	0.349	0.290	3.146	0.000	0.509	1.580

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	237	236	186	177	367	269	334
N.S.	1	1.00	0.85	0.85	0.67	0.63	1.32	0.96	1.20
time (sec)	N/A	0.185	0.744	0.425	0.273	3.302	1.330	0.622	2.001

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	222	250	199	184	736	245	343
N.S.	1	1.00	0.58	0.66	0.52	0.48	1.93	0.64	0.90
time (sec)	N/A	0.266	0.647	0.362	0.281	2.881	0.972	0.579	1.688

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	204	206	154	149	286	229	291
N.S.	1	1.00	0.93	0.94	0.70	0.68	1.30	1.04	1.32
time (sec)	N/A	0.164	0.543	0.287	0.285	2.468	0.652	0.583	1.277

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	178	219	170	151	563	187	471
N.S.	1	1.00	0.59	0.73	0.56	0.50	1.87	0.62	1.56
time (sec)	N/A	0.225	0.464	0.253	0.279	2.261	0.480	0.523	2.306

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	146	142	123	123	206	165	204
N.S.	1	1.00	0.88	0.86	0.75	0.75	1.25	1.00	1.24
time (sec)	N/A	0.130	0.402	0.204	0.275	2.405	0.288	0.510	0.828

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	107	153	136	121	381	122	320
N.S.	1	1.00	0.99	1.42	1.26	1.12	3.53	1.13	2.96
time (sec)	N/A	0.033	0.413	0.180	0.270	3.332	0.212	0.450	1.959

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	181	116	126	121	0	217	190
N.S.	1	1.00	1.21	0.77	0.84	0.81	0.00	1.45	1.27
time (sec)	N/A	0.115	1.028	0.281	0.271	3.770	0.000	0.511	2.750

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	477	155	135	136	0	128	255
N.S.	1	1.00	4.01	1.30	1.13	1.14	0.00	1.08	2.14
time (sec)	N/A	0.125	6.291	0.303	0.504	2.652	0.000	0.510	1.231

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	268	157	142	153	0	206	221
N.S.	1	1.00	1.77	1.04	0.94	1.01	0.00	1.36	1.46
time (sec)	N/A	0.114	2.386	0.361	0.281	2.665	0.000	0.546	2.962

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	105	101	116	119	0	104	546
N.S.	1	1.00	1.02	0.98	1.13	1.16	0.00	1.01	5.30
time (sec)	N/A	0.113	0.448	0.359	0.498	2.353	0.000	0.521	1.794

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	936	225	192	163	0	325	278
N.S.	1	1.00	5.57	1.34	1.14	0.97	0.00	1.93	1.65
time (sec)	N/A	0.134	6.281	0.400	0.280	2.531	0.000	0.553	4.215

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	131	96	103	109	0	73	139
N.S.	1	1.00	4.37	3.20	3.43	3.63	0.00	2.43	4.63
time (sec)	N/A	0.034	1.428	0.297	0.281	2.401	0.000	0.545	0.800

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	1342	296	251	187	0	536	419
N.S.	1	1.00	5.20	1.15	0.97	0.72	0.00	2.08	1.62
time (sec)	N/A	0.205	6.298	0.379	0.274	2.059	0.000	0.594	4.266

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	54	171	151	142	0	144	186
N.S.	1	1.00	0.38	1.20	1.06	0.99	0.00	1.01	1.30
time (sec)	N/A	0.087	0.583	0.326	0.274	1.908	0.000	0.549	1.098

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	330	1732	363	322	214	0	706	566
N.S.	1	1.00	5.25	1.10	0.98	0.65	0.00	2.14	1.72
time (sec)	N/A	0.242	6.423	0.425	0.279	2.205	0.000	0.598	4.400

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	115	236	193	167	0	214	447
N.S.	1	1.00	0.57	1.17	0.96	0.83	0.00	1.06	2.22
time (sec)	N/A	0.121	0.849	0.349	0.288	2.093	0.000	0.581	4.284

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	408	408	242	427	382	251	0	880	703
N.S.	1	1.00	0.59	1.05	0.94	0.62	0.00	2.16	1.72
time (sec)	N/A	0.282	1.419	0.471	0.284	3.194	0.000	0.608	5.147

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	175	300	233	194	0	284	560
N.S.	1	1.00	0.69	1.18	0.92	0.76	0.00	1.12	2.20
time (sec)	N/A	0.159	1.782	0.374	0.290	3.267	0.000	0.590	4.818

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	515	515	307	335	290	250	979	342	801
N.S.	1	1.00	0.60	0.65	0.56	0.49	1.90	0.66	1.56
time (sec)	N/A	0.348	1.301	0.451	0.299	2.870	2.039	0.608	2.513

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	278	291	224	217	440	313	495
N.S.	1	1.00	0.82	0.86	0.66	0.64	1.31	0.93	1.47
time (sec)	N/A	0.211	1.069	0.425	0.303	2.728	1.441	0.652	4.342

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	426	426	259	305	228	220	821	278	650
N.S.	1	1.00	0.61	0.72	0.54	0.52	1.93	0.65	1.53
time (sec)	N/A	0.286	0.911	0.379	0.299	2.934	1.093	0.704	2.480

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	236	261	194	186	357	259	372
N.S.	1	1.00	0.86	0.95	0.71	0.68	1.30	0.94	1.35
time (sec)	N/A	0.197	0.802	0.297	0.290	2.918	0.650	0.646	4.421

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	188	236	187	182	609	211	472
N.S.	1	1.00	1.49	1.87	1.48	1.44	4.83	1.67	3.75
time (sec)	N/A	0.064	0.661	0.256	0.286	2.380	0.492	0.588	2.319

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	156	175	172	155	267	187	248
N.S.	1	1.00	1.66	1.86	1.83	1.65	2.84	1.99	2.64
time (sec)	N/A	0.033	0.502	0.237	0.275	2.252	0.339	0.471	0.938

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	711	192	170	160	0	199	297
N.S.	1	1.00	4.18	1.13	1.00	0.94	0.00	1.17	1.75
time (sec)	N/A	0.150	6.482	0.325	0.279	3.039	0.000	0.597	2.648

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	632	169	162	177	0	283	277
N.S.	1	1.00	3.08	0.82	0.79	0.86	0.00	1.38	1.35
time (sec)	N/A	0.162	6.322	0.371	0.279	2.401	0.000	0.608	3.978

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	571	209	179	177	0	173	354
N.S.	1	1.00	3.38	1.24	1.06	1.05	0.00	1.02	2.09
time (sec)	N/A	0.175	6.379	0.395	0.497	2.887	0.000	0.650	2.533

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	397	230	181	190	0	281	302
N.S.	1	1.00	1.95	1.13	0.89	0.93	0.00	1.38	1.48
time (sec)	N/A	0.143	6.004	0.463	0.283	4.765	0.000	0.644	4.047

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	126	139	174	155	0	144	971
N.S.	1	1.00	0.86	0.95	1.18	1.05	0.00	0.98	6.61
time (sec)	N/A	0.160	0.785	0.332	0.484	2.847	0.000	0.653	3.447

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	1219	316	230	196	0	410	345
N.S.	1	1.00	5.44	1.41	1.03	0.88	0.00	1.83	1.54
time (sec)	N/A	0.162	6.365	0.378	0.278	3.175	0.000	0.668	4.256

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	370	120	166	144	0	89	169
N.S.	1	1.00	12.33	4.00	5.53	4.80	0.00	2.97	5.63
time (sec)	N/A	0.033	6.244	0.369	0.285	2.174	0.000	0.677	0.976

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	1677	405	289	227	0	680	514
N.S.	1	1.00	5.27	1.27	0.91	0.71	0.00	2.14	1.62
time (sec)	N/A	0.239	6.417	0.450	0.277	2.929	0.000	0.682	4.205

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	54	217	223	176	0	176	419
N.S.	1	1.00	0.31	1.23	1.26	0.99	0.00	0.99	2.37
time (sec)	N/A	0.107	0.478	0.392	0.274	2.277	0.000	0.734	4.272

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	391	331	489	360	257	0	888	675
N.S.	1	1.00	0.85	1.25	0.92	0.66	0.00	2.27	1.73
time (sec)	N/A	0.274	2.460	0.499	0.283	2.421	0.000	0.764	4.709

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	115	299	275	207	0	262	548
N.S.	1	1.00	0.48	1.24	1.14	0.86	0.00	1.08	2.26
time (sec)	N/A	0.145	1.230	0.417	0.298	3.005	0.000	0.720	4.442

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	472	472	374	572	420	287	0	1096	831
N.S.	1	1.00	0.79	1.21	0.89	0.61	0.00	2.32	1.76
time (sec)	N/A	0.311	1.930	0.559	0.272	2.905	0.000	0.744	5.924

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	218	197	564	208	0	322	2500
N.S.	1	1.00	0.96	0.87	2.48	0.92	0.00	1.42	11.01
time (sec)	N/A	0.163	0.455	0.349	0.486	3.492	0.000	0.438	11.158

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	137	221	379	262	0	286	342
N.S.	1	1.00	0.83	1.33	2.28	1.58	0.00	1.72	2.06
time (sec)	N/A	0.139	1.081	0.397	0.478	3.555	0.000	0.499	3.316

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	143	120	284	119	0	182	2500
N.S.	1	1.00	1.20	1.01	2.39	1.00	0.00	1.53	21.01
time (sec)	N/A	0.095	0.254	0.258	0.481	2.878	0.000	0.468	6.163

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	79	90	142	187	1034	118	110
N.S.	1	1.00	0.87	0.99	1.56	2.05	11.36	1.30	1.21
time (sec)	N/A	0.059	0.190	0.292	0.492	2.542	117.793	0.462	0.631

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	62	124	61	291	74	1069
N.S.	1	1.00	0.91	1.38	2.76	1.36	6.47	1.64	23.76
time (sec)	N/A	0.045	0.072	0.203	0.475	2.166	0.901	0.441	1.156

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	43	80	131	0	74	39
N.S.	1	1.00	0.96	0.91	1.70	2.79	0.00	1.57	0.83
time (sec)	N/A	0.018	0.042	0.207	0.474	2.698	0.000	0.433	0.467

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	18	19	103	59	0	19	62
N.S.	1	1.00	0.44	0.46	2.51	1.44	0.00	0.46	1.51
time (sec)	N/A	0.058	0.021	0.283	0.292	2.568	0.000	0.441	0.723

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	109	129	163	191	0	136	310
N.S.	1	1.00	1.36	1.61	2.04	2.39	0.00	1.70	3.88
time (sec)	N/A	0.061	0.153	0.447	0.487	3.232	0.000	0.495	0.713

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	71	53	238	117	0	54	300
N.S.	1	1.00	0.81	0.60	2.70	1.33	0.00	0.61	3.41
time (sec)	N/A	0.103	0.323	0.353	0.266	2.523	0.000	0.447	1.496

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	321	269	361	259	0	278	724
N.S.	1	1.00	2.10	1.76	2.36	1.69	0.00	1.82	4.73
time (sec)	N/A	0.119	2.205	0.622	0.501	1.241	0.000	0.487	2.213

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	119	106	462	183	0	120	575
N.S.	1	1.00	0.75	0.67	2.92	1.16	0.00	0.76	3.64
time (sec)	N/A	0.160	1.271	0.477	0.275	1.928	0.000	0.472	3.682

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	661	479	625	346	0	554	2500
N.S.	1	1.00	2.52	1.83	2.39	1.32	0.00	2.11	9.54
time (sec)	N/A	0.196	5.415	0.811	0.493	2.528	0.000	0.497	2.841

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	149	154	282	279	0	250	2500
N.S.	1	1.00	1.03	1.06	1.94	1.92	0.00	1.72	17.24
time (sec)	N/A	0.230	1.022	0.526	0.495	2.238	0.000	0.444	11.457

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	231	130	172	348	302	0	286	286
N.S.	1	1.67	0.94	1.25	2.52	2.19	0.00	2.07	2.07
time (sec)	N/A	0.782	0.839	0.532	0.484	2.395	0.000	0.503	2.813

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	192	97	131	173	1583	159	3114
N.S.	1	1.00	2.34	1.18	1.60	2.11	19.30	1.94	37.98
time (sec)	N/A	0.095	0.443	0.339	0.471	3.239	3.061	0.442	4.867

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	79	118	182	215	0	138	136
N.S.	1	1.00	0.95	1.42	2.19	2.59	0.00	1.66	1.64
time (sec)	N/A	0.046	0.250	0.358	0.495	3.113	0.000	0.459	0.840

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	21	21	57	0	20	47
N.S.	1	1.00	1.00	0.66	0.66	1.78	0.00	0.62	1.47
time (sec)	N/A	0.012	0.039	0.253	0.284	2.589	0.000	0.400	0.486

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	120	135	212	293	0	166	383
N.S.	1	1.00	1.30	1.47	2.30	3.18	0.00	1.80	4.16
time (sec)	N/A	0.057	0.866	0.523	0.501	2.758	0.000	0.506	1.171

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	121	57	60	178	0	71	382
N.S.	1	1.00	1.61	0.76	0.80	2.37	0.00	0.95	5.09
time (sec)	N/A	0.064	0.367	0.438	0.272	2.700	0.000	0.464	2.360

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	709	259	471	355	0	280	585
N.S.	1	1.00	3.96	1.45	2.63	1.98	0.00	1.56	3.27
time (sec)	N/A	0.180	6.129	0.684	0.505	3.128	0.000	0.540	1.900

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	207	114	115	281	0	149	1132
N.S.	1	1.00	1.47	0.81	0.82	1.99	0.00	1.06	8.03
time (sec)	N/A	0.102	1.628	0.558	0.298	3.497	0.000	0.451	4.203

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	C	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	492	211	283	658	480	0	399	610
N.S.	1	2.28	0.98	1.31	3.05	2.22	0.00	1.85	2.82
time (sec)	N/A	1.224	1.146	1.016	0.507	2.745	0.000	0.558	4.253

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	154	140	481	341	0	265	2500
N.S.	1	1.00	1.26	1.15	3.94	2.80	0.00	2.17	20.49
time (sec)	N/A	0.149	1.356	0.552	0.503	2.827	0.000	0.515	8.539

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	225	119	280	412	352	0	293	443
N.S.	1	1.89	1.00	2.35	3.46	2.96	0.00	2.46	3.72
time (sec)	N/A	0.430	0.727	0.576	0.500	1.962	0.000	0.548	1.718

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	30	57	21	171	142	0	20	85
N.S.	1	1.36	2.59	0.95	7.77	6.45	0.00	0.91	3.86
time (sec)	N/A	0.022	0.143	0.384	0.275	2.608	0.000	0.516	0.615

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	132	191	326	294	0	221	260
N.S.	1	1.00	1.28	1.85	3.17	2.85	0.00	2.15	2.52
time (sec)	N/A	0.039	0.303	0.406	0.487	1.889	0.000	0.443	2.733

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	115	63	315	284	0	62	396
N.S.	1	1.00	1.34	0.73	3.66	3.30	0.00	0.72	4.60
time (sec)	N/A	0.071	0.665	0.539	0.319	2.198	0.000	0.490	2.595

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	396	269	518	513	0	314	1311
N.S.	1	1.00	1.52	1.03	1.99	1.97	0.00	1.21	5.04
time (sec)	N/A	0.204	2.562	0.798	0.503	3.499	0.000	0.537	2.620

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	191	115	652	354	0	140	1204
N.S.	1	1.00	1.19	0.71	4.05	2.20	0.00	0.87	7.48
time (sec)	N/A	0.126	2.394	0.727	0.327	2.281	0.000	0.514	4.744

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	688	444	902	564	0	510	1203
N.S.	1	1.00	1.80	1.16	2.36	1.47	0.00	1.33	3.14
time (sec)	N/A	0.581	2.582	1.056	0.509	2.492	0.000	0.594	3.822

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	302	195	1053	476	0	243	1712
N.S.	1	1.00	1.30	0.84	4.54	2.05	0.00	1.05	7.38
time (sec)	N/A	0.171	5.294	0.953	0.307	2.099	0.000	0.530	7.638

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	419	183	385	575	0	370	2500
N.S.	1	1.00	2.54	1.11	2.33	3.48	0.00	2.24	15.15
time (sec)	N/A	0.206	6.240	0.944	0.503	3.092	0.000	0.506	12.555

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	B	C	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	362	165	494	724	524	0	524	764
N.S.	1	2.31	1.05	3.15	4.61	3.34	0.00	3.34	4.87
time (sec)	N/A	0.837	1.202	0.908	0.515	1.941	0.000	0.563	2.691

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	124	21	53	255	0	20	224
N.S.	1	1.00	4.13	0.70	1.77	8.50	0.00	0.67	7.47
time (sec)	N/A	0.035	0.709	0.644	0.310	2.483	0.000	0.481	1.292

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	128	383	606	420	0	426	505
N.S.	1	1.00	0.91	2.72	4.30	2.98	0.00	3.02	3.58
time (sec)	N/A	0.082	0.805	0.658	0.511	2.451	0.000	0.512	3.821

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	85	64	85	217	0	50	222
N.S.	1	1.00	0.87	0.65	0.87	2.21	0.00	0.51	2.27
time (sec)	N/A	0.030	0.321	0.454	0.291	3.310	0.000	0.430	1.249

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	290	411	661	745	0	527	2848
N.S.	1	1.00	1.26	1.78	2.86	3.23	0.00	2.28	12.33
time (sec)	N/A	0.137	3.482	1.072	0.493	3.116	0.000	0.519	4.830

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	175	118	144	537	0	138	666
N.S.	1	1.00	1.27	0.86	1.04	3.89	0.00	1.00	4.83
time (sec)	N/A	0.115	1.309	0.848	0.271	3.422	0.000	0.483	4.438

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	538	452	936	820	0	548	1961
N.S.	1	1.00	1.34	1.13	2.34	2.05	0.00	1.37	4.90
time (sec)	N/A	0.575	3.683	1.347	0.526	2.640	0.000	0.530	4.593

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	366	195	217	553	0	249	1599
N.S.	1	1.00	1.58	0.84	0.94	2.38	0.00	1.07	6.89
time (sec)	N/A	0.175	1.893	1.168	0.279	3.163	0.000	0.495	7.970

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	82	102	0	76	219	116	164
N.S.	1	1.00	0.83	1.03	0.00	0.77	2.21	1.17	1.66
time (sec)	N/A	0.108	0.125	0.701	0.000	2.079	0.217	0.408	5.137

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	111	141	0	63	196	119	134
N.S.	1	1.00	1.59	2.01	0.00	0.90	2.80	1.70	1.91
time (sec)	N/A	0.088	0.061	0.493	0.000	1.790	0.229	0.439	2.087

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	60	75	0	54	151	99	111
N.S.	1	1.00	0.80	1.00	0.00	0.72	2.01	1.32	1.48
time (sec)	N/A	0.091	0.097	0.381	0.000	2.727	0.146	0.433	3.433

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	73	75	0	41	126	67	78
N.S.	1	1.00	1.40	1.44	0.00	0.79	2.42	1.29	1.50
time (sec)	N/A	0.080	0.060	0.274	0.000	2.825	0.164	0.422	0.772

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	38	48	0	32	60	60	39
N.S.	1	1.00	0.83	1.04	0.00	0.70	1.30	1.30	0.85
time (sec)	N/A	0.021	0.063	0.202	0.000	4.123	0.077	0.411	0.712

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	23	29	17	31	21	25
N.S.	1	1.00	1.00	0.79	1.00	0.59	1.07	0.72	0.86
time (sec)	N/A	0.012	0.037	0.165	0.270	3.979	0.062	0.435	0.613

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	22	101	26	0	57	41
N.S.	1	1.00	1.00	0.96	4.39	1.13	0.00	2.48	1.78
time (sec)	N/A	0.049	0.062	0.243	0.280	4.196	0.000	0.433	0.739

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	35	70	83	80	0	58	43
N.S.	1	1.00	1.13	2.26	2.68	2.58	0.00	1.87	1.39
time (sec)	N/A	0.068	0.237	0.278	0.283	2.469	0.000	0.435	0.670

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	35	30	108	33	0	27	25
N.S.	1	1.00	1.03	0.88	3.18	0.97	0.00	0.79	0.74
time (sec)	N/A	0.077	0.221	0.302	0.276	3.065	0.000	0.451	0.684

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	138	186	174	0	99	116
N.S.	1	1.00	0.90	2.30	3.10	2.90	0.00	1.65	1.93
time (sec)	N/A	0.086	0.275	0.351	0.286	3.245	0.000	0.439	2.552

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	53	51	211	72	0	47	99
N.S.	1	1.00	1.02	0.98	4.06	1.38	0.00	0.90	1.90
time (sec)	N/A	0.082	0.306	0.366	0.284	2.416	0.000	0.463	1.291

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	66	206	288	266	0	138	193
N.S.	1	1.00	0.79	2.45	3.43	3.17	0.00	1.64	2.30
time (sec)	N/A	0.097	0.524	0.434	0.292	1.629	0.000	0.433	4.246

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	67	72	313	109	0	67	139
N.S.	1	1.00	0.96	1.03	4.47	1.56	0.00	0.96	1.99
time (sec)	N/A	0.088	0.387	0.448	0.290	1.887	0.000	0.442	1.889

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	149	174	0	74	231	145	161
N.S.	1	1.00	1.75	2.05	0.00	0.87	2.72	1.71	1.89
time (sec)	N/A	0.129	0.095	0.709	0.000	2.049	0.317	0.423	4.114

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	82	88	0	65	189	103	138
N.S.	1	1.00	0.81	0.87	0.00	0.64	1.87	1.02	1.37
time (sec)	N/A	0.071	0.112	0.487	0.000	2.137	0.201	0.425	4.736

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	111	108	0	52	163	93	90
N.S.	1	1.00	1.63	1.59	0.00	0.76	2.40	1.37	1.32
time (sec)	N/A	0.127	0.075	0.398	0.000	2.403	0.274	0.420	1.015

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	60	62	0	43	117	72	69
N.S.	1	1.00	0.67	0.70	0.00	0.48	1.31	0.81	0.78
time (sec)	N/A	0.056	0.099	0.296	0.000	3.455	0.129	0.433	1.756

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	73	57	45	30	92	47	79
N.S.	1	1.00	1.40	1.10	0.87	0.58	1.77	0.90	1.52
time (sec)	N/A	0.080	0.052	0.260	0.271	2.544	0.121	0.425	0.647

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	23	22	17	44	30	31
N.S.	1	1.00	1.00	0.74	0.71	0.55	1.42	0.97	1.00
time (sec)	N/A	0.012	0.043	0.205	0.265	3.254	0.093	0.404	0.611

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	184	54	117	64	0	57	44
N.S.	1	1.00	4.00	1.17	2.54	1.39	0.00	1.24	0.96
time (sec)	N/A	0.077	0.267	0.296	0.470	2.070	0.000	0.455	0.672

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	71	30	30	70	0	100	83
N.S.	1	1.00	1.29	0.55	0.55	1.27	0.00	1.82	1.51
time (sec)	N/A	0.049	0.479	0.323	0.278	2.766	0.000	0.423	0.782

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	146	102	167	134	0	95	104
N.S.	1	1.00	2.61	1.82	2.98	2.39	0.00	1.70	1.86
time (sec)	N/A	0.105	0.465	0.373	0.280	4.035	0.000	0.451	1.181

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	68	20	35	54	0	35	49
N.S.	1	1.00	2.00	0.59	1.03	1.59	0.00	1.03	1.44
time (sec)	N/A	0.043	0.262	0.376	0.303	3.245	0.000	0.450	0.725

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	215	170	295	230	0	151	136
N.S.	1	1.00	2.56	2.02	3.51	2.74	0.00	1.80	1.62
time (sec)	N/A	0.135	1.048	0.444	0.285	2.750	0.000	0.452	3.208

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	77	47	47	97	0	47	76
N.S.	1	1.00	1.10	0.67	0.67	1.39	0.00	0.67	1.09
time (sec)	N/A	0.054	0.433	0.456	0.296	2.144	0.000	0.470	0.915

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	106	102	0	76	224	119	164
N.S.	1	1.00	0.85	0.82	0.00	0.61	1.79	0.95	1.31
time (sec)	N/A	0.077	0.179	0.704	0.000	2.539	0.227	0.472	4.785

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	149	141	0	63	197	119	134
N.S.	1	1.00	1.41	1.33	0.00	0.59	1.86	1.12	1.26
time (sec)	N/A	0.167	0.075	0.553	0.000	2.307	0.259	0.456	3.169

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	84	75	0	54	155	80	96
N.S.	1	1.00	0.64	0.57	0.00	0.41	1.18	0.61	0.73
time (sec)	N/A	0.096	0.111	0.396	0.000	2.570	0.166	0.469	3.487

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	111	90	69	41	131	73	133
N.S.	1	1.00	1.23	1.00	0.77	0.46	1.46	0.81	1.48
time (sec)	N/A	0.153	0.068	0.402	0.292	2.960	0.183	0.454	0.867

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	77	23	51	30	95	57	100
N.S.	1	1.00	2.41	0.72	1.59	0.94	2.97	1.78	3.12
time (sec)	N/A	0.022	0.056	0.324	0.282	2.818	0.126	0.465	0.754

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	57	29	17	44	36	68
N.S.	1	1.00	1.00	1.84	0.94	0.55	1.42	1.16	2.19
time (sec)	N/A	0.013	0.046	0.277	0.300	2.394	0.081	0.422	0.632

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	91	35	99	55	0	100	101
N.S.	1	1.00	1.49	0.57	1.62	0.90	0.00	1.64	1.66
time (sec)	N/A	0.043	0.329	0.419	0.489	3.501	0.000	0.448	0.773

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	109	86	319	112	0	110	105
N.S.	1	1.00	1.76	1.39	5.15	1.81	0.00	1.77	1.69
time (sec)	N/A	0.111	0.367	0.409	0.498	3.716	0.000	0.465	0.992

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	110	41	301	113	0	128	104
N.S.	1	1.00	1.47	0.55	4.01	1.51	0.00	1.71	1.39
time (sec)	N/A	0.056	0.688	0.438	0.508	2.711	0.000	0.463	0.924

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	64	138	215	182	0	112	135
N.S.	1	1.00	0.84	1.82	2.83	2.39	0.00	1.47	1.78
time (sec)	N/A	0.121	0.520	0.521	0.294	2.192	0.000	0.465	2.720

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	90	21	240	69	0	47	55
N.S.	1	1.00	2.65	0.62	7.06	2.03	0.00	1.38	1.62
time (sec)	N/A	0.041	0.496	0.467	0.290	3.534	0.000	0.475	0.883

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	115	206	341	278	0	164	150
N.S.	1	1.00	1.11	1.98	3.28	2.67	0.00	1.58	1.44
time (sec)	N/A	0.167	0.469	0.520	0.299	2.680	0.000	0.461	3.293

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	90	0	0	0	0	0	-1
N.S.	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.042	2.338	0.167	0.000	0.000	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	16	6	31	5	17	22	21
N.S.	1	1.00	3.20	1.20	6.20	1.00	3.40	4.40	4.20
time (sec)	N/A	0.016	0.022	0.104	0.267	2.659	0.058	0.399	1.108

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	19	11	54	10	0	10	21
N.S.	1	1.00	1.90	1.10	5.40	1.00	0.00	1.00	2.10
time (sec)	N/A	0.046	0.027	0.145	0.467	2.490	0.000	0.415	0.596

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	4	21	30	4	0	14	4
N.S.	1	1.00	1.00	5.25	7.50	1.00	0.00	3.50	1.00
time (sec)	N/A	0.038	0.024	0.139	0.469	2.973	0.000	0.395	0.554

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	25	19	28	24	0	12	12
N.S.	1	1.00	2.27	1.73	2.55	2.18	0.00	1.09	1.09
time (sec)	N/A	0.038	0.040	0.094	0.462	2.212	0.000	0.413	0.583

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	20	14	23	22	0	10	23
N.S.	1	1.00	2.22	1.56	2.56	2.44	0.00	1.11	2.56
time (sec)	N/A	0.056	0.028	0.158	0.462	2.897	0.000	0.405	0.593

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	23	11	15	18	0	10	10
N.S.	1	1.00	2.30	1.10	1.50	1.80	0.00	1.00	1.00
time (sec)	N/A	0.018	0.022	0.098	0.286	2.882	0.000	0.386	0.555

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	20	8	25	13	0	12	15
N.S.	1	1.00	1.82	0.73	2.27	1.18	0.00	1.09	1.36
time (sec)	N/A	0.038	0.027	0.146	0.267	2.861	0.000	0.404	0.574

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	18	8	29	9	17	20	19
N.S.	1	1.00	2.00	0.89	3.22	1.00	1.89	2.22	2.11
time (sec)	N/A	0.019	0.025	0.101	0.273	3.051	0.064	0.418	0.957

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	23	13	54	14	0	14	23
N.S.	1	1.00	1.64	0.93	3.86	1.00	0.00	1.00	1.64
time (sec)	N/A	0.050	0.028	0.140	0.459	3.036	0.000	0.413	0.608

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	21	30	6	0	14	6
N.S.	1	1.00	1.00	3.50	5.00	1.00	0.00	2.33	1.00
time (sec)	N/A	0.041	0.026	0.138	0.484	2.686	0.000	0.410	0.577

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	29	19	28	28	0	14	14
N.S.	1	1.00	1.93	1.27	1.87	1.87	0.00	0.93	0.93
time (sec)	N/A	0.044	0.042	0.093	0.465	2.955	0.000	0.402	0.577

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	18	14	23	20	0	8	23
N.S.	1	1.00	2.57	2.00	3.29	2.86	0.00	1.14	3.29
time (sec)	N/A	0.061	0.028	0.158	0.470	3.444	0.000	0.420	0.610

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	25	11	15	17	0	10	10
N.S.	1	1.00	2.27	1.00	1.36	1.55	0.00	0.91	0.91
time (sec)	N/A	0.020	0.022	0.097	0.262	2.046	0.000	0.398	0.563

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	22	8	25	15	0	14	15
N.S.	1	1.00	1.69	0.62	1.92	1.15	0.00	1.08	1.15
time (sec)	N/A	0.042	0.028	0.148	0.275	3.089	0.000	0.404	0.564

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	15	24	22	21	27	16	14
N.S.	1	1.00	0.65	1.04	0.96	0.91	1.17	0.70	0.61
time (sec)	N/A	0.065	0.021	0.050	0.266	2.514	1.184	0.412	0.589

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	14	25	38	6	0	18	6
N.S.	1	1.00	2.33	4.17	6.33	1.00	0.00	3.00	1.00
time (sec)	N/A	0.046	0.014	0.125	0.473	2.174	0.000	0.398	1.047

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	20	11	34	12	0	10	24
N.S.	1	1.00	2.00	1.10	3.40	1.20	0.00	1.00	2.40
time (sec)	N/A	0.043	0.011	0.125	0.460	2.666	0.000	0.399	0.591

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	36	25	39	20	0	22	11
N.S.	1	1.00	5.14	3.57	5.57	2.86	0.00	3.14	1.57
time (sec)	N/A	0.055	0.033	0.148	0.476	3.186	0.000	0.398	0.575

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	10	15	23	17	0	8	8
N.S.	1	1.00	0.83	1.25	1.92	1.42	0.00	0.67	0.67
time (sec)	N/A	0.038	0.023	0.085	0.477	2.578	0.000	0.405	0.573

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	25	6	29	15	0	12	11
N.S.	1	1.00	2.27	0.55	2.64	1.36	0.00	1.09	1.00
time (sec)	N/A	0.040	0.011	0.132	0.274	2.151	0.000	0.397	0.663

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	9	6	5	9	9	0	4	4
N.S.	1	1.00	0.67	0.56	1.00	1.00	0.00	0.44	0.44
time (sec)	N/A	0.018	0.024	0.078	0.267	2.713	0.000	0.419	0.542

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	4	4	14	25	38	4	0	18	4
N.S.	1	1.00	3.50	6.25	9.50	1.00	0.00	4.50	1.00
time (sec)	N/A	0.048	0.012	0.132	0.464	3.083	0.000	0.406	1.026

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	20	9	46	10	0	10	31
N.S.	1	1.00	2.00	0.90	4.60	1.00	0.00	1.00	3.10
time (sec)	N/A	0.045	0.015	0.133	0.477	3.182	0.000	0.403	0.618

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	46	25	39	18	0	20	9
N.S.	1	1.00	9.20	5.00	7.80	3.60	0.00	4.00	1.80
time (sec)	N/A	0.066	0.020	0.147	0.466	1.537	0.000	0.407	0.570

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	17	23	14	0	12	10
N.S.	1	1.00	1.00	1.06	1.44	0.88	0.00	0.75	0.62
time (sec)	N/A	0.043	0.020	0.092	0.472	1.659	0.000	0.409	0.578

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	25	6	41	15	0	14	19
N.S.	1	1.00	1.92	0.46	3.15	1.15	0.00	1.08	1.46
time (sec)	N/A	0.043	0.013	0.133	0.273	2.235	0.000	0.408	0.610

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	8	9	10	10	0	8	6
N.S.	1	1.00	0.67	0.75	0.83	0.83	0.00	0.67	0.50
time (sec)	N/A	0.021	0.012	0.076	0.268	2.614	0.000	0.423	0.534

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	61	21	176	44	0	68	42
N.S.	1	1.00	2.65	0.91	7.65	1.91	0.00	2.96	1.83
time (sec)	N/A	0.026	0.216	0.184	0.524	2.654	0.000	0.430	0.656

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	30	29	252	52	0	82	62
N.S.	1	1.00	0.59	0.57	4.94	1.02	0.00	1.61	1.22
time (sec)	N/A	0.126	0.085	0.190	0.505	2.512	0.000	0.426	0.619

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	17	16	18	0	16	16
N.S.	1	1.00	1.11	0.94	0.89	1.00	0.00	0.89	0.89
time (sec)	N/A	0.023	0.123	0.109	0.478	2.027	0.000	0.415	0.059

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	24	37	35	37	0	37	61
N.S.	1	1.00	0.83	1.28	1.21	1.28	0.00	1.28	2.10
time (sec)	N/A	0.027	0.041	0.224	0.500	2.442	0.000	0.464	0.695

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	13	11	11	0	11	45
N.S.	1	1.00	1.00	1.18	1.00	1.00	0.00	1.00	4.09
time (sec)	N/A	0.018	0.044	0.108	0.477	2.991	0.000	0.423	0.671

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	30	19	39	30	0	79	14
N.S.	1	1.00	1.88	1.19	2.44	1.88	0.00	4.94	0.88
time (sec)	N/A	0.022	0.055	0.132	0.466	3.609	0.000	0.442	0.607

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	22	20	245	46	0	72	56
N.S.	1	1.00	0.46	0.42	5.10	0.96	0.00	1.50	1.17
time (sec)	N/A	0.085	0.034	0.166	0.502	2.766	0.000	0.409	0.709

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	13	28	12	0	28	20
N.S.	1	1.00	1.00	1.30	2.80	1.20	0.00	2.80	2.00
time (sec)	N/A	0.017	0.013	0.145	0.261	2.263	0.000	0.415	0.663

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	23	21	64	31	0	18	33
N.S.	1	1.00	1.64	1.50	4.57	2.21	0.00	1.29	2.36
time (sec)	N/A	0.103	0.011	0.156	0.474	2.548	0.000	0.437	0.620

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	13	24	14	0	26	14
N.S.	1	1.00	1.00	1.08	2.00	1.17	0.00	2.17	1.17
time (sec)	N/A	0.021	0.011	0.109	0.271	2.081	0.000	0.444	0.059

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	52	46	61	0	48	69
N.S.	1	1.00	1.00	1.53	1.35	1.79	0.00	1.41	2.03
time (sec)	N/A	0.027	0.019	0.224	0.264	2.657	0.000	0.458	1.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	26	28	0	28	15
N.S.	1	1.00	1.00	1.09	2.36	2.55	0.00	2.55	1.36
time (sec)	N/A	0.020	0.004	0.151	0.281	3.554	0.000	0.418	0.639

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	17	13	0	46	13
N.S.	1	1.00	1.00	0.93	1.13	0.87	0.00	3.07	0.87
time (sec)	N/A	0.022	0.015	0.117	0.266	2.207	0.000	0.414	0.043

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	44	18	0	10	29
N.S.	1	1.00	1.00	1.10	4.40	1.80	0.00	1.00	2.90
time (sec)	N/A	0.070	0.007	0.139	0.264	2.542	0.000	0.402	0.580

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	29	28	28	0	11588	29
N.S.	1	1.00	1.00	0.88	0.85	0.85	0.00	351.15	0.88
time (sec)	N/A	0.044	0.018	0.122	0.266	2.404	0.000	2.232	0.681

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	38	29	28	28	0	99	49
N.S.	1	1.00	1.15	0.88	0.85	0.85	0.00	3.00	1.48
time (sec)	N/A	0.039	0.139	0.095	0.254	3.587	0.000	0.480	0.644

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	29	40	26	25	25	0	102	28
N.S.	1	1.32	1.82	1.18	1.14	1.14	0.00	4.64	1.27
time (sec)	N/A	0.022	0.017	0.067	0.263	3.378	0.000	0.437	0.626

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	37	31	25	25	37	27	40
N.S.	1	1.00	1.42	1.19	0.96	0.96	1.42	1.04	1.54
time (sec)	N/A	0.009	0.027	0.060	0.258	3.841	0.071	0.412	0.655

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	23	32	34	0	107	40
N.S.	1	1.00	1.00	0.92	1.28	1.36	0.00	4.28	1.60
time (sec)	N/A	0.019	0.022	0.086	0.265	2.488	0.000	0.466	0.664

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	27	24	0	71	24
N.S.	1	1.00	1.00	0.89	0.96	0.86	0.00	2.54	0.86
time (sec)	N/A	0.036	0.026	0.108	0.258	1.260	0.000	0.462	0.635

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	32	26	0	97	29
N.S.	1	1.00	1.00	0.85	0.97	0.79	0.00	2.94	0.88
time (sec)	N/A	0.040	0.031	0.137	0.273	2.043	0.000	0.493	0.693

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	77	100	68	85	0	43089	101
N.S.	1	1.00	0.73	0.94	0.64	0.80	0.00	406.50	0.95
time (sec)	N/A	0.258	0.445	0.124	0.267	2.833	0.000	138.265	0.793

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	82	86	66	74	0	5161	76
N.S.	1	1.00	0.95	1.00	0.77	0.86	0.00	60.01	0.88
time (sec)	N/A	0.136	0.247	0.115	0.265	3.560	0.000	2.896	0.715

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	117	72	76	83	0	5713	121
N.S.	1	1.00	1.34	0.83	0.87	0.95	0.00	65.67	1.39
time (sec)	N/A	0.222	0.169	0.108	0.275	2.255	0.000	2.223	0.840

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	116	77	84	108	0	2752	143
N.S.	1	1.00	1.51	1.00	1.09	1.40	0.00	35.74	1.86
time (sec)	N/A	0.084	0.670	0.126	0.481	1.206	0.000	0.849	0.797

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	75	98	102	126	0	171	147
N.S.	1	1.00	0.83	1.09	1.13	1.40	0.00	1.90	1.63
time (sec)	N/A	0.303	0.220	0.133	0.467	2.236	0.000	0.994	0.788

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	201	92	82	115	0	158	227
N.S.	1	1.00	2.03	0.93	0.83	1.16	0.00	1.60	2.29
time (sec)	N/A	0.325	1.253	0.151	0.469	2.715	0.000	1.019	1.026

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	336	138	129	129	0	226	177
N.S.	1	1.00	2.69	1.10	1.03	1.03	0.00	1.81	1.42
time (sec)	N/A	0.306	0.682	0.173	0.279	2.185	0.000	1.054	3.269

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	114	109	95	100	0	0	149
N.S.	1	1.00	1.48	1.42	1.23	1.30	0.00	0.00	1.94
time (sec)	N/A	0.140	0.259	0.148	0.271	2.827	0.000	0.000	0.800

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	106	99	94	101	0	0	237
N.S.	1	1.00	0.88	0.82	0.78	0.84	0.00	0.00	1.98
time (sec)	N/A	0.132	0.211	0.154	0.269	3.149	0.000	0.000	0.792

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	98	106	87	128	0	0	225
N.S.	1	1.00	0.88	0.95	0.78	1.14	0.00	0.00	2.01
time (sec)	N/A	0.117	0.299	0.125	0.263	2.136	0.000	0.000	4.182

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	102	116	113	123	0	0	219
N.S.	1	1.00	0.88	1.00	0.97	1.06	0.00	0.00	1.89
time (sec)	N/A	0.079	0.350	0.141	0.267	3.788	0.000	0.000	4.524

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	100	154	109	122	0	0	219
N.S.	1	1.00	0.87	1.34	0.95	1.06	0.00	0.00	1.90
time (sec)	N/A	0.168	0.563	0.187	0.267	2.350	0.000	0.000	4.447

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	97	151	96	107	0	0	223
N.S.	1	1.00	0.87	1.36	0.86	0.96	0.00	0.00	2.01
time (sec)	N/A	0.172	1.981	0.231	0.279	3.332	0.000	0.000	4.217

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	99	188	128	109	0	0	220
N.S.	1	1.00	0.83	1.58	1.08	0.92	0.00	0.00	1.85
time (sec)	N/A	0.149	0.358	0.253	0.283	2.497	0.000	0.000	4.153

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	100	100	188	123	0	303	161
N.S.	1	1.00	0.88	0.88	1.66	1.09	0.00	2.68	1.42
time (sec)	N/A	0.236	0.408	0.342	0.481	3.267	0.000	0.517	1.160

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	80	85	129	98	0	190	117
N.S.	1	1.00	0.87	0.92	1.40	1.07	0.00	2.07	1.27
time (sec)	N/A	0.191	0.246	0.302	0.470	3.034	0.000	0.538	0.823

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	70	75	102	75	0	257	93
N.S.	1	1.00	0.88	0.94	1.28	0.94	0.00	3.21	1.16
time (sec)	N/A	0.167	0.120	0.240	0.463	2.973	0.000	0.550	0.868

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	63	70	71	64	0	100	67
N.S.	1	1.00	0.85	0.95	0.96	0.86	0.00	1.35	0.91
time (sec)	N/A	0.058	0.098	0.215	0.273	2.763	0.000	0.478	0.900

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	64	70	73	65	0	101	68
N.S.	1	1.00	0.85	0.93	0.97	0.87	0.00	1.35	0.91
time (sec)	N/A	0.119	0.075	0.280	0.266	2.417	0.000	0.488	0.708

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	103	87	125	96	0	253	93
N.S.	1	1.00	1.10	0.93	1.33	1.02	0.00	2.69	0.99
time (sec)	N/A	0.171	0.181	0.341	0.284	2.111	0.000	0.559	0.843

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	92	99	158	147	0	190	118
N.S.	1	1.00	0.85	0.92	1.46	1.36	0.00	1.76	1.09
time (sec)	N/A	0.199	0.319	0.388	0.274	3.560	0.000	0.549	0.827

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	164	215	0	857	0	1362	2500
N.S.	1	1.00	0.67	0.88	0.00	3.53	0.00	5.60	10.29
time (sec)	N/A	0.463	3.131	0.429	0.000	2.212	0.000	0.851	5.445

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	151	184	0	705	0	331	2500
N.S.	1	1.00	0.67	0.81	0.00	3.11	0.00	1.46	11.01
time (sec)	N/A	0.366	2.205	0.419	0.000	1.725	0.000	0.688	5.245

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	131	155	0	518	0	282	213
N.S.	1	1.00	0.60	0.71	0.00	2.37	0.00	1.29	0.97
time (sec)	N/A	0.294	1.415	0.315	0.000	2.595	0.000	0.689	1.126

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	128	162	0	526	0	289	245
N.S.	1	1.00	0.63	0.80	0.00	2.59	0.00	1.42	1.21
time (sec)	N/A	0.315	1.327	0.381	0.000	2.373	0.000	0.507	1.193

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	127	162	0	532	0	288	245
N.S.	1	1.00	0.93	1.19	0.00	3.91	0.00	2.12	1.80
time (sec)	N/A	0.219	1.219	0.380	0.000	1.888	0.000	0.618	1.207

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	121	155	0	516	0	284	215
N.S.	1	1.00	0.92	1.18	0.00	3.94	0.00	2.17	1.64
time (sec)	N/A	0.241	0.842	0.377	0.000	1.415	0.000	0.600	1.084

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	196	199	0	864	0	354	2500
N.S.	1	1.00	0.85	0.86	0.00	3.74	0.00	1.53	10.82
time (sec)	N/A	0.326	2.091	0.645	0.000	4.102	0.000	0.684	5.207

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	713	213	684	1180	0	848	527
N.S.	1	1.00	2.88	0.86	2.76	4.76	0.00	3.42	2.12
time (sec)	N/A	0.746	6.367	0.714	0.547	3.969	0.000	1.049	2.049

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	204	199	589	1045	0	676	491
N.S.	1	1.00	0.88	0.86	2.54	4.50	0.00	2.91	2.12
time (sec)	N/A	0.621	6.196	0.739	0.319	3.524	0.000	1.029	1.212

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	184	176	593	994	0	690	490
N.S.	1	1.00	0.87	0.83	2.81	4.71	0.00	3.27	2.32
time (sec)	N/A	0.532	5.623	0.688	0.316	3.210	0.000	0.968	1.109

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	696	196	601	1071	0	800	494
N.S.	1	1.00	3.04	0.86	2.62	4.68	0.00	3.49	2.16
time (sec)	N/A	0.383	6.336	0.693	0.314	4.458	0.000	0.537	1.130

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	703	196	602	1076	0	801	496
N.S.	1	1.00	3.04	0.85	2.61	4.66	0.00	3.47	2.15
time (sec)	N/A	0.493	6.333	0.618	0.308	3.830	0.000	0.776	1.122

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	217	181	596	939	0	689	492
N.S.	1	1.00	1.02	0.85	2.81	4.43	0.00	3.25	2.32
time (sec)	N/A	0.279	6.299	0.604	0.303	2.883	0.000	0.864	1.129

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	217	184	591	971	0	675	490
N.S.	1	1.00	0.95	0.81	2.59	4.26	0.00	2.96	2.15
time (sec)	N/A	0.292	6.249	0.593	0.325	2.817	0.000	0.937	1.133

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	246	0	180	412	0	0	861
N.S.	1	1.00	1.59	0.00	1.16	2.66	0.00	0.00	5.55
time (sec)	N/A	0.295	1.605	0.618	0.280	2.769	0.000	0.000	7.512

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	6669	0	0	0	0	0	-1
N.S.	1	1.00	25.26	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.518	29.923	0.417	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	35	40	36	35	0	0	35
N.S.	1	1.00	0.90	1.03	0.92	0.90	0.00	0.00	0.90
time (sec)	N/A	0.049	0.084	1.295	0.279	4.658	0.000	0.000	0.979

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	106	0	0	0	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.299	0.658	0.141	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	61	82	105	142	699	94	93
N.S.	1	1.00	0.94	1.26	1.62	2.18	10.75	1.45	1.43
time (sec)	N/A	0.052	0.147	0.201	0.473	2.076	108.264	0.458	1.381

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	153	98	211	94	0	152	2500
N.S.	1	1.00	1.66	1.07	2.29	1.02	0.00	1.65	27.17
time (sec)	N/A	0.093	0.344	0.174	0.490	1.896	0.000	0.426	6.233

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	113	163	278	210	0	190	286
N.S.	1	1.00	0.93	1.34	2.28	1.72	0.00	1.56	2.34
time (sec)	N/A	0.120	1.050	0.286	0.482	1.767	0.000	0.451	1.195

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	82	98	212	94	0	156	2500
N.S.	1	1.00	0.88	1.05	2.28	1.01	0.00	1.68	26.88
time (sec)	N/A	0.093	0.336	0.177	0.475	1.620	0.000	0.450	6.187

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	115	165	281	215	0	192	277
N.S.	1	1.00	1.03	1.47	2.51	1.92	0.00	1.71	2.47
time (sec)	N/A	0.136	0.677	0.292	0.487	1.484	0.000	0.469	1.001

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	178	163	431	174	0	275	2500
N.S.	1	1.00	1.01	0.93	2.45	0.99	0.00	1.56	14.20
time (sec)	N/A	0.194	0.560	0.258	0.497	1.725	0.000	0.432	11.949

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	112	170	281	213	0	201	291
N.S.	1	1.00	0.91	1.38	2.28	1.73	0.00	1.63	2.37
time (sec)	N/A	0.106	0.560	0.291	0.484	1.429	0.000	0.467	1.265

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	287	163	424	175	0	273	2500
N.S.	1	1.00	1.64	0.93	2.42	1.00	0.00	1.56	14.29
time (sec)	N/A	0.192	0.810	0.252	0.516	1.994	0.000	0.457	11.144

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	223	302	521	307	0	361	600
N.S.	1	1.00	1.16	1.56	2.70	1.59	0.00	1.87	3.11
time (sec)	N/A	0.252	1.631	0.411	0.494	1.214	0.000	0.475	1.643

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	C	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	87	144	84	118	138	991	144	1017
N.S.	1	1.24	2.06	1.20	1.69	1.97	14.16	2.06	14.53
time (sec)	N/A	0.116	0.271	0.223	0.475	2.319	0.659	0.430	5.161

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	152	111	142	265	252	0	209	249
N.S.	1	1.38	1.01	1.29	2.41	2.29	0.00	1.90	2.26
time (sec)	N/A	0.174	0.638	0.420	0.501	2.336	0.000	0.478	1.232

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	198	226	138	259	236	0	223	2500
N.S.	1	1.53	1.75	1.07	2.01	1.83	0.00	1.73	19.38
time (sec)	N/A	0.297	1.574	0.401	0.473	1.968	0.000	0.391	7.682

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	151	110	143	264	252	0	204	253
N.S.	1	1.39	1.01	1.31	2.42	2.31	0.00	1.87	2.32
time (sec)	N/A	0.181	0.733	0.424	0.479	1.916	0.000	0.465	1.158

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	186	145	143	257	244	0	219	2500
N.S.	1	1.42	1.11	1.09	1.96	1.86	0.00	1.67	19.08
time (sec)	N/A	0.371	1.758	0.398	0.467	1.933	0.000	0.446	12.131

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	238	200	271	611	360	0	342	594
N.S.	1	1.38	1.16	1.58	3.55	2.09	0.00	1.99	3.45
time (sec)	N/A	0.495	1.340	0.588	0.503	1.676	0.000	0.452	2.924

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	196	221	141	256	252	0	214	2500
N.S.	1	1.53	1.73	1.10	2.00	1.97	0.00	1.67	19.53
time (sec)	N/A	0.301	1.383	0.411	0.486	2.014	0.000	0.445	8.158

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	238	198	265	606	369	0	335	586
N.S.	1	1.35	1.12	1.51	3.44	2.10	0.00	1.90	3.33
time (sec)	N/A	0.494	1.339	0.588	0.493	1.724	0.000	0.464	2.875

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	289	409	232	456	371	0	435	2500
N.S.	1	1.38	1.95	1.10	2.17	1.77	0.00	2.07	11.90
time (sec)	N/A	0.863	3.025	0.543	0.487	2.687	0.000	0.442	13.920

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	76	63	98	140	0	90	408
N.S.	1	1.00	1.62	1.34	2.09	2.98	0.00	1.91	8.68
time (sec)	N/A	0.057	0.126	0.273	0.484	2.787	0.000	0.447	1.476

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	60	49	79	142	0	75	123
N.S.	1	1.00	1.25	1.02	1.65	2.96	0.00	1.56	2.56
time (sec)	N/A	0.055	0.090	0.267	0.484	2.339	0.000	0.446	0.984

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [29] had the largest ratio of [33]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	7	5	1.00	14	0.357
2	A	6	4	1.00	14	0.286
3	A	6	5	1.00	12	0.417
4	A	3	2	1.00	9	0.222
5	A	3	2	1.00	12	0.167
6	A	5	4	1.00	14	0.286
7	A	6	5	1.00	14	0.357
8	A	5	5	1.00	16	0.312
9	A	4	4	1.00	16	0.250
10	A	2	2	1.00	14	0.143
11	A	2	2	1.00	11	0.182
12	A	3	3	1.00	14	0.214
13	A	4	4	1.00	16	0.250
14	A	6	6	1.00	16	0.375
15	B	19	11	2.64	16	0.688
16	A	4	4	1.00	16	0.250
17	A	3	3	1.00	14	0.214
18	A	1	1	1.00	11	0.091
19	A	4	4	1.00	14	0.286
20	A	3	2	1.00	16	0.125
21	A	11	7	1.00	16	0.438
22	A	5	5	1.00	16	0.312
23	B	13	7	3.26	16	0.438
24	A	2	2	1.27	14	0.143
25	A	3	3	1.00	11	0.273

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	3	2	1.00	14	0.143
27	A	12	7	1.00	16	0.438
28	A	3	2	1.00	16	0.125
29	A	1	1	1.00	33	0.030
30	A	8	5	1.00	26	0.192
31	A	6	4	1.00	26	0.154
32	A	7	5	1.00	26	0.192
33	A	6	4	1.00	26	0.154
34	A	6	5	1.00	24	0.208
35	A	3	2	1.00	17	0.118
36	A	3	2	1.00	24	0.083
37	A	5	4	1.00	26	0.154
38	A	6	5	1.00	26	0.192
39	A	6	5	1.00	26	0.192
40	A	6	4	1.00	26	0.154
41	A	7	5	1.00	26	0.192
42	A	6	4	1.00	26	0.154
43	A	9	6	1.00	28	0.214
44	A	12	6	1.00	28	0.214
45	A	9	6	1.00	28	0.214
46	A	10	6	1.00	28	0.214
47	A	8	5	1.00	26	0.192
48	A	2	2	1.00	19	0.105
49	A	7	6	1.00	26	0.231
50	A	3	3	1.00	28	0.107
51	A	7	5	1.00	28	0.179
52	A	2	2	1.00	28	0.071
53	A	9	6	1.00	28	0.214
54	A	3	2	1.00	28	0.071
55	A	11	6	1.00	28	0.214
56	A	3	2	1.00	28	0.071
57	A	17	7	1.00	28	0.250
58	A	12	7	1.00	28	0.250
59	A	15	8	1.00	28	0.286
60	A	12	6	1.00	28	0.214

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	4	4	1.00	26	0.154
62	A	2	1	1.00	19	0.053
63	A	7	6	1.00	26	0.231
64	A	10	8	1.00	28	0.286
65	A	4	4	1.00	28	0.143
66	A	9	5	1.00	28	0.179
67	A	2	2	1.00	28	0.071
68	A	12	7	1.00	28	0.250
69	A	3	2	1.00	28	0.071
70	A	14	7	1.00	28	0.250
71	A	3	2	1.00	28	0.071
72	A	16	7	1.00	28	0.250
73	A	3	2	1.00	28	0.071
74	A	15	7	1.00	28	0.250
75	A	22	7	1.00	28	0.250
76	A	15	7	1.00	28	0.250
77	A	19	8	1.00	28	0.286
78	A	14	6	1.00	26	0.231
79	A	3	2	1.00	19	0.105
80	A	14	8	1.00	26	0.308
81	A	7	6	1.00	28	0.214
82	A	14	9	1.00	28	0.321
83	A	5	5	1.00	28	0.179
84	A	12	5	1.00	28	0.179
85	A	2	2	1.00	28	0.071
86	A	16	7	1.00	28	0.250
87	A	3	2	1.00	28	0.071
88	A	19	7	1.00	28	0.250
89	A	3	2	1.00	28	0.071
90	A	22	7	1.00	28	0.250
91	A	3	2	1.00	28	0.071
92	A	29	10	1.00	28	0.357
93	A	18	7	1.00	28	0.250
94	A	25	8	1.00	28	0.286
95	A	18	7	1.00	28	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	5	4	1.00	26	0.154
97	A	3	2	1.00	19	0.105
98	A	8	6	1.00	26	0.231
99	A	17	10	1.00	28	0.357
100	A	7	6	1.00	28	0.214
101	A	17	10	1.00	28	0.357
102	A	6	5	1.00	28	0.179
103	A	15	6	1.00	28	0.214
104	A	2	2	1.00	28	0.071
105	A	19	8	1.00	28	0.286
106	A	3	2	1.00	28	0.071
107	A	22	8	1.00	28	0.286
108	A	3	2	1.00	28	0.071
109	A	25	8	1.00	28	0.286
110	A	9	5	1.00	28	0.179
111	A	7	5	1.00	28	0.179
112	A	5	5	1.00	28	0.179
113	A	4	4	1.00	28	0.143
114	A	2	2	1.00	26	0.077
115	A	2	2	1.00	19	0.105
116	A	3	3	1.00	26	0.115
117	A	4	4	1.00	28	0.143
118	A	6	6	1.00	28	0.214
119	A	7	5	1.00	28	0.179
120	A	9	6	1.00	28	0.214
121	A	11	5	1.00	28	0.179
122	A	7	6	1.00	28	0.214
123	A	11	6	1.67	28	0.214
124	A	4	4	1.00	28	0.143
125	A	3	3	1.00	26	0.115
126	A	1	1	1.00	19	0.053
127	A	4	4	1.00	26	0.154
128	A	3	2	1.00	28	0.071
129	A	11	7	1.00	28	0.250
130	A	3	2	1.00	28	0.071

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	B	15	7	2.28	28	0.250
132	A	5	5	1.00	28	0.179
133	A	6	4	1.89	28	0.143
134	A	2	2	1.36	26	0.077
135	A	3	3	1.00	19	0.158
136	A	3	2	1.00	26	0.077
137	A	12	7	1.00	28	0.250
138	A	3	2	1.00	28	0.071
139	A	31	8	1.00	28	0.286
140	A	3	2	1.00	28	0.071
141	A	6	5	1.00	28	0.179
142	B	7	4	2.31	28	0.143
143	A	2	2	1.00	28	0.071
144	A	5	5	1.00	26	0.192
145	A	2	2	1.00	19	0.105
146	A	8	5	1.00	26	0.192
147	A	3	2	1.00	28	0.071
148	A	32	8	1.00	28	0.286
149	A	3	2	1.00	28	0.071
150	A	9	6	1.00	31	0.194
151	A	7	5	1.00	31	0.161
152	A	8	6	1.00	31	0.194
153	A	7	5	1.00	31	0.161
154	A	2	2	1.00	29	0.069
155	A	1	1	1.00	22	0.045
156	A	4	3	1.00	29	0.103
157	A	6	5	1.00	31	0.161
158	A	7	6	1.00	31	0.194
159	A	7	6	1.00	31	0.194
160	A	7	5	1.00	31	0.161
161	A	8	6	1.00	31	0.194
162	A	7	5	1.00	31	0.161
163	A	10	7	1.00	31	0.226
164	A	5	4	1.00	31	0.129
165	A	10	7	1.00	31	0.226

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	3	2	1.00	31	0.065
167	A	9	6	1.00	29	0.207
168	A	1	1	1.00	22	0.045
169	A	8	7	1.00	29	0.241
170	A	4	3	1.00	31	0.097
171	A	8	6	1.00	31	0.194
172	A	3	3	1.00	31	0.097
173	A	10	7	1.00	31	0.226
174	A	4	3	1.00	31	0.097
175	A	5	4	1.00	31	0.129
176	A	13	8	1.00	31	0.258
177	A	4	2	1.00	31	0.065
178	A	13	7	1.00	31	0.226
179	A	2	2	1.00	29	0.069
180	A	1	1	1.00	22	0.045
181	A	4	3	1.00	29	0.103
182	A	11	9	1.00	31	0.290
183	A	4	3	1.00	31	0.097
184	A	10	6	1.00	31	0.194
185	A	3	3	1.00	31	0.097
186	A	13	8	1.00	31	0.258
187	A	1	1	1.00	33	0.030
188	A	3	3	1.00	7	0.429
189	A	4	3	1.00	10	0.300
190	A	3	3	1.00	10	0.300
191	A	3	3	1.00	10	0.300
192	A	4	4	1.00	10	0.400
193	A	2	2	1.00	10	0.200
194	A	5	5	1.00	10	0.500
195	A	3	3	1.00	9	0.333
196	A	4	3	1.00	12	0.250
197	A	3	3	1.00	12	0.250
198	A	3	3	1.00	12	0.250
199	A	4	4	1.00	12	0.333
200	A	2	2	1.00	12	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	5	5	1.00	12	0.417
202	A	4	4	1.00	20	0.200
203	A	3	3	1.00	10	0.300
204	A	4	3	1.00	10	0.300
205	A	4	4	1.00	10	0.400
206	A	3	3	1.00	10	0.300
207	A	5	5	1.00	10	0.500
208	A	2	2	1.00	10	0.200
209	A	3	3	1.00	12	0.250
210	A	4	3	1.00	12	0.250
211	A	4	4	1.00	12	0.333
212	A	3	3	1.00	12	0.250
213	A	5	5	1.00	12	0.417
214	A	2	2	1.00	12	0.167
215	A	3	3	1.00	15	0.200
216	A	4	2	1.00	22	0.091
217	A	2	2	1.00	22	0.091
218	A	4	3	1.00	22	0.136
219	A	2	2	1.00	22	0.091
220	A	3	2	1.00	22	0.091
221	A	2	1	1.00	22	0.045
222	A	3	3	1.00	17	0.176
223	A	3	2	1.00	24	0.083
224	A	2	2	1.00	24	0.083
225	A	3	2	1.00	24	0.083
226	A	2	2	1.00	24	0.083
227	A	2	1	1.00	24	0.042
228	A	2	1	1.00	24	0.042
229	A	6	4	1.00	26	0.154
230	A	6	5	1.00	26	0.192
231	A	5	5	1.32	24	0.208
232	A	3	2	1.00	17	0.118
233	A	5	5	1.00	24	0.208
234	A	6	5	1.00	26	0.192
235	A	6	4	1.00	26	0.154

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	6	5	1.00	28	0.179
237	A	5	5	1.00	28	0.179
238	A	7	7	1.00	26	0.269
239	A	10	8	1.00	19	0.421
240	A	7	7	1.00	26	0.269
241	A	7	7	1.00	28	0.250
242	A	9	9	1.00	28	0.321
243	A	4	3	1.00	28	0.107
244	A	5	4	1.00	28	0.143
245	A	5	4	1.00	26	0.154
246	A	4	3	1.00	19	0.158
247	A	5	4	1.00	26	0.154
248	A	5	4	1.00	28	0.143
249	A	5	4	1.00	28	0.143
250	A	5	4	1.00	28	0.143
251	A	5	4	1.00	28	0.143
252	A	5	4	1.00	26	0.154
253	A	4	3	1.00	19	0.158
254	A	7	5	1.00	26	0.192
255	A	5	4	1.00	28	0.143
256	A	5	4	1.00	28	0.143
257	A	12	8	1.00	28	0.286
258	A	11	7	1.00	28	0.250
259	A	11	7	1.00	26	0.269
260	A	11	7	1.00	19	0.368
261	A	6	6	1.00	26	0.231
262	A	6	6	1.00	28	0.214
263	A	12	8	1.00	28	0.286
264	A	6	5	1.00	28	0.179
265	A	6	5	1.00	28	0.179
266	A	6	5	1.00	26	0.192
267	A	5	4	1.00	19	0.210
268	A	6	5	1.00	26	0.192
269	A	6	5	1.00	28	0.179
270	A	5	4	1.00	28	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	4	3	1.00	28	0.107
272	A	8	7	1.00	28	0.250
273	A	6	4	1.00	26	0.154
274	A	7	4	1.00	28	0.143
275	A	5	5	1.00	16	0.312
276	A	7	7	1.00	18	0.389
277	A	9	8	1.00	18	0.444
278	A	7	7	1.00	18	0.389
279	A	10	8	1.00	20	0.400
280	A	13	8	1.00	20	0.400
281	A	9	8	1.00	18	0.444
282	A	13	9	1.00	20	0.450
283	A	17	9	1.00	20	0.450
284	A	6	5	1.24	16	0.312
285	A	13	8	1.38	18	0.444
286	A	17	13	1.53	18	0.722
287	A	13	8	1.39	18	0.444
288	A	21	10	1.42	20	0.500
289	A	33	12	1.38	20	0.600
290	A	17	13	1.53	18	0.722
291	A	33	12	1.35	20	0.600
292	A	48	12	1.38	20	0.600
293	A	5	4	1.00	14	0.286
294	A	5	4	1.00	14	0.286

Chapter 3

Listing of integrals

Local contents

3.1 $\int \sin^3(x)(a\cos(x)+b\sin(x))\,dx$

Optimal. Leaf size=36

$$\frac{3bx}{8}-\frac{3}{8}b\cos(x)\sin(x)-\frac{1}{4}b\cos(x)\sin^3(x)+\frac{1}{4}a\sin^4(x)$$

```
[Out] 3/8*b*x-3/8*b*cos(x)*sin(x)-1/4*b*cos(x)*sin(x)^3+1/4*a*sin(x)^4
```

Rubi [A]
time = 0.03, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.357$, Rules used = {3168, 2644, 30, 2715, 8}

$$\frac{1}{4}a\sin^4(x)+\frac{3bx}{8}-\frac{1}{4}b\sin^3(x)\cos(x)-\frac{3}{8}b\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3*(a*Cos[x] + b*Sin[x]),x]
[Out] (3*b*x)/8 - (3*b*Cos[x]*Sin[x])/8 - (b*Cos[x]*Sin[x]^3)/4 + (a*Sin[x]^4)/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3168

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[sin[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(x)(a\cos(x)+b\sin(x))\,dx &= \int \left(a\cos(x)\sin^3(x)+b\sin^4(x)\right)\,dx\\ &= a\int \cos(x)\sin^3(x)\,dx+b\int \sin^4(x)\,dx\\ &= -\frac{1}{4}b\cos(x)\sin^3(x)+a\text{Subst}\left(\int x^3\,dx,x,\sin(x)\right)+\frac{1}{4}(3b)\int \sin^2(x)\,dx\\ &= -\frac{3}{8}b\cos(x)\sin(x)-\frac{1}{4}b\cos(x)\sin^3(x)+\frac{1}{4}a\sin^4(x)+\frac{1}{8}(3b)\int 1\,dx\\ &= \frac{3bx}{8}-\frac{3}{8}b\cos(x)\sin(x)-\frac{1}{4}b\cos(x)\sin^3(x)+\frac{1}{4}a\sin^4(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 34, normalized size = 0.94

$$\frac{3bx}{8}+\frac{1}{4}a\sin^4(x)-\frac{1}{4}b\sin(2x)+\frac{1}{32}b\sin(4x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3*(a*Cos[x] + b*Sin[x]),x]
[Out] (3*b*x)/8 + (a*Sin[x]^4)/4 - (b*Sin[2*x])/4 + (b*Sin[4*x])/32
```

Maple [A]
time = 0.13, size = 28, normalized size = 0.78

method	result
default	$b\left(-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}\right)+\frac{a\left(\sin^4(x)\right)}{4}$
risch	$\frac{3bx}{8}+\frac{a\cos(4x)}{32}+\frac{b\sin(4x)}{32}-\frac{a\cos(2x)}{8}-\frac{b\sin(2x)}{4}$
norman	$\frac{4a\left(\tan^4\left(\frac{x}{2}\right)\right)+\frac{3bx}{8}-\frac{3b\tan\left(\frac{x}{2}\right)}{4}-\frac{11b\left(\tan^3\left(\frac{x}{2}\right)\right)}{4}+\frac{11b\left(\tan^5\left(\frac{x}{2}\right)\right)}{4}+\frac{3b\left(\tan^7\left(\frac{x}{2}\right)\right)}{4}+\frac{3bx\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+\frac{9bx\left(\tan^4\left(\frac{x}{2}\right)\right)}{4}+\frac{3bx\left(\tan^6\left(\frac{x}{2}\right)\right)}{2}+\frac{3bx(}{}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3*(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] b*(-1/4*(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x)+1/4*a*sin(x)^4
```

Maxima [A]
time = 0.27, size = 25, normalized size = 0.69

$$\frac{1}{4}\,a\sin\left(x\right)^{4}+\frac{1}{32}\,b(12\,x+\sin\left(4\,x\right)-8\,\sin\left(2\,x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3*(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*sin(x)^4 + 1/32*b*(12*x + sin(4*x) - 8*sin(2*x))
```

Fricas [A]
time = 2.60, size = 36, normalized size = 1.00

$$\frac{1}{4}\,a\cos\left(x\right)^{4}-\frac{1}{2}\,a\cos\left(x\right)^{2}+\frac{3}{8}\,bx+\frac{1}{8}\left(2\,b\cos\left(x\right)^{3}-5\,b\cos\left(x\right)\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3*(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/4*a*cos(x)^4 - 1/2*a*cos(x)^2 + 3/8*b*x + 1/8*(2*b*cos(x)^3 - 5*b*cos(x))
*sin(x)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(37) = 74$.
time = 0.14, size = 75, normalized size = 2.08

$$\frac{a\sin^{4}\left(x\right)}{4}+\frac{3bx\sin^{4}\left(x\right)}{8}+\frac{3bx\sin^{2}\left(x\right)\cos^{2}\left(x\right)}{4}+\frac{3bx\cos^{4}\left(x\right)}{8}-\frac{5b\sin^{3}\left(x\right)\cos\left(x\right)}{8}-\frac{3b\sin\left(x\right)\cos^{3}\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3*(a*cos(x)+b*sin(x)),x)
```

```
[Out] a*sin(x)**4/4 + 3*b*x*sin(x)**4/8 + 3*b*x*sin(x)**2*cos(x)**2/4 + 3*b*x*cos
(x)**4/8 - 5*b*sin(x)**3*cos(x)/8 - 3*b*sin(x)*cos(x)**3/8
```

Giac [A]
time = 0.40, size = 33, normalized size = 0.92

$$\frac{3}{8}\,bx+\frac{1}{32}\,a\cos\left(4\,x\right)-\frac{1}{8}\,a\cos\left(2\,x\right)+\frac{1}{32}\,b\sin\left(4\,x\right)-\frac{1}{4}\,b\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3*(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] 3/8*b*x + 1/32*a*cos(4*x) - 1/8*a*cos(2*x) + 1/32*b*sin(4*x) - 1/4*b*sin(2*
x)
```

Mupad [B]
time = 0.53, size = 35, normalized size = 0.97

$$\frac{a\cos(x)^4}{4} + \frac{b\sin(x)\cos(x)^3}{4} - \frac{a\cos(x)^2}{2} - \frac{5b\sin(x)\cos(x)}{8} + \frac{3bx}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3*(a*cos(x) + b*sin(x)),x)
[Out] (3*b*x)/8 - (a*cos(x)^2)/2 + (a*cos(x)^4)/4 - (5*b*cos(x)*sin(x))/8 + (b*co
s(x)^3*sin(x))/4
```

3.2 $\int \sin^2(x)(a\cos(x)+b\sin(x))\,dx$

Optimal. Leaf size=24

$$-b\cos(x)+\frac{1}{3}b\cos^3(x)+\frac{1}{3}a\sin^3(x)$$

```
[Out] -b*cos(x)+1/3*b*cos(x)^3+1/3*a*sin(x)^3
```

Rubi [A]
time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3168, 2644, 30, 2713}

$$\frac{1}{3}a\sin^3(x)+\frac{1}{3}b\cos^3(x)-b\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2*(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -(b*Cos[x]) + (b*Cos[x]^3)/3 + (a*Sin[x]^3)/3
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3168

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[sin[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \sin^2(x)(a\cos(x)+b\sin(x))\,dx = \int \left(a\cos(x)\sin^2(x)+b\sin^3(x)\right)\,dx$$
$$= a\int \cos(x)\sin^2(x)\,dx + b\int \sin^3(x)\,dx$$
$$= a\text{Subst}\left(\int x^2\,dx, x, \sin(x)\right) - b\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, \cos(x)\right)$$
$$= -b\cos(x)+\frac{1}{3}b\cos^3(x)+\frac{1}{3}a\sin^3(x)$$

Mathematica [A]
time = 0.01, size = 26, normalized size = 1.08

$$-\frac{3}{4}b\cos(x)+\frac{1}{12}b\cos(3x)+\frac{1}{3}a\sin^3(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2*(a*Cos[x] + b*Sin[x]),x]
[Out] (-3*b*Cos[x])/4 + (b*Cos[3*x])/12 + (a*Sin[x]^3)/3
```

Maple [A]
time = 0.05, size = 20, normalized size = 0.83

method	result	size
default	$-\frac{b(2+\sin^2(x))\cos(x)}{3}+\frac{a(\sin^3(x))}{3}$	20
risch	$-\frac{3b\cos(x)}{4}+\frac{a\sin(x)}{4}+\frac{b\cos(3x)}{12}-\frac{a\sin(3x)}{12}$	26
norman	$\frac{-4b\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{8a\left(\tan^3\left(\frac{x}{2}\right)\right)}{3}-\frac{4b}{3}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}$	34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2*(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
[Out] -1/3*b*(2+sin(x)^2)*cos(x)+1/3*a*sin(x)^3
```

Maxima [A]
time = 0.27, size = 20, normalized size = 0.83

$$\frac{1}{3}a\sin(x)^3+\frac{1}{3}\left(\cos(x)^3-3\cos(x)\right)b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2*(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*sin(x)^3 + 1/3*(cos(x)^3 - 3*cos(x))*b
```

Fricas [A]
time = 2.23, size = 27, normalized size = 1.12

$$\frac{1}{3}\,b\cos\left(x\right)^{3}-b\cos\left(x\right)-\frac{1}{3}\,\left(a\cos\left(x\right)^{2}-a\right)\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2*(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/3*b*cos(x)^3 - b*cos(x) - 1/3*(a*cos(x)^2 - a)*sin(x)
```

Sympy [A]
time = 0.08, size = 27, normalized size = 1.12

$$\frac{a\sin^{3}\left(x\right)}{3}-b\sin^{2}\left(x\right)\cos\left(x\right)-\frac{2b\cos^{3}\left(x\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2*(a*cos(x)+b*sin(x)),x)
```

```
[Out] a*sin(x)**3/3 - b*sin(x)**2*cos(x) - 2*b*cos(x)**3/3
```

Giac [A]
time = 0.41, size = 25, normalized size = 1.04

$$\frac{1}{12}\,b\cos\left(3\,x\right)-\frac{3}{4}\,b\cos\left(x\right)-\frac{1}{12}\,a\sin\left(3\,x\right)+\frac{1}{4}\,a\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2*(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] 1/12*b*cos(3*x) - 3/4*b*cos(x) - 1/12*a*sin(3*x) + 1/4*a*sin(x)
```

Mupad [B]
time = 0.45, size = 32, normalized size = 1.33

$$-\frac{4\left(-2\,a\tan\left(\frac{x}{2}\right)^{3}+3\,b\tan\left(\frac{x}{2}\right)^{2}+b\right)}{3\left(\tan\left(\frac{x}{2}\right)^{2}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2*(a*cos(x) + b*sin(x)),x)
```

```
[Out] -(4*(b - 2*a*tan(x/2)^3 + 3*b*tan(x/2)^2))/(3*(tan(x/2)^2 + 1)^3)
```

3.3 $\int \sin(x)(a\cos(x)+b\sin(x))\,dx$

Optimal. Leaf size=25

$$\frac{bx}{2}-\frac{1}{2}b\cos(x)\sin(x)+\frac{1}{2}a\sin^2(x)$$

```
[Out] 1/2*b*x-1/2*b*cos(x)*sin(x)+1/2*a*sin(x)^2
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3168, 2644, 30, 2715, 8}

$$\frac{1}{2}a\sin^2(x)+\frac{bx}{2}-\frac{1}{2}b\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (b*x)/2 - (b*Cos[x]*Sin[x])/2 + (a*Sin[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3168

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[sin[c + d*x]^m*(a
```

```
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(x)(a\cos(x)+b\sin(x))\,dx &= \int \left(a\cos(x)\sin(x)+b\sin^2(x)\right)\,dx\\ &= a\int \cos(x)\sin(x)\,dx+b\int \sin^2(x)\,dx\\ &= -\frac{1}{2}b\cos(x)\sin(x)+a\text{Subst}\Big(\int x\,dx,x,\sin(x)\Big)+\frac{1}{2}b\int 1\,dx\\ &= \frac{bx}{2}-\frac{1}{2}b\cos(x)\sin(x)+\frac{1}{2}a\sin^2(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.00

$$\frac{bx}{2}-\frac{1}{2}a\cos^2(x)-\frac{1}{4}b\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*(a*Cos[x] + b*Sin[x]),x]
[Out] (b*x)/2 - (a*Cos[x]^2)/2 - (b*Sin[2*x])/4
```

Maple [A]
time = 0.04, size = 21, normalized size = 0.84

method	result	size
risch	$\frac{bx}{2}-\frac{a\cos(2x)}{4}-\frac{b\sin(2x)}{4}$	20
default	$-\frac{(\cos^2(x))a}{2}+b\left(-\frac{\cos(x)\sin(x)}{2}+\frac{x}{2}\right)$	21
meijerg	$\frac{a\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(2x)}{\sqrt{\pi}}\right)}{4}+\frac{b\sqrt{\pi}\left(\frac{2x}{\sqrt{\pi}}-\frac{\sin(2x)}{\sqrt{\pi}}\right)}{4}$	43
norman	$\frac{b\left(\tan^3\left(\frac{x}{2}\right)\right)+2a\left(\tan^2\left(\frac{x}{2}\right)\right)+bx\left(\tan^2\left(\frac{x}{2}\right)\right)+\frac{bx}{2}-b\tan\left(\frac{x}{2}\right)+\frac{bx\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
[Out] -1/2*cos(x)^2*a+b*(-1/2*cos(x)*sin(x)+1/2*x)
```

Maxima [A]
time = 0.35, size = 21, normalized size = 0.84

$$-\frac{1}{2}\,a\cos\left(x\right)^2+\frac{1}{4}\,b(2\,x-\sin\left(2\,x\right))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(a*cos(x)+b*sin(x)),x, algorithm="maxima")
[Out] -1/2*a*cos(x)^2 + 1/4*b*(2*x - sin(2*x))
```

Fricas [A]
time = 3.04, size = 19, normalized size = 0.76

$$-\frac{1}{2}\,a\cos\left(x\right)^2-\frac{1}{2}\,b\cos\left(x\right)\sin\left(x\right)+\frac{1}{2}\,bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(a*cos(x)+b*sin(x)),x, algorithm="fricas")
[Out] -1/2*a*cos(x)^2 - 1/2*b*cos(x)*sin(x) + 1/2*b*x
```

Sympy [A]
time = 0.05, size = 37, normalized size = 1.48

$$\frac{a\sin^2\left(x\right)}{2}+\frac{bx\sin^2\left(x\right)}{2}+\frac{bx\cos^2\left(x\right)}{2}-\frac{b\sin\left(x\right)\cos\left(x\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(a*cos(x)+b*sin(x)),x)
[Out] a*sin(x)**2/2 + b*x*sin(x)**2/2 + b*x*cos(x)**2/2 - b*sin(x)*cos(x)/2
```

Giac [A]
time = 0.40, size = 19, normalized size = 0.76

$$\frac{1}{2}\,bx-\frac{1}{4}\,a\cos\left(2\,x\right)-\frac{1}{4}\,b\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(a*cos(x)+b*sin(x)),x, algorithm="giac")
[Out] 1/2*b*x - 1/4*a*cos(2*x) - 1/4*b*sin(2*x)
```

Mupad [B]
time = 0.41, size = 19, normalized size = 0.76

$$\frac{a\sin\left(x\right)^2}{2}-\frac{b\,\cos\left(x\right)\,\sin\left(x\right)}{2}+\frac{b\,x}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*(a*cos(x) + b*sin(x)),x)
[Out] (a*sin(x)^2)/2 + (b*x)/2 - (b*cos(x)*sin(x))/2
```

3.4 $\int (a \cos(x) + b \sin(x)) \, dx$

Optimal. Leaf size=10

$$-b \cos(x) + a \sin(x)$$

```
[Out] -b*cos(x)+a*sin(x)
```

Rubi [A]
time = 0.01, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {2717, 2718}

$$a \sin(x) - b \cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[a*Cos[x] + b*Sin[x],x]
```

```
[Out] -(b*Cos[x]) + a*Sin[x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int (a \cos(x) + b \sin(x)) \, dx &= a \int \cos(x) \, dx + b \int \sin(x) \, dx \\ &= -b \cos(x) + a \sin(x) \end{aligned}$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 1.00

$$-b \cos(x) + a \sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Cos[x] + b*Sin[x],x]
```

```
[Out] -(b*Cos[x]) + a*Sin[x]
```

Maple [A]
time = 0.03, size = 11, normalized size = 1.10

method	result	size
default	$-b\cos(x)+a\sin(x)$	11
risch	$-b\cos(x)+a\sin(x)$	11
meijerg	$a\sin(x)+b\sqrt{\pi}\,\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(x)}{\sqrt{\pi}}\right)$	22
norman	$\frac{2a\tan(\frac{x}{2})-2b}{1+\tan^2(\frac{x}{2})}$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cos(x)+b*sin(x),x,method=_RETURNVERBOSE)
[Out] -b*cos(x)+a*sin(x)
```

Maxima [A]
time = 0.29, size = 10, normalized size = 1.00

$$-b\cos(x)+a\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(x)+b*sin(x),x, algorithm="maxima")
[Out] -b*cos(x) + a*sin(x)
```

Fricas [A]
time = 3.12, size = 10, normalized size = 1.00

$$-b\cos(x)+a\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(x)+b*sin(x),x, algorithm="fricas")
[Out] -b*cos(x) + a*sin(x)
```

Sympy [A]
time = 0.01, size = 8, normalized size = 0.80

$$a\sin(x)-b\cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(x)+b*sin(x),x)
```

```
[Out] a*sin(x) - b*cos(x)
```

Giac [**A**]
time = 0.40, size = 10, normalized size = 1.00

$$-b\cos\left(x\right) + a\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(x)+b*sin(x),x, algorithm="giac")
```

```
[Out] -b*cos(x) + a*sin(x)
```

Mupad [**B**]
time = 0.40, size = 10, normalized size = 1.00

$$a\,\sin\left(x\right)-b\,\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cos(x) + b*sin(x),x)
```

```
[Out] a*sin(x) - b*cos(x)
```

3.5 $\int \csc(x)(a\cos(x)+b\sin(x))\,dx$

Optimal. Leaf size=9

$$bx + a\log(\sin(x))$$

```
[Out] b*x+a*ln(sin(x))
```

Rubi [A]
time = 0.01, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3164, 3556}

$$a\log(\sin(x)) + bx$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]*(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] b*x + a*Log[Sin[x]]
```

Rule 3164

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cot[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \csc(x)(a\cos(x)+b\sin(x))\,dx &= \int (b+a\cot(x))\,dx \\ &= bx + a\int \cot(x)\,dx \\ &= bx + a\log(\sin(x)) \end{aligned}$$

Mathematica [A]
time = 0.01, size = 9, normalized size = 1.00

$$bx + a\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]*(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] b*x + a*Log[Sin[x]]
```

Maple [A]
time = 0.06, size = 10, normalized size = 1.11

method	result	size
default	$bx + a\ln(\sin(x))$	10
risch	$bx - iax + a\ln(e^{2ix} - 1)$	20
norman	$\frac{bx+bx(\tan^2(\frac{x}{2}))}{1+\tan^2(\frac{x}{2})} + a\ln\left(\tan\left(\frac{x}{2}\right)\right) - a\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)*(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] b*x+a*ln(sin(x))
```

Maxima [A]
time = 0.29, size = 9, normalized size = 1.00

$$bx + a\log(\sin(x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] b*x + a*log(sin(x))
```

Fricas [A]
time = 3.26, size = 11, normalized size = 1.22

$$bx + a\log\left(\frac{1}{2}\sin(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] b*x + a*log(1/2*sin(x))
```

Sympy [A]
time = 0.66, size = 8, normalized size = 0.89

$$a\log(\sin(x)) + bx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*(a*cos(x)+b*sin(x)),x)
```

```
[Out] a*log(sin(x)) + b*x
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(9) = 18$.
time = 0.43, size = 24, normalized size = 2.67

$$bx - a\log\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right) + a\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)*(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] b*x - a*log(tan(1/2*x)^2 + 1) + a*log(abs(tan(1/2*x)))
```

Mupad [B]
time = 0.54, size = 54, normalized size = 6.00

$$a\ln\left(\tan\left(\frac{x}{2}\right)\right) - a\ln\left(\tan\left(\frac{x}{2}\right) - \mathrm{i}\right) - a\ln\left(\tan\left(\frac{x}{2}\right) + 1\mathrm{i}\right) - b\ln\left(\tan\left(\frac{x}{2}\right) - \mathrm{i}\right) 1\mathrm{i} + b\ln\left(\tan\left(\frac{x}{2}\right) + 1\mathrm{i}\right) 1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x) + b*sin(x))/sin(x),x)
```

```
[Out] a*log(tan(x/2)) - a*log(tan(x/2) - 1i) - a*log(tan(x/2) + 1i) - b*log(tan(x
/2) - 1i)*1i + b*log(tan(x/2) + 1i)*1i
```

3.6 $\int \csc^2(x)(a\cos(x)+b\sin(x))\,dx$

Optimal. Leaf size=12

$$-b\tanh^{-1}(\cos(x))-a\csc(x)$$

```
[Out] -b*arctanh(cos(x))-a*csc(x)
```

Rubi [A]
time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.286$, Rules used = {3168, 3855, 2686, 8}

$$-a\csc(x)-b\tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2*(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -(b*ArcTanh[Cos[x]]) - a*Csc[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3168

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[sin[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^2(x)(a\cos(x)+b\sin(x))\,dx &= \int (b\csc(x)+a\cot(x)\csc(x))\,dx\\ &= a\int \cot(x)\csc(x)\,dx + b\int \csc(x)\,dx\\ &= -b\tanh^{-1}(\cos(x)) - a\text{Subst}(\int 1\,dx, x, \csc(x))\\ &= -b\tanh^{-1}(\cos(x)) - a\csc(x)\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(12) = 24.
time = 0.01, size = 25, normalized size = 2.08

$$-a\csc(x) - b\log\left(\cos\left(\frac{x}{2}\right)\right) + b\log\left(\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2*(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -(a*Csc[x]) - b*Log[Cos[x/2]] + b*Log[Sin[x/2]]
```

Maple [**A**]
time = 0.06, size = 19, normalized size = 1.58

method	result	size
default	$-\frac{a}{\sin(x)} + b\ln\left(-\cot\left(x\right)+\csc\left(x\right)\right)$	19
risch	$-\frac{2ia\,\mathrm{e}^{ix}}{\mathrm{e}^{2ix}-1} - b\ln\left(\mathrm{e}^{ix}+1\right) + b\ln\left(\mathrm{e}^{ix}-1\right)$	41
norman	$\frac{-\frac{a}{2}-a\left(\tan^2\left(\frac{x}{2}\right)\right)-\frac{a\left(\tan^4\left(\frac{x}{2}\right)\right)}{2}}{\tan\left(\frac{x}{2}\right)\left(1+\tan^2\left(\frac{x}{2}\right)\right)} + b\ln\left(\tan\left(\frac{x}{2}\right)\right)$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2*(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -a/sin(x)+b*ln(-cot(x)+csc(x))
```

Maxima [**A**]
time = 0.28, size = 24, normalized size = 2.00

$$-\frac{1}{2}\,b(\log\left(\cos\left(x\right)+1\right) - \log\left(\cos\left(x\right)-1\right)) - \frac{a}{\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/2*b*(log(cos(x) + 1) - log(cos(x) - 1)) - a/sin(x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(12) = 24$.
time = 2.44, size = 33, normalized size = 2.75

$$-\frac{b\log\left(\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)\sin\left(x\right)-b\log\left(-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)\sin\left(x\right)+2\,a}{2\,\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(b*log(1/2*cos(x) + 1/2)*sin(x) - b*log(-1/2*cos(x) + 1/2)*sin(x) + 2*
a)/sin(x)
```

Sympy [A]
time = 1.02, size = 24, normalized size = 2.00

$$-\frac{a}{\sin\left(x\right)}+\frac{b\log\left(\cos\left(x\right)-1\right)}{2}-\frac{b\log\left(\cos\left(x\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2*(a*cos(x)+b*sin(x)),x)
```

```
[Out] -a/sin(x) + b*log(cos(x) - 1)/2 - b*log(cos(x) + 1)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(12) = 24$.
time = 0.41, size = 33, normalized size = 2.75

$$b\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)-\frac{1}{2}\,a\tan\left(\frac{1}{2}\,x\right)-\frac{2\,b\tan\left(\frac{1}{2}\,x\right)+a}{2\,\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2*(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] b*log(abs(tan(1/2*x))) - 1/2*a*tan(1/2*x) - 1/2*(2*b*tan(1/2*x) + a)/tan(1/
2*x)
```

Mupad [B]
time = 0.40, size = 24, normalized size = 2.00

$$b\,\ln\left(\tan\left(\frac{x}{2}\right)\right)-\frac{a}{2\tan\left(\frac{x}{2}\right)}-\frac{a\tan\left(\frac{x}{2}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x) + b*sin(x))/sin(x)^2,x)
```

```
[Out] b*log(tan(x/2)) - a/(2*tan(x/2)) - (a*tan(x/2))/2
```

3.7 $\int \csc^3(x)(a\cos(x)+b\sin(x))\,dx$

Optimal. Leaf size=15

$$-b\cot(x)-\frac{1}{2}a\csc^2(x)$$

```
[Out] -b*cot(x)-1/2*a*csc(x)^2
```

Rubi [A]
time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.357$, Rules used = {3168, 3852, 8, 2686, 30}

$$-\frac{1}{2}a\csc^2(x)-b\cot(x)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3*(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -(b*Cot[x]) - (a*Csc[x]^2)/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3168

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[sin[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^3(x)(a\cos(x)+b\sin(x))\,dx &= \int \left(b\csc^2(x)+a\cot(x)\csc^2(x)\right)\,dx\\ &= a\int \cot(x)\csc^2(x)\,dx+b\int \csc^2(x)\,dx\\ &= -(a\text{Subst}(\int x\,dx,x,\csc(x)))-b\text{Subst}(\int 1\,dx,x,\cot(x))\\ &= -b\cot(x)-\frac{1}{2}a\csc^2(x)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 15, normalized size = 1.00

$$-b\cot(x)-\frac{1}{2}a\csc^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3*(a*Cos[x] + b*Sin[x]),x]
[Out] -(b*Cot[x]) - (a*Csc[x]^2)/2
```

Maple [A]
time = 0.07, size = 14, normalized size = 0.93

method	result	size
default	$-\frac{a}{2\sin(x)^2}-b\cot(x)$	14
risch	$-\frac{2i\left(ia\,e^{2ix}+b\,e^{2ix}-b\right)}{\left(e^{2ix}-1\right)^2}$	33
norman	$\frac{-\frac{a}{8}-\frac{a\left(\tan^6\left(\frac{x}{2}\right)\right)}{8}-\frac{b\tan\left(\frac{x}{2}\right)}{2}+\frac{b\left(\tan^5\left(\frac{x}{2}\right)\right)}{2}}{\tan\left(\frac{x}{2}\right)^2\left(1+\tan^2\left(\frac{x}{2}\right)\right)}$	47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3*(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
[Out] -1/2*a/sin(x)^2-b*cot(x)
```

Maxima [A]
time = 0.28, size = 15, normalized size = 1.00

$$-\frac{b}{\tan(x)}-\frac{a}{2\,\sin(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3*(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] -b/tan(x) - 1/2*a/sin(x)^2
```

Fricas [A]
time = 1.51, size = 19, normalized size = 1.27

$$\frac{2\,b\cos\left(x\right)\sin\left(x\right)+a}{2\,\left(\cos\left(x\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3*(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*cos(x)*sin(x) + a)/(cos(x)^2 - 1)
```

Sympy [A]
time = 2.18, size = 17, normalized size = 1.13

$$-\frac{a}{2\sin^{2}\left(x\right)}-\frac{b\cos\left(x\right)}{\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3*(a*cos(x)+b*sin(x)),x)
```

```
[Out] -a/(2*sin(x)**2) - b*cos(x)/sin(x)
```

Giac [A]
time = 0.41, size = 13, normalized size = 0.87

$$-\frac{2\,b\tan\left(x\right)+a}{2\,\tan\left(x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3*(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*tan(x) + a)/tan(x)^2
```

Mupad [B]
time = 0.41, size = 14, normalized size = 0.93

$$-\frac{a+b\,\sin\left(2\,x\right)}{2\sin\left(x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(x) + b*sin(x))/sin(x)^3,x)
```

```
[Out] -(a + b*sin(2*x))/(2*sin(x)^2)
```

3.8 $\int \frac{\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=91

$$\frac{a^2bx}{(a^2+b^2)^2}+\frac{bx}{2(a^2+b^2)}-\frac{a^3\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^2}-\frac{b\cos(x)\sin(x)}{2(a^2+b^2)}-\frac{a\sin^2(x)}{2(a^2+b^2)}$$

```
[Out] a^2*b*x/(a^2+b^2)^2+1/2*b*x/(a^2+b^2)-a^3*ln(a*cos(x)+b*sin(x))/(a^2+b^2)^2
-1/2*b*cos(x)*sin(x)/(a^2+b^2)-1/2*a*sin(x)^2/(a^2+b^2)
```

Rubi [A]
time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3178, 3176, 3212, 2715, 8}

$$\frac{a^2bx}{(a^2+b^2)^2}+\frac{bx}{2(a^2+b^2)}-\frac{a\sin^2(x)}{2(a^2+b^2)}-\frac{b\sin(x)\cos(x)}{2(a^2+b^2)}-\frac{a^3\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (a^2*b*x)/(a^2 + b^2)^2 + (b*x)/(2*(a^2 + b^2)) - (a^3*Log[a*Cos[x] + b*Sin
[x]])/(a^2 + b^2)^2 - (b*Cos[x]*Sin[x])/(2*(a^2 + b^2)) - (a*Sin[x]^2)/(2*(
a^2 + b^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3176

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3178

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
```

```
 + b^2)*(m - 1))), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*
Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*
x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m
, 1]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx &= -\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{a^2\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}+\frac{b\int \sin^2(x)\,dx}{a^2+b^2}\\ &= \frac{a^2bx}{(a^2+b^2)^2}-\frac{b\cos(x)\sin(x)}{2(a^2+b^2)}-\frac{a\sin^2(x)}{2(a^2+b^2)}-\frac{a^3\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{(a^2+b^2)^2}+\frac{b\int 1\,dx}{2(a^2+b^2)}\\ &= \frac{a^2bx}{(a^2+b^2)^2}+\frac{bx}{2(a^2+b^2)}-\frac{a^3\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^2}-\frac{b\cos(x)\sin(x)}{2(a^2+b^2)}-\frac{a\sin^2(x)}{2(a^2}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.20, size = 94, normalized size = 1.03

$$\frac{-4ia^3x+6a^2bx+2b^3x+4ia^3\text{ArcTan}(\tan(x))+a(a^2+b^2)\cos(2x)-2a^3\log\left((a\cos(x)+b\sin(x))^2\right)-a^2b\sin(2x)-b^3\sin(2x)}{4(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] ((-4*I)*a^3*x + 6*a^2*b*x + 2*b^3*x + (4*I)*a^3*ArcTan[Tan[x]] + a*(a^2 + b
^2)*Cos[2*x] - 2*a^3*Log[(a*Cos[x] + b*Sin[x])^2] - a^2*b*Sin[2*x] - b^3*Si
n[2*x])/(4*(a^2 + b^2)^2)
```

Maple [A]
time = 0.12, size = 97, normalized size = 1.07

method	result

default	$\frac{\frac{\left(-\frac{1}{2}a^2b-\frac{1}{2}b^3\right)\tan(x)+\frac{a^3}{2}+\frac{a\,b^2}{2}}{\tan^2(x)+1}+\frac{a^3\ln\left(\tan^2(x)+1\right)}{2}+\frac{\left(3a^2b+b^3\right)\arctan(\tan(x))}{2}}{\left(a^2+b^2\right)^2}-\frac{a^3\ln(a+b\tan(x))}{\left(a^2+b^2\right)^2}$
risch	$\frac{bx}{4iab-2a^2+2b^2}+\frac{ixa}{2iab-a^2+b^2}+\frac{\mathrm{e}^{2ix}}{-8ib+8a}+\frac{\mathrm{e}^{-2ix}}{8ib+8a}+\frac{2ia^3x}{a^4+2a^2b^2+b^4}-\frac{a^3\ln\left(\mathrm{e}^{2ix}-\frac{ib+a}{ib-a}\right)}{a^4+2a^2b^2+b^4}$
norman	$\frac{\frac{b\left(\tan^5\left(\frac{x}{2}\right)\right)}{a^2+b^2}-\frac{2a\left(\tan^2\left(\frac{x}{2}\right)\right)}{a^2+b^2}-\frac{2a\left(\tan^4\left(\frac{x}{2}\right)\right)}{a^2+b^2}-\frac{b\tan\left(\frac{x}{2}\right)}{a^2+b^2}+\frac{b\left(3a^2+b^2\right)x}{2a^4+4a^2b^2+2b^4}+\frac{3b\left(3a^2+b^2\right)x\left(\tan^2\left(\frac{x}{2}\right)\right)}{2\left(a^4+2a^2b^2+b^4\right)}+\frac{3b\left(3a^2+b^2\right)x\left(\tan^4\left(\frac{x}{2}\right)\right)}{2\left(a^4+2a^2b^2+b^4\right)}+\frac{b\left(3a^2+b^2\right)}{2a^4+4a}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a^2+b^2)^2*(((-1/2*a^2*b-1/2*b^3)*tan(x)+1/2*a^3+1/2*a*b^2)/(tan(x)^2+1)
+1/2*a^3*ln(tan(x)^2+1)+1/2*(3*a^2*b+b^3)*arctan(tan(x)))-a^3/(a^2+b^2)^2*l
n(a+b*tan(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(85) = 170$.
time = 0.52, size = 209, normalized size = 2.30

$$-\frac{a^3\log\left(-a-\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^4+2\,a^2b^2+b^4}+\frac{a^3\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{(3\,a^2b+b^3)\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^4+2\,a^2b^2+b^4}-\frac{\frac{b\sin(x)}{\cos(x)+1}+\frac{2\,a\sin(x)^2}{(\cos(x)+1)^2}-\frac{b\sin(x)^3}{(\cos(x)+1)^3}}{a^2+b^2+\frac{2\,(a^2+b^2)\sin(x)^2}{(\cos(x)+1)^2}+\frac{(a^2+b^2)\sin(x)^4}{(\cos(x)+1)^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] -a^3*log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/(a^4 + 2
*a^2*b^2 + b^4) + a^3*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(a^4 + 2*a^2*b^2 + b
^4) + (3*a^2*b + b^3)*arctan(sin(x)/(cos(x) + 1))/(a^4 + 2*a^2*b^2 + b^4) -
 (b*sin(x)/(cos(x) + 1) + 2*a*sin(x)^2/(cos(x) + 1)^2 - b*sin(x)^3/(cos(x)
+ 1)^3)/(a^2 + b^2 + 2*(a^2 + b^2)*sin(x)^2/(cos(x) + 1)^2 + (a^2 + b^2)*si
n(x)^4/(cos(x) + 1)^4)
```

Fricas [A]
time = 2.71, size = 93, normalized size = 1.02

$$-\frac{a^3\log\left(2\,ab\cos\left(x\right)\sin\left(x\right)+(a^2-b^2)\cos\left(x\right)^2+b^2\right)-(a^3+ab^2)\cos\left(x\right)^2+(a^2b+b^3)\cos\left(x\right)\sin\left(x\right)-(3\,a^2b+b^3)x}{2\,(a^4+2\,a^2b^2+b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(a^3*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - (a^3 + a*
b^2)*cos(x)^2 + (a^2*b + b^3)*cos(x)*sin(x) - (3*a^2*b + b^3)*x)/(a^4 + 2*a
^2*b^2 + b^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.40, size = 148, normalized size = 1.63

$$-\frac{a^{3}b\log\left(|b\tan\left(x\right)+a|\right)}{a^{4}b+2\,a^{2}b^{3}+b^{5}}+\frac{a^{3}\log\left(\tan\left(x\right)^{2}+1\right)}{2\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)}+\frac{\left(3\,a^{2}b+b^{3}\right)x}{2\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)}-\frac{a^{3}\tan\left(x\right)^{2}+a^{2}b\tan\left(x\right)+b^{3}\tan\left(x\right)-ab^{2}}{2\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\left(\tan\left(x\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] -a^3*b*log(abs(b*tan(x) + a))/(a^4*b + 2*a^2*b^3 + b^5) + 1/2*a^3*log(tan(x
)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 1/2*(3*a^2*b + b^3)*x/(a^4 + 2*a^2*b^2 +
 b^4) - 1/2*(a^3*tan(x)^2 + a^2*b*tan(x) + b^3*tan(x) - a*b^2)/((a^4 + 2*a^
2*b^2 + b^4)*(tan(x)^2 + 1))
```

Mupad [**B**]
time = 7.50, size = 2500, normalized size = 27.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a*cos(x) + b*sin(x)),x)
```

```
[Out] (4*a^3*log(1/(cos(x) + 1)))/(4*a^4 + 4*b^4 + 8*a^2*b^2) - ((b*tan(x/2))/(a^
2 + b^2) + (2*a*tan(x/2)^2)/(a^2 + b^2) - (b*tan(x/2)^3)/(a^2 + b^2))/(2*ta
n(x/2)^2 + tan(x/2)^4 + 1) - (a^3*log(a + 2*b*tan(x/2) - a*tan(x/2)^2))/(a^
4 + b^4 + 2*a^2*b^2) - (b*atan((tan(x/2)*((((4*a^3*((b*((8*(4*a^2*b^8 - 8*a
^10 + 16*a^4*b^6 + 12*a^6*b^4 - 8*a^8*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*
b^2) + (32*a^3*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b
^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))*(3*
a^2 + b^2))/(2*(a^4 + b^4 + 2*a^2*b^2)) + (16*a^3*b*(3*a^2 + b^2)*(12*a*b^1
0 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8
*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4
*a^4 + 4*b^4 + 8*a^2*b^2) + (b*(3*a^2 + b^2)*((8*(2*a*b^8 + 13*a^3*b^6 + 32
*a^5*b^4 + 21*a^7*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (4*a^3*((8*(4
*a^2*b^8 - 8*a^10 + 16*a^4*b^6 + 12*a^6*b^4 - 8*a^8*b^2))/(a^6 + b^6 + 3*a^
2*b^4 + 3*a^4*b^2) + (32*a^3*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*
```

```
b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*
a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2)))/(2*(a^4 + b^4 + 2*a^2*b^2)) - (b^
3*(3*a^2 + b^2)^3*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^
9*b^2))/((a^4 + b^4 + 2*a^2*b^2)^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(1
6*a^8 + b^8 + 5*a^2*b^6 - 13*a^4*b^4 - 73*a^6*b^2))/(16*a^8 + b^8 + 7*a^2*b
^6 + 15*a^4*b^4 + 25*a^6*b^2)^2 + (2*a*b*(b^6 - 28*a^6 + 10*a^2*b^4 + 17*a^
4*b^2)*((8*(8*a^8 + 2*a^4*b^4 + 9*a^6*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*
b^2) + (4*a^3*((8*(2*a*b^8 + 13*a^3*b^6 + 32*a^5*b^4 + 21*a^7*b^2))/(a^6 +
b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (4*a^3*((8*(4*a^2*b^8 - 8*a^10 + 16*a^4*b^6
+ 12*a^6*b^4 - 8*a^8*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (32*a^3*(1
2*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*
b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*
a^2*b^2)))/(4*a^4 + 4*b^4 + 8*a^2*b^2) - (b*(3*a^2 + b^2)*((b*((8*(4*a^2*b^
8 - 8*a^10 + 16*a^4*b^6 + 12*a^6*b^4 - 8*a^8*b^2))/(a^6 + b^6 + 3*a^2*b^4 +
 3*a^4*b^2) + (32*a^3*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 1
2*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2
)))*(3*a^2 + b^2))/(2*(a^4 + b^4 + 2*a^2*b^2)) + (16*a^3*b*(3*a^2 + b^2)*(1
2*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*
b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2
))))/(2*(a^4 + b^4 + 2*a^2*b^2)) - (8*a^3*b^2*(3*a^2 + b^2)^2*(12*a*b^10 +
48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2
*b^2)*(a^4 + b^4 + 2*a^2*b^2)^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(16*
a^8 + b^8 + 7*a^2*b^6 + 15*a^4*b^4 + 25*a^6*b^2)^2)*(a^10 + b^10 + 5*a^2*b^
8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2))/(4*a*b^3 + 12*a^3*b) - (((4*a^3*(
(b*(3*a^2 + b^2)*((8*(2*a*b^9 - 10*a^9*b + 8*a^3*b^7 - 16*a^7*b^3))/(a^6 +
b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*a^3*(12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7
 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^
2*b^4 + 3*a^4*b^2))))/(2*(a^4 + b^4 + 2*a^2*b^2)) - (16*a^3*b*(3*a^2 + b^2)
*(12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 +
 4*b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*
b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2) - (b*((8*(a^2*b^7 + 2*a^4*b^5 + a^6*b^3
))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (4*a^3*((8*(2*a*b^9 - 10*a^9*b + 8
*a^3*b^7 - 16*a^7*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*a^3*(12*a
^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4
 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2
*b^2))*(3*a^2 + b^2))/(2*(a^4 + b^4 + 2*a^2*b^2)) + (b^3*(3*a^2 + b^2)^3*(1
2*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((a^4 + b^4
+ 2*a^2*b^2)^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(16*a^8 + b^8 + 5*a^2*
b^6 - 13*a^4*b^4 - 73*a^6*b^2)*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a
^6*b^4 + 5*a^8*b^2))/((4*a*b^3 + 12*a^3*b)*(16*a^8 + b^8 + 7*a^2*b^6 + 15*a
^4*b^4 + 25*a^6*b^2)^2) + (2*a*b*(b^6 - 28*a^6 + 10*a^2*b^4 + 17*a^4*b^2)*(
(8*(2*a^7*b + a^5*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (4*a^3*((8*(a
^2*b^7 + 2*a^4*b^5 + a^6*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (4*a^3
*((8*(2*a*b^9 - 10*a^9*b + 8*a^3*b^7 - 16*a^7*b^3))/(a^6 + b^6 + 3*a^2*b^4
+ 3*a^4*b^2) - (32*a^3*(12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 +
```

```
48*a^8*b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^
2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2)))/(4*a^4 + 4*b^4 + 8*a^2*b^2) + (b*((b*(3
*a^2 + b^2)*((8*(2*a*b^9 - 10*a^9*b + 8*a^3*b^7 - 16*a^7*b^3))/(a^6 + b^6 +
 3*a^2*b^4 + 3*a^4*b^2) - (32*a^3*(12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72
*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4
 + 3*a^4*b^2))))/(2*(a^4 + b^4 + 2*a^2*b^2)) - (16*a^3*b*(3*a^2 + b^2)*(12*
a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^
4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)*(a^6 + b...
```

3.9 $\int \frac{\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=68

$$-\frac{a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{b\cos(x)}{a^2+b^2}-\frac{a\sin(x)}{a^2+b^2}$$

```
[Out] -a^2*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)-b*cos(x)/
(a^2+b^2)-a*sin(x)/(a^2+b^2)
```

Rubi [A]
time = 0.06, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3178, 3153, 212, 2718}

$$-\frac{a\sin(x)}{a^2+b^2}-\frac{b\cos(x)}{a^2+b^2}-\frac{a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -((a^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2)) -
 (b*Cos[x])/(a^2 + b^2) - (a*Sin[x])/(a^2 + b^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3178

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
```

```
 + b^2)*(m - 1))), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*
Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*
x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m
, 1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx &= -\frac{a\sin(x)}{a^2+b^2}+\frac{a^2\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}+\frac{b\int \sin(x)\,dx}{a^2+b^2}\\
&= -\frac{b\cos(x)}{a^2+b^2}-\frac{a\sin(x)}{a^2+b^2}-\frac{a^2\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,b\cos(x)-a\sin(x)\right)}{a^2+b^2}\\
&= -\frac{a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{b\cos(x)}{a^2+b^2}-\frac{a\sin(x)}{a^2+b^2}
\end{aligned}
$$

Mathematica [A]
time = 0.17, size = 62, normalized size = 0.91

$$
\frac{2a^2\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{b\cos(x)+a\sin(x)}{a^2+b^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a*Cos[x] + b*Sin[x]),x]
[Out] (2*a^2*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) - (b*C
os[x] + a*Sin[x])/(a^2 + b^2)
```

Maple [A]
time = 0.12, size = 84, normalized size = 1.24

method	result	size
default	$\frac{-2a\tan\left(\frac{x}{2}\right)-2b}{(a^2+b^2)\left(1+\tan^2\left(\frac{x}{2}\right)\right)}+\frac{8a^2\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(4a^2+4b^2)\sqrt{a^2+b^2}}$	84
risch	$\frac{i\mathrm{e}^{ix}}{-2ib+2a}-\frac{i\mathrm{e}^{-ix}}{2(ib+a)}-\frac{a^2\ln\left(\mathrm{e}^{ix}-\frac{ia^3+ia\,b^2-a^2b-b^3}{(a^2+b^2)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}}+\frac{a^2\ln\left(\mathrm{e}^{ix}+\frac{ia^3+ia\,b^2-a^2b-b^3}{(a^2+b^2)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}}$	146

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(a^2+b^2)*(-a*tan(1/2*x)-b)/(1+tan(1/2*x)^2)+8*a^2/(4*a^2+4*b^2)/(a^2+b^2
)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [A]
time = 0.49, size = 106, normalized size = 1.56

$$-\frac{a^2\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\left(b+\frac{a\sin(x)}{\cos(x)+1}\right)}{a^2+b^2+\frac{(a^2+b^2)\sin(x)^2}{(\cos(x)+1)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] -a^2*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(cos(x
) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b + a*sin(x)/(cos(x) + 1)
)/(a^2 + b^2 + (a^2 + b^2)*sin(x)^2/(cos(x) + 1)^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(64) = 128$.
time = 2.12, size = 144, normalized size = 2.12

$$\frac{\sqrt{a^2+b^2}\,a^2\log\left(-\frac{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\,(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2}\right)-2\,(a^2b+b^3)\cos(x)-2\,(a^3+ab^2)\sin(x)}{2\,(a^4+2\,a^2b^2+b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + b^2)*a^2*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 -
 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x
) + (a^2 - b^2)*cos(x)^2 + b^2)) - 2*(a^2*b + b^3)*cos(x) - 2*(a^3 + a*b^2)
*sin(x))/(a^4 + 2*a^2*b^2 + b^4)
```

Sympy [C] Result contains complex when optimal does not.
time = 110.15, size = 706, normalized size = 10.38

$$\begin{cases} \tilde{\infty}\cos(x) & \text{for}\: a=0\wedge b=0 \\ -\frac{\cos(x)}{b} & \text{for}\: a=0 \\ -\frac{\sin^2(x)}{3ib\sin(x)+3b\cos(x)}-\frac{2i\sin(x)\cos(x)}{3ib\sin(x)+3b\cos(x)}-\frac{2\cos^2(x)}{3ib\sin(x)+3b\cos(x)} & \text{for}\: a=-ib \\ -\frac{\sin^2(x)}{-3ib\sin(x)+3b\cos(x)}+\frac{2i\sin(x)\cos(x)}{-3ib\sin(x)+3b\cos(x)}-\frac{2\cos^2(x)}{-3ib\sin(x)+3b\cos(x)} & \text{for}\: a=ib \\ -\frac{a^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)\tan^2\left(\frac{x}{2}\right)}{a^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+a^2\sqrt{a^2+b^2}+b^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+b^2\sqrt{a^2+b^2}}-\frac{a^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{a^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+a^2\sqrt{a^2+b^2}+b^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+b^2\sqrt{a^2+b^2}}+\frac{a^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)\tan^2\left(\frac{x}{2}\right)}{a^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+a^2\sqrt{a^2+b^2}+b^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+b^2\sqrt{a^2+b^2}}+\frac{a^2\log\left(\tan\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{a^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+a^2\sqrt{a^2+b^2}+b^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+b^2\sqrt{a^2+b^2}}-\frac{2a\sqrt{a^2+b^2}\tan\left(\frac{x}{2}\right)}{a^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+a^2\sqrt{a^2+b^2}+b^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+b^2\sqrt{a^2+b^2}}-\frac{2b\sqrt{a^2+b^2}}{a^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+a^2\sqrt{a^2+b^2}+b^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+b^2\sqrt{a^2+b^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Piecewise((zoo*cos(x), Eq(a, 0) & Eq(b, 0)), (-cos(x)/b, Eq(a, 0)), (-sin(x
)**2/(3*I*b*sin(x) + 3*b*cos(x)) - 2*I*sin(x)*cos(x)/(3*I*b*sin(x) + 3*b*co
```

```
s(x)) - 2*cos(x)**2/(3*I*b*sin(x) + 3*b*cos(x)), Eq(a, -I*b)), (-sin(x)**2/
(-3*I*b*sin(x) + 3*b*cos(x)) + 2*I*sin(x)*cos(x)/(-3*I*b*sin(x) + 3*b*cos(x
)) - 2*cos(x)**2/(-3*I*b*sin(x) + 3*b*cos(x)), Eq(a, I*b)), (-a**2*log(tan(
x/2) - b/a - sqrt(a**2 + b**2)/a)*tan(x/2)**2/(a**2*sqrt(a**2 + b**2)*tan(x
/2)**2 + a**2*sqrt(a**2 + b**2) + b**2*sqrt(a**2 + b**2)*tan(x/2)**2 + b**2
*sqrt(a**2 + b**2)) - a**2*log(tan(x/2) - b/a - sqrt(a**2 + b**2)/a)/(a**2*
sqrt(a**2 + b**2)*tan(x/2)**2 + a**2*sqrt(a**2 + b**2) + b**2*sqrt(a**2 + b
**2)*tan(x/2)**2 + b**2*sqrt(a**2 + b**2)) + a**2*log(tan(x/2) - b/a + sqrt
(a**2 + b**2)/a)*tan(x/2)**2/(a**2*sqrt(a**2 + b**2)*tan(x/2)**2 + a**2*sqr
t(a**2 + b**2) + b**2*sqrt(a**2 + b**2)*tan(x/2)**2 + b**2*sqrt(a**2 + b**2
)) + a**2*log(tan(x/2) - b/a + sqrt(a**2 + b**2)/a)/(a**2*sqrt(a**2 + b**2)
*tan(x/2)**2 + a**2*sqrt(a**2 + b**2) + b**2*sqrt(a**2 + b**2)*tan(x/2)**2
+ b**2*sqrt(a**2 + b**2)) - 2*a*sqrt(a**2 + b**2)*tan(x/2)/(a**2*sqrt(a**2
+ b**2)*tan(x/2)**2 + a**2*sqrt(a**2 + b**2) + b**2*sqrt(a**2 + b**2)*tan(x
/2)**2 + b**2*sqrt(a**2 + b**2)) - 2*b*sqrt(a**2 + b**2)/(a**2*sqrt(a**2 +
b**2)*tan(x/2)**2 + a**2*sqrt(a**2 + b**2) + b**2*sqrt(a**2 + b**2)*tan(x/2
)**2 + b**2*sqrt(a**2 + b**2)), True))
```

Giac [A]
time = 0.47, size = 94, normalized size = 1.38

$$-\frac{a^2 \log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\,\left(a\tan\left(\frac{1}{2}\,x\right)+b\right)}{\left(a^2+b^2\right)\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] -a^2*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) -
 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a*tan(1/2*x) + b)/((a^2 +
 b^2)*(tan(1/2*x)^2 + 1))
```

Mupad [B]
time = 0.58, size = 94, normalized size = 1.38

$$-\frac{\frac{2\,b}{a^2+b^2}+\frac{2\,a\tan\left(\frac{x}{2}\right)}{a^2+b^2}}{\tan\left(\frac{x}{2}\right)^2+1}-\frac{2\,a^2\,\mathrm{atanh}\left(\frac{2\,a^2\,b+2\,b^3-2\,a\tan\left(\frac{x}{2}\right)\,\left(a^2+b^2\right)}{2\,{\left(a^2+b^2\right)}^{3/2}}\right)}{{\left(a^2+b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a*cos(x) + b*sin(x)),x)
```

```
[Out] - ((2*b)/(a^2 + b^2) + (2*a*tan(x/2))/(a^2 + b^2))/(tan(x/2)^2 + 1) - (2*a^
2*atanh((2*a^2*b + 2*b^3 - 2*a*tan(x/2)*(a^2 + b^2))/(2*(a^2 + b^2)^(3/2)))
)/(a^2 + b^2)^(3/2)
```

3.10 $\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=35

$$\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}$$

```
[Out] b*x/(a^2+b^2)-a*ln(a*cos(x)+b*sin(x))/(a^2+b^2)
```

Rubi [A]
time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3176, 3212}

$$\frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a*Cos[x] + b*Sin[x]),x]
[Out] (b*x)/(a^2 + b^2) - (a*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)
```

Rule 3176

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx &= \frac{bx}{a^2+b^2}-\frac{a\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}\\ &= \frac{bx}{a^2+b^2}-\frac{a\log(a\cos(x)+b\sin(x))}{a^2+b^2}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.06, size = 47, normalized size = 1.34

$$\frac{2(-ia+b)x+2ia\text{ArcTan}(\tan(x))-a\log\left((a\cos(x)+b\sin(x))^2\right)}{2\left(a^2+b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (2*((-I)*a + b)*x + (2*I)*a*ArcTan[Tan[x]] - a*Log[(a*Cos[x] + b*Sin[x])^2]
)/(2*(a^2 + b^2))
```

Maple [A]
time = 0.07, size = 47, normalized size = 1.34

method	result	size
default	$\frac{\frac{a\ln\left(\tan^2(x)+1\right)}{2}+b\arctan(\tan(x))}{a^2+b^2}-\frac{a\ln(a+b\tan(x))}{a^2+b^2}$	47
risch	$\frac{ix}{ib-a}+\frac{2iax}{a^2+b^2}-\frac{a\ln\left(\mathrm{e}^{2ix}-\frac{ib+a}{ib-a}\right)}{a^2+b^2}$	67
norman	$\frac{\frac{bx}{a^2+b^2}+\frac{bx\left(\tan^2\left(\frac{x}{2}\right)\right)}{a^2+b^2}}{1+\tan^2\left(\frac{x}{2}\right)}+\frac{a\ln\left(1+\tan^2\left(\frac{x}{2}\right)\right)}{a^2+b^2}-\frac{a\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-2b\tan\left(\frac{x}{2}\right)-a\right)}{a^2+b^2}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a^2+b^2)*(1/2*a*ln(tan(x)^2+1)+b*arctan(tan(x)))-a/(a^2+b^2)*ln(a+b*tan(
x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 88 vs. 2(35) = 70.
time = 0.48, size = 88, normalized size = 2.51

$$\frac{2\,b\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2+b^2}-\frac{a\log\left(-a-\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^2+b^2}+\frac{a\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] 2*b*arctan(sin(x)/(cos(x) + 1))/(a^2 + b^2) - a*log(-a - 2*b*sin(x)/(cos(x)
 + 1) + a*sin(x)^2/(cos(x) + 1)^2)/(a^2 + b^2) + a*log(sin(x)^2/(cos(x) + 1
)^2 + 1)/(a^2 + b^2)
```

Fricas [A]
time = 1.57, size = 46, normalized size = 1.31

$$\frac{2\,bx - a\log\left(2\,ab\cos\left(x\right)\sin\left(x\right) + \left(a^2 - b^2\right)\cos\left(x\right)^2 + b^2\right)}{2\,\left(a^2 + b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*x - a*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2))/(a^2
+ b^2)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.35, size = 165, normalized size = 4.71

$$\begin{cases} \tilde{\infty}x & \text{for}\: a = 0 \wedge b = 0 \\ -\frac{\log\left(\cos\left(x\right)\right)}{a} & \text{for}\: b = 0 \\ \frac{ix\sin\left(x\right)}{2ib\sin\left(x\right) + 2b\cos\left(x\right)} + \frac{x\cos\left(x\right)}{2ib\sin\left(x\right) + 2b\cos\left(x\right)} - \frac{\sin\left(x\right)}{2ib\sin\left(x\right) + 2b\cos\left(x\right)} & \text{for}\: a = -ib \\ -\frac{ix\sin\left(x\right)}{-2ib\sin\left(x\right) + 2b\cos\left(x\right)} + \frac{x\cos\left(x\right)}{-2ib\sin\left(x\right) + 2b\cos\left(x\right)} - \frac{\sin\left(x\right)}{-2ib\sin\left(x\right) + 2b\cos\left(x\right)} & \text{for}\: a = ib \\ -\frac{a\log\left(\frac{a\cos\left(x\right)}{b} + \sin\left(x\right)\right)}{a^2 + b^2} + \frac{bx}{a^2 + b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (-log(cos(x))/a, Eq(b, 0)), (I*x*si
n(x)/(2*I*b*sin(x) + 2*b*cos(x)) + x*cos(x)/(2*I*b*sin(x) + 2*b*cos(x)) - s
in(x)/(2*I*b*sin(x) + 2*b*cos(x)), Eq(a, -I*b)), (-I*x*sin(x)/(-2*I*b*sin(x
) + 2*b*cos(x)) + x*cos(x)/(-2*I*b*sin(x) + 2*b*cos(x)) - sin(x)/(-2*I*b*si
n(x) + 2*b*cos(x)), Eq(a, I*b)), (-a*log(a*cos(x)/b + sin(x))/(a**2 + b**2)
 + b*x/(a**2 + b**2), True))
```

Giac [A]
time = 0.42, size = 55, normalized size = 1.57

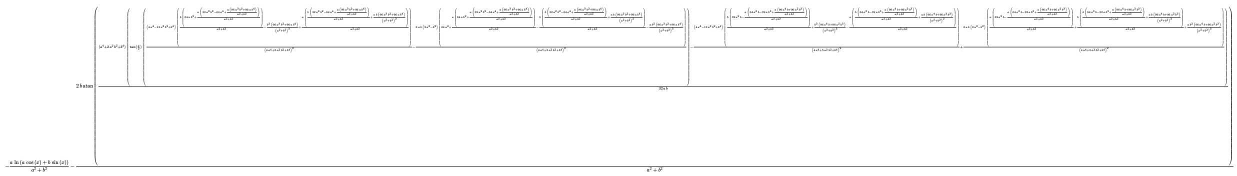
$$-\frac{ab\log\left(\left|b\tan\left(x\right) + a\right|\right)}{a^2b + b^3} + \frac{bx}{a^2 + b^2} + \frac{a\log\left(\tan\left(x\right)^2 + 1\right)}{2\,\left(a^2 + b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] -a*b*log(abs(b*tan(x) + a))/(a^2*b + b^3) + b*x/(a^2 + b^2) + 1/2*a*log(tan
(x)^2 + 1)/(a^2 + b^2)
```

Mupad [B]
time = 2.14, size = 970, normalized size = 27.71



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a*cos(x) + b*sin(x)),x)
```

```
[Out] - (a*log(a*cos(x) + b*sin(x)))/(a^2 + b^2) - (2*b*atan(((a^4 + b^4 + 2*a^2*
b^2)*(tan(x/2)*(((4*a^4 + b^4 - 13*a^2*b^2)*((b*(64*a*b^2 + (a*(32*a^2*b^2
- 64*a^4 + (a*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)))/(a^2 + b^2)))/(a^2 + b
^2) - (b^3*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)^3 + (a*((b*(32*a^2*b^2 - 64
*a^4 + (a*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)))/(a^2 + b^2) + (a*b*(96*a*b
^4 + 96*a^3*b^2))/(a^2 + b^2)^2))/(a^2 + b^2)))/(4*a^4 + b^4 + 5*a^2*b^2)^2
 - (6*a*b*(2*a^2 - b^2)*(64*a^2 + (a*(64*a*b^2 + (a*(32*a^2*b^2 - 64*a^4 +
(a*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)))/(a^2 + b^2)))/(a^2 + b^2) - (b*((
b*(32*a^2*b^2 - 64*a^4 + (a*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)))/(a^2 + b
^2) + (a*b*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)^2))/(a^2 + b^2) - (a*b^2*(9
6*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)^3))/(4*a^4 + b^4 + 5*a^2*b^2)^2) - ((4*a
^4 + b^4 - 13*a^2*b^2)*((b*(32*a^2*b - (a*(64*a^3*b - 32*a*b^3 + (a*(96*a^4
*b + 96*a^2*b^3))/(a^2 + b^2)))/(a^2 + b^2)))/(a^2 + b^2) + (b^3*(96*a^4*b
+ 96*a^2*b^3))/(a^2 + b^2)^3 - (a*((b*(64*a^3*b - 32*a*b^3 + (a*(96*a^4*b +
 96*a^2*b^3))/(a^2 + b^2)))/(a^2 + b^2) + (a*b*(96*a^4*b + 96*a^2*b^3))/(a^
2 + b^2)^2))/(a^2 + b^2)))/(4*a^4 + b^4 + 5*a^2*b^2)^2 + (6*a*b*(2*a^2 - b^
2)*((a*(32*a^2*b - (a*(64*a^3*b - 32*a*b^3 + (a*(96*a^4*b + 96*a^2*b^3))/(a
^2 + b^2)))/(a^2 + b^2)))/(a^2 + b^2) + (b*((b*(64*a^3*b - 32*a*b^3 + (a*(9
6*a^4*b + 96*a^2*b^3))/(a^2 + b^2)))/(a^2 + b^2) + (a*b*(96*a^4*b + 96*a^2*
b^3))/(a^2 + b^2)^2))/(a^2 + b^2) + (a*b^2*(96*a^4*b + 96*a^2*b^3))/(a^2 +
b^2)^3))/(4*a^4 + b^4 + 5*a^2*b^2)^2))/(32*a*b)))/(a^2 + b^2)
```

3.11 $\int \frac{1}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=36

$$-\frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

```
[Out] -arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3153, 212}

$$-\frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x] + b*Sin[x])^(-1),x]
```

```
[Out] -(ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]]/Sqrt[a^2 + b^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a\cos(x)+b\sin(x)}\,dx &= -\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, b\cos(x)-a\sin(x)\right)\\ &= -\frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 38, normalized size = 1.06

$$\frac{2\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x] + b*Sin[x])^(-1),x]
[Out] (2*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2]
```

Maple [A]
time = 0.08, size = 35, normalized size = 0.97

method	result	size
default	$\frac{2\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$	35
risch	$\frac{\ln\left(e^{ix}+\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\frac{\ln\left(e^{ix}-\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
[Out] 2/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [A]
time = 0.50, size = 61, normalized size = 1.69

$$-\frac{\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
[Out] -log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(cos(x) +
1) - sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(32) = 64$.
time = 2.40, size = 96, normalized size = 2.67

$$\frac{\log\left(-\frac{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\,(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2}\right)}{2\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2 + 2*sqrt
(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x
)^2 + b^2))/sqrt(a^2 + b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a\cos(x) + b\sin(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Integral(1/(a*cos(x) + b*sin(x)), x)
```

Giac [A]
time = 0.41, size = 61, normalized size = 1.69

$$-\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] -log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) - 2*b
 + 2*sqrt(a^2 + b^2)))/sqrt(a^2 + b^2)
```

Mupad [B]
time = 1.05, size = 31, normalized size = 0.86

$$-\frac{2\,\mathrm{atanh}\left(\frac{b-a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x) + b*sin(x)),x)
```

```
[Out] -(2*atanh((b - a*tan(x/2))/(a^2 + b^2)^(1/2)))/(a^2 + b^2)^(1/2)
```

3.12 $\int \frac{\csc(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=23

$$\frac{\log(\sin(x))}{a} - \frac{\log(a\cos(x)+b\sin(x))}{a}$$

[Out] ln(sin(x))/a-ln(a*cos(x)+b*sin(x))/a

Rubi [**A**]
time = 0.05, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3180, 3556, 3212}

$$\frac{\log(\sin(x))}{a} - \frac{\log(a\cos(x)+b\sin(x))}{a}$$

Antiderivative was successfully verified.

[In] Int[Csc[x]/(a*Cos[x] + b*Sin[x]),x]

[Out] Log[Sin[x]]/a - Log[a*Cos[x] + b*Sin[x]]/a

Rule 3180

```
Int[1/(sin[(c_.) + (d_.)*(x_)]*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(
c_.) + (d_.)*(x_)])), x_Symbol] :> Dist[1/a, Int[Cot[c + d*x], x], x] - Dis
t[1/a, Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*
x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{\int \cot(x)\,dx}{a} - \frac{\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a}$$
$$= \frac{\log(\sin(x))}{a} - \frac{\log(a\cos(x)+b\sin(x))}{a}$$

Mathematica [A]
time = 0.05, size = 20, normalized size = 0.87

$$\frac{\log(\sin(x)) - \log(a\cos(x)+b\sin(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (Log[Sin[x]] - Log[a*Cos[x] + b*Sin[x]])/a
```

Maple [A]
time = 0.10, size = 21, normalized size = 0.91

method	result	size
default	$\frac{\ln(\tan(x))}{a} - \frac{\ln(a+b\tan(x))}{a}$	21
norman	$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a} - \frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-2b\tan\left(\frac{x}{2}\right)-a\right)}{a}$	36
risch	$-\frac{\ln\left(\mathrm{e}^{2ix}-\frac{ib+a}{ib-a}\right)}{a} + \frac{\ln\left(\mathrm{e}^{2ix}-1\right)}{a}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*ln(tan(x))-1/a*ln(a+b*tan(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 48 vs. 2(23) = 46.
time = 0.27, size = 48, normalized size = 2.09

$$-\frac{\log\left(-a - \frac{2\,b\sin(x)}{\cos(x)+1} + \frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a} + \frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] -log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/a + log(sin(
x)/(cos(x) + 1))/a
```

Fricas [A]
time = 1.97, size = 44, normalized size = 1.91

$$-\frac{\log\left(2\,ab\cos\left(x\right)\sin\left(x\right)+\left(a^{2}-b^{2}\right)\cos\left(x\right)^{2}+b^{2}\right)-\log\left(-\frac{1}{4}\,\cos\left(x\right)^{2}+\frac{1}{4}\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - log(-1/4*cos(
x)^2 + 1/4))/a
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc\left(x\right)}{a\cos\left(x\right)+b\sin\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Integral(csc(x)/(a*cos(x) + b*sin(x)), x)
```

Giac [A]
time = 0.39, size = 22, normalized size = 0.96

$$-\frac{\log\left(\left|b\tan\left(x\right)+a\right|\right)}{a}+\frac{\log\left(\left|\tan\left(x\right)\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] -log(abs(b*tan(x) + a))/a + log(abs(tan(x)))/a
```

Mupad [B]
time = 0.58, size = 32, normalized size = 1.39

$$-\frac{\ln\left(-a\tan\left(\frac{x}{2}\right)^{2}+2\,b\tan\left(\frac{x}{2}\right)+a\right)-\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(a*cos(x) + b*sin(x))),x)
```

```
[Out] -(log(a + 2*b*tan(x/2) - a*tan(x/2)^2) - log(tan(x/2)))/a
```

3.13 $\int \frac{\csc^2(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=55

$$\frac{b\tanh^{-1}(\cos(x))}{a^2}-\frac{\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2}-\frac{\csc(x)}{a}$$

```
[Out] b*arctanh(cos(x))/a^2-csc(x)/a-arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))
*(a^2+b^2)^(1/2)/a^2
```

Rubi [A]
time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {3182, 3855, 3153, 212}

$$-\frac{\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2}+\frac{b\tanh^{-1}(\cos(x))}{a^2}-\frac{\csc(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (b*ArcTanh[Cos[x]])/a^2 - (Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[x] - a*Sin[x])/Sq
rt[a^2 + b^2]])/a^2 - Csc[x]/a
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3182

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[Sin[c + d*x]^(m + 1)/(a*d*(m + 1))
, x] + (-Dist[b/a^2, Int[Sin[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/a^
2, Int[Sin[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(x)}{a\cos(x)+b\sin(x)}\,dx &= -\frac{\csc(x)}{a}-\frac{b\int \csc(x)\,dx}{a^2}+\frac{(a^2+b^2)\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{a^2}\\ &= \frac{b\tanh^{-1}(\cos(x))}{a^2}-\frac{\csc(x)}{a}-\frac{(a^2+b^2)\,\mathrm{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,b\cos(x)-a\sin(x\right.}{a^2}\\ &= \frac{b\tanh^{-1}(\cos(x))}{a^2}-\frac{\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2}-\frac{\csc(x)}{a}\end{aligned}$$

Mathematica [**A**]
time = 0.14, size = 67, normalized size = 1.22

$$\frac{2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)-a\csc(x)+b\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (2*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]] - a*Csc[x] +
b*(Log[Cos[x/2]] - Log[Sin[x/2]]))/a^2
```

Maple [**A**]
time = 0.16, size = 81, normalized size = 1.47

method	result
default	$-\frac{\tan\left(\frac{x}{2}\right)}{2a}-\frac{\left(-4a^2-4b^2\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{2a^2\sqrt{a^2+b^2}}-\frac{1}{2a\tan\left(\frac{x}{2}\right)}-\frac{b\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}$
risch	$-\frac{2i e^{ix}}{a\left(e^{2ix}-1\right)}-\frac{\sqrt{a^2+b^2}\,\ln\left(e^{ix}-\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{a^2}+\frac{\sqrt{a^2+b^2}\,\ln\left(e^{ix}+\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{a^2}-\frac{b\ln\left(e^{ix}-1\right)}{a^2}+\frac{b\ln\left(e^{ix}+\right.}{a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a*tan(1/2*x)-1/2/a^2*(-4*a^2-4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*t
an(1/2*x)-2*b)/(a^2+b^2)^(1/2))-1/2/a/tan(1/2*x)-b/a^2*ln(tan(1/2*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(51) = 102$.
time = 0.49, size = 107, normalized size = 1.95

$$-\frac{b\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2} - \frac{\sqrt{a^2+b^2}\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{a^2} - \frac{\cos(x)+1}{2\,a\sin(x)} - \frac{\sin(x)}{2\,a(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] -b*log(sin(x)/(cos(x) + 1))/a^2 - sqrt(a^2 + b^2)*log((b - a*sin(x)/(cos(x)
 + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(cos(x) + 1) - sqrt(a^2 + b^2)))/a^2
 - 1/2*(cos(x) + 1)/(a*sin(x)) - 1/2*sin(x)/(a*(cos(x) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(51) = 102$.
time = 2.17, size = 133, normalized size = 2.42

$$\frac{b\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)\sin(x) - b\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)\sin(x) + \sqrt{a^2+b^2}\log\left(-\frac{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2}\right)\sin(x) - 2\,a}{2\,a^2\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*log(1/2*cos(x) + 1/2)*sin(x) - b*log(-1/2*cos(x) + 1/2)*sin(x) + sqr
t(a^2 + b^2)*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2
 + 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b
^2)*cos(x)^2 + b^2))*sin(x) - 2*a)/(a^2*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(x)}{a\cos(x)+b\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Integral(csc(x)**2/(a*cos(x) + b*sin(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(51) = 102$.

time = 0.45, size = 108, normalized size = 1.96

$$-\frac{b\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{a^2}-\frac{\tan\left(\frac{1}{2}\,x\right)}{2\,a}-\frac{\sqrt{a^2+b^2}\,\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{a^2}+\frac{2\,b\tan\left(\frac{1}{2}\,x\right)-a}{2\,a^2\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] -b*log(abs(tan(1/2*x)))/a^2 - 1/2*tan(1/2*x)/a - sqrt(a^2 + b^2)*log(abs(2*
a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) - 2*b + 2*sqrt(a
^2 + b^2)))/a^2 + 1/2*(2*b*tan(1/2*x) - a)/(a^2*tan(1/2*x))
```

Mupad [B]
time = 0.68, size = 170, normalized size = 3.09

$$\frac{2\,\mathrm{atanh}\left(\frac{a^3\,\cos\left(\frac{x}{2}\right)\,\sqrt{a^2+b^2}+4\,b^3\,\sin\left(\frac{x}{2}\right)\,\sqrt{a^2+b^2}+3\,a^2\,b\,\sin\left(\frac{x}{2}\right)\,\sqrt{a^2+b^2}+2\,a\,b^2\,\cos\left(\frac{x}{2}\right)\,\sqrt{a^2+b^2}}{\sin\left(\frac{x}{2}\right)\,a^4+2\,\cos\left(\frac{x}{2}\right)\,a^3\,b+5\,\sin\left(\frac{x}{2}\right)\,a^2\,b^2+2\,\cos\left(\frac{x}{2}\right)\,a\,b^3+4\,\sin\left(\frac{x}{2}\right)\,b^4}\right)\,\sqrt{a^2+b^2}}{a^2}-\frac{1}{a\,\sin\left(x\right)}-\frac{b\,\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a*cos(x) + b*sin(x))),x)
```

```
[Out] (2*atanh((a^3*cos(x/2)*(a^2 + b^2)^(1/2) + 4*b^3*sin(x/2)*(a^2 + b^2)^(1/2)
 + 3*a^2*b*sin(x/2)*(a^2 + b^2)^(1/2) + 2*a*b^2*cos(x/2)*(a^2 + b^2)^(1/2))
/(a^4*sin(x/2) + 4*b^4*sin(x/2) + 5*a^2*b^2*sin(x/2) + 2*a*b^3*cos(x/2) + 2
*a^3*b*cos(x/2)))*(a^2 + b^2)^(1/2))/a^2 - 1/(a*sin(x)) - (b*log(sin(x/2)/c
os(x/2)))/a^2
```

3.14 $\int \frac{\csc^3(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=55

$$\frac{b\cot(x)}{a^2}-\frac{\csc^2(x)}{2a}+\frac{(a^2+b^2)\log(\sin(x))}{a^3}-\frac{(a^2+b^2)\log(a\cos(x)+b\sin(x))}{a^3}$$

```
[Out] b*cot(x)/a^2-1/2*csc(x)^2/a+(a^2+b^2)*ln(sin(x))/a^3-(a^2+b^2)*ln(a*cos(x)+
b*sin(x))/a^3
```

Rubi [A]
time = 0.09, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3182, 3852, 8, 3180, 3556, 3212}

$$\frac{b\cot(x)}{a^2}+\frac{(a^2+b^2)\log(\sin(x))}{a^3}-\frac{(a^2+b^2)\log(a\cos(x)+b\sin(x))}{a^3}-\frac{\csc^2(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (b*Cot[x])/a^2 - Csc[x]^2/(2*a) + ((a^2 + b^2)*Log[Sin[x]])/a^3 - ((a^2 + b
^2)*Log[a*Cos[x] + b*Sin[x]])/a^3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3180

```
Int[1/(sin[(c_.) + (d_.)*(x_)]*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(
c_.) + (d_.)*(x_)])), x_Symbol] :> Dist[1/a, Int[Cot[c + d*x], x], x] - Dis
t[1/a, Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*
x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3182

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[Sin[c + d*x]^(m + 1)/(a*d*(m + 1))
, x] + (-Dist[b/a^2, Int[Sin[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/a^
2, Int[Sin[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
```

```
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^3(x)}{a\cos(x)+b\sin(x)}\,dx &= -\frac{\csc^2(x)}{2a}-\frac{b\int \csc^2(x)\,dx}{a^2}+\frac{(a^2+b^2)\int \frac{\csc(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2}\\ &= -\frac{\csc^2(x)}{2a}+\frac{b\text{Subst}(\int 1\,dx,x,\cot(x))}{a^2}+\frac{(a^2+b^2)\int \cot(x)\,dx}{a^3}-\frac{(a^2+b^2)\int \frac{b\cos}{a\cos}}{a^3}\\ &= \frac{b\cot(x)}{a^2}-\frac{\csc^2(x)}{2a}+\frac{(a^2+b^2)\log(\sin(x))}{a^3}-\frac{(a^2+b^2)\log(a\cos(x)+b\sin(x))}{a^3}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 48, normalized size = 0.87

$$\frac{2ab\cot(x)-a^2\csc^2(x)+2(a^2+b^2)\left(\log(\sin(x))-\log(a\cos(x)+b\sin(x))\right)}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (2*a*b*Cot[x] - a^2*Csc[x]^2 + 2*(a^2 + b^2)*(Log[Sin[x]] - Log[a*Cos[x] +
b*Sin[x]]))/(2*a^3)
```

Maple [A]
time = 0.12, size = 53, normalized size = 0.96

method	result	size

default	$-\frac{1}{2a\tan(x)^2}+\frac{(a^2+b^2)\ln(\tan(x))}{a^3}+\frac{b}{a^2\tan(x)}-\frac{(a^2+b^2)\ln(a+b\tan(x))}{a^3}$	53
norman	$\frac{-\frac{1}{8a}-\frac{\tan^4(\frac{x}{2})}{8a}+\frac{b\tan(\frac{x}{2})}{2a^2}-\frac{b(\tan^3(\frac{x}{2}))}{2a^2}}{\tan(\frac{x}{2})^2}+\frac{(a^2+b^2)\ln(\tan(\frac{x}{2}))}{a^3}-\frac{(a^2+b^2)\ln(a(\tan^2(\frac{x}{2}))-2b\tan(\frac{x}{2})-a)}{a^3}$	96
risch	$\frac{2i(-ia\,e^{2ix}+b\,e^{2ix}-b)}{(e^{2ix}-1)^2a^2}-\frac{\ln\left(e^{2ix}-\frac{ib+a}{ib-a}\right)}{a}-\frac{\ln\left(e^{2ix}-\frac{ib+a}{ib-a}\right)b^2}{a^3}+\frac{\ln(e^{2ix}-1)}{a}+\frac{\ln(e^{2ix}-1)b^2}{a^3}$	127

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a/tan(x)^2+(a^2+b^2)/a^3*ln(tan(x))+b/a^2/tan(x)-(a^2+b^2)/a^3*ln(a+b*
tan(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(53) = 106$.
time = 0.28, size = 119, normalized size = 2.16

$$-\frac{\frac{4\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}}{8\,a^2}-\frac{(a^2+b^2)\log\left(-a-\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^3}+\frac{(a^2+b^2)\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^3}-\frac{\left(a-\frac{4\,b\sin(x)}{\cos(x)+1}\right)(\cos(x)+1)^2}{8\,a^2\sin(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] -1/8*(4*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/a^2 - (a^2 + b^2
)*log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/a^3 + (a^2
+ b^2)*log(sin(x)/(cos(x) + 1))/a^3 - 1/8*(a - 4*b*sin(x)/(cos(x) + 1))*(co
s(x) + 1)^2/(a^2*sin(x)^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(53) = 106$.
time = 1.66, size = 117, normalized size = 2.13

$$-\frac{2\,ab\cos(x)\sin(x)-a^2+\left((a^2+b^2)\cos(x)^2-a^2-b^2\right)\log\left(2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2\right)-\left((a^2+b^2)\cos(x)^2-a^2-b^2\right)\log\left(-\frac{1}{4}\cos(x)^2+\frac{1}{4}\right)}{2\left(a^3\cos(x)^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*cos(x)*sin(x) - a^2 + ((a^2 + b^2)*cos(x)^2 - a^2 - b^2)*log(2*
a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - ((a^2 + b^2)*cos(x)^2 - a
^2 - b^2)*log(-1/4*cos(x)^2 + 1/4))/(a^3*cos(x)^2 - a^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^3(x)}{a\cos(x)+b\sin(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Integral(csc(x)**3/(a*cos(x) + b*sin(x)), x)
```

Giac [A]
time = 0.41, size = 78, normalized size = 1.42

$$\frac{(a^2+b^2)\log(|\tan(x)|)}{a^3} - \frac{(a^2b+b^3)\log(|b\tan(x)+a|)}{a^3b} - \frac{3a^2\tan(x)^2+3b^2\tan(x)^2-2ab\tan(x)+a^2}{2a^3\tan(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] (a^2 + b^2)*log(abs(tan(x)))/a^3 - (a^2*b + b^3)*log(abs(b*tan(x) + a))/(a^
3*b) - 1/2*(3*a^2*tan(x)^2 + 3*b^2*tan(x)^2 - 2*a*b*tan(x) + a^2)/(a^3*tan(
x)^2)
```

Mupad [B]
time = 0.57, size = 91, normalized size = 1.65

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)(a^2+b^2)}{a^3} - \frac{\ln\left(-a\tan\left(\frac{x}{2}\right)^2+2b\tan\left(\frac{x}{2}\right)+a\right)(a^2+b^2)}{a^3} - \frac{\tan\left(\frac{x}{2}\right)^2}{8a} - \frac{b\tan\left(\frac{x}{2}\right)}{2a^2} - \frac{\frac{a}{2}-2b\tan\left(\frac{x}{2}\right)}{4a^2\tan\left(\frac{x}{2}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^3*(a*cos(x) + b*sin(x))),x)
```

```
[Out] (log(tan(x/2))*(a^2 + b^2))/a^3 - (log(a + 2*b*tan(x/2) - a*tan(x/2)^2)*(a^
2 + b^2))/a^3 - tan(x/2)^2/(8*a) - (b*tan(x/2))/(2*a^2) - (a/2 - 2*b*tan(x/
2))/(4*a^2*tan(x/2)^2)
```

3.15 $\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=107

$$\frac{6a^2b\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}+\frac{3a(a^2-b^2)+a(a^2+b^2)\cos(2x)-b(a^2+b^2)\sin(2x)}{2\,(a^2+b^2)^2\,(a\cos(x)+b\sin(x))}$$

```
[Out] 6*a^2*b*arctanh((-b+a*tan(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)+1/2*(3*a
*(a^2-b^2)+a*(a^2+b^2)*cos(2*x)-b*(a^2+b^2)*sin(2*x))/(a^2+b^2)^2/(a*cos(x)
+b*sin(x))
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 283 vs. $2(107) = 214$.
time = 0.87, antiderivative size = 283, normalized size of antiderivative = 2.64, number of steps used = 19, number of rules used = 11, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$,
Rules used = {4486, 2717, 2718, 6874, 653, 209, 652, 632, 212, 3179, 3153}

$$\frac{3a^2\cos(x)}{b^2(a^2+b^2)}+\frac{2a^2\left(a+b\tan\left(\frac{x}{2}\right)\right)}{(a^2+b^2)^2\left(-a\tan^2\left(\frac{x}{2}\right)+a+2b\tan\left(\frac{x}{2}\right)\right)}+\frac{2a^2(3a^2+b^2)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{5/2}}-\frac{2a^2b\tanh^{-1}\left(\frac{b-a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}-\frac{3a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b\,(a^2+b^2)^{3/2}}+\frac{3a^3\sin(x)}{b^3(a^2+b^2)}-\frac{2a^3\cos^2\left(\frac{x}{2}\right)\left((a^2-b^2)\tan\left(\frac{x}{2}\right)+2ab\right)}{b^3\,(a^2+b^2)^2}-\frac{2a\sin(x)}{b^3}-\frac{\cos(x)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (-3*a^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(b*(a^2 + b^2)^(3/2
)) - (2*a^2*b*ArcTanh[(b - a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2)
+ (2*a^2*(3*a^2 + b^2)*ArcTanh[(b - a*Tan[x/2])/Sqrt[a^2 + b^2]])/(b*(a^2 +
 b^2)^(5/2)) - Cos[x]/b^2 + (3*a^2*Cos[x])/(b^2*(a^2 + b^2)) - (2*a*Sin[x])
/b^3 + (3*a^3*Sin[x])/(b^3*(a^2 + b^2)) - (2*a^3*Cos[x/2]^2*(2*a*b + (a^2 -
 b^2)*Tan[x/2]))/(b^3*(a^2 + b^2)^2) + (2*a^2*(a + b*Tan[x/2]))/((a^2 + b^2
)^2*(a + 2*b*Tan[x/2] - a*Tan[x/2]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3179

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \int \left(-\frac{2a\cos(x)}{b^3}+\frac{\sin(x)}{b^2}-\frac{a^3\cos^3(x)}{b^3(a\cos(x)+b\sin(x))^2}+\frac{3a^2\cos^2(x)}{b^3(a\cos(x)+b\sin(x))}\right.$$

$$= -\frac{(2a)\int\cos(x)\,dx}{b^3}+\frac{(3a^2)\int\frac{\cos^2(x)}{a\cos(x)+b\sin(x)}\,dx}{b^3}-\frac{a^3\int\frac{\cos^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{b^3}+\frac{\int \mathrm{si}}{}$$

$$= -\frac{\cos(x)}{b^2}+\frac{3a^2\cos(x)}{b^2(a^2+b^2)}-\frac{2a\sin(x)}{b^3}-\frac{(2a^3)\,\mathrm{Subst}\left(\int\frac{(1-x^2)^3}{(1+x^2)^2(a+2bx-ax^2)^2}\,dx,x,\mathrm{t}\right.}{b^3}$$

$$= -\frac{\cos(x)}{b^2}+\frac{3a^2\cos(x)}{b^2(a^2+b^2)}-\frac{2a\sin(x)}{b^3}+\frac{3a^3\sin(x)}{b^3(a^2+b^2)}-\frac{(2a^3)\,\mathrm{Subst}\left(\int\left(\frac{2(a^2-b^2-}{(a^2+b^2)^2(1}\right.\right.}{}$$

$$= -\frac{3a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}}-\frac{\cos(x)}{b^2}+\frac{3a^2\cos(x)}{b^2(a^2+b^2)}-\frac{2a\sin(x)}{b^3}+\frac{3a^3\sin(}{b^3(a^2+}$$

$$= \frac{a^3(a^2-b^2)\,x}{b^3(a^2+b^2)^2}-\frac{3a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}}-\frac{\cos(x)}{b^2}+\frac{3a^2\cos(x)}{b^2(a^2+b^2)}-\frac{2a\sin(}{b^3}$$

$$= -\frac{3a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}}+\frac{2a^2(3a^2+b^2)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{5/2}}-\frac{\cos(x)}{b^2}$$

$$= -\frac{3a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b(a^2+b^2)^{3/2}}-\frac{2a^2b\tanh^{-1}\left(\frac{b-a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}+\frac{2a^2(3a^2+b^2)\,\mathrm{ta}}{b(a^2}$$

Mathematica [A]
time = 0.48, size = 107, normalized size = 1.00

$$\frac{6a^2b\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}+\frac{3a(a^2-b^2)+a(a^2+b^2)\cos(2x)-b(a^2+b^2)\sin(2x)}{2(a^2+b^2)^2(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (6*a^2*b*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) + (3
*a*(a^2 - b^2) + a*(a^2 + b^2)*Cos[2*x] - b*(a^2 + b^2)*Sin[2*x])/(2*(a^2 +
 b^2)^2*(a*Cos[x] + b*Sin[x]))
```

Maple [A]
time = 0.24, size = 140, normalized size = 1.31

method	result
default	$-\frac{4\left(ab\tan\left(\frac{x}{2}\right)-\frac{a^2}{2}+\frac{b^2}{2}\right)}{(a^4+2a^2b^2+b^4)\left(1+\tan^2\left(\frac{x}{2}\right)\right)}+\frac{4a^2\left(\frac{-\frac{b\tan\left(\frac{x}{2}\right)}{2}-\frac{a}{2}}{a\left(\tan^2\left(\frac{x}{2}\right)\right)-2b\tan\left(\frac{x}{2}\right)-a}+\frac{3b\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}}\right)}{a^4+2a^2b^2+b^4}$
risch	$\frac{e^{ix}}{-4iab+2a^2-2b^2}+\frac{e^{-ix}}{4iab+2a^2-2b^2}+\frac{2a^3e^{ix}}{(-ib\,e^{2ix}+a\,e^{2ix}+ib+a)(ib+a)^2(-ib+a)^2}-\frac{3iba^2\ln\left(e^{ix}+\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,(a^2+b^2)^2}+\frac{3iba^2}{\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -4/(a^4+2*a^2*b^2+b^4)*(a*b*tan(1/2*x)-1/2*a^2+1/2*b^2)/(1+tan(1/2*x)^2)+4*
a^2/(a^4+2*a^2*b^2+b^4)*((-1/2*b*tan(1/2*x)-1/2*a)/(a*tan(1/2*x)^2-2*b*tan(
1/2*x)-a)+3/2*b/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^
(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(99) = 198$.
time = 0.50, size = 253, normalized size = 2.36

$$-\frac{3a^2b\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}}+\frac{2\left(2a^3-ab^2-\frac{3ab^2\sin(x)^2}{(\cos(x)+1)^2}+\frac{3a^2b\sin(x)^3}{(\cos(x)+1)^3}+\frac{(a^2b-2b^3)\sin(x)}{\cos(x)+1}\right)}{a^5+2a^3b^2+ab^4+\frac{2(a^4b+2a^2b^3+b^5)\sin(x)}{\cos(x)+1}+\frac{2(a^4b+2a^2b^3+b^5)\sin(x)^3}{(\cos(x)+1)^3}-\frac{(a^5+2a^3b^2+ab^4)\sin(x)^4}{(\cos(x)+1)^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -3*a^2*b*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(c
os(x) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) +
2*(2*a^3 - a*b^2 - 3*a*b^2*sin(x)^2/(cos(x) + 1)^2 + 3*a^2*b*sin(x)^3/(cos(
x) + 1)^3 + (a^2*b - 2*b^3)*sin(x)/(cos(x) + 1))/(a^5 + 2*a^3*b^2 + a*b^4 +
 2*(a^4*b + 2*a^2*b^3 + b^5)*sin(x)/(cos(x) + 1) + 2*(a^4*b + 2*a^2*b^3 + b
^5)*sin(x)^3/(cos(x) + 1)^3 - (a^5 + 2*a^3*b^2 + a*b^4)*sin(x)^4/(cos(x) +
1)^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(99) = 198$.

time = 1.70, size = 240, normalized size = 2.24

$$\frac{2\,a^5-2\,a^3b^2-4\,ab^4+2\,(a^5+2\,a^3b^2+ab^4)\cos(x)^2-2\,(a^4b+2\,a^2b^3+b^5)\cos(x)\sin(x)+3\,(a^3b\cos(x)+a^2b^2\sin(x))\sqrt{a^2+b^2}\,\log\left(-\frac{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\,(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2}\right)}{2\,((a^7+3\,a^5b^2+3\,a^3b^4+ab^6)\cos(x)+(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^5 - 2*a^3*b^2 - 4*a*b^4 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^2 - 2
*(a^4*b + 2*a^2*b^3 + b^5)*cos(x)*sin(x) + 3*(a^3*b*cos(x) + a^2*b^2*sin(x)
)*sqrt(a^2 + b^2)*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2
- b^2 + 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^
2 - b^2)*cos(x)^2 + b^2)))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(x) +
(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.42, size = 186, normalized size = 1.74

$$-\frac{3\,a^2b\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}}-\frac{2\left(3\,a^2b\tan\left(\frac{1}{2}\,x\right)^3-3\,ab^2\tan\left(\frac{1}{2}\,x\right)^2+a^2b\tan\left(\frac{1}{2}\,x\right)-2\,b^3\tan\left(\frac{1}{2}\,x\right)+2\,a^3-ab^2\right)}{\left(a\tan\left(\frac{1}{2}\,x\right)^4-2\,b\tan\left(\frac{1}{2}\,x\right)^3-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)(a^4+2\,a^2b^2+b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -3*a^2*b*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*
x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) -
2*(3*a^2*b*tan(1/2*x)^3 - 3*a*b^2*tan(1/2*x)^2 + a^2*b*tan(1/2*x) - 2*b^3*t
an(1/2*x) + 2*a^3 - a*b^2)/((a*tan(1/2*x)^4 - 2*b*tan(1/2*x)^3 - 2*b*tan(1/
2*x) - a)*(a^4 + 2*a^2*b^2 + b^4))
```

Mupad [B]
time = 0.84, size = 224, normalized size = 2.09

$$-\frac{\frac{2\,(a\,b^2-2\,a^3)}{a^4+2\,a^2\,b^2+b^4}-\frac{2\tan\left(\frac{x}{2}\right)\,(a^2\,b-2\,b^3)}{a^4+2\,a^2\,b^2+b^4}+\frac{6\,a\,b^2\tan\left(\frac{x}{2}\right)^2}{a^4+2\,a^2\,b^2+b^4}-\frac{6\,a^2\,b\tan\left(\frac{x}{2}\right)^3}{a^4+2\,a^2\,b^2+b^4}}{-a\tan\left(\frac{x}{2}\right)^4+2\,b\tan\left(\frac{x}{2}\right)^3+2\,b\tan\left(\frac{x}{2}\right)+a}-\frac{6\,a^2\,b\,\mathrm{atanh}\left(\frac{2\,a^4\,b+2\,b^5+4\,a^2\,b^3-2\,a\tan\left(\frac{x}{2}\right)\,(a^4+2\,a^2\,b^2+b^4)}{2\,(a^2+b^2)^{5/2}}\right)}{(a^2+b^2)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a*cos(x) + b*sin(x))^2,x)
```

```
[Out] - ((2*(a*b^2 - 2*a^3))/(a^4 + b^4 + 2*a^2*b^2) - (2*tan(x/2)*(a^2*b - 2*b^3
))/(a^4 + b^4 + 2*a^2*b^2) + (6*a*b^2*tan(x/2)^2)/(a^4 + b^4 + 2*a^2*b^2) -
 (6*a^2*b*tan(x/2)^3)/(a^4 + b^4 + 2*a^2*b^2))/(a + 2*b*tan(x/2) - a*tan(x/
2)^4 + 2*b*tan(x/2)^3) - (6*a^2*b*atanh((2*a^4*b + 2*b^5 + 4*a^2*b^3 - 2*a*
tan(x/2)*(a^4 + b^4 + 2*a^2*b^2))/(2*(a^2 + b^2)^(5/2))))/(a^2 + b^2)^(5/2)
```

3.16 $\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=64

$$-\frac{(a^2-b^2)\,x}{(a^2+b^2)^2}+\frac{a}{(a^2+b^2)\,(b+a\cot(x))}-\frac{2ab\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^2}$$

```
[Out] -(a^2-b^2)*x/(a^2+b^2)^2+a/(a^2+b^2)/(b+a*cot(x))-2*a*b*ln(a*cos(x)+b*sin(x
))/(a^2+b^2)^2
```

Rubi [A]
time = 0.09, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3164, 3564, 3612, 3611}

$$-\frac{x(a^2-b^2)}{(a^2+b^2)^2}+\frac{a}{(a^2+b^2)\,(a\cot(x)+b)}-\frac{2ab\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] -(((a^2 - b^2)*x)/(a^2 + b^2)^2) + a/((a^2 + b^2)*(b + a*Cot[x])) - (2*a*b*
Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^2
```

Rule 3164

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cot[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx &= \int \frac{1}{(b+a\cot(x))^2}\,dx\\
&= \frac{a}{(a^2+b^2)\,(b+a\cot(x))} + \frac{\int \frac{b-a\cot(x)}{b+a\cot(x)}\,dx}{a^2+b^2}\\
&= -\frac{(a^2-b^2)\,x}{(a^2+b^2)^2} + \frac{a}{(a^2+b^2)\,(b+a\cot(x))} - \frac{(2ab)\int \frac{-a+b\cot(x)}{b+a\cot(x)}\,dx}{(a^2+b^2)^2}\\
&= -\frac{(a^2-b^2)\,x}{(a^2+b^2)^2} + \frac{a}{(a^2+b^2)\,(b+a\cot(x))} - \frac{2ab\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^2}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.28, size = 121, normalized size = 1.89

$$\frac{-a\cos(x)\left((a+ib)^2x+ab\log\left((a\cos(x)+b\sin(x))^2\right)\right)+\left(a^3+ab^2(1-2ix)-a^2bx+b^3x-ab^2\log\left((a\cos(x)+b\sin(x))^2\right)\right)\sin(x)+2iab\text{ArcTan}(\tan(x))(a\cos(x)+b\sin(x))}{(a^2+b^2)^2\,(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (-(a*Cos[x]*((a + I*b)^2*x + a*b*Log[(a*Cos[x] + b*Sin[x])^2])) + (a^3 + a*
b^2*(1 - (2*I)*x) - a^2*b*x + b^3*x - a*b^2*Log[(a*Cos[x] + b*Sin[x])^2])*S
in[x] + (2*I)*a*b*ArcTan[Tan[x]]*(a*Cos[x] + b*Sin[x]))/((a^2 + b^2)^2*(a*C
os[x] + b*Sin[x]))
```

Maple [A]
time = 0.13, size = 81, normalized size = 1.27

method	result
default	$\frac{ab\ln\left(\tan^2(x)+1\right)+\left(-a^2+b^2\right)\arctan(\tan(x))}{\left(a^2+b^2\right)^2}-\frac{a^2}{(a^2+b^2)b(a+b\tan(x))}-\frac{2ba\ln(a+b\tan(x))}{\left(a^2+b^2\right)^2}$
risch	$\frac{x}{2iab-a^2+b^2}+\frac{4iabx}{a^4+2a^2b^2+b^4}+\frac{2ia^2}{(-ib\,e^{2ix}+a\,e^{2ix}+ib+a)(ib+a)(-ib+a)^2}-\frac{2ab\ln\left(e^{2ix}-\frac{ib+a}{ib-a}\right)}{a^4+2a^2b^2+b^4}$

norman	$\frac{\frac{a\left(a^2-b^2\right)x}{a^4+2a^2b^2+b^4}+\frac{a\left(a^2-b^2\right)x\left(\tan^2\left(\frac{x}{2}\right)\right)}{a^4+2a^2b^2+b^4}-\frac{2a\tan\left(\frac{x}{2}\right)}{a^2+b^2}-\frac{4a\left(\tan^3\left(\frac{x}{2}\right)\right)}{a^2+b^2}-\frac{2a\left(\tan^5\left(\frac{x}{2}\right)\right)}{a^2+b^2}-\frac{a\left(a^2-b^2\right)x\left(\tan^4\left(\frac{x}{2}\right)\right)}{a^4+2a^2b^2+b^4}-\frac{a\left(a^2-b^2\right)x\left(\tan^6\left(\frac{x}{2}\right)\right)}{a^4+2a^2b^2+b^4}+\frac{2b\left(a^2\right.}{a^4}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)^2\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-2b\tan\left(\frac{x}{2}\right)-a\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a^2+b^2)^2*(a*b*ln(tan(x)^2+1)+(-a^2+b^2)*arctan(tan(x)))-a^2/(a^2+b^2)/
b/(a+b*tan(x))-2*b*a/(a^2+b^2)^2*ln(a+b*tan(x))
```

Maxima [A]
time = 0.47, size = 117, normalized size = 1.83

$$-\frac{2\,ab\log\left(b\tan\left(x\right)+a\right)}{a^4+2\,a^2b^2+b^4}+\frac{ab\log\left(\tan\left(x\right)^2+1\right)}{a^4+2\,a^2b^2+b^4}-\frac{a^2}{a^3b+ab^3+\left(a^2b^2+b^4\right)\tan\left(x\right)}-\frac{(a^2-b^2)x}{a^4+2\,a^2b^2+b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -2*a*b*log(b*tan(x) + a)/(a^4 + 2*a^2*b^2 + b^4) + a*b*log(tan(x)^2 + 1)/(a
^4 + 2*a^2*b^2 + b^4) - a^2/(a^3*b + a*b^3 + (a^2*b^2 + b^4)*tan(x)) - (a^2
 - b^2)*x/(a^4 + 2*a^2*b^2 + b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(64) = 128.
time = 1.94, size = 132, normalized size = 2.06

$$-\frac{(a^2b+(a^3-ab^2)x)\cos\left(x\right)+(a^2b\cos\left(x\right)+ab^2\sin\left(x\right))\log\left(2\,ab\cos\left(x\right)\sin\left(x\right)+(a^2-b^2)\cos\left(x\right)^2+b^2\right)-(a^3-(a^2b-b^3)x)\sin\left(x\right)}{(a^5+2\,a^3b^2+ab^4)\cos\left(x\right)+(a^4b+2\,a^2b^3+b^5)\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -((a^2*b + (a^3 - a*b^2)*x)*cos(x) + (a^2*b*cos(x) + a*b^2*sin(x))*log(2*a*
b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - (a^3 - (a^2*b - b^3)*x)*sin
(x))/((a^5 + 2*a^3*b^2 + a*b^4)*cos(x) + (a^4*b + 2*a^2*b^3 + b^5)*sin(x))
```

Sympy [C] Result contains complex when optimal does not.
time = 0.66, size = 976, normalized size = 15.25

$$\begin{cases} \tilde{\infty}x & \text{for}\: a = 0 \wedge b = 0 \\ \frac{-x+\frac{\sin\left(x\right)}{\cos\left(x\right)}}{a^2} & \text{for}\: b = 0 \\ -\frac{2x\sin^2\left(x\right)}{-8b^2\sin^2\left(x\right)+16ib^2\sin\left(x\right)\cos\left(x\right)+8b^2\cos^2\left(x\right)}+\frac{4ix\sin\left(x\right)\cos\left(x\right)}{-8b^2\sin^2\left(x\right)+16ib^2\sin\left(x\right)\cos\left(x\right)+8b^2\cos^2\left(x\right)}+\frac{2x\cos^2\left(x\right)}{-8b^2\sin^2\left(x\right)+16ib^2\sin\left(x\right)\cos\left(x\right)+8b^2\cos^2\left(x\right)}-\frac{3i\sin^2\left(x\right)}{-8b^2\sin^2\left(x\right)+16ib^2\sin\left(x\right)\cos\left(x\right)+8b^2\cos^2\left(x\right)}-\frac{i\cos^2\left(x\right)}{-8b^2\sin^2\left(x\right)+16ib^2\sin\left(x\right)\cos\left(x\right)+8b^2\cos^2\left(x\right)} & \text{for}\: a = -ib \\ -\frac{2x\sin^2\left(x\right)}{-8b^2\sin^2\left(x\right)-16ib^2\sin\left(x\right)\cos\left(x\right)+8b^2\cos^2\left(x\right)}-\frac{4ix\sin\left(x\right)\cos\left(x\right)}{-8b^2\sin^2\left(x\right)-16ib^2\sin\left(x\right)\cos\left(x\right)+8b^2\cos^2\left(x\right)}+\frac{2x\cos^2\left(x\right)}{-8b^2\sin^2\left(x\right)-16ib^2\sin\left(x\right)\cos\left(x\right)+8b^2\cos^2\left(x\right)}+\frac{3i\sin^2\left(x\right)}{-8b^2\sin^2\left(x\right)-16ib^2\sin\left(x\right)\cos\left(x\right)+8b^2\cos^2\left(x\right)}+\frac{i\cos^2\left(x\right)}{-8b^2\sin^2\left(x\right)-16ib^2\sin\left(x\right)\cos\left(x\right)+8b^2\cos^2\left(x\right)} & \text{for}\: a = ib \\ -\frac{a^3x\cos\left(x\right)}{D}+\frac{a^3\sin\left(x\right)}{D}-\frac{a^2bx\sin\left(x\right)}{D}-\frac{2a^2b\log\left(\frac{a\cos\left(x\right)}{b}+\sin\left(x\right)\right)\cos\left(x\right)}{D}+\frac{ab^2x\cos\left(x\right)}{D}-\frac{2ab^2\log\left(\frac{a\cos\left(x\right)}{b}+\sin\left(x\right)\right)\sin\left(x\right)}{D}+\frac{ab^2\sin\left(x\right)}{D}+\frac{b^3x\sin\left(x\right)}{D} & \text{otherwise} \end{cases}$$

where $D = a^5\cos\left(x\right)+a^4b\sin\left(x\right)+2a^3b^2\cos\left(x\right)+2a^2b^3\sin\left(x\right)+ab^4\cos\left(x\right)+b^5\sin\left(x\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), ((-x + sin(x)/cos(x))/a**2, Eq(b, 0
)), (-2*x*sin(x)**2/(-8*b**2*sin(x)**2 + 16*I*b**2*sin(x)*cos(x) + 8*b**2*c
```

```
os(x)**2) + 4*I*x*sin(x)*cos(x)/(-8*b**2*sin(x)**2 + 16*I*b**2*sin(x)*cos(x
) + 8*b**2*cos(x)**2) + 2*x*cos(x)**2/(-8*b**2*sin(x)**2 + 16*I*b**2*sin(x)
*cos(x) + 8*b**2*cos(x)**2) - 3*I*sin(x)**2/(-8*b**2*sin(x)**2 + 16*I*b**2*
sin(x)*cos(x) + 8*b**2*cos(x)**2) - I*cos(x)**2/(-8*b**2*sin(x)**2 + 16*I*b
**2*sin(x)*cos(x) + 8*b**2*cos(x)**2), Eq(a, -I*b)), (-2*x*sin(x)**2/(-8*b*
*2*sin(x)**2 - 16*I*b**2*sin(x)*cos(x) + 8*b**2*cos(x)**2) - 4*I*x*sin(x)*c
os(x)/(-8*b**2*sin(x)**2 - 16*I*b**2*sin(x)*cos(x) + 8*b**2*cos(x)**2) + 2*
x*cos(x)**2/(-8*b**2*sin(x)**2 - 16*I*b**2*sin(x)*cos(x) + 8*b**2*cos(x)**2
) + 3*I*sin(x)**2/(-8*b**2*sin(x)**2 - 16*I*b**2*sin(x)*cos(x) + 8*b**2*cos
(x)**2) + I*cos(x)**2/(-8*b**2*sin(x)**2 - 16*I*b**2*sin(x)*cos(x) + 8*b**2
*cos(x)**2), Eq(a, I*b)), (-a**3*x*cos(x)/(a**5*cos(x) + a**4*b*sin(x) + 2*
a**3*b**2*cos(x) + 2*a**2*b**3*sin(x) + a*b**4*cos(x) + b**5*sin(x)) + a**3
*sin(x)/(a**5*cos(x) + a**4*b*sin(x) + 2*a**3*b**2*cos(x) + 2*a**2*b**3*sin
(x) + a*b**4*cos(x) + b**5*sin(x)) - a**2*b*x*sin(x)/(a**5*cos(x) + a**4*b*
sin(x) + 2*a**3*b**2*cos(x) + 2*a**2*b**3*sin(x) + a*b**4*cos(x) + b**5*sin
(x)) - 2*a**2*b*log(a*cos(x)/b + sin(x))*cos(x)/(a**5*cos(x) + a**4*b*sin(x
) + 2*a**3*b**2*cos(x) + 2*a**2*b**3*sin(x) + a*b**4*cos(x) + b**5*sin(x))
+ a*b**2*x*cos(x)/(a**5*cos(x) + a**4*b*sin(x) + 2*a**3*b**2*cos(x) + 2*a**
2*b**3*sin(x) + a*b**4*cos(x) + b**5*sin(x)) - 2*a*b**2*log(a*cos(x)/b + si
n(x))*sin(x)/(a**5*cos(x) + a**4*b*sin(x) + 2*a**3*b**2*cos(x) + 2*a**2*b**
3*sin(x) + a*b**4*cos(x) + b**5*sin(x)) + a*b**2*sin(x)/(a**5*cos(x) + a**4
*b*sin(x) + 2*a**3*b**2*cos(x) + 2*a**2*b**3*sin(x) + a*b**4*cos(x) + b**5*
sin(x)) + b**3*x*sin(x)/(a**5*cos(x) + a**4*b*sin(x) + 2*a**3*b**2*cos(x) +
 2*a**2*b**3*sin(x) + a*b**4*cos(x) + b**5*sin(x)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(64) = 128.

time = 0.41, size = 139, normalized size = 2.17

$$-\frac{2\,ab^{2}\log\left(|b\tan\left(x\right)+a|\right)}{a^{4}b+2\,a^{2}b^{3}+b^{5}}+\frac{ab\log\left(\tan\left(x\right)^{2}+1\right)}{a^{4}+2\,a^{2}b^{2}+b^{4}}-\frac{(a^{2}-b^{2})x}{a^{4}+2\,a^{2}b^{2}+b^{4}}+\frac{2\,ab^{3}\tan\left(x\right)-a^{4}+a^{2}b^{2}}{(a^{4}b+2\,a^{2}b^{3}+b^{5})(b\tan\left(x\right)+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -2*a*b^2*log(abs(b*tan(x) + a))/(a^4*b + 2*a^2*b^3 + b^5) + a*b*log(tan(x)^
2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - (a^2 - b^2)*x/(a^4 + 2*a^2*b^2 + b^4) + (2
*a*b^3*tan(x) - a^4 + a^2*b^2)/((a^4*b + 2*a^2*b^3 + b^5)*(b*tan(x) + a))
```

Mupad [B]

time = 7.06, size = 626, normalized size = 9.78

$$\frac{a^{3}\sin\left(x\right)+a\,b^{2}\sin\left(x\right)-2\,a^{3}\,\mathrm{atan}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)\cos\left(x\right)+2\,b^{3}\,\mathrm{atan}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)\sin\left(x\right)+2\,a\,b^{2}\,\mathrm{atan}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)\cos\left(x\right)-2\,a^{2}\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)\sin\left(x\right)+2\,a^{2}\,b\cos\left(x\right)\ln\left(\frac{1024\,a^{14}+26624\,a^{12}\,b^{2}+146432\,a^{10}\,b^{4}-348160\,a^{8}\,b^{6}+146432\,a^{6}\,b^{8}+26624\,a^{4}\,b^{10}+1024\,a^{2}\,b^{12}}{\frac{a^{16}}{2}+\frac{b^{16}}{2}+4\,a^{2}\,b^{14}+14\,a^{4}\,b^{12}+28\,a^{6}\,b^{10}+35\,a^{8}\,b^{8}+28\,a^{10}\,b^{6}+14\,a^{12}\,b^{4}+4\,a^{14}\,b^{2}+\frac{a^{16}\cos\left(x\right)}{2}+\frac{b^{16}\cos\left(x\right)}{2}+4\,a^{2}\,b^{14}\cos\left(x\right)+14\,a^{4}\,b^{12}\cos\left(x\right)+28\,a^{6}\,b^{10}\cos\left(x\right)+35\,a^{8}\,b^{8}\cos\left(x\right)+28\,a^{10}\,b^{6}\cos\left(x\right)+14\,a^{12}\,b^{4}\cos\left(x\right)+4\,a^{14}\,b^{2}\cos\left(x\right)}\right)+2\,a\,b^{2}\ln\left(\frac{1024\,a^{14}+26624\,a^{12}\,b^{2}+146432\,a^{10}\,b^{4}-348160\,a^{8}\,b^{6}+146432\,a^{6}\,b^{8}+26624\,a^{4}\,b^{10}+1024\,a^{2}\,b^{12}}{\frac{a^{16}}{2}+\frac{b^{16}}{2}+4\,a^{2}\,b^{14}+14\,a^{4}\,b^{12}+28\,a^{6}\,b^{10}+35\,a^{8}\,b^{8}+28\,a^{10}\,b^{6}+14\,a^{12}\,b^{4}+4\,a^{14}\,b^{2}+\frac{a^{16}\cos\left(x\right)}{2}+\frac{b^{16}\cos\left(x\right)}{2}+4\,a^{2}\,b^{14}\cos\left(x\right)+14\,a^{4}\,b^{12}\cos\left(x\right)+28\,a^{6}\,b^{10}\cos\left(x\right)+35\,a^{8}\,b^{8}\cos\left(x\right)+28\,a^{10}\,b^{6}\cos\left(x\right)+14\,a^{12}\,b^{4}\cos\left(x\right)+4\,a^{14}\,b^{2}\cos\left(x\right)}\right)\sin\left(x\right)-2\,a^{2}\,b\ln\left(\frac{a\cos\left(x\right)+b\sin\left(x\right)}{{\cos\left(\frac{x}{2}\right)}^{2}}\right)\cos\left(x\right)-2\,a\,b^{2}\ln\left(\frac{a\cos\left(x\right)+b\sin\left(x\right)}{{\cos\left(\frac{x}{2}\right)}^{2}}\right)\sin\left(x\right)}{\cos\left(x\right)\,a^{5}+\sin\left(x\right)\,a^{4}\,b+2\,\cos\left(x\right)\,a^{3}\,b^{2}+2\,\sin\left(x\right)\,a^{2}\,b^{3}+\cos\left(x\right)\,a\,b^{4}+\sin\left(x\right)\,b^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a*cos(x) + b*sin(x))^2,x)
```

```
[Out] (a^3*sin(x) + a*b^2*sin(x) - 2*a^3*atan(sin(x/2)/cos(x/2))*cos(x) + 2*b^3*a
tan(sin(x/2)/cos(x/2))*sin(x) + 2*a*b^2*atan(sin(x/2)/cos(x/2))*cos(x) - 2*
a^2*b*atan(sin(x/2)/cos(x/2))*sin(x) + 2*a^2*b*cos(x)*log((1024*a^14 + 1024
*a^2*b^12 + 26624*a^4*b^10 + 146432*a^6*b^8 - 348160*a^8*b^6 + 146432*a^10*
b^4 + 26624*a^12*b^2)/(a^16/2 + b^16/2 + 4*a^2*b^14 + 14*a^4*b^12 + 28*a^6*
b^10 + 35*a^8*b^8 + 28*a^10*b^6 + 14*a^12*b^4 + 4*a^14*b^2 + (a^16*cos(x))/
2 + (b^16*cos(x))/2 + 4*a^2*b^14*cos(x) + 14*a^4*b^12*cos(x) + 28*a^6*b^10*
cos(x) + 35*a^8*b^8*cos(x) + 28*a^10*b^6*cos(x) + 14*a^12*b^4*cos(x) + 4*a^
14*b^2*cos(x))) + 2*a*b^2*log((1024*a^14 + 1024*a^2*b^12 + 26624*a^4*b^10 +
 146432*a^6*b^8 - 348160*a^8*b^6 + 146432*a^10*b^4 + 26624*a^12*b^2)/(a^16/
2 + b^16/2 + 4*a^2*b^14 + 14*a^4*b^12 + 28*a^6*b^10 + 35*a^8*b^8 + 28*a^10*
b^6 + 14*a^12*b^4 + 4*a^14*b^2 + (a^16*cos(x))/2 + (b^16*cos(x))/2 + 4*a^2*
b^14*cos(x) + 14*a^4*b^12*cos(x) + 28*a^6*b^10*cos(x) + 35*a^8*b^8*cos(x) +
 28*a^10*b^6*cos(x) + 14*a^12*b^4*cos(x) + 4*a^14*b^2*cos(x)))*sin(x) - 2*a
^2*b*log((a*cos(x) + b*sin(x))/cos(x/2)^2)*cos(x) - 2*a*b^2*log((a*cos(x) +
 b*sin(x))/cos(x/2)^2)*sin(x))/(b^5*sin(x) + a^5*cos(x) + a*b^4*cos(x) + a^
4*b*sin(x) + 2*a^3*b^2*cos(x) + 2*a^2*b^3*sin(x))
```

3.17 $\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=60

$$-\frac{b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a}{(a^2+b^2)\,(a\cos(x)+b\sin(x))}$$

```
[Out] -b*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)+a/(a^2+b^2)
/(a*cos(x)+b*sin(x))
```

Rubi [A]
time = 0.03, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3233, 3153, 212}

$$\frac{a}{(a^2+b^2)\,(a\cos(x)+b\sin(x))}-\frac{b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] -((b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2)) + a
/((a^2 + b^2)*(a*Cos[x] + b*Sin[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3233

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[-(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
```

```
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rubi steps

$$\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{a}{(a^2+b^2)(a\cos(x)+b\sin(x))}+\frac{b\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}$$
$$=\frac{a}{(a^2+b^2)(a\cos(x)+b\sin(x))}-\frac{b\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,b\cos(x)-a\sin(x)\right)}{a^2+b^2}$$
$$=-\frac{b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a}{(a^2+b^2)(a\cos(x)+b\sin(x))}$$

Mathematica [A]
time = 0.16, size = 62, normalized size = 1.03

$$\frac{2b\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{a}{(a^2+b^2)(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a*Cos[x] + b*Sin[x])^2,x]
[Out] (2*b*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) + a/((a^
2 + b^2)*(a*Cos[x] + b*Sin[x]))
```

Maple [A]
time = 0.13, size = 97, normalized size = 1.62

method	result	size
default	$\frac{8b\tan\left(\frac{x}{2}\right)+8a}{(-4a^2-4b^2)\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-2b\tan\left(\frac{x}{2}\right)-a\right)}-\frac{8b\,\text{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(-4a^2-4b^2)\sqrt{a^2+b^2}}$	97
risch	$\frac{2a\,e^{ix}}{(-ib\,e^{2ix}+a\,e^{2ix}+ib+a)(-ib+a)(ib+a)}+\frac{ib\ln\left(e^{ix}-\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,(a^2+b^2)}-\frac{ib\ln\left(e^{ix}+\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,(a^2+b^2)}$	157

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
[Out] 4*(2*b*tan(1/2*x)+2*a)/(-4*a^2-4*b^2)/(a*tan(1/2*x)^2-2*b*tan(1/2*x)-a)-8*b
/(-4*a^2-4*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^
(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(56) = 112$.
time = 0.50, size = 128, normalized size = 2.13

$$-\frac{b\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}+\frac{2\left(a+\frac{b\sin(x)}{\cos(x)+1}\right)}{a^3+ab^2+\frac{2\,(a^2b+b^3)\sin(x)}{\cos(x)+1}-\frac{(a^3+ab^2)\sin(x)^2}{(\cos(x)+1)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -b*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(cos(x)
+ 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(a + b*sin(x)/(cos(x) + 1))/
(a^3 + a*b^2 + 2*(a^2*b + b^3)*sin(x)/(cos(x) + 1) - (a^3 + a*b^2)*sin(x)^2
/(cos(x) + 1)^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(56) = 112$.
time = 2.68, size = 164, normalized size = 2.73

$$\frac{2\,a^3+2\,ab^2+\left(ab\cos\left(x\right)+b^2\sin\left(x\right)\right)\sqrt{a^2+b^2}\,\log\left(-\frac{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\,(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2}\right)}{2\left(\left(a^5+2\,a^3b^2+ab^4\right)\cos\left(x\right)+\left(a^4b+2\,a^2b^3+b^5\right)\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3 + 2*a*b^2 + (a*b*cos(x) + b^2*sin(x))*sqrt(a^2 + b^2)*log(-(2*a*
b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b
*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)))/(
(a^5 + 2*a^3*b^2 + a*b^4)*cos(x) + (a^4*b + 2*a^2*b^3 + b^5)*sin(x))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [A]
time = 0.44, size = 103, normalized size = 1.72

$$-\frac{b\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\,\left(b\tan\left(\frac{1}{2}\,x\right)+a\right)}{\left(a\tan\left(\frac{1}{2}\,x\right)^2-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)\left(a^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -b*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) - 2
*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b*tan(1/2*x) + a)/((a*tan(1
/2*x)^2 - 2*b*tan(1/2*x) - a)*(a^2 + b^2))
```

Mupad [B]
time = 0.61, size = 86, normalized size = 1.43

$$\frac{\frac{2\,a}{a^2+b^2}+\frac{2\,b\tan\left(\frac{x}{2}\right)}{a^2+b^2}}{-a\tan\left(\frac{x}{2}\right)^2+2\,b\tan\left(\frac{x}{2}\right)+a}-\frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b-2\,a\tan\left(\frac{x}{2}\right)}{2\,\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a*cos(x) + b*sin(x))^2,x)
```

```
[Out] ((2*a)/(a^2 + b^2) + (2*b*tan(x/2))/(a^2 + b^2))/(a + 2*b*tan(x/2) - a*tan(
x/2)^2) - (2*b*atanh((2*b - 2*a*tan(x/2))/(2*(a^2 + b^2)^(1/2))))/(a^2 + b^
2)^(3/2)
```

3.18 $\int \frac{1}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=17

$$\frac{\sin(x)}{a(a\cos(x)+b\sin(x))}$$

```
[Out] sin(x)/a/(a*cos(x)+b*sin(x))
```

Rubi [A]
time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3154}

$$\frac{\sin(x)}{a(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x] + b*Sin[x])^(-2),x]
[Out] Sin[x]/(a*(a*Cos[x] + b*Sin[x]))
```

Rule 3154

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{\sin(x)}{a(a\cos(x)+b\sin(x))}$$

Mathematica [A]
time = 0.02, size = 17, normalized size = 1.00

$$\frac{\sin(x)}{a(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x] + b*Sin[x])^(-2),x]
[Out] Sin[x]/(a*(a*Cos[x] + b*Sin[x]))
```

Maple [A]
time = 0.08, size = 14, normalized size = 0.82

method	result	size
default	$-\frac{1}{b(a+b\tan(x))}$	14
norman	$-\frac{2\tan(\frac{x}{2})}{a(a(\tan^2(\frac{x}{2}))-2b\tan(\frac{x}{2})-a)}$	31
risch	$\frac{2i}{(-ib+a)(-ib\,e^{2ix}+a\,e^{2ix}+ib+a)}$	36

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b/(a+b*tan(x))
```

Maxima [A]
time = 0.27, size = 14, normalized size = 0.82

$$-\frac{1}{b^2\tan(x)+ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -1/(b^2*tan(x) + a*b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(17) = 34.
time = 2.19, size = 39, normalized size = 2.29

$$-\frac{b\cos(x)-a\sin(x)}{(a^3+ab^2)\cos(x)+(a^2b+b^3)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -(b*cos(x) - a*sin(x))/((a^3 + a*b^2)*cos(x) + (a^2*b + b^3)*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(a\cos(x)+b\sin(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Integral((a*cos(x) + b*sin(x))**(-2), x)
```

Giac [**A**]
time = 0.40, size = 13, normalized size = 0.76

$$-\frac{1}{(b\tan(x)+a)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -1/((b*tan(x) + a)*b)
```

Mupad [**B**]
time = 0.43, size = 29, normalized size = 1.71

$$\frac{2\tan\left(\frac{x}{2}\right)}{a\left(-a\tan\left(\frac{x}{2}\right)^2+2\,b\tan\left(\frac{x}{2}\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x) + b*sin(x))^2,x)
```

```
[Out] (2*tan(x/2))/(a*(a + 2*b*tan(x/2) - a*tan(x/2)^2))
```

3.19 $\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=63

$$-\frac{\tanh^{-1}(\cos(x))}{a^2}+\frac{b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}+\frac{1}{a(a\cos(x)+b\sin(x))}$$

```
[Out] -arctanh(cos(x))/a^2+1/a/(a*cos(x)+b*sin(x))+b*arctanh((b*cos(x)-a*sin(x))/
(a^2+b^2)^(1/2))/a^2/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3172, 3855, 3153, 212}

$$\frac{b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}-\frac{\tanh^{-1}(\cos(x))}{a^2}+\frac{1}{a(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] -(ArcTanh[Cos[x]]/a^2) + (b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])
/(a^2*Sqrt[a^2 + b^2]) + 1/(a*(a*Cos[x] + b*Sin[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3172

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/si
n[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-(a*Cos[c + d*x] + b*Sin[c + d*x])
^(n + 1)/(a*d*(n + 1)), x] + (Dist[1/a^2, Int[(a*Cos[c + d*x] + b*Sin[c + d
*x])^(n + 2)/Sin[c + d*x], x], x] - Dist[b/a^2, Int[(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& LtQ[n, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^2}\,dx &= \frac{1}{a(a\cos(x)+b\sin(x))}+\frac{\int \csc(x)\,dx}{a^2}-\frac{b\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{a^2}\\ &= -\frac{\tanh^{-1}(\cos(x))}{a^2}+\frac{1}{a(a\cos(x)+b\sin(x))}+\frac{b\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,b\cos(x\right.}{a^2}\\ &= -\frac{\tanh^{-1}(\cos(x))}{a^2}+\frac{b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^2\sqrt{a^2+b^2}}+\frac{1}{a(a\cos(x)+b\sin(x))}\end{aligned}$$

Mathematica [A]
time = 0.35, size = 72, normalized size = 1.14

$$\frac{-\frac{2b\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\frac{a\csc(x)}{b+a\cot(x)}-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] ((-2*b*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] + (a*Csc
[x])/(b + a*Cot[x]) - Log[Cos[x/2]] + Log[Sin[x/2]])/a^2
```

Maple [A]
time = 0.20, size = 85, normalized size = 1.35

method	result	siz
default	$\frac{\frac{4\left(-\frac{b\tan\left(\frac{x}{2}\right)}{2}-\frac{a}{2}\right)}{a\left(\tan^2\left(\frac{x}{2}\right)\right)-2b\tan\left(\frac{x}{2}\right)-a}-\frac{2b\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{a^2}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}$	85
risch	$\frac{2\,e^{ix}}{a(-ib\,e^{2ix}+a\,e^{2ix}+ib+a)}-\frac{ib\ln\left(e^{ix}-\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,a^2}+\frac{ib\ln\left(e^{ix}+\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,a^2}-\frac{\ln\left(e^{ix}+1\right)}{a^2}+\frac{\ln\left(e^{ix}-1\right)}{a^2}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4/a^2*((-1/2*b*tan(1/2*x)-1/2*a)/(a*tan(1/2*x)^2-2*b*tan(1/2*x)-a)-1/2*b/(a
^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2)))+1/a^2*ln(t
an(1/2*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(59) = 118$.
time = 0.49, size = 128, normalized size = 2.03

$$\frac{2\left(a+\frac{b\sin(x)}{\cos(x)+1}\right)}{a^3+\frac{2a^2b\sin(x)}{\cos(x)+1}-\frac{a^3\sin(x)^2}{(\cos(x)+1)^2}}+\frac{b\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a^2}+\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] 2*(a + b*sin(x)/(cos(x) + 1))/(a^3 + 2*a^2*b*sin(x)/(cos(x) + 1) - a^3*sin(
x)^2/(cos(x) + 1)^2) + b*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/
(b - a*sin(x)/(cos(x) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^2) + log(
sin(x)/(cos(x) + 1))/a^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(59) = 118$.
time = 3.39, size = 220, normalized size = 3.49

$$\frac{2a^3+2ab^2+(ab\cos(x)+b^2\sin(x))\sqrt{a^2+b^2}\log\left(\frac{2ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2-2a^2-b^2-2\sqrt{a^2+b^2}(b\cos(x)-a\sin(x))}{2ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2}\right)-((a^3+ab^2)\cos(x)+(a^2b+b^3)\sin(x))\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+((a^3+ab^2)\cos(x)+(a^2b+b^3)\sin(x))\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)}{2((a^5+a^3b^2)\cos(x)+(a^4b+a^2b^3)\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^3 + 2*a*b^2 + (a*b*cos(x) + b^2*sin(x))*sqrt(a^2 + b^2)*log((2*a*b
*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*
cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)) - (
(a^3 + a*b^2)*cos(x) + (a^2*b + b^3)*sin(x))*log(1/2*cos(x) + 1/2) + ((a^3
+ a*b^2)*cos(x) + (a^2*b + b^3)*sin(x))*log(-1/2*cos(x) + 1/2))/((a^5 + a^3
*b^2)*cos(x) + (a^4*b + a^2*b^3)*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Integral(csc(x)/(a*cos(x) + b*sin(x))**2, x)
```

Giac [A]
time = 0.45, size = 109, normalized size = 1.73

$$\frac{b\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,a^2}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{a^2}-\frac{2\,\left(b\tan\left(\frac{1}{2}\,x\right)+a\right)}{\left(a\tan\left(\frac{1}{2}\,x\right)^2-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] b*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) - 2*
b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^2) + log(abs(tan(1/2*x)))/a^2 -
2*(b*tan(1/2*x) + a)/((a*tan(1/2*x)^2 - 2*b*tan(1/2*x) - a)*a^2)
```

Mupad [B]
time = 0.81, size = 492, normalized size = 7.81

$$\frac{\frac{2}{a}+\frac{2\,b\tan\left(\frac{x}{2}\right)}{a^2}}{-a\tan\left(\frac{x}{2}\right)^2+2\,b\tan\left(\frac{x}{2}\right)+a}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^2}+\frac{b\,\mathrm{atan}\left(\frac{\frac{b\,\sqrt{a^2+b^2}\,\left(4\,b+\frac{2\tan\left(\frac{x}{2}\right)\left(a^2+4\,b^2\right)}{a}+\frac{b\,\left(2\,a^2\,b+\frac{2\tan\left(\frac{x}{2}\right)\left(3\,a^4+4\,a^2\,b^2\right)}{a}\right)\sqrt{a^2+b^2}}{a^4+a^2\,b^2}\right)1{}\mathrm{i}}{a^4+a^2\,b^2}+\frac{b\,\sqrt{a^2+b^2}\,\left(4\,b+\frac{2\tan\left(\frac{x}{2}\right)\left(a^2+4\,b^2\right)}{a}-\frac{b\,\left(2\,a^2\,b+\frac{2\tan\left(\frac{x}{2}\right)\left(3\,a^4+4\,a^2\,b^2\right)}{a}\right)\sqrt{a^2+b^2}}{a^4+a^2\,b^2}\right)1{}\mathrm{i}}{a^4+a^2\,b^2}}{\frac{4\,b}{a^2}+\frac{b\,\sqrt{a^2+b^2}\,\left(4\,b+\frac{2\tan\left(\frac{x}{2}\right)\left(a^2+4\,b^2\right)}{a}+\frac{b\,\left(2\,a^2\,b+\frac{2\tan\left(\frac{x}{2}\right)\left(3\,a^4+4\,a^2\,b^2\right)}{a}\right)\sqrt{a^2+b^2}}{a^4+a^2\,b^2}\right)}{a^4+a^2\,b^2}-\frac{b\,\sqrt{a^2+b^2}\,\left(4\,b+\frac{2\tan\left(\frac{x}{2}\right)\left(a^2+4\,b^2\right)}{a}-\frac{b\,\left(2\,a^2\,b+\frac{2\tan\left(\frac{x}{2}\right)\left(3\,a^4+4\,a^2\,b^2\right)}{a}\right)\sqrt{a^2+b^2}}{a^4+a^2\,b^2}\right)}{a^4+a^2\,b^2}}\right)\sqrt{a^2+b^2}\,2{}\mathrm{i}}{a^4+a^2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(a*cos(x) + b*sin(x))^2),x)
```

```
[Out] (2/a + (2*b*tan(x/2))/a^2)/(a + 2*b*tan(x/2) - a*tan(x/2)^2) + log(tan(x/2)
)/a^2 + (b*atan(((b*(a^2 + b^2)^(1/2)*(4*b + (2*tan(x/2)*(a^2 + 4*b^2))/a +
 (b*(2*a^2*b + (2*tan(x/2)*(3*a^4 + 4*a^2*b^2))/a)*(a^2 + b^2)^(1/2))/(a^4
+ a^2*b^2))*1i)/(a^4 + a^2*b^2) + (b*(a^2 + b^2)^(1/2)*(4*b + (2*tan(x/2)*(
a^2 + 4*b^2))/a - (b*(2*a^2*b + (2*tan(x/2)*(3*a^4 + 4*a^2*b^2))/a)*(a^2 +
b^2)^(1/2))/(a^4 + a^2*b^2))*1i)/(a^4 + a^2*b^2))/((4*b)/a^2 + (b*(a^2 + b^
2)^(1/2)*(4*b + (2*tan(x/2)*(a^2 + 4*b^2))/a + (b*(2*a^2*b + (2*tan(x/2)*(3
*a^4 + 4*a^2*b^2))/a)*(a^2 + b^2)^(1/2))/(a^4 + a^2*b^2)))/(a^4 + a^2*b^2)
- (b*(a^2 + b^2)^(1/2)*(4*b + (2*tan(x/2)*(a^2 + 4*b^2))/a - (b*(2*a^2*b +
(2*tan(x/2)*(3*a^4 + 4*a^2*b^2))/a)*(a^2 + b^2)^(1/2))/(a^4 + a^2*b^2)))/(a
^4 + a^2*b^2)))*(a^2 + b^2)^(1/2)*2i)/(a^4 + a^2*b^2)
```

3.20 $\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=49

$$-\frac{\cot(x)}{a^2}-\frac{2b\log(\tan(x))}{a^3}+\frac{2b\log(a+b\tan(x))}{a^3}-\frac{\frac{1}{b}+\frac{b}{a^2}}{a+b\tan(x)}$$

```
[Out] -cot(x)/a^2-2*b*ln(tan(x))/a^3+2*b*ln(a+b*tan(x))/a^3+(-1/b-b/a^2)/(a+b*tan
(x))
```

Rubi [A]
time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3166, 908}

$$-\frac{2b\log(\tan(x))}{a^3}+\frac{2b\log(a+b\tan(x))}{a^3}-\frac{\frac{b}{a^2}+\frac{1}{b}}{a+b\tan(x)}-\frac{\cot(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] -(Cot[x]/a^2) - (2*b*Log[Tan[x]])/a^3 + (2*b*Log[a + b*Tan[x]])/a^3 - (b^(-
1) + b/a^2)/(a + b*Tan[x])
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3166

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[1/d, Subst[Int[x^m*((a + b*x
)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n, 0]
 && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \text{Subst}\left(\int \frac{1+x^2}{x^2(a+bx)^2}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{a^2x^2} - \frac{2b}{a^3x} + \frac{a^2+b^2}{a^2(a+bx)^2} + \frac{2b^2}{a^3(a+bx)}\right) dx, x, \tan(x)\right)$$
$$= -\frac{\cot(x)}{a^2} - \frac{2b\log(\tan(x))}{a^3} + \frac{2b\log(a+b\tan(x))}{a^3} - \frac{\frac{1}{b}+\frac{b}{a^2}}{a+b\tan(x)}$$

Mathematica [A]
time = 0.22, size = 76, normalized size = 1.55

$$\frac{a^2+b^2-a^2\cot^2(x)-2b^2\log(\sin(x))-ab\cot(x)(1+2\log(\sin(x))-2\log(a\cos(x)+b\sin(x)))+2b^2\log(a\cos(x)+b\sin(x))}{a^3(b+a\cot(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (a^2 + b^2 - a^2*Cot[x]^2 - 2*b^2*Log[Sin[x]] - a*b*Cot[x]*(1 + 2*Log[Sin[x
]] - 2*Log[a*Cos[x] + b*Sin[x]]) + 2*b^2*Log[a*Cos[x] + b*Sin[x]])/(a^3*(b
+ a*Cot[x]))
```

Maple [A]
time = 0.14, size = 56, normalized size = 1.14

method	result	size
default	$-\frac{1}{a^2\tan(x)} - \frac{2b\ln(\tan(x))}{a^3} - \frac{a^2+b^2}{a^2b(a+b\tan(x))} + \frac{2b\ln(a+b\tan(x))}{a^3}$	56
risch	$-\frac{4(b\,e^{2ix}-b+ia)}{(e^{2ix}-1)(-ib\,e^{2ix}+a\,e^{2ix}+ib+a)a^2} - \frac{2b\ln(e^{2ix}-1)}{a^3} + \frac{2b\ln\left(e^{2ix}-\frac{ib+a}{ib-a}\right)}{a^3}$	100
norman	$\frac{\frac{1}{2a}+\frac{\tan^4(\frac{x}{2})}{2a}-\frac{(3a^2+4b^2)(\tan^2(\frac{x}{2}))}{a^3}}{\tan(\frac{x}{2})(a(\tan^2(\frac{x}{2}))-2b\tan(\frac{x}{2})-a)} - \frac{2b\ln(\tan(\frac{x}{2}))}{a^3} + \frac{2b\ln(a(\tan^2(\frac{x}{2}))-2b\tan(\frac{x}{2})-a)}{a^3}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2/tan(x)-2*b*ln(tan(x))/a^3-(a^2+b^2)/a^2/b/(a+b*tan(x))+2*b*ln(a+b*ta
n(x))/a^3
```

Maxima [A]
time = 0.28, size = 62, normalized size = 1.27

$$-\frac{ab+(a^2+2\,b^2)\tan(x)}{a^2b^2\tan(x)^2+a^3b\tan(x)} + \frac{2\,b\log(b\tan(x)+a)}{a^3} - \frac{2\,b\log(\tan(x))}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -(a*b + (a^2 + 2*b^2)*tan(x))/(a^2*b^2*tan(x)^2 + a^3*b*tan(x)) + 2*b*log(b
*tan(x) + a)/a^3 - 2*b*log(tan(x))/a^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(49) = 98$.
time = 3.22, size = 134, normalized size = 2.73

$$\frac{2\,a^{2}\cos\left(x\right)^{2}+2\,ab\cos\left(x\right)\sin\left(x\right)-a^{2}+\left(b^{2}\cos\left(x\right)^{2}-ab\cos\left(x\right)\sin\left(x\right)-b^{2}\right)\log\left(2\,ab\cos\left(x\right)\sin\left(x\right)+\left(a^{2}-b^{2}\right)\cos\left(x\right)^{2}+b^{2}\right)-\left(b^{2}\cos\left(x\right)^{2}-ab\cos\left(x\right)\sin\left(x\right)-b^{2}\right)\log\left(-\frac{1}{4}\,\cos\left(x\right)^{2}+\frac{1}{4}\right)}{a^{3}b\cos\left(x\right)^{2}-a^{4}\cos\left(x\right)\sin\left(x\right)-a^{3}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] (2*a^2*cos(x)^2 + 2*a*b*cos(x)*sin(x) - a^2 + (b^2*cos(x)^2 - a*b*cos(x)*si
n(x) - b^2)*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - (b^2*co
s(x)^2 - a*b*cos(x)*sin(x) - b^2)*log(-1/4*cos(x)^2 + 1/4))/(a^3*b*cos(x)^2
 - a^4*cos(x)*sin(x) - a^3*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{2}\left(x\right)}{\left(a\cos\left(x\right)+b\sin\left(x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Integral(csc(x)**2/(a*cos(x) + b*sin(x))**2, x)
```

Giac [A]
time = 0.40, size = 63, normalized size = 1.29

$$\frac{2\,b\log\left(\left|b\tan\left(x\right)+a\right|\right)}{a^{3}}-\frac{2\,b\log\left(\left|\tan\left(x\right)\right|\right)}{a^{3}}-\frac{a^{2}\tan\left(x\right)+2\,b^{2}\tan\left(x\right)+ab}{\left(b\tan\left(x\right)^{2}+a\tan\left(x\right)\right)a^{2}b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] 2*b*log(abs(b*tan(x) + a))/a^3 - 2*b*log(abs(tan(x)))/a^3 - (a^2*tan(x) + 2
*b^2*tan(x) + a*b)/((b*tan(x)^2 + a*tan(x))*a^2*b)
```

Mupad [B]
time = 0.62, size = 114, normalized size = 2.33

$$\frac{\tan\left(\frac{x}{2}\right)}{2\,a^{2}}-\frac{a+2\,b\tan\left(\frac{x}{2}\right)-\frac{\tan\left(\frac{x}{2}\right)^{2}\left(5\,a^{2}+4\,b^{2}\right)}{a}}{-2\,a^{3}\tan\left(\frac{x}{2}\right)^{3}+2\,a^{3}\tan\left(\frac{x}{2}\right)+4\,b\,a^{2}\tan\left(\frac{x}{2}\right)^{2}}+\frac{2\,b\,\ln\left(-a\tan\left(\frac{x}{2}\right)^{2}+2\,b\tan\left(\frac{x}{2}\right)+a\right)}{a^{3}}-\frac{2\,b\,\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a*cos(x) + b*sin(x))^2),x)
```

```
[Out] tan(x/2)/(2*a^2) - (a + 2*b*tan(x/2) - (tan(x/2)^2*(5*a^2 + 4*b^2))/a)/(2*a
^3*tan(x/2) - 2*a^3*tan(x/2)^3 + 4*a^2*b*tan(x/2)^2) + (2*b*log(a + 2*b*tan
(x/2) - a*tan(x/2)^2))/a^3 - (2*b*log(tan(x/2)))/a^3
```

3.21 $\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=118

$$-\frac{\tanh^{-1}(\cos(x))}{2a^2}-\frac{2b^2\tanh^{-1}(\cos(x))}{a^4}-\frac{(a^2+b^2)\tanh^{-1}(\cos(x))}{a^4}+\frac{3b\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^4}+\frac{2b}{}$$

```
[Out] -1/2*arctanh(cos(x))/a^2-2*b^2*arctanh(cos(x))/a^4-(a^2+b^2)*arctanh(cos(x)
)/a^4+2*b*csc(x)/a^3-1/2*cot(x)*csc(x)/a^2+(a^2+b^2)/a^3/(a*cos(x)+b*sin(x)
)+3*b*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/a^4
```

Rubi [A]
time = 0.13, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3184, 3172, 3855, 3153, 212, 3853, 3182}

$$-\frac{2b^2\tanh^{-1}(\cos(x))}{a^4}+\frac{2b\csc(x)}{a^3}-\frac{\tanh^{-1}(\cos(x))}{2a^2}-\frac{\cot(x)\csc(x)}{2a^2}-\frac{(a^2+b^2)\tanh^{-1}(\cos(x))}{a^4}+\frac{3b\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^4}+\frac{a^2+b^2}{a^3(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] -1/2*ArcTanh[Cos[x]]/a^2 - (2*b^2*ArcTanh[Cos[x]])/a^4 - ((a^2 + b^2)*ArcTa
nh[Cos[x]])/a^4 + (3*b*Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a
^2 + b^2]])/a^4 + (2*b*Csc[x])/a^3 - (Cot[x]*Csc[x])/(2*a^2) + (a^2 + b^2)/
(a^3*(a*Cos[x] + b*Sin[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3172

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/si
n[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-(a*Cos[c + d*x] + b*Sin[c + d*x])
^(n + 1)/(a*d*(n + 1)), x] + (Dist[1/a^2, Int[(a*Cos[c + d*x] + b*Sin[c + d
*x])^(n + 2)/Sin[c + d*x], x], x] - Dist[b/a^2, Int[(a*Cos[c + d*x] + b*Sin
```

```
[c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& LtQ[n, -1]
```

Rule 3182

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[Sin[c + d*x]^(m + 1)/(a*d*(m + 1))
, x] + (-Dist[b/a^2, Int[Sin[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/a^
2, Int[Sin[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3184

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2 + b^2)/a^2, Int[Sin[c +
d*x]^(m + 2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^n, x], x] + (Dist[1/a^2, Int
[Sin[c + d*x]^m*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] - Dist[2*
(b/a^2), Int[Sin[c + d*x]^(m + 1)*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 1)
, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && L
tQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{\int \csc^3(x)\,dx}{a^2} - \frac{(2b)\int \frac{\csc^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2} + \frac{(a^2+b^2)\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2}$$
$$= \frac{2b\csc(x)}{a^3} - \frac{\cot(x)\csc(x)}{2a^2} + \frac{a^2+b^2}{a^3(a\cos(x)+b\sin(x))} + \frac{\int \csc(x)\,dx}{2a^2} + \frac{(2b^2)\int \text{cs}}{a^4}$$
$$= -\frac{\tanh^{-1}(\cos(x))}{2a^2} - \frac{2b^2\tanh^{-1}(\cos(x))}{a^4} - \frac{(a^2+b^2)\tanh^{-1}(\cos(x))}{a^4} + \frac{2b\csc(x}{a^3}$$
$$= -\frac{\tanh^{-1}(\cos(x))}{2a^2} - \frac{2b^2\tanh^{-1}(\cos(x))}{a^4} - \frac{(a^2+b^2)\tanh^{-1}(\cos(x))}{a^4} + \frac{3b\sqrt{a^2\ -}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 270 vs. $2(118) = 236$.
time = 2.03, size = 270, normalized size = 2.29

$$\frac{-48b\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)(b+a\cot(x))+8a^3\csc(x)+8ab^2\csc(x)-12a^2b\log\left(\cos\left(\frac{x}{2}\right)\right)-24b^3\log\left(\cos\left(\frac{x}{2}\right)\right)-12a^3\cot(x)\log\left(\cos\left(\frac{x}{2}\right)\right)-24ab^2\cot(x)\log\left(\cos\left(\frac{x}{2}\right)\right)+12a^2b\log\left(\sin\left(\frac{x}{2}\right)\right)+24b^3\log\left(\sin\left(\frac{x}{2}\right)\right)+12a^3\cot(x)\log\left(\sin\left(\frac{x}{2}\right)\right)+24ab^2\cot(x)\log\left(\sin\left(\frac{x}{2}\right)\right)+a^2b\sec^2\left(\frac{x}{2}\right)+a^3\cot(x)\sec^2\left(\frac{x}{2}\right)-a\csc^2\left(\frac{x}{2}\right)\left(-4ab\cos(x)+a^2\cot(x)+b(a-4b\sin(x))\right)+8ab^2\tan\left(\frac{x}{2}\right)+8a^2b\cot(x)\tan\left(\frac{x}{2}\right)}{8a^4(b+a\cot(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (-48*b*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]]*(b + a*Co
t[x]) + 8*a^3*Csc[x] + 8*a*b^2*Csc[x] - 12*a^2*b*Log[Cos[x/2]] - 24*b^3*Log
[Cos[x/2]] - 12*a^3*Cot[x]*Log[Cos[x/2]] - 24*a*b^2*Cot[x]*Log[Cos[x/2]] +
12*a^2*b*Log[Sin[x/2]] + 24*b^3*Log[Sin[x/2]] + 12*a^3*Cot[x]*Log[Sin[x/2]]
 + 24*a*b^2*Cot[x]*Log[Sin[x/2]] + a^2*b*Sec[x/2]^2 + a^3*Cot[x]*Sec[x/2]^2
 - a*Csc[x/2]^2*(-4*a*b*Cos[x] + a^2*Cot[x] + b*(a - 4*b*Sin[x])) + 8*a*b^2
*Tan[x/2] + 8*a^2*b*Cot[x]*Tan[x/2])/(8*a^4*(b + a*Cot[x]))
```

Maple [A]
time = 0.25, size = 158, normalized size = 1.34

method	result
default	$\frac{\frac{a\left(\tan^2\left(\frac{x}{2}\right)\right)}{2}+4b\tan\left(\frac{x}{2}\right)}{4a^3} + \frac{4\left(\frac{\left(-\frac{1}{2}a^2b-\frac{1}{2}b^3\right)\tan\left(\frac{x}{2}\right)-\frac{\left(a^2+b^2\right)a}{2}}{a\left(\tan^2\left(\frac{x}{2}\right)\right)-2b\tan\left(\frac{x}{2}\right)-a}\right)-6b\sqrt{a^2+b^2}\,\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{a^4} - \frac{1}{8a^2\tan\left(\frac{x}{2}\right)^2} +$
risch	$\frac{3iab\,e^{5ix}+3a^2e^{5ix}+6b^2e^{5ix}-2a^2e^{3ix}-12b^2e^{3ix}-3ie^{ix}ab+3\,e^{ix}a^2+6\,e^{ix}b^2}{\left(e^{2ix}-1\right)^2\left(-ib\,e^{2ix}+a\,e^{2ix}+ib+a\right)a^3} + \frac{3\ln\left(e^{ix}-1\right)}{2a^2} + \frac{3\ln\left(e^{ix}-1\right)b^2}{a^4} - \frac{3\ln\left(e^{ix}+1\right)}{2a^2} - 3\ln$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^3*(1/2*a*tan(1/2*x)^2+4*b*tan(1/2*x))+4/a^4*(((-1/2*a^2*b-1/2*b^3)*ta
n(1/2*x)-1/2*(a^2+b^2)*a)/(a*tan(1/2*x)^2-2*b*tan(1/2*x)-a)-3/2*b*(a^2+b^2)
^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2)))-1/8/a^2/tan(1/2*x
)^2+1/4/a^4*(6*a^2+12*b^2)*ln(tan(1/2*x))+b/a^3/tan(1/2*x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(110) = 220$.
time = 0.49, size = 242, normalized size = 2.05

$$-\frac{a^{3}-\frac{6\,a^{2}b\sin\left(x\right)}{\cos\left(x\right)+1}-\frac{\left(17\,a^{3}+32\,ab^{2}\right)\sin\left(x\right)^{2}}{\left(\cos\left(x\right)+1\right)^{2}}-\frac{8\,\left(a^{2}b+2\,b^{3}\right)\sin\left(x\right)^{3}}{\left(\cos\left(x\right)+1\right)^{3}}}{8\,\left(\frac{a^{5}\sin\left(x\right)^{2}}{\left(\cos\left(x\right)+1\right)^{2}}+\frac{2\,a^{4}b\sin\left(x\right)^{3}}{\left(\cos\left(x\right)+1\right)^{3}}-\frac{a^{5}\sin\left(x\right)^{4}}{\left(\cos\left(x\right)+1\right)^{4}}\right)}+\frac{\frac{8\,b\sin\left(x\right)}{\cos\left(x\right)+1}+\frac{a\sin\left(x\right)^{2}}{\left(\cos\left(x\right)+1\right)^{2}}}{8\,a^{3}}+\frac{3\,\left(a^{2}+2\,b^{2}\right)\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)}{2\,a^{4}}+\frac{3\,\left(a^{2}b+b^{3}\right)\log\left(\frac{b-\frac{a\sin\left(x\right)}{\cos\left(x\right)+1}+\sqrt{a^{2}+b^{2}}}{b-\frac{a\sin\left(x\right)}{\cos\left(x\right)+1}-\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(a^3 - 6*a^2*b*sin(x)/(cos(x) + 1) - (17*a^3 + 32*a*b^2)*sin(x)^2/(cos
(x) + 1)^2 - 8*(a^2*b + 2*b^3)*sin(x)^3/(cos(x) + 1)^3)/(a^5*sin(x)^2/(cos(
x) + 1)^2 + 2*a^4*b*sin(x)^3/(cos(x) + 1)^3 - a^5*sin(x)^4/(cos(x) + 1)^4)
+ 1/8*(8*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/a^3 + 3/2*(a^2
+ 2*b^2)*log(sin(x)/(cos(x) + 1))/a^4 + 3*(a^2*b + b^3)*log((b - a*sin(x)/(
cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(cos(x) + 1) - sqrt(a^2 + b^2)
))/(sqrt(a^2 + b^2)*a^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(110) = 220$.
time = 3.43, size = 345, normalized size = 2.92

$$-\frac{6\,a^{2}b\cos\left(x\right)\sin\left(x\right)+4\,a^{3}+12\,ab^{2}-6\,\left(a^{3}+2\,ab^{2}\right)\cos\left(x\right)^{2}-6\,\left(ab\cos\left(x\right)^{3}-ab\cos\left(x\right)+\left(b^{2}\cos\left(x\right)^{2}-b^{2}\right)\sin\left(x\right)\right)\sqrt{a^{2}+b^{2}}\log\left(\frac{2\,ab\cos\left(x\right)\sin\left(x\right)+\left(a^{2}-b^{2}\right)\cos\left(x\right)^{2}-2\,a^{2}-b^{2}-2\,\sqrt{a^{2}+b^{2}}\left(b\cos\left(x\right)-a\sin\left(x\right)\right)}{2\,ab\cos\left(x\right)\sin\left(x\right)+\left(a^{2}-b^{2}\right)\cos\left(x\right)^{2}+b^{2}}\right)+3\,\left(\left(a^{3}+2\,ab^{2}\right)\cos\left(x\right)^{3}-\left(a^{3}+2\,ab^{2}\right)\cos\left(x\right)-\left(a^{2}b+2\,b^{3}-\left(a^{2}b+2\,b^{3}\right)\cos\left(x\right)^{2}\right)\sin\left(x\right)\right)\log\left(\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)-3\,\left(\left(a^{3}+2\,ab^{2}\right)\cos\left(x\right)^{3}-\left(a^{3}+2\,ab^{2}\right)\cos\left(x\right)-\left(a^{2}b+2\,b^{3}-\left(a^{2}b+2\,b^{3}\right)\cos\left(x\right)^{2}\right)\sin\left(x\right)\right)\log\left(-\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)}{4\,\left(a^{5}\cos\left(x\right)^{3}-a^{5}\cos\left(x\right)+\left(a^{4}b\cos\left(x\right)^{2}-a^{4}b\right)\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(6*a^2*b*cos(x)*sin(x) + 4*a^3 + 12*a*b^2 - 6*(a^3 + 2*a*b^2)*cos(x)^2
 - 6*(a*b*cos(x)^3 - a*b*cos(x) + (b^2*cos(x)^2 - b^2)*sin(x))*sqrt(a^2 + b
^2)*log((2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2 - 2*sqrt(
a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)
^2 + b^2)) + 3*((a^3 + 2*a*b^2)*cos(x)^3 - (a^3 + 2*a*b^2)*cos(x) - (a^2*b
+ 2*b^3 - (a^2*b + 2*b^3)*cos(x)^2)*sin(x))*log(1/2*cos(x) + 1/2) - 3*((a^3
 + 2*a*b^2)*cos(x)^3 - (a^3 + 2*a*b^2)*cos(x) - (a^2*b + 2*b^3 - (a^2*b + 2
*b^3)*cos(x)^2)*sin(x))*log(-1/2*cos(x) + 1/2))/(a^5*cos(x)^3 - a^5*cos(x)
+ (a^4*b*cos(x)^2 - a^4*b)*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{3}\left(x\right)}{\left(a\cos\left(x\right)+b\sin\left(x\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Integral(csc(x)**3/(a*cos(x) + b*sin(x))**2, x)
```

Giac [A]
time = 0.45, size = 215, normalized size = 1.82

$$\frac{3\left(a^{2}+2b^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)}{2a^{4}}+\frac{3\left(a^{2}b+b^{3}\right)\log\left(\frac{\left|2a\tan\left(\frac{1}{2}x\right)-2b-2\sqrt{a^{2}+b^{2}}\right|}{\left|2a\tan\left(\frac{1}{2}x\right)-2b+2\sqrt{a^{2}+b^{2}}\right|}\right)}{\sqrt{a^{2}+b^{2}}\,a^{4}}+\frac{a^{2}\tan\left(\frac{1}{2}x\right)^{2}+8ab\tan\left(\frac{1}{2}x\right)}{8a^{4}}-\frac{2\left(a^{2}b\tan\left(\frac{1}{2}x\right)+b^{3}\tan\left(\frac{1}{2}x\right)+a^{3}+ab^{2}\right)}{\left(a\tan\left(\frac{1}{2}x\right)^{2}-2b\tan\left(\frac{1}{2}x\right)-a\right)a^{4}}-\frac{18a^{2}\tan\left(\frac{1}{2}x\right)^{2}+36b^{2}\tan\left(\frac{1}{2}x\right)^{2}-8ab\tan\left(\frac{1}{2}x\right)+a^{2}}{8a^{4}\tan\left(\frac{1}{2}x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] 3/2*(a^2 + 2*b^2)*log(abs(tan(1/2*x)))/a^4 + 3*(a^2*b + b^3)*log(abs(2*a*ta
n(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) - 2*b + 2*sqrt(a^2 +
 b^2)))/(sqrt(a^2 + b^2)*a^4) + 1/8*(a^2*tan(1/2*x)^2 + 8*a*b*tan(1/2*x))/a
^4 - 2*(a^2*b*tan(1/2*x) + b^3*tan(1/2*x) + a^3 + a*b^2)/((a*tan(1/2*x)^2 -
 2*b*tan(1/2*x) - a)*a^4) - 1/8*(18*a^2*tan(1/2*x)^2 + 36*b^2*tan(1/2*x)^2
- 8*a*b*tan(1/2*x) + a^2)/(a^4*tan(1/2*x)^2)
```

Mupad [B]
time = 0.74, size = 511, normalized size = 4.33

$$\frac{\tan\left(\frac{x}{2}\right)^{2}\left(\frac{17a^{2}}{2}+16b^{2}\right)-\frac{a^{2}}{2}+3ab\tan\left(\frac{x}{2}\right)+\frac{4\tan\left(\frac{x}{2}\right)^{3}\left(a^{2}b+2b^{3}\right)}{a}}{-4a^{4}\tan\left(\frac{x}{2}\right)^{4}+4a^{4}\tan\left(\frac{x}{2}\right)^{2}+8ba^{3}\tan\left(\frac{x}{2}\right)^{3}}+\frac{\tan\left(\frac{x}{2}\right)^{2}}{8a^{2}}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)\left(3a^{2}+6b^{2}\right)}{2a^{4}}+\frac{b\tan\left(\frac{x}{2}\right)}{a^{3}}-\frac{6b\,\mathrm{atanh}\left(\frac{54b^{2}\sqrt{a^{2}+b^{2}}}{18a^{2}b+90b^{3}+\frac{72b^{5}}{a^{2}}+\frac{216b^{4}\tan\left(\frac{x}{2}\right)}{a}+\frac{144b^{6}\tan\left(\frac{x}{2}\right)}{a^{3}}+72ab^{2}\tan\left(\frac{x}{2}\right)}+\frac{72b^{4}\sqrt{a^{2}+b^{2}}}{18a^{4}b+72b^{5}+90a^{2}b^{3}+72a^{3}b^{2}\tan\left(\frac{x}{2}\right)+\frac{144b^{6}\tan\left(\frac{x}{2}\right)}{a}+216ab^{4}\tan\left(\frac{x}{2}\right)}+\frac{144b^{3}\tan\left(\frac{x}{2}\right)\sqrt{a^{2}+b^{2}}}{216b^{4}\tan\left(\frac{x}{2}\right)+90ab^{3}+18a^{3}b+\frac{72b^{5}}{a}+72a^{2}b^{2}\tan\left(\frac{x}{2}\right)+\frac{144b^{6}\tan\left(\frac{x}{2}\right)}{a^{2}}}+\frac{144b^{5}\tan\left(\frac{x}{2}\right)\sqrt{a^{2}+b^{2}}}{18a^{5}b+72\tan\left(\frac{x}{2}\right)a^{4}b^{2}+90a^{3}b^{3}+216\tan\left(\frac{x}{2}\right)a^{2}b^{4}+72ab^{5}+144\tan\left(\frac{x}{2}\right)b^{6}}+\frac{18b\tan\left(\frac{x}{2}\right)\sqrt{a^{2}+b^{2}}}{18ab+72b^{2}\tan\left(\frac{x}{2}\right)+\frac{90b^{3}}{a}+\frac{72b^{5}}{a^{3}}+\frac{216b^{4}\tan\left(\frac{x}{2}\right)}{a^{2}}+\frac{144b^{6}\tan\left(\frac{x}{2}\right)}{a^{4}}}\right)\sqrt{a^{2}+b^{2}}}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^3*(a*cos(x) + b*sin(x))^2),x)
```

```
[Out] (tan(x/2)^2*((17*a^2)/2 + 16*b^2) - a^2/2 + 3*a*b*tan(x/2) + (4*tan(x/2)^3*
(a^2*b + 2*b^3))/a)/(4*a^4*tan(x/2)^2 - 4*a^4*tan(x/2)^4 + 8*a^3*b*tan(x/2)
^3) + tan(x/2)^2/(8*a^2) + (log(tan(x/2))*(3*a^2 + 6*b^2))/(2*a^4) + (b*tan
(x/2))/a^3 - (6*b*atanh((54*b^2*(a^2 + b^2)^(1/2))/(18*a^2*b + 90*b^3 + (72
*b^5)/a^2 + (216*b^4*tan(x/2))/a + (144*b^6*tan(x/2))/a^3 + 72*a*b^2*tan(x/
2)) + (72*b^4*(a^2 + b^2)^(1/2))/(18*a^4*b + 72*b^5 + 90*a^2*b^3 + 72*a^3*b
^2*tan(x/2) + (144*b^6*tan(x/2))/a + 216*a*b^4*tan(x/2)) + (144*b^3*tan(x/2
)*(a^2 + b^2)^(1/2))/(216*b^4*tan(x/2) + 90*a*b^3 + 18*a^3*b + (72*b^5)/a +
 72*a^2*b^2*tan(x/2) + (144*b^6*tan(x/2))/a^2) + (144*b^5*tan(x/2)*(a^2 + b
^2)^(1/2))/(144*b^6*tan(x/2) + 72*a*b^5 + 18*a^5*b + 90*a^3*b^3 + 216*a^2*b
^4*tan(x/2) + 72*a^4*b^2*tan(x/2)) + (18*b*tan(x/2)*(a^2 + b^2)^(1/2))/(18*
a*b + 72*b^2*tan(x/2) + (90*b^3)/a + (72*b^5)/a^3 + (216*b^4*tan(x/2))/a^2
+ (144*b^6*tan(x/2))/a^4))*(a^2 + b^2)^(1/2))/a^4
```

3.22 $\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

Optimal. Leaf size=98

$$-\frac{b(3a^2-b^2)\,x}{(a^2+b^2)^3}+\frac{a}{2\,(a^2+b^2)\,(b+a\cot(x))^2}+\frac{2ab}{(a^2+b^2)^2\,(b+a\cot(x))}+\frac{a(a^2-3b^2)\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^3}$$

```
[Out] -b*(3*a^2-b^2)*x/(a^2+b^2)^3+1/2*a/(a^2+b^2)/(b+a*cot(x))^2+2*a*b/(a^2+b^2)
^2/(b+a*cot(x))+a*(a^2-3*b^2)*ln(a*cos(x)+b*sin(x))/(a^2+b^2)^3
```

Rubi [A]
time = 0.13, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3164, 3564, 3610, 3612, 3611}

$$-\frac{bx(3a^2-b^2)}{(a^2+b^2)^3}+\frac{2ab}{(a^2+b^2)^2\,(a\cot(x)+b)}+\frac{a}{2\,(a^2+b^2)\,(a\cot(x)+b)^2}+\frac{a(a^2-3b^2)\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^3/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] -((b*(3*a^2 - b^2)*x)/(a^2 + b^2)^3) + a/(2*(a^2 + b^2)*(b + a*Cot[x])^2) +
 (2*a*b)/((a^2 + b^2)^2*(b + a*Cot[x])) + (a*(a^2 - 3*b^2)*Log[a*Cos[x] + b
*Sin[x]])/(a^2 + b^2)^3
```

Rule 3164

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cot[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\sin^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \int \frac{1}{(b+a\cot(x))^3}\,dx$$

$$= \frac{a}{2\left(a^2+b^2\right)(b+a\cot(x))^2} + \frac{\int \frac{b-a\cot(x)}{(b+a\cot(x))^2}\,dx}{a^2+b^2}$$

$$= \frac{a}{2\left(a^2+b^2\right)(b+a\cot(x))^2} + \frac{2ab}{\left(a^2+b^2\right)^2(b+a\cot(x))} + \frac{\int \frac{-a^2+b^2-2ab\cot(x)}{b+a\cot(x)}\,dx}{\left(a^2+b^2\right)^2}$$

$$= -\frac{b(3a^2-b^2)\,x}{\left(a^2+b^2\right)^3} + \frac{a}{2\left(a^2+b^2\right)(b+a\cot(x))^2} + \frac{2ab}{\left(a^2+b^2\right)^2(b+a\cot(x))} + \frac{(a(a}{}$$

$$= -\frac{b(3a^2-b^2)\,x}{\left(a^2+b^2\right)^3} + \frac{a}{2\left(a^2+b^2\right)(b+a\cot(x))^2} + \frac{2ab}{\left(a^2+b^2\right)^2(b+a\cot(x))} + \frac{a(a^2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.89, size = 114, normalized size = 1.16

$$\frac{b(-3a^2+b^2)\,x}{\left(a^2+b^2\right)^3} + \frac{a(a^2-3b^2)\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^3} + \frac{a^3}{2(a-ib)^2(a+ib)^2(a\cos(x)+b\sin(x))^2} + \frac{3ab\sin(x)}{\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^3/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] (b*(-3*a^2 + b^2)*x)/(a^2 + b^2)^3 + (a*(a^2 - 3*b^2)*Log[a*Cos[x] + b*Sin[
x]])/(a^2 + b^2)^3 + a^3/(2*(a - I*b)^2*(a + I*b)^2*(a*Cos[x] + b*Sin[x])^2
) + (3*a*b*Sin[x])/((a^2 + b^2)^2*(a*Cos[x] + b*Sin[x]))
```

Maple [A]
time = 0.33, size = 134, normalized size = 1.37

method	result
default	$\frac{\frac{\left(-a^{3}+3 a\, b^{2}\right) \ln \left(\tan^{2}\left(x\right)+1\right)}{2}+\left(-3 a^{2} b+b^{3}\right) \arctan \left(\tan \left(x\right)\right)}{\left(a^{2}+b^{2}\right)^{3}}+\frac{a \left(a^{2}-3 b^{2}\right) \ln \left(a+b \tan \left(x\right)\right)}{\left(a^{2}+b^{2}\right)^{3}}-\frac{a^{2} \left(a^{2}+3 b^{2}\right)}{\left(a^{2}+b^{2}\right)^{2} b^{2} \left(a+b \tan \left(x\right)\right)}+\frac{}{2 b^{2} \left(a^{2}+b^{2}\right.}$
risch	$-\frac{i x}{3 i a^{2} b-i b^{3}-a^{3}+3 a\, b^{2}}-\frac{2 i a^{3} x}{a^{6}+3 a^{4} b^{2}+3 a^{2} b^{4}+b^{6}}+\frac{6 i a x\, b^{2}}{a^{6}+3 a^{4} b^{2}+3 a^{2} b^{4}+b^{6}}+\frac{2 a^{2} \left(2 i a b\, {\mathrm e}^{2 i x}+a^{2} {\mathrm e}^{2 i x}+3 b^{2} {\mathrm e}^{2 i x}+3 i a b-3 b^{2}\right)}{\left(-i b\, {\mathrm e}^{2 i x}+a\, {\mathrm e}^{2 i x}+i b+a\right)^{2} \left(i b+a\right)^{2} \left(-i b+a\right)^{3}}+\frac{a}{a^{6}}$
norman	$\frac{\frac{\left(2 a^{5}+10 a^{3} b^{2}\right) \left(\tan^{2}\left(\frac{x}{2}\right)\right)}{a^{2} \left(a^{4}+2 a^{2} b^{2}+b^{4}\right)}+\frac{\left(2 a^{5}+10 a^{3} b^{2}\right) \left(\tan^{8}\left(\frac{x}{2}\right)\right)}{a^{2} \left(a^{4}+2 a^{2} b^{2}+b^{4}\right)}-\frac{2 \left(-3 a^{5}-15 a^{3} b^{2}\right) \left(\tan^{4}\left(\frac{x}{2}\right)\right)}{a^{2} \left(a^{4}+2 a^{2} b^{2}+b^{4}\right)}-\frac{2 \left(-3 a^{5}-15 a^{3} b^{2}\right) \left(\tan^{6}\left(\frac{x}{2}\right)\right)}{a^{2} \left(a^{4}+2 a^{2} b^{2}+b^{4}\right)}-\frac{\left(3 a^{2}-b^{2}\right) a^{2} b x}{a^{6}+3 a^{4} b^{2}+3 a^{2} b^{4}+b^{6}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a*cos(x)+b*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/(a^2+b^2)^3*(1/2*(-a^3+3*a*b^2)*ln(tan(x)^2+1)+(-3*a^2*b+b^3)*arctan(tan(
x)))+a*(a^2-3*b^2)/(a^2+b^2)^3*ln(a+b*tan(x))-a^2*(a^2+3*b^2)/(a^2+b^2)^2/b
^2/(a+b*tan(x))+1/2*a^3/b^2/(a^2+b^2)/(a+b*tan(x))^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(96) = 192$.
time = 0.50, size = 359, normalized size = 3.66

$$-\frac{2 \, {\left(3 \, a^{2} b - b^{3}\right)} \arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right) + 1}\right)}{a^{6} + 3 \, a^{4} b^{2} + 3 \, a^{2} b^{4} + b^{6}} + \frac{{\left(a^{3} - 3 \, a b^{2}\right)} \log\left(-a - \frac{2 \, b \sin\left(x\right)}{\cos\left(x\right) + 1} + \frac{a \sin\left(x\right)^{2}}{{\left(\cos\left(x\right) + 1\right)}^{2}}\right)}{a^{6} + 3 \, a^{4} b^{2} + 3 \, a^{2} b^{4} + b^{6}} - \frac{{\left(a^{3} - 3 \, a b^{2}\right)} \log\left(\frac{\sin\left(x\right)^{2}}{{\left(\cos\left(x\right) + 1\right)}^{2}} + 1\right)}{a^{6} + 3 \, a^{4} b^{2} + 3 \, a^{2} b^{4} + b^{6}} + \frac{2 \, {\left(\frac{2 \, a^{2} b \sin\left(x\right)}{\cos\left(x\right) + 1} - \frac{2 \, a^{2} b \sin\left(x\right)^{3}}{{\left(\cos\left(x\right) + 1\right)}^{3}} + \frac{{\left(a^{3} + 5 \, a b^{2}\right)} \sin\left(x\right)^{2}}{{\left(\cos\left(x\right) + 1\right)}^{2}}\right)}}{a^{6} + 2 \, a^{4} b^{2} + a^{2} b^{4} + \frac{4 \, {\left(a^{5} b + 2 \, a^{3} b^{3} + a b^{5}\right)} \sin\left(x\right)}{\cos\left(x\right) + 1} - \frac{2 \, {\left(a^{6} - 3 \, a^{2} b^{4} - 2 \, b^{6}\right)} \sin\left(x\right)^{2}}{{\left(\cos\left(x\right) + 1\right)}^{2}} - \frac{4 \, {\left(a^{5} b + 2 \, a^{3} b^{3} + a b^{5}\right)} \sin\left(x\right)^{3}}{{\left(\cos\left(x\right) + 1\right)}^{3}} + \frac{{\left(a^{6} + 2 \, a^{4} b^{2} + a^{2} b^{4}\right)} \sin\left(x\right)^{4}}{{\left(\cos\left(x\right) + 1\right)}^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a*cos(x)+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -2*(3*a^2*b - b^3)*arctan(sin(x)/(cos(x) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4
 + b^6) + (a^3 - 3*a*b^2)*log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(co
s(x) + 1)^2)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^3 - 3*a*b^2)*log(sin(
x)^2/(cos(x) + 1)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(2*a^2*b*s
in(x)/(cos(x) + 1) - 2*a^2*b*sin(x)^3/(cos(x) + 1)^3 + (a^3 + 5*a*b^2)*sin(
x)^2/(cos(x) + 1)^2)/(a^6 + 2*a^4*b^2 + a^2*b^4 + 4*(a^5*b + 2*a^3*b^3 + a*
b^5)*sin(x)/(cos(x) + 1) - 2*(a^6 - 3*a^2*b^4 - 2*b^6)*sin(x)^2/(cos(x) + 1
)^2 - 4*(a^5*b + 2*a^3*b^3 + a*b^5)*sin(x)^3/(cos(x) + 1)^3 + (a^6 + 2*a^4*
b^2 + a^2*b^4)*sin(x)^4/(cos(x) + 1)^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(96) = 192$.
time = 2.79, size = 282, normalized size = 2.88

$$\frac{a^{5} + 7 \, a^{3} b^{2} - 2 \, {\left(6 \, a^{3} b^{2} + {\left(3 \, a^{4} b - 4 \, a^{2} b^{3} + b^{5}\right)} x\right)} \cos\left(x\right)^{2} + 2 \, {\left(3 \, a^{4} b - 3 \, a^{2} b^{3} - 2 \, {\left(3 \, a^{3} b^{2} - a b^{4}\right)} x\right)} \cos\left(x\right) \sin\left(x\right) - 2 \, {\left(3 \, a^{2} b^{3} - b^{5}\right)} x + {\left(a^{3} b^{2} - 3 \, a b^{4} + {\left(a^{5} - 4 \, a^{3} b^{2} + 3 \, a b^{4}\right)} \cos\left(x\right)^{2} + 2 \, {\left(a^{4} b - 3 \, a^{2} b^{3}\right)} \cos\left(x\right) \sin\left(x\right)\right)} \log\left(2 \, a b \cos\left(x\right) \sin\left(x\right) + {\left(a^{2} - b^{2}\right)} \cos\left(x\right)^{2} + b^{2}\right)}{2 \, {\left(a^{6} b^{2} + 3 \, a^{4} b^{4} + 3 \, a^{2} b^{6} + b^{8} + {\left(a^{8} + 2 \, a^{6} b^{2} - 2 \, a^{2} b^{6} - b^{8}\right)} \cos\left(x\right)^{2} + 2 \, {\left(a^{7} b + 3 \, a^{5} b^{3} + 3 \, a^{3} b^{5} + a b^{7}\right)} \cos\left(x\right) \sin\left(x\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a*cos(x)+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(a^5 + 7*a^3*b^2 - 2*(6*a^3*b^2 + (3*a^4*b - 4*a^2*b^3 + b^5)*x)*cos(x)
^2 + 2*(3*a^4*b - 3*a^2*b^3 - 2*(3*a^3*b^2 - a*b^4)*x)*cos(x)*sin(x) - 2*(3
*a^2*b^3 - b^5)*x + (a^3*b^2 - 3*a*b^4 + (a^5 - 4*a^3*b^2 + 3*a*b^4)*cos(x)
^2 + 2*(a^4*b - 3*a^2*b^3)*cos(x)*sin(x))*log(2*a*b*cos(x)*sin(x) + (a^2 -
b^2)*cos(x)^2 + b^2))/(a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8 + (a^8 + 2*a^6
*b^2 - 2*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7
)*cos(x)*sin(x))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**3/(a*cos(x)+b*sin(x))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 242 vs. 2(96) = 192.
time = 0.44, size = 242, normalized size = 2.47

$$-\frac{(3\,a^2b-b^3)x}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{(a^3-3\,ab^2)\log\left(\tan\left(x\right)^2+1\right)}{2\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)}+\frac{(a^3b-3\,ab^3)\log\left(\left|b\tan\left(x\right)+a\right|\right)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}-\frac{3\,a^3b^4\tan\left(x\right)^2-9\,ab^6\tan\left(x\right)^2+2\,a^6b\tan\left(x\right)+14\,a^4b^3\tan\left(x\right)-12\,a^2b^5\tan\left(x\right)+a^7+9\,a^5b^2-4\,a^3b^4}{2\,(a^6b^2+3\,a^4b^4+3\,a^2b^6+b^8)(b\tan\left(x\right)+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^3/(a*cos(x)+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -(3*a^2*b - b^3)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/2*(a^3 - 3*a*b^2
)*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (a^3*b - 3*a*b^3)
*log(abs(b*tan(x) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) - 1/2*(3*a^3*
b^4*tan(x)^2 - 9*a*b^6*tan(x)^2 + 2*a^6*b*tan(x) + 14*a^4*b^3*tan(x) - 12*a
^2*b^5*tan(x) + a^7 + 9*a^5*b^2 - 4*a^3*b^4)/((a^6*b^2 + 3*a^4*b^4 + 3*a^2*
b^6 + b^8)*(b*tan(x) + a)^2)
```

Mupad [**B**]
time = 8.60, size = 2500, normalized size = 25.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^3/(a*cos(x) + b*sin(x))^3,x)
```

```
[Out] ((2*tan(x/2)^2*(5*a*b^2 + a^3))/(a^4 + b^4 + 2*a^2*b^2) - (4*a^2*b*tan(x/2)
^3)/(a^4 + b^4 + 2*a^2*b^2) + (4*a^2*b*tan(x/2))/(a^4 + b^4 + 2*a^2*b^2))/(
```

a^2 - tan(x/2)^2*(2*a^2 - 4*b^2) + a^2*tan(x/2)^4 + 4*a*b*tan(x/2) - 4*a*b*
tan(x/2)^3) - (log(a + 2*b*tan(x/2) - a*tan(x/2)^2)*(3*a*b^2 - a^3))/(a^6 +
b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (log(1/(cos(x) + 1))*(6*a*b^2 - 2*a^3))/(2*
(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (2*b*atan((tan(x/2)*(((((6*a*b^2 - 2
a^3)((b*((32*(a^2*b^12 - 2*a^14 + 15*a^4*b^10 + 48*a^6*b^8 + 62*a^8*b^6 +
33*a^10*b^4 + 3*a^12*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6
*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (16*(6*a*b^2 - 2*a^3)*(3*a*b^16 + 21*a^3*
b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4
+ 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b
^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(3*a^2 - b^2))/(
a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (16*b*(6*a*b^2 - 2*a^3)*(3*a^2 - b^2)*
(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^1
1*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^2*(
a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^
2))))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (b*((32*(2*a*b^10 - 24*a^3*
b^8 - 36*a^5*b^6 + 8*a^7*b^4 + 18*a^9*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*
a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - ((6*a*b^2 - 2*a^3)*((32*(
a^2*b^12 - 2*a^14 + 15*a^4*b^10 + 48*a^6*b^8 + 62*a^8*b^6 + 33*a^10*b^4 + 3
*a^12*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^
4 + 6*a^10*b^2) - (16*(6*a*b^2 - 2*a^3)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^
12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/
((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8
+ 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a
^4*b^2)))*(3*a^2 - b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (32*b^3*(3*a
^2 - b^2)^3*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*
b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*
a^4*b^2)^3*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4
+ 6*a^10*b^2)))*(4*a^8 + b^8 - 67*a^2*b^6 + 155*a^4*b^4 - 61*a^6*b^2))/(4*
a^8 + b^8 + 31*a^2*b^6 + 15*a^4*b^4 - 11*a^6*b^2)^2 + (2*a*b*(10*a^6 - 7*b^
6 + 68*a^2*b^4 - 59*a^4*b^2)*((32*(2*a^8 - 6*a^2*b^6 + 2*a^4*b^4 - 6*a^6*b^
2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^
10*b^2) - ((6*a*b^2 - 2*a^3)*((32*(2*a*b^10 - 24*a^3*b^8 - 36*a^5*b^6 + 8*a
^7*b^4 + 18*a^9*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 +
15*a^8*b^4 + 6*a^10*b^2) - ((6*a*b^2 - 2*a^3)*((32*(a^2*b^12 - 2*a^14 + 15
*a^4*b^10 + 48*a^6*b^8 + 62*a^8*b^6 + 33*a^10*b^4 + 3*a^12*b^2))/(a^12 + b^
12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (16*
(6*a*b^2 - 2*a^3)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 10
5*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^
4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8
*b^4 + 6*a^10*b^2))))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(2*(a^6 + b
^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (b*((b*((32*(a^2*b^12 - 2*a^14 + 15*a^4*b^10
+ 48*a^6*b^8 + 62*a^8*b^6 + 33*a^10*b^4 + 3*a^12*b^2))/(a^12 + b^12 + 6*a^
2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (16*(6*a*b^2
- 2*a^3)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8
+ 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4

```
*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*
a^10*b^2)))*(3*a^2 - b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (16*b*(6*a
*b^2 - 2*a^3)*(3*a^2 - b^2)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7
*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6
+ 3*a^2*b^4 + 3*a^4*b^2)^2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*
b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(3*a^2 - b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*
a^4*b^2) + (16*b^2*(6*a*b^2 - 2*a^3)*(3*a^2 - b^2)^2*(3*a*b^16 + 21*a^3*b^1
4 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 +
3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^3*(a^12 + b^12 + 6*a^2*b^
10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(4*a^8 + b^8 + 3
1*a^2*b^6 + 15*a^4*b^4 - 11*a^6*b^2)^2)*(a^16 + b^16 + 8*a^2*b^14 + 28*a^4*
b^12 + 56*a^6*b^10 + 70*a^8*b^8 + 56*a^10*b^6 + 28*a^12*b^4 + 8*a^14*b^2))/
(32*a*b^3 - 96*a^3*b) + (((b*((32*(5*a^2*b^9 - 3*a^10*b + 12*a^4*b^7 + 6*a^
6*b^5 - 4*a^8*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 1
5*a^8*b^4 + 6*a^10*b^2) - ((6*a*b^2 - 2*a^3)*((32*(3*a^3*b^11 - 4*a^13*b -
a*b^13 + 18*a^5*b^9 + 22*a^7*b^7 + 3*a^9*b^5 - 9*a^11*b^3))/(a^12 + b^12 +
6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + (16*(6*a*
b^2 - 2*a^3)*(3*a^16*b + 3*a^2*b^15 + 21*a^4*b^13 + 63*a^6*b^11 + 105*a^8*b
^9 + 105*a^10*b^7 + 63*a^12*b^5 + 21*a^14*b^3))/((a^6 + b^6 + 3*a^2*b^4 + 3
*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^...
```

3.23 $\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

Optimal. Leaf size=92

$$-\frac{(a^2-2b^2)\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}+\frac{a(3ab\cos(x)+(a^2+4b^2)\sin(x))}{2\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))^2}$$

```
[Out] -(a^2-2*b^2)*arctanh((-b+a*tan(1/2*x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)+1/2
*a*(3*a*b*cos(x)+(a^2+4*b^2)*sin(x))/(a^2+b^2)^2/(a*cos(x)+b*sin(x))^2
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 300 vs. $2(92) = 184$.
time = 0.51, antiderivative size = 300, normalized size of antiderivative = 3.26, number of steps used = 13, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$,
Rules used = {4486, 1674, 12, 632, 212, 3234, 3153}

$$\frac{2\left((a^2+2b^2)\tan\left(\frac{x}{2}\right)+ab\right)}{a\left(a^2+b^2\right)\left(-a\tan^2\left(\frac{x}{2}\right)+a+2b\tan\left(\frac{x}{2}\right)\right)^2}+\frac{2a}{b\left(a^2+b^2\right)(a\cos(x)+b\sin(x))}-\frac{a^2(2a^2-b^2)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)^{5/2}}+\frac{2a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2\left(a^2+b^2\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}}-\frac{4a^4+ab(5a^2+2b^2)\tan\left(\frac{x}{2}\right)+3a^2b^2+2b^4}{ab\left(a^2+b^2\right)^2\left(-a\tan^2\left(\frac{x}{2}\right)+a+2b\tan\left(\frac{x}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]^2/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] (2*a^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(b^2*(a^2 + b^2)^(3/
2)) - ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]]/(b^2*Sqrt[a^2 + b^2])
- (a^2*(2*a^2 - b^2)*ArcTanh[(b - a*Tan[x/2])/Sqrt[a^2 + b^2]])/(b^2*(a^2 +
 b^2)^(5/2)) + (2*a)/(b*(a^2 + b^2)*(a*Cos[x] + b*Sin[x])) + (2*(a*b + (a^2
 + 2*b^2)*Tan[x/2]))/(a*(a^2 + b^2)*(a + 2*b*Tan[x/2] - a*Tan[x/2]^2)^2) -
(4*a^4 + 3*a^2*b^2 + 2*b^4 + a*b*(5*a^2 + 2*b^2)*Tan[x/2])/(a*b*(a^2 + b^2)
^2*(a + 2*b*Tan[x/2] - a*Tan[x/2]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3234

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rule 4486

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \int \left(\frac{a^2\cos^2(x)}{b^2(a\cos(x)+b\sin(x))^3} - \frac{2a\cos(x)}{b^2(a\cos(x)+b\sin(x))^2} + \frac{1}{b^2(a\cos(x)+b\sin(\right.$$

$$= \frac{\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{b^2} - \frac{(2a)\int \frac{\cos(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{b^2} + \frac{a^2\int \frac{\cos^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx}{b^2}$$

$$= \frac{2a}{b(a^2+b^2)(a\cos(x)+b\sin(x))} - \frac{\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, b\cos(x)-a\sin(x)\right)}{b^2}$$

$$= -\frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} + \frac{2a}{b(a^2+b^2)(a\cos(x)+b\sin(x))} + \frac{2(ab-}{a(a^2+b^2)(a}$$

$$= \frac{2a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2(a^2+b^2)^{3/2}} - \frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} + \frac{2a}{b(a^2+b^2)(a\cos(x}$$

$$= \frac{2a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2(a^2+b^2)^{3/2}} - \frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} + \frac{2a}{b(a^2+b^2)(a\cos(x}$$

$$= \frac{2a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2(a^2+b^2)^{3/2}} - \frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} + \frac{2a}{b(a^2+b^2)(a\cos(x}$$

$$= \frac{2a^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2(a^2+b^2)^{3/2}} - \frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}} - \frac{a^2(2a^2-b^2)\tanh}{b^2(a^2+}$$

Mathematica [A]
time = 0.45, size = 92, normalized size = 1.00

$$-\frac{(a^2-2b^2)\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} + \frac{a(3ab\cos(x)+(a^2+4b^2)\sin(x))}{2(a^2+b^2)^2(a\cos(x)+b\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]^2/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] -(((a^2 - 2*b^2)*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5
/2)) + (a*(3*a*b*Cos[x] + (a^2 + 4*b^2)*Sin[x]))/(2*(a^2 + b^2)^2*(a*Cos[x]
 + b*Sin[x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(84) = 168$.
time = 0.41, size = 212, normalized size = 2.30

method	result
default	$-\frac{8\left(-\frac{a\left(a^{2}-2b^{2}\right)\left(\tan^{3}\left(\frac{x}{2}\right)\right)}{8\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}+\frac{3b\left(a^{2}-2b^{2}\right)\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{8\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}-\frac{\left(a^{2}+10b^{2}\right)a\tan\left(\frac{x}{2}\right)}{8\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}-\frac{3a^{2}b}{8\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}\right)}{\left(a\left(\tan^{2}\left(\frac{x}{2}\right)\right)-2b\tan\left(\frac{x}{2}\right)-a\right)^{2}}-\frac{\left(a^{2}-2b^{2}\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^{2}+b^{2}}}\right.}{\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}+b^{2}}}$
risch	$-\frac{ia\left(3iab\,e^{3ix}+a^{2}e^{3ix}+4b^{2}e^{3ix}+3ie^{ix}ab-e^{ix}a^{2}-4\,e^{ix}b^{2}\right)}{\left(-ib\,e^{2ix}+a\,e^{2ix}+ib+a\right)^{2}\left(ib+a\right)^{2}\left(-ib+a\right)^{2}}-\frac{\ln\left(e^{ix}+\frac{ia^{5}+2ia^{3}b^{2}+ia\,b^{4}-b\,a^{4}-2b^{3}a^{2}-b^{5}}{\left(a^{2}+b^{2}\right)^{\frac{5}{2}}}\right)a^{2}}{2\left(a^{2}+b^{2}\right)^{\frac{5}{2}}}+\frac{\ln\left(e^{ix}+\frac{ia^{5}+2ia^{3}b^{2}}{}\right.}{(a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a*cos(x)+b*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -8*(-1/8*a*(a^2-2*b^2)/(a^4+2*a^2*b^2+b^4)*tan(1/2*x)^3+3/8*b*(a^2-2*b^2)/(
a^4+2*a^2*b^2+b^4)*tan(1/2*x)^2-1/8*(a^2+10*b^2)*a/(a^4+2*a^2*b^2+b^4)*tan(
1/2*x)-3/8*a^2*b/(a^4+2*a^2*b^2+b^4))/(a*tan(1/2*x)^2-2*b*tan(1/2*x)-a)^2-(
a^2-2*b^2)/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-
2*b)/(a^2+b^2)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(84) = 168$.
time = 0.49, size = 299, normalized size = 3.25

$$\frac{\left(a^{2}-2\,b^{2}\right)\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^{2}+b^{2}}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^{2}+b^{2}}}\right)}{2\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}+b^{2}}}+\frac{3\,a^{2}b+\frac{\left(a^{3}+10\,ab^{2}\right)\sin(x)}{\cos(x)+1}-\frac{3\left(a^{2}b-2\,b^{3}\right)\sin(x)^{2}}{(\cos(x)+1)^{2}}+\frac{\left(a^{3}-2\,ab^{2}\right)\sin(x)^{3}}{(\cos(x)+1)^{3}}}{a^{6}+2\,a^{4}b^{2}+a^{2}b^{4}+\frac{4\left(a^{5}b+2\,a^{3}b^{3}+ab^{5}\right)\sin(x)}{\cos(x)+1}-\frac{2\left(a^{6}-3\,a^{2}b^{4}-2\,b^{6}\right)\sin(x)^{2}}{(\cos(x)+1)^{2}}-\frac{4\left(a^{5}b+2\,a^{3}b^{3}+ab^{5}\right)\sin(x)^{3}}{(\cos(x)+1)^{3}}+\frac{\left(a^{6}+2\,a^{4}b^{2}+a^{2}b^{4}\right)\sin(x)^{4}}{(\cos(x)+1)^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2 - 2*b^2)*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*
sin(x)/(cos(x) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 +
 b^2)) + (3*a^2*b + (a^3 + 10*a*b^2)*sin(x)/(cos(x) + 1) - 3*(a^2*b - 2*b^3
)*sin(x)^2/(cos(x) + 1)^2 + (a^3 - 2*a*b^2)*sin(x)^3/(cos(x) + 1)^3)/(a^6 +
 2*a^4*b^2 + a^2*b^4 + 4*(a^5*b + 2*a^3*b^3 + a*b^5)*sin(x)/(cos(x) + 1) -
2*(a^6 - 3*a^2*b^4 - 2*b^6)*sin(x)^2/(cos(x) + 1)^2 - 4*(a^5*b + 2*a^3*b^3
+ a*b^5)*sin(x)^3/(cos(x) + 1)^3 + (a^6 + 2*a^4*b^2 + a^2*b^4)*sin(x)^4/(co
s(x) + 1)^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(84) = 168$.
time = 2.56, size = 282, normalized size = 3.07

$$-\frac{\left(a^{2}b^{2}-2\,b^{4}+\left(a^{4}-3\,a^{2}b^{2}+2\,b^{4}\right)\cos\left(x\right)^{2}+2\left(a^{3}b-2\,ab^{3}\right)\cos\left(x\right)\sin\left(x\right)\right)\sqrt{a^{2}+b^{2}}\log\left(-\frac{2\,ab\cos(x)\sin(x)+\left(a^{2}-b^{2}\right)\cos(x)^{2}-2\,a^{2}-b^{2}+2\,\sqrt{a^{2}+b^{2}}\left(b\cos(x)-a\sin(x)\right)}{2\,ab\cos(x)\sin(x)+\left(a^{2}-b^{2}\right)\cos(x)^{2}+b^{2}}\right)-6\left(a^{4}b+a^{2}b^{3}\right)\cos\left(x\right)-2\left(a^{5}+5\,a^{3}b^{2}+4\,ab^{4}\right)\sin\left(x\right)}{4\left(a^{6}b^{2}+3\,a^{4}b^{4}+3\,a^{2}b^{6}+b^{8}+\left(a^{8}+2\,a^{6}b^{2}-2\,a^{2}b^{6}-b^{8}\right)\cos\left(x\right)^{2}+2\left(a^{7}b+3\,a^{5}b^{3}+3\,a^{3}b^{5}+ab^{7}\right)\cos\left(x\right)\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/4*((a^2*b^2 - 2*b^4 + (a^4 - 3*a^2*b^2 + 2*b^4)*cos(x)^2 + 2*(a^3*b - 2*
a*b^3)*cos(x)*sin(x))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^
2)*cos(x)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b
*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)) - 6*(a^4*b + a^2*b^3)*cos(x)
- 2*(a^5 + 5*a^3*b^2 + 4*a*b^4)*sin(x))/(a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 +
b^8 + (a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*cos(x)^2 + 2*(a^7*b + 3*a^5*b^3 +
 3*a^3*b^5 + a*b^7)*cos(x)*sin(x))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)**2/(a*cos(x)+b*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(84) = 168.
time = 0.46, size = 197, normalized size = 2.14

$$\frac{(a^2-2\,b^2)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{2\,(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}}+\frac{a^3\tan\left(\frac{1}{2}\,x\right)^3-2\,ab^2\tan\left(\frac{1}{2}\,x\right)^3-3\,a^2b\tan\left(\frac{1}{2}\,x\right)^2+6\,b^3\tan\left(\frac{1}{2}\,x\right)^2+a^3\tan\left(\frac{1}{2}\,x\right)+10\,ab^2\tan\left(\frac{1}{2}\,x\right)+3\,a^2b}{(a^4+2\,a^2b^2+b^4)\left(a\tan\left(\frac{1}{2}\,x\right)^2-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)^2/(a*cos(x)+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] 1/2*(a^2 - 2*b^2)*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a
*tan(1/2*x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 +
 b^2)) + (a^3*tan(1/2*x)^3 - 2*a*b^2*tan(1/2*x)^3 - 3*a^2*b*tan(1/2*x)^2 +
6*b^3*tan(1/2*x)^2 + a^3*tan(1/2*x) + 10*a*b^2*tan(1/2*x) + 3*a^2*b)/((a^4
+ 2*a^2*b^2 + b^4)*(a*tan(1/2*x)^2 - 2*b*tan(1/2*x) - a)^2)
```

Mupad [**B**]
time = 0.80, size = 263, normalized size = 2.86

$$\frac{\frac{3\,a^2\,b}{a^4+2\,a^2\,b^2+b^4}+\frac{a\tan\left(\frac{x}{2}\right)\,(a^2+10\,b^2)}{a^4+2\,a^2\,b^2+b^4}+\frac{a\tan\left(\frac{x}{2}\right)^3\,(a^2-2\,b^2)}{a^4+2\,a^2\,b^2+b^4}-\frac{3\,b\tan\left(\frac{x}{2}\right)^2\,(a^2-2\,b^2)}{a^4+2\,a^2\,b^2+b^4}}{a^2-\tan\left(\frac{x}{2}\right)^2\,(2\,a^2-4\,b^2)+a^2\tan\left(\frac{x}{2}\right)^4+4\,a\,b\tan\left(\frac{x}{2}\right)-4\,a\,b\tan\left(\frac{x}{2}\right)^3}+\frac{\mathrm{atanh}\left(\frac{2\,a^4\,b+4\,a^2\,b^3+2\,b^5}{2\,(a^2+b^2)^{5/2}}-\frac{a\tan\left(\frac{x}{2}\right)\,(a^4+2\,a^2\,b^2+b^4)}{(a^2+b^2)^{5/2}}\right)\,(a^2-2\,b^2)}{(a^2+b^2)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)^2/(a*cos(x) + b*sin(x))^3,x)
```

```
[Out] ((3*a^2*b)/(a^4 + b^4 + 2*a^2*b^2) + (a*tan(x/2)*(a^2 + 10*b^2))/(a^4 + b^4
 + 2*a^2*b^2) + (a*tan(x/2)^3*(a^2 - 2*b^2))/(a^4 + b^4 + 2*a^2*b^2) - (3*b
```

```
*tan(x/2)^2*(a^2 - 2*b^2))/(a^4 + b^4 + 2*a^2*b^2))/(a^2 - tan(x/2)^2*(2*a^
2 - 4*b^2) + a^2*tan(x/2)^4 + 4*a*b*tan(x/2) - 4*a*b*tan(x/2)^3) + (atanh((
2*a^4*b + 2*b^5 + 4*a^2*b^3)/(2*(a^2 + b^2)^(5/2)) - (a*tan(x/2)*(a^4 + b^4
 + 2*a^2*b^2))/(a^2 + b^2)^(5/2))*(a^2 - 2*b^2))/(a^2 + b^2)^(5/2)
```

3.24 $\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

Optimal. Leaf size=15

$$\frac{1}{2a(b+a\cot(x))^2}$$

```
[Out] 1/2/a/(b+a*cot(x))^2
```

Rubi [A]
time = 0.02, antiderivative size = 19, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3166, 37}

$$\frac{\tan^2(x)}{2a(a+b\tan(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] Tan[x]^2/(2*a*(a + b*Tan[x])^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3166

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[1/d, Subst[Int[x^m*((a + b*x
)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n, 0]
 && GtQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^3}\,dx &= \text{Subst}\left(\int \frac{x}{(a+bx)^3}\,dx, x, \tan(x)\right)\\ &= \frac{\tan^2(x)}{2a(a+b\tan(x))^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 47 vs. $2(15) = 30$.
time = 0.11, size = 47, normalized size = 3.13

$$\frac{2b^2\sin^2(x) + a(a + b\sin(2x))}{2a\,(a^2 + b^2)\,(a\cos(x) + b\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] (2*b^2*Sin[x]^2 + a*(a + b*Sin[2*x]))/(2*a*(a^2 + b^2)*(a*Cos[x] + b*Sin[x]
)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(13) = 26$.
time = 0.25, size = 29, normalized size = 1.93

method	result	size
default	$-\frac{1}{b^2(a+b\tan(x))} + \frac{a}{2b^2(a+b\tan(x))^2}$	29
norman	$\frac{\frac{2\left(\tan^2\left(\frac{x}{2}\right)\right)}{a}+\frac{2\left(\tan^4\left(\frac{x}{2}\right)\right)}{a}}{\left(1+\tan^2\left(\frac{x}{2}\right)\right)\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-2b\tan\left(\frac{x}{2}\right)-a\right)^2}$	56
risch	$-\frac{2i\left(ia\,e^{2ix}+b\,e^{2ix}-b\right)}{\left(b\,e^{2ix}+ia\,e^{2ix}-b+ia\right)^2(ia+b)^2}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a*cos(x)+b*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/b^2/(a+b*tan(x))+1/2*a/b^2/(a+b*tan(x))^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(13) = 26$.
time = 0.28, size = 84, normalized size = 5.60

$$\frac{2\,\sin\left(x\right)^2}{\left(a^3 + \frac{4\,a^2b\sin(x)}{\cos(x)+1} - \frac{4\,a^2b\sin(x)^3}{(\cos(x)+1)^3} + \frac{a^3\sin(x)^4}{(\cos(x)+1)^4} - \frac{2\,(a^3-2\,ab^2)\sin(x)^2}{(\cos(x)+1)^2}\right)(\cos\left(x\right)+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] 2*sin(x)^2/((a^3 + 4*a^2*b*sin(x)/(cos(x) + 1) - 4*a^2*b*sin(x)^3/(cos(x) +
 1)^3 + a^3*sin(x)^4/(cos(x) + 1)^4 - 2*(a^3 - 2*a*b^2)*sin(x)^2/(cos(x) +
1)^2)*(cos(x) + 1)^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(13) = 26.
time = 2.17, size = 116, normalized size = 7.73

$$-\frac{4\,ab^2\cos(x)^2 - a^3 - 3\,ab^2 - 2\,(a^2b - b^3)\cos(x)\sin(x)}{2\,\left(a^4b^2 + 2\,a^2b^4 + b^6 + (a^6 + a^4b^2 - a^2b^4 - b^6)\cos(x)^2 + 2\,(a^5b + 2\,a^3b^3 + ab^5)\cos(x)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a*b^2*cos(x)^2 - a^3 - 3*a*b^2 - 2*(a^2*b - b^3)*cos(x)*sin(x))/(a^
4*b^2 + 2*a^2*b^4 + b^6 + (a^6 + a^4*b^2 - a^2*b^4 - b^6)*cos(x)^2 + 2*(a^5
*b + 2*a^3*b^3 + a*b^5)*cos(x)*sin(x))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.41, size = 20, normalized size = 1.33

$$-\frac{2\,b\tan(x) + a}{2\,(b\tan(x) + a)^2 b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(a*cos(x)+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*b*tan(x) + a)/((b*tan(x) + a)^2*b^2)
```

Mupad [B]
time = 0.49, size = 48, normalized size = 3.20

$$\frac{\tan\left(\frac{x}{2}\right)^2\left(a - \frac{2\,a^2-4\,b^2}{2\,a}\right)}{b^2\left(-a\tan\left(\frac{x}{2}\right)^2 + 2\,b\tan\left(\frac{x}{2}\right) + a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(a*cos(x) + b*sin(x))^3,x)
```

```
[Out] (tan(x/2)^2*(a - (2*a^2 - 4*b^2)/(2*a)))/(b^2*(a + 2*b*tan(x/2) - a*tan(x/2
)^2)^2)
```

3.25 $\int \frac{1}{(a\cos(x)+b\sin(x))^3}\,dx$

Optimal. Leaf size=73

$$-\frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}}-\frac{b\cos(x)-a\sin(x)}{2\left(a^2+b^2\right)\left(a\cos(x)+b\sin(x)\right)^2}$$

```
[Out] -1/2*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)+1/2*(-b*c
os(x)+a*sin(x))/(a^2+b^2)/(a*cos(x)+b*sin(x))^2
```

Rubi [A]
time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3155, 3153, 212}

$$-\frac{b\cos(x)-a\sin(x)}{2\left(a^2+b^2\right)\left(a\cos(x)+b\sin(x)\right)^2}-\frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[x] + b*Sin[x])^(-3),x]
```

```
[Out] -1/2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]]/(a^2 + b^2)^(3/2) - (b*
Cos[x] - a*Sin[x])/(2*(a^2 + b^2)*(a*Cos[x] + b*Sin[x])^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3155

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(a\cos(x)+b\sin(x))^3}\,dx = -\frac{b\cos(x)-a\sin(x)}{2(a^2+b^2)(a\cos(x)+b\sin(x))^2}+\frac{\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{2(a^2+b^2)}$$
$$= -\frac{b\cos(x)-a\sin(x)}{2(a^2+b^2)(a\cos(x)+b\sin(x))^2}-\frac{\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,b\cos(x)-a\sin(\right.}{2(a^2+b^2)}$$
$$= -\frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{3/2}}-\frac{b\cos(x)-a\sin(x)}{2(a^2+b^2)(a\cos(x)+b\sin(x))^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.18, size = 101, normalized size = 1.38

$$\frac{(a^2+b^2)(-b\cos(x)+a\sin(x))+2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)(a\cos(x)+b\sin(x))^2}{2(a-ib)^2(a+ib)^2(a\cos(x)+b\sin(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[x] + b*Sin[x])^(-3),x]
```

```
[Out] ((a^2 + b^2)*(-(b*Cos[x]) + a*Sin[x]) + 2*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*T
an[x/2])/Sqrt[a^2 + b^2]]*(a*Cos[x] + b*Sin[x])^2)/(2*(a - I*b)^2*(a + I*b)
^2*(a*Cos[x] + b*Sin[x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(65) = 130.
time = 0.28, size = 157, normalized size = 2.15

method	result	size
default	$-\frac{2\left(-\frac{(a^2+2b^2)\left(\tan^3\left(\frac{x}{2}\right)\right)}{2(a^2+b^2)a}-\frac{b(a^2-2b^2)\left(\tan^2\left(\frac{x}{2}\right)\right)}{2(a^2+b^2)a^2}-\frac{(a^2-2b^2)\tan\left(\frac{x}{2}\right)}{2(a^2+b^2)a}+\frac{b}{2a^2+2b^2}\right)}{\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-2b\tan\left(\frac{x}{2}\right)-a\right)^2}+\frac{\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}$	157
risch	$\frac{ia\,e^{3ix}+b\,e^{3ix}-ia\,e^{ix}+b\,e^{ix}}{(b\,e^{2ix}+ia\,e^{2ix}-b+ia)^2(-ia+b)(ia+b)}+\frac{\ln\left(e^{ix}+\frac{ia^3+ia\,b^2-a^2b-b^3}{(a^2+b^2)^{\frac{3}{2}}}\right)}{2(a^2+b^2)^{\frac{3}{2}}}-\frac{\ln\left(e^{ix}-\frac{ia^3+ia\,b^2-a^2b-b^3}{(a^2+b^2)^{\frac{3}{2}}}\right)}{2(a^2+b^2)^{\frac{3}{2}}}$	185

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x)+b*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-1/2*(a^2+2*b^2)/(a^2+b^2)/a*tan(1/2*x)^3-1/2*b*(a^2-2*b^2)/(a^2+b^2)/a
^2*tan(1/2*x)^2-1/2*(a^2-2*b^2)/(a^2+b^2)/a*tan(1/2*x)+1/2*b/(a^2+b^2))/(a*
```

```
tan(1/2*x)^2-2*b*tan(1/2*x)-a)^2+1/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1/2
*x)-2*b)/(a^2+b^2)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(65) = 130$.
time = 0.49, size = 250, normalized size = 3.42

$$-\frac{a^2b - \frac{(a^3-2ab^2)\sin(x)}{\cos(x)+1} - \frac{(a^2b-2b^3)\sin(x)^2}{(\cos(x)+1)^2} - \frac{(a^3+2ab^2)\sin(x)^3}{(\cos(x)+1)^3}}{a^6 + a^4b^2 + \frac{4(a^5b+a^3b^3)\sin(x)}{\cos(x)+1} - \frac{2(a^6-a^4b^2-2a^2b^4)\sin(x)^2}{(\cos(x)+1)^2} - \frac{4(a^5b+a^3b^3)\sin(x)^3}{(\cos(x)+1)^3} + \frac{(a^6+a^4b^2)\sin(x)^4}{(\cos(x)+1)^4}} - \frac{\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{2(a^2+b^2)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -(a^2*b - (a^3 - 2*a*b^2)*sin(x)/(cos(x) + 1) - (a^2*b - 2*b^3)*sin(x)^2/(c
os(x) + 1)^2 - (a^3 + 2*a*b^2)*sin(x)^3/(cos(x) + 1)^3)/(a^6 + a^4*b^2 + 4*
(a^5*b + a^3*b^3)*sin(x)/(cos(x) + 1) - 2*(a^6 - a^4*b^2 - 2*a^2*b^4)*sin(x
)^2/(cos(x) + 1)^2 - 4*(a^5*b + a^3*b^3)*sin(x)^3/(cos(x) + 1)^3 + (a^6 + a
^4*b^2)*sin(x)^4/(cos(x) + 1)^4) - 1/2*log((b - a*sin(x)/(cos(x) + 1) + sqr
t(a^2 + b^2))/(b - a*sin(x)/(cos(x) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3
/2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(65) = 130$.
time = 2.92, size = 225, normalized size = 3.08

$$\frac{(2\,ab\cos(x)\sin(x) + (a^2-b^2)\cos(x)^2 + b^2)\sqrt{a^2+b^2}\,\log\left(-\frac{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2-2\,a^2-b^2+2\sqrt{a^2+b^2}\,(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2}\right) - 2\,(a^2b+b^3)\cos(x) + 2\,(a^3+ab^2)\sin(x)}{4\,(a^4b^2 + 2\,a^2b^4 + b^6 + (a^6+a^4b^2-a^2b^4-b^6)\cos(x)^2 + 2\,(a^5b+2\,a^3b^3+ab^5)\cos(x)\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)*sqrt(a^2 + b^2)*log
(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 +
b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b
^2)) - 2*(a^2*b + b^3)*cos(x) + 2*(a^3 + a*b^2)*sin(x))/(a^4*b^2 + 2*a^2*b^
4 + b^6 + (a^6 + a^4*b^2 - a^2*b^4 - b^6)*cos(x)^2 + 2*(a^5*b + 2*a^3*b^3 +
 a*b^5)*cos(x)*sin(x))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x))**3,x)
```

[Out] Timed out

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. 2(65) = 130.

time = 0.45, size = 166, normalized size = 2.27

$$-\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{2\,(a^2+b^2)^{\frac{3}{2}}}+\frac{a^3\tan\left(\frac{1}{2}\,x\right)^3+2\,ab^2\tan\left(\frac{1}{2}\,x\right)^3+a^2b\tan\left(\frac{1}{2}\,x\right)^2-2\,b^3\tan\left(\frac{1}{2}\,x\right)^2+a^3\tan\left(\frac{1}{2}\,x\right)-2\,ab^2\tan\left(\frac{1}{2}\,x\right)-a^2b}{(a^4+a^2b^2)\left(a\tan\left(\frac{1}{2}\,x\right)^2-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(x)+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -1/2*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) -
 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + (a^3*tan(1/2*x)^3 + 2*a*b^2*
tan(1/2*x)^3 + a^2*b*tan(1/2*x)^2 - 2*b^3*tan(1/2*x)^2 + a^3*tan(1/2*x) - 2
*a*b^2*tan(1/2*x) - a^2*b)/((a^4 + a^2*b^2)*(a*tan(1/2*x)^2 - 2*b*tan(1/2*x
) - a)^2)
```

Mupad [B]

time = 0.76, size = 216, normalized size = 2.96

$$\frac{\frac{\tan\left(\frac{x}{2}\right)^3\,(a^2+2\,b^2)}{a\,(a^2+b^2)}-\frac{b}{a^2+b^2}+\frac{\tan\left(\frac{x}{2}\right)\,(a^2-2\,b^2)}{a\,(a^2+b^2)}+\frac{b\tan\left(\frac{x}{2}\right)^2\,(a^2-2\,b^2)}{a^2\,(a^2+b^2)}}{a^2-\tan\left(\frac{x}{2}\right)^2\,(2\,a^2-4\,b^2)+a^2\tan\left(\frac{x}{2}\right)^4+4\,a\,b\tan\left(\frac{x}{2}\right)-4\,a\,b\tan\left(\frac{x}{2}\right)^3}-\frac{\mathrm{atanh}\left(-\frac{\left(2\,a\tan\left(\frac{x}{2}\right)-\frac{2\,a^2\,b+2\,b^3}{a^2+b^2}\right)\left(\frac{a^2}{2}+\frac{b^2}{2}\right)}{(a^2+b^2)^{3/2}}\right)}{(a^2+b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(x) + b*sin(x))^3,x)
```

```
[Out] ((tan(x/2)^3*(a^2 + 2*b^2))/(a*(a^2 + b^2)) - b/(a^2 + b^2) + (tan(x/2)*(a^
2 - 2*b^2))/(a*(a^2 + b^2)) + (b*tan(x/2)^2*(a^2 - 2*b^2))/(a^2*(a^2 + b^2)
))/(a^2 - tan(x/2)^2*(2*a^2 - 4*b^2) + a^2*tan(x/2)^4 + 4*a*b*tan(x/2) - 4*
a*b*tan(x/2)^3) - atanh(-((2*a*tan(x/2) - (2*a^2*b + 2*b^3)/(a^2 + b^2))*(a
^2/2 + b^2/2))/(a^2 + b^2)^(3/2))/(a^2 + b^2)^(3/2)
```

3.26 $\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

Optimal. Leaf size=59

$$\frac{\log(\tan(x))}{a^3}-\frac{\log(a+b\tan(x))}{a^3}+\frac{\frac{1}{a}+\frac{a}{b^2}}{2(a+b\tan(x))^2}+\frac{\frac{1}{a^2}-\frac{1}{b^2}}{a+b\tan(x)}$$

```
[Out] ln(tan(x))/a^3-ln(a+b*tan(x))/a^3+1/2*(1/a+a/b^2)/(a+b*tan(x))^2+(1/a^2-1/b
^2)/(a+b*tan(x))
```

Rubi [A]
time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3166, 908}

$$-\frac{\log(a+b\tan(x))}{a^3}+\frac{\log(\tan(x))}{a^3}+\frac{\frac{1}{a^2}-\frac{1}{b^2}}{a+b\tan(x)}+\frac{\frac{a}{b^2}+\frac{1}{a}}{2(a+b\tan(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] Log[Tan[x]]/a^3 - Log[a + b*Tan[x]]/a^3 + (a^(-1) + a/b^2)/(2*(a + b*Tan[x]
)^2) + (a^(-2) - b^(-2))/(a + b*Tan[x])
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3166

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[1/d, Subst[Int[x^m*((a + b*x
)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n, 0]
 && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \text{Subst}\left(\int \frac{1+x^2}{x(a+bx)^3}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{a^3x}+\frac{-a^2-b^2}{ab(a+bx)^3}+\frac{a^2-b^2}{a^2b(a+bx)^2}-\frac{b}{a^3(a+bx)}\right)dx, x, \tan(x)\right)$$
$$= \frac{\log(\tan(x))}{a^3}-\frac{\log(a+b\tan(x))}{a^3}+\frac{\frac{1}{a}+\frac{a}{b^2}}{2(a+b\tan(x))^2}+\frac{\frac{1}{a^2}-\frac{1}{b^2}}{a+b\tan(x)}$$

Mathematica [A]
time = 0.25, size = 96, normalized size = 1.63

$$\frac{a^2\csc^2(x)+2ab\cot(x)(-1+2\log(\sin(x))-2\log(a\cos(x)+b\sin(x)))+2b^2(-1+\log(\sin(x))-\log(a\cos(x)+b\sin(x)))+2a^2\cot^2(x)(\log(\sin(x))-\log(a\cos(x)+b\sin(x)))}{2a^3(b+a\cot(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] (a^2*Csc[x]^2 + 2*a*b*Cot[x]*(-1 + 2*Log[Sin[x]] - 2*Log[a*Cos[x] + b*Sin[x
]]) + 2*b^2*(-1 + Log[Sin[x]] - Log[a*Cos[x] + b*Sin[x]]) + 2*a^2*Cot[x]^2*
(Log[Sin[x]] - Log[a*Cos[x] + b*Sin[x]]))/(2*a^3*(b + a*Cot[x])^2)
```

Maple [A]
time = 0.32, size = 73, normalized size = 1.24

method	result	size
default	$\frac{\ln(\tan(x))}{a^3}-\frac{-a^2-b^2}{2a\,b^2(a+b\tan(x))^2}-\frac{a^2-b^2}{a^2b^2(a+b\tan(x))}-\frac{\ln(a+b\tan(x))}{a^3}$	73
norman	$\frac{-\frac{2\left(-a^2+3b^2\right)\left(\tan^2\left(\frac{x}{2}\right)\right)}{a^3}-\frac{4b\tan\left(\frac{x}{2}\right)}{a^2}+\frac{4b\left(\tan^3\left(\frac{x}{2}\right)\right)}{a^2}}{\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-2b\tan\left(\frac{x}{2}\right)-a\right)^2}+\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^3}-\frac{\ln\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-2b\tan\left(\frac{x}{2}\right)-a\right)}{a^3}$	103
risch	$\frac{2a^2e^{2ix}-2b^2e^{2ix}-4iab\,e^{2ix}+2b^2-2iab}{\left(-ib\,e^{2ix}+a\,e^{2ix}+ib+a\right)^2a^2(-ib+a)}+\frac{\ln\left(e^{2ix}-1\right)}{a^3}-\frac{\ln\left(e^{2ix}-\frac{ib+a}{ib-a}\right)}{a^3}$	119

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(a*cos(x)+b*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] ln(tan(x))/a^3-1/2*(-a^2-b^2)/a/b^2/(a+b*tan(x))^2-(a^2-b^2)/a^2/b^2/(a+b*t
an(x))-ln(a+b*tan(x))/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(57) = 114$.
time = 0.29, size = 172, normalized size = 2.92

$$-\frac{2\left(\frac{2\,ab\sin(x)}{\cos(x)+1}-\frac{2\,ab\sin(x)^3}{(\cos(x)+1)^3}-\frac{(a^2-3\,b^2)\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^5+\frac{4\,a^4b\sin(x)}{\cos(x)+1}-\frac{4\,a^4b\sin(x)^3}{(\cos(x)+1)^3}+\frac{a^5\sin(x)^4}{(\cos(x)+1)^4}-\frac{2\,(a^5-2\,a^3b^2)\sin(x)^2}{(\cos(x)+1)^2}}-\frac{\log\left(-a-\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^3}+\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -2*(2*a*b*sin(x)/(cos(x) + 1) - 2*a*b*sin(x)^3/(cos(x) + 1)^3 - (a^2 - 3*b^
2)*sin(x)^2/(cos(x) + 1)^2)/(a^5 + 4*a^4*b*sin(x)/(cos(x) + 1) - 4*a^4*b*si
n(x)^3/(cos(x) + 1)^3 + a^5*sin(x)^4/(cos(x) + 1)^4 - 2*(a^5 - 2*a^3*b^2)*s
in(x)^2/(cos(x) + 1)^2) - log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(co
s(x) + 1)^2)/a^3 + log(sin(x)/(cos(x) + 1))/a^3
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 220 vs. $2(57) = 114$.
time = 2.55, size = 220, normalized size = 3.73

$$\frac{4\,a^{2}b^{2}\cos\left(x\right)^{2}+a^{4}-a^{2}b^{2}-2\left(a^{3}b-ab^{3}\right)\cos\left(x\right)\sin\left(x\right)-\left(a^{2}b^{2}+b^{4}+\left(a^{4}-b^{4}\right)\cos\left(x\right)^{2}+2\left(a^{3}b+ab^{3}\right)\cos\left(x\right)\sin\left(x\right)\right)\log\left(2\,ab\cos\left(x\right)\sin\left(x\right)+\left(a^{2}-b^{2}\right)\cos\left(x\right)^{2}+b^{2}\right)+\left(a^{2}b^{2}+b^{4}+\left(a^{4}-b^{4}\right)\cos\left(x\right)^{2}+2\left(a^{3}b+ab^{3}\right)\cos\left(x\right)\sin\left(x\right)\right)\log\left(-\frac{1}{4}\cos\left(x\right)^{2}+\frac{1}{4}\right)}{2\left(a^{5}b^{2}+a^{3}b^{4}+\left(a^{7}-a^{3}b^{4}\right)\cos\left(x\right)^{2}+2\left(a^{6}b+a^{4}b^{3}\right)\cos\left(x\right)\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*b^2*cos(x)^2 + a^4 - a^2*b^2 - 2*(a^3*b - a*b^3)*cos(x)*sin(x) -
 (a^2*b^2 + b^4 + (a^4 - b^4)*cos(x)^2 + 2*(a^3*b + a*b^3)*cos(x)*sin(x))*l
og(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) + (a^2*b^2 + b^4 + (a^
4 - b^4)*cos(x)^2 + 2*(a^3*b + a*b^3)*cos(x)*sin(x))*log(-1/4*cos(x)^2 + 1/
4))/(a^5*b^2 + a^3*b^4 + (a^7 - a^3*b^4)*cos(x)^2 + 2*(a^6*b + a^4*b^3)*cos
(x)*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(x\right)}{\left(a\cos\left(x\right)+b\sin\left(x\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x))**3,x)
```

```
[Out] Integral(csc(x)/(a*cos(x) + b*sin(x))**3, x)
```

Giac [A]
time = 0.43, size = 77, normalized size = 1.31

$$-\frac{\log\left(\left|b\tan\left(x\right)+a\right|\right)}{a^{3}}+\frac{\log\left(\left|\tan\left(x\right)\right|\right)}{a^{3}}+\frac{3\,b^{4}\tan\left(x\right)^{2}-2\,a^{3}b\tan\left(x\right)+8\,ab^{3}\tan\left(x\right)-a^{4}+6\,a^{2}b^{2}}{2\left(b\tan\left(x\right)+a\right)^{2}a^{3}b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(a*cos(x)+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -log(abs(b*tan(x) + a))/a^3 + log(abs(tan(x)))/a^3 + 1/2*(3*b^4*tan(x)^2 -
2*a^3*b*tan(x) + 8*a*b^3*tan(x) - a^4 + 6*a^2*b^2)/((b*tan(x) + a)^2*a^3*b^
2)
```

Mupad [B]
time = 0.71, size = 131, normalized size = 2.22

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^3}-\frac{\ln\left(-a\tan\left(\frac{x}{2}\right)^2+2\,b\tan\left(\frac{x}{2}\right)+a\right)}{a^3}+\frac{\frac{2\tan\left(\frac{x}{2}\right)^2\,\left(a^2-3\,b^2\right)}{a^3}+\frac{4\,b\tan\left(\frac{x}{2}\right)^3}{a^2}-\frac{4\,b\tan\left(\frac{x}{2}\right)}{a^2}}{a^2-\tan\left(\frac{x}{2}\right)^2\,\left(2\,a^2-4\,b^2\right)+a^2\tan\left(\frac{x}{2}\right)^4+4\,a\,b\tan\left(\frac{x}{2}\right)-4\,a\,b\tan\left(\frac{x}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(a*cos(x) + b*sin(x))^3),x)
```

```
[Out] log(tan(x/2))/a^3 - log(a + 2*b*tan(x/2) - a*tan(x/2)^2)/a^3 + ((2*tan(x/2)
^2*(a^2 - 3*b^2))/a^3 + (4*b*tan(x/2)^3)/a^2 - (4*b*tan(x/2))/a^2)/(a^2 - t
an(x/2)^2*(2*a^2 - 4*b^2) + a^2*tan(x/2)^4 + 4*a*b*tan(x/2) - 4*a*b*tan(x/2
)^3)
```

3.27 $\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

Optimal. Leaf size=184

$$\frac{3b\tanh^{-1}(\cos(x))}{a^4}-\frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2a^2\sqrt{a^2+b^2}}-\frac{2b^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}}-\frac{\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right.}{a^4}$$

```
[Out] 3*b*arctanh(cos(x))/a^4-csc(x)/a^3+1/2*(-b*cos(x)+a*sin(x))/a^2/(a*cos(x)+b
*sin(x))^2-2*b/a^3/(a*cos(x)+b*sin(x))-1/2*arctanh((b*cos(x)-a*sin(x))/(a^2
+b^2)^(1/2))/a^2/(a^2+b^2)^(1/2)-2*b^2*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2
)^(1/2))/a^4/(a^2+b^2)^(1/2)-arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))*(
a^2+b^2)^(1/2)/a^4
```

Rubi [A]
time = 0.16, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3184, 3155, 3153, 212, 3182, 3855, 3172}

$$\frac{3b\tanh^{-1}(\cos(x))}{a^4}-\frac{2b}{a^3(a\cos(x)+b\sin(x))}-\frac{\csc(x)}{a^3}-\frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2a^2\sqrt{a^2+b^2}}-\frac{b\cos(x)-a\sin(x)}{2a^2(a\cos(x)+b\sin(x))^2}-\frac{2b^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}}-\frac{\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^2/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] (3*b*ArcTanh[Cos[x]])/a^4 - ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]]/
(2*a^2*Sqrt[a^2 + b^2]) - (2*b^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b
^2]])/(a^4*Sqrt[a^2 + b^2]) - (Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[x] - a*Sin[x]
)/Sqrt[a^2 + b^2]])/a^4 - Csc[x]/a^3 - (b*Cos[x] - a*Sin[x])/(2*a^2*(a*Cos[
x] + b*Sin[x])^2) - (2*b)/(a^3*(a*Cos[x] + b*Sin[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3155

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
```

```
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3172

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/si
n[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-(a*Cos[c + d*x] + b*Sin[c + d*x])
^(n + 1)/(a*d*(n + 1)), x] + (Dist[1/a^2, Int[(a*Cos[c + d*x] + b*Sin[c + d
*x])^(n + 2)/Sin[c + d*x], x], x] - Dist[b/a^2, Int[(a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
&& LtQ[n, -1]
```

Rule 3182

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[Sin[c + d*x]^(m + 1)/(a*d*(m + 1))
, x] + (-Dist[b/a^2, Int[Sin[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/a^
2, Int[Sin[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /; F
reeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3184

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2 + b^2)/a^2, Int[Sin[c +
d*x]^(m + 2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^n, x], x] + (Dist[1/a^2, Int
[Sin[c + d*x]^m*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] - Dist[2*
(b/a^2), Int[Sin[c + d*x]^(m + 1)*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 1)
, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && L
tQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \frac{\int \frac{\csc^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2} - \frac{(2b)\int \frac{\csc(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2} + \frac{(a^2+b^2)\int \frac{1}{(a\cos(x)+b\sin(x))^3}\,dx}{a^2}$$

$$= -\frac{\csc(x)}{a^3} - \frac{b\cos(x)-a\sin(x)}{2a^2(a\cos(x)+b\sin(x))^2} - \frac{2b}{a^3(a\cos(x)+b\sin(x))} + \frac{\int \frac{1}{a\cos(x)+b\sin(}}{2a^2}$$

$$= \frac{3b\tanh^{-1}(\cos(x))}{a^4} - \frac{\csc(x)}{a^3} - \frac{b\cos(x)-a\sin(x)}{2a^2(a\cos(x)+b\sin(x))^2} - \frac{2b}{a^3(a\cos(x)+b\sin(}$$

$$= \frac{3b\tanh^{-1}(\cos(x))}{a^4} - \frac{\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{2a^2\sqrt{a^2+b^2}} - \frac{2b^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{a^4\sqrt{a^2+b^2}} -$$

Mathematica [A]
time = 0.83, size = 193, normalized size = 1.05

$$\frac{\csc^3(x)(a\cos(x)+b\sin(x))\left(a(a^2+b^2)\sin(x)-5ab(a\cos(x)+b\sin(x))+\frac{6(a^2+2b^2)\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)(a\cos(x)+b\sin(x))^2}{\sqrt{a^2+b^2}}-a\cot\left(\frac{x}{2}\right)(a\cos(x)+b\sin(x))^2+6b\log\left(\cos\left(\frac{x}{2}\right)\right)(a\cos(x)+b\sin(x))^2-6b\log\left(\sin\left(\frac{x}{2}\right)\right)(a\cos(x)+b\sin(x))^2-a(a\cos(x)+b\sin(x))^2\tan\left(\frac{x}{2}\right)\right)}{2a^4(b+a\cot(x))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^2/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] (Csc[x]^3*(a*Cos[x] + b*Sin[x])*(a*(a^2 + b^2)*Sin[x] - 5*a*b*(a*Cos[x] + b
*Sin[x]) + (6*(a^2 + 2*b^2)*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]]*(a*C
os[x] + b*Sin[x])^2)/Sqrt[a^2 + b^2] - a*Cot[x/2]*(a*Cos[x] + b*Sin[x])^2 +
 6*b*Log[Cos[x/2]]*(a*Cos[x] + b*Sin[x])^2 - 6*b*Log[Sin[x/2]]*(a*Cos[x] +
b*Sin[x])^2 - a*(a*Cos[x] + b*Sin[x])^2*Tan[x/2]))/(2*a^4*(b + a*Cot[x])^3)
```

Maple [A]
time = 0.50, size = 163, normalized size = 0.89

method	result
default	$-\frac{\tan\left(\frac{x}{2}\right)}{2a^3} - \frac{4\left(\frac{-\frac{a\left(a^2+6b^2\right)\left(\tan^3\left(\frac{x}{2}\right)\right)}{4}-\frac{5b\left(a^2-2b^2\right)\left(\tan^2\left(\frac{x}{2}\right)\right)}{4}-\frac{a\left(a^2-14b^2\right)\tan\left(\frac{x}{2}\right)}{4}+\frac{5a^2b}{4}}{\left(a\left(\tan^2\left(\frac{x}{2}\right)\right)-2b\tan\left(\frac{x}{2}\right)-a\right)^2}-\frac{3\left(a^2+2b^2\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{4\sqrt{a^2+b^2}}\right)}{a^4}$
risch	$-\frac{i(9iab\,e^{5ix}-3a^2e^{5ix}+6b^2e^{5ix}-2a^2e^{3ix}-12b^2e^{3ix}-9ie^{ix}ab-3\,e^{ix}a^2+6\,e^{ix}b^2)}{(e^{2ix}-1)(b\,e^{2ix}+ia\,e^{2ix}-b+ia)^2a^3} - \frac{3\ln\left(e^{ix}-\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}\,a^2} - \frac{3\ln\left(e^{ix}-\frac{ia-}{\sqrt{a^2-}}\right)}{\sqrt{a^2+b^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^2/(a*cos(x)+b*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^3*tan(1/2*x)-4/a^4*((-1/4*a*(a^2+6*b^2)*tan(1/2*x)^3-5/4*b*(a^2-2*b^
2)*tan(1/2*x)^2-1/4*a*(a^2-14*b^2)*tan(1/2*x)+5/4*a^2*b)/(a*tan(1/2*x)^2-2*
b*tan(1/2*x)-a)^2-3/4*(a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*
x)-2*b)/(a^2+b^2)^(1/2)))-1/2/a^3/tan(1/2*x)-3/a^4*b*ln(tan(1/2*x))
```

Maxima [A]
time = 0.51, size = 276, normalized size = 1.50

$$-\frac{a^3+\frac{14\,a^2b\sin(x)}{\cos(x)+1}-\frac{4\,(a^3-8\,ab^2)\sin(x)^2}{(\cos(x)+1)^2}-\frac{2\,(7\,a^2b-10\,b^3)\sin(x)^3}{(\cos(x)+1)^3}-\frac{(a^3+12\,ab^2)\sin(x)^4}{(\cos(x)+1)^4}}{2\left(\frac{a^6\sin(x)}{\cos(x)+1}+\frac{4\,a^5b\sin(x)^2}{(\cos(x)+1)^2}-\frac{4\,a^5b\sin(x)^4}{(\cos(x)+1)^4}+\frac{a^6\sin(x)^5}{(\cos(x)+1)^5}-\frac{2\,(a^6-2\,a^4b^2)\sin(x)^3}{(\cos(x)+1)^3}\right)}-\frac{3\,b\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^4}-\frac{3\,(a^2+2\,b^2)\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{2\,\sqrt{a^2+b^2}\,a^4}-\frac{\sin(x)}{2\,a^3(\cos(x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a*cos(x)+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(a^3 + 14*a^2*b*sin(x)/(cos(x) + 1) - 4*(a^3 - 8*a*b^2)*sin(x)^2/(cos(
x) + 1)^2 - 2*(7*a^2*b - 10*b^3)*sin(x)^3/(cos(x) + 1)^3 - (a^3 + 12*a*b^2)
*sin(x)^4/(cos(x) + 1)^4)/(a^6*sin(x)/(cos(x) + 1) + 4*a^5*b*sin(x)^2/(cos(
x) + 1)^2 - 4*a^5*b*sin(x)^4/(cos(x) + 1)^4 + a^6*sin(x)^5/(cos(x) + 1)^5 -
 2*(a^6 - 2*a^4*b^2)*sin(x)^3/(cos(x) + 1)^3) - 3*b*log(sin(x)/(cos(x) + 1)
)/a^4 - 3/2*(a^2 + 2*b^2)*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))
/(b - a*sin(x)/(cos(x) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^4) - 1/2
*sin(x)/(a^3*(cos(x) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 463 vs. 2(168) = 336.
time = 2.58, size = 463, normalized size = 2.52

$$-\frac{2\,a^5-10\,a^3b^2-12\,ab^4-6\,(a^5-a^3b^2-2\,ab^4)\cos(x)^2-18\,(a^4b+a^2b^3)\cos(x)\sin(x)-3\left(2\,(a^3b+2\,ab^3)\cos(x)^3-2\,(a^3b+2\,ab^3)\cos(x)-(a^2b^2+2\,b^4+(a^4+a^2b^2-2\,b^4)\cos(x)^2)\sin(x)\right)\sqrt{a^2+b^2}\log\left(-\frac{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\,(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2}\right)-6\left(2\,(a^3b^2+ab^4)\cos(x)^3-2\,(a^3b^2+ab^4)\cos(x)-(a^2b^3+b^5+(a^4b-b^5)\cos(x)^2)\sin(x)\right)\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)+6\left(2\,(a^3b^2+ab^4)\cos(x)^3-2\,(a^3b^2+ab^4)\cos(x)-(a^2b^3+b^5+(a^4b-b^5)\cos(x)^2)\sin(x)\right)\log\left(-\frac{1}{2}\cos(x)+\frac{1}{2}\right)}{4\left(2\,(a^7b+a^5b^3)\cos(x)^3-2\,(a^7b+a^5b^3)\cos(x)-(a^6b^2+a^4b^4+(a^8-a^4b^4)\cos(x)^2)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a*cos(x)+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^5 - 10*a^3*b^2 - 12*a*b^4 - 6*(a^5 - a^3*b^2 - 2*a*b^4)*cos(x)^2
- 18*(a^4*b + a^2*b^3)*cos(x)*sin(x) - 3*(2*(a^3*b + 2*a*b^3)*cos(x)^3 - 2*
(a^3*b + 2*a*b^3)*cos(x) - (a^2*b^2 + 2*b^4 + (a^4 + a^2*b^2 - 2*b^4)*cos(x
)^2)*sin(x))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)
^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*s
in(x) + (a^2 - b^2)*cos(x)^2 + b^2)) - 6*(2*(a^3*b^2 + a*b^4)*cos(x)^3 - 2*
(a^3*b^2 + a*b^4)*cos(x) - (a^2*b^3 + b^5 + (a^4*b - b^5)*cos(x)^2)*sin(x))
*log(1/2*cos(x) + 1/2) + 6*(2*(a^3*b^2 + a*b^4)*cos(x)^3 - 2*(a^3*b^2 + a*b
^4)*cos(x) - (a^2*b^3 + b^5 + (a^4*b - b^5)*cos(x)^2)*sin(x))*log(-1/2*cos(
x) + 1/2))/(2*(a^7*b + a^5*b^3)*cos(x)^3 - 2*(a^7*b + a^5*b^3)*cos(x) - (a^
6*b^2 + a^4*b^4 + (a^8 - a^4*b^4)*cos(x)^2)*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^2(x)}{(a\cos(x)+b\sin(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**2/(a*cos(x)+b*sin(x))**3,x)
```

```
[Out] Integral(csc(x)**2/(a*cos(x) + b*sin(x))**3, x)
```

Giac [A]
time = 0.45, size = 212, normalized size = 1.15

$$-\frac{3\,b\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{a^4}-\frac{\tan\left(\frac{1}{2}\,x\right)}{2\,a^3}-\frac{3\,(a^2+2\,b^2)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{2\,\sqrt{a^2+b^2}\,a^4}+\frac{6\,b\tan\left(\frac{1}{2}\,x\right)-a}{2\,a^4\tan\left(\frac{1}{2}\,x\right)}+\frac{a^3\tan\left(\frac{1}{2}\,x\right)^3+6\,ab^2\tan\left(\frac{1}{2}\,x\right)^3+5\,a^2b\tan\left(\frac{1}{2}\,x\right)^2-10\,b^3\tan\left(\frac{1}{2}\,x\right)^2+a^3\tan\left(\frac{1}{2}\,x\right)-14\,ab^2\tan\left(\frac{1}{2}\,x\right)-5\,a^2b}{\left(a\tan\left(\frac{1}{2}\,x\right)^2-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)^2a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^2/(a*cos(x)+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] -3*b*log(abs(tan(1/2*x)))/a^4 - 1/2*tan(1/2*x)/a^3 - 3/2*(a^2 + 2*b^2)*log(
abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) - 2*b + 2*
sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a^4) + 1/2*(6*b*tan(1/2*x) - a)/(a^4*tan
(1/2*x)) + (a^3*tan(1/2*x)^3 + 6*a*b^2*tan(1/2*x)^3 + 5*a^2*b*tan(1/2*x)^2
- 10*b^3*tan(1/2*x)^2 + a^3*tan(1/2*x) - 14*a*b^2*tan(1/2*x) - 5*a^2*b)/((a
*tan(1/2*x)^2 - 2*b*tan(1/2*x) - a)^2*a^4)
```

Mupad [B]
time = 1.01, size = 813, normalized size = 4.42

$$\frac{\tan\left(\frac{x}{2}\right)^4\left(a^2+12\,b^2\right)+\tan\left(\frac{x}{2}\right)^2\left(4\,a^2-32\,b^2\right)-a^2-14\,a\,b\tan\left(\frac{x}{2}\right)+\frac{2\tan\left(\frac{x}{2}\right)^3\left(7\,a^2b-10\,b^3\right)}{a}}{2\,a^5\tan\left(\frac{x}{2}\right)-\tan\left(\frac{x}{2}\right)^3\left(4\,a^5-8\,a^3b^2\right)+2\,a^5\tan\left(\frac{x}{2}\right)^5+8\,a^4b\tan\left(\frac{x}{2}\right)^2-8\,a^4b\tan\left(\frac{x}{2}\right)^4}-\frac{\tan\left(\frac{x}{2}\right)}{2\,a^3}-\frac{3\,b\ln\left(\tan\left(\frac{x}{2}\right)\right)}{a^4}-\frac{\mathrm{atan}\left(\frac{\frac{(a^2+2\,b^2)\sqrt{a^2+b^2}\left(\frac{3\,a^6+12\,a^4b^2}{a^6}+\frac{\tan\left(\frac{x}{2}\right)(12\,a^4b+24\,a^2b^3)}{a^5}-\frac{3\,(a^2+2\,b^2)\left(2\,a^2b+\frac{\tan\left(\frac{x}{2}\right)(6\,a^8+8\,a^6b^2)}{a^5}\right)\sqrt{a^2+b^2}}{2\,(a^6+a^4b^2)}\right)3i}{2\,(a^6+a^4b^2)}+\frac{(a^2+2\,b^2)\sqrt{a^2+b^2}\left(\frac{3\,a^6+12\,a^4b^2}{a^6}+\frac{\tan\left(\frac{x}{2}\right)(12\,a^4b+24\,a^2b^3)}{a^5}+\frac{3\,(a^2+2\,b^2)\left(2\,a^2b+\frac{\tan\left(\frac{x}{2}\right)(6\,a^8+8\,a^6b^2)}{a^5}\right)\sqrt{a^2+b^2}}{2\,(a^6+a^4b^2)}\right)3i}{2\,(a^6+a^4b^2)}}{\frac{2\,(9\,a^2b+18\,b^3)}{a^6}-\frac{2\tan\left(\frac{x}{2}\right)(9\,a^2+18\,b^2)}{a^5}-\frac{3\,(a^2+2\,b^2)\sqrt{a^2+b^2}\left(\frac{3\,a^6+12\,a^4b^2}{a^6}+\frac{\tan\left(\frac{x}{2}\right)(12\,a^4b+24\,a^2b^3)}{a^5}-\frac{3\,(a^2+2\,b^2)\left(2\,a^2b+\frac{\tan\left(\frac{x}{2}\right)(6\,a^8+8\,a^6b^2)}{a^5}\right)\sqrt{a^2+b^2}}{2\,(a^6+a^4b^2)}\right)}{2\,(a^6+a^4b^2)}+\frac{3\,(a^2+2\,b^2)\sqrt{a^2+b^2}\left(\frac{3\,a^6+12\,a^4b^2}{a^6}+\frac{\tan\left(\frac{x}{2}\right)(12\,a^4b+24\,a^2b^3)}{a^5}+\frac{3\,(a^2+2\,b^2)\left(2\,a^2b+\frac{\tan\left(\frac{x}{2}\right)(6\,a^8+8\,a^6b^2)}{a^5}\right)\sqrt{a^2+b^2}}{2\,(a^6+a^4b^2)}\right)}{2\,(a^6+a^4b^2)}}\right)(a^2+2\,b^2)\sqrt{a^2+b^2}\,3i}{a^6+a^4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^2*(a*cos(x) + b*sin(x))^3),x)
```

```
[Out] (tan(x/2)^4*(a^2 + 12*b^2) + tan(x/2)^2*(4*a^2 - 32*b^2) - a^2 - 14*a*b*tan
(x/2) + (2*tan(x/2)^3*(7*a^2*b - 10*b^3))/a)/(2*a^5*tan(x/2) - tan(x/2)^3*(
4*a^5 - 8*a^3*b^2) + 2*a^5*tan(x/2)^5 + 8*a^4*b*tan(x/2)^2 - 8*a^4*b*tan(x/
2)^4) - tan(x/2)/(2*a^3) - (3*b*log(tan(x/2)))/a^4 - (atan((((a^2 + 2*b^2)*
(a^2 + b^2)^(1/2)*((3*a^6 + 12*a^4*b^2)/a^6 + (tan(x/2)*(12*a^4*b + 24*a^2*
b^3))/a^5 - (3*(a^2 + 2*b^2)*(2*a^2*b + (tan(x/2)*(6*a^8 + 8*a^6*b^2))/a^5)
*(a^2 + b^2)^(1/2))/(2*(a^6 + a^4*b^2)))*3i)/(2*(a^6 + a^4*b^2)) + ((a^2 +
2*b^2)*(a^2 + b^2)^(1/2)*((3*a^6 + 12*a^4*b^2)/a^6 + (tan(x/2)*(12*a^4*b +
24*a^2*b^3))/a^5 + (3*(a^2 + 2*b^2)*(2*a^2*b + (tan(x/2)*(6*a^8 + 8*a^6*b^2
))/a^5)*(a^2 + b^2)^(1/2))/(2*(a^6 + a^4*b^2)))*3i)/(2*(a^6 + a^4*b^2)))/((
2*(9*a^2*b + 18*b^3))/a^6 - (2*tan(x/2)*(9*a^2 + 18*b^2))/a^5 - (3*(a^2 + 2
*b^2)*(a^2 + b^2)^(1/2)*((3*a^6 + 12*a^4*b^2)/a^6 + (tan(x/2)*(12*a^4*b + 2
4*a^2*b^3))/a^5 - (3*(a^2 + 2*b^2)*(2*a^2*b + (tan(x/2)*(6*a^8 + 8*a^6*b^2)
)/a^5)*(a^2 + b^2)^(1/2))/(2*(a^6 + a^4*b^2))))/(2*(a^6 + a^4*b^2)) + (3*(a
^2 + 2*b^2)*(a^2 + b^2)^(1/2)*((3*a^6 + 12*a^4*b^2)/a^6 + (tan(x/2)*(12*a^4
*b + 24*a^2*b^3))/a^5 + (3*(a^2 + 2*b^2)*(2*a^2*b + (tan(x/2)*(6*a^8 + 8*a^
6*b^2))/a^5)*(a^2 + b^2)^(1/2))/(2*(a^6 + a^4*b^2))))/(2*(a^6 + a^4*b^2))))
*(a^2 + 2*b^2)*(a^2 + b^2)^(1/2)*3i)/(a^6 + a^4*b^2)
```

3.28 $\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx$

Optimal. Leaf size=117

$$\frac{3b\cot(x)}{a^4}-\frac{\cot^2(x)}{2a^3}+\frac{2(a^2+3b^2)\log(\tan(x))}{a^5}-\frac{2(a^2+3b^2)\log(a+b\tan(x))}{a^5}+\frac{(a^2+b^2)^2}{2a^3b^2(a+b\tan(x))^2}-\frac{(a^2-3}{a^4b^2(a}$$

```
[Out] 3*b*cot(x)/a^4-1/2*cot(x)^2/a^3+2*(a^2+3*b^2)*ln(tan(x))/a^5-2*(a^2+3*b^2)*
ln(a+b*tan(x))/a^5+1/2*(a^2+b^2)^2/a^3/b^2/(a+b*tan(x))^2-(a^2-3*b^2)*(a^2+
b^2)/a^4/b^2/(a+b*tan(x))
```

Rubi [A]
time = 0.10, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3166, 908}

$$\frac{3b\cot(x)}{a^4}-\frac{\cot^2(x)}{2a^3}+\frac{2(a^2+3b^2)\log(\tan(x))}{a^5}-\frac{2(a^2+3b^2)\log(a+b\tan(x))}{a^5}-\frac{(a^2-3b^2)(a^2+b^2)}{a^4b^2(a+b\tan(x))}+\frac{(a^2+b^2)^2}{2a^3b^2(a+b\tan(x))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]^3/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] (3*b*Cot[x])/a^4 - Cot[x]^2/(2*a^3) + (2*(a^2 + 3*b^2)*Log[Tan[x]])/a^5 - (
2*(a^2 + 3*b^2)*Log[a + b*Tan[x]])/a^5 + (a^2 + b^2)^2/(2*a^3*b^2*(a + b*Ta
n[x])^2) - ((a^2 - 3*b^2)*(a^2 + b^2))/(a^4*b^2*(a + b*Tan[x]))
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3166

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[1/d, Subst[Int[x^m*((a + b*x
)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Tan[c + d*x]], x] /; FreeQ[{a, b, c,
 d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n, 0]
 && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx = \text{Subst}\left(\int \frac{(1+x^2)^2}{x^3(a+bx)^3}\,dx, x, \tan(x)\right)$$

$$= \text{Subst}\left(\int \left(\frac{1}{a^3x^3} - \frac{3b}{a^4x^2} + \frac{2(a^2+3b^2)}{a^5x} - \frac{(a^2+b^2)^2}{a^3b(a+bx)^3} + \frac{a^4-2a^2b^2-3b^4}{a^4b(a+bx)^2} - \right.\right.$$

$$= \frac{3b\cot(x)}{a^4} - \frac{\cot^2(x)}{2a^3} + \frac{2(a^2+3b^2)\log(\tan(x))}{a^5} - \frac{2(a^2+3b^2)\log(a+b\tan(x))}{a^5}$$

Mathematica [A]
time = 0.86, size = 208, normalized size = 1.78

$$\frac{6a^3b\cot^3(x)+a^4\csc^2(x)-2ab\cot(x)\left(3a^2+a^2\csc^2(x)-4(a^2+3b^2)\log(\sin(x))+4a^2\log(a\cos(x)+b\sin(x))+12b^2\log(a\cos(x)+b\sin(x))\right)+2b^2\left(-3(a^2+b^2)+2(a^2+3b^2)\log(\sin(x))-2(a^2+3b^2)\log(a\cos(x)+b\sin(x))\right)+\cot^2(x)\left(-a^4\csc^2(x)+4a^2(3b^2+(a^2+3b^2)\log(\sin(x))-(a^2+3b^2)\log(a\cos(x)+b\sin(x)))\right)}{2a^5(b+a\cot(x))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]^3/(a*Cos[x] + b*Sin[x])^3,x]
```

```
[Out] (6*a^3*b*Cot[x]^3 + a^4*Csc[x]^2 - 2*a*b*Cot[x]*(3*a^2 + a^2*Csc[x]^2 - 4*(
a^2 + 3*b^2)*Log[Sin[x]] + 4*a^2*Log[a*Cos[x] + b*Sin[x]] + 12*b^2*Log[a*Co
s[x] + b*Sin[x]]) + 2*b^2*(-3*(a^2 + b^2) + 2*(a^2 + 3*b^2)*Log[Sin[x]] - 2
*(a^2 + 3*b^2)*Log[a*Cos[x] + b*Sin[x]]) + Cot[x]^2*(-(a^4*Csc[x]^2) + 4*a^
2*(3*b^2 + (a^2 + 3*b^2)*Log[Sin[x]] - (a^2 + 3*b^2)*Log[a*Cos[x] + b*Sin[x
]])))/(2*a^5*(b + a*Cot[x])^2)
```

Maple [A]
time = 0.40, size = 128, normalized size = 1.09

method	result
default	$-\frac{1}{2a^3\tan(x)^2}+\frac{(2a^2+6b^2)\ln(\tan(x))}{a^5}+\frac{3b}{a^4\tan(x)}-\frac{-a^4-2a^2b^2-b^4}{2a^3b^2(a+b\tan(x))^2}-\frac{a^4-2a^2b^2-3b^4}{a^4b^2(a+b\tan(x))}-\frac{2(a^2+3b^2)\ln(a+b\tan(x))}{a^5}$
norman	$\frac{\frac{b\tan(\frac{x}{2})}{a^2}+\frac{b(-13a^2-24b^2)(\tan^3(\frac{x}{2}))}{a^4}-\frac{1}{8a}-\frac{\tan^8(\frac{x}{2})}{8a}-\frac{(-9a^4+56a^2b^2+144b^4)(\tan^4(\frac{x}{2}))}{4a^5}-\frac{b(\tan^7(\frac{x}{2}))}{a^2}-\frac{b(-13a^2-24b^2)(\tan^5(\frac{x}{2}))}{a^4}}{\tan(\frac{x}{2})^2(a(\tan^2(\frac{x}{2}))-2b\tan(\frac{x}{2})-a)^2}+$
risch	$\frac{4i(6iab^2+3a^2b-3b^3-9iab^2e^{2ix}+3iab^2e^{6ix}+a^2be^{6ix}-3a^2be^{4ix}+ia^3e^{6ix}+ia^3e^{2ix}-a^2be^{2ix}+3b^3e^{6ix}-9b^3e^{4ix}+9b^3e^{2ix})}{(e^{2ix}-1)^2(be^{2ix}+iae^{2ix}-b+ia)^2a^4}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)^3/(a*cos(x)+b*sin(x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^3/tan(x)^2+(2*a^2+6*b^2)/a^5*ln(tan(x))+3/a^4*b/tan(x)-1/2*(-a^4-2*a
^2*b^2-b^4)/a^3/b^2/(a+b*tan(x))^2-(a^4-2*a^2*b^2-3*b^4)/a^4/b^2/(a+b*tan(x
))-2*(a^2+3*b^2)*ln(a+b*tan(x))/a^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(113) = 226$.
time = 0.30, size = 308, normalized size = 2.63

$$-\frac{a^4 - \frac{8a^3 b\sin(x)}{\cos(x)+1} - \frac{2(a^4+22a^2b^2)\sin(x)^2}{(\cos(x)+1)^2} + \frac{4(21a^3b+4ab^3)\sin(x)^3}{(\cos(x)+1)^3} - \frac{(15a^4-144a^2b^2-112b^4)\sin(x)^4}{(\cos(x)+1)^4} - \frac{4(19a^3b+16ab^3)\sin(x)^5}{(\cos(x)+1)^5}}{8\left(\frac{a^7\sin(x)^2}{(\cos(x)+1)^2} + \frac{4a^6b\sin(x)^3}{(\cos(x)+1)^3} - \frac{4a^6b\sin(x)^5}{(\cos(x)+1)^5} + \frac{a^7\sin(x)^6}{(\cos(x)+1)^6} - \frac{2(a^7-2a^5b^2)\sin(x)^4}{(\cos(x)+1)^4}\right)} - \frac{\frac{12b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}}{8a^4} - \frac{2(a^2+3b^2)\log\left(-a-\frac{2b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^5} + \frac{2(a^2+3b^2)\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a*cos(x)+b*sin(x))^3,x, algorithm="maxima")
```

```
[Out] -1/8*(a^4 - 8*a^3*b*sin(x)/(cos(x) + 1) - 2*(a^4 + 22*a^2*b^2)*sin(x)^2/(co
s(x) + 1)^2 + 4*(21*a^3*b + 4*a*b^3)*sin(x)^3/(cos(x) + 1)^3 - (15*a^4 - 14
4*a^2*b^2 - 112*b^4)*sin(x)^4/(cos(x) + 1)^4 - 4*(19*a^3*b + 16*a*b^3)*sin(
x)^5/(cos(x) + 1)^5)/(a^7*sin(x)^2/(cos(x) + 1)^2 + 4*a^6*b*sin(x)^3/(cos(x
) + 1)^3 - 4*a^6*b*sin(x)^5/(cos(x) + 1)^5 + a^7*sin(x)^6/(cos(x) + 1)^6 -
2*(a^7 - 2*a^5*b^2)*sin(x)^4/(cos(x) + 1)^4) - 1/8*(12*b*sin(x)/(cos(x) + 1
) + a*sin(x)^2/(cos(x) + 1)^2)/a^4 - 2*(a^2 + 3*b^2)*log(-a - 2*b*sin(x)/(c
os(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/a^5 + 2*(a^2 + 3*b^2)*log(sin(x)/(c
os(x) + 1))/a^5
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(113) = 226$.
time = 2.21, size = 385, normalized size = 3.29

$$-\frac{24a^2b^2\cos(x)^4 - a^4 + 6a^2b^2 + 2(a^4-15a^2b^2)\cos(x)^2 - 2\left((a^4+2a^2b^2-3b^4)\cos(x)^4 - a^2b^2 - 3b^4 - (a^4+a^2b^2-6b^4)\cos(x)^2 + 2\left((a^3b+3ab^3)\cos(x)^3 - (a^3b+3ab^3)\cos(x)\right)\sin(x)\right)\log\left(2ab\cos(x)\sin(x) + (a^2-b^2)\cos(x)^2 + b^2\right) + 2\left((a^4+2a^2b^2-3b^4)\cos(x)^4 - a^2b^2 - 3b^4 - (a^4+a^2b^2-6b^4)\cos(x)^2 + 2\left((a^3b+3ab^3)\cos(x)^3 - (a^3b+3ab^3)\cos(x)\right)\sin(x)\right)\log\left(-\frac{1}{4}\cos(x)^2+\frac{1}{4}\right) - 4\left(3(a^3b-ab^3)\cos(x)^3 - (2a^3b-3ab^3)\cos(x)\right)\sin(x)}{2\left(a^5b^2 - (a^7-a^5b^2)\cos(x)^4 + (a^7-2a^5b^2)\cos(x)^2 - 2(a^6b\cos(x)^3 - a^6b\cos(x))\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a*cos(x)+b*sin(x))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(24*a^2*b^2*cos(x)^4 - a^4 + 6*a^2*b^2 + 2*(a^4 - 15*a^2*b^2)*cos(x)^2
 - 2*((a^4 + 2*a^2*b^2 - 3*b^4)*cos(x)^4 - a^2*b^2 - 3*b^4 - (a^4 + a^2*b^2
 - 6*b^4)*cos(x)^2 + 2*((a^3*b + 3*a*b^3)*cos(x)^3 - (a^3*b + 3*a*b^3)*cos(
x))*sin(x))*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) + 2*((a^4
 + 2*a^2*b^2 - 3*b^4)*cos(x)^4 - a^2*b^2 - 3*b^4 - (a^4 + a^2*b^2 - 6*b^4)*
cos(x)^2 + 2*((a^3*b + 3*a*b^3)*cos(x)^3 - (a^3*b + 3*a*b^3)*cos(x))*sin(x)
)*log(-1/4*cos(x)^2 + 1/4) - 4*(3*(a^3*b - a*b^3)*cos(x)^3 - (2*a^3*b - 3*a
*b^3)*cos(x))*sin(x))/(a^5*b^2 - (a^7 - a^5*b^2)*cos(x)^4 + (a^7 - 2*a^5*b^
2)*cos(x)^2 - 2*(a^6*b*cos(x)^3 - a^6*b*cos(x))*sin(x))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(x)}{(a\cos(x)+b\sin(x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)**3/(a*cos(x)+b*sin(x))**3,x)
```

```
[Out] Integral(csc(x)**3/(a*cos(x) + b*sin(x))**3, x)
```

Giac [A]
time = 0.43, size = 146, normalized size = 1.25

$$\frac{2\,(a^2+3\,b^2)\log\left(|\tan\left(x\right)|\right)}{a^5}-\frac{2\,(a^2b+3\,b^3)\log\left(|b\tan\left(x\right)+a|\right)}{a^5b}-\frac{2\,a^4b\tan\left(x\right)^3-4\,a^2b^3\tan\left(x\right)^3-12\,b^5\tan\left(x\right)^3+a^5\tan\left(x\right)^2-6\,a^3b^2\tan\left(x\right)^2-18\,ab^4\tan\left(x\right)^2-4\,a^2b^3\tan\left(x\right)+a^3b^2}{2\,\left(b\tan\left(x\right)^2+a\tan\left(x\right)\right)^2a^4b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)^3/(a*cos(x)+b*sin(x))^3,x, algorithm="giac")
```

```
[Out] 2*(a^2 + 3*b^2)*log(abs(tan(x)))/a^5 - 2*(a^2*b + 3*b^3)*log(abs(b*tan(x) +
 a))/(a^5*b) - 1/2*(2*a^4*b*tan(x)^3 - 4*a^2*b^3*tan(x)^3 - 12*b^5*tan(x)^3
 + a^5*tan(x)^2 - 6*a^3*b^2*tan(x)^2 - 18*a*b^4*tan(x)^2 - 4*a^2*b^3*tan(x)
 + a^3*b^2)/((b*tan(x)^2 + a*tan(x))^2*a^4*b^2)
```

Mupad [B]
time = 0.86, size = 253, normalized size = 2.16

$$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)\,\left(2\,a^2+6\,b^2\right)}{a^5}-\frac{{\tan\left(\frac{x}{2}\right)}^3\,\left(42\,a^2\,b+8\,b^3\right)-{\tan\left(\frac{x}{2}\right)}^5\,\left(38\,a^2\,b+32\,b^3\right)-{\tan\left(\frac{x}{2}\right)}^2\,\left(a^3+22\,a\,b^2\right)+\frac{a^3}{2}+\frac{{\tan\left(\frac{x}{2}\right)}^4\,\left(-15\,a^4+144\,a^2\,b^2+112\,b^4\right)}{2\,a}-4\,a^2\,b\,\tan\left(\frac{x}{2}\right)}{4\,a^6\,{\tan\left(\frac{x}{2}\right)}^2-{\tan\left(\frac{x}{2}\right)}^4\,\left(8\,a^6-16\,a^4\,b^2\right)+4\,a^6\,{\tan\left(\frac{x}{2}\right)}^6+16\,a^5\,b\,{\tan\left(\frac{x}{2}\right)}^3-16\,a^5\,b\,{\tan\left(\frac{x}{2}\right)}^5}-\frac{\ln\left(-a\,{\tan\left(\frac{x}{2}\right)}^2+2\,b\,\tan\left(\frac{x}{2}\right)+a\right)\,\left(2\,a^2+6\,b^2\right)}{a^5}-\frac{{\tan\left(\frac{x}{2}\right)}^2}{8\,a^3}-\frac{3\,b\,\tan\left(\frac{x}{2}\right)}{2\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^3*(a*cos(x) + b*sin(x))^3),x)
```

```
[Out] (log(tan(x/2))*(2*a^2 + 6*b^2))/a^5 - (tan(x/2)^3*(42*a^2*b + 8*b^3) - tan(
x/2)^5*(38*a^2*b + 32*b^3) - tan(x/2)^2*(22*a*b^2 + a^3) + a^3/2 + (tan(x/2
)^4*(112*b^4 - 15*a^4 + 144*a^2*b^2))/(2*a) - 4*a^2*b*tan(x/2))/(4*a^6*tan(
x/2)^2 - tan(x/2)^4*(8*a^6 - 16*a^4*b^2) + 4*a^6*tan(x/2)^6 + 16*a^5*b*tan(
x/2)^3 - 16*a^5*b*tan(x/2)^5) - (log(a + 2*b*tan(x/2) - a*tan(x/2)^2)*(2*a^
2 + 6*b^2))/a^5 - tan(x/2)^2/(8*a^3) - (3*b*tan(x/2))/(2*a^4)
```

3.29 $\int \sin^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx$

Optimal. Leaf size=66

$$-\frac{i\,{}_2F_1\left(1,n;1+n;-\frac{1}{2}i(i+\cot(c+dx))\right)\sin^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n}{2dn}$$

```
[Out] -1/2*I*hypergeom([1, n],[1+n],-1/2*I*(I+cot(d*x+c)))*(a*cos(d*x+c)+I*a*sin(
d*x+c))^n/d/n/(sin(d*x+c)^n)
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = $\{3162\}$

$$-\frac{i\sin^{-n}(c+dx)\,{}_2F_1\left(1,n;n+1;-\frac{1}{2}i(\cot(c+dx)+i)\right)(a\cos(c+dx)+ia\sin(c+dx))^n}{2dn}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n/Sin[c + d*x]^n,x]
```

```
[Out] ((-1/2*I)*Hypergeometric2F1[1, n, 1 + n, (-1/2*I)*(I + Cot[c + d*x])]*(a*Co
s[c + d*x] + I*a*Sin[c + d*x])^n)/(d*n*Sin[c + d*x]^n)
```

Rule 3162

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c
 + d*x])^n/(2*b*d*n*Sin[c + d*x]^n))*Hypergeometric2F1[1, n, n + 1, (b + a*
Cot[c + d*x])/(2*b)], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[m + n, 0] && E
qQ[a^2 + b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \sin^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx = -\frac{i\,{}_2F_1\left(1,n;1+n;-\frac{1}{2}i(i+\cot(c+dx))\right)\sin^{-n}(c+}{2dn}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 3.86, size = 367, normalized size = 5.56

$$-\frac{4\cos\left(\frac{1}{2}(c+dx)\right)\left(F_1\left(1-n;-2n,1;2-n;-i\tan\left(\frac{1}{2}(c+dx)\right),i\tan\left(\frac{1}{2}(c+dx)\right)\right)+{}_2F_1\left(1-2n,1-n;2-n;-i\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin\left(\frac{1}{2}(c+dx)\right)(a(\cos(c+dx)+i\sin(c+dx)))^n\sin^{-n}(c+dx)}{d(-1+n)\left(2F_1\left(1-n;-2n,1;2-n;-i\tan\left(\frac{1}{2}(c+dx)\right),i\tan\left(\frac{1}{2}(c+dx)\right)\right)+\frac{\left(-2nF_1\left(2-n;1-2n,1;3-n;-i\tan\left(\frac{1}{2}(c+dx)\right),i\tan\left(\frac{1}{2}(c+dx)\right)\right)(-1+\cos(c+dx)+i\sin(c+dx))-F_1\left(2-n;-2n,2;3-n;-i\tan\left(\frac{1}{2}(c+dx)\right),i\tan\left(\frac{1}{2}(c+dx)\right)\right)(-1+\cos(c+dx)+i\sin(c+dx))+(-2+n)(1+\cos(c+dx))\left(1+i\tan\left(\frac{1}{2}(c+dx)\right)\right)^{2n}\right)\left(1-i\tan\left(\frac{1}{2}(c+dx)\right)\right)}{-2+n}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n/Sin[c + d*x]^n,x]
```

```
[Out] (-4*Cos[(c + d*x)/2]*(AppellF1[1 - n, -2*n, 1, 2 - n, (-I)*Tan[(c + d*x)/2]
, I*Tan[(c + d*x)/2]] + Hypergeometric2F1[1 - 2*n, 1 - n, 2 - n, (-I)*Tan[(
c + d*x)/2]])*Sin[(c + d*x)/2]*(a*(Cos[c + d*x] + I*Sin[c + d*x]))^n)/(d*(-
1 + n)*Sin[c + d*x]^n*(2*AppellF1[1 - n, -2*n, 1, 2 - n, (-I)*Tan[(c + d*x)
/2], I*Tan[(c + d*x)/2]] + ((-2*n*AppellF1[2 - n, 1 - 2*n, 1, 3 - n, (-I)*T
an[(c + d*x)/2], I*Tan[(c + d*x)/2]]*(-1 + Cos[c + d*x] + I*Sin[c + d*x]) -
 AppellF1[2 - n, -2*n, 2, 3 - n, (-I)*Tan[(c + d*x)/2], I*Tan[(c + d*x)/2]]
*(-1 + Cos[c + d*x] + I*Sin[c + d*x]) + (-2 + n)*(1 + Cos[c + d*x])*(1 + I*
Tan[(c + d*x)/2])^(2*n))*(1 - I*Tan[(c + d*x)/2]))/(-2 + n)))
```

Maple [**F**]
time = 0.16, size = 0, normalized size = 0.00

$$\int \left(a \cos \left(d x+c\right)+i a \sin \left(d x+c\right)\right)^{n} \left(\sin^{-n}\left(d x+c\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(sin(d*x+c)^n),x)
```

```
[Out] int((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(sin(d*x+c)^n),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(sin(d*x+c)^n),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*cos(d*x + c) + I*a*sin(d*x + c))^n*sin(d*x + c)^(-n), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(sin(d*x+c)^n),x, algorithm="fric
as")
```

```
[Out] integral(e^(I*d*n*x + I*c*n + n*log(a))/(1/2*(-I*e^(2*I*d*x + 2*I*c) + I)*e
^(-I*d*x - I*c))^n, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(i\sin (c+dx)+\cos (c+dx)))^n\sin ^{-n}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**n/(sin(d*x+c)**n),x)
```

```
[Out] Integral((a*(I*sin(c + d*x) + cos(c + d*x)))**n/sin(c + d*x)**n, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(sin(d*x+c)^n),x, algorithm="giac
")
```

```
[Out] integrate((a*cos(d*x + c) + I*a*sin(d*x + c))^n/sin(d*x + c)^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{{\left(a\,\cos \left(c+d\,x\right)+a\,\sin \left(c+d\,x\right)\,1{}\mathrm{i}\right)}^n}{{\sin \left(c+d\,x\right)}^n} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^n/sin(c + d*x)^n,x)
```

```
[Out] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^n/sin(c + d*x)^n, x)
```

3.30 $\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=87

$$\frac{5ax}{16}-\frac{b\cos^6(c+dx)}{6d}+\frac{5a\cos(c+dx)\sin(c+dx)}{16d}+\frac{5a\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{a\cos^5(c+dx)\sin(c+dx)}{6d}$$

```
[Out] 5/16*a*x-1/6*b*cos(d*x+c)^6/d+5/16*a*cos(d*x+c)*sin(d*x+c)/d+5/24*a*cos(d*x
+c)^3*sin(d*x+c)/d+1/6*a*cos(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3169, 2715, 8, 2645, 30}

$$\frac{a\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a\sin(c+dx)\cos(c+dx)}{16d}+\frac{5ax}{16}-\frac{b\cos^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (5*a*x)/16 - (b*Cos[c + d*x]^6)/(6*d) + (5*a*Cos[c + d*x]*Sin[c + d*x])/(16
*d) + (5*a*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*Cos[c + d*x]^5*Sin[c +
d*x])/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx &= \int \left(a\cos^6(c+dx)+b\cos^5(c+dx)\sin(c+dx)\right)\,dx \\
&= a\int \cos^6(c+dx)\,dx+b\int \cos^5(c+dx)\sin(c+dx)\,dx \\
&= \frac{a\cos^5(c+dx)\sin(c+dx)}{6d}+\frac{1}{6}(5a)\int \cos^4(c+dx)\,dx - \\
&= -\frac{b\cos^6(c+dx)}{6d}+\frac{5a\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{a\cos^5}{} \\
&= -\frac{b\cos^6(c+dx)}{6d}+\frac{5a\cos(c+dx)\sin(c+dx)}{16d}+\frac{5a\cos^3}{} \\
&= \frac{5ax}{16}-\frac{b\cos^6(c+dx)}{6d}+\frac{5a\cos(c+dx)\sin(c+dx)}{16d}+\frac{5a}{}
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 57, normalized size = 0.66

$$\frac{-32b\cos^6(c+dx)+a(60c+60dx+45\sin(2(c+dx))+9\sin(4(c+dx))+\sin(6(c+dx)))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
[Out] (-32*b*Cos[c + d*x]^6 + a*(60*c + 60*d*x + 45*Sin[2*(c + d*x)] + 9*Sin[4*(c
 + d*x)] + Sin[6*(c + d*x)]))/(192*d)
```

Maple [A]
time = 0.22, size = 62, normalized size = 0.71

method	result
derivativedivides	$\frac{a\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)-\frac{b\left(\cos^6(dx+c)\right)}{6}}{d}$

default	$\frac{a\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)-\frac{b\left(\cos^6(dx+c)\right)}{6}}{d}$
risch	$\frac{5ax}{16}-\frac{b\cos(6dx+6c)}{192d}+\frac{a\sin(6dx+6c)}{192d}-\frac{b\cos(4dx+4c)}{32d}+\frac{3a\sin(4dx+4c)}{64d}-\frac{5b\cos(2dx+2c)}{64d}+\frac{15a\sin(2dx+2c)}{64d}$
norman	$\frac{5ax}{16}+\frac{11a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d}-\frac{5a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}+\frac{15a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{15a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{5a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{11a\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16
*d*x+5/16*c)-1/6*b*cos(d*x+c)^6)
```

Maxima [A]
time = 0.30, size = 62, normalized size = 0.71

$$-\frac{32\,b\cos\left(dx+c\right)^6+\left(4\,\sin\left(2\,dx+2\,c\right)^3-60\,dx-60\,c-9\,\sin\left(4\,dx+4\,c\right)-48\,\sin\left(2\,dx+2\,c\right)\right)a}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/192*(32*b*cos(d*x + c)^6 + (4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin
(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a)/d
```

Fricas [A]
time = 2.12, size = 62, normalized size = 0.71

$$-\frac{8\,b\cos\left(dx+c\right)^6-15\,adx-\left(8\,a\cos\left(dx+c\right)^5+10\,a\cos\left(dx+c\right)^3+15\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/48*(8*b*cos(d*x + c)^6 - 15*a*d*x - (8*a*cos(d*x + c)^5 + 10*a*cos(d*x +
 c)^3 + 15*a*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(82) = 164$.
time = 0.44, size = 175, normalized size = 2.01

$$\begin{cases} \frac{5ax\sin^6(c+dx)}{16}+\frac{15ax\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{15ax\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{5ax\cos^6(c+dx)}{16}+\frac{5a\sin^5(c+dx)\cos(c+dx)}{16d}+\frac{5a\sin^3(c+dx)\cos^3(c+dx)}{6d}+\frac{11a\sin(c+dx)\cos^5(c+dx)}{16d}-\frac{b\cos^6(c+dx)}{6d} & \text{for}\ d\neq 0 \\ x(a\cos(c)+b\sin(c))\cos^5(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((5*a*x*sin(c + d*x)**6/16 + 15*a*x*sin(c + d*x)**4*cos(c + d*x)**
2/16 + 15*a*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a*x*cos(c + d*x)**6/16
 + 5*a*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a*sin(c + d*x)**3*cos(c + d*
x)**3/(6*d) + 11*a*sin(c + d*x)*cos(c + d*x)**5/(16*d) - b*cos(c + d*x)**6/
(6*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))*cos(c)**5, True))
```

Giac [A]
time = 0.45, size = 95, normalized size = 1.09

$$\frac{5}{16}\,ax-\frac{b\cos\left(6\,dx+6\,c\right)}{192\,d}-\frac{b\cos\left(4\,dx+4\,c\right)}{32\,d}-\frac{5\,b\cos\left(2\,dx+2\,c\right)}{64\,d}+\frac{a\sin\left(6\,dx+6\,c\right)}{192\,d}+\frac{3\,a\sin\left(4\,dx+4\,c\right)}{64\,d}+\frac{15\,a\sin\left(2\,dx+2\,c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 5/16*a*x - 1/192*b*cos(6*d*x + 6*c)/d - 1/32*b*cos(4*d*x + 4*c)/d - 5/64*b*
cos(2*d*x + 2*c)/d + 1/192*a*sin(6*d*x + 6*c)/d + 3/64*a*sin(4*d*x + 4*c)/d
 + 15/64*a*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 4.20, size = 149, normalized size = 1.71

$$\frac{5\,a\,x}{16}+\frac{-\frac{11\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}-\frac{15\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{4}+\frac{20\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{3}+\frac{15\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{4}-\frac{5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24}+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+\frac{11\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

```
[Out] (5*a*x)/16 + ((11*a*tan(c/2 + (d*x)/2))/8 - (5*a*tan(c/2 + (d*x)/2)^3)/24 +
 (15*a*tan(c/2 + (d*x)/2)^5)/4 - (15*a*tan(c/2 + (d*x)/2)^7)/4 + (5*a*tan(c
/2 + (d*x)/2)^9)/24 - (11*a*tan(c/2 + (d*x)/2)^11)/8 + 2*b*tan(c/2 + (d*x)/
2)^2 + (20*b*tan(c/2 + (d*x)/2)^6)/3 + 2*b*tan(c/2 + (d*x)/2)^10)/(d*(tan(c
/2 + (d*x)/2)^2 + 1)^6)
```

3.31 $\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=60

$$-\frac{b\cos^5(c+dx)}{5d}+\frac{a\sin(c+dx)}{d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin^5(c+dx)}{5d}$$

```
[Out] -1/5*b*cos(d*x+c)^5/d+a*sin(d*x+c)/d-2/3*a*sin(d*x+c)^3/d+1/5*a*sin(d*x+c)^
5/d
```

Rubi [A]
time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {3169, 2713, 2645, 30}

$$\frac{a\sin^5(c+dx)}{5d}-\frac{2a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] -1/5*(b*Cos[c + d*x]^5)/d + (a*Sin[c + d*x])/d - (2*a*Sin[c + d*x]^3)/(3*d)
 + (a*Sin[c + d*x]^5)/(5*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
```

```
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \int \left(a\cos^5(c+dx)+b\cos^4(c+dx)\sin(c+dx)\right)\,dx$$
$$= a\int \cos^5(c+dx)\,dx + b\int \cos^4(c+dx)\sin(c+dx)\,dx$$
$$= -\frac{a\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, -\sin(c+dx)\right)}{d} - \frac{b\text{Su}}{}$$
$$= -\frac{b\cos^5(c+dx)}{5d} + \frac{a\sin(c+dx)}{d} - \frac{2a\sin^3(c+dx)}{3d} + \frac{a}{}$$

Mathematica [A]
time = 0.02, size = 63, normalized size = 1.05

$$-\frac{b\cos^5(c+dx)}{5d} + \frac{5a\sin(c+dx)}{8d} + \frac{5a\sin(3(c+dx))}{48d} + \frac{a\sin(5(c+dx))}{80d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] -1/5*(b*Cos[c + d*x]^5)/d + (5*a*Sin[c + d*x])/(8*d) + (5*a*Sin[3*(c + d*x)
])/(48*d) + (a*Sin[5*(c + d*x)])/(80*d)
```

Maple [A]
time = 0.17, size = 46, normalized size = 0.77

method	result
derivativedivides	$\frac{\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}-\frac{b\left(\cos^5(dx+c)\right)}{5}}{d}$
default	$\frac{\frac{a\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}-\frac{b\left(\cos^5(dx+c)\right)}{5}}{d}$
risch	$-\frac{b\cos(dx+c)}{8d}+\frac{5a\sin(dx+c)}{8d}-\frac{b\cos(5dx+5c)}{80d}+\frac{a\sin(5dx+5c)}{80d}-\frac{b\cos(3dx+3c)}{16d}+\frac{5a\sin(3dx+3c)}{48d}$
norman	$\frac{-\frac{2b}{5d}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{8a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{116a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{8a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)-1/5*b*cos(d*x+c)^
5)
```

Maxima [A]
time = 0.28, size = 49, normalized size = 0.82

$$-\frac{3\,b\cos\left(dx+c\right)^{5}-\left(3\,\sin\left(dx+c\right)^{5}-10\,\sin\left(dx+c\right)^{3}+15\,\sin\left(dx+c\right)\right)a}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/15*(3*b*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(
d*x + c))*a)/d
```

Fricas [A]
time = 2.51, size = 51, normalized size = 0.85

$$-\frac{3\,b\cos\left(dx+c\right)^{5}-\left(3\,a\cos\left(dx+c\right)^{4}+4\,a\cos\left(dx+c\right)^{2}+8\,a\right)\sin\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/15*(3*b*cos(d*x + c)^5 - (3*a*cos(d*x + c)^4 + 4*a*cos(d*x + c)^2 + 8*a)
*sin(d*x + c))/d
```

Sympy [A]
time = 0.25, size = 87, normalized size = 1.45

$$\begin{cases} \frac{8a\sin^{5}\left(c+dx\right)}{15d}+\frac{4a\sin^{3}\left(c+dx\right)\cos^{2}\left(c+dx\right)}{3d}+\frac{a\sin\left(c+dx\right)\cos^{4}\left(c+dx\right)}{d}-\frac{b\cos^{5}\left(c+dx\right)}{5d} & \text{for}\ d\neq 0 \\ x\left(a\cos\left(c\right)+b\sin\left(c\right)\right)\cos^{4}\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Piecewise(((8*a*sin(c + d*x)**5/(15*d) + 4*a*sin(c + d*x)**3*cos(c + d*x)**2
/(3*d) + a*sin(c + d*x)*cos(c + d*x)**4/d - b*cos(c + d*x)**5/(5*d), Ne(d,
0)), (x*(a*cos(c) + b*sin(c))*cos(c)**4, True))
```

Giac [A]
time = 0.44, size = 85, normalized size = 1.42

$$-\frac{b\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{b\cos\left(3\,dx+3\,c\right)}{16\,d}-\frac{b\cos\left(dx+c\right)}{8\,d}+\frac{a\sin\left(5\,dx+5\,c\right)}{80\,d}+\frac{5\,a\sin\left(3\,dx+3\,c\right)}{48\,d}+\frac{5\,a\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/80*b*cos(5*d*x + 5*c)/d - 1/16*b*cos(3*d*x + 3*c)/d - 1/8*b*cos(d*x + c)
/d + 1/80*a*sin(5*d*x + 5*c)/d + 5/48*a*sin(3*d*x + 3*c)/d + 5/8*a*sin(d*x
+ c)/d
```

Mupad [B]
time = 0.46, size = 67, normalized size = 1.12

$$\frac{8\,a\,\sin\left(c+d\,x\right)}{15\,d}-\frac{b\cos\left(c+d\,x\right)^5}{5\,d}+\frac{4\,a\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{15\,d}+\frac{a\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

```
[Out] (8*a*sin(c + d*x))/(15*d) - (b*cos(c + d*x)^5)/(5*d) + (4*a*cos(c + d*x)^2*
sin(c + d*x))/(15*d) + (a*cos(c + d*x)^4*sin(c + d*x))/(5*d)
```

3.32 $\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=65

$$\frac{3ax}{8}-\frac{b\cos^4(c+dx)}{4d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(c+dx)\sin(c+dx)}{4d}$$

```
[Out] 3/8*a*x-1/4*b*cos(d*x+c)^4/d+3/8*a*cos(d*x+c)*sin(d*x+c)/d+1/4*a*cos(d*x+c)
^3*sin(d*x+c)/d
```

Rubi [A]
time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3169, 2715, 8, 2645, 30}

$$\frac{a\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}-\frac{b\cos^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (3*a*x)/8 - (b*Cos[c + d*x]^4)/(4*d) + (3*a*Cos[c + d*x]*Sin[c + d*x])/(8*d
) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx &= \int \left(a\cos^4(c+dx)+b\cos^3(c+dx)\sin(c+dx)\right)\,dx\\ &= a\int \cos^4(c+dx)\,dx+b\int \cos^3(c+dx)\sin(c+dx)\,dx\\ &= \frac{a\cos^3(c+dx)\sin(c+dx)}{4d}+\frac{1}{4}(3a)\int \cos^2(c+dx)\,dx -\\ &= -\frac{b\cos^4(c+dx)}{4d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a\cos^3(\\ &= \frac{3ax}{8}-\frac{b\cos^4(c+dx)}{4d}+\frac{3a\cos(c+dx)\sin(c+dx)}{8d}+\frac{a}{}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 62, normalized size = 0.95

$$\frac{3a(c+dx)}{8d}-\frac{b\cos^4(c+dx)}{4d}+\frac{a\sin(2(c+dx))}{4d}+\frac{a\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
[Out] (3*a*(c + d*x))/(8*d) - (b*Cos[c + d*x]^4)/(4*d) + (a*Sin[2*(c + d*x)])/(4*
d) + (a*Sin[4*(c + d*x)])/(32*d)
```

Maple [A]
time = 0.15, size = 52, normalized size = 0.80

method	result
derivativedivides	$\frac{a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{b\left(\cos^4(dx+c)\right)}{4}}{d}$
default	$\frac{a\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{b\left(\cos^4(dx+c)\right)}{4}}{d}$

risch	$\frac{3ax}{8}-\frac{b\cos(4dx+4c)}{32d}+\frac{a\sin(4dx+4c)}{32d}-\frac{b\cos(2dx+2c)}{8d}+\frac{a\sin(2dx+2c)}{4d}$
norman	$\frac{\frac{3ax}{8}+\frac{5a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{3a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{3a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{5a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{3ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{9ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4}+}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)-1/4*b*c
os(d*x+c)^4)
```

Maxima [A]
time = 0.28, size = 48, normalized size = 0.74

$$-\frac{8\,b\cos\left(dx+c\right)^4-\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/32*(8*b*cos(d*x + c)^4 - (12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x
 + 2*c))*a)/d
```

Fricas [A]
time = 3.12, size = 51, normalized size = 0.78

$$-\frac{2\,b\cos\left(dx+c\right)^4-3\,adx-\left(2\,a\cos\left(dx+c\right)^3+3\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/8*(2*b*cos(d*x + c)^4 - 3*a*d*x - (2*a*cos(d*x + c)^3 + 3*a*cos(d*x + c)
)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(60) = 120$.
time = 0.18, size = 128, normalized size = 1.97

$$\begin{cases}\frac{3ax\sin^4\left(c+dx\right)}{8}+\frac{3ax\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{4}+\frac{3ax\cos^4\left(c+dx\right)}{8}+\frac{3a\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{8d}+\frac{5a\sin\left(c+dx\right)\cos^3\left(c+dx\right)}{8d}-\frac{b\cos^4\left(c+dx\right)}{4d} & \text{for}\ d\neq 0\\ x\left(a\cos\left(c\right)+b\sin\left(c\right)\right)\cos^3\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**4/8 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + 3*a*x*cos(c + d*x)**4/8 + 3*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*a*
sin(c + d*x)*cos(c + d*x)**3/(8*d) - b*cos(c + d*x)**4/(4*d), Ne(d, 0)), (x
*(a*cos(c) + b*sin(c))*cos(c)**3, True))
```

Giac [A]
time = 0.43, size = 65, normalized size = 1.00

$$\frac{3}{8}\,ax - \frac{b\cos\left(4\,dx + 4\,c\right)}{32\,d} - \frac{b\cos\left(2\,dx + 2\,c\right)}{8\,d} + \frac{a\sin\left(4\,dx + 4\,c\right)}{32\,d} + \frac{a\sin\left(2\,dx + 2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 3/8*a*x - 1/32*b*cos(4*d*x + 4*c)/d - 1/8*b*cos(2*d*x + 2*c)/d + 1/32*a*sin
(4*d*x + 4*c)/d + 1/4*a*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 4.11, size = 107, normalized size = 1.65

$$\frac{3\,a\,x}{8} + \frac{-\frac{5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4} + 2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 + \frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4} - \frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4} + 2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + \frac{5\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

```
[Out] (3*a*x)/8 + ((5*a*tan(c/2 + (d*x)/2))/4 - (3*a*tan(c/2 + (d*x)/2)^3)/4 + (3
*a*tan(c/2 + (d*x)/2)^5)/4 - (5*a*tan(c/2 + (d*x)/2)^7)/4 + 2*b*tan(c/2 + (
d*x)/2)^2 + 2*b*tan(c/2 + (d*x)/2)^6)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.33 $\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=44

$$-\frac{b\cos^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{a\sin^3(c+dx)}{3d}$$

```
[Out] -1/3*b*cos(d*x+c)^3/d+a*sin(d*x+c)/d-1/3*a*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3169, 2713, 2645, 30}

$$-\frac{a\sin^3(c+dx)}{3d}+\frac{a\sin(c+dx)}{d}-\frac{b\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] -1/3*(b*Cos[c + d*x]^3)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \int \left(a\cos^3(c+dx)+b\cos^2(c+dx)\sin(c+dx)\right)\,dx$$
$$= a\int \cos^3(c+dx)\,dx + b\int \cos^2(c+dx)\sin(c+dx)\,dx$$
$$= -\frac{a\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(c+dx)\right)}{d} - \frac{b\text{Subst}\left(\int \right.}{}$$
$$= -\frac{b\cos^3(c+dx)}{3d} + \frac{a\sin(c+dx)}{d} - \frac{a\sin^3(c+dx)}{3d}$$

Mathematica [A]
time = 0.02, size = 44, normalized size = 1.00

$$-\frac{b\cos^3(c+dx)}{3d} + \frac{a\sin(c+dx)}{d} - \frac{a\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
[Out] -1/3*(b*Cos[c + d*x]^3)/d + (a*Sin[c + d*x])/d - (a*Sin[c + d*x]^3)/(3*d)
```

Maple [A]
time = 0.13, size = 36, normalized size = 0.82

method	result	size
derivativedivides	$\frac{\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}-\frac{b\left(\cos^3(dx+c)\right)}{3}}{d}$	36
default	$\frac{\frac{a\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}-\frac{b\left(\cos^3(dx+c)\right)}{3}}{d}$	36
risch	$-\frac{b\cos(dx+c)}{4d}+\frac{3a\sin(dx+c)}{4d}-\frac{b\cos(3dx+3c)}{12d}+\frac{a\sin(3dx+3c)}{12d}$	56
norman	$\frac{-\frac{2b}{3d}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{4a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$	90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/3*a*(2+cos(d*x+c)^2)*sin(d*x+c)-1/3*b*cos(d*x+c)^3)
```

Maxima [A]
time = 0.29, size = 35, normalized size = 0.80

$$-\frac{b\cos\left(dx+c\right)^3+\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/3*(b*cos(d*x + c)^3 + (sin(d*x + c)^3 - 3*sin(d*x + c))*a)/d
```

Fricas [A]
time = 3.31, size = 38, normalized size = 0.86

$$-\frac{b\cos\left(dx+c\right)^{3}-\left(a\cos\left(dx+c\right)^{2}+2\,a\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(b*cos(d*x + c)^3 - (a*cos(d*x + c)^2 + 2*a)*sin(d*x + c))/d
```

Sympy [A]
time = 0.11, size = 63, normalized size = 1.43

$$\begin{cases}\frac{2a\sin^{3}\left(c+dx\right)}{3d}+\frac{a\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)}{d}-\frac{b\cos^{3}\left(c+dx\right)}{3d} & \text{for}\: d\neq 0\\ x\left(a\cos\left(c\right)+b\sin\left(c\right)\right)\cos^{2}\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((2*a*sin(c + d*x)**3/(3*d) + a*sin(c + d*x)*cos(c + d*x)**2/d - b
*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))*cos(c)**2, True
))
```

Giac [A]
time = 0.40, size = 55, normalized size = 1.25

$$-\frac{b\cos\left(3\,dx+3\,c\right)}{12\,d}-\frac{b\cos\left(dx+c\right)}{4\,d}+\frac{a\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{3\,a\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*b*cos(3*d*x + 3*c)/d - 1/4*b*cos(d*x + c)/d + 1/12*a*sin(3*d*x + 3*c)
/d + 3/4*a*sin(d*x + c)/d
```

Mupad [B]
time = 0.44, size = 47, normalized size = 1.07

$$\frac{2\,a\,\sin\left(c+d\,x\right)}{3\,d}-\frac{b\cos\left(c+d\,x\right)^{3}}{3\,d}+\frac{a\cos\left(c+d\,x\right)^{2}\,\sin\left(c+d\,x\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

```
[Out] (2*a*sin(c + d*x))/(3*d) - (b*cos(c + d*x)^3)/(3*d) + (a*cos(c + d*x)^2*sin
(c + d*x))/(3*d)
```

3.34 $\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=43

$$\frac{ax}{2}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{b\sin^2(c+dx)}{2d}$$

```
[Out] 1/2*a*x+1/2*a*cos(d*x+c)*sin(d*x+c)/d+1/2*b*sin(d*x+c)^2/d
```

Rubi [A]
time = 0.03, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {3169, 2715, 8, 2644, 30}

$$\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}+\frac{b\sin^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (a*x)/2 + (a*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (b*Sin[c + d*x]^2)/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx &= \int \left(a\cos^2(c+dx)+b\cos(c+dx)\sin(c+dx)\right)\,dx\\ &= a\int \cos^2(c+dx)\,dx+b\int \cos(c+dx)\sin(c+dx)\,dx\\ &= \frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}a\int 1\,dx+\frac{b\text{Subst}(\int x\,dx,}{d}\\ &= \frac{ax}{2}+\frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{b\sin^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 46, normalized size = 1.07

$$\frac{a(c+dx)}{2d}-\frac{b\cos^2(c+dx)}{2d}+\frac{a\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
[Out] (a*(c + d*x))/(2*d) - (b*Cos[c + d*x]^2)/(2*d) + (a*Sin[2*(c + d*x)])/(4*d)
```

Maple [A]
time = 0.09, size = 41, normalized size = 0.95

method	result	size
risch	$\frac{ax}{2}-\frac{b\cos(2dx+2c)}{4d}+\frac{a\sin(2dx+2c)}{4d}$	36
derivativedivides	$\frac{a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{\left(\cos^2(dx+c)\right)b}{2}}{d}$	41
default	$\frac{a\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{\left(\cos^2(dx+c)\right)b}{2}}{d}$	41
norman	$\frac{\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{ax}{2}-\frac{a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-1/2*cos(d*x+c)^2*b)
```

Maxima [A]
time = 0.27, size = 37, normalized size = 0.86

$$-\frac{2\,b\cos\left(dx+c\right)^{2}-\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(2*b*cos(d*x + c)^2 - (2*d*x + 2*c + sin(2*d*x + 2*c))*a)/d
```

Fricas [A]
time = 4.57, size = 35, normalized size = 0.81

$$\frac{adx-b\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*x - b*cos(d*x + c)^2 + a*cos(d*x + c)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(36) = 72$.
time = 0.08, size = 73, normalized size = 1.70

$$\begin{cases}\frac{ax\sin^{2}\left(c+dx\right)}{2}+\frac{ax\cos^{2}\left(c+dx\right)}{2}+\frac{a\sin\left(c+dx\right)\cos\left(c+dx\right)}{2d}+\frac{b\sin^{2}\left(c+dx\right)}{2d} & \text{for}\ d\neq 0\\ x\left(a\cos\left(c\right)+b\sin\left(c\right)\right)\cos\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 + a*sin(c + d*x)*c
os(c + d*x)/(2*d) + b*sin(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a*cos(c) + b*si
n(c))*cos(c), True))
```

Giac [A]
time = 0.41, size = 35, normalized size = 0.81

$$\frac{1}{2}\,ax-\frac{b\cos\left(2\,dx+2\,c\right)}{4\,d}+\frac{a\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*a*x - 1/4*b*cos(2*d*x + 2*c)/d + 1/4*a*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 0.43, size = 35, normalized size = 0.81

$$\frac{a\,x}{2}-\frac{b\,\cos\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{a\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

```
[Out] (a*x)/2 - (b*cos(2*c + 2*d*x))/(4*d) + (a*sin(2*c + 2*d*x))/(4*d)
```

3.35 $\int (a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=24

$$-\frac{b\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d}$$

```
[Out] -b*cos(d*x+c)/d+a*sin(d*x+c)/d
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2717, 2718}

$$\frac{a\sin(c+dx)}{d}-\frac{b\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a*Cos[c + d*x] + b*Sin[c + d*x],x]
```

```
[Out] -((b*Cos[c + d*x])/d) + (a*Sin[c + d*x])/d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\cos(c+dx)+b\sin(c+dx))\,dx &= a\int \cos(c+dx)\,dx+b\int \sin(c+dx)\,dx\\ &= -\frac{b\cos(c+dx)}{d}+\frac{a\sin(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 46, normalized size = 1.92

$$-\frac{b\cos(c)\cos(dx)}{d}+\frac{a\cos(dx)\sin(c)}{d}+\frac{a\cos(c)\sin(dx)}{d}+\frac{b\sin(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[a*Cos[c + d*x] + b*Sin[c + d*x],x]

[Out] -((b*Cos[c]*Cos[d*x])/d) + (a*Cos[d*x]*Sin[c])/d + (a*Cos[c]*Sin[d*x])/d + (b*Sin[c]*Sin[d*x])/d

Maple [A]
time = 0.07, size = 25, normalized size = 1.04

method	result	size
derivativedivides	$\frac{-b\cos(dx+c)+a\sin(dx+c)}{d}$	23
default	$-\frac{b\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$	25
risch	$-\frac{b\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$	25
norman	$\frac{\frac{2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$	50
meijerg	$\frac{\left(\cos(c)\sqrt{\pi}\,a+\sqrt{\pi}\,\sin(c)b\right)\sin(dx)}{\sqrt{\pi}\,d}+\frac{\left(\cos(c)\sqrt{\pi}\,b-\sqrt{\pi}\,\sin(c)a\right)\left(\frac{1}{\sqrt{\pi}}-\frac{\cos(dx)}{\sqrt{\pi}}\right)}{d}$	61

Verification of antiderivative is not currently implemented for this CAS.

[In] int(a*cos(d*x+c)+b*sin(d*x+c),x,method=_RETURNVERBOSE)

[Out] -b*cos(d*x+c)/d+a*sin(d*x+c)/d

Maxima [A]
time = 0.27, size = 24, normalized size = 1.00

$$-\frac{b\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*cos(d*x+c)+b*sin(d*x+c),x, algorithm="maxima")

[Out] -b*cos(d*x + c)/d + a*sin(d*x + c)/d

Fricas [A]
time = 2.85, size = 23, normalized size = 0.96

$$-\frac{b\cos(dx+c)-a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(a*cos(d*x+c)+b*sin(d*x+c),x, algorithm="fricas")

[Out] -(b*cos(d*x + c) - a*sin(d*x + c))/d

Sympy [A]
time = 0.05, size = 31, normalized size = 1.29

$$a\left(\begin{cases}\frac{\sin(c+dx)}{d} & \text{for } d\neq 0\\ x\cos(c) & \text{otherwise}\end{cases}\right)+b\left(\begin{cases}-\frac{\cos(c+dx)}{d} & \text{for } d\neq 0\\ x\sin(c) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+b*sin(d*x+c),x)
```

```
[Out] a*Piecewise((sin(c + d*x)/d, Ne(d, 0)), (x*cos(c), True)) + b*Piecewise((-c
os(c + d*x)/d, Ne(d, 0)), (x*sin(c), True))
```

Giac [A]
time = 0.42, size = 24, normalized size = 1.00

$$-\frac{b\cos(dx+c)}{d}+\frac{a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*cos(d*x+c)+b*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -b*cos(d*x + c)/d + a*sin(d*x + c)/d
```

Mupad [B]
time = 0.36, size = 38, normalized size = 1.58

$$-\frac{2\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\left(b\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-a\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*cos(c + d*x) + b*sin(c + d*x),x)
```

```
[Out] -(2*cos(c/2 + (d*x)/2)*(b*cos(c/2 + (d*x)/2) - a*sin(c/2 + (d*x)/2)))/d
```

3.36 $\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=17

$$ax-\frac{b\log(\cos(c+dx))}{d}$$

```
[Out] a*x-b*ln(cos(d*x+c))/d
```

Rubi [A]
time = 0.02, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {3165, 3556}

$$ax-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] a*x - (b*Log[Cos[c + d*x]])/d
```

Rule 3165

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx &= \int (a+b\tan(c+dx))\,dx\\ &= ax+b\int \tan(c+dx)\,dx\\ &= ax-\frac{b\log(\cos(c+dx))}{d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 17, normalized size = 1.00

$$ax-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
[Out] a*x - (b*Log[Cos[c + d*x]])/d
```

Maple [A]
time = 0.14, size = 23, normalized size = 1.35

method	result	size
derivativedivides	$\frac{a(dx+c)-b\ln(\cos(dx+c))}{d}$	23
default	$\frac{a(dx+c)-b\ln(\cos(dx+c))}{d}$	23
risch	$ibx+ax+\frac{2ibc}{d}-\frac{b\ln\left(\mathrm{e}^{2i(dx+c)}+1\right)}{d}$	36
norman	$\frac{ax+ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{b\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*(d*x+c)-b*ln(cos(d*x+c)))
```

Maxima [A]
time = 0.29, size = 30, normalized size = 1.76

$$\frac{2\,(dx+c)a-b\log\left(-\sin\left(dx+c\right)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
[Out] 1/2*(2*(d*x + c)*a - b*log(-sin(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 3.05, size = 21, normalized size = 1.24

$$\frac{adx-b\log\left(-\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
[Out] (a*d*x - b*log(-cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))*sec(c + d*x), x)
```

Giac [A]
time = 0.42, size = 27, normalized size = 1.59

$$\frac{2\,(dx+c)a+b\log\left(\tan\left(dx+c\right)^{2}+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*a + b*log(tan(d*x + c)^2 + 1))/d
```

Mupad [B]
time = 0.57, size = 70, normalized size = 4.12

$$\frac{b\,\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}}\right)}{d}+\frac{2\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{b\,\ln\left(\frac{\cos(c+d\,x)}{\cos(c+d\,x)+1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))/cos(c + d*x),x)
```

```
[Out] (b*log(1/cos(c/2 + (d*x)/2)^2))/d + (2*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2)))/d - (b*log(cos(c + d*x)/(cos(c + d*x) + 1)))/d
```

3.37 $\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=24

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{b\sec(c+dx)}{d}$$

```
[Out] a*arctanh(sin(d*x+c))/d+b*sec(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3169, 3855, 2686, 8}

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d}+\frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (b*Sec[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx &= \int (a\sec(c+dx)+b\sec(c+dx)\tan(c+dx))\,dx\\ &= a\int \sec(c+dx)\,dx + b\int \sec(c+dx)\tan(c+dx)\,dx\\ &= \frac{a\tanh^{-1}(\sin(c+dx))}{d} + \frac{b\text{Subst}(\int 1\,dx, x, \sec(c+dx))}{d}\\ &= \frac{a\tanh^{-1}(\sin(c+dx))}{d} + \frac{b\sec(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 24, normalized size = 1.00

$$\frac{a\tanh^{-1}(\sin(c+dx))}{d} + \frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
[Out] (a*ArcTanh[Sin[c + d*x]])/d + (b*Sec[c + d*x])/d
```

Maple [A]
time = 0.16, size = 32, normalized size = 1.33

method	result	size
derivativedivides	$\frac{a\ln(\sec(dx+c)+\tan(dx+c))+\frac{b}{\cos(dx+c)}}{d}$	32
default	$\frac{a\ln(\sec(dx+c)+\tan(dx+c))+\frac{b}{\cos(dx+c)}}{d}$	32
risch	$\frac{2b\,e^{i(dx+c)}}{d(e^{2i(dx+c)}+1)} + \frac{a\ln(e^{i(dx+c)}+i)}{d} - \frac{a\ln(e^{i(dx+c)}-i)}{d}$	67
norman	$\frac{-\frac{2b}{d}-\frac{2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)} + \frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d} - \frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$	92

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*ln(sec(d*x+c)+tan(d*x+c))+b/cos(d*x+c))
```

Maxima [A]
time = 0.28, size = 40, normalized size = 1.67

$$\frac{a(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+\frac{2\,b}{\cos(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(a*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 2*b/cos(d*x + c))/
d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(24) = 48$.
time = 2.93, size = 54, normalized size = 2.25

$$\frac{a\cos(dx+c)\log(\sin(dx+c)+1)-a\cos(dx+c)\log(-\sin(dx+c)+1)+2\,b}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x + c
) + 1) + 2*b)/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(c+dx)+b\sin(c+dx))\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))*sec(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(24) = 48$.
time = 0.42, size = 54, normalized size = 2.25

$$\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,b}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) - 2*b/(tan(1/2*d*x + 1/2*c)^2 - 1))/d
```

Mupad [B]
time = 0.41, size = 38, normalized size = 1.58

$$\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{2\,b}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))/cos(c + d*x)^2,x)
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)))/d - (2*b)/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.38 $\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=28

$$\frac{b\sec^2(c+dx)}{2d}+\frac{a\tan(c+dx)}{d}$$

```
[Out] 1/2*b*sec(d*x+c)^2/d+a*tan(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.192$, Rules used = {3169, 3852, 8, 2686, 30}

$$\frac{a\tan(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^2)/(2*d) + (a*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx &= \int \left(a\sec^2(c+dx)+b\sec^2(c+dx)\tan(c+dx)\right)\,dx\\ &= a\int \sec^2(c+dx)\,dx+b\int \sec^2(c+dx)\tan(c+dx)\,dx\\ &= -\frac{a\text{Subst}(\int 1\,dx,x,-\tan(c+dx))}{d}+\frac{b\text{Subst}(\int x\,dx,x,s}{d}\\ &= \frac{b\sec^2(c+dx)}{2d}+\frac{a\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 28, normalized size = 1.00

$$\frac{b\sec^2(c+dx)}{2d}+\frac{a\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
[Out] (b*Sec[c + d*x]^2)/(2*d) + (a*Tan[c + d*x])/d
```

Maple [A]
time = 0.19, size = 25, normalized size = 0.89

method	result	size
derivativedivides	$\frac{a\tan(dx+c)+\frac{b}{2\cos(dx+c)^2}}{d}$	25
default	$\frac{a\tan(dx+c)+\frac{b}{2\cos(dx+c)^2}}{d}$	25
risch	$\frac{2ia\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}+2ia}{d\left(e^{2i(dx+c)}+1\right)^2}$	48
norman	$\frac{\frac{2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(a*tan(d*x+c)+1/2*b/cos(d*x+c)^2)
```

Maxima [A]
time = 0.28, size = 30, normalized size = 1.07

$$\frac{2\,a\tan\left(dx+c\right)-\frac{b}{\sin\left(dx+c\right)^{2}-1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
[Out] 1/2*(2*a*tan(d*x + c) - b/(sin(d*x + c)^2 - 1))/d
```

Fricas [A]
time = 2.94, size = 30, normalized size = 1.07

$$\frac{2\,a\cos\left(dx+c\right)\sin\left(dx+c\right)+b}{2\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(2*a*cos(d*x + c)*sin(d*x + c) + b)/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c)),x)
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))*sec(c + d*x)**3, x)
```

Giac [A]
time = 0.44, size = 25, normalized size = 0.89

$$\frac{b\tan\left(dx+c\right)^{2}+2\,a\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
[Out] 1/2*(b*tan(d*x + c)^2 + 2*a*tan(d*x + c))/d
```

Mupad [B]
time = 0.40, size = 23, normalized size = 0.82

$$\frac{\tan(c+d\,x)\,\left(2\,a+b\tan(c+d\,x)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))/cos(c + d*x)^3,x)
[Out] (tan(c + d*x)*(2*a + b*tan(c + d*x)))/(2*d)
```

3.39 $\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=52

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] 1/2*a*arctanh(sin(d*x+c))/d+1/3*b*sec(d*x+c)^3/d+1/2*a*sec(d*x+c)*tan(d*x+c
)/d
```

Rubi [A]
time = 0.05, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3169, 3853, 3855, 2686, 30}

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a\tan(c+dx)\sec(c+dx)}{2d}+\frac{b\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]^3)/(3*d) + (a*Sec[c + d*x
]*Tan[c + d*x])/(2*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx &= \int \left(a\sec^3(c+dx)+b\sec^3(c+dx)\tan(c+dx)\right)\,dx\\ &= a\int \sec^3(c+dx)\,dx+b\int \sec^3(c+dx)\tan(c+dx)\,dx\\ &= \frac{a\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}a\int \sec(c+dx)\,dx+\frac{b\text{Su}}{}\\ &= \frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)}{2c}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 52, normalized size = 1.00

$$\frac{a\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b\sec^3(c+dx)}{3d}+\frac{a\sec(c+dx)\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/(2*d) + (b*Sec[c + d*x]^3)/(3*d) + (a*Sec[c + d*x
]*Tan[c + d*x])/(2*d)
```

Maple [A]
time = 0.24, size = 50, normalized size = 0.96

method	result
derivativedivides	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{b}{3\cos(dx+c)^3}}{d}$
default	$\frac{a\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{b}{3\cos(dx+c)^3}}{d}$
risch	$\frac{-3ia\,e^{5i(dx+c)}+8b\,e^{3i(dx+c)}+3ia\,e^{i(dx+c)}}{3d\left(e^{2i(dx+c)}+1\right)^3}+\frac{a\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)}{2d}$

norman	$\frac{\frac{a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2b}{3d}-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2b\left(\tan^6 \right.}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+1/3*b/cos(
d*x+c)^3)
```

Maxima [A]
time = 0.29, size = 61, normalized size = 1.17

$$-\frac{3\,a\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-\frac{4\,b}{\cos(dx+c)^3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)) - 4*b/cos(d*x + c)^3)/d
```

Fricas [A]
time = 1.93, size = 74, normalized size = 1.42

$$\frac{3\,a\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,a\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+6\,a\cos\left(dx+c\right)\sin\left(dx+c\right)+4\,b}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(3*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*a*cos(d*x + c)^3*log(-si
n(d*x + c) + 1) + 6*a*cos(d*x + c)*sin(d*x + c) + 4*b)/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)\sec^4\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))*sec(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(46) = 92$.
time = 0.43, size = 99, normalized size = 1.90

$$\frac{3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-6\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) + 2*(3*a*tan(1/2*d*x + 1/2*c)^5 - 6*b*tan(1/2*d*x + 1/2*c)^4 - 3*
a*tan(1/2*d*x + 1/2*c) - 2*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B]
time = 2.21, size = 105, normalized size = 2.02

$$\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{2\,b}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))/cos(c + d*x)^4,x)
```

```
[Out] (a*atanh(tan(c/2 + (d*x)/2)))/d - ((2*b)/3 + a*tan(c/2 + (d*x)/2) - a*tan(c
/2 + (d*x)/2)^5 + 2*b*tan(c/2 + (d*x)/2)^4)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*
tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.40 $\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=44

$$\frac{b\sec^4(c+dx)}{4d}+\frac{a\tan(c+dx)}{d}+\frac{a\tan^3(c+dx)}{3d}$$

```
[Out] 1/4*b*sec(d*x+c)^4/d+a*tan(d*x+c)/d+1/3*a*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3169, 3852, 2686, 30}

$$\frac{a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^4)/(4*d) + (a*Tan[c + d*x])/d + (a*Tan[c + d*x]^3)/(3*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \int \left(a\sec^4(c+dx)+b\sec^4(c+dx)\tan(c+dx)\right)\,dx$$
$$= a\int \sec^4(c+dx)\,dx + b\int \sec^4(c+dx)\tan(c+dx)\,dx$$
$$= -\frac{a\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -\tan(c+dx)\right)}{d} + \frac{b\text{Subst}\left(\int\right.}{}$$
$$= \frac{b\sec^4(c+dx)}{4d} + \frac{a\tan(c+dx)}{d} + \frac{a\tan^3(c+dx)}{3d}$$

Mathematica [**A**]
time = 0.10, size = 41, normalized size = 0.93

$$\frac{b\sec^4(c+dx)}{4d} + \frac{a\left(\tan(c+dx)+\frac{1}{3}\tan^3(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
[Out] (b*Sec[c + d*x]^4)/(4*d) + (a*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d
```

Maple [**A**]
time = 0.23, size = 38, normalized size = 0.86

method	result
derivativedivides	$\frac{-a\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{b}{4\cos(dx+c)^4}}{d}$
default	$\frac{-a\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{b}{4\cos(dx+c)^4}}{d}$
risch	$\frac{4ia\,e^{4i(dx+c)}+4b\,e^{4i(dx+c)}+\frac{16ia\,e^{2i(dx+c)}}{3}+\frac{4ia}{3}}{d\left(e^{2i(dx+c)}+1\right)^4}$
norman	$\frac{\frac{2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2b\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{4a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{4a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2a\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2b\left(\tan\right.}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+1/4*b/cos(d*x+c)^4)
```

Maxima [A]
time = 0.27, size = 41, normalized size = 0.93

$$\frac{4 \left(\tan \left(dx+c\right)^{3}+3 \tan \left(dx+c\right)\right)a+\frac{3\, b}{\left(\sin(dx+c)^{2}-1\right)^{2}}}{12\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*a + 3*b/(sin(d*x + c)^2 - 1)^2)/d
```

Fricas [A]
time = 2.81, size = 45, normalized size = 1.02

$$\frac{4 \left(2\, a \cos \left(dx+c\right)^{3}+a \cos \left(dx+c\right)\right) \sin \left(dx+c\right)+3\, b}{12\, d \cos \left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(4*(2*a*cos(d*x + c)^3 + a*cos(d*x + c))*sin(d*x + c) + 3*b)/(d*cos(d*
x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \cos \left(c+dx\right)+b \sin \left(c+dx\right)\right) \sec^{5}\left(c+dx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))*sec(c + d*x)**5, x)
```

Giac [A]
time = 0.45, size = 48, normalized size = 1.09

$$\frac{3\, b \tan \left(dx+c\right)^{4}+4\, a \tan \left(dx+c\right)^{3}+6\, b \tan \left(dx+c\right)^{2}+12\, a \tan \left(dx+c\right)}{12\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(3*b*tan(d*x + c)^4 + 4*a*tan(d*x + c)^3 + 6*b*tan(d*x + c)^2 + 12*a*t
an(d*x + c))/d
```

Mupad [B]
time = 0.52, size = 40, normalized size = 0.91

$$\frac{\frac{b}{4}+\frac{a\,\sin\left(2\,c+2\,d\,x\right)}{3}+\frac{a\,\sin\left(4\,c+4\,d\,x\right)}{12}}{d\,{\cos\left(c+d\,x\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))/cos(c + d*x)^5,x)
[Out] (b/4 + (a*sin(2*c + 2*d*x))/3 + (a*sin(4*c + 4*d*x))/12)/(d*cos(c + d*x)^4)
```

3.41 $\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=74

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\sec^5(c+dx)}{5d}+\frac{3a\sec(c+dx)\tan(c+dx)}{8d}+\frac{a\sec^3(c+dx)\tan(c+dx)}{4d}$$

```
[Out] 3/8*a*arctanh(sin(d*x+c))/d+1/5*b*sec(d*x+c)^5/d+3/8*a*sec(d*x+c)*tan(d*x+c
)/d+1/4*a*sec(d*x+c)^3*tan(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3169, 3853, 3855, 2686, 30}

$$\frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a\tan(c+dx)\sec(c+dx)}{8d}+\frac{b\sec^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (3*a*ArcTanh[Sin[c + d*x]])/(8*d) + (b*Sec[c + d*x]^5)/(5*d) + (3*a*Sec[c +
 d*x]*Tan[c + d*x])/(8*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx &= \int \left(a\sec^5(c+dx)+b\sec^5(c+dx)\tan(c+dx)\right)\,dx\\ &= a\int \sec^5(c+dx)\,dx+b\int \sec^5(c+dx)\tan(c+dx)\,dx\\ &= \frac{a\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}(3a)\int \sec^3(c+dx)\,dx+\\ &= \frac{b\sec^5(c+dx)}{5d}+\frac{3a\sec(c+dx)\tan(c+dx)}{8d}+\frac{a\sec^3(c\\ &= \frac{3a\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b\sec^5(c+dx)}{5d}+\frac{3a\sec(c+d\end{aligned}$$

Mathematica [A]
time = 0.24, size = 68, normalized size = 0.92

$$\frac{b\sec^5(c+dx)}{5d}+\frac{a\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{3a\left(\tanh^{-1}(\sin(c+dx))+\sec(c+dx)\tan(c+dx)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
[Out] (b*Sec[c + d*x]^5)/(5*d) + (a*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (3*a*(Ar
cTanh[Sin[c + d*x]] + Sec[c + d*x]*Tan[c + d*x]))/(8*d)
```

Maple [A]
time = 0.29, size = 63, normalized size = 0.85

method	result
derivativedivides	$\frac{a\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{b}{5\cos(dx+c)^5}}{d}$
default	$\frac{a\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{b}{5\cos(dx+c)^5}}{d}$

risch	$\frac{-15ia\,\mathrm{e}^{9i(dx+c)}-70ia\,\mathrm{e}^{7i(dx+c)}+128b\,\mathrm{e}^{5i(dx+c)}+70ia\,\mathrm{e}^{3i(dx+c)}+15ia\,\mathrm{e}^{i(dx+c)}}{20d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{5}}-\frac{3a\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{8d}+\frac{3a\ln\left(\mathrm{e}^{i(dx+c)}+\right.}{8d}$
norman	$\frac{-\frac{2b}{5d}-\frac{5a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{3a\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{a\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{a\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{3a\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{5a\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{4b}{}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}\left(1+\tan^{2}\left(\frac{dx}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+ta
n(d*x+c)))+1/5*b/cos(d*x+c)^5)
```

Maxima [A]
time = 0.28, size = 86, normalized size = 1.16

$$-\frac{5\,a\left(\frac{2\left(3\,\sin(dx+c)^{3}-5\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-\frac{16\,b}{\cos(dx+c)^{5}}}{80\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/80*(5*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d
*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 16*b/
cos(d*x + c)^5)/d
```

Fricas [A]
time = 2.32, size = 88, normalized size = 1.19

$$\frac{15\,a\cos\left(dx+c\right)^{5}\log\left(\sin\left(dx+c\right)+1\right)-15\,a\cos\left(dx+c\right)^{5}\log\left(-\sin\left(dx+c\right)+1\right)+10\left(3\,a\cos\left(dx+c\right)^{3}+2\,a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+16\,b}{80\,d\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/80*(15*a*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*a*cos(d*x + c)^5*log(-
sin(d*x + c) + 1) + 10*(3*a*cos(d*x + c)^3 + 2*a*cos(d*x + c))*sin(d*x + c)
 + 16*b)/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. 2(66) = 132.
time = 0.45, size = 141, normalized size = 1.91

$$\frac{15\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(25\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-40\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-80\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+10\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-25\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-8\,b\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{5}}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/40*(15*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a*log(abs(tan(1/2*d*x +
1/2*c) - 1)) + 2*(25*a*tan(1/2*d*x + 1/2*c)^9 - 40*b*tan(1/2*d*x + 1/2*c)^8
 - 10*a*tan(1/2*d*x + 1/2*c)^7 - 80*b*tan(1/2*d*x + 1/2*c)^4 + 10*a*tan(1/2
*d*x + 1/2*c)^3 - 25*a*tan(1/2*d*x + 1/2*c) - 8*b)/(tan(1/2*d*x + 1/2*c)^2
- 1)^5)/d
```

Mupad [B]
time = 4.12, size = 175, normalized size = 2.36

$$\frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,d}-\frac{-\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{4}+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{2}+4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{2}+\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{2\,b}{5}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}-10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))/cos(c + d*x)^6,x)
```

```
[Out] (3*a*atanh(tan(c/2 + (d*x)/2)))/(4*d) - ((2*b)/5 + (5*a*tan(c/2 + (d*x)/2))
/4 - (a*tan(c/2 + (d*x)/2)^3)/2 + (a*tan(c/2 + (d*x)/2)^7)/2 - (5*a*tan(c/2
 + (d*x)/2)^9)/4 + 4*b*tan(c/2 + (d*x)/2)^4 + 2*b*tan(c/2 + (d*x)/2)^8)/(d*
(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6
 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.42 $\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx$

Optimal. Leaf size=60

$$\frac{b\sec^6(c+dx)}{6d}+\frac{a\tan(c+dx)}{d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan^5(c+dx)}{5d}$$

```
[Out] 1/6*b*sec(d*x+c)^6/d+a*tan(d*x+c)/d+2/3*a*tan(d*x+c)^3/d+1/5*a*tan(d*x+c)^5
/d
```

Rubi [A]
time = 0.05, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3169, 3852, 2686, 30}

$$\frac{a\tan^5(c+dx)}{5d}+\frac{2a\tan^3(c+dx)}{3d}+\frac{a\tan(c+dx)}{d}+\frac{b\sec^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (b*Sec[c + d*x]^6)/(6*d) + (a*Tan[c + d*x])/d + (2*a*Tan[c + d*x]^3)/(3*d)
+ (a*Tan[c + d*x]^5)/(5*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))\,dx = \int \left(a\sec^6(c+dx)+b\sec^6(c+dx)\tan(c+dx)\right)\,dx$$
$$= a\int \sec^6(c+dx)\,dx + b\int \sec^6(c+dx)\tan(c+dx)\,dx$$
$$= -\frac{a\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx, x, -\tan(c+dx)\right)}{d} + \frac{b\text{Su}}{}$$
$$= \frac{b\sec^6(c+dx)}{6d} + \frac{a\tan(c+dx)}{d} + \frac{2a\tan^3(c+dx)}{3d} + \frac{a\,\text{t}}{}$$

Mathematica [A]
time = 0.19, size = 53, normalized size = 0.88

$$\frac{b\sec^6(c+dx)}{6d} + \frac{a\left(\tan(c+dx)+\frac{2}{3}\tan^3(c+dx)+\frac{1}{5}\tan^5(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
[Out] (b*Sec[c + d*x]^6)/(6*d) + (a*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c
+ d*x]^5/5))/d
```

Maple [A]
time = 0.28, size = 48, normalized size = 0.80

method	result
derivativedivides	$\frac{-a\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{b}{6\cos(dx+c)^6}}{d}$
default	$\frac{-a\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{b}{6\cos(dx+c)^6}}{d}$
risch	$\frac{\frac{32ia\,e^{6i(dx+c)}}{3}+\frac{32b\,e^{6i(dx+c)}}{3}+16ia\,e^{4i(dx+c)}+\frac{32ia\,e^{2i(dx+c)}}{5}+\frac{16ia}{15}}{d\left(e^{2i(dx+c)}+1\right)^6}$
norman	$\frac{\frac{2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2b\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2b\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{8a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{86a(}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^6(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+1/6*b/cos(d*x
+c)^6)
```

Maxima [A]
time = 0.30, size = 53, normalized size = 0.88

$$\frac{2\left(3\tan\left(dx+c\right)^{5}+10\tan\left(dx+c\right)^{3}+15\tan\left(dx+c\right)\right)a-\frac{5b}{\left(\sin\left(dx+c\right)^{2}-1\right)^{3}}}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/30*(2*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a - 5*b/(s
in(d*x + c)^2 - 1)^3)/d
```

Fricas [A]
time = 2.46, size = 57, normalized size = 0.95

$$\frac{2\left(8a\cos\left(dx+c\right)^{5}+4a\cos\left(dx+c\right)^{3}+3a\cos\left(dx+c\right)\right)\sin\left(dx+c\right)+5b}{30d\cos\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/30*(2*(8*a*cos(d*x + c)^5 + 4*a*cos(d*x + c)^3 + 3*a*cos(d*x + c))*sin(d*
x + c) + 5*b)/(d*cos(d*x + c)^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.45, size = 70, normalized size = 1.17

$$\frac{5b\tan\left(dx+c\right)^{6}+6a\tan\left(dx+c\right)^{5}+15b\tan\left(dx+c\right)^{4}+20a\tan\left(dx+c\right)^{3}+15b\tan\left(dx+c\right)^{2}+30a\tan\left(dx+c\right)}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/30*(5*b*tan(d*x + c)^6 + 6*a*tan(d*x + c)^5 + 15*b*tan(d*x + c)^4 + 20*a*
tan(d*x + c)^3 + 15*b*tan(d*x + c)^2 + 30*a*tan(d*x + c))/d
```

Mupad [B]
time = 0.67, size = 65, normalized size = 1.08

$$\frac{\frac{8\,a\,\sin(c+d\,x)\,\cos(c+d\,x)^5}{15}+\frac{4\,a\,\sin(c+d\,x)\,\cos(c+d\,x)^3}{15}+\frac{a\,\sin(c+d\,x)\,\cos(c+d\,x)}{5}+\frac{b}{6}}{d\cos\left(c+d\,x\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))/cos(c + d*x)^7,x)
```

```
[Out] (b/6 + (a*cos(c + d*x)*sin(c + d*x))/5 + (4*a*cos(c + d*x)^3*sin(c + d*x))/
15 + (8*a*cos(c + d*x)^5*sin(c + d*x))/15)/(d*cos(c + d*x)^6)
```

3.43 $\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=137

$$-\frac{2ab\cos^7(c+dx)}{7d}+\frac{a^2\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{d}+\frac{b^2\sin^3(c+dx)}{3d}+\frac{3a^2\sin^5(c+dx)}{5d}-\frac{2b^2\sin^5(c+dx)}{5d}-a^2$$

```
[Out] -2/7*a*b*cos(d*x+c)^7/d+a^2*sin(d*x+c)/d-a^2*sin(d*x+c)^3/d+1/3*b^2*sin(d*x
+c)^3/d+3/5*a^2*sin(d*x+c)^5/d-2/5*b^2*sin(d*x+c)^5/d-1/7*a^2*sin(d*x+c)^7/
d+1/7*b^2*sin(d*x+c)^7/d
```

Rubi [A]
time = 0.10, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3169, 2713, 2645, 30, 2644, 276}

$$-\frac{a^2\sin^7(c+dx)}{7d}+\frac{3a^2\sin^5(c+dx)}{5d}-\frac{a^2\sin^3(c+dx)}{d}+\frac{a^2\sin(c+dx)}{d}-\frac{2ab\cos^7(c+dx)}{7d}+\frac{b^2\sin^7(c+dx)}{7d}-\frac{2b^2\sin^5(c+dx)}{5d}+\frac{b^2\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*b*Cos[c + d*x]^7)/(7*d) + (a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^3)
/d + (b^2*Sin[c + d*x]^3)/(3*d) + (3*a^2*Sin[c + d*x]^5)/(5*d) - (2*b^2*Sin
[c + d*x]^5)/(5*d) - (a^2*Sin[c + d*x]^7)/(7*d) + (b^2*Sin[c + d*x]^7)/(7*d
)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \int \left(a^2\cos^7(c+dx)+2ab\cos^6(c+dx)\sin(c+dx)+b^2\right.$$

$$= a^2\int \cos^7(c+dx)\,dx+(2ab)\int \cos^6(c+dx)\sin(c+dx$$

$$= -\frac{a^2\text{Subst}\left(\int \left(1-3x^2+3x^4-x^6\right)\,dx,x,-\sin(c+dx)\right)}{d}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d}+\frac{a^2\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{d}$$

$$= -\frac{2ab\cos^7(c+dx)}{7d}+\frac{a^2\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{d}$$

Mathematica [A]
time = 0.41, size = 154, normalized size = 1.12

$$-\frac{1050ab\cos(c+dx)+630ab\cos(3(c+dx))+210ab\cos(5(c+dx))+30ab\cos(7(c+dx))-3675a^2\sin(c+dx)-525b^2\sin(c+dx)-735a^2\sin(3(c+dx))+35b^2\sin(3(c+dx))-147a^2\sin(5(c+dx))+63b^2\sin(5(c+dx))-15a^2\sin(7(c+dx))+15b^2\sin(7(c+dx))}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] -1/6720*(1050*a*b*Cos[c + d*x] + 630*a*b*Cos[3*(c + d*x)] + 210*a*b*Cos[5*(
c + d*x)] + 30*a*b*Cos[7*(c + d*x)] - 3675*a^2*Sin[c + d*x] - 525*b^2*Sin[c
 + d*x] - 735*a^2*Sin[3*(c + d*x)] + 35*b^2*Sin[3*(c + d*x)] - 147*a^2*Sin[
5*(c + d*x)] + 63*b^2*Sin[5*(c + d*x)] - 15*a^2*Sin[7*(c + d*x)] + 15*b^2*S
in[7*(c + d*x)])/d
```

Maple [A]
time = 0.28, size = 108, normalized size = 0.79

method	result
derivativedivides	$\frac{b^{2}\left(-\frac{\sin(dx+c)\left(\cos^{6}(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^{4}(dx+c)+\frac{4\left(\cos^{2}(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{2ab\left(\cos^{7}(dx+c)\right)}{7}+\frac{a^{2}\left(\frac{16}{5}+\cos^{6}(dx+c)+\frac{6\left(\cos^{4}\right.}{}\right.}{}}{d}$
default	$\frac{b^{2}\left(-\frac{\sin(dx+c)\left(\cos^{6}(dx+c)\right)}{7}+\frac{\left(\frac{8}{3}+\cos^{4}(dx+c)+\frac{4\left(\cos^{2}(dx+c)\right)}{3}\right)\sin(dx+c)}{35}\right)-\frac{2ab\left(\cos^{7}(dx+c)\right)}{7}+\frac{a^{2}\left(\frac{16}{5}+\cos^{6}(dx+c)+\frac{6\left(\cos^{4}\right.}{}\right.}{}}{d}$
risch	$-\frac{5ab\cos(dx+c)}{32d}+\frac{35a^{2}\sin(dx+c)}{64d}+\frac{5b^{2}\sin(dx+c)}{64d}-\frac{ab\cos(7dx+7c)}{224d}+\frac{\sin(7dx+7c)a^{2}}{448d}-\frac{\sin(7dx+7c)b^{2}}{448d}-\frac{ab}{}$
norman	$\frac{-\frac{4ab}{7d}+\frac{2a^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^{2}\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4\left(3a^{2}+2b^{2}\right)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{4\left(3a^{2}+2b^{2}\right)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{8\left(53a^{2}+38b^{2}\right)\left(\tan\right.}{35d}}{\left(\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c
)^2)*sin(d*x+c))-2/7*a*b*cos(d*x+c)^7+1/7*a^2*(16/5+cos(d*x+c)^6+6/5*cos(d*
x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 98, normalized size = 0.72

$$-\frac{30\,ab\cos\left(dx+c\right)^{7}+3\left(5\,\sin\left(dx+c\right)^{7}-21\,\sin\left(dx+c\right)^{5}+35\,\sin\left(dx+c\right)^{3}-35\,\sin\left(dx+c\right)\right)a^{2}-\left(15\,\sin\left(dx+c\right)^{7}-42\,\sin\left(dx+c\right)^{5}+35\,\sin\left(dx+c\right)^{3}\right)b^{2}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/105*(30*a*b*cos(d*x + c)^7 + 3*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 3
5*sin(d*x + c)^3 - 35*sin(d*x + c))*a^2 - (15*sin(d*x + c)^7 - 42*sin(d*x +
 c)^5 + 35*sin(d*x + c)^3)*b^2)/d
```

Fricas [A]
time = 2.86, size = 94, normalized size = 0.69

$$-\frac{30\,ab\cos\left(dx+c\right)^{7}-\left(15\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{6}+3\left(6\,a^{2}+b^{2}\right)\cos\left(dx+c\right)^{4}+4\left(6\,a^{2}+b^{2}\right)\cos\left(dx+c\right)^{2}+48\,a^{2}+8\,b^{2}\right)\sin\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/105*(30*a*b*cos(d*x + c)^7 - (15*(a^2 - b^2)*cos(d*x + c)^6 + 3*(6*a^2 +
 b^2)*cos(d*x + c)^4 + 4*(6*a^2 + b^2)*cos(d*x + c)^2 + 48*a^2 + 8*b^2)*sin
(d*x + c))/d
```

Sympy [A]
time = 0.65, size = 187, normalized size = 1.36

$$\begin{cases} \frac{16a^2\sin^7(c+dx)}{35d} + \frac{8a^2\sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{2a^2\sin^3(c+dx)\cos^4(c+dx)}{d} + \frac{a^2\sin(c+dx)\cos^6(c+dx)}{d} - \frac{2ab\cos^7(c+dx)}{7d} + \frac{8b^2\sin^7(c+dx)}{105d} + \frac{4b^2\sin^5(c+dx)\cos^2(c+dx)}{15d} + \frac{b^2\sin^3(c+dx)\cos^4(c+dx)}{3d} & \text{for } d \neq 0 \\ x(a\cos(c) + b\sin(c))^2\cos^5(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((16*a**2*sin(c + d*x)**7/(35*d) + 8*a**2*sin(c + d*x)**5*cos(c +
d*x)**2/(5*d) + 2*a**2*sin(c + d*x)**3*cos(c + d*x)**4/d + a**2*sin(c + d*x
)*cos(c + d*x)**6/d - 2*a*b*cos(c + d*x)**7/(7*d) + 8*b**2*sin(c + d*x)**7/
(105*d) + 4*b**2*sin(c + d*x)**5*cos(c + d*x)**2/(15*d) + b**2*sin(c + d*x)
**3*cos(c + d*x)**4/(3*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**2*cos(c)**5
, True))
```

Giac [A]
time = 0.48, size = 155, normalized size = 1.13

$$-\frac{ab\cos(7dx+7c)}{224d} - \frac{ab\cos(5dx+5c)}{32d} - \frac{3ab\cos(3dx+3c)}{32d} - \frac{5ab\cos(dx+c)}{32d} + \frac{(a^2-b^2)\sin(7dx+7c)}{448d} + \frac{(7a^2-3b^2)\sin(5dx+5c)}{320d} + \frac{(21a^2-b^2)\sin(3dx+3c)}{192d} + \frac{5(7a^2+b^2)\sin(dx+c)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/224*a*b*cos(7*d*x + 7*c)/d - 1/32*a*b*cos(5*d*x + 5*c)/d - 3/32*a*b*cos(
3*d*x + 3*c)/d - 5/32*a*b*cos(d*x + c)/d + 1/448*(a^2 - b^2)*sin(7*d*x + 7*
c)/d + 1/320*(7*a^2 - 3*b^2)*sin(5*d*x + 5*c)/d + 1/192*(21*a^2 - b^2)*sin(
3*d*x + 3*c)/d + 5/64*(7*a^2 + b^2)*sin(d*x + c)/d
```

Mupad [B]
time = 0.69, size = 176, normalized size = 1.28

$$\frac{16a^2\sin(c+dx)}{35d} + \frac{8b^2\sin(c+dx)}{105d} + \frac{8a^2\cos(c+dx)^2\sin(c+dx)}{35d} + \frac{6a^2\cos(c+dx)^4\sin(c+dx)}{35d} + \frac{a^2\cos(c+dx)^6\sin(c+dx)}{7d} + \frac{4b^2\cos(c+dx)^2\sin(c+dx)}{105d} + \frac{b^2\cos(c+dx)^4\sin(c+dx)}{35d} - \frac{b^2\cos(c+dx)^6\sin(c+dx)}{7d} - \frac{2ab\cos(c+dx)^7}{7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

```
[Out] (16*a^2*sin(c + d*x))/(35*d) + (8*b^2*sin(c + d*x))/(105*d) + (8*a^2*cos(c
+ d*x)^2*sin(c + d*x))/(35*d) + (6*a^2*cos(c + d*x)^4*sin(c + d*x))/(35*d)
+ (a^2*cos(c + d*x)^6*sin(c + d*x))/(7*d) + (4*b^2*cos(c + d*x)^2*sin(c + d
*x))/(105*d) + (b^2*cos(c + d*x)^4*sin(c + d*x))/(35*d) - (b^2*cos(c + d*x)
^6*sin(c + d*x))/(7*d) - (2*a*b*cos(c + d*x)^7)/(7*d)
```

3.44 $\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=174

$$\frac{5a^2x}{16}+\frac{b^2x}{16}-\frac{ab\cos^6(c+dx)}{3d}+\frac{5a^2\cos(c+dx)\sin(c+dx)}{16d}+\frac{b^2\cos(c+dx)\sin(c+dx)}{16d}+\frac{5a^2\cos^3(c+dx)\sin(c}{24d}$$

```
[Out] 5/16*a^2*x+1/16*b^2*x-1/3*a*b*cos(d*x+c)^6/d+5/16*a^2*cos(d*x+c)*sin(d*x+c)
/d+1/16*b^2*cos(d*x+c)*sin(d*x+c)/d+5/24*a^2*cos(d*x+c)^3*sin(d*x+c)/d+1/24
*b^2*cos(d*x+c)^3*sin(d*x+c)/d+1/6*a^2*cos(d*x+c)^5*sin(d*x+c)/d-1/6*b^2*co
s(d*x+c)^5*sin(d*x+c)/d
```

Rubi [A]
time = 0.12, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3169, 2715, 8, 2645, 30, 2648}

$$\frac{a^2\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a^2\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a^2\sin(c+dx)\cos(c+dx)}{16d}+\frac{5a^2x}{16}-\frac{ab\cos^6(c+dx)}{3d}-\frac{b^2\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{b^2\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{b^2\sin(c+dx)\cos(c+dx)}{16d}+\frac{b^2x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (5*a^2*x)/16 + (b^2*x)/16 - (a*b*Cos[c + d*x]^6)/(3*d) + (5*a^2*Cos[c + d*x
]*Sin[c + d*x])/(16*d) + (b^2*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (5*a^2*Co
s[c + d*x]^3*Sin[c + d*x])/(24*d) + (b^2*Cos[c + d*x]^3*Sin[c + d*x])/(24*d
) + (a^2*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (b^2*Cos[c + d*x]^5*Sin[c + d
*x])/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
```

```
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx &= \int \left(a^2\cos^6(c+dx)+2ab\cos^5(c+dx)\sin(c+dx)+b^2 \right.\\
&= a^2\int \cos^6(c+dx)\,dx+(2ab)\int \cos^5(c+dx)\sin(c+dx \\
&= \frac{a^2\cos^5(c+dx)\sin(c+dx)}{6d}-\frac{b^2\cos^5(c+dx)\sin(c+d}{6d} \\
&= -\frac{ab\cos^6(c+dx)}{3d}+\frac{5a^2\cos^3(c+dx)\sin(c+dx)}{24d}+\frac{b^2 c}{} \\
&= -\frac{ab\cos^6(c+dx)}{3d}+\frac{5a^2\cos(c+dx)\sin(c+dx)}{16d}+\frac{b^2 c}{} \\
&= \frac{5a^2x}{16}+\frac{b^2x}{16}-\frac{ab\cos^6(c+dx)}{3d}+\frac{5a^2\cos(c+dx)\sin(c}{16d}
\end{aligned}$$

Mathematica [A]
time = 0.27, size = 147, normalized size = 0.84

$$\frac{(5a^2+b^2)(c+dx)}{16d}-\frac{5ab\cos(2(c+dx))}{32d}-\frac{ab\cos(4(c+dx))}{16d}-\frac{ab\cos(6(c+dx))}{96d}+\frac{(15a^2+b^2)\sin(2(c+dx))}{64d}+\frac{(3a^2-b^2)\sin(4(c+dx))}{64d}+\frac{(a^2-b^2)\sin(6(c+dx))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

[Out] ((5*a^2 + b^2)*(c + d*x))/(16*d) - (5*a*b*Cos[2*(c + d*x)])/(32*d) - (a*b*Cos[4*(c + d*x)])/(16*d) - (a*b*Cos[6*(c + d*x)])/(96*d) + ((15*a^2 + b^2)*Sin[2*(c + d*x)])/(64*d) + ((3*a^2 - b^2)*Sin[4*(c + d*x)])/(64*d) + ((a^2 - b^2)*Sin[6*(c + d*x)])/(192*d)

Maple [A]
time = 0.24, size = 118, normalized size = 0.68

method	result
derivativedivides	$\frac{b^{2}\left(-\frac{\sin(dx+c)\left(\cos^{5}(dx+c)\right)}{6}+\frac{\left(\cos^{3}(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)-\frac{ab\left(\cos^{6}(dx+c)\right)}{3}+a^{2}\left(\frac{\left(\cos^{5}(dx+c)+\frac{5\left(\cos^{3}(\right.}{4}\right.}{}\right.}{d}$
default	$\frac{b^{2}\left(-\frac{\sin(dx+c)\left(\cos^{5}(dx+c)\right)}{6}+\frac{\left(\cos^{3}(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}+\frac{dx}{16}+\frac{c}{16}\right)-\frac{ab\left(\cos^{6}(dx+c)\right)}{3}+a^{2}\left(\frac{\left(\cos^{5}(dx+c)+\frac{5\left(\cos^{3}(\right.}{4}\right.}{}\right.}{d}$
risch	$\frac{5a^{2}x}{16}+\frac{b^{2}x}{16}-\frac{ab\cos(6dx+6c)}{96d}+\frac{\sin(6dx+6c)a^{2}}{192d}-\frac{\sin(6dx+6c)b^{2}}{192d}-\frac{ab\cos(4dx+4c)}{16d}+\frac{3\sin(4dx+4c)a^{2}}{64d}-\sin$
norman	$\frac{\left(\frac{5a^{2}}{16}+\frac{b^{2}}{16}\right)x+\left(\frac{5a^{2}}{16}+\frac{b^{2}}{16}\right)x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{15a^{2}}{8}+\frac{3b^{2}}{8}\right)x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{15a^{2}}{8}+\frac{3b^{2}}{8}\right)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{25a^{2}}{4}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(b^2*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)-1/3*a*b*cos(d*x+c)^6+a^2*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))

Maxima [A]
time = 0.29, size = 102, normalized size = 0.59

$$-\frac{64\,ab\cos\left(dx+c\right)^{6}+\left(4\,\sin\left(2\,dx+2\,c\right)^{3}-60\,dx-60\,c-9\,\sin\left(4\,dx+4\,c\right)-48\,\sin\left(2\,dx+2\,c\right)\right)a^{2}-\left(4\,\sin\left(2\,dx+2\,c\right)^{3}+12\,dx+12\,c-3\,\sin\left(4\,dx+4\,c\right)\right)b^{2}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] -1/192*(64*a*b*cos(d*x + c)^6 + (4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^2 - (4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*b^2)/d

Fricas [A]
time = 2.44, size = 95, normalized size = 0.55

$$-\frac{16\,ab\cos\left(dx+c\right)^{6}-3\left(5\,a^{2}+b^{2}\right)dx-\left(8\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{5}+2\left(5\,a^{2}+b^{2}\right)\cos\left(dx+c\right)^{3}+3\left(5\,a^{2}+b^{2}\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/48*(16*a*b*cos(d*x + c)^6 - 3*(5*a^2 + b^2)*d*x - (8*(a^2 - b^2)*cos(d*x
 + c)^5 + 2*(5*a^2 + b^2)*cos(d*x + c)^3 + 3*(5*a^2 + b^2)*cos(d*x + c))*si
n(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(162) = 324$.
time = 0.45, size = 340, normalized size = 1.95

$$\begin{cases} \frac{5a^2x\sin^6(c+dx)}{16} + \frac{15a^2x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{15a^2x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{5a^2x\cos^6(c+dx)}{16} + \frac{5a^2\sin^5(c+dx)\cos(c+dx)}{16d} + \frac{5a^2\sin^3(c+dx)\cos^3(c+dx)}{6d} + \frac{11a^2\sin(c+dx)\cos^5(c+dx)}{16d} - \frac{ab\cos^6(c+dx)}{3d} + \frac{b^2x\sin^6(c+dx)}{16} + \frac{3b^2x\sin^4(c+dx)\cos^2(c+dx)}{16} + \frac{3b^2x\sin^2(c+dx)\cos^4(c+dx)}{16} + \frac{b^2x\cos^6(c+dx)}{16} + \frac{b^2\sin^5(c+dx)\cos(c+dx)}{16d} + \frac{b^2\sin^3(c+dx)\cos^3(c+dx)}{6d} - \frac{b^2\sin(c+dx)\cos^5(c+dx)}{16d} & \text{for } d \neq 0 \\ x(a\cos(c) + b\sin(c))^2\cos^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((5*a**2*x*sin(c + d*x)**6/16 + 15*a**2*x*sin(c + d*x)**4*cos(c +
d*x)**2/16 + 15*a**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a**2*x*cos(c
+ d*x)**6/16 + 5*a**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a**2*sin(c +
d*x)**3*cos(c + d*x)**3/(6*d) + 11*a**2*sin(c + d*x)*cos(c + d*x)**5/(16*d)
 - a*b*cos(c + d*x)**6/(3*d) + b**2*x*sin(c + d*x)**6/16 + 3*b**2*x*sin(c +
 d*x)**4*cos(c + d*x)**2/16 + 3*b**2*x*sin(c + d*x)**2*cos(c + d*x)**4/16 +
 b**2*x*cos(c + d*x)**6/16 + b**2*sin(c + d*x)**5*cos(c + d*x)/(16*d) + b**
2*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) - b**2*sin(c + d*x)*cos(c + d*x)**5
/(16*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**2*cos(c)**4, True))
```

Giac [A]
time = 0.46, size = 132, normalized size = 0.76

$$\frac{1}{16}\left(5a^2+b^2\right)x - \frac{ab\cos(6dx+6c)}{96d} - \frac{ab\cos(4dx+4c)}{16d} - \frac{5ab\cos(2dx+2c)}{32d} + \frac{(a^2-b^2)\sin(6dx+6c)}{192d} + \frac{(3a^2-b^2)\sin(4dx+4c)}{64d} + \frac{(15a^2+b^2)\sin(2dx+2c)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(5*a^2 + b^2)*x - 1/96*a*b*cos(6*d*x + 6*c)/d - 1/16*a*b*cos(4*d*x + 4
*c)/d - 5/32*a*b*cos(2*d*x + 2*c)/d + 1/192*(a^2 - b^2)*sin(6*d*x + 6*c)/d
+ 1/64*(3*a^2 - b^2)*sin(4*d*x + 4*c)/d + 1/64*(15*a^2 + b^2)*sin(2*d*x + 2
*c)/d
```

Mupad [B]
time = 0.61, size = 156, normalized size = 0.90

$$\frac{5a^2x}{16} + \frac{b^2x}{16} + \frac{5a^2\cos(c+dx)^3\sin(c+dx)}{24d} + \frac{a^2\cos(c+dx)^5\sin(c+dx)}{6d} + \frac{b^2\cos(c+dx)^3\sin(c+dx)}{24d} - \frac{b^2\cos(c+dx)^5\sin(c+dx)}{6d} - \frac{ab\cos(c+dx)^6}{3d} + \frac{5a^2\cos(c+dx)\sin(c+dx)}{16d} + \frac{b^2\cos(c+dx)\sin(c+dx)}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

```
[Out] (5*a^2*x)/16 + (b^2*x)/16 + (5*a^2*cos(c + d*x)^3*sin(c + d*x))/(24*d) + (a
^2*cos(c + d*x)^5*sin(c + d*x))/(6*d) + (b^2*cos(c + d*x)^3*sin(c + d*x))/(
24*d) - (b^2*cos(c + d*x)^5*sin(c + d*x))/(6*d) - (a*b*cos(c + d*x)^6)/(3*d
) + (5*a^2*cos(c + d*x)*sin(c + d*x))/(16*d) + (b^2*cos(c + d*x)*sin(c + d*
x))/(16*d)
```

3.45 $\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=103

$$-\frac{2ab\cos^5(c+dx)}{5d}+\frac{a^2\sin(c+dx)}{d}-\frac{2a^2\sin^3(c+dx)}{3d}+\frac{b^2\sin^3(c+dx)}{3d}+\frac{a^2\sin^5(c+dx)}{5d}-\frac{b^2\sin^5(c+dx)}{5d}$$

```
[Out] -2/5*a*b*cos(d*x+c)^5/d+a^2*sin(d*x+c)/d-2/3*a^2*sin(d*x+c)^3/d+1/3*b^2*sin
(d*x+c)^3/d+1/5*a^2*sin(d*x+c)^5/d-1/5*b^2*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.09, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3169, 2713, 2645, 30, 2644, 14}

$$\frac{a^2\sin^5(c+dx)}{5d}-\frac{2a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{2ab\cos^5(c+dx)}{5d}-\frac{b^2\sin^5(c+dx)}{5d}+\frac{b^2\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*b*Cos[c + d*x]^5)/(5*d) + (a^2*Sin[c + d*x])/d - (2*a^2*Sin[c + d*x]^
3)/(3*d) + (b^2*Sin[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*x]^5)/(5*d) - (b^2*S
in[c + d*x]^5)/(5*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \int \left(a^2\cos^5(c+dx)+2ab\cos^4(c+dx)\sin(c+dx)+b^2 c\right.$$
$$= a^2\int \cos^5(c+dx)\,dx+(2ab)\int \cos^4(c+dx)\sin(c+dx)$$
$$= -\frac{a^2\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,-\sin(c+dx)\right)}{d}-\frac{(2a}{}$$
$$= -\frac{2ab\cos^5(c+dx)}{5d}+\frac{a^2\sin(c+dx)}{d}-\frac{2a^2\sin^3(c+dx)}{3d}$$
$$= -\frac{2ab\cos^5(c+dx)}{5d}+\frac{a^2\sin(c+dx)}{d}-\frac{2a^2\sin^3(c+dx)}{3d}$$

Mathematica [A]
time = 0.19, size = 116, normalized size = 1.13

$$\frac{-60ab\cos(c+dx)-30ab\cos(3(c+dx))-6ab\cos(5(c+dx))+150a^2\sin(c+dx)+30b^2\sin(c+dx)+25a^2\sin(3(c+dx))-5b^2\sin(3(c+dx))+3a^2\sin(5(c+dx))-3b^2\sin(5(c+dx))}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (-60*a*b*Cos[c + d*x] - 30*a*b*Cos[3*(c + d*x)] - 6*a*b*Cos[5*(c + d*x)] +
150*a^2*Sin[c + d*x] + 30*b^2*Sin[c + d*x] + 25*a^2*Sin[3*(c + d*x)] - 5*b^
2*Sin[3*(c + d*x)] + 3*a^2*Sin[5*(c + d*x)] - 3*b^2*Sin[5*(c + d*x)])/(240*
d)
```

Maple [A]
time = 0.19, size = 88, normalized size = 0.85

method	result
derivativedivides	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{2ab\left(\cos^5(dx+c)\right)}{5}+\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+}{5}}{d}$
default	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{2ab\left(\cos^5(dx+c)\right)}{5}+\frac{a^2\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+}{5}}{d}$
risch	$-\frac{ab\cos(dx+c)}{4d}+\frac{5a^2\sin(dx+c)}{8d}+\frac{b^2\sin(dx+c)}{8d}-\frac{ab\cos(5dx+5c)}{40d}+\frac{\sin(5dx+5c)a^2}{80d}-\frac{\sin(5dx+5c)b^2}{80d}-\frac{ab\,c}{}$
norman	$\frac{-\frac{4ab}{5d}+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{8\left(a^2+b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{8\left(a^2+b^2\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{4\left(29a^2-4b^2\right)\left(\tan^5\left(\frac{d}{}\right.\right.}{15d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-2/
5*a*b*cos(d*x+c)^5+1/5*a^2*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 77, normalized size = 0.75

$$-\frac{6\,ab\cos\left(dx+c\right)^5-\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\sin\left(dx+c\right)\right)a^2+\left(3\,\sin\left(dx+c\right)^5-5\,\sin\left(dx+c\right)^3\right)b^2}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/15*(6*a*b*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*si
n(d*x + c))*a^2 + (3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*b^2)/d
```

Fricas [A]
time = 2.23, size = 74, normalized size = 0.72

$$-\frac{6\,ab\cos\left(dx+c\right)^5-\left(3\left(a^2-b^2\right)\cos\left(dx+c\right)^4+\left(4\,a^2+b^2\right)\cos\left(dx+c\right)^2+8\,a^2+2\,b^2\right)\sin\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(6*a*b*cos(d*x + c)^5 - (3*(a^2 - b^2)*cos(d*x + c)^4 + (4*a^2 + b^2)
*cos(d*x + c)^2 + 8*a^2 + 2*b^2)*sin(d*x + c))/d
```

Sympy [A]
time = 0.29, size = 138, normalized size = 1.34

$$\begin{cases} \frac{8a^2\sin^5(c+dx)}{15d} + \frac{4a^2\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{a^2\sin(c+dx)\cos^4(c+dx)}{d} - \frac{2ab\cos^5(c+dx)}{5d} + \frac{2b^2\sin^5(c+dx)}{15d} + \frac{b^2\sin^3(c+dx)\cos^2(c+dx)}{3d} & \text{for } d \neq 0 \\ x(a\cos(c) + b\sin(c))^2\cos^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((8*a**2*sin(c + d*x)**5/(15*d) + 4*a**2*sin(c + d*x)**3*cos(c + d
*x)**2/(3*d) + a**2*sin(c + d*x)*cos(c + d*x)**4/d - 2*a*b*cos(c + d*x)**5/
(5*d) + 2*b**2*sin(c + d*x)**5/(15*d) + b**2*sin(c + d*x)**3*cos(c + d*x)**
2/(3*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**2*cos(c)**3, True))
```

Giac [A]
time = 0.43, size = 114, normalized size = 1.11

$$-\frac{ab\cos(5\,dx+5\,c)}{40\,d} - \frac{ab\cos(3\,dx+3\,c)}{8\,d} - \frac{ab\cos(dx+c)}{4\,d} + \frac{(a^2-b^2)\sin(5\,dx+5\,c)}{80\,d} + \frac{(5\,a^2-b^2)\sin(3\,dx+3\,c)}{48\,d} + \frac{(5\,a^2+b^2)\sin(dx+c)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/40*a*b*cos(5*d*x + 5*c)/d - 1/8*a*b*cos(3*d*x + 3*c)/d - 1/4*a*b*cos(d*x
 + c)/d + 1/80*(a^2 - b^2)*sin(5*d*x + 5*c)/d + 1/48*(5*a^2 - b^2)*sin(3*d*
x + 3*c)/d + 1/8*(5*a^2 + b^2)*sin(d*x + c)/d
```

Mupad [B]
time = 0.61, size = 115, normalized size = 1.12

$$\frac{2\left(\frac{3\sin(c+d\,x)\,a^2\cos(c+d\,x)^4}{2} + 2\,\sin(c+d\,x)\,a^2\cos(c+d\,x)^2 + 4\,\sin(c+d\,x)\,a^2 - 3\,a\,b\cos(c+d\,x)^5 - \frac{3\sin(c+d\,x)\,b^2\cos(c+d\,x)^4}{2} + \frac{\sin(c+d\,x)\,b^2\cos(c+d\,x)^2}{2} + \sin(c+d\,x)\,b^2\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

```
[Out] (2*(4*a^2*sin(c + d*x) + b^2*sin(c + d*x) + 2*a^2*cos(c + d*x)^2*sin(c + d*
x) + (3*a^2*cos(c + d*x)^4*sin(c + d*x))/2 + (b^2*cos(c + d*x)^2*sin(c + d*
x))/2 - (3*b^2*cos(c + d*x)^4*sin(c + d*x))/2 - 3*a*b*cos(c + d*x)^5))/(15*
d)
```

3.46 $\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=126

$$\frac{3a^2x}{8}+\frac{b^2x}{8}-\frac{ab\cos^4(c+dx)}{2d}+\frac{3a^2\cos(c+dx)\sin(c+dx)}{8d}+\frac{b^2\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2\cos^3(c+dx)\sin(c}{4d}$$

```
[Out] 3/8*a^2*x+1/8*b^2*x-1/2*a*b*cos(d*x+c)^4/d+3/8*a^2*cos(d*x+c)*sin(d*x+c)/d+
1/8*b^2*cos(d*x+c)*sin(d*x+c)/d+1/4*a^2*cos(d*x+c)^3*sin(d*x+c)/d-1/4*b^2*c
os(d*x+c)^3*sin(d*x+c)/d
```

Rubi [A]

time = 0.10, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3169, 2715, 8, 2645, 30, 2648}

$$\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{3a^2x}{8}-\frac{ab\cos^4(c+dx)}{2d}-\frac{b^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{b^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{b^2x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (3*a^2*x)/8 + (b^2*x)/8 - (a*b*Cos[c + d*x]^4)/(2*d) + (3*a^2*Cos[c + d*x]*
Sin[c + d*x])/(8*d) + (b^2*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*Cos[c +
d*x]^3*Sin[c + d*x])/(4*d) - (b^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
```

```
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \int \left(a^2\cos^4(c+dx)+2ab\cos^3(c+dx)\sin(c+dx)+b^2 \mathrm{c}\right.$$

$$= a^2\int \cos^4(c+dx)\,dx+(2ab)\int \cos^3(c+dx)\sin(c+dx)$$

$$= \frac{a^2\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{b^2\cos^3(c+dx)\sin(c+dx}{4d}$$

$$= -\frac{ab\cos^4(c+dx)}{2d}+\frac{3a^2\cos(c+dx)\sin(c+dx)}{8d}+\frac{b^2\cos}{}$$

$$= \frac{3a^2x}{8}+\frac{b^2x}{8}-\frac{ab\cos^4(c+dx)}{2d}+\frac{3a^2\cos(c+dx)\sin(c+}{8d}$$

Mathematica [A]
time = 0.25, size = 98, normalized size = 0.78

$$\frac{(3a^2+b^2)\,(c+dx)}{8d}-\frac{ab\cos(2(c+dx))}{4d}-\frac{ab\cos(4(c+dx))}{16d}+\frac{a^2\sin(2(c+dx))}{4d}+\frac{(a^2-b^2)\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((3*a^2 + b^2)*(c + d*x))/(8*d) - (a*b*Cos[2*(c + d*x)])/(4*d) - (a*b*Cos[4
*(c + d*x)])/(16*d) + (a^2*Sin[2*(c + d*x)])/(4*d) + ((a^2 - b^2)*Sin[4*(c
+ d*x)])/(32*d)
```

Maple [A]
time = 0.17, size = 97, normalized size = 0.77

method	result
derivativedivides	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{ab\left(\cos^4(dx+c)\right)}{2}+a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\right.}{d}$
default	$\frac{b^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{ab\left(\cos^4(dx+c)\right)}{2}+a^2\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\right.}{d}$
risch	$\frac{3a^2x}{8}+\frac{b^2x}{8}-\frac{ab\cos(4dx+4c)}{16d}+\frac{\sin(4dx+4c)a^2}{32d}-\frac{\sin(4dx+4c)b^2}{32d}-\frac{ab\cos(2dx+2c)}{4d}+\frac{\sin(2dx+2c)a^2}{4d}$
norman	$\left(\frac{3a^2}{8}+\frac{b^2}{8}\right)x+\left(\frac{3a^2}{2}+\frac{b^2}{2}\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{3a^2}{2}+\frac{b^2}{2}\right)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{3a^2}{8}+\frac{b^2}{8}\right)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{9a^2}{4}+\frac{3b^2}{4}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/
8*c)-1/2*a*b*cos(d*x+c)^4+a^2*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)
+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.28, size = 75, normalized size = 0.60

$$-\frac{16\,ab\cos\left(dx+c\right)^4-\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^2-\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)b^2}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/32*(16*a*b*cos(d*x + c)^4 - (12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*
d*x + 2*c))*a^2 - (4*d*x + 4*c - sin(4*d*x + 4*c))*b^2)/d
```

Fricas [A]
time = 2.24, size = 75, normalized size = 0.60

$$-\frac{4\,ab\cos\left(dx+c\right)^4-\left(3\,a^2+b^2\right)dx-\left(2\,\left(a^2-b^2\right)\cos\left(dx+c\right)^3+\left(3\,a^2+b^2\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/8*(4*a*b*cos(d*x + c)^4 - (3*a^2 + b^2)*d*x - (2*(a^2 - b^2)*cos(d*x + c
)^3 + (3*a^2 + b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(116)=232$.
time = 0.20, size = 238, normalized size = 1.89

$$\begin{cases}\frac{3a^2x\sin^4\left(c+dx\right)}{8}+\frac{3a^2x\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{4}+\frac{3a^2x\cos^4\left(c+dx\right)}{8}+\frac{3a^2\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{8d}+\frac{5a^2\sin\left(c+dx\right)\cos^3\left(c+dx\right)}{8d}-\frac{ab\cos^4\left(c+dx\right)}{2d}+\frac{b^2x\sin^4\left(c+dx\right)}{8}+\frac{b^2x\sin^2\left(c+dx\right)\cos^2\left(c+dx\right)}{4}+\frac{b^2x\cos^4\left(c+dx\right)}{8}+\frac{b^2\sin^3\left(c+dx\right)\cos\left(c+dx\right)}{8d}-\frac{b^2\sin\left(c+dx\right)\cos^3\left(c+dx\right)}{8d} & \text{for}\; d\neq 0\\ x\left(a\cos\left(c\right)+b\sin\left(c\right)\right)^2\cos^2\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(c + d*x)**4/8 + 3*a**2*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + 3*a**2*x*cos(c + d*x)**4/8 + 3*a**2*sin(c + d*x)**3*cos(c + d*x)/
(8*d) + 5*a**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) - a*b*cos(c + d*x)**4/(2*
d) + b**2*x*sin(c + d*x)**4/8 + b**2*x*sin(c + d*x)**2*cos(c + d*x)**2/4 +
b**2*x*cos(c + d*x)**4/8 + b**2*sin(c + d*x)**3*cos(c + d*x)/(8*d) - b**2*s
in(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**2*c
os(c)**2, True))
```

Giac [A]
time = 0.44, size = 85, normalized size = 0.67

$$\frac{1}{8}\left(3\,a^{2}+b^{2}\right)x-\frac{ab\cos\left(4\,dx+4\,c\right)}{16\,d}-\frac{ab\cos\left(2\,dx+2\,c\right)}{4\,d}+\frac{a^{2}\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{\left(a^{2}-b^{2}\right)\sin\left(4\,dx+4\,c\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*a^2 + b^2)*x - 1/16*a*b*cos(4*d*x + 4*c)/d - 1/4*a*b*cos(2*d*x + 2*c
)/d + 1/4*a^2*sin(2*d*x + 2*c)/d + 1/32*(a^2 - b^2)*sin(4*d*x + 4*c)/d
```

Mupad [B]
time = 0.58, size = 89, normalized size = 0.71

$$\frac{4\,a^2\,\sin\left(2\,c+2\,d\,x\right)+\frac{a^2\,\sin\left(4\,c+4\,d\,x\right)}{2}-\frac{b^2\,\sin\left(4\,c+4\,d\,x\right)}{2}+2\,a\,b\,{\sin\left(2\,c+2\,d\,x\right)}^2+8\,a\,b\,{\sin\left(c+d\,x\right)}^2+6\,a^2\,d\,x+2\,b^2\,d\,x}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

```
[Out] (4*a^2*sin(2*c + 2*d*x) + (a^2*sin(4*c + 4*d*x))/2 - (b^2*sin(4*c + 4*d*x))
/2 + 2*a*b*sin(2*c + 2*d*x)^2 + 8*a*b*sin(c + d*x)^2 + 6*a^2*d*x + 2*b^2*d*
x)/(16*d)
```

3.47 $\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=67

$$-\frac{2ab\cos^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{3d}+\frac{b^2\sin^3(c+dx)}{3d}$$

```
[Out] -2/3*a*b*cos(d*x+c)^3/d+a^2*sin(d*x+c)/d-1/3*a^2*sin(d*x+c)^3/d+1/3*b^2*sin
(d*x+c)^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3169, 2713, 2645, 30, 2644}

$$-\frac{a^2\sin^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{2ab\cos^3(c+dx)}{3d}+\frac{b^2\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*b*Cos[c + d*x]^3)/(3*d) + (a^2*Sin[c + d*x])/d - (a^2*Sin[c + d*x]^3)
/(3*d) + (b^2*Sin[c + d*x]^3)/(3*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx &= \int \left(a^2\cos^3(c+dx)+2ab\cos^2(c+dx)\sin(c+dx)+b^2\,\mathrm{co}\right.\\ &= a^2\int \cos^3(c+dx)\,dx+(2ab)\int \cos^2(c+dx)\sin(c+dx)\,\mathrm{c}\\ &= -\frac{a^2\mathrm{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+dx)\right)}{d}-\frac{(2ab)\mathrm{Subst}}{}\\ &= -\frac{2ab\cos^3(c+dx)}{3d}+\frac{a^2\sin(c+dx)}{d}-\frac{a^2\sin^3(c+dx)}{3d}+\end{aligned}$$

Mathematica [A]
time = 0.43, size = 64, normalized size = 0.96

$$\frac{-3ab\cos(c+dx)-ab\cos(3(c+dx))+\left(5a^2+b^2+\left(a^2-b^2\right)\cos(2(c+dx))\right)\sin(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (-3*a*b*Cos[c + d*x] - a*b*Cos[3*(c + d*x)] + (5*a^2 + b^2 + (a^2 - b^2)*Co
s[2*(c + d*x)])*Sin[c + d*x])/(6*d)
```

Maple [A]
time = 0.14, size = 52, normalized size = 0.78

method	result	size
derivativedivides	$\frac{\frac{b^2\left(\sin^3(dx+c)\right)}{3}-\frac{2ab\left(\cos^3(dx+c)\right)}{3}+\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$	52
default	$\frac{\frac{b^2\left(\sin^3(dx+c)\right)}{3}-\frac{2ab\left(\cos^3(dx+c)\right)}{3}+\frac{a^2\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$	52
risch	$-\frac{ab\cos(dx+c)}{2d}+\frac{3a^2\sin(dx+c)}{4d}+\frac{b^2\sin(dx+c)}{4d}-\frac{ab\cos(3dx+3c)}{6d}+\frac{\sin(3dx+3c)a^2}{12d}-\frac{\sin(3dx+3c)b^2}{12d}$	93

norman	$\frac{-\frac{4ab}{3d}+\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4\left(a^2+2b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{4ab\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$	104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*b^2*sin(d*x+c)^3-2/3*a*b*cos(d*x+c)^3+1/3*a^2*(2+cos(d*x+c)^2)*sin
(d*x+c))
```

Maxima [A]
time = 0.28, size = 52, normalized size = 0.78

$$-\frac{2\,ab\cos\left(dx+c\right)^3-b^2\sin\left(dx+c\right)^3+\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^2}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(2*a*b*cos(d*x + c)^3 - b^2*sin(d*x + c)^3 + (sin(d*x + c)^3 - 3*sin(d
*x + c))*a^2)/d
```

Fricas [A]
time = 2.32, size = 53, normalized size = 0.79

$$-\frac{2\,ab\cos\left(dx+c\right)^3-\left(\left(a^2-b^2\right)\cos\left(dx+c\right)^2+2\,a^2+b^2\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(2*a*b*cos(d*x + c)^3 - ((a^2 - b^2)*cos(d*x + c)^2 + 2*a^2 + b^2)*sin
(d*x + c))/d
```

Sympy [A]
time = 0.13, size = 85, normalized size = 1.27

$$\begin{cases} \frac{2a^2\sin^3\left(c+dx\right)}{3d}+\frac{a^2\sin\left(c+dx\right)\cos^2\left(c+dx\right)}{d}-\frac{2ab\cos^3\left(c+dx\right)}{3d}+\frac{b^2\sin^3\left(c+dx\right)}{3d} & \text{for } d\neq 0 \\ x\left(a\cos\left(c\right)+b\sin\left(c\right)\right)^2\cos\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((2*a**2*sin(c + d*x)**3/(3*d) + a**2*sin(c + d*x)*cos(c + d*x)**2
/d - 2*a*b*cos(c + d*x)**3/(3*d) + b**2*sin(c + d*x)**3/(3*d), Ne(d, 0)), (
x*(a*cos(c) + b*sin(c))**2*cos(c), True))
```

Giac [A]
time = 0.42, size = 73, normalized size = 1.09

$$-\frac{ab\cos\left(3\,dx+3\,c\right)}{6\,d}-\frac{ab\cos\left(dx+c\right)}{2\,d}+\frac{\left(a^{2}-b^{2}\right)\sin\left(3\,dx+3\,c\right)}{12\,d}+\frac{\left(3\,a^{2}+b^{2}\right)\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/6*a*b*cos(3*d*x + 3*c)/d - 1/2*a*b*cos(d*x + c)/d + 1/12*(a^2 - b^2)*sin
(3*d*x + 3*c)/d + 1/4*(3*a^2 + b^2)*sin(d*x + c)/d
```

Mupad [B]
time = 0.51, size = 77, normalized size = 1.15

$$\frac{2\left(\frac{\sin(c+d\,x)\,a^{2}\cos(c+d\,x)^{2}}{2}+\sin\left(c+d\,x\right)\,a^{2}-a\,b\cos\left(c+d\,x\right)^{3}-\frac{\sin(c+d\,x)\,b^{2}\cos(c+d\,x)^{2}}{2}+\frac{\sin(c+d\,x)\,b^{2}}{2}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

```
[Out] (2*(a^2*sin(c + d*x) + (b^2*sin(c + d*x))/2 + (a^2*cos(c + d*x)^2*sin(c + d
*x))/2 - (b^2*cos(c + d*x)^2*sin(c + d*x))/2 - a*b*cos(c + d*x)^3))/(3*d)
```

3.48 $\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=55

$$\frac{1}{2}\left(a^2+b^2\right)x-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}$$

```
[Out] 1/2*(a^2+b^2)*x-1/2*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+c))
/d
```

Rubi [A]
time = 0.01, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3152, 8}

$$\frac{1}{2}x\left(a^2+b^2\right)-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2 + b^2)*x)/2 - ((b*Cos[c + d*x] - a*Sin[c + d*x])*(a*Cos[c + d*x] + b*
Sin[c + d*x]))/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3152

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[(n - 1)*((a^2 + b^2)/n), Int[(a*Cos
[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int (a\cos(c+dx)+b\sin(c+dx))^2\,dx &= -\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{2d}+\frac{1}{2}\ldots\\ &= \frac{1}{2}\left(a^2+b^2\right)x-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin\ldots}{2d}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 52, normalized size = 0.95

$$\frac{2\left(a^2+b^2\right)(c+dx)-2ab\cos(2(c+dx))+\left(a^2-b^2\right)\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*(a^2 + b^2)*(c + d*x) - 2*a*b*Cos[2*(c + d*x)] + (a^2 - b^2)*Sin[2*(c +
d*x)])/(4*d)
```

Maple [A]
time = 0.12, size = 70, normalized size = 1.27

method	result
risch	$\frac{a^2 x}{2}+\frac{b^2 x}{2}-\frac{ab\cos(2dx+2c)}{2d}+\frac{\sin(2dx+2c)a^2}{4d}-\frac{\sin(2dx+2c)b^2}{4d}$
derivativedivides	$\frac{b^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-ab\left(\cos^2(dx+c)\right)+a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
default	$\frac{b^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-ab\left(\cos^2(dx+c)\right)+a^2\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
norman	$\frac{\left(\frac{a^2}{2}+\frac{b^2}{2}\right)x+\left(\frac{a^2}{2}+\frac{b^2}{2}\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{\left(a^2-b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\left(a^2+b^2\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{\left(a^2-b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^2*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-a*b*cos(d*x+c)^2+a^2*(1
/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.27, size = 68, normalized size = 1.24

$$-\frac{ab\cos\left(dx+c\right)^2}{d}+\frac{\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^2}{4\,d}+\frac{\left(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right)\right)b^2}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -a*b*cos(d*x + c)^2/d + 1/4*(2*d*x + 2*c + sin(2*d*x + 2*c))*a^2/d + 1/4*(2
*d*x + 2*c - sin(2*d*x + 2*c))*b^2/d
```

Fricas [A]
time = 2.17, size = 52, normalized size = 0.95

$$-\frac{2\,ab\cos\left(dx+c\right)^2-\left(a^2+b^2\right)dx-\left(a^2-b^2\right)\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*b*cos(d*x + c)^2 - (a^2 + b^2)*d*x - (a^2 - b^2)*cos(d*x + c)*sin
(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(49) = 98$.
time = 0.09, size = 128, normalized size = 2.33

$$\begin{cases} \frac{a^2x\sin^2(c+dx)}{2} + \frac{a^2x\cos^2(c+dx)}{2} + \frac{a^2\sin(c+dx)\cos(c+dx)}{2d} + \frac{ab\sin^2(c+dx)}{d} + \frac{b^2x\sin^2(c+dx)}{2} + \frac{b^2x\cos^2(c+dx)}{2} - \frac{b^2\sin(c+dx)\cos(c+dx)}{2d} & \text{for } d \neq 0 \\ x(a\cos(c) + b\sin(c))^2 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((a**2*x*sin(c + d*x)**2/2 + a**2*x*cos(c + d*x)**2/2 + a**2*sin(c
 + d*x)*cos(c + d*x)/(2*d) + a*b*sin(c + d*x)**2/d + b**2*x*sin(c + d*x)**2
/2 + b**2*x*cos(c + d*x)**2/2 - b**2*sin(c + d*x)*cos(c + d*x)/(2*d), Ne(d,
 0)), (x*(a*cos(c) + b*sin(c))**2, True))
```

Giac [A]
time = 0.40, size = 50, normalized size = 0.91

$$\frac{1}{2}\left(a^2+b^2\right)x - \frac{ab\cos(2\,dx+2\,c)}{2\,d} + \frac{\left(a^2-b^2\right)\sin(2\,dx+2\,c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^2 + b^2)*x - 1/2*a*b*cos(2*d*x + 2*c)/d + 1/4*(a^2 - b^2)*sin(2*d*x
+ 2*c)/d
```

Mupad [B]
time = 0.48, size = 63, normalized size = 1.15

$$\frac{a^2\,x}{2} + \frac{b^2\,x}{2} + \frac{a^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d} - \frac{b^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d} - \frac{a\,b\,\cos\left(2\,c+2\,d\,x\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

```
[Out] (a^2*x)/2 + (b^2*x)/2 + (a^2*sin(2*c + 2*d*x))/(4*d) - (b^2*sin(2*c + 2*d*x
))/(4*d) - (a*b*cos(2*c + 2*d*x))/(2*d)
```

3.49 $\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=55

$$\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ab\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)}{d}-\frac{b^2\sin(c+dx)}{d}$$

```
[Out] b^2*arctanh(sin(d*x+c))/d-2*a*b*cos(d*x+c)/d+a^2*sin(d*x+c)/d-b^2*sin(d*x+c
)/d
```

Rubi [A]
time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3169, 2717, 2718, 2672, 327, 212}

$$\frac{a^2\sin(c+dx)}{d}-\frac{2ab\cos(c+dx)}{d}-\frac{b^2\sin(c+dx)}{d}+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (b^2*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Cos[c + d*x])/d + (a^2*Sin[c + d*x])
/d - (b^2*Sin[c + d*x])/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \int \left(a^2\cos(c+dx)+2ab\sin(c+dx)+b^2\sin(c+dx)\tan\right.$$
$$= a^2\int\cos(c+dx)\,dx+(2ab)\int\sin(c+dx)\,dx+b^2\int \sin$$
$$= -\frac{2ab\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)}{d}+\frac{b^2\text{Subst}\left(\int\frac{x^2}{1-x^2}\,dx\right.}{}$$
$$= -\frac{2ab\cos(c+dx)}{d}+\frac{a^2\sin(c+dx)}{d}-\frac{b^2\sin(c+dx)}{d}+$$
$$= \frac{b^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ab\cos(c+dx)}{d}+\frac{a^2\sin(c+}{d}$$

Mathematica [A]
time = 0.17, size = 84, normalized size = 1.53

$$\frac{-2ab\cos(c+dx)+b^2\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+(a^2-b^2)\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (-2*a*b*Cos[c + d*x] + b^2*(-Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (a^2 - b^2)*Sin[c + d*x])/d
```

Maple [A]
time = 0.20, size = 53, normalized size = 0.96

method	result
derivativedivides	$\frac{\sin(dx+c)a^2-2ab\cos(dx+c)+b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
default	$\frac{\sin(dx+c)a^2-2ab\cos(dx+c)+b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
norman	$\frac{\frac{4ab\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2\left(a^2-b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2\left(a^2-b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{b^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-$
risch	$-\frac{e^{i(dx+c)}ab}{d}-\frac{ie^{i(dx+c)}a^2}{2d}+\frac{ie^{i(dx+c)}b^2}{2d}-\frac{e^{-i(dx+c)}ab}{d}+\frac{ie^{-i(dx+c)}a^2}{2d}-\frac{ie^{-i(dx+c)}b^2}{2d}+\frac{b^2\ln\left(e^{i(dx+c)}+i\right)}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(sin(d*x+c)*a^2-2*a*b*cos(d*x+c)+b^2*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x
+c))))
```

Maxima [A]
time = 0.27, size = 60, normalized size = 1.09

$$\frac{b^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\sin(dx+c))-4ab\cos(dx+c)+2a^2\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) -
 4*a*b*cos(d*x + c) + 2*a^2*sin(d*x + c))/d
```

Fricas [A]
time = 2.79, size = 62, normalized size = 1.13

$$-\frac{4ab\cos(dx+c)-b^2\log(\sin(dx+c)+1)+b^2\log(-\sin(dx+c)+1)-2(a^2-b^2)\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a*b*cos(d*x + c) - b^2*log(sin(d*x + c) + 1) + b^2*log(-sin(d*x + c
) + 1) - 2*(a^2 - b^2)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(a\cos(c+dx)+b\sin(c+dx))^2\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))**2*sec(c + d*x), x)
```

Giac [A]
time = 0.44, size = 89, normalized size = 1.62

$$\frac{b^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-b^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,ab\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (b^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - b^2*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + 2*(a^2*tan(1/2*d*x + 1/2*c) - b^2*tan(1/2*d*x + 1/2*c) - 2*a*b)/(ta
n(1/2*d*x + 1/2*c)^2 + 1))/d
```

Mupad [B]
time = 0.49, size = 66, normalized size = 1.20

$$\frac{2\,b^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{4\,a\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2-2\,b^2\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^2/cos(c + d*x),x)
```

```
[Out] (2*b^2*atanh(tan(c/2 + (d*x)/2)))/d - (4*a*b - tan(c/2 + (d*x)/2)*(2*a^2 -
2*b^2))/(d*(tan(c/2 + (d*x)/2)^2 + 1))
```

3.50 $\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=39

$$\left(a^2-b^2\right)x-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

```
[Out] (a^2-b^2)*x-2*a*b*ln(cos(d*x+c))/d+b^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.04, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {3165, 3558, 3556}

$$x\left(a^2-b^2\right)-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2 - b^2)*x - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Tan[c + d*x])/d
```

Rule 3165

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3558

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(a^2 - b^2)
*x, x] + (Dist[2*a*b, Int[Tan[c + d*x], x], x] + Simp[b^2*(Tan[c + d*x]/d),
 x]) /; FreeQ[{a, b, c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx &= \int (a+b\tan(c+dx))^2\,dx\\ &= \left(a^2-b^2\right)x+\frac{b^2\tan(c+dx)}{d}+(2ab)\int \tan(c+dx)\,dx\\ &= \left(a^2-b^2\right)x-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.15, size = 69, normalized size = 1.77

$$\frac{-i((a+ib)^2\log(i-\tan(c+dx))-(a-ib)^2\log(i+\tan(c+dx)))+2b^2\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((-I)*((a + I*b)^2*Log[I - Tan[c + d*x]] - (a - I*b)^2*Log[I + Tan[c + d*x]
]) + 2*b^2*Tan[c + d*x])/(2*d)
```

Maple [A]
time = 0.22, size = 44, normalized size = 1.13

method	result
derivativedivides	$\frac{a^2(dx+c)-2ab\ln(\cos(dx+c))+b^2(\tan(dx+c)-dx-c)}{d}$
default	$\frac{a^2(dx+c)-2ab\ln(\cos(dx+c))+b^2(\tan(dx+c)-dx-c)}{d}$
risch	$2iabx+a^2x-b^2x+\frac{4iabc}{d}+\frac{2ib^2}{d(e^{2i(dx+c)}+1)}-\frac{2ab\ln(e^{2i(dx+c)}+1)}{d}$
norman	$\frac{(-a^2+b^2)x+(-a^2+b^2)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(a^2-b^2)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(a^2-b^2)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-4}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(d*x+c)-2*a*b*ln(cos(d*x+c))+b^2*(tan(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.49, size = 49, normalized size = 1.26

$$\frac{(dx+c)a^2-(dx+c-\tan(dx+c))b^2-ab\log\left(-\sin(dx+c)^2+1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] ((d*x + c)*a^2 - (d*x + c - tan(d*x + c))*b^2 - a*b*log(-sin(d*x + c)^2 + 1
))/d
```

Fricas [A]
time = 2.66, size = 60, normalized size = 1.54

$$\frac{(a^2-b^2)dx\cos(dx+c)-2ab\cos(dx+c)\log(-\cos(dx+c))+b^2\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] ((a^2 - b^2)*d*x*cos(d*x + c) - 2*a*b*cos(d*x + c)*log(-cos(d*x + c)) + b^2
*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(c+dx)+b\sin(c+dx))^2\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))**2*sec(c + d*x)**2, x)
```

Giac [A]
time = 0.44, size = 44, normalized size = 1.13

$$\frac{ab\log\left(\tan(dx+c)^2+1\right)+b^2\tan(dx+c)+(a^2-b^2)(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (a*b*log(tan(d*x + c)^2 + 1) + b^2*tan(d*x + c) + (a^2 - b^2)*(d*x + c))/d
```

Mupad [B]
time = 0.68, size = 118, normalized size = 3.03

$$\frac{b^2\tan(c+d\,x)}{d}+\frac{2\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{2\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{2\,a\,b\,\ln\left(\frac{\cos(c+d\,x)}{\cos(c+d\,x)+1}\right)}{d}+\frac{2\,a\,b\,\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^2/cos(c + d*x)^2,x)
```

```
[Out] (b^2*tan(c + d*x))/d + (2*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/
d - (2*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (2*a*b*log(cos(
c + d*x)/(cos(c + d*x) + 1)))/d + (2*a*b*log(1/cos(c/2 + (d*x)/2)^2))/d
```

3.51 $\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=67

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{2ab\sec(c+dx)}{d}+\frac{b^2\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] a^2*arctanh(sin(d*x+c))/d-1/2*b^2*arctanh(sin(d*x+c))/d+2*a*b*sec(d*x+c)/d+
1/2*b^2*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3169, 3855, 2686, 8, 2691}

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{2ab\sec(c+dx)}{d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b^2\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/d - (b^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a*b*
Sec[c + d*x])/d + (b^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
```

```
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = \int \left(a^2\sec(c+dx)+2ab\sec(c+dx)\tan(c+dx)+b^2\sec\right.$$
$$= a^2\int \sec(c+dx)\,dx+(2ab)\int \sec(c+dx)\tan(c+dx)\,d$$
$$= \frac{a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\sec(c+dx)\tan(c+dx)}{2d}-\frac{1}{2}$$
$$= \frac{a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{2ab}{}$$

Mathematica [A]
time = 0.05, size = 67, normalized size = 1.00

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{2ab\sec(c+dx)}{d}+\frac{b^2\sec(c+dx)\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/d - (b^2*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a*b*
Sec[c + d*x])/d + (b^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A]
time = 0.25, size = 83, normalized size = 1.24

method	result
derivativedivides	$\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))+\frac{2ab}{\cos(dx+c)}+b^2\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))+\frac{2ab}{\cos(dx+c)}+b^2\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
risch	$\frac{b\left(-ib\,e^{3i(dx+c)}+4a\,e^{3i(dx+c)}+ib\,e^{i(dx+c)}+4a\,e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{\ln\left(e^{i(dx+c)}-i\right)a^2}{d}+\frac{b^2\ln\left(e^{i(dx+c)}-i\right)}{2d}+\frac{\ln\left(e^{i(dx+c)}+i\right)a}{d}$

norman	$\frac{\frac{4ab}{d}+\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{b^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{3b^2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4ab\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4ab\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*ln(sec(d*x+c)+tan(d*x+c))+2*a*b/cos(d*x+c)+b^2*(1/2*sin(d*x+c)^3/c
os(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.27, size = 89, normalized size = 1.33

$$-\frac{b^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)-2\,a^2(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))-\frac{8\,ab}{\cos(dx+c)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - lo
g(sin(d*x + c) - 1)) - 2*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)
) - 8*a*b/cos(d*x + c))/d
```

Fricas [A]
time = 3.07, size = 96, normalized size = 1.43

$$\frac{(2\,a^2-b^2)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-(2\,a^2-b^2)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+8\,ab\cos\left(dx+c\right)+2\,b^2\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^2 - b^2)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*a^2 - b^2)*cos
(d*x + c)^2*log(-sin(d*x + c) + 1) + 8*a*b*cos(d*x + c) + 2*b^2*sin(d*x + c
))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)^2\sec^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))**2*sec(c + d*x)**3, x)
```

Giac [A]
time = 0.47, size = 122, normalized size = 1.82

$$\frac{\left(2\,a^{2}-b^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(2\,a^{2}-b^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,ab\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*((2*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (2*a^2 - b^2)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(b^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1
/2*d*x + 1/2*c)^2 + b^2*tan(1/2*d*x + 1/2*c) + 4*a*b)/(tan(1/2*d*x + 1/2*c)
^2 - 1)^2)/d
```

Mupad [B]
time = 0.90, size = 106, normalized size = 1.58

$$\frac{b^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+b^{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+4\,a\,b}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}+\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a^{2}-b^{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^2/cos(c + d*x)^3,x)
```

```
[Out] (4*a*b + b^2*tan(c/2 + (d*x)/2)^3 + b^2*tan(c/2 + (d*x)/2) - 4*a*b*tan(c/2
+ (d*x)/2)^2)/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1)) + (at
anh(tan(c/2 + (d*x)/2))*(2*a^2 - b^2))/d
```

3.52 $\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=30

$$\frac{(b+a\cot(c+dx))^3\tan^3(c+dx)}{3bd}$$

```
[Out] 1/3*(b+a*cot(d*x+c))^3*tan(d*x+c)^3/b/d
```

Rubi [A]
time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3167, 37}

$$\frac{\tan^3(c+dx)(a\cot(c+dx)+b)^3}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((b + a*Cot[c + d*x])^3*Tan[c + d*x]^3)/(3*b*d)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx &= -\frac{\text{Subst}\left(\int \frac{(b+ax)^2}{x^4}\,dx, x, \cot(c+dx)\right)}{d}\\ &= \frac{(b+a\cot(c+dx))^3\tan^3(c+dx)}{3bd}\end{aligned}$$

Mathematica [A]
time = 0.74, size = 56, normalized size = 1.87

$$\frac{\sec^2(c+dx)\left(6ab+(3a^2+b^2+(3a^2-b^2)\cos(2(c+dx)))\tan(c+dx)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^2*(6*a*b + (3*a^2 + b^2 + (3*a^2 - b^2)*Cos[2*(c + d*x)])*Tan
[c + d*x]))/(6*d)
```

Maple [A]
time = 0.26, size = 48, normalized size = 1.60

method	result
derivativedivides	$\frac{a^2\tan(dx+c)+\frac{ab}{\cos(dx+c)^2}+\frac{b^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}}{d}$
default	$\frac{a^2\tan(dx+c)+\frac{ab}{\cos(dx+c)^2}+\frac{b^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}}{d}$
risch	$-\frac{2i\left(6iab\,e^{4i(dx+c)}-3a^2e^{4i(dx+c)}+3b^2e^{4i(dx+c)}+6iab\,e^{2i(dx+c)}-6a^2e^{2i(dx+c)}-3a^2+b^2\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}$
norman	$\frac{\frac{4ab\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a^2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{8b^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{8b^2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{4\left(3a^2-4b^2\right)\left(\tan^5\left(\frac{dx}{2}-\right.\right.}{3d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*tan(d*x+c)+a*b/cos(d*x+c)^2+1/3*b^2*sin(d*x+c)^3/cos(d*x+c)^3)
```

Maxima [A]
time = 0.27, size = 45, normalized size = 1.50

$$\frac{b^2\tan(dx+c)^3+3a^2\tan(dx+c)-\frac{3ab}{\sin(dx+c)^2-1}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(b^2*tan(d*x + c)^3 + 3*a^2*tan(d*x + c) - 3*a*b/(sin(d*x + c)^2 - 1))/
d
```

Fricas [A]
time = 3.29, size = 55, normalized size = 1.83

$$\frac{3ab\cos(dx+c)+\left((3a^2-b^2)\cos(dx+c)^2+b^2\right)\sin(dx+c)}{3d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a*b*cos(d*x + c) + ((3*a^2 - b^2)*cos(d*x + c)^2 + b^2)*sin(d*x + c)
)/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos(c+dx)+b\sin(c+dx))^2\sec^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))**2*sec(c + d*x)**4, x)
```

Giac [A]
time = 0.46, size = 41, normalized size = 1.37

$$\frac{b^2\tan(dx+c)^3+3\,ab\tan(dx+c)^2+3\,a^2\tan(dx+c)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3*(b^2*tan(d*x + c)^3 + 3*a*b*tan(d*x + c)^2 + 3*a^2*tan(d*x + c))/d
```

Mupad [B]
time = 0.49, size = 68, normalized size = 2.27

$$\frac{\frac{b^2\sin(c+d\,x)}{3}+\frac{\cos(c+d\,x)^2\sin(c+d\,x)\,(3\,a^2-b^2)}{3}+a\,b\,\cos(c+d\,x)\,\sin(c+d\,x)^2}{d\cos(c+d\,x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^2/cos(c + d*x)^4,x)
```

```
[Out] ((b^2*sin(c + d*x))/3 + (cos(c + d*x)^2*sin(c + d*x)*(3*a^2 - b^2))/3 + a*b
*cos(c + d*x)*sin(c + d*x)^2)/(d*cos(c + d*x)^3)
```

3.53 $\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=120

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{2d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{2ab\sec^3(c+dx)}{3d}+\frac{a^2\sec(c+dx)\tan(c+dx)}{2d}-\frac{b^2\sec(c+d}{8}$$

```
[Out] 1/2*a^2*arctanh(sin(d*x+c))/d-1/8*b^2*arctanh(sin(d*x+c))/d+2/3*a*b*sec(d*x
+c)^3/d+1/2*a^2*sec(d*x+c)*tan(d*x+c)/d-1/8*b^2*sec(d*x+c)*tan(d*x+c)/d+1/4
*b^2*sec(d*x+c)^3*tan(d*x+c)/d
```

Rubi [A]
time = 0.10, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3169, 3853, 3855, 2686, 30, 2691}

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2\tan(c+dx)\sec(c+dx)}{2d}+\frac{2ab\sec^3(c+dx)}{3d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b^2\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{b^2\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/(2*d) - (b^2*ArcTanh[Sin[c + d*x]])/(8*d) + (2*
a*b*Sec[c + d*x]^3)/(3*d) + (a^2*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (b^2*Se
c[c + d*x]*Tan[c + d*x])/(8*d) + (b^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx &= \int \left(a^2\sec^3(c+dx)+2ab\sec^3(c+dx)\tan(c+dx)+b^2\right.\\ &= a^2\int \sec^3(c+dx)\,dx+(2ab)\int \sec^3(c+dx)\tan(c+dx\\ &= \frac{a^2\sec(c+dx)\tan(c+dx)}{2d}+\frac{b^2\sec^3(c+dx)\tan(c+dx}{4d}\\ &= \frac{a^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{2ab\sec^3(c+dx)}{3d}+\frac{a^2\sec(c-}{}\\ &= \frac{a^2\tanh^{-1}(\sin(c+dx))}{2d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{2a}{}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 120, normalized size = 1.00

$$\frac{a^2\tanh^{-1}(\sin(c+dx))}{2d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{2ab\sec^3(c+dx)}{3d}+\frac{a^2\sec(c+dx)\tan(c+dx)}{2d}-\frac{b^2\sec(c+dx)\tan(c+dx)}{8d}+\frac{b^2\sec^3(c+dx)\tan(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*ArcTanh[Sin[c + d*x]])/(2*d) - (b^2*ArcTanh[Sin[c + d*x]])/(8*d) + (2*
a*b*Sec[c + d*x]^3)/(3*d) + (a^2*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (b^2*Se
c[c + d*x]*Tan[c + d*x])/(8*d) + (b^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Maple [A]
time = 0.32, size = 118, normalized size = 0.98

method	result
derivativedivides	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{2ab}{3\cos(dx+c)^3}+b^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)}{}\right.}{d}$
default	$\frac{a^2\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{2ab}{3\cos(dx+c)^3}+b^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+c)}{}\right.}{d}$
risch	$-\frac{i(12a^2e^{7i(dx+c)}-3b^2e^{7i(dx+c)}+12a^2e^{5i(dx+c)}+21b^2e^{5i(dx+c)}+64iab\,e^{5i(dx+c)}-12a^2e^{3i(dx+c)}-21b^2e^{3i(dx+c)}+64iab\,e}{12d\left(e^{2i(dx+c)}+1\right)^4}$
norman	$\frac{\frac{4ab}{3d}-\frac{(4a^2-11b^2)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}-\frac{(4a^2-11b^2)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{(4a^2+b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{(4a^2+b^2)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{(4a^2+9}{}}{\left(\tan^2\left(\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+2/3*a*b/
cos(d*x+c)^3+b^2*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)
^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.28, size = 129, normalized size = 1.08

$$\frac{3\,b^2\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-12\,a^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{32\,ab}{\cos(dx+c)^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(3*b^2*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x
+ c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 12*a^2*(2*si
n(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c)
- 1)) + 32*a*b/cos(d*x + c)^3)/d
```

Fricas [A]
time = 3.74, size = 120, normalized size = 1.00

$$\frac{3\,(4\,a^2-b^2)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\,(4\,a^2-b^2)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+32\,ab\cos\left(dx+c\right)+6\left(\left(4\,a^2-b^2\right)\cos\left(dx+c\right)^2+2\,b^2\right)\sin\left(dx+c\right)}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(4*a^2 - b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*a^2 - b^2
)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 32*a*b*cos(d*x + c) + 6*((4*a^2 -
 b^2)*cos(d*x + c)^2 + 2*b^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(108) = 216$.
time = 0.49, size = 249, normalized size = 2.08

$$\frac{3\,(4\,a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(4\,a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-48\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+21\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+48\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+21\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-16\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+16\,ab\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*(4*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*a^2 - b^2)*
log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(12*a^2*tan(1/2*d*x + 1/2*c)^7 + 3*b
^2*tan(1/2*d*x + 1/2*c)^7 - 48*a*b*tan(1/2*d*x + 1/2*c)^6 - 12*a^2*tan(1/2*
d*x + 1/2*c)^5 + 21*b^2*tan(1/2*d*x + 1/2*c)^5 + 48*a*b*tan(1/2*d*x + 1/2*c
)^4 - 12*a^2*tan(1/2*d*x + 1/2*c)^3 + 21*b^2*tan(1/2*d*x + 1/2*c)^3 - 16*a*
b*tan(1/2*d*x + 1/2*c)^2 + 12*a^2*tan(1/2*d*x + 1/2*c) + 3*b^2*tan(1/2*d*x
+ 1/2*c) + 16*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [**B**]
time = 3.16, size = 216, normalized size = 1.80

$$\frac{\left(a^2+\frac{b^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(\frac{7\,b^2}{4}-a^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(\frac{7\,b^2}{4}-a^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\frac{4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\left(a^2+\frac{b^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{4\,a\,b}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2-\frac{b^2}{4}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^2/cos(c + d*x)^5,x)
```

```
[Out] ((4*a*b)/3 + tan(c/2 + (d*x)/2)*(a^2 + b^2/4) + tan(c/2 + (d*x)/2)^7*(a^2 +
 b^2/4) - tan(c/2 + (d*x)/2)^3*(a^2 - (7*b^2)/4) - tan(c/2 + (d*x)/2)^5*(a^
2 - (7*b^2)/4) - (4*a*b*tan(c/2 + (d*x)/2)^2)/3 + 4*a*b*tan(c/2 + (d*x)/2)^
4 - 4*a*b*tan(c/2 + (d*x)/2)^6)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d
*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (atanh(tan
(c/2 + (d*x)/2))*(a^2 - b^2/4))/d
```

3.54 $\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=85

$$\frac{a^2\tan(c+dx)}{d}+\frac{ab\tan^2(c+dx)}{d}+\frac{(a^2+b^2)\tan^3(c+dx)}{3d}+\frac{ab\tan^4(c+dx)}{2d}+\frac{b^2\tan^5(c+dx)}{5d}$$

```
[Out] a^2*tan(d*x+c)/d+a*b*tan(d*x+c)^2/d+1/3*(a^2+b^2)*tan(d*x+c)^3/d+1/2*a*b*ta
n(d*x+c)^4/d+1/5*b^2*tan(d*x+c)^5/d
```

Rubi [A]
time = 0.05, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3167, 908}

$$\frac{(a^2+b^2)\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}+\frac{ab\tan^4(c+dx)}{2d}+\frac{ab\tan^2(c+dx)}{d}+\frac{b^2\tan^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Tan[c + d*x])/d + (a*b*Tan[c + d*x]^2)/d + ((a^2 + b^2)*Tan[c + d*x]^3
)/(3*d) + (a*b*Tan[c + d*x]^4)/(2*d) + (b^2*Tan[c + d*x]^5)/(5*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^2(1+x^2)}{x^6}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{x^6}+\frac{2ab}{x^5}+\frac{a^2+b^2}{x^4}+\frac{2ab}{x^3}+\frac{a^2}{x^2}\right)dx, x, \cot(c-\right.}{d}$$
$$= \frac{a^2\tan(c+dx)}{d}+\frac{ab\tan^2(c+dx)}{d}+\frac{(a^2+b^2)\tan^3(c+}{3d}$$

Mathematica [A]
time = 0.22, size = 54, normalized size = 0.64

$$\frac{(a+b\tan(c+dx))^3\left(a^2+10b^2-3ab\tan(c+dx)+6b^2\tan^2(c+dx)\right)}{30b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a + b*Tan[c + d*x])^3*(a^2 + 10*b^2 - 3*a*b*Tan[c + d*x] + 6*b^2*Tan[c +
d*x]^2))/(30*b^3*d)
```

Maple [A]
time = 0.31, size = 82, normalized size = 0.96

method	result
derivativedivides	$\frac{-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{ab}{2\cos(dx+c)^4}+b^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)}{d}$
default	$\frac{-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{ab}{2\cos(dx+c)^4}+b^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)}{d}$
risch	$\frac{4i(-30iab\,e^{6i(dx+c)}+15a^2e^{6i(dx+c)}-15b^2e^{6i(dx+c)}-30iab\,e^{4i(dx+c)}+35a^2e^{4i(dx+c)}+5b^2e^{4i(dx+c)}+25a^2e^{2i(dx+c)}-5b^2}{15d\left(e^{2i(dx+c)}+1\right)^5}$
norman	$\frac{\frac{4ab\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4ab\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a^2\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4\left(a^2-2b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{4\left(a^2-2b^2\right.}{}}{\left(\tan^2\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+1/2*a*b/cos(d*x+c)^4+b^2*(1/5*
sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3))
```

Maxima [A]
time = 0.27, size = 70, normalized size = 0.82

$$\frac{10\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)a^2+2\left(3\,\tan\left(dx+c\right)^5+5\,\tan\left(dx+c\right)^3\right)b^2+\frac{15\,ab}{\left(\sin(dx+c)^2-1\right)^2}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(10*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^2 + 2*(3*tan(d*x + c)^5 + 5*ta
n(d*x + c)^3)*b^2 + 15*a*b/(sin(d*x + c)^2 - 1)^2)/d
```

Fricas [A]
time = 3.14, size = 79, normalized size = 0.93

$$\frac{15\,ab\cos\left(dx+c\right)+2\left(2\left(5\,a^2-b^2\right)\cos\left(dx+c\right)^4+\left(5\,a^2-b^2\right)\cos\left(dx+c\right)^2+3\,b^2\right)\sin\left(dx+c\right)}{30\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/30*(15*a*b*cos(d*x + c) + 2*(2*(5*a^2 - b^2)*cos(d*x + c)^4 + (5*a^2 - b^
2)*cos(d*x + c)^2 + 3*b^2)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 80, normalized size = 0.94

$$\frac{6\,b^2\tan\left(dx+c\right)^5+15\,ab\tan\left(dx+c\right)^4+10\,a^2\tan\left(dx+c\right)^3+10\,b^2\tan\left(dx+c\right)^3+30\,ab\tan\left(dx+c\right)^2+30\,a^2\tan\left(dx+c\right)}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(6*b^2*tan(d*x + c)^5 + 15*a*b*tan(d*x + c)^4 + 10*a^2*tan(d*x + c)^3
+ 10*b^2*tan(d*x + c)^3 + 30*a*b*tan(d*x + c)^2 + 30*a^2*tan(d*x + c))/d
```

Mupad [B]
time = 0.62, size = 98, normalized size = 1.15

$$\frac{\frac{b^2\,\sin(c+d\,x)}{5}+\cos\left(c+d\,x\right)^2\,\left(\frac{a^2\,\sin(c+d\,x)}{3}-\frac{b^2\,\sin(c+d\,x)}{15}\right)+\cos\left(c+d\,x\right)^4\,\left(\frac{2\,a^2\,\sin(c+d\,x)}{3}-\frac{2\,b^2\,\sin(c+d\,x)}{15}\right)+\frac{a\,b\,\cos(c+d\,x)}{2}}{d\cos\left(c+d\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^2/cos(c + d*x)^6,x)
```

```
[Out] ((b^2*sin(c + d*x))/5 + cos(c + d*x)^2*((a^2*sin(c + d*x))/3 - (b^2*sin(c +
 d*x))/15) + cos(c + d*x)^4*((2*a^2*sin(c + d*x))/3 - (2*b^2*sin(c + d*x))/
15) + (a*b*cos(c + d*x))/2)/(d*cos(c + d*x)^5)
```

3.55 $\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=168

$$\frac{3a^2\tanh^{-1}(\sin(c+dx))}{8d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{16d}+\frac{2ab\sec^5(c+dx)}{5d}+\frac{3a^2\sec(c+dx)\tan(c+dx)}{8d}-\frac{b^2\sec(c+$$

```
[Out] 3/8*a^2*arctanh(sin(d*x+c))/d-1/16*b^2*arctanh(sin(d*x+c))/d+2/5*a*b*sec(d*
x+c)^5/d+3/8*a^2*sec(d*x+c)*tan(d*x+c)/d-1/16*b^2*sec(d*x+c)*tan(d*x+c)/d+1
/4*a^2*sec(d*x+c)^3*tan(d*x+c)/d-1/24*b^2*sec(d*x+c)^3*tan(d*x+c)/d+1/6*b^2
*sec(d*x+c)^5*tan(d*x+c)/d
```

Rubi [A]
time = 0.12, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3169, 3853, 3855, 2686, 30, 2691}

$$\frac{3a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a^2\tan(c+dx)\sec(c+dx)}{8d}+\frac{2ab\sec^5(c+dx)}{5d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{16d}+\frac{b^2\tan(c+dx)\sec^5(c+dx)}{6d}-\frac{b^2\tan(c+dx)\sec^3(c+dx)}{24d}-\frac{b^2\tan(c+dx)\sec(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (3*a^2*ArcTanh[Sin[c + d*x]])/(8*d) - (b^2*ArcTanh[Sin[c + d*x]])/(16*d) +
(2*a*b*Sec[c + d*x]^5)/(5*d) + (3*a^2*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (b
^2*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (a^2*Sec[c + d*x]^3*Tan[c + d*x])/(4
*d) - (b^2*Sec[c + d*x]^3*Tan[c + d*x])/(24*d) + (b^2*Sec[c + d*x]^5*Tan[c
+ d*x])/(6*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
```

```
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx &= \int \left(a^2\sec^5(c+dx)+2ab\sec^5(c+dx)\tan(c+dx)+b^2\right.\\ &= a^2\int \sec^5(c+dx)\,dx+(2ab)\int \sec^5(c+dx)\tan(c+d\\ &= \frac{a^2\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{b^2\sec^5(c+dx)\tan(c+}{6d}\\ &= \frac{2ab\sec^5(c+dx)}{5d}+\frac{3a^2\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^2\mathrm{se}}{}\\ &= \frac{3a^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{2ab\sec^5(c+dx)}{5d}+\frac{3a^2\sec(}{}\\ &= \frac{3a^2\tanh^{-1}(\sin(c+dx))}{8d}-\frac{b^2\tanh^{-1}(\sin(c+dx))}{16d}+\frac{2}{}\end{aligned}$$

Mathematica [A]
time = 0.67, size = 104, normalized size = 0.62

$$\frac{15(6a^2-b^2)\tanh^{-1}(\sin(c+dx))+15(6a^2-b^2)\sec(c+dx)\tan(c+dx)+10(6a^2-b^2)\sec^3(c+dx)\tan(c+dx)+8b\sec^5(c+dx)(12a+5b\tan(c+dx))}{240d}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]

[Out] (15*(6*a^2 - b^2)*ArcTanh[Sin[c + d*x]] + 15*(6*a^2 - b^2)*Sec[c + d*x]*Tan[c + d*x] + 10*(6*a^2 - b^2)*Sec[c + d*x]^3*Tan[c + d*x] + 8*b*Sec[c + d*x]^5*(12*a + 5*b*Tan[c + d*x]))/(240*d)

Maple [A]
time = 0.39, size = 149, normalized size = 0.89

method	result
derivativedivides	$\frac{a^2\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{2ab}{5\cos(dx+c)^5}+b^2\left(\frac{\sin^3(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^3(dx+c}{8\cos(dx+c}\right.}{d}$
default	$\frac{a^2\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{2ab}{5\cos(dx+c)^5}+b^2\left(\frac{\sin^3(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^3(dx+c}{8\cos(dx+c}\right.}{d}$
risch	$-\frac{i(90a^2e^{11i(dx+c)}-15b^2e^{11i(dx+c)}+510a^2e^{9i(dx+c)}-85b^2e^{9i(dx+c)}+420a^2e^{7i(dx+c)}+570b^2e^{7i(dx+c)}+1536iab\,e^{7i(dx+c}}{120d(e^{2i(d}}$
norman	$\frac{4ab}{5d}-\frac{\left(6a^2-281b^2\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{\left(6a^2-281b^2\right)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{7\left(6a^2-25b^2\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d}-\frac{7\left(6a^2-25b^2\right)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{24d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+2/5*a*b/cos(d*x+c)^5+b^2*(1/6*sin(d*x+c)^3/cos(d*x+c)^6+1/8*sin(d*x+c)^3/cos(d*x+c)^4+1/16*sin(d*x+c)^3/cos(d*x+c)^2+1/16*sin(d*x+c)-1/16*ln(sec(d*x+c)+tan(d*x+c))))

Maxima [A]
time = 0.28, size = 180, normalized size = 1.07

$$\frac{5\,b^2\left(\frac{2\left(3\,\sin(dx+c)^5-8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log(\sin(dx+c)+1)+3\,\log(\sin(dx+c)-1)\right)-30\,a^2\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log(\sin(dx+c)+1)+3\,\log(\sin(dx+c)-1)\right)+\frac{192\,ab}{\cos(dx+c)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] 1/480*(5*b^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 30*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) + 192*a*b/cos(d*x + c)^5)/d

Fricas [A]
time = 3.26, size = 142, normalized size = 0.85

$$\frac{15\,(6\,a^2-b^2)\cos(dx+c)^6\log(\sin(dx+c)+1)-15\,(6\,a^2-b^2)\cos(dx+c)^6\log(-\sin(dx+c)+1)+192\,ab\cos(dx+c)+10\,\left(3\,(6\,a^2-b^2)\cos(dx+c)^4+2\,(6\,a^2-b^2)\cos(dx+c)^2+8\,b^2\right)\sin(dx+c)}{480\,d\cos(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/480*(15*(6*a^2 - b^2)*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 15*(6*a^2 -
b^2)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 192*a*b*cos(d*x + c) + 10*(3*(
6*a^2 - b^2)*cos(d*x + c)^4 + 2*(6*a^2 - b^2)*cos(d*x + c)^2 + 8*b^2)*sin(d
*x + c))/(d*cos(d*x + c)^6)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(152) = 304.
time = 0.46, size = 343, normalized size = 2.04

$$\frac{15\,(6\,a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,(6\,a^2-b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(150\,a^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{11}+15\,b^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{11}-480\,ab\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{10}-210\,a^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{9}+235\,b^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{9}+480\,ab\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{8}+60\,a^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{7}+390\,b^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{7}-960\,ab\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{6}+60\,a^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{5}+390\,b^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{5}+960\,ab\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{4}-210\,a^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{3}+235\,b^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{3}-96\,ab\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{2}+150\,a^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)+15\,b^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)+96\,ab\right)}{\left(\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2-1\right)^6}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(15*(6*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(6*a^2 - b^
2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(150*a^2*tan(1/2*d*x + 1/2*c)^11
+ 15*b^2*tan(1/2*d*x + 1/2*c)^11 - 480*a*b*tan(1/2*d*x + 1/2*c)^10 - 210*a^
2*tan(1/2*d*x + 1/2*c)^9 + 235*b^2*tan(1/2*d*x + 1/2*c)^9 + 480*a*b*tan(1/2
*d*x + 1/2*c)^8 + 60*a^2*tan(1/2*d*x + 1/2*c)^7 + 390*b^2*tan(1/2*d*x + 1/2
*c)^7 - 960*a*b*tan(1/2*d*x + 1/2*c)^6 + 60*a^2*tan(1/2*d*x + 1/2*c)^5 + 39
0*b^2*tan(1/2*d*x + 1/2*c)^5 + 960*a*b*tan(1/2*d*x + 1/2*c)^4 - 210*a^2*tan
(1/2*d*x + 1/2*c)^3 + 235*b^2*tan(1/2*d*x + 1/2*c)^3 - 96*a*b*tan(1/2*d*x +
 1/2*c)^2 + 150*a^2*tan(1/2*d*x + 1/2*c) + 15*b^2*tan(1/2*d*x + 1/2*c) + 96
*a*b)/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

Mupad [**B**]
time = 3.26, size = 328, normalized size = 1.95

$$\frac{\left(\frac{5\,a^2}{4}+\frac{b^2}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\left(\frac{47\,b^2}{24}-\frac{7\,a^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+\left(\frac{a^2}{2}+\frac{13\,b^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}-8\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+\left(\frac{a^2}{2}+\frac{13\,b^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}+8\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+\left(\frac{47\,b^2}{24}-\frac{7\,a^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}-\frac{4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{5}+\left(\frac{5\,a^2}{4}+\frac{b^2}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{4\,a\,b}{5}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-20\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}+\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{3\,a^2}{4}-\frac{b^2}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^2/cos(c + d*x)^7,x)
```

```
[Out] ((4*a*b)/5 + tan(c/2 + (d*x)/2)^5*(a^2/2 + (13*b^2)/4) + tan(c/2 + (d*x)/2)
^7*(a^2/2 + (13*b^2)/4) + tan(c/2 + (d*x)/2)^11*((5*a^2)/4 + b^2/8) - tan(c
/2 + (d*x)/2)^3*((7*a^2)/4 - (47*b^2)/24) - tan(c/2 + (d*x)/2)^9*((7*a^2)/4
 - (47*b^2)/24) + tan(c/2 + (d*x)/2)*((5*a^2)/4 + b^2/8) - (4*a*b*tan(c/2 +
 (d*x)/2)^2)/5 + 8*a*b*tan(c/2 + (d*x)/2)^4 - 8*a*b*tan(c/2 + (d*x)/2)^6 +
4*a*b*tan(c/2 + (d*x)/2)^8 - 4*a*b*tan(c/2 + (d*x)/2)^10)/(d*(15*tan(c/2 +
(d*x)/2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2
+ (d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) + (ata
nh(tan(c/2 + (d*x)/2))*((3*a^2)/4 - b^2/8))/d
```

3.56 $\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx$

Optimal. Leaf size=125

$$\frac{a^2\tan(c+dx)}{d}+\frac{ab\tan^2(c+dx)}{d}+\frac{(2a^2+b^2)\tan^3(c+dx)}{3d}+\frac{ab\tan^4(c+dx)}{d}+\frac{(a^2+2b^2)\tan^5(c+dx)}{5d}+\frac{ab\,\text{ta}}{}$$

```
[Out] a^2*tan(d*x+c)/d+a*b*tan(d*x+c)^2/d+1/3*(2*a^2+b^2)*tan(d*x+c)^3/d+a*b*tan(
d*x+c)^4/d+1/5*(a^2+2*b^2)*tan(d*x+c)^5/d+1/3*a*b*tan(d*x+c)^6/d+1/7*b^2*ta
n(d*x+c)^7/d
```

Rubi [A]
time = 0.07, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3167, 962}

$$\frac{(a^2+2b^2)\tan^5(c+dx)}{5d}+\frac{(2a^2+b^2)\tan^3(c+dx)}{3d}+\frac{a^2\tan(c+dx)}{d}+\frac{ab\tan^6(c+dx)}{3d}+\frac{ab\tan^4(c+dx)}{d}+\frac{ab\tan^2(c+dx)}{d}+\frac{b^2\tan^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Tan[c + d*x])/d + (a*b*Tan[c + d*x]^2)/d + ((2*a^2 + b^2)*Tan[c + d*x]
^3)/(3*d) + (a*b*Tan[c + d*x]^4)/d + ((a^2 + 2*b^2)*Tan[c + d*x]^5)/(5*d) +
 (a*b*Tan[c + d*x]^6)/(3*d) + (b^2*Tan[c + d*x]^7)/(7*d)
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^2\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^2(1+x^2)^2}{x^8}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{b^2}{x^8}+\frac{2ab}{x^7}+\frac{a^2+2b^2}{x^6}+\frac{4ab}{x^5}+\frac{2a^2+b^2}{x^4}+\frac{2ab}{x^3}+\frac{a^2}{x^2}\right.\right.}{d}$$

$$= \frac{a^2\tan(c+dx)}{d}+\frac{ab\tan^2(c+dx)}{d}+\frac{(2a^2+b^2)\tan^3(c+}{3d}$$

Mathematica [A]
time = 0.76, size = 104, normalized size = 0.83

$$\frac{\tan(c+dx)\left(105a^2+105ab\tan(c+dx)+35(2a^2+b^2)\tan^2(c+dx)+105ab\tan^3(c+dx)+21(a^2+2b^2)\tan^4(c+dx)+35ab\tan^5(c+dx)+15b^2\tan^6(c+dx)\right)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (Tan[c + d*x]*(105*a^2 + 105*a*b*Tan[c + d*x] + 35*(2*a^2 + b^2)*Tan[c + d*
x]^2 + 105*a*b*Tan[c + d*x]^3 + 21*(a^2 + 2*b^2)*Tan[c + d*x]^4 + 35*a*b*Ta
n[c + d*x]^5 + 15*b^2*Tan[c + d*x]^6))/(105*d)
```

Maple [A]
time = 0.25, size = 110, normalized size = 0.88

method	result
derivativedivides	$\frac{-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{ab}{3\cos(dx+c)^6}+b^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right)}{d}$
default	$\frac{-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{ab}{3\cos(dx+c)^6}+b^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c)\right)}{105\cos(dx+c)^3}\right)}{d}$
risch	$\frac{16i\left(-140iab\,e^{8i(dx+c)}+70a^2e^{8i(dx+c)}-70b^2e^{8i(dx+c)}-140iab\,e^{6i(dx+c)}+175a^2e^{6i(dx+c)}+35b^2e^{6i(dx+c)}+147a^2e^{4i(dx+c)}\right.}{105d\left(e^{2i(dx+c)}+1\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^2*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+1/3*a*b/cos
(d*x+c)^6+b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*x+c)^5
+8/105*sin(d*x+c)^3/cos(d*x+c)^3))
```

Maxima [A]
time = 0.28, size = 91, normalized size = 0.73

$$\frac{7\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)a^2+\left(15\tan(dx+c)^7+42\tan(dx+c)^5+35\tan(dx+c)^3\right)b^2-\frac{35\,ab}{\left(\sin(dx+c)^2-1\right)^3}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")

[Out] 1/105*(7*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^2 + (15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*b^2 - 35*a*b/(sin(d*x + c)^2 - 1)^3)/d

Fricas [A]
time = 2.75, size = 100, normalized size = 0.80

$$\frac{35\,ab\cos\left(dx+c\right)+\left(8\left(7\,a^2-b^2\right)\cos\left(dx+c\right)^6+4\left(7\,a^2-b^2\right)\cos\left(dx+c\right)^4+3\left(7\,a^2-b^2\right)\cos\left(dx+c\right)^2+15\,b^2\right)\sin\left(dx+c\right)}{105\,d\cos\left(dx+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")

[Out] 1/105*(35*a*b*cos(d*x + c) + (8*(7*a^2 - b^2)*cos(d*x + c)^6 + 4*(7*a^2 - b^2)*cos(d*x + c)^4 + 3*(7*a^2 - b^2)*cos(d*x + c)^2 + 15*b^2)*sin(d*x + c))/(d*cos(d*x + c)^7)

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**8*(a*cos(d*x+c)+b*sin(d*x+c))**2,x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep

Giac [A]
time = 0.49, size = 118, normalized size = 0.94

$$\frac{15\,b^2\tan\left(dx+c\right)^7+35\,ab\tan\left(dx+c\right)^6+21\,a^2\tan\left(dx+c\right)^5+42\,b^2\tan\left(dx+c\right)^5+105\,ab\tan\left(dx+c\right)^4+70\,a^2\tan\left(dx+c\right)^3+35\,b^2\tan\left(dx+c\right)^3+105\,ab\tan\left(dx+c\right)^2+105\,a^2\tan\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")

[Out] 1/105*(15*b^2*tan(d*x + c)^7 + 35*a*b*tan(d*x + c)^6 + 21*a^2*tan(d*x + c)^5 + 42*b^2*tan(d*x + c)^5 + 105*a*b*tan(d*x + c)^4 + 70*a^2*tan(d*x + c)^3 + 35*b^2*tan(d*x + c)^3 + 105*a*b*tan(d*x + c)^2 + 105*a^2*tan(d*x + c))/d

Mupad [B]
time = 0.83, size = 130, normalized size = 1.04

$$\frac{\frac{b^2\,\sin(c+d\,x)}{7}+\cos\left(c+d\,x\right)^2\,\left(\frac{a^2\,\sin(c+d\,x)}{5}-\frac{b^2\,\sin(c+d\,x)}{35}\right)+\cos\left(c+d\,x\right)^4\,\left(\frac{4\,a^2\,\sin(c+d\,x)}{15}-\frac{4\,b^2\,\sin(c+d\,x)}{105}\right)+\cos\left(c+d\,x\right)^6\,\left(\frac{8\,a^2\,\sin(c+d\,x)}{15}-\frac{8\,b^2\,\sin(c+d\,x)}{105}\right)+\frac{a\,b\,\cos(c+d\,x)}{3}}{d\,\cos\left(c+d\,x\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^2/cos(c + d*x)^8,x)
```

```
[Out] ((b^2*sin(c + d*x))/7 + cos(c + d*x)^2*((a^2*sin(c + d*x))/5 - (b^2*sin(c +
 d*x))/35) + cos(c + d*x)^4*((4*a^2*sin(c + d*x))/15 - (4*b^2*sin(c + d*x))
/105) + cos(c + d*x)^6*((8*a^2*sin(c + d*x))/15 - (8*b^2*sin(c + d*x))/105)
 + (a*b*cos(c + d*x))/3)/(d*cos(c + d*x)^7)
```

3.57 $\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=265

$$\frac{35a^3x}{128}+\frac{15}{128}ab^2x-\frac{b^3\cos^6(c+dx)}{6d}-\frac{3a^2b\cos^8(c+dx)}{8d}+\frac{b^3\cos^8(c+dx)}{8d}+\frac{35a^3\cos(c+dx)\sin(c+dx)}{128d}+\frac{15ab}{}$$

```
[Out] 35/128*a^3*x+15/128*a*b^2*x-1/6*b^3*cos(d*x+c)^6/d-3/8*a^2*b*cos(d*x+c)^8/d
+1/8*b^3*cos(d*x+c)^8/d+35/128*a^3*cos(d*x+c)*sin(d*x+c)/d+15/128*a*b^2*cos
(d*x+c)*sin(d*x+c)/d+35/192*a^3*cos(d*x+c)^3*sin(d*x+c)/d+5/64*a*b^2*cos(d*
x+c)^3*sin(d*x+c)/d+7/48*a^3*cos(d*x+c)^5*sin(d*x+c)/d+1/16*a*b^2*cos(d*x+c
)^5*sin(d*x+c)/d+1/8*a^3*cos(d*x+c)^7*sin(d*x+c)/d-3/8*a*b^2*cos(d*x+c)^7*s
in(d*x+c)/d
```

Rubi [A]
time = 0.18, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3169, 2715, 8, 2645, 30, 2648, 14}

$$\frac{a^3\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{7a^3\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{35a^3\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{35a^3\sin(c+dx)\cos(c+dx)}{128d}+\frac{35a^3x}{128}-\frac{3a^2b\cos^8(c+dx)}{8d}-\frac{3ab^2\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{ab^2\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{5ab^2\sin(c+dx)\cos^3(c+dx)}{64d}+\frac{15ab^2\sin(c+dx)\cos(c+dx)}{128d}+\frac{15}{128}ab^2x+\frac{b^3\cos^8(c+dx)}{8d}-\frac{b^3\cos^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (35*a^3*x)/128 + (15*a*b^2*x)/128 - (b^3*Cos[c + d*x]^6)/(6*d) - (3*a^2*b*C
os[c + d*x]^8)/(8*d) + (b^3*Cos[c + d*x]^8)/(8*d) + (35*a^3*Cos[c + d*x]*Si
n[c + d*x])/(128*d) + (15*a*b^2*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (35*a^
3*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + (5*a*b^2*Cos[c + d*x]^3*Sin[c + d*
x])/(64*d) + (7*a^3*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) + (a*b^2*Cos[c + d*
x]^5*Sin[c + d*x])/(16*d) + (a^3*Cos[c + d*x]^7*Sin[c + d*x])/(8*d) - (3*a*
b^2*Cos[c + d*x]^7*Sin[c + d*x])/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \int \left(a^3\cos^8(c+dx)+3a^2b\cos^7(c+dx)\sin(c+dx)+3\right.$$

$$= a^3\int \cos^8(c+dx)\,dx + \left(3a^2b\right)\int \cos^7(c+dx)\sin(c+$$

$$= \frac{a^3\cos^7(c+dx)\sin(c+dx)}{8d} - \frac{3ab^2\cos^7(c+dx)\sin(c+}{8d}$$

$$= -\frac{3a^2b\cos^8(c+dx)}{8d} + \frac{7a^3\cos^5(c+dx)\sin(c+dx)}{48d} +$$

$$= -\frac{b^3\cos^6(c+dx)}{6d} - \frac{3a^2b\cos^8(c+dx)}{8d} + \frac{b^3\cos^8(c+dx}{8d}$$

$$= -\frac{b^3\cos^6(c+dx)}{6d} - \frac{3a^2b\cos^8(c+dx)}{8d} + \frac{b^3\cos^8(c+dx}{8d}$$

$$= \frac{35a^3x}{128} + \frac{15}{128}ab^2x - \frac{b^3\cos^6(c+dx)}{6d} - \frac{3a^2b\cos^8(c+d}{8d}$$

Mathematica [A]
time = 0.51, size = 235, normalized size = 0.89

$$\frac{5a(7a^2+3b^2)(c+dx)}{128d} - \frac{3b(7a^2+b^2)\cos(2(c+dx))}{128d} - \frac{b(21a^2+b^2)\cos(4(c+dx))}{256d} - \frac{b(9a^2-b^2)\cos(6(c+dx))}{384d} - \frac{b(3a^2-b^2)\cos(8(c+dx))}{1024d} + \frac{a(14a^2+3b^2)\sin(2(c+dx))}{64d} + \frac{a(7a^2-3b^2)\sin(4(c+dx))}{128d} + \frac{a(2a^2-3b^2)\sin(6(c+dx))}{192d} + \frac{a(a^2-3b^2)\sin(8(c+dx))}{1024d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (5*a*(7*a^2 + 3*b^2)*(c + d*x))/(128*d) - (3*b*(7*a^2 + b^2)*Cos[2*(c + d*x
)])/(128*d) - (b*(21*a^2 + b^2)*Cos[4*(c + d*x)])/(256*d) - (b*(9*a^2 - b^2
)*Cos[6*(c + d*x)])/(384*d) - (b*(3*a^2 - b^2)*Cos[8*(c + d*x)])/(1024*d) +
 (a*(14*a^2 + 3*b^2)*Sin[2*(c + d*x)])/(64*d) + (a*(7*a^2 - 3*b^2)*Sin[4*(c
 + d*x)])/(128*d) + (a*(2*a^2 - 3*b^2)*Sin[6*(c + d*x)])/(192*d) + (a*(a^2
- 3*b^2)*Sin[8*(c + d*x)])/(1024*d)
```

Maple [A]
time = 0.38, size = 175, normalized size = 0.66 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+3*a*b^2*(-1/8*s
in(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))
*sin(d*x+c)+5/128*d*x+5/128*c)-3/8*a^2*b*cos(d*x+c)^8+a^3*(1/8*(cos(d*x+c)^
7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/128*d
*x+35/128*c))
```

Maxima [A]
time = 0.28, size = 163, normalized size = 0.62

$$-\frac{1152 a^2 b \cos\left(dx+c\right)^8 + \left(128 \sin\left(2 dx+2 c\right)^3 - 840 dx - 840 c - 3 \sin\left(8 dx+8 c\right) - 168 \sin\left(4 dx+4 c\right) - 768 \sin\left(2 dx+2 c\right)\right) a^3 - 3 \left(64 \sin\left(2 dx+2 c\right)^3 + 120 dx + 120 c - 3 \sin\left(8 dx+8 c\right) - 24 \sin\left(4 dx+4 c\right)\right) a b^2 - 128 \left(3 \sin\left(dx+c\right)^8 - 8 \sin\left(dx+c\right)^6 + 6 \sin\left(dx+c\right)^4\right) b^3}{3072 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/3072*(1152*a^2*b*cos(d*x + c)^8 + (128*sin(2*d*x + 2*c)^3 - 840*d*x - 84
0*c - 3*sin(8*d*x + 8*c) - 168*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*a^3
 - 3*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*sin
(4*d*x + 4*c))*a*b^2 - 128*(3*sin(d*x + c)^8 - 8*sin(d*x + c)^6 + 6*sin(d*x
 + c)^4)*b^3)/d
```

Fricas [A]
time = 3.06, size = 150, normalized size = 0.57

$$-\frac{64 b^3 \cos\left(dx+c\right)^6 + 48 \left(3 a^2 b - b^3\right) \cos\left(dx+c\right)^8 - 15 \left(7 a^3 + 3 a b^2\right) dx - \left(48 \left(a^3 - 3 a b^2\right) \cos\left(dx+c\right)^7 + 8 \left(7 a^3 + 3 a b^2\right) \cos\left(dx+c\right)^5 + 10 \left(7 a^3 + 3 a b^2\right) \cos\left(dx+c\right)^3 + 15 \left(7 a^3 + 3 a b^2\right) \cos\left(dx+c\right)\right) \sin\left(dx+c\right)}{384 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/384*(64*b^3*cos(d*x + c)^6 + 48*(3*a^2*b - b^3)*cos(d*x + c)^8 - 15*(7*a
^3 + 3*a*b^2)*d*x - (48*(a^3 - 3*a*b^2)*cos(d*x + c)^7 + 8*(7*a^3 + 3*a*b^2
)*cos(d*x + c)^5 + 10*(7*a^3 + 3*a*b^2)*cos(d*x + c)^3 + 15*(7*a^3 + 3*a*b^
2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A]
time = 1.01, size = 508, normalized size = 1.92

$$\begin{cases} \frac{35 a^3 x \sin^8(c+dx)}{128} + \frac{35 a^3 x \sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{105 a^3 x \sin^4(c+dx)\cos^4(c+dx)}{64} + \frac{35 a^3 x \sin^2(c+dx)\cos^6(c+dx)}{32} + \frac{35 a^3 x \cos^8(c+dx)}{128} + \frac{35 a^3 \sin^7(c+dx)\cos(c+dx)}{128 d} + \frac{385 a^3 \sin^5(c+dx)\cos^3(c+dx)}{384 d} + \frac{511 a^3 \sin^3(c+dx)\cos^5(c+dx)}{384 d} + \frac{93 a^3 \sin(c+dx)\cos^7(c+dx)}{128 d} - \frac{3 a^2 b \cos^8(c+dx)}{8 d} + \frac{15 a b^2 x \sin^8(c+dx)}{128} + \frac{15 a b^2 x \sin^6(c+dx)\cos^2(c+dx)}{32} + \frac{45 a b^2 x \sin^4(c+dx)\cos^4(c+dx)}{64} + \frac{15 a b^2 x \sin^2(c+dx)\cos^6(c+dx)}{32} + \frac{15 a b^2 x \cos^8(c+dx)}{128} + \frac{15 a b^2 \sin^7(c+dx)\cos(c+dx)}{128 d} + \frac{55 a b^2 \sin^5(c+dx)\cos^3(c+dx)}{128 d} + \frac{73 a b^2 \sin^3(c+dx)\cos^5(c+dx)}{128 d} - \frac{15 a b^2 \sin(c+dx)\cos^7(c+dx)}{128 d} - \frac{b^3 \sin^2(c+dx)\cos^6(c+dx)}{6 d} - \frac{b^3 \cos^8(c+dx)}{24 d} & \text{for}\ d \neq 0 \\ x\left(a\cos(c) + b\sin(c)\right)^3 \cos^5(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((35*a**3*x*sin(c + d*x)**8/128 + 35*a**3*x*sin(c + d*x)**6*cos(c
+ d*x)**2/32 + 105*a**3*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 35*a**3*x*si
n(c + d*x)**2*cos(c + d*x)**6/32 + 35*a**3*x*cos(c + d*x)**8/128 + 35*a**3*
sin(c + d*x)**7*cos(c + d*x)/(128*d) + 385*a**3*sin(c + d*x)**5*cos(c + d*x
)**3/(384*d) + 511*a**3*sin(c + d*x)**3*cos(c + d*x)**5/(384*d) + 93*a**3*s
in(c + d*x)*cos(c + d*x)**7/(128*d) - 3*a**2*b*cos(c + d*x)**8/(8*d) + 15*a
*b**2*x*sin(c + d*x)**8/128 + 15*a*b**2*x*sin(c + d*x)**6*cos(c + d*x)**2/3
2 + 45*a*b**2*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 15*a*b**2*x*sin(c + d*
x)**2*cos(c + d*x)**6/32 + 15*a*b**2*x*cos(c + d*x)**8/128 + 15*a*b**2*sin(
c + d*x)**7*cos(c + d*x)/(128*d) + 55*a*b**2*sin(c + d*x)**5*cos(c + d*x)**
```

```
3/(128*d) + 73*a*b**2*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) - 15*a*b**2*s
in(c + d*x)*cos(c + d*x)**7/(128*d) - b**3*sin(c + d*x)**2*cos(c + d*x)**6/
(6*d) - b**3*cos(c + d*x)**8/(24*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**3
*cos(c)**5, True))
```

Giac [A]
time = 0.53, size = 218, normalized size = 0.82

$$\frac{5}{128}\left(7a^3+3ab^2\right)x-\frac{\left(3a^2b-b^3\right)\cos\left(8dx+8c\right)}{1024d}-\frac{\left(9a^2b-b^3\right)\cos\left(6dx+6c\right)}{384d}-\frac{\left(21a^2b+b^3\right)\cos\left(4dx+4c\right)}{256d}-\frac{3\left(7a^2b+b^3\right)\cos\left(2dx+2c\right)}{128d}+\frac{\left(a^3-3ab^2\right)\sin\left(8dx+8c\right)}{1024d}+\frac{\left(2a^3-3ab^2\right)\sin\left(6dx+6c\right)}{192d}+\frac{\left(7a^3-3ab^2\right)\sin\left(4dx+4c\right)}{128d}+\frac{\left(14a^3+3ab^2\right)\sin\left(2dx+2c\right)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 5/128*(7*a^3 + 3*a*b^2)*x - 1/1024*(3*a^2*b - b^3)*cos(8*d*x + 8*c)/d - 1/3
84*(9*a^2*b - b^3)*cos(6*d*x + 6*c)/d - 1/256*(21*a^2*b + b^3)*cos(4*d*x +
4*c)/d - 3/128*(7*a^2*b + b^3)*cos(2*d*x + 2*c)/d + 1/1024*(a^3 - 3*a*b^2)*
sin(8*d*x + 8*c)/d + 1/192*(2*a^3 - 3*a*b^2)*sin(6*d*x + 6*c)/d + 1/128*(7*
a^3 - 3*a*b^2)*sin(4*d*x + 4*c)/d + 1/64*(14*a^3 + 3*a*b^2)*sin(2*d*x + 2*c
)/d
```

Mupad [B]
time = 2.29, size = 523, normalized size = 1.97

$$\frac{4b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{15ab^2}{64}-\frac{93a^3}{64}\right)+\frac{40b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{3}+4b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}\left(\frac{15ab^2}{64}-\frac{93a^3}{64}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{397ab^2}{64}+\frac{91a^3}{192}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\left(\frac{397ab^2}{64}+\frac{91a^3}{192}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(\frac{895ab^2}{64}-\frac{1799a^3}{192}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\left(\frac{895ab^2}{64}-\frac{1799a^3}{192}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(\frac{1765ab^2}{64}-\frac{1085a^3}{192}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9\left(\frac{1765ab^2}{64}-\frac{1085a^3}{192}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(42a^2b-\frac{16b^3}{3}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\left(42a^2b-\frac{16b^3}{3}\right)+6a^2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+6a^2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+28\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+56\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+70\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+56\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+28\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}+\frac{5a\,\mathrm{atan}\left(\frac{5a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(7a^2+3b^2\right)}{64\left(\frac{15ab^2}{64}+\frac{35a^3}{64}\right)}\right)\left(7a^2+3b^2\right)}{64d}-\frac{5a\left(7a^2+3b^2\right)\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\frac{dx}{2}\right)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

```
[Out] (4*b^3*tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)*((15*a*b^2)/64 - (93*a^3)/
64) + (40*b^3*tan(c/2 + (d*x)/2)^8)/3 + 4*b^3*tan(c/2 + (d*x)/2)^12 + tan(c
/2 + (d*x)/2)^15*((15*a*b^2)/64 - (93*a^3)/64) + tan(c/2 + (d*x)/2)^3*((397
*a*b^2)/64 + (91*a^3)/192) - tan(c/2 + (d*x)/2)^13*((397*a*b^2)/64 + (91*a^
3)/192) - tan(c/2 + (d*x)/2)^5*((895*a*b^2)/64 - (1799*a^3)/192) + tan(c/2
+ (d*x)/2)^11*((895*a*b^2)/64 - (1799*a^3)/192) + tan(c/2 + (d*x)/2)^7*((17
65*a*b^2)/64 - (1085*a^3)/192) - tan(c/2 + (d*x)/2)^9*((1765*a*b^2)/64 - (1
085*a^3)/192) + tan(c/2 + (d*x)/2)^6*(42*a^2*b - (16*b^3)/3) + tan(c/2 + (d
*x)/2)^10*(42*a^2*b - (16*b^3)/3) + 6*a^2*b*tan(c/2 + (d*x)/2)^2 + 6*a^2*b*
tan(c/2 + (d*x)/2)^14)/(d*(8*tan(c/2 + (d*x)/2)^2 + 28*tan(c/2 + (d*x)/2)^4
 + 56*tan(c/2 + (d*x)/2)^6 + 70*tan(c/2 + (d*x)/2)^8 + 56*tan(c/2 + (d*x)/2
)^10 + 28*tan(c/2 + (d*x)/2)^12 + 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)
/2)^16 + 1)) + (5*a*atan((5*a*tan(c/2 + (d*x)/2)*(7*a^2 + 3*b^2))/(64*((15*
a*b^2)/64 + (35*a^3)/64)))*(7*a^2 + 3*b^2))/(64*d) - (5*a*(7*a^2 + 3*b^2)*(
atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(64*d)
```

3.58 $\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=175

$$-\frac{b^3\cos^5(c+dx)}{5d}-\frac{3a^2b\cos^7(c+dx)}{7d}+\frac{b^3\cos^7(c+dx)}{7d}+\frac{a^3\sin(c+dx)}{d}-\frac{a^3\sin^3(c+dx)}{d}+\frac{ab^2\sin^3(c+dx)}{d}+3a$$

```
[Out] -1/5*b^3*cos(d*x+c)^5/d-3/7*a^2*b*cos(d*x+c)^7/d+1/7*b^3*cos(d*x+c)^7/d+a^3
*sin(d*x+c)/d-a^3*sin(d*x+c)^3/d+a*b^2*sin(d*x+c)^3/d+3/5*a^3*sin(d*x+c)^5/
d-6/5*a*b^2*sin(d*x+c)^5/d-1/7*a^3*sin(d*x+c)^7/d+3/7*a*b^2*sin(d*x+c)^7/d
```

Rubi [A]
time = 0.13, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3169, 2713, 2645, 30, 2644, 276, 14}

$$-\frac{a^3\sin^7(c+dx)}{7d}+\frac{3a^3\sin^5(c+dx)}{5d}-\frac{a^3\sin^3(c+dx)}{d}+\frac{a^3\sin(c+dx)}{d}-\frac{3a^2b\cos^7(c+dx)}{7d}+\frac{3ab^2\sin^7(c+dx)}{7d}-\frac{6ab^2\sin^5(c+dx)}{5d}+\frac{ab^2\sin^3(c+dx)}{d}+\frac{b^3\cos^7(c+dx)}{7d}-\frac{b^3\cos^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/5*(b^3*Cos[c + d*x]^5)/d - (3*a^2*b*Cos[c + d*x]^7)/(7*d) + (b^3*Cos[c +
 d*x]^7)/(7*d) + (a^3*Sin[c + d*x])/d - (a^3*Sin[c + d*x]^3)/d + (a*b^2*Sin
[c + d*x]^3)/d + (3*a^3*Sin[c + d*x]^5)/(5*d) - (6*a*b^2*Sin[c + d*x]^5)/(5
*d) - (a^3*Sin[c + d*x]^7)/(7*d) + (3*a*b^2*Sin[c + d*x]^7)/(7*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
```

```
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \int \left(a^3\cos^7(c+dx)+3a^2b\cos^6(c+dx)\sin(c+dx)+3$$

$$= a^3\int \cos^7(c+dx)\,dx + \left(3a^2b\right)\int \cos^6(c+dx)\sin(c+$$

$$= -\frac{a^3\text{Subst}\left(\int \left(1-3x^2+3x^4-x^6\right)\,dx, x, -\sin(c+dx)\right)}{d}$$

$$= -\frac{3a^2b\cos^7(c+dx)}{7d}+\frac{a^3\sin(c+dx)}{d}-\frac{a^3\sin^3(c+dx)}{d}$$

$$= -\frac{b^3\cos^5(c+dx)}{5d}-\frac{3a^2b\cos^7(c+dx)}{7d}+\frac{b^3\cos^7(c+dx}{7d}$$

Mathematica [A]
time = 0.44, size = 204, normalized size = 1.17

$$\frac{-105b(5a^2+b^2)\cos(c+dx)-35(9a^2b+b^3)\cos(3(c+dx))-105a^2b\cos(5(c+dx))+7b^3\cos(5(c+dx))-15a^2b\cos(7(c+dx))+5b^3\cos(7(c+dx))+1225a^3\sin(c+dx)+525ab^2\sin(c+dx)+245a^3\sin(3(c+dx))-35ab^2\sin(3(c+dx))+49a^3\sin(5(c+dx))-63ab^2\sin(5(c+dx))+5a^3\sin(7(c+dx))-15ab^2\sin(7(c+dx))}{2240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (-105*b*(5*a^2 + b^2)*Cos[c + d*x] - 35*(9*a^2*b + b^3)*Cos[3*(c + d*x)] -
105*a^2*b*Cos[5*(c + d*x)] + 7*b^3*Cos[5*(c + d*x)] - 15*a^2*b*Cos[7*(c + d
*x)] + 5*b^3*Cos[7*(c + d*x)] + 1225*a^3*Sin[c + d*x] + 525*a*b^2*Sin[c + d
*x] + 245*a^3*Sin[3*(c + d*x)] - 35*a*b^2*Sin[3*(c + d*x)] + 49*a^3*Sin[5*(
c + d*x)] - 63*a*b^2*Sin[5*(c + d*x)] + 5*a^3*Sin[7*(c + d*x)] - 15*a*b^2*S
in[7*(c + d*x)])/(2240*d)
```

Maple [A]
time = 0.28, size = 145, normalized size = 0.83

method	result
derivativedivides	$\frac{b^3\left(-\frac{(\sin^2(dx+c))(\cos^5(dx+c))}{7}-\frac{2(\cos^5(dx+c))}{35}\right)+3a\,b^2\left(-\frac{\sin(dx+c)(\cos^6(dx+c))}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos^2(dx+c))}{3}\right)\sin(d}{35}\right.}{d}$
default	$\frac{b^3\left(-\frac{(\sin^2(dx+c))(\cos^5(dx+c))}{7}-\frac{2(\cos^5(dx+c))}{35}\right)+3a\,b^2\left(-\frac{\sin(dx+c)(\cos^6(dx+c))}{7}+\frac{\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4(\cos^2(dx+c))}{3}\right)\sin(d}{35}\right.}{d}$
risch	$-\frac{15a^2b\cos(dx+c)}{64d}-\frac{3b^3\cos(dx+c)}{64d}+\frac{35a^3\sin(dx+c)}{64d}+\frac{15a\,b^2\sin(dx+c)}{64d}-\frac{3b\cos(7dx+7c)a^2}{448d}+\frac{b^3\cos(7dx+7c}{448d}$
norman	$-\frac{30a^2b+4b^3}{35d}+\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^3\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4b^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}-\frac{4b^3\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{8b^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{6a^2b(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*cos(d*x+c)^5)+3*a*b^2*(-1/7*s
in(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
-3/7*a^2*b*cos(d*x+c)^7+1/7*a^3*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos
(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 126, normalized size = 0.72

$$-\frac{15\,a^2b\cos\left(dx+c\right)^7+\left(5\,\sin\left(dx+c\right)^7-21\,\sin\left(dx+c\right)^5+35\,\sin\left(dx+c\right)^3-35\,\sin\left(dx+c\right)\right)a^3-\left(15\,\sin\left(dx+c\right)^7-42\,\sin\left(dx+c\right)^5+35\,\sin\left(dx+c\right)^3\right)ab^2-\left(5\,\cos\left(dx+c\right)^7-7\,\cos\left(dx+c\right)^5\right)b^3}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/35*(15*a^2*b*cos(d*x + c)^7 + (5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35
*sin(d*x + c)^3 - 35*sin(d*x + c))*a^3 - (15*sin(d*x + c)^7 - 42*sin(d*x +
c)^5 + 35*sin(d*x + c)^3)*a*b^2 - (5*cos(d*x + c)^7 - 7*cos(d*x + c)^5)*b^3
)/d
```

Fricas [A]
time = 3.17, size = 123, normalized size = 0.70

$$-\frac{7\,b^3\cos\left(dx+c\right)^5+5\,\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^7-\left(5\,\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^6+3\,\left(2\,a^3+ab^2\right)\cos\left(dx+c\right)^4+16\,a^3+8\,ab^2+4\,\left(2\,a^3+ab^2\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/35*(7*b^3*cos(d*x + c)^5 + 5*(3*a^2*b - b^3)*cos(d*x + c)^7 - (5*(a^3 -
3*a*b^2)*cos(d*x + c)^6 + 3*(2*a^3 + a*b^2)*cos(d*x + c)^4 + 16*a^3 + 8*a*b
^2 + 4*(2*a^3 + a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [A]
time = 0.66, size = 233, normalized size = 1.33

$$\begin{cases} \frac{16a^3\sin^7(c+dx)}{35d}+\frac{8a^3\sin^5(c+dx)\cos^2(c+dx)}{5d}+\frac{2a^3\sin^3(c+dx)\cos^4(c+dx)}{d}+\frac{a^3\sin(c+dx)\cos^6(c+dx)}{d}-\frac{3a^2b\cos^7(c+dx)}{7d}+\frac{8ab^2\sin^7(c+dx)}{35d}+\frac{4ab^2\sin^5(c+dx)\cos^2(c+dx)}{5d}+\frac{ab^2\sin^3(c+dx)\cos^4(c+dx)}{d}-\frac{b^3\sin^2(c+dx)\cos^5(c+dx)}{5d}-\frac{2b^3\cos^7(c+dx)}{35d} & \text{for } d\neq 0 \\ x\left(a\cos(c)+b\sin(c)\right)^3\cos^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((16*a**3*sin(c + d*x)**7/(35*d) + 8*a**3*sin(c + d*x)**5*cos(c +
d*x)**2/(5*d) + 2*a**3*sin(c + d*x)**3*cos(c + d*x)**4/d + a**3*sin(c + d*x
)*cos(c + d*x)**6/d - 3*a**2*b*cos(c + d*x)**7/(7*d) + 8*a*b**2*sin(c + d*x
)**7/(35*d) + 4*a*b**2*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + a*b**2*sin(c
 + d*x)**3*cos(c + d*x)**4/d - b**3*sin(c + d*x)**2*cos(c + d*x)**5/(5*d) -
 2*b**3*cos(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**3*cos(
c)**4, True))
```

Giac [A]
time = 0.51, size = 197, normalized size = 1.13

$$-\frac{\left(3\,a^2b-b^3\right)\cos\left(7\,dx+7\,c\right)}{448\,d}-\frac{\left(15\,a^2b-b^3\right)\cos\left(5\,dx+5\,c\right)}{320\,d}-\frac{\left(9\,a^2b+b^3\right)\cos\left(3\,dx+3\,c\right)}{64\,d}-\frac{3\,\left(5\,a^2b+b^3\right)\cos\left(dx+c\right)}{64\,d}+\frac{\left(a^3-3\,ab^2\right)\sin\left(7\,dx+7\,c\right)}{448\,d}+\frac{\left(7\,a^3-9\,ab^2\right)\sin\left(5\,dx+5\,c\right)}{320\,d}+\frac{\left(7\,a^3-ab^2\right)\sin\left(3\,dx+3\,c\right)}{64\,d}+\frac{5\,\left(7\,a^3+3\,ab^2\right)\sin\left(dx+c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/448*(3*a^2*b - b^3)*cos(7*d*x + 7*c)/d - 1/320*(15*a^2*b - b^3)*cos(5*d*
x + 5*c)/d - 1/64*(9*a^2*b + b^3)*cos(3*d*x + 3*c)/d - 3/64*(5*a^2*b + b^3)
*cos(d*x + c)/d + 1/448*(a^3 - 3*a*b^2)*sin(7*d*x + 7*c)/d + 1/320*(7*a^3 -
 9*a*b^2)*sin(5*d*x + 5*c)/d + 1/64*(7*a^3 - a*b^2)*sin(3*d*x + 3*c)/d + 5/
64*(7*a^3 + 3*a*b^2)*sin(d*x + c)/d
```

Mupad [B]
time = 0.78, size = 214, normalized size = 1.22

$$\frac{16\,a^3\,\sin\left(c+d\,x\right)}{35\,d}-\frac{b^3\cos\left(c+d\,x\right)^5}{5\,d}+\frac{b^3\cos\left(c+d\,x\right)^7}{7\,d}-\frac{3\,a^2\,b\cos\left(c+d\,x\right)^7}{7\,d}+\frac{8\,a^3\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{35\,d}+\frac{6\,a^3\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{35\,d}+\frac{a^3\cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)}{7\,d}+\frac{8\,a\,b^2\,\sin\left(c+d\,x\right)}{35\,d}+\frac{4\,a\,b^2\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{35\,d}+\frac{3\,a\,b^2\cos\left(c+d\,x\right)^4\,\sin\left(c+d\,x\right)}{35\,d}-\frac{3\,a\,b^2\cos\left(c+d\,x\right)^6\,\sin\left(c+d\,x\right)}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^4*(a*cos(c + d*x) + b*sin(c + d*x))^3,x)

[Out] (16*a^3*sin(c + d*x))/(35*d) - (b^3*cos(c + d*x)^5)/(5*d) + (b^3*cos(c + d*x)^7)/(7*d) - (3*a^2*b*cos(c + d*x)^7)/(7*d) + (8*a^3*cos(c + d*x)^2*sin(c + d*x))/(35*d) + (6*a^3*cos(c + d*x)^4*sin(c + d*x))/(35*d) + (a^3*cos(c + d*x)^6*sin(c + d*x))/(7*d) + (8*a*b^2*sin(c + d*x))/(35*d) + (4*a*b^2*cos(c + d*x)^2*sin(c + d*x))/(35*d) + (3*a*b^2*cos(c + d*x)^4*sin(c + d*x))/(35*d) - (3*a*b^2*cos(c + d*x)^6*sin(c + d*x))/(7*d)

3.59 $\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=216

$$\frac{5a^3x}{16}+\frac{3}{16}ab^2x-\frac{a^2b\cos^6(c+dx)}{2d}+\frac{5a^3\cos(c+dx)\sin(c+dx)}{16d}+\frac{3ab^2\cos(c+dx)\sin(c+dx)}{16d}+\frac{5a^3\cos^3(c+}{2}$$

```
[Out] 5/16*a^3*x+3/16*a*b^2*x-1/2*a^2*b*cos(d*x+c)^6/d+5/16*a^3*cos(d*x+c)*sin(d*
x+c)/d+3/16*a*b^2*cos(d*x+c)*sin(d*x+c)/d+5/24*a^3*cos(d*x+c)^3*sin(d*x+c)/
d+1/8*a*b^2*cos(d*x+c)^3*sin(d*x+c)/d+1/6*a^3*cos(d*x+c)^5*sin(d*x+c)/d-1/2
*a*b^2*cos(d*x+c)^5*sin(d*x+c)/d+1/4*b^3*sin(d*x+c)^4/d-1/6*b^3*sin(d*x+c)^
6/d
```

Rubi [A]

time = 0.16, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3169, 2715, 8, 2645, 30, 2648, 2644, 14}

$$\frac{a^3\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a^3\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a^3\sin(c+dx)\cos(c+dx)}{16d}+\frac{5a^3x}{16}-\frac{a^2b\cos^6(c+dx)}{2d}-\frac{ab^2\sin(c+dx)\cos^5(c+dx)}{2d}+\frac{ab^2\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{3ab^2\sin(c+dx)\cos(c+dx)}{16d}+\frac{3}{16}ab^2x-\frac{b^3\sin^6(c+dx)}{6d}+\frac{b^3\sin^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (5*a^3*x)/16 + (3*a*b^2*x)/16 - (a^2*b*Cos[c + d*x]^6)/(2*d) + (5*a^3*Cos[c
 + d*x]*Sin[c + d*x])/(16*d) + (3*a*b^2*Cos[c + d*x]*Sin[c + d*x])/(16*d) +
 (5*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) + (a*b^2*Cos[c + d*x]^3*Sin[c +
 d*x])/(8*d) + (a^3*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (a*b^2*Cos[c + d*x
]^5*Sin[c + d*x])/(2*d) + (b^3*Sin[c + d*x]^4)/(4*d) - (b^3*Sin[c + d*x]^6)
/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \int \left(a^3\cos^6(c+dx)+3a^2b\cos^5(c+dx)\sin(c+dx)+3\cdots\right.$$

$$= a^3\int \cos^6(c+dx)\,dx + \left(3a^2b\right)\int \cos^5(c+dx)\sin(c+\cdots$$

$$= \frac{a^3\cos^5(c+dx)\sin(c+dx)}{6d} - \frac{ab^2\cos^5(c+dx)\sin(c+\cdots}{2d}$$

$$= -\frac{a^2b\cos^6(c+dx)}{2d} + \frac{5a^3\cos^3(c+dx)\sin(c+dx)}{24d} + \frac{ab\cdots}{}$$

$$= -\frac{a^2b\cos^6(c+dx)}{2d} + \frac{5a^3\cos(c+dx)\sin(c+dx)}{16d} + \frac{3ab\cdots}{}$$

$$= \frac{5a^3x}{16} + \frac{3}{16}ab^2x - \frac{a^2b\cos^6(c+dx)}{2d} + \frac{5a^3\cos(c+dx)\,\mathrm{s}\cdots}{16d}$$

Mathematica [A]
time = 0.34, size = 171, normalized size = 0.79

$$\frac{a(5a^2+3b^2)\,(c+dx)}{16d} - \frac{3b(5a^2+b^2)\cos(2(c+dx))}{64d} - \frac{3a^2b\cos(4(c+dx))}{32d} - \frac{b(3a^2-b^2)\cos(6(c+dx))}{192d} + \frac{3a(5a^2+b^2)\sin(2(c+dx))}{64d} + \frac{3a(a^2-b^2)\sin(4(c+dx))}{64d} + \frac{a(a^2-3b^2)\sin(6(c+dx))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*(5*a^2 + 3*b^2)*(c + d*x))/(16*d) - (3*b*(5*a^2 + b^2)*Cos[2*(c + d*x)])
/(64*d) - (3*a^2*b*Cos[4*(c + d*x)])/(32*d) - (b*(3*a^2 - b^2)*Cos[6*(c + d
*x)])/(192*d) + (3*a*(5*a^2 + b^2)*Sin[2*(c + d*x)])/(64*d) + (3*a*(a^2 - b
^2)*Sin[4*(c + d*x)])/(64*d) + (a*(a^2 - 3*b^2)*Sin[6*(c + d*x)])/(192*d)
```

Maple [A]
time = 0.26, size = 155, normalized size = 0.72

method	result
derivativedivides	$\frac{b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^4(dx+c)\right)}{6}-\frac{\left(\cos^4(dx+c)\right)}{12}\right)+3a\,b^2\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}\cdots\right.}{d}$
default	$\frac{b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^4(dx+c)\right)}{6}-\frac{\left(\cos^4(dx+c)\right)}{12}\right)+3a\,b^2\left(-\frac{\sin(dx+c)\left(\cos^5(dx+c)\right)}{6}+\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{24}\cdots\right.}{d}$
risch	$\frac{5a^3x}{16}+\frac{3a\,b^2x}{16}-\frac{b\cos(6dx+6c)a^2}{64d}+\frac{b^3\cos(6dx+6c)}{192d}+\frac{a^3\sin(6dx+6c)}{192d}-\frac{a\sin(6dx+6c)b^2}{64d}-\frac{3b\cos(4dx+4c)a\cdots}{32d}$

norman	$\frac{\left(\frac{5}{16}a^3+\frac{3}{16}a\,b^2\right)x+\left(\frac{5}{16}a^3+\frac{3}{16}a\,b^2\right)x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{15}{8}a^3+\frac{9}{8}a\,b^2\right)x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{15}{8}a^3+\frac{9}{8}a\,b^2\right)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+3*a*b^2*(-1/6*s
in(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d
*x+1/16*c)-1/2*a^2*b*cos(d*x+c)^6+a^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+1
5/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A]
time = 0.28, size = 131, normalized size = 0.61

$$-\frac{96\,a^2b\cos\left(dx+c\right)^6+\left(4\,\sin\left(2\,dx+2\,c\right)^3-60\,dx-60\,c-9\,\sin\left(4\,dx+4\,c\right)-48\,\sin\left(2\,dx+2\,c\right)\right)a^3-3\left(4\,\sin\left(2\,dx+2\,c\right)^3+12\,dx+12\,c-3\,\sin\left(4\,dx+4\,c\right)\right)ab^2+16\left(2\,\sin\left(dx+c\right)^6-3\,\sin\left(dx+c\right)^4\right)b^3}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/192*(96*a^2*b*cos(d*x + c)^6 + (4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9
*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^3 - 3*(4*sin(2*d*x + 2*c)^3 + 12
*d*x + 12*c - 3*sin(4*d*x + 4*c))*a*b^2 + 16*(2*sin(d*x + c)^6 - 3*sin(d*x
+ c)^4)*b^3)/d
```

Fricas [A]
time = 2.87, size = 128, normalized size = 0.59

$$-\frac{12\,b^3\cos\left(dx+c\right)^4+8\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^6-3\left(5\,a^3+3\,ab^2\right)dx-\left(8\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^5+2\left(5\,a^3+3\,ab^2\right)\cos\left(dx+c\right)^3+3\left(5\,a^3+3\,ab^2\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/48*(12*b^3*cos(d*x + c)^4 + 8*(3*a^2*b - b^3)*cos(d*x + c)^6 - 3*(5*a^3
+ 3*a*b^2)*d*x - (8*(a^3 - 3*a*b^2)*cos(d*x + c)^5 + 2*(5*a^3 + 3*a*b^2)*co
s(d*x + c)^3 + 3*(5*a^3 + 3*a*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A]
time = 0.44, size = 400, normalized size = 1.85

$$\begin{cases} \frac{5a^3x\sin^6(c+dx)}{16}+\frac{15a^3x\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{15a^3x\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{5a^3x\cos^6(c+dx)}{16}+\frac{5a^3\sin^5(c+dx)\cos(c+dx)}{16d}+\frac{5a^3\sin^3(c+dx)\cos^3(c+dx)}{6d}+\frac{11a^3\sin(c+dx)\cos^5(c+dx)}{16d}-\frac{a^2b\cos^6(c+dx)}{2d}+\frac{3ab^2x\sin^6(c+dx)}{16}+\frac{9ab^2x\sin^4(c+dx)\cos^2(c+dx)}{16}+\frac{9ab^2x\sin^2(c+dx)\cos^4(c+dx)}{16}+\frac{3ab^2x\cos^6(c+dx)}{16}+\frac{3ab^2\sin^5(c+dx)\cos(c+dx)}{16d}+\frac{ab^2\sin^3(c+dx)\cos^3(c+dx)}{2d}-\frac{3ab^2\sin(c+dx)\cos^5(c+dx)}{16d}-\frac{b^3\sin^2(c+dx)\cos^4(c+dx)}{4d}-\frac{b^3\cos^6(c+dx)}{12d} & \text{for}\ d\neq 0 \\ x(a\cos(c)+b\sin(c))^3\cos^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((5*a**3*x*sin(c + d*x)**6/16 + 15*a**3*x*sin(c + d*x)**4*cos(c +
d*x)**2/16 + 15*a**3*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a**3*x*cos(c
+ d*x)**6/16 + 5*a**3*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a**3*sin(c +
d*x)**3*cos(c + d*x)**3/(6*d) + 11*a**3*sin(c + d*x)*cos(c + d*x)**5/(16*d)
 - a**2*b*cos(c + d*x)**6/(2*d) + 3*a*b**2*x*sin(c + d*x)**6/16 + 9*a*b**2*
x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 9*a*b**2*x*sin(c + d*x)**2*cos(c + d
*x)**4/16 + 3*a*b**2*x*cos(c + d*x)**6/16 + 3*a*b**2*sin(c + d*x)**5*cos(c
+ d*x)/(16*d) + a*b**2*sin(c + d*x)**3*cos(c + d*x)**3/(2*d) - 3*a*b**2*sin
(c + d*x)*cos(c + d*x)**5/(16*d) - b**3*sin(c + d*x)**2*cos(c + d*x)**4/(4*
d) - b**3*cos(c + d*x)**6/(12*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**3*co
s(c)**3, True))
```

Giac [A]
time = 0.50, size = 157, normalized size = 0.73

$$-\frac{3\,a^2 b\cos\left(4\,dx+4\,c\right)}{32\,d}+\frac{1}{16}\left(5\,a^3+3\,ab^2\right)x-\frac{\left(3\,a^2 b-b^3\right)\cos\left(6\,dx+6\,c\right)}{192\,d}-\frac{3\left(5\,a^2 b+b^3\right)\cos\left(2\,dx+2\,c\right)}{64\,d}+\frac{\left(a^3-3\,ab^2\right)\sin\left(6\,dx+6\,c\right)}{192\,d}+\frac{3\left(a^3-ab^2\right)\sin\left(4\,dx+4\,c\right)}{64\,d}+\frac{3\left(5\,a^3+ab^2\right)\sin\left(2\,dx+2\,c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -3/32*a^2*b*cos(4*d*x + 4*c)/d + 1/16*(5*a^3 + 3*a*b^2)*x - 1/192*(3*a^2*b
- b^3)*cos(6*d*x + 6*c)/d - 3/64*(5*a^2*b + b^3)*cos(2*d*x + 2*c)/d + 1/192
*(a^3 - 3*a*b^2)*sin(6*d*x + 6*c)/d + 3/64*(a^3 - a*b^2)*sin(4*d*x + 4*c)/d
 + 3/64*(5*a^3 + a*b^2)*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 2.03, size = 407, normalized size = 1.88

$$\frac{4\,b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3\,a\,b^2}{8}-\frac{11\,a^3}{8}\right)+4\,b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\left(\frac{3\,a\,b^2}{8}-\frac{11\,a^3}{8}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(\frac{39\,a\,b^2}{4}-\frac{15\,a^3}{4}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(\frac{39\,a\,b^2}{4}-\frac{15\,a^3}{4}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{47\,a\,b^2}{8}-\frac{5\,a^3}{24}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9\left(\frac{47\,a\,b^2}{8}-\frac{5\,a^3}{24}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(20\,a^2\,b-\frac{8\,b^3}{3}\right)+6\,a^2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+6\,a^2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}+\frac{a\,\mathrm{atan}\left(\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(5\,a^2+3\,b^2\right)}{8\left(\frac{5\,a^3}{8}+\frac{3\,a\,b^2}{8}\right)}\right)\left(5\,a^2+3\,b^2\right)}{8\,d}-\frac{a\left(5\,a^2+3\,b^2\right)\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\frac{dx}{2}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

```
[Out] (4*b^3*tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)*((3*a*b^2)/8 - (11*a^3)/8)
 + 4*b^3*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^11*((3*a*b^2)/8 - (11*a^
3)/8) - tan(c/2 + (d*x)/2)^5*((39*a*b^2)/4 - (15*a^3)/4) + tan(c/2 + (d*x)/
2)^7*((39*a*b^2)/4 - (15*a^3)/4) + tan(c/2 + (d*x)/2)^3*((47*a*b^2)/8 - (5*
a^3)/24) - tan(c/2 + (d*x)/2)^9*((47*a*b^2)/8 - (5*a^3)/24) + tan(c/2 + (d*
x)/2)^6*(20*a^2*b - (8*b^3)/3) + 6*a^2*b*tan(c/2 + (d*x)/2)^2 + 6*a^2*b*tan
(c/2 + (d*x)/2)^10)/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 +
20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10
 + tan(c/2 + (d*x)/2)^12 + 1)) + (a*atan((a*tan(c/2 + (d*x)/2)*(5*a^2 + 3*b
^2))/(8*((3*a*b^2)/8 + (5*a^3)/8)))*(5*a^2 + 3*b^2))/(8*d) - (a*(5*a^2 + 3*
b^2)*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2))/(8*d)
```

3.60 $\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=140

$$-\frac{b^3\cos^3(c+dx)}{3d}-\frac{3a^2b\cos^5(c+dx)}{5d}+\frac{b^3\cos^5(c+dx)}{5d}+\frac{a^3\sin(c+dx)}{d}-\frac{2a^3\sin^3(c+dx)}{3d}+\frac{ab^2\sin^3(c+dx)}{d}+\cdots$$

```
[Out] -1/3*b^3*cos(d*x+c)^3/d-3/5*a^2*b*cos(d*x+c)^5/d+1/5*b^3*cos(d*x+c)^5/d+a^3
*sin(d*x+c)/d-2/3*a^3*sin(d*x+c)^3/d+a*b^2*sin(d*x+c)^3/d+1/5*a^3*sin(d*x+c
)^5/d-3/5*a*b^2*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.11, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3169, 2713, 2645, 30, 2644, 14}

$$\frac{a^3\sin^5(c+dx)}{5d}-\frac{2a^3\sin^3(c+dx)}{3d}+\frac{a^3\sin(c+dx)}{d}-\frac{3a^2b\cos^5(c+dx)}{5d}-\frac{3ab^2\sin^5(c+dx)}{5d}+\frac{ab^2\sin^3(c+dx)}{d}+\frac{b^3\cos^5(c+dx)}{5d}-\frac{b^3\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/3*(b^3*Cos[c + d*x]^3)/d - (3*a^2*b*Cos[c + d*x]^5)/(5*d) + (b^3*Cos[c +
 d*x]^5)/(5*d) + (a^3*Sin[c + d*x])/d - (2*a^3*Sin[c + d*x]^3)/(3*d) + (a*b
^2*Sin[c + d*x]^3)/d + (a^3*Sin[c + d*x]^5)/(5*d) - (3*a*b^2*Sin[c + d*x]^5
)/(5*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \int \left(a^3\cos^5(c+dx)+3a^2b\cos^4(c+dx)\sin(c+dx)+3\right.$$

$$= a^3\int \cos^5(c+dx)\,dx+\left(3a^2b\right)\int \cos^4(c+dx)\sin(c+$$

$$= -\frac{a^3\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,-\sin(c+dx)\right)}{d}-\frac{(3}{}$$

$$= -\frac{3a^2b\cos^5(c+dx)}{5d}+\frac{a^3\sin(c+dx)}{d}-\frac{2a^3\sin^3(c+dx}{3d}$$

$$= -\frac{b^3\cos^3(c+dx)}{3d}-\frac{3a^2b\cos^5(c+dx)}{5d}+\frac{b^3\cos^5(c+dx}{5d}$$

Mathematica [A]
time = 0.32, size = 150, normalized size = 1.07

$$\frac{-30b(3a^2+b^2)\cos(c+dx)-5(9a^2b+b^3)\cos(3(c+dx))-9a^2b\cos(5(c+dx))+3b^3\cos(5(c+dx))+150a^3\sin(c+dx)+90ab^2\sin(c+dx)+25a^3\sin(3(c+dx))-15ab^2\sin(3(c+dx))+3a^3\sin(5(c+dx))-9ab^2\sin(5(c+dx))}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (-30*b*(3*a^2 + b^2)*Cos[c + d*x] - 5*(9*a^2*b + b^3)*Cos[3*(c + d*x)] - 9*
a^2*b*Cos[5*(c + d*x)] + 3*b^3*Cos[5*(c + d*x)] + 150*a^3*Sin[c + d*x] + 90
*a*b^2*Sin[c + d*x] + 25*a^3*Sin[3*(c + d*x)] - 15*a*b^2*Sin[3*(c + d*x)] +
 3*a^3*Sin[5*(c + d*x)] - 9*a*b^2*Sin[5*(c + d*x)])/(240*d)
```

Maple [A]
time = 0.21, size = 125, normalized size = 0.89

method	result
derivativedivides	$\frac{b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+3a\,b^2\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{3a^2b\left(\cos^5\right.}{5}}{d}$
default	$\frac{b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+3a\,b^2\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)-\frac{3a^2b\left(\cos^5\right.}{5}}{d}$
risch	$-\frac{3a^2b\cos(dx+c)}{8d}-\frac{b^3\cos(dx+c)}{8d}+\frac{5a^3\sin(dx+c)}{8d}+\frac{3a\,b^2\sin(dx+c)}{8d}-\frac{3b\cos(5dx+5c)a^2}{80d}+\frac{b^3\cos(5dx+5c)}{80d}+$
norman	$\frac{-\frac{18a^2b+4b^3}{15d}+\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^3\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4b^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4b^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{6a^2b\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2\left(18a^2\right.}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+3*a*b^2*(-1/5*s
in(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-3/5*a^2*b*cos(d*x+
c)^5+1/5*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 107, normalized size = 0.76

$$-\frac{9\,a^2b\cos\left(dx+c\right)^5-\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\sin\left(dx+c\right)\right)a^3+3\left(3\,\sin\left(dx+c\right)^5-5\,\sin\left(dx+c\right)^3\right)ab^2-\left(3\,\cos\left(dx+c\right)^5-5\,\cos\left(dx+c\right)^3\right)b^3}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/15*(9*a^2*b*cos(d*x + c)^5 - (3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*
sin(d*x + c))*a^3 + 3*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a*b^2 - (3*cos(
d*x + c)^5 - 5*cos(d*x + c)^3)*b^3)/d
```

Fricas [A]
time = 3.96, size = 102, normalized size = 0.73

$$-\frac{5\,b^3\cos\left(dx+c\right)^3+3\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^5-\left(3\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^4+8\,a^3+6\,ab^2+\left(4\,a^3+3\,ab^2\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/15*(5*b^3*cos(d*x + c)^3 + 3*(3*a^2*b - b^3)*cos(d*x + c)^5 - (3*(a^3 -
3*a*b^2)*cos(d*x + c)^4 + 8*a^3 + 6*a*b^2 + (4*a^3 + 3*a*b^2)*cos(d*x + c)^
2)*sin(d*x + c))/d
```

Sympy [A]
time = 0.28, size = 182, normalized size = 1.30

$$\begin{cases} \frac{8a^3\sin^5(c+dx)}{15d} + \frac{4a^3\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{a^3\sin(c+dx)\cos^4(c+dx)}{d} - \frac{3a^2b\cos^5(c+dx)}{5d} + \frac{2ab^2\sin^5(c+dx)}{5d} + \frac{ab^2\sin^3(c+dx)\cos^2(c+dx)}{d} - \frac{b^3\sin^2(c+dx)\cos^3(c+dx)}{3d} - \frac{2b^3\cos^5(c+dx)}{15d} & \text{for } d \neq 0 \\ x(a\cos(c) + b\sin(c))^3\cos^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((8*a**3*sin(c + d*x)**5/(15*d) + 4*a**3*sin(c + d*x)**3*cos(c + d
*x)**2/(3*d) + a**3*sin(c + d*x)*cos(c + d*x)**4/d - 3*a**2*b*cos(c + d*x)*
*5/(5*d) + 2*a*b**2*sin(c + d*x)**5/(5*d) + a*b**2*sin(c + d*x)**3*cos(c +
d*x)**2/d - b**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 2*b**3*cos(c + d*x
)**5/(15*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**3*cos(c)**2, True))
```

Giac [A]
time = 0.48, size = 145, normalized size = 1.04

$$-\frac{(3a^2b - b^3)\cos(5dx + 5c)}{80d} - \frac{(9a^2b + b^3)\cos(3dx + 3c)}{48d} - \frac{(3a^2b + b^3)\cos(dx + c)}{8d} + \frac{(a^3 - 3ab^2)\sin(5dx + 5c)}{80d} + \frac{(5a^3 - 3ab^2)\sin(3dx + 3c)}{48d} + \frac{(5a^3 + 3ab^2)\sin(dx + c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/80*(3*a^2*b - b^3)*cos(5*d*x + 5*c)/d - 1/48*(9*a^2*b + b^3)*cos(3*d*x +
 3*c)/d - 1/8*(3*a^2*b + b^3)*cos(d*x + c)/d + 1/80*(a^3 - 3*a*b^2)*sin(5*d
*x + 5*c)/d + 1/48*(5*a^3 - 3*a*b^2)*sin(3*d*x + 3*c)/d + 1/8*(5*a^3 + 3*a*
b^2)*sin(d*x + c)/d
```

Mupad [B]
time = 0.70, size = 147, normalized size = 1.05

$$\frac{2\left(\frac{3\sin(c+dx)\,a^3\cos(c+dx)^4}{2} + 2\sin(c+dx)\,a^3\cos(c+dx)^2 + 4\sin(c+dx)\,a^3 - \frac{9a^2b\cos(c+dx)^5}{2} - \frac{9\sin(c+dx)\,ab^2\cos(c+dx)^4}{2} + \frac{3\sin(c+dx)\,ab^2\cos(c+dx)^2}{2} + 3\sin(c+dx)\,ab^2 + \frac{3b^3\cos(c+dx)^5}{2} - \frac{5b^3\cos(c+dx)^3}{2}\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

```
[Out] (2*(4*a^3*sin(c + d*x) - (5*b^3*cos(c + d*x)^3)/2 + (3*b^3*cos(c + d*x)^5)/
2 - (9*a^2*b*cos(c + d*x)^5)/2 + 2*a^3*cos(c + d*x)^2*sin(c + d*x) + (3*a^3
*cos(c + d*x)^4*sin(c + d*x))/2 + 3*a*b^2*sin(c + d*x) + (3*a*b^2*cos(c + d
*x)^2*sin(c + d*x))/2 - (9*a*b^2*cos(c + d*x)^4*sin(c + d*x))/2))/(15*d)
```

3.61 $\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=78

$$\frac{3}{8}a\left(a^2+b^2\right)x+\frac{3a(b+a\cot(c+dx))(a-b\cot(c+dx))\sin^2(c+dx)}{8d}+\frac{(b+a\cot(c+dx))^3\sin^4(c+dx)}{4d}$$

```
[Out] 3/8*a*(a^2+b^2)*x+3/8*a*(b+a*cot(d*x+c))*(a-b*cot(d*x+c))*sin(d*x+c)^2/d+1/
4*(b+a*cot(d*x+c))^3*sin(d*x+c)^4/d
```

Rubi [A]
time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3167, 819, 737, 209}

$$\frac{3}{8}ax\left(a^2+b^2\right)+\frac{\sin^4(c+dx)(a\cot(c+dx)+b)^3}{4d}+\frac{3a\sin^2(c+dx)(a\cot(c+dx)+b)(a-b\cot(c+dx))}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*a*(a^2 + b^2)*x)/8 + (3*a*(b + a*Cot[c + d*x])*(a - b*Cot[c + d*x])*Sin[
c + d*x]^2)/(8*d) + ((b + a*Cot[c + d*x])^3*Sin[c + d*x]^4)/(4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[(2*p + 3)*((c*d^2 + a*e^2)/(2*a*c*(p + 1))), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Dist[m*((c*d*f + a*e*g)/(2*a*c*(p + 1))), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = -\frac{\text{Subst}\left(\int \frac{x(b+ax)^3}{(1+x^2)^3}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= \frac{(b+a\cot(c+dx))^3\sin^4(c+dx)}{4d} - \frac{(3a)\text{Subst}\left(\int \frac{(b+ax)^2}{(1+x^2)^2}\right.}{4}$$
$$= \frac{3a(b+a\cot(c+dx))(a-b\cot(c+dx))\sin^2(c+dx)}{8d} +$$
$$= \frac{3}{8}a\left(a^2+b^2\right)x + \frac{3a(b+a\cot(c+dx))(a-b\cot(c+dx}{8d}$$

Mathematica [A]
time = 0.44, size = 94, normalized size = 1.21

$$\frac{12a(a^2+b^2)(c+dx) - 4(3a^2b+b^3)\cos(2(c+dx)) + (-3a^2b+b^3)\cos(4(c+dx)) + 8a^3\sin(2(c+dx)) + a(a^2-3b^2)\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
[Out] (12*a*(a^2 + b^2)*(c + d*x) - 4*(3*a^2*b + b^3)*Cos[2*(c + d*x)] + (-3*a^2*
b + b^3)*Cos[4*(c + d*x)] + 8*a^3*Sin[2*(c + d*x)] + a*(a^2 - 3*b^2)*Sin[4*
(c + d*x)])/(32*d)
```

Maple [A]
time = 0.18, size = 114, normalized size = 1.46

method	result
derivativedivides	$\frac{\frac{b^3\left(\sin^4(dx+c)\right)}{4}+3ab^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{3a^2b\left(\cos^4(dx+c)\right)}{4}+a^3\left(\frac{\left(\cos^3(dx+c)+3\right.}{}\right.}{d}$
default	$\frac{\frac{b^3\left(\sin^4(dx+c)\right)}{4}+3ab^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)-\frac{3a^2b\left(\cos^4(dx+c)\right)}{4}+a^3\left(\frac{\left(\cos^3(dx+c)+3\right.}{}\right.}{d}$
risch	$\frac{3a^3x}{8}+\frac{3ab^2x}{8}-\frac{3b\cos(4dx+4c)a^2}{32d}+\frac{b^3\cos(4dx+4c)}{32d}+\frac{a^3\sin(4dx+4c)}{32d}-\frac{3a\sin(4dx+4c)b^2}{32d}-\frac{3b\cos(2dx+2c}{8d}$

norman	$\frac{\left(\frac{3}{8}a^3+\frac{3}{8}a\,b^2\right)x+\left(\frac{3}{2}a^3+\frac{3}{2}a\,b^2\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{3}{2}a^3+\frac{3}{2}a\,b^2\right)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{3}{8}a^3+\frac{3}{8}a\,b^2\right)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{9}{4}a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*b^3*sin(d*x+c)^4+3*a*b^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x
+c)*sin(d*x+c)+1/8*d*x+1/8*c)-3/4*a^2*b*cos(d*x+c)^4+a^3*(1/4*(cos(d*x+c)^3
+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.28, size = 91, normalized size = 1.17

$$-\frac{24\,a^2b\cos\left(dx+c\right)^4-8\,b^3\sin\left(dx+c\right)^4-\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^3-3\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)ab^2}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/32*(24*a^2*b*cos(d*x + c)^4 - 8*b^3*sin(d*x + c)^4 - (12*d*x + 12*c + si
n(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^3 - 3*(4*d*x + 4*c - sin(4*d*x + 4*c
))*a*b^2)/d
```

Fricas [A]
time = 3.22, size = 100, normalized size = 1.28

$$-\frac{4\,b^3\cos\left(dx+c\right)^2+2\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^4-3\left(a^3+ab^2\right)dx-\left(2\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^3+3\left(a^3+ab^2\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/8*(4*b^3*cos(d*x + c)^2 + 2*(3*a^2*b - b^3)*cos(d*x + c)^4 - 3*(a^3 + a*
b^2)*d*x - (2*(a^3 - 3*a*b^2)*cos(d*x + c)^3 + 3*(a^3 + a*b^2)*cos(d*x + c)
)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(71) = 142$.
time = 0.20, size = 272, normalized size = 3.49

$$\begin{cases} \frac{3a^3x\sin^4(c+dx)}{8}+\frac{3a^3x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3a^3x\cos^4(c+dx)}{8}+\frac{3a^3\sin^3(c+dx)\cos(c+dx)}{8d}+\frac{5a^3\sin(c+dx)\cos^3(c+dx)}{8d}-\frac{3a^2b\cos^4(c+dx)}{4d}+\frac{3ab^2x\sin^4(c+dx)}{8}+\frac{3ab^2x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3ab^2x\cos^4(c+dx)}{8}+\frac{3ab^2\sin^3(c+dx)\cos(c+dx)}{8d}-\frac{3ab^2\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{b^3\sin^4(c+dx)}{4d} & \text{for}\ d\neq 0 \\ x(a\cos(c)+b\sin(c))^3\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((3*a**3*x*sin(c + d*x)**4/8 + 3*a**3*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + 3*a**3*x*cos(c + d*x)**4/8 + 3*a**3*sin(c + d*x)**3*cos(c + d*x)/
```

```
(8*d) + 5*a**3*sin(c + d*x)*cos(c + d*x)**3/(8*d) - 3*a**2*b*cos(c + d*x)**
4/(4*d) + 3*a*b**2*x*sin(c + d*x)**4/8 + 3*a*b**2*x*sin(c + d*x)**2*cos(c +
 d*x)**2/4 + 3*a*b**2*x*cos(c + d*x)**4/8 + 3*a*b**2*sin(c + d*x)**3*cos(c
+ d*x)/(8*d) - 3*a*b**2*sin(c + d*x)*cos(c + d*x)**3/(8*d) + b**3*sin(c + d
*x)**4/(4*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**3*cos(c), True))
```

Giac [A]
time = 0.45, size = 104, normalized size = 1.33

$$\frac{a^3\sin\left(2\,dx+2\,c\right)}{4\,d}+\frac{3}{8}\left(a^3+ab^2\right)x-\frac{\left(3\,a^2b-b^3\right)\cos\left(4\,dx+4\,c\right)}{32\,d}-\frac{\left(3\,a^2b+b^3\right)\cos\left(2\,dx+2\,c\right)}{8\,d}+\frac{\left(a^3-3\,ab^2\right)\sin\left(4\,dx+4\,c\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*a^3*sin(2*d*x + 2*c)/d + 3/8*(a^3 + a*b^2)*x - 1/32*(3*a^2*b - b^3)*cos
(4*d*x + 4*c)/d - 1/8*(3*a^2*b + b^3)*cos(2*d*x + 2*c)/d + 1/32*(a^3 - 3*a*
b^2)*sin(4*d*x + 4*c)/d
```

Mupad [B]
time = 1.63, size = 281, normalized size = 3.60

$$\frac{4\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{3\,a\,b^2}{4}-\frac{5\,a^3}{4}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(\frac{3\,a\,b^2}{4}-\frac{5\,a^3}{4}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{21\,a\,b^2}{4}-\frac{3\,a^3}{4}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(\frac{21\,a\,b^2}{4}-\frac{3\,a^3}{4}\right)+6\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+6\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}-\frac{3\,a\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)\,\left(a^2+b^2\right)}{4\,d}+\frac{3\,a\,\mathrm{atan}\left(\frac{3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2+b^2\right)}{4\,\left(\frac{3\,a^3}{4}+\frac{3\,a\,b^2}{4}\right)}\right)\,\left(a^2+b^2\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

```
[Out] (4*b^3*tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)*((3*a*b^2)/4 - (5*a^3)/4)
+ tan(c/2 + (d*x)/2)^7*((3*a*b^2)/4 - (5*a^3)/4) + tan(c/2 + (d*x)/2)^3*((2
1*a*b^2)/4 - (3*a^3)/4) - tan(c/2 + (d*x)/2)^5*((21*a*b^2)/4 - (3*a^3)/4) +
 6*a^2*b*tan(c/2 + (d*x)/2)^2 + 6*a^2*b*tan(c/2 + (d*x)/2)^6)/(d*(4*tan(c/2
 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 +
 (d*x)/2)^8 + 1)) - (3*a*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2)*(a^2 + b^2))/
(4*d) + (3*a*atan((3*a*tan(c/2 + (d*x)/2)*(a^2 + b^2))/(4*((3*a*b^2)/4 + (3
*a^3)/4)))*(a^2 + b^2))/(4*d)
```

3.62 $\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=58

$$-\frac{(a^2+b^2)\,(b\cos(c+dx)-a\sin(c+dx))}{d}+\frac{(b\cos(c+dx)-a\sin(c+dx))^3}{3d}$$

```
[Out] -(a^2+b^2)*(b*cos(d*x+c)-a*sin(d*x+c))/d+1/3*(b*cos(d*x+c)-a*sin(d*x+c))^3/
d
```

Rubi [A]
time = 0.02, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {3151}

$$\frac{(b\cos(c+dx)-a\sin(c+dx))^3}{3d}-\frac{(a^2+b^2)\,(b\cos(c+dx)-a\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] -(((a^2 + b^2)*(b*Cos[c + d*x] - a*Sin[c + d*x]))/d) + (b*Cos[c + d*x] - a*
Sin[c + d*x])^3/(3*d)
```

Rule 3151

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[-d^(-1), Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x, b
*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
b^2, 0] && IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a\cos(c+dx)+b\sin(c+dx))^3\,dx &= -\frac{\text{Subst}\left(\int \left(a^2+b^2-x^2\right)\,dx,x,b\cos(c+dx)-a\sin(c+dx)\right)}{d}\\ &= -\frac{(a^2+b^2)\,(b\cos(c+dx)-a\sin(c+dx))}{d}+\frac{(b\cos(c+dx)-a\sin(c+dx))^3}{3d}\end{aligned}$$

Mathematica [A]
time = 0.39, size = 81, normalized size = 1.40

$$\frac{-9b(a^2+b^2)\cos(c+dx)+(-3a^2b+b^3)\cos(3(c+dx))+2a(5a^2+3b^2+(a^2-3b^2)\cos(2(c+dx)))\sin(c+dx)}{12d}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]

[Out] (-9*b*(a^2 + b^2)*Cos[c + d*x] + (-3*a^2*b + b^3)*Cos[3*(c + d*x)] + 2*a*(5*a^2 + 3*b^2 + (a^2 - 3*b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])/(12*d)

Maple [A]
time = 0.16, size = 75, normalized size = 1.29

method	result
derivativedivides	$\frac{-\frac{b^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+a\,b^2\left(\sin^3(dx+c)\right)-a^2b\left(\cos^3(dx+c)\right)+\frac{a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$
default	$\frac{-\frac{b^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+a\,b^2\left(\sin^3(dx+c)\right)-a^2b\left(\cos^3(dx+c)\right)+\frac{a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{d}$
risch	$-\frac{3a^2b\cos(dx+c)}{4d}-\frac{3b^3\cos(dx+c)}{4d}+\frac{3a^3\sin(dx+c)}{4d}+\frac{3a\,b^2\sin(dx+c)}{4d}-\frac{b\cos(3dx+3c)a^2}{4d}+\frac{b^3\cos(3dx+3c)}{12d}-$
norman	$\frac{-\frac{6a^2b+4b^3}{3d}+\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{4b^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{6a^2b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{4a\left(a^2+6b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/3*b^3*(2+sin(d*x+c)^2)*cos(d*x+c)+a*b^2*sin(d*x+c)^3-a^2*b*cos(d*x+c)^3+1/3*a^3*(2+cos(d*x+c)^2)*sin(d*x+c))

Maxima [A]
time = 0.28, size = 84, normalized size = 1.45

$$-\frac{a^2b\cos\left(dx+c\right)^3}{d}+\frac{ab^2\sin\left(dx+c\right)^3}{d}-\frac{\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)a^3}{3\,d}+\frac{\left(\cos\left(dx+c\right)^3-3\,\cos\left(dx+c\right)\right)b^3}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")

[Out] -a^2*b*cos(d*x + c)^3/d + a*b^2*sin(d*x + c)^3/d - 1/3*(sin(d*x + c)^3 - 3*sin(d*x + c))*a^3/d + 1/3*(cos(d*x + c)^3 - 3*cos(d*x + c))*b^3/d

Fricas [A]
time = 3.15, size = 77, normalized size = 1.33

$$-\frac{3\,b^3\cos\left(dx+c\right)+\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^3-\left(2\,a^3+3\,ab^2+\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")

[Out] -1/3*(3*b^3*cos(d*x + c) + (3*a^2*b - b^3)*cos(d*x + c)^3 - (2*a^3 + 3*a*b^2 + (a^3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/d

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(48) = 96$.
time = 0.13, size = 117, normalized size = 2.02

$$\begin{cases} \frac{2a^3\sin^3(c+dx)}{3d} + \frac{a^3\sin(c+dx)\cos^2(c+dx)}{d} - \frac{a^2b\cos^3(c+dx)}{d} + \frac{ab^2\sin^3(c+dx)}{d} - \frac{b^3\sin^2(c+dx)\cos(c+dx)}{d} - \frac{2b^3\cos^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x(a\cos(c) + b\sin(c))^3 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((2*a**3*sin(c + d*x)**3/(3*d) + a**3*sin(c + d*x)*cos(c + d*x)**2
/d - a**2*b*cos(c + d*x)**3/d + a*b**2*sin(c + d*x)**3/d - b**3*sin(c + d*x
)**2*cos(c + d*x)/d - 2*b**3*cos(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a*cos(c)
 + b*sin(c))**3, True))
```

Giac [A]
time = 0.43, size = 91, normalized size = 1.57

$$-\frac{(3a^2b - b^3)\cos(3dx + 3c)}{12d} - \frac{3(a^2b + b^3)\cos(dx + c)}{4d} + \frac{(a^3 - 3ab^2)\sin(3dx + 3c)}{12d} + \frac{3(a^3 + ab^2)\sin(dx + c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/12*(3*a^2*b - b^3)*cos(3*d*x + 3*c)/d - 3/4*(a^2*b + b^3)*cos(d*x + c)/d
 + 1/12*(a^3 - 3*a*b^2)*sin(3*d*x + 3*c)/d + 3/4*(a^3 + a*b^2)*sin(d*x + c)
/d
```

Mupad [B]
time = 0.57, size = 104, normalized size = 1.79

$$\frac{\frac{\sin(c+dx)\,a^3\cos(c+dx)^2}{3} + \frac{2\sin(c+dx)\,a^3}{3} - a^2b\cos(c+dx)^3 - \sin(c+dx)\,ab^2\cos(c+dx)^2 + \sin(c+dx)\,ab^2 + \frac{b^3\cos(c+dx)^3}{3} - b^3\cos(c+dx)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

```
[Out] ((2*a^3*sin(c + d*x))/3 - b^3*cos(c + d*x) + (b^3*cos(c + d*x)^3)/3 - a^2*b
*cos(c + d*x)^3 + (a^3*cos(c + d*x)^2*sin(c + d*x))/3 + a*b^2*sin(c + d*x)
- a*b^2*cos(c + d*x)^2*sin(c + d*x))/d
```

3.63 $\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=91

$$\frac{1}{2}a\left(a^2+3b^2\right)x-\frac{b^3\log(\sin(c+dx))}{d}+\frac{b^3\log(\tan(c+dx))}{d}+\frac{\left(b\left(3a^2-b^2\right)+a\left(a^2-3b^2\right)\cot(c+dx)\right)\sin^2(c+$$

```
[Out] 1/2*a*(a^2+3*b^2)*x-b^3*ln(sin(d*x+c))/d+b^3*ln(tan(d*x+c))/d+1/2*(b*(3*a^2
-b^2)+a*(a^2-3*b^2)*cot(d*x+c))*sin(d*x+c)^2/d
```

Rubi [A]
time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3167, 1819, 815, 649, 209, 266}

$$\frac{\sin^2(c+dx)\left(a\left(a^2-3b^2\right)\cot(c+dx)+b\left(3a^2-b^2\right)\right)}{2d}+\frac{1}{2}ax\left(a^2+3b^2\right)-\frac{b^3\log(\sin(c+dx))}{d}+\frac{b^3\log(\tan(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*(a^2 + 3*b^2)*x)/2 - (b^3*Log[Sin[c + d*x]])/d + (b^3*Log[Tan[c + d*x]])
/d + ((b*(3*a^2 - b^2) + a*(a^2 - 3*b^2)*Cot[c + d*x])*Sin[c + d*x]^2)/(2*d
)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^3}{x(1+x^2)^2}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= \frac{(b(3a^2-b^2)+a(a^2-3b^2)\cot(c+dx))\sin^2(c+dx)}{2d} + \cdots$$

$$= \frac{(b(3a^2-b^2)+a(a^2-3b^2)\cot(c+dx))\sin^2(c+dx)}{2d} + \cdots$$

$$= \frac{b^3\log(\tan(c+dx))}{d} + \frac{(b(3a^2-b^2)+a(a^2-3b^2)\cot(c+ \cdots}{2d}$$

$$= \frac{b^3\log(\tan(c+dx))}{d} + \frac{(b(3a^2-b^2)+a(a^2-3b^2)\cot(c+ \cdots}{2d}$$

$$= \frac{1}{2}a\left(a^2+3b^2\right)x - \frac{b^3\log(\sin(c+dx))}{d} + \frac{b^3\log(\tan(c+dx \cdots}{d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 401 vs. 2(91) = 182.
time = 0.84, size = 401, normalized size = 4.41

$$\frac{5a^4b^2+2a^2b^4-b^6+(-3a^4b^2-2a^2b^4+b^6)\cos(2(c+dx))+2a^2b^4\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+2b^6\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)-a^5\sqrt{-b^2}\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+4a^3(-b^2)^{3/2}\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)-3a(-b^2)^{5/2}\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+2a^2b^4\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)+2b^6\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)+a^5\sqrt{-b^2}\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)+3ab^4\sqrt{-b^2}\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)-4a^3(-b^2)^{3/2}\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)+ab(a^4-2a^2b^2-3b^4)\sin(2(c+dx))}{4b(a^2+b^2)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (5*a^4*b^2 + 2*a^2*b^4 - b^6 + (-3*a^4*b^2 - 2*a^2*b^4 + b^6)*Cos[2*(c + d*
x)] + 2*a^2*b^4*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 2*b^6*Log[Sqrt[-b^2] - b
*Tan[c + d*x]] - a^5*Sqrt[-b^2]*Log[Sqrt[-b^2] - b*Tan[c + d*x]] + 4*a^3*(-
b^2)^(3/2)*Log[Sqrt[-b^2] - b*Tan[c + d*x]] - 3*a*(-b^2)^(5/2)*Log[Sqrt[-b^
2] - b*Tan[c + d*x]] + 2*a^2*b^4*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + 2*b^6*L
og[Sqrt[-b^2] + b*Tan[c + d*x]] + a^5*Sqrt[-b^2]*Log[Sqrt[-b^2] + b*Tan[c +
 d*x]] + 3*a*b^4*Sqrt[-b^2]*Log[Sqrt[-b^2] + b*Tan[c + d*x]] - 4*a^3*(-b^2)
^(3/2)*Log[Sqrt[-b^2] + b*Tan[c + d*x]] + a*b*(a^4 - 2*a^2*b^2 - 3*b^4)*Sin
[2*(c + d*x)])/(4*b*(a^2 + b^2)*d)
```

Maple [A]
time = 0.24, size = 98, normalized size = 1.08

method	result
derivativedivides	$\frac{a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{3a^2\left(\cos^2(dx+c)\right)b}{2}+3a\,b^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b^3\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos\right.}{d}$
default	$\frac{a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-\frac{3a^2\left(\cos^2(dx+c)\right)b}{2}+3a\,b^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b^3\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos\right.}{d}$
risch	$ix\,b^3+\frac{a^3x}{2}+\frac{3a\,b^2x}{2}-\frac{3\,e^{2i(dx+c)}a^2b}{8d}+\frac{e^{2i(dx+c)}b^3}{8d}-\frac{ie^{2i(dx+c)}a^3}{8d}+\frac{3ie^{2i(dx+c)}a\,b^2}{8d}-\frac{3\,e^{-2i(dx+c)}a^2b}{8d}$
norman	$\frac{(\frac{1}{2}a^3+\frac{3}{2}a\,b^2)x+(\frac{1}{2}a^3+\frac{3}{2}a\,b^2)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{3}{2}a^3+\frac{9}{2}a\,b^2)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{3}{2}a^3+\frac{9}{2}a\,b^2)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-3/2*a^2*cos(d*x+c)^2*b+3
*a*b^2*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+b^3*(-1/2*sin(d*x+c)^2-ln
(cos(d*x+c))))
```

Maxima [A]
time = 0.28, size = 91, normalized size = 1.00

$$\frac{6\,a^2b\sin\left(dx+c\right)^2+\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^3+3\left(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right)\right)ab^2-2\left(\sin\left(dx+c\right)^2+\log\left(\sin\left(dx+c\right)^2-1\right)\right)b^3}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(6*a^2*b*sin(d*x + c)^2 + (2*d*x + 2*c + sin(2*d*x + 2*c))*a^3 + 3*(2*d
*x + 2*c - sin(2*d*x + 2*c))*a*b^2 - 2*(sin(d*x + c)^2 + log(sin(d*x + c)^2
 - 1))*b^3)/d
```

Fricas [A]
time = 2.35, size = 79, normalized size = 0.87

$$-\frac{2\,b^3\log\left(-\cos\left(dx+c\right)\right)-\left(a^3+3\,ab^2\right)dx+\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^2-\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^3*log(-cos(d*x + c)) - (a^3 + 3*a*b^2)*d*x + (3*a^2*b - b^3)*cos(
d*x + c)^2 - (a^3 - 3*a*b^2)*cos(d*x + c)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)^3\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))**3*sec(c + d*x), x)
```

Giac [A]
time = 0.50, size = 93, normalized size = 1.02

$$\frac{b^3\log\left(\tan\left(dx+c\right)^2+1\right)+\left(a^3+3\,ab^2\right)\left(dx+c\right)-\frac{b^3\tan\left(dx+c\right)^2-a^3\tan\left(dx+c\right)+3\,ab^2\tan\left(dx+c\right)+3\,a^2b}{\tan\left(dx+c\right)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*log(tan(d*x + c)^2 + 1) + (a^3 + 3*a*b^2)*(d*x + c) - (b^3*tan(d*x
 + c)^2 - a^3*tan(d*x + c) + 3*a*b^2*tan(d*x + c) + 3*a^2*b)/(tan(d*x + c)^
2 + 1))/d
```

Mupad [B]
time = 1.24, size = 156, normalized size = 1.71

$$\frac{b^3\,\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)-b^3\,\ln\left(\frac{\cos\left(c+d\,x\right)}{\cos\left(c+d\,x\right)+1}\right)+a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+\frac{b^3\,\cos\left(2\,c+2\,d\,x\right)}{4}+\frac{a^3\,\sin\left(2\,c+2\,d\,x\right)}{4}+3\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-\frac{3\,a^2\,b\,\cos\left(2\,c+2\,d\,x\right)}{4}-\frac{3\,a\,b^2\,\sin\left(2\,c+2\,d\,x\right)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x),x)
```

```
[Out] (b^3*log(1/cos(c/2 + (d*x)/2)^2) - b^3*log(cos(c + d*x)/(cos(c + d*x) + 1))
 + a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + (b^3*cos(2*c + 2*d*x))
/4 + (a^3*sin(2*c + 2*d*x))/4 + 3*a*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2)) - (3*a^2*b*cos(2*c + 2*d*x))/4 - (3*a*b^2*sin(2*c + 2*d*x))/4)/d
```

3.64 $\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=86

$$\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{3a^2b\cos(c+dx)}{d}+\frac{b^3\cos(c+dx)}{d}+\frac{b^3\sec(c+dx)}{d}+\frac{a^3\sin(c+dx)}{d}-\frac{3ab^2\sin(c+dx)}{d}$$

```
[Out] 3*a*b^2*arctanh(sin(d*x+c))/d-3*a^2*b*cos(d*x+c)/d+b^3*cos(d*x+c)/d+b^3*sec
(d*x+c)/d+a^3*sin(d*x+c)/d-3*a*b^2*sin(d*x+c)/d
```

Rubi [A]
time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3169, 2717, 2718, 2672, 327, 212, 2670, 14}

$$\frac{a^3\sin(c+dx)}{d}-\frac{3a^2b\cos(c+dx)}{d}-\frac{3ab^2\sin(c+dx)}{d}+\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^3\cos(c+dx)}{d}+\frac{b^3\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*a*b^2*ArcTanh[Sin[c + d*x]])/d - (3*a^2*b*Cos[c + d*x])/d + (b^3*Cos[c +
 d*x])/d + (b^3*Sec[c + d*x])/d + (a^3*Sin[c + d*x])/d - (3*a*b^2*Sin[c + d
*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \int \left(a^3\cos(c+dx)+3a^2b\sin(c+dx)+3ab^2\sin(c+dx)\right.$$

$$= a^3\int \cos(c+dx)\,dx + \left(3a^2b\right)\int \sin(c+dx)\,dx + \left(3ab^2\right)$$

$$= -\frac{3a^2b\cos(c+dx)}{d} + \frac{a^3\sin(c+dx)}{d} + \frac{\left(3ab^2\right)\mathrm{Subst}\left(\int \right.}{}$$

$$= -\frac{3a^2b\cos(c+dx)}{d} + \frac{a^3\sin(c+dx)}{d} - \frac{3ab^2\sin(c+dx)}{d}$$

$$= \frac{3ab^2\tanh^{-1}(\sin(c+dx))}{d} - \frac{3a^2b\cos(c+dx)}{d} + \frac{b^3\cos(c}{d}$$

Mathematica [**A**]

time = 1.12, size = 131, normalized size = 1.52

$$\frac{\sec(c+dx)\left(-3a^2b+3b^3+\left(-3a^2b+b^3\right)\cos(2(c+dx))-6ab^2\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+a^3\sin(2(c+dx))-3ab^2\sin(2(c+dx))\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]*(-3*a^2*b + 3*b^3 + (-3*a^2*b + b^3)*Cos[2*(c + d*x)] - 6*a*b
^2*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]]) + a^3*Sin[2*(c + d*x)] - 3*a*b^2*Sin[2*(c + d*x)
]))/(2*d)
```

Maple [A]
time = 0.28, size = 96, normalized size = 1.12

method	result
derivativedivides	$\frac{\sin(dx+c)a^3-3a^2b\cos(dx+c)+3a\,b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+b^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(d\right.}{d}$
default	$\frac{\sin(dx+c)a^3-3a^2b\cos(dx+c)+3a\,b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+b^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(d\right.}{d}$
risch	$-\frac{3\,\mathrm{e}^{i(dx+c)}a^2b}{2d}+\frac{\mathrm{e}^{i(dx+c)}b^3}{2d}-\frac{i\mathrm{e}^{i(dx+c)}a^3}{2d}+\frac{3i\mathrm{e}^{i(dx+c)}a\,b^2}{2d}-\frac{3\,\mathrm{e}^{-i(dx+c)}a^2b}{2d}+\frac{\mathrm{e}^{-i(dx+c)}b^3}{2d}+\frac{i\mathrm{e}^{-i(dx+c)}a^3}{2d}$
norman	$\frac{\frac{6a^2b-4b^3}{d}-\frac{6a^2b\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2\left(3a^2b-4b^3\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2a\left(a^2-3b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2a\left(a^2-3b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(sin(d*x+c)*a^3-3*a^2*b*cos(d*x+c)+3*a*b^2*(-sin(d*x+c)+ln(sec(d*x+c)+t
an(d*x+c)))+b^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.29, size = 84, normalized size = 0.98

$$\frac{2\,b^3\left(\frac{1}{\cos(dx+c)}+\cos(dx+c)\right)+3\,ab^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\,\sin(dx+c))-6\,a^2b\cos(dx+c)+2\,a^3\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^3*(1/cos(d*x + c) + cos(d*x + c)) + 3*a*b^2*(log(sin(d*x + c) + 1)
 - log(sin(d*x + c) - 1) - 2*sin(d*x + c)) - 6*a^2*b*cos(d*x + c) + 2*a^3*s
in(d*x + c))/d
```

Fricas [A]
time = 2.23, size = 109, normalized size = 1.27

$$\frac{3\,ab^2\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-3\,ab^2\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,b^3-2\,\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^2+2\,\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*a*b^2*cos(d*x + c)*log(sin(d*x + c) + 1) - 3*a*b^2*cos(d*x + c)*log(
-sin(d*x + c) + 1) + 2*b^3 - 2*(3*a^2*b - b^3)*cos(d*x + c)^2 + 2*(a^3 - 3*
a*b^2)*cos(d*x + c)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)^3\sec^2\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))**3*sec(c + d*x)**2, x)
```

Giac [A]
time = 0.47, size = 150, normalized size = 1.74

$$\frac{3\,ab^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,ab^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\,\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^2b-2\,b^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (3*a*b^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*b^2*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + 2*(a^3*tan(1/2*d*x + 1/2*c)^3 - 3*a*b^2*tan(1/2*d*x + 1/2*c
)^3 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^2 - a^3*tan(1/2*d*x + 1/2*c) + 3*a*b^2*t
an(1/2*d*x + 1/2*c) + 3*a^2*b - 2*b^3)/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

Mupad [B]
time = 1.05, size = 116, normalized size = 1.35

$$\frac{6\,a\,b^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(6\,a\,b^2-2\,a^3\right)-6\,a^2\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(6\,a\,b^2-2\,a^3\right)+4\,b^3+6\,a^2\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^2,x)
```

```
[Out] (6*a*b^2*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)^3*(6*a*b^2 - 2*
a^3) - 6*a^2*b - tan(c/2 + (d*x)/2)*(6*a*b^2 - 2*a^3) + 4*b^3 + 6*a^2*b*tan
(c/2 + (d*x)/2)^2)/(d*(tan(c/2 + (d*x)/2)^4 - 1))
```

3.65 $\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=72

$$a\left(a^2-3b^2\right)x-\frac{b(3a^2-b^2)\log(\cos(c+dx))}{d}+\frac{2ab^2\tan(c+dx)}{d}+\frac{b(a+b\tan(c+dx))^2}{2d}$$

```
[Out] a*(a^2-3*b^2)*x-b*(3*a^2-b^2)*ln(cos(d*x+c))/d+2*a*b^2*tan(d*x+c)/d+1/2*b*(
a+b*tan(d*x+c))^2/d
```

Rubi [A]
time = 0.07, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3165, 3563, 3606, 3556}

$$-\frac{b(3a^2-b^2)\log(\cos(c+dx))}{d}+ax\left(a^2-3b^2\right)+\frac{2ab^2\tan(c+dx)}{d}+\frac{b(a+b\tan(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] a*(a^2 - 3*b^2)*x - (b*(3*a^2 - b^2)*Log[Cos[c + d*x]])/d + (2*a*b^2*Tan[c
+ d*x]])/d + (b*(a + b*Tan[c + d*x])^2)/(2*d)
```

Rule 3165

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
```

```
f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \int (a+b\tan(c+dx))^3\,dx$$
$$= \frac{b(a+b\tan(c+dx))^2}{2d} + \int (a+b\tan(c+dx))\left(a^2-b^2+\right.$$
$$= a\left(a^2-3b^2\right)x + \frac{2ab^2\tan(c+dx)}{d} + \frac{b(a+b\tan(c+dx))}{2d}$$
$$= a\left(a^2-3b^2\right)x - \frac{b(3a^2-b^2)\log(\cos(c+dx))}{d} + \frac{2ab^2\tan}{d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.29, size = 79, normalized size = 1.10

$$\frac{(ia-b)^3\log(i-\tan(c+dx))-(ia+b)^3\log(i+\tan(c+dx))+6ab^2\tan(c+dx)+b^3\tan^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
[Out] ((I*a - b)^3*Log[I - Tan[c + d*x]] - (I*a + b)^3*Log[I + Tan[c + d*x]] + 6*
a*b^2*Tan[c + d*x] + b^3*Tan[c + d*x]^2)/(2*d)
```

Maple [A]
time = 0.29, size = 70, normalized size = 0.97

method	result
derivativedivides	$\frac{a^3(dx+c)-3a^2b\ln(\cos(dx+c))+3a\,b^2(\tan(dx+c)-dx-c)+b^3\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)}{d}$
default	$\frac{a^3(dx+c)-3a^2b\ln(\cos(dx+c))+3a\,b^2(\tan(dx+c)-dx-c)+b^3\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)}{d}$
risch	$3ia^2bx-ix\,b^3+a^3x-3a\,b^2x+\frac{6ib\,a^2c}{d}-\frac{2ib^3c}{d}+\frac{2b^2\left(3ia\,e^{2i(dx+c)}+b\,e^{2i(dx+c)}+3ia\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{3b\ln(e^{2i(dx+c)}}{d}$
norman	$\frac{(a^3-3a\,b^2)x+(-2a^3+6a\,b^2)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-2a^3+6a\,b^2)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(a^3-3a\,b^2)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(a^3-3a}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(d*x+c)-3*a^2*b*ln(cos(d*x+c))+3*a*b^2*(tan(d*x+c)-d*x-c)+b^3*(1/2
*tan(d*x+c)^2+ln(cos(d*x+c))))
```

Maxima [A]
time = 0.49, size = 85, normalized size = 1.18

$$\frac{2\,(dx+c)a^3-6\,(dx+c-\tan\,(dx+c))ab^2-b^3\left(\frac{1}{\sin(dx+c)^2-1}-\log\,(\sin\,(dx+c)^2-1)\right)-3\,a^2b\log\,(-\sin\,(dx+c)^2+1)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(d*x + c)*a^3 - 6*(d*x + c - tan(d*x + c))*a*b^2 - b^3*(1/(sin(d*x +
 c)^2 - 1) - log(sin(d*x + c)^2 - 1)) - 3*a^2*b*log(-sin(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 2.62, size = 88, normalized size = 1.22

$$\frac{2\,(a^3-3\,ab^2)dx\cos\,(dx+c)^2+6\,ab^2\cos\,(dx+c)\sin\,(dx+c)-2\,(3\,a^2b-b^3)\cos\,(dx+c)^2\log\,(-\cos\,(dx+c))+b^3}{2\,d\cos\,(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(a^3 - 3*a*b^2)*d*x*cos(d*x + c)^2 + 6*a*b^2*cos(d*x + c)*sin(d*x +
c) - 2*(3*a^2*b - b^3)*cos(d*x + c)^2*log(-cos(d*x + c)) + b^3)/(d*cos(d*x
+ c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\cos\,(c+dx)+b\sin\,(c+dx))^3\sec^3\,(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))**3*sec(c + d*x)**3, x)
```

Giac [A]
time = 0.48, size = 71, normalized size = 0.99

$$\frac{b^3\tan\,(dx+c)^2+6\,ab^2\tan\,(dx+c)+2\,(a^3-3\,ab^2)(dx+c)+(3\,a^2b-b^3)\log\,(\tan\,(dx+c)^2+1)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(b^3*tan(d*x + c)^2 + 6*a*b^2*tan(d*x + c) + 2*(a^3 - 3*a*b^2)*(d*x + c
) + (3*a^2*b - b^3)*log(tan(d*x + c)^2 + 1))/d
```

Mupad [B]
time = 1.86, size = 183, normalized size = 2.54

$$\frac{2\left(\frac{b^3\,\ln\left(\frac{\cos(c+d\,x)}{\cos(c+d\,x)+1}\right)}{2}-\frac{b^3\,\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)}{2}+a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+\frac{3\,a^2\,b\,\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)}{2}-3\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-\frac{3\,a^2\,b\,\ln\left(\frac{\cos(c+d\,x)}{\cos(c+d\,x)+1}\right)}{2}\right)}{d}+\frac{\frac{b^3}{2}+\frac{3\,a\,\sin(2\,c+2\,d\,x)\,b^2}{2}}{d\,\left(\frac{\cos(2\,c+2\,d\,x)}{2}+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^3,x)
```

```
[Out] (2*((b^3*log(cos(c + d*x)/(cos(c + d*x) + 1)))/2 - (b^3*log(1/cos(c/2 + (d*
x)/2)^2))/2 + a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + (3*a^2*b*lo
g(1/cos(c/2 + (d*x)/2)^2))/2 - 3*a*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d
*x)/2)) - (3*a^2*b*log(cos(c + d*x)/(cos(c + d*x) + 1)))/2))/d + (b^3/2 + (
3*a*b^2*sin(2*c + 2*d*x))/2)/(d*(cos(2*c + 2*d*x)/2 + 1/2))
```

3.66 $\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=103

$$\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3a^2b\sec(c+dx)}{d}-\frac{b^3\sec(c+dx)}{d}+\frac{b^3\sec^3(c+dx)}{3d}+\frac{3ab}{}$$

```
[Out] a^3*arctanh(sin(d*x+c))/d-3/2*a*b^2*arctanh(sin(d*x+c))/d+3*a^2*b*sec(d*x+c
)/d-b^3*sec(d*x+c)/d+1/3*b^3*sec(d*x+c)^3/d+3/2*a*b^2*sec(d*x+c)*tan(d*x+c)
/d
```

Rubi [A]
time = 0.09, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3169, 3855, 2686, 8, 2691}

$$\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{3a^2b\sec(c+dx)}{d}-\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3ab^2\tan(c+dx)\sec(c+dx)}{2d}+\frac{b^3\sec^3(c+dx)}{3d}-\frac{b^3\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*ArcTanh[Sin[c + d*x]])/d - (3*a*b^2*ArcTanh[Sin[c + d*x]])/(2*d) + (3*
a^2*b*Sec[c + d*x])/d - (b^3*Sec[c + d*x])/d + (b^3*Sec[c + d*x]^3)/(3*d) +
 (3*a*b^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
```

```
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx &= \int \left(a^3\sec(c+dx)+3a^2b\sec(c+dx)\tan(c+dx)+3ab^2\right.\\ &= a^3\int \sec(c+dx)\,dx+\left(3a^2b\right)\int \sec(c+dx)\tan(c+dx)\\ &= \frac{a^3\tanh^{-1}(\sin(c+dx))}{d}+\frac{3ab^2\sec(c+dx)\tan(c+dx)}{2d}\\ &= \frac{a^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{2d}+\frac{3a}{}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 293 vs. 2(103) = 206.
time = 1.70, size = 293, normalized size = 2.84

$$\frac{36a^2b-10b^3-6a(2a^2-3b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12a^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-18ab^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{9ab^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{b^3}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+2b(18a^2-b^2+2b^2\cos(c+dx)+(18a^2-5b^2)\cos(2(c+dx)))\sec^3(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)-\frac{9ab^2}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{b^3}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (36*a^2*b - 10*b^3 - 6*a*(2*a^2 - 3*b^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 12*a^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 18*a*b^2*Log[Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]] + (9*a*b^2)/(Cos[(c + d*x)/2] - Sin[(c +
d*x)/2])^2 + b^3/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + 2*b*(18*a^2 - b^
2 + 2*b^2*Cos[c + d*x] + (18*a^2 - 5*b^2)*Cos[2*(c + d*x)])*Sec[c + d*x]^3*
Sin[(c + d*x)/2]^2 - (9*a*b^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + b^
3/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2)/(12*d)
```

Maple [A]
time = 0.34, size = 146, normalized size = 1.42

method	result

derivativedivides	$\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))+\frac{3a^2b}{\cos(dx+c)}+3ab^2\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+b^3\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}\right.}{d}$
default	$\frac{a^3\ln(\sec(dx+c)+\tan(dx+c))+\frac{3a^2b}{\cos(dx+c)}+3ab^2\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+b^3\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}\right.}{d}$
risch	$-\frac{b(9iab\,e^{5i(dx+c)}-18a^2e^{5i(dx+c)}+6b^2e^{5i(dx+c)}-36a^2e^{3i(dx+c)}+4b^2e^{3i(dx+c)}-9iab\,e^{i(dx+c)}-18a^2e^{i(dx+c)}+6b^2e^{i(dx}}{3d\left(e^{2i(dx+c)}+1\right)^3}$
norman	$\frac{\frac{(12a^2b-8b^3)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{18a^2b-4b^3}{3d}-\frac{(6a^2b+4b^3)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{3ab^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{9ab^2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{6ab^2(\text{ta}}{}}{(\tan}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^2*b/cos(d*x+c)+3*a*b^2*(1/2*sin(d*x+
c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))+b^3*(1/3*si
n(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+c)^2)*co
s(d*x+c)))
```

Maxima [A]
time = 0.33, size = 118, normalized size = 1.15

$$-\frac{9\,ab^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)-6\,a^3\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)-\frac{36\,a^2b}{\cos(dx+c)}+\frac{4\left(3\,\cos(dx+c)^2-1\right)b^3}{\cos(dx+c)^3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/12*(9*a*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1)
 - log(sin(d*x + c) - 1)) - 6*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
 - 1)) - 36*a^2*b/cos(d*x + c) + 4*(3*cos(d*x + c)^2 - 1)*b^3/cos(d*x + c)^
3)/d
```

Fricas [A]
time = 1.87, size = 123, normalized size = 1.19

$$\frac{3\left(2\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\left(2\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+18\,ab^2\cos\left(dx+c\right)\sin\left(dx+c\right)+4\,b^3+12\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^2}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(3*(2*a^3 - 3*a*b^2)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*a^3 -
 3*a*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 18*a*b^2*cos(d*x + c)*sin
(d*x + c) + 4*b^3 + 12*(3*a^2*b - b^3)*cos(d*x + c)^2)/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 171, normalized size = 1.66

$$\frac{3\,(2\,a^3-3\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(2\,a^3-3\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(9\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-18\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+36\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-18\,a^2b+4\,b^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(3*(2*a^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*a^3 - 3*
a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(9*a*b^2*tan(1/2*d*x + 1/2*c)
^5 - 18*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 36*a^2*b*tan(1/2*d*x + 1/2*c)^2 - 12
*b^3*tan(1/2*d*x + 1/2*c)^2 - 9*a*b^2*tan(1/2*d*x + 1/2*c) - 18*a^2*b + 4*b
^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B]
time = 2.38, size = 160, normalized size = 1.55

$$-\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(3\,a\,b^2-2\,a^3\right)}{d}-\frac{6\,a^2\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(12\,a^2\,b-4\,b^3\right)-\frac{4\,b^3}{3}+3\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-3\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^4,x)
```

```
[Out] - (atanh(tan(c/2 + (d*x)/2))*(3*a*b^2 - 2*a^3))/d - (6*a^2*b - tan(c/2 + (d
*x)/2)^2*(12*a^2*b - 4*b^3) - (4*b^3)/3 + 3*a*b^2*tan(c/2 + (d*x)/2) + 6*a^
2*b*tan(c/2 + (d*x)/2)^4 - 3*a*b^2*tan(c/2 + (d*x)/2)^5)/(d*(3*tan(c/2 + (d
*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.67 $\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=30

$$\frac{(b+a\cot(c+dx))^4\tan^4(c+dx)}{4bd}$$

```
[Out] 1/4*(b+a*cot(d*x+c))^4*tan(d*x+c)^4/b/d
```

Rubi [A]

time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3167, 37}

$$\frac{\tan^4(c+dx)(a\cot(c+dx)+b)^4}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] ((b + a*Cot[c + d*x])^4*Tan[c + d*x]^4)/(4*b*d)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx &= -\frac{\text{Subst}\left(\int \frac{(b+ax)^3}{x^5}\,dx, x, \cot(c+dx)\right)}{d}\\ &= \frac{(b+a\cot(c+dx))^4\tan^4(c+dx)}{4bd}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 79 vs. 2(30) = 60.
time = 0.64, size = 79, normalized size = 2.63

$$\frac{\sec^4(c+dx)\left((6a^2b-2b^3)\cos(2(c+dx))+a(6ab+2(a^2+b^2)\sin(2(c+dx))+(a^2-b^2)\sin(4(c+dx)))\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^4*((6*a^2*b - 2*b^3)*Cos[2*(c + d*x)] + a*(6*a*b + 2*(a^2 + b
^2)*Sin[2*(c + d*x)] + (a^2 - b^2)*Sin[4*(c + d*x)])))/(8*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(28) = 56.
time = 0.34, size = 72, normalized size = 2.40

method	result
derivativedivides	$\frac{a^3\tan(dx+c)+\frac{3a^2b}{2\cos(dx+c)^2}+\frac{a\,b^2\left(\sin^3(dx+c)\right)}{\cos(dx+c)^3}+\frac{b^3\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}}{d}$
default	$\frac{a^3\tan(dx+c)+\frac{3a^2b}{2\cos(dx+c)^2}+\frac{a\,b^2\left(\sin^3(dx+c)\right)}{\cos(dx+c)^3}+\frac{b^3\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}}{d}$
risch	$-\frac{2(-ia^3e^{6i(dx+c)}+3ia\,b^2e^{6i(dx+c)}-3a^2b\,e^{6i(dx+c)}+b^3e^{6i(dx+c)}-3ia^3e^{4i(dx+c)}+3ia\,b^2e^{4i(dx+c)}-6a^2b\,e^{4i(dx+c)}-3ia^3e}{d\left(e^{2i(dx+c)}+1\right)^4}$
norman	$\frac{\frac{6a^2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{6a^2b\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(6a^2b+4b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(6a^2b+4b^3\right)\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2a^3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*tan(d*x+c)+3/2*a^2*b/cos(d*x+c)^2+a*b^2*sin(d*x+c)^3/cos(d*x+c)^3+
1/4*b^3*sin(d*x+c)^4/cos(d*x+c)^4)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(28) = 56.
time = 0.28, size = 87, normalized size = 2.90

$$\frac{4\,ab^2\tan\left(dx+c\right)^3+4\,a^3\tan\left(dx+c\right)+\frac{\left(2\,\sin(dx+c)^2-1\right)b^3}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-\frac{6\,a^2b}{\sin(dx+c)^2-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*a*b^2*tan(d*x + c)^3 + 4*a^3*tan(d*x + c) + (2*sin(d*x + c)^2 - 1)*b
^3/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 6*a^2*b/(sin(d*x + c)^2 - 1))/
d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(28) = 56$.

time = 1.92, size = 78, normalized size = 2.60

$$\frac{b^3 + 2\,(3\,a^2b - b^3)\cos{(dx+c)}^2 + 4\,\left(ab^2\cos{(dx+c)} + (a^3 - ab^2)\cos{(dx+c)}^3\right)\sin{(dx+c)}}{4\,d\cos{(dx+c)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(b^3 + 2*(3*a^2*b - b^3)*cos(d*x + c)^2 + 4*(a*b^2*cos(d*x + c) + (a^3
- a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(28) = 56$.

time = 0.48, size = 57, normalized size = 1.90

$$\frac{b^3\tan{(dx+c)}^4 + 4\,ab^2\tan{(dx+c)}^3 + 6\,a^2b\tan{(dx+c)}^2 + 4\,a^3\tan{(dx+c)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^3*tan(d*x + c)^4 + 4*a*b^2*tan(d*x + c)^3 + 6*a^2*b*tan(d*x + c)^2 +
 4*a^3*tan(d*x + c))/d
```

Mupad [B]

time = 0.64, size = 88, normalized size = 2.93

$$\frac{\cos{(c+d\,x)}^3\,(a^3\,\sin{(c+d\,x)} - a\,b^2\,\sin{(c+d\,x)}) + \cos{(c+d\,x)}^2\,\left(\frac{3\,a^2\,b}{2} - \frac{b^3}{2}\right) + \frac{b^3}{4} + a\,b^2\,\cos{(c+d\,x)}\,\sin{(c+d\,x)}}{d\cos{(c+d\,x)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^5,x)
```

```
[Out] (cos(c + d*x)^3*(a^3*sin(c + d*x) - a*b^2*sin(c + d*x)) + cos(c + d*x)^2*((
3*a^2*b)/2 - b^3/2) + b^3/4 + a*b^2*cos(c + d*x)*sin(c + d*x))/(d*cos(c + d
*x)^4)
```

3.68 $\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=158

$$\frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2b\sec^3(c+dx)}{d}-\frac{b^3\sec^3(c+dx)}{3d}+\frac{b^3\sec^5(c+dx)}{5d}+\frac{a^3\,\mathrm{se}}{}$$

```
[Out] 1/2*a^3*arctanh(sin(d*x+c))/d-3/8*a*b^2*arctanh(sin(d*x+c))/d+a^2*b*sec(d*x
+c)^3/d-1/3*b^3*sec(d*x+c)^3/d+1/5*b^3*sec(d*x+c)^5/d+1/2*a^3*sec(d*x+c)*ta
n(d*x+c)/d-3/8*a*b^2*sec(d*x+c)*tan(d*x+c)/d+3/4*a*b^2*sec(d*x+c)^3*tan(d*x
+c)/d
```

Rubi [A]
time = 0.13, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3169, 3853, 3855, 2686, 30, 2691, 14}

$$\frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^3\tan(c+dx)\sec(c+dx)}{2d}+\frac{a^2b\sec^3(c+dx)}{d}-\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3ab^2\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{3ab^2\tan(c+dx)\sec(c+dx)}{8d}+\frac{b^3\sec^5(c+dx)}{5d}-\frac{b^3\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (3*a*b^2*ArcTanh[Sin[c + d*x]])/(8*d) +
 (a^2*b*Sec[c + d*x]^3)/d - (b^3*Sec[c + d*x]^3)/(3*d) + (b^3*Sec[c + d*x]^
5)/(5*d) + (a^3*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (3*a*b^2*Sec[c + d*x]*Ta
n[c + d*x])/(8*d) + (3*a*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \int \left(a^3\sec^3(c+dx)+3a^2b\sec^3(c+dx)\tan(c+dx)+3\right.$$

$$= a^3\int \sec^3(c+dx)\,dx + \left(3a^2b\right)\int \sec^3(c+dx)\tan(c+$$

$$= \frac{a^3\sec(c+dx)\tan(c+dx)}{2d} + \frac{3ab^2\sec^3(c+dx)\tan(c+}{4d}$$

$$= \frac{a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a^2b\sec^3(c+dx)}{d} + \frac{a^3\sec(c+}{}$$

$$= \frac{a^3\tanh^{-1}(\sin(c+dx))}{2d} - \frac{3ab^2\tanh^{-1}(\sin(c+dx))}{8d} +$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 464 vs. 2(158) = 316.
time = 1.40, size = 464, normalized size = 2.94

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^5*(960*a^2*b + 64*b^3 + 320*(3*a^2*b - b^3)*Cos[2*(c + d*x)]
- 300*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 225*a
*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 60*a^3*Cos
[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 45*a*b^2*Cos[5*(c
+ d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 150*a*(4*a^2 - 3*b^2)*Co
s[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]]) + 300*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin
[(c + d*x)/2]] - 225*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]] + 60*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
] - 45*a*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 24
0*a^3*Sin[2*(c + d*x)] + 540*a*b^2*Sin[2*(c + d*x)] + 120*a^3*Sin[4*(c + d*
x)] - 90*a*b^2*Sin[4*(c + d*x)]))/(1920*d)
```

Maple [A]

time = 0.42, size = 198, normalized size = 1.25

method	result
derivativedivides	$\frac{a^{3}\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{a^{2}b}{\cos(dx+c)^{3}}+3a\,b^{2}\left(\frac{\sin^{3}(dx+c)}{4\cos(dx+c)^{4}}+\frac{\sin^{3}(dx+c)}{8\cos(dx+c)^{2}}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+}{}\right.}{d}$
default	$\frac{a^{3}\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{a^{2}b}{\cos(dx+c)^{3}}+3a\,b^{2}\left(\frac{\sin^{3}(dx+c)}{4\cos(dx+c)^{4}}+\frac{\sin^{3}(dx+c)}{8\cos(dx+c)^{2}}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx+}{}\right.}{d}$
risch	$-\frac{60ia^{3}e^{9i(dx+c)}-45ia\,b^{2}e^{9i(dx+c)}+120ia^{3}e^{7i(dx+c)}+270ia\,b^{2}e^{7i(dx+c)}-480a^{2}b\,e^{7i(dx+c)}+160b^{3}e^{7i(dx+c)}-960a^{2}b\,e^{5i(}}{60d(e}$
norman	$-\frac{30a^{2}b-4b^{3}}{15d}-\frac{6a^{2}b\left(\tan^{14}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{2\left(3a^{2}b+2b^{3}\right)\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2\left(15a^{2}b-20b^{3}\right)\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{2\left(15a^{2}b+4b^{3}\right)\left(\tan^{2}\left(\frac{d}{}\right.\right.}{15d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+a^2*b/co
s(d*x+c)^3+3*a*b^2*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/cos(d*x+
c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+b^3*(1/5*sin(d*x+c)^4/co
s(d*x+c)^5+1/15*sin(d*x+c)^4/cos(d*x+c)^3-1/15*sin(d*x+c)^4/cos(d*x+c)-1/15
*(2+sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]

time = 0.27, size = 157, normalized size = 0.99

$$\frac{45\,ab^{2}\left(\frac{2\left(\sin(dx+c)^{3}+\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-60\,a^{3}\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{240\,a^{2}b}{\cos(dx+c)^{3}}-\frac{16\left(5\,\cos(dx+c)^{2}-3\right)b^{3}}{\cos(dx+c)^{5}}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/240*(45*a*b^2*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(
d*x + c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 60*a^3*(
2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
 c) - 1)) + 240*a^2*b/cos(d*x + c)^3 - 16*(5*cos(d*x + c)^2 - 3)*b^3/cos(d*
x + c)^5)/d
```

Fricas [**A**]
time = 3.10, size = 147, normalized size = 0.93

$$\frac{15\,(4\,a^3-3\,ab^2)\cos(dx+c)^5\log(\sin(dx+c)+1)-15\,(4\,a^3-3\,ab^2)\cos(dx+c)^5\log(-\sin(dx+c)+1)+48\,b^3+80\,(3\,a^2b-b^3)\cos(dx+c)^2+30\,\left(6\,ab^2\cos(dx+c)+(4\,a^3-3\,ab^2)\cos(dx+c)^3\right)\sin(dx+c)}{240\,d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/240*(15*(4*a^3 - 3*a*b^2)*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(4*a^
3 - 3*a*b^2)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 48*b^3 + 80*(3*a^2*b -
 b^3)*cos(d*x + c)^2 + 30*(6*a*b^2*cos(d*x + c) + (4*a^3 - 3*a*b^2)*cos(d*x
 + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 333 vs. 2(144) = 288.
time = 0.52, size = 333, normalized size = 2.11

$$\frac{15\,(4\,a^3-3\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,(4\,a^3-3\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(60\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+45\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-360\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-120\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+270\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+720\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-240\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-480\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-80\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+120\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-270\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+240\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-80\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-60\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-45\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-120\,a^2b+16\,b^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(15*(4*a^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*a^3
- 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(60*a^3*tan(1/2*d*x + 1/2
*c)^9 + 45*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 360*a^2*b*tan(1/2*d*x + 1/2*c)^8
- 120*a^3*tan(1/2*d*x + 1/2*c)^7 + 270*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 720*a
^2*b*tan(1/2*d*x + 1/2*c)^6 - 240*b^3*tan(1/2*d*x + 1/2*c)^6 - 480*a^2*b*ta
n(1/2*d*x + 1/2*c)^4 - 80*b^3*tan(1/2*d*x + 1/2*c)^4 + 120*a^3*tan(1/2*d*x
+ 1/2*c)^3 - 270*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 240*a^2*b*tan(1/2*d*x + 1/2
```

```
*c)^2 - 80*b^3*tan(1/2*d*x + 1/2*c)^2 - 60*a^3*tan(1/2*d*x + 1/2*c) - 45*a*
b^2*tan(1/2*d*x + 1/2*c) - 120*a^2*b + 16*b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)
^5)/d
```

Mupad [B]
time = 4.28, size = 293, normalized size = 1.85

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,\left(a^3+\frac{3\,a\,b^2}{4}\right)-2\,a^2\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{9\,a\,b^2}{2}-2\,a^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(\frac{9\,a\,b^2}{2}-2\,a^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(4\,a^2\,b-\frac{4\,b^3}{3}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(8\,a^2\,b+\frac{4\,b^3}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(12\,a^2\,b-4\,b^3\right)+\frac{4\,b^3}{15}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^3+\frac{3\,a\,b^2}{4}\right)-6\,a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}-\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{3\,a\,b^2}{4}-a^3\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^6,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^9*((3*a*b^2)/4 + a^3) - 2*a^2*b - tan(c/2 + (d*x)/2)^3*
((9*a*b^2)/2 - 2*a^3) + tan(c/2 + (d*x)/2)^7*((9*a*b^2)/2 - 2*a^3) + tan(c/
2 + (d*x)/2)^2*(4*a^2*b - (4*b^3)/3) - tan(c/2 + (d*x)/2)^4*(8*a^2*b + (4*b
^3)/3) + tan(c/2 + (d*x)/2)^6*(12*a^2*b - 4*b^3) + (4*b^3)/15 - tan(c/2 + (
d*x)/2)*((3*a*b^2)/4 + a^3) - 6*a^2*b*tan(c/2 + (d*x)/2)^8)/(d*(5*tan(c/2 +
 (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2
 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1)) - (atanh(tan(c/2 + (d*x)/2))*((
3*a*b^2)/4 - a^3))/d
```

3.69 $\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=120

$$\frac{a^3\tan(c+dx)}{d}+\frac{3a^2b\tan^2(c+dx)}{2d}+\frac{a(a^2+3b^2)\tan^3(c+dx)}{3d}+\frac{b(3a^2+b^2)\tan^4(c+dx)}{4d}+\frac{3ab^2\tan^5(c+dx)}{5d}$$

```
[Out] a^3*tan(d*x+c)/d+3/2*a^2*b*tan(d*x+c)^2/d+1/3*a*(a^2+3*b^2)*tan(d*x+c)^3/d+
1/4*b*(3*a^2+b^2)*tan(d*x+c)^4/d+3/5*a*b^2*tan(d*x+c)^5/d+1/6*b^3*tan(d*x+c
)^6/d
```

Rubi [A]

time = 0.07, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3167, 908}

$$\frac{a^3\tan(c+dx)}{d}+\frac{b(3a^2+b^2)\tan^4(c+dx)}{4d}+\frac{a(a^2+3b^2)\tan^3(c+dx)}{3d}+\frac{3a^2b\tan^2(c+dx)}{2d}+\frac{3ab^2\tan^5(c+dx)}{5d}+\frac{b^3\tan^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Tan[c + d*x])/d + (3*a^2*b*Tan[c + d*x]^2)/(2*d) + (a*(a^2 + 3*b^2)*Ta
n[c + d*x]^3)/(3*d) + (b*(3*a^2 + b^2)*Tan[c + d*x]^4)/(4*d) + (3*a*b^2*Tan
[c + d*x]^5)/(5*d) + (b^3*Tan[c + d*x]^6)/(6*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^3(1+x^2)}{x^7}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b^3}{x^7}+\frac{3ab^2}{x^6}+\frac{3a^2b+b^3}{x^5}+\frac{a^3+3ab^2}{x^4}+\frac{3a^2b}{x^3}+\frac{a^3}{x^2}\right) a}{d}$$
$$= \frac{a^3\tan(c+dx)}{d}+\frac{3a^2b\tan^2(c+dx)}{2d}+\frac{a(a^2+3b^2)\tan^3(}{3d}$$

Mathematica [A]
time = 0.39, size = 54, normalized size = 0.45

$$\frac{(a+b\tan(c+dx))^4\left(a^2+15b^2-4ab\tan(c+dx)+10b^2\tan^2(c+dx)\right)}{60b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
[Out] ((a + b*Tan[c + d*x])^4*(a^2 + 15*b^2 - 4*a*b*Tan[c + d*x] + 10*b^2*Tan[c +
 d*x]^2))/(60*b^3*d)
```

Maple [A]
time = 0.29, size = 127, normalized size = 1.06

method	result
derivativedivides	$\frac{-a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{3a^2b}{4\cos(dx+c)^4}+3ab^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+b^3\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+c}{12\cos(dx+c}\right.}{d}$
default	$\frac{-a^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{3a^2b}{4\cos(dx+c)^4}+3ab^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+b^3\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx+c}{12\cos(dx+c}\right.}{d}$
risch	$-\frac{4\left(-15ia^3e^{8i(dx+c)}+45iab^2e^{8i(dx+c)}-45a^2be^{8i(dx+c)}+15b^3e^{8i(dx+c)}-50ia^3e^{6i(dx+c)}+30iab^2e^{6i(dx+c)}-90a^2be^{6i(dx\right.}{15d\left(e^{2i(d\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 1/d*(-a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+3/4*a^2*b/cos(d*x+c)^4+3*a*b^2
*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+b^3*(1/6*si
n(d*x+c)^4/cos(d*x+c)^6+1/12*sin(d*x+c)^4/cos(d*x+c)^4))
```

Maxima [A]
time = 0.30, size = 122, normalized size = 1.02

$$\frac{20\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^3+12\left(3\tan(dx+c)^5+5\tan(dx+c)^3\right)ab^2-\frac{5\left(3\sin(dx+c)^2-1\right)b^3}{\sin(dx+c)^6-3\sin(dx+c)^4+3\sin(dx+c)^2-1}+\frac{45a^2b}{\left(\sin(dx+c)^2-1\right)^2}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(20*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^3 + 12*(3*tan(d*x + c)^5 + 5*t
an(d*x + c)^3)*a*b^2 - 5*(3*sin(d*x + c)^2 - 1)*b^3/(sin(d*x + c)^6 - 3*sin
(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) + 45*a^2*b/(sin(d*x + c)^2 - 1)^2)/d
```

Fricas [A]
time = 3.59, size = 105, normalized size = 0.88

$$\frac{10\,b^3 + 15\,(3\,a^2b - b^3)\cos{(dx+c)}^2 + 4\,\left(2\,(5\,a^3 - 3\,ab^2)\cos{(dx+c)}^5 + 9\,ab^2\cos{(dx+c)} + (5\,a^3 - 3\,ab^2)\cos{(dx+c)}^3\right)\sin{(dx+c)}}{60\,d\cos{(dx+c)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(10*b^3 + 15*(3*a^2*b - b^3)*cos(d*x + c)^2 + 4*(2*(5*a^3 - 3*a*b^2)*c
os(d*x + c)^5 + 9*a*b^2*cos(d*x + c) + (5*a^3 - 3*a*b^2)*cos(d*x + c)^3)*si
n(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.50, size = 112, normalized size = 0.93

$$\frac{10\,b^3\tan{(dx+c)}^6 + 36\,ab^2\tan{(dx+c)}^5 + 45\,a^2b\tan{(dx+c)}^4 + 15\,b^3\tan{(dx+c)}^4 + 20\,a^3\tan{(dx+c)}^3 + 60\,ab^2\tan{(dx+c)}^3 + 90\,a^2b\tan{(dx+c)}^2 + 60\,a^3\tan{(dx+c)}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(10*b^3*tan(d*x + c)^6 + 36*a*b^2*tan(d*x + c)^5 + 45*a^2*b*tan(d*x +
c)^4 + 15*b^3*tan(d*x + c)^4 + 20*a^3*tan(d*x + c)^3 + 60*a*b^2*tan(d*x + c
)^3 + 90*a^2*b*tan(d*x + c)^2 + 60*a^3*tan(d*x + c))/d
```

Mupad [B]
time = 0.84, size = 123, normalized size = 1.02

$$\frac{\cos{(c+d\,x)}^3\,\left(\frac{a^3\,\sin(c+d\,x)}{3}-\frac{a\,b^2\,\sin(c+d\,x)}{5}\right)+\cos{(c+d\,x)}^5\,\left(\frac{2\,a^3\,\sin(c+d\,x)}{3}-\frac{2\,a\,b^2\,\sin(c+d\,x)}{5}\right)+\cos{(c+d\,x)}^2\,\left(\frac{3\,a^2\,b}{4}-\frac{b^3}{4}\right)+\frac{b^3}{6}+\frac{3\,a\,b^2\,\cos(c+d\,x)\,\sin(c+d\,x)}{5}}{d\,\cos{(c+d\,x)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^7,x)
```

```
[Out] (cos(c + d*x)^3*((a^3*sin(c + d*x))/3 - (a*b^2*sin(c + d*x))/5) + cos(c + d
*x)^5*((2*a^3*sin(c + d*x))/3 - (2*a*b^2*sin(c + d*x))/5) + cos(c + d*x)^2*
((3*a^2*b)/4 - b^3/4) + b^3/6 + (3*a*b^2*cos(c + d*x)*sin(c + d*x))/5)/(d*c
os(c + d*x)^6)
```

3.70 $\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=210

$$\frac{3a^3\tanh^{-1}(\sin(c+dx))}{8d}-\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{16d}+\frac{3a^2b\sec^5(c+dx)}{5d}-\frac{b^3\sec^5(c+dx)}{5d}+\frac{b^3\sec^7(c+dx)}{7d}+$$

```
[Out] 3/8*a^3*arctanh(sin(d*x+c))/d-3/16*a*b^2*arctanh(sin(d*x+c))/d+3/5*a^2*b*se
c(d*x+c)^5/d-1/5*b^3*sec(d*x+c)^5/d+1/7*b^3*sec(d*x+c)^7/d+3/8*a^3*sec(d*x+
c)*tan(d*x+c)/d-3/16*a*b^2*sec(d*x+c)*tan(d*x+c)/d+1/4*a^3*sec(d*x+c)^3*tan
(d*x+c)/d-1/8*a*b^2*sec(d*x+c)^3*tan(d*x+c)/d+1/2*a*b^2*sec(d*x+c)^5*tan(d*
x+c)/d
```

Rubi [A]
time = 0.16, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3169, 3853, 3855, 2686, 30, 2691, 14}

$$\frac{3a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a^3\tan(c+dx)\sec(c+dx)}{8d}+\frac{3a^2b\sec^5(c+dx)}{5d}-\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{16d}+\frac{ab^2\tan(c+dx)\sec^5(c+dx)}{2d}-\frac{ab^2\tan(c+dx)\sec^3(c+dx)}{8d}-\frac{3ab^2\tan(c+dx)\sec(c+dx)}{16d}+\frac{b^3\sec^7(c+dx)}{7d}-\frac{b^3\sec^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (3*a^3*ArcTanh[Sin[c + d*x]])/(8*d) - (3*a*b^2*ArcTanh[Sin[c + d*x]])/(16*d
) + (3*a^2*b*Sec[c + d*x]^5)/(5*d) - (b^3*Sec[c + d*x]^5)/(5*d) + (b^3*Sec[
c + d*x]^7)/(7*d) + (3*a^3*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (3*a*b^2*Sec[
c + d*x]*Tan[c + d*x])/(16*d) + (a^3*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) - (
a*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(8*d) + (a*b^2*Sec[c + d*x]^5*Tan[c + d*
x])/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
```

```
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx &= \int \left(a^3\sec^5(c+dx)+3a^2b\sec^5(c+dx)\tan(c+dx)+3a\right.\\
&= a^3\int \sec^5(c+dx)\,dx+\left(3a^2b\right)\int \sec^5(c+dx)\tan(c+dx\\
&= \frac{a^3\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{ab^2\sec^5(c+dx)\tan(c+d}{2d}\\
&= \frac{3a^2b\sec^5(c+dx)}{5d}+\frac{3a^3\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^3\sec}{}\\
&= \frac{3a^3\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a^2b\sec^5(c+dx)}{5d}-\frac{b^3\sec^5(c}{5d}\\
&= \frac{3a^3\tanh^{-1}(\sin(c+dx))}{8d}-\frac{3ab^2\tanh^{-1}(\sin(c+dx))}{16d}+\frac{3}{}
\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 637 vs. $2(210) = 420$.
time = 2.13, size = 637, normalized size = 3.03

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^7*(10752*a^2*b + 1536*b^3 + 3584*(3*a^2*b - b^3)*Cos[2*(c + d
*x)] - 4410*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
 2205*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 147
0*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 735*a*b^2
*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 210*a^3*Cos[7*
(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 105*a*b^2*Cos[7*(c +
d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 3675*a*(2*a^2 - b^2)*Cos[c
 + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]]) + 4410*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]] - 2205*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + 1470*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]] - 735*a*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] +
210*a^3*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 105*a*b
^2*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4340*a^3*Sin
[2*(c + d*x)] + 6790*a*b^2*Sin[2*(c + d*x)] + 2800*a^3*Sin[4*(c + d*x)] - 1
400*a*b^2*Sin[4*(c + d*x)] + 420*a^3*Sin[6*(c + d*x)] - 210*a*b^2*Sin[6*(c
+ d*x)]))/(35840*d)
```

Maple [**A**]
time = 0.36, size = 248, normalized size = 1.18

method	result
derivativedivides	$\frac{a^3\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{3a^2b}{5\cos(dx+c)^5}+3a\,b^2\left(\frac{\sin^3(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^3}{8\cos(}\right.}{}$
default	$\frac{a^3\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+\frac{3a^2b}{5\cos(dx+c)^5}+3a\,b^2\left(\frac{\sin^3(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^3}{8\cos(}\right.}{}$
risch	$-\frac{105ia\,b^2e^{i(dx+c)}-210ia^3e^{i(dx+c)}-1400ia^3e^{3i(dx+c)}-2170ia^3e^{5i(dx+c)}+1400ia^3e^{11i(dx+c)}+2170ia^3e^{9i(dx+c)}-5376}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+
tan(d*x+c)))+3/5*a^2*b/cos(d*x+c)^5+3*a*b^2*(1/6*sin(d*x+c)^3/cos(d*x+c)^6+
1/8*sin(d*x+c)^3/cos(d*x+c)^4+1/16*sin(d*x+c)^3/cos(d*x+c)^2+1/16*sin(d*x+c
```

```
)-1/16*ln(sec(d*x+c)+tan(d*x+c)))+b^3*(1/7*sin(d*x+c)^4/cos(d*x+c)^7+3/35*s
in(d*x+c)^4/cos(d*x+c)^5+1/35*sin(d*x+c)^4/cos(d*x+c)^3-1/35*sin(d*x+c)^4/c
os(d*x+c)-1/35*(2+sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.29, size = 208, normalized size = 0.99

$$\frac{35\,ab^2\left(\frac{2\left(3\,\sin(dx+c)^5-8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-70\,a^3\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{672\,a^2b}{\cos(dx+c)^5}-\frac{32\left(7\,\cos(dx+c)^2-5\right)b^3}{\cos(dx+c)^7}}{1120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/1120*(35*a*b^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x + c))/
(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x
+ c) + 1) + 3*log(sin(d*x + c) - 1)) - 70*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(
d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1)
 + 3*log(sin(d*x + c) - 1)) + 672*a^2*b/cos(d*x + c)^5 - 32*(7*cos(d*x + c)
^2 - 5)*b^3/cos(d*x + c)^7)/d
```

Fricas [A]
time = 3.94, size = 170, normalized size = 0.81

$$\frac{105\,(2\,a^3-ab^2)\cos\left(dx+c\right)^7\log\left(\sin\left(dx+c\right)+1\right)-105\,(2\,a^3-ab^2)\cos\left(dx+c\right)^7\log\left(-\sin\left(dx+c\right)+1\right)+160\,b^3+224\,(3\,a^2b-b^3)\cos\left(dx+c\right)^2+70\,\left(3\,(2\,a^3-ab^2)\cos\left(dx+c\right)^5+8\,ab^2\cos\left(dx+c\right)+2\,(2\,a^3-ab^2)\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)}{1120\,d\cos\left(dx+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/1120*(105*(2*a^3 - a*b^2)*cos(d*x + c)^7*log(sin(d*x + c) + 1) - 105*(2*a
^3 - a*b^2)*cos(d*x + c)^7*log(-sin(d*x + c) + 1) + 160*b^3 + 224*(3*a^2*b
- b^3)*cos(d*x + c)^2 + 70*(3*(2*a^3 - a*b^2)*cos(d*x + c)^5 + 8*a*b^2*cos(
d*x + c) + 2*(2*a^3 - a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^
7)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 465 vs. 2(190) = 380.
time = 0.50, size = 465, normalized size = 2.21

$$\frac{105\,(2\,a^3-ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\,(2\,a^3-ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(350\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{13}+105\,ab^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{13}-1680\,a^2b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{12}-840\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{11}+1540\,ab^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{11}+3360\,a^2b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{10}-1120\,b^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^{10}+630\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^9+1085\,ab^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^9-5040\,a^2b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^8-1120\,b^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^8+6720\,a^2b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^6-2240\,b^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^6-630\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^5-1085\,ab^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^5-3696\,a^2b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^4-448\,b^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^4+840\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3-1540\,ab^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3+672\,a^2b\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2-224\,b^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2-350\,a^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)-105\,ab^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)-336\,a^2b+32\,b^3\right)}{\left(\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2-1\right)^7}}{560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/560*(105*(2*a^3 - a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(2*a^3
- a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(350*a^3*tan(1/2*d*x + 1/2*
c)^13 + 105*a*b^2*tan(1/2*d*x + 1/2*c)^13 - 1680*a^2*b*tan(1/2*d*x + 1/2*c)
^12 - 840*a^3*tan(1/2*d*x + 1/2*c)^11 + 1540*a*b^2*tan(1/2*d*x + 1/2*c)^11
+ 3360*a^2*b*tan(1/2*d*x + 1/2*c)^10 - 1120*b^3*tan(1/2*d*x + 1/2*c)^10 + 6
30*a^3*tan(1/2*d*x + 1/2*c)^9 + 1085*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 5040*a^
2*b*tan(1/2*d*x + 1/2*c)^8 - 1120*b^3*tan(1/2*d*x + 1/2*c)^8 + 6720*a^2*b*t
an(1/2*d*x + 1/2*c)^6 - 2240*b^3*tan(1/2*d*x + 1/2*c)^6 - 630*a^3*tan(1/2*d
*x + 1/2*c)^5 - 1085*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3696*a^2*b*tan(1/2*d*x
+ 1/2*c)^4 - 448*b^3*tan(1/2*d*x + 1/2*c)^4 + 840*a^3*tan(1/2*d*x + 1/2*c)^
3 - 1540*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 672*a^2*b*tan(1/2*d*x + 1/2*c)^2 -
224*b^3*tan(1/2*d*x + 1/2*c)^2 - 350*a^3*tan(1/2*d*x + 1/2*c) - 105*a*b^2*t
an(1/2*d*x + 1/2*c) - 336*a^2*b + 32*b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)^7)/d
```

Mupad [B]
time = 4.28, size = 423, normalized size = 2.01

$$\frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a^2-b^2\right)}{8\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{5\,a^3}{4}+\frac{3\,a\,b^2}{8}\right)+\frac{6\,a^2\,b}{5}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{11\,a\,b^2}{2}-3\,a^3\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}\,\left(\frac{11\,a\,b^2}{2}-3\,a^3\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}\,\left(\frac{5\,a^3}{4}+\frac{3\,a\,b^2}{8}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(\frac{9\,a^3}{4}+\frac{31\,a\,b^2}{8}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9\,\left(\frac{9\,a^3}{4}+\frac{31\,a\,b^2}{8}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\,\left(12\,a^2\,b-4\,b^3\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{12\,a^2\,b}{5}-\frac{4\,b^3}{5}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(18\,a^2\,b+4\,b^3\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(24\,a^2\,b-8\,b^3\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{66\,a^2\,b}{5}+\frac{8\,b^3}{5}\right)-\frac{4\,b^3}{35}+6\,a^2\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}-7\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+21\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-35\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+35\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-21\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+7\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^8,x)
```

```
[Out] (3*a*atanh(tan(c/2 + (d*x)/2))*(2*a^2 - b^2))/(8*d) - (tan(c/2 + (d*x)/2)*(
(3*a*b^2)/8 + (5*a^3)/4) + (6*a^2*b)/5 + tan(c/2 + (d*x)/2)^3*((11*a*b^2)/2
 - 3*a^3) - tan(c/2 + (d*x)/2)^11*((11*a*b^2)/2 - 3*a^3) - tan(c/2 + (d*x)/
2)^13*((3*a*b^2)/8 + (5*a^3)/4) + tan(c/2 + (d*x)/2)^5*((31*a*b^2)/8 + (9*a
^3)/4) - tan(c/2 + (d*x)/2)^9*((31*a*b^2)/8 + (9*a^3)/4) - tan(c/2 + (d*x)/
2)^10*(12*a^2*b - 4*b^3) - tan(c/2 + (d*x)/2)^2*((12*a^2*b)/5 - (4*b^3)/5)
+ tan(c/2 + (d*x)/2)^8*(18*a^2*b + 4*b^3) - tan(c/2 + (d*x)/2)^6*(24*a^2*b
- 8*b^3) + tan(c/2 + (d*x)/2)^4*((66*a^2*b)/5 + (8*b^3)/5) - (4*b^3)/35 + 6
*a^2*b*tan(c/2 + (d*x)/2)^12)/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*
x)/2)^4 + 35*tan(c/2 + (d*x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 +
(d*x)/2)^10 - 7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1))
```

3.71 $\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=174

$$\frac{a^3\tan(c+dx)}{d}+\frac{3a^2b\tan^2(c+dx)}{2d}+\frac{a(2a^2+3b^2)\tan^3(c+dx)}{3d}+\frac{b(6a^2+b^2)\tan^4(c+dx)}{4d}+\frac{a(a^2+6b^2)\tan^5(c}{5d}$$

```
[Out] a^3*tan(d*x+c)/d+3/2*a^2*b*tan(d*x+c)^2/d+1/3*a*(2*a^2+3*b^2)*tan(d*x+c)^3/
d+1/4*b*(6*a^2+b^2)*tan(d*x+c)^4/d+1/5*a*(a^2+6*b^2)*tan(d*x+c)^5/d+1/6*b*(
3*a^2+2*b^2)*tan(d*x+c)^6/d+3/7*a*b^2*tan(d*x+c)^7/d+1/8*b^3*tan(d*x+c)^8/d
```

Rubi [A]
time = 0.10, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3167, 962}

$$\frac{a^3\tan(c+dx)}{d}+\frac{b(3a^2+2b^2)\tan^6(c+dx)}{6d}+\frac{a(a^2+6b^2)\tan^5(c+dx)}{5d}+\frac{b(6a^2+b^2)\tan^4(c+dx)}{4d}+\frac{a(2a^2+3b^2)\tan^3(c+dx)}{3d}+\frac{3a^2b\tan^2(c+dx)}{2d}+\frac{3ab^2\tan^7(c+dx)}{7d}+\frac{b^3\tan^8(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Tan[c + d*x])/d + (3*a^2*b*Tan[c + d*x]^2)/(2*d) + (a*(2*a^2 + 3*b^2)*
Tan[c + d*x]^3)/(3*d) + (b*(6*a^2 + b^2)*Tan[c + d*x]^4)/(4*d) + (a*(a^2 +
6*b^2)*Tan[c + d*x]^5)/(5*d) + (b*(3*a^2 + 2*b^2)*Tan[c + d*x]^6)/(6*d) + (
3*a*b^2*Tan[c + d*x]^7)/(7*d) + (b^3*Tan[c + d*x]^8)/(8*d)
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^3(1+x^2)^2}{x^9}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b^3}{x^9}+\frac{3ab^2}{x^8}+\frac{3a^2b+2b^3}{x^7}+\frac{a^3+6ab^2}{x^6}+\frac{6a^2b+b^3}{x^5}+\right.\right.}{d}$$
$$= \frac{a^3\tan(c+dx)}{d}+\frac{3a^2b\tan^2(c+dx)}{2d}+\frac{a(2a^2+3b^2)\tan}{3d}$$

Mathematica [A]

time = 0.63, size = 115, normalized size = 0.66

$$\frac{\frac{1}{4}(a^2+b^2)^2(a+b\tan(c+dx))^4-\frac{4}{5}a(a^2+b^2)(a+b\tan(c+dx))^5+\frac{1}{3}(3a^2+b^2)(a+b\tan(c+dx))^6-\frac{4}{7}a(a+b\tan(c+dx))^7+\frac{1}{8}(a+b\tan(c+dx))^8}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (((a^2 + b^2)^2*(a + b*Tan[c + d*x])^4)/4 - (4*a*(a^2 + b^2)*(a + b*Tan[c +
 d*x])^5)/5 + ((3*a^2 + b^2)*(a + b*Tan[c + d*x])^6)/3 - (4*a*(a + b*Tan[c
+ d*x])^7)/7 + (a + b*Tan[c + d*x])^8/8)/(b^5*d)
```

Maple [A]

time = 0.32, size = 173, normalized size = 0.99

method	result
derivativedivides	$\frac{-a^3\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{a^2b}{2\cos(dx+c)^6}+3a\,b^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c\right.}{105\cos(dx+c}\right.}{d}$
default	$\frac{-a^3\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{a^2b}{2\cos(dx+c)^6}+3a\,b^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c\right.}{105\cos(dx+c}\right.}{d}$
risch	$-\frac{16(210ia\,b^2e^{10i(dx+c)}+24ia\,b^2e^{2i(dx+c)}-210a^2b\,e^{10i(dx+c)}+70b^3e^{10i(dx+c)}+3ia\,b^2-56ia^3e^{2i(dx+c)}-420a^2b\,e^{8i(dx+}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+1/2*a^2*b/c
os(d*x+c)^6+3*a*b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(d*
x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+b^3*(1/8*sin(d*x+c)^4/cos(d*x+c)^8+
1/12*sin(d*x+c)^4/cos(d*x+c)^6+1/24*sin(d*x+c)^4/cos(d*x+c)^4))
```

Maxima [A]

time = 0.28, size = 154, normalized size = 0.89

$$\frac{56\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)a^3+24\left(15\tan(dx+c)^7+42\tan(dx+c)^5+35\tan(dx+c)^3\right)ab^2+\frac{35\left(4\sin(dx+c)^2-1\right)b^3}{\sin(dx+c)^8-4\sin(dx+c)^6+6\sin(dx+c)^4-4\sin(dx+c)^2+1}-\frac{420a^2b}{\left(\sin(dx+c)^2-1\right)^3}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/840*(56*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^3 + 24
*(15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*a*b^2 + 35*(4*
sin(d*x + c)^2 - 1)*b^3/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)
^4 - 4*sin(d*x + c)^2 + 1) - 420*a^2*b/(sin(d*x + c)^2 - 1)^3)/d
```

Fricas [A]
time = 2.92, size = 128, normalized size = 0.74

$$\frac{105\,b^3+140\,(3\,a^2b-b^3)\cos{(dx+c)}^2+8\,\big(8\,(7\,a^3-3\,ab^2)\cos{(dx+c)}^7+4\,(7\,a^3-3\,ab^2)\cos{(dx+c)}^5+45\,ab^2\cos{(dx+c)}+3\,(7\,a^3-3\,ab^2)\cos{(dx+c)}^3\big)\sin{(dx+c)}}{840\,d\cos{(dx+c)}^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/840*(105*b^3 + 140*(3*a^2*b - b^3)*cos(d*x + c)^2 + 8*(8*(7*a^3 - 3*a*b^2
)*cos(d*x + c)^7 + 4*(7*a^3 - 3*a*b^2)*cos(d*x + c)^5 + 45*a*b^2*cos(d*x +
c) + 3*(7*a^3 - 3*a*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^8)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [A]
time = 0.53, size = 166, normalized size = 0.95

$$\frac{105\,b^3\tan{(dx+c)}^8+360\,ab^2\tan{(dx+c)}^7+420\,a^2b\tan{(dx+c)}^6+280\,b^3\tan{(dx+c)}^6+168\,a^3\tan{(dx+c)}^5+1008\,ab^2\tan{(dx+c)}^5+1260\,a^2b\tan{(dx+c)}^4+210\,b^3\tan{(dx+c)}^4+560\,a^3\tan{(dx+c)}^3+840\,ab^2\tan{(dx+c)}^3+1260\,a^2b\tan{(dx+c)}^2+840\,a^3\tan{(dx+c)}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/840*(105*b^3*tan(d*x + c)^8 + 360*a*b^2*tan(d*x + c)^7 + 420*a^2*b*tan(d*
x + c)^6 + 280*b^3*tan(d*x + c)^6 + 168*a^3*tan(d*x + c)^5 + 1008*a*b^2*tan
(d*x + c)^5 + 1260*a^2*b*tan(d*x + c)^4 + 210*b^3*tan(d*x + c)^4 + 560*a^3*
tan(d*x + c)^3 + 840*a*b^2*tan(d*x + c)^3 + 1260*a^2*b*tan(d*x + c)^2 + 840
*a^3*tan(d*x + c))/d
```

Mupad [B]
time = 1.15, size = 156, normalized size = 0.90

$$\frac{\cos{(c+d\,x)}^3\left(\frac{a^3\sin(c+d\,x)}{5}-\frac{3\,a\,b^2\sin(c+d\,x)}{35}\right)+\cos{(c+d\,x)}^5\left(\frac{4\,a^3\sin(c+d\,x)}{15}-\frac{4\,a\,b^2\sin(c+d\,x)}{35}\right)+\cos{(c+d\,x)}^7\left(\frac{8\,a^3\sin(c+d\,x)}{15}-\frac{8\,a\,b^2\sin(c+d\,x)}{35}\right)+\cos{(c+d\,x)}^2\left(\frac{a^2\,b}{2}-\frac{b^3}{6}\right)+\frac{b^3}{8}+\frac{3\,a\,b^2\cos(c+d\,x)\sin(c+d\,x)}{7}}{d\cos{(c+d\,x)}^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^9,x)
```

```
[Out] (cos(c + d*x)^3*((a^3*sin(c + d*x))/5 - (3*a*b^2*sin(c + d*x))/35) + cos(c
+ d*x)^5*((4*a^3*sin(c + d*x))/15 - (4*a*b^2*sin(c + d*x))/35) + cos(c + d*
x)^7*((8*a^3*sin(c + d*x))/15 - (8*a*b^2*sin(c + d*x))/35) + cos(c + d*x)^2
*((a^2*b)/2 - b^3/6) + b^3/8 + (3*a*b^2*cos(c + d*x)*sin(c + d*x))/7)/(d*co
s(c + d*x)^8)
```

3.72 $\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=259

$$\frac{5a^3\tanh^{-1}(\sin(c+dx))}{16d}-\frac{15ab^2\tanh^{-1}(\sin(c+dx))}{128d}+\frac{3a^2b\sec^7(c+dx)}{7d}-\frac{b^3\sec^7(c+dx)}{7d}+\frac{b^3\sec^9(c+dx)}{9d}+\frac{5}{}$$

```
[Out] 5/16*a^3*arctanh(sin(d*x+c))/d-15/128*a*b^2*arctanh(sin(d*x+c))/d+3/7*a^2*b
*sec(d*x+c)^7/d-1/7*b^3*sec(d*x+c)^7/d+1/9*b^3*sec(d*x+c)^9/d+5/16*a^3*sec(
d*x+c)*tan(d*x+c)/d-15/128*a*b^2*sec(d*x+c)*tan(d*x+c)/d+5/24*a^3*sec(d*x+c
)^3*tan(d*x+c)/d-5/64*a*b^2*sec(d*x+c)^3*tan(d*x+c)/d+1/6*a^3*sec(d*x+c)^5*
tan(d*x+c)/d-1/16*a*b^2*sec(d*x+c)^5*tan(d*x+c)/d+3/8*a*b^2*sec(d*x+c)^7*ta
n(d*x+c)/d
```

Rubi [A]

time = 0.19, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3169, 3853, 3855, 2686, 30, 2691, 14}

$$\frac{5a^3\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^3\tan(c+dx)\sec^5(c+dx)}{6d}+\frac{5a^3\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{5a^3\tan(c+dx)\sec(c+dx)}{16d}+\frac{3a^2b\sec^7(c+dx)}{7d}-\frac{15ab^2\tanh^{-1}(\sin(c+dx))}{128d}+\frac{3ab^2\tan(c+dx)\sec^7(c+dx)}{8d}-\frac{ab^2\tan(c+dx)\sec^5(c+dx)}{16d}-\frac{5ab^2\tan(c+dx)\sec^3(c+dx)}{64d}-\frac{15ab^2\tan(c+dx)\sec(c+dx)}{128d}+\frac{b^3\sec^9(c+dx)}{9d}-\frac{b^3\sec^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (5*a^3*ArcTanh[Sin[c + d*x]])/(16*d) - (15*a*b^2*ArcTanh[Sin[c + d*x]])/(12
8*d) + (3*a^2*b*Sec[c + d*x]^7)/(7*d) - (b^3*Sec[c + d*x]^7)/(7*d) + (b^3*S
ec[c + d*x]^9)/(9*d) + (5*a^3*Sec[c + d*x]*Tan[c + d*x])/(16*d) - (15*a*b^2
*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (5*a^3*Sec[c + d*x]^3*Tan[c + d*x])/(
24*d) - (5*a*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(64*d) + (a^3*Sec[c + d*x]^5*
Tan[c + d*x])/(6*d) - (a*b^2*Sec[c + d*x]^5*Tan[c + d*x])/(16*d) + (3*a*b^2
*Sec[c + d*x]^7*Tan[c + d*x])/(8*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = \int \left(a^3\sec^7(c+dx)+3a^2b\sec^7(c+dx)\tan(c+dx)+3a\right.$$
$$= a^3\int \sec^7(c+dx)\,dx + \left(3a^2b\right)\int \sec^7(c+dx)\tan(c+d$$
$$= \frac{a^3\sec^5(c+dx)\tan(c+dx)}{6d} + \frac{3ab^2\sec^7(c+dx)\tan(c+}{8d}$$
$$= \frac{3a^2b\sec^7(c+dx)}{7d} + \frac{5a^3\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{a^3 s}{}$$
$$= \frac{3a^2b\sec^7(c+dx)}{7d} - \frac{b^3\sec^7(c+dx)}{7d} + \frac{b^3\sec^9(c+dx)}{9d} +$$
$$= \frac{5a^3\tanh^{-1}(\sin(c+dx))}{16d} + \frac{3a^2b\sec^7(c+dx)}{7d} - \frac{b^3\sec^7(}{7}$$
$$= \frac{5a^3\tanh^{-1}(\sin(c+dx))}{16d} - \frac{15ab^2\tanh^{-1}(\sin(c+dx))}{128d} +$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 810 vs. $2(259) = 518$.
time = 4.19, size = 810, normalized size = 3.13

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^9*(442368*a^2*b + 81920*b^3 + 147456*(3*a^2*b - b^3)*Cos[2*(c
 + d*x)] - 211680*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] + 79380*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
] - 90720*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3
4020*a*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2268
0*a^3*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 8505*a*b^
2*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 2520*a^3*Cos[
9*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 945*a*b^2*Cos[9*(c
+ d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 39690*a*(8*a^2 - 3*b^2)*
Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/
2] + Sin[(c + d*x)/2]]) + 211680*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]
+ Sin[(c + d*x)/2]] - 79380*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + S
in[(c + d*x)/2]] + 90720*a^3*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]] - 34020*a*b^2*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + 22680*a^3*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]] - 8505*a*b^2*Cos[7*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]
+ 2520*a^3*Cos[9*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 945*
```

```
a*b^2*Cos[9*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 223776*a^
3*Sin[2*(c + d*x)] + 303156*a*b^2*Sin[2*(c + d*x)] + 167328*a^3*Sin[4*(c +
d*x)] - 62748*a*b^2*Sin[4*(c + d*x)] + 43680*a^3*Sin[6*(c + d*x)] - 16380*a
*b^2*Sin[6*(c + d*x)] + 5040*a^3*Sin[8*(c + d*x)] - 1890*a*b^2*Sin[8*(c + d
*x)]))/(2064384*d)
```

Maple [A]
time = 0.45, size = 294, normalized size = 1.14

method	result
derivativedivides	$a^3\left(-\left(\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{3a^2b}{7\cos(dx+c)^7}+3ab^2\left(\frac{\sin\cdots}{8\cos\cdots}\right.$
default	$a^3\left(-\left(\left(-\frac{\left(\sec^5(dx+c)\right)}{6}-\frac{5\left(\sec^3(dx+c)\right)}{24}-\frac{5\sec(dx+c)}{16}\right)\tan(dx+c)+\frac{5\ln(\sec(dx+c)+\tan(dx+c))}{16}\right)+\frac{3a^2b}{7\cos(dx+c)^7}+3ab^2\left(\frac{\sin\cdots}{8\cos\cdots}\right.$
risch	$-\frac{-83664ia^3e^{5i(dx+c)}+8190iab^2e^{3i(dx+c)}+2520ia^3e^{17i(dx+c)}+83664ia^3e^{13i(dx+c)}+21840ia^3e^{15i(dx+c)}+151578iab^2\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x+c))*tan(d*x+c)
+5/16*ln(sec(d*x+c)+tan(d*x+c)))+3/7*a^2*b/cos(d*x+c)^7+3*a*b^2*(1/8*sin(d*
x+c)^3/cos(d*x+c)^8+5/48*sin(d*x+c)^3/cos(d*x+c)^6+5/64*sin(d*x+c)^3/cos(d*
x+c)^4+5/128*sin(d*x+c)^3/cos(d*x+c)^2+5/128*sin(d*x+c)-5/128*ln(sec(d*x+c)
+tan(d*x+c)))+b^3*(1/9*sin(d*x+c)^4/cos(d*x+c)^9+5/63*sin(d*x+c)^4/cos(d*x+
c)^7+1/21*sin(d*x+c)^4/cos(d*x+c)^5+1/63*sin(d*x+c)^4/cos(d*x+c)^3-1/63*sin
(d*x+c)^4/cos(d*x+c)-1/63*(2+sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.29, size = 248, normalized size = 0.96

$$\frac{63\,ab^2\left(\frac{2\left(15\,\sin(dx+c)^7-55\,\sin(dx+c)^5+73\,\sin(dx+c)^3+15\,\sin(dx+c)\right)}{\sin(dx+c)^8-4\,\sin(dx+c)^6+6\,\sin(dx+c)^4-4\,\sin(dx+c)^2+1}-15\,\log\left(\sin\left(dx+c\right)+1\right)+15\,\log\left(\sin\left(dx+c\right)-1\right)\right)-168\,a^3\left(\frac{2\left(15\,\sin(dx+c)^5-40\,\sin(dx+c)^3+33\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-15\,\log\left(\sin\left(dx+c\right)+1\right)+15\,\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{6912\,a^2b}{\cos(dx+c)^7}-\frac{256\left(9\,\cos(dx+c)^2-7\right)b^3}{\cos(dx+c)^9}}{16128\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima"
)
```

```
[Out] 1/16128*(63*a*b^2*(2*(15*sin(d*x + c)^7 - 55*sin(d*x + c)^5 + 73*sin(d*x +
c)^3 + 15*sin(d*x + c))/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)
^4 - 4*sin(d*x + c)^2 + 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c)
 - 1)) - 168*a^3*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c
))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(
d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 6912*a^2*b/cos(d*x + c)^7 - 256
*(9*cos(d*x + c)^2 - 7)*b^3/cos(d*x + c)^9)/d
```

Fricas [A]
time = 4.22, size = 192, normalized size = 0.74

$$\frac{315 \, {\left(8 \, a^{3} - 3 \, a b^{2}\right)} \cos \left(d x + c\right)^{9} \log \left(\sin \left(d x + c\right) + 1\right) - 315 \, {\left(8 \, a^{3} - 3 \, a b^{2}\right)} \cos \left(d x + c\right)^{9} \log \left(-\sin \left(d x + c\right) + 1\right) + 1792 \, b^{3} + 2304 \, {\left(3 \, a^{2} b - b^{3}\right)} \cos \left(d x + c\right)^{2} + 42 \, {\left(15 \, {\left(8 \, a^{3} - 3 \, a b^{2}\right)} \cos \left(d x + c\right)^{7} + 10 \, {\left(8 \, a^{3} - 3 \, a b^{2}\right)} \cos \left(d x + c\right)^{5} + 144 \, a b^{2} \cos \left(d x + c\right) + 8 \, {\left(8 \, a^{3} - 3 \, a b^{2}\right)} \cos \left(d x + c\right)^{3}\right)} \sin \left(d x + c\right)}{16128 \, d \cos \left(d x + c\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas"
)
```

```
[Out] 1/16128*(315*(8*a^3 - 3*a*b^2)*cos(d*x + c)^9*log(sin(d*x + c) + 1) - 315*(
8*a^3 - 3*a*b^2)*cos(d*x + c)^9*log(-sin(d*x + c) + 1) + 1792*b^3 + 2304*(3
*a^2*b - b^3)*cos(d*x + c)^2 + 42*(15*(8*a^3 - 3*a*b^2)*cos(d*x + c)^7 + 10
*(8*a^3 - 3*a*b^2)*cos(d*x + c)^5 + 144*a*b^2*cos(d*x + c) + 8*(8*a^3 - 3*a
*b^2)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^9)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 597 vs. 2(235) = 470.
time = 0.53, size = 597, normalized size = 2.31

$$\frac{315 \, {\left(8 \, a^{3} - 3 \, a b^{2}\right)} \log \left({\left| \tan \left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + 1 \right|}\right) - 315 \, {\left(8 \, a^{3} - 3 \, a b^{2}\right)} \log \left({\left| \tan \left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) - 1 \right|}\right) + \frac{2 \, {\left(5544 \, a^{3} \tan \left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{17} + 945 \, a b^{2} \tan \left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{17} - 24192 \, a^{2} b \tan \left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{16} - \ldots [illegible]}{[illegible]}}{[illegible]}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8064*(315*(8*a^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 315*(8*a
^3 - 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(5544*a^3*tan(1/2*d*x
+ 1/2*c)^17 + 945*a*b^2*tan(1/2*d*x + 1/2*c)^17 - 24192*a^2*b*tan(1/2*d*x +
 1/2*c)^16 - 15792*a^3*tan(1/2*d*x + 1/2*c)^15 + 24066*a*b^2*tan(1/2*d*x +
1/2*c)^15 + 48384*a^2*b*tan(1/2*d*x + 1/2*c)^14 - 16128*b^3*tan(1/2*d*x + 1
/2*c)^14 + 29232*a^3*tan(1/2*d*x + 1/2*c)^13 + 31374*a*b^2*tan(1/2*d*x + 1/
2*c)^13 - 145152*a^2*b*tan(1/2*d*x + 1/2*c)^12 - 26880*b^3*tan(1/2*d*x + 1/
2*c)^12 - 33264*a^3*tan(1/2*d*x + 1/2*c)^11 + 54810*a*b^2*tan(1/2*d*x + 1/2
*c)^11 + 241920*a^2*b*tan(1/2*d*x + 1/2*c)^10 - 80640*b^3*tan(1/2*d*x + 1/2
*c)^10 - 193536*a^2*b*tan(1/2*d*x + 1/2*c)^8 - 48384*b^3*tan(1/2*d*x + 1/2*
c)^8 + 33264*a^3*tan(1/2*d*x + 1/2*c)^7 - 54810*a*b^2*tan(1/2*d*x + 1/2*c)^
```

```
7 + 145152*a^2*b*tan(1/2*d*x + 1/2*c)^6 - 48384*b^3*tan(1/2*d*x + 1/2*c)^6
- 29232*a^3*tan(1/2*d*x + 1/2*c)^5 - 31374*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 7
6032*a^2*b*tan(1/2*d*x + 1/2*c)^4 - 6912*b^3*tan(1/2*d*x + 1/2*c)^4 + 15792
*a^3*tan(1/2*d*x + 1/2*c)^3 - 24066*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6912*a^2
*b*tan(1/2*d*x + 1/2*c)^2 - 2304*b^3*tan(1/2*d*x + 1/2*c)^2 - 5544*a^3*tan(
1/2*d*x + 1/2*c) - 945*a*b^2*tan(1/2*d*x + 1/2*c) - 3456*a^2*b + 256*b^3)/(
tan(1/2*d*x + 1/2*c)^2 - 1)^9)/d
```

Mupad [B]

time = 4.56, size = 547, normalized size = 2.11

$$-\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{15\,a\,b^2}{64}-\frac{5\,a^3}{8}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{11\,a^3}{8}+\frac{15\,a\,b^2}{64}\right)+\frac{6\,a^2\,b}{7}-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{17}\,\left(\frac{11\,a^3}{8}+\frac{15\,a\,b^2}{64}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{191\,a\,b^2}{32}-\frac{47\,a^3}{12}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{15}\,\left(\frac{191\,a\,b^2}{32}-\frac{47\,a^3}{12}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(\frac{29\,a^3}{4}+\frac{249\,a\,b^2}{32}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}\,\left(\frac{29\,a^3}{4}+\frac{249\,a\,b^2}{32}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(\frac{435\,a\,b^2}{32}-\frac{33\,a^3}{4}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}\,\left(\frac{435\,a\,b^2}{32}-\frac{33\,a^3}{4}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}\,\left(12\,a^2\,b-4\,b^3\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{12\,a^2\,b}{7}-\frac{4\,b^3}{7}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(36\,a^2\,b-12\,b^3\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(48\,a^2\,b+12\,b^3\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}\,\left(36\,a^2\,b+\frac{20\,b^3}{3}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\,\left(60\,a^2\,b-20\,b^3\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(\frac{132\,a^2\,b}{7}+\frac{12\,b^3}{7}\right)-\frac{4\,b^3}{63}+6\,a^2\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{16}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{18}-9\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{16}+36\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}-84\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+126\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-126\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+84\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-36\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+9\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^10,x)
```

```
[Out] - (atanh(tan(c/2 + (d*x)/2))*((15*a*b^2)/64 - (5*a^3)/8))/d - (tan(c/2 + (d
*x)/2)*((15*a*b^2)/64 + (11*a^3)/8) + (6*a^2*b)/7 - tan(c/2 + (d*x)/2)^17*(
(15*a*b^2)/64 + (11*a^3)/8) + tan(c/2 + (d*x)/2)^3*((191*a*b^2)/32 - (47*a^
3)/12) - tan(c/2 + (d*x)/2)^15*((191*a*b^2)/32 - (47*a^3)/12) + tan(c/2 + (
d*x)/2)^5*((249*a*b^2)/32 + (29*a^3)/4) - tan(c/2 + (d*x)/2)^13*((249*a*b^2
)/32 + (29*a^3)/4) + tan(c/2 + (d*x)/2)^7*((435*a*b^2)/32 - (33*a^3)/4) - t
an(c/2 + (d*x)/2)^11*((435*a*b^2)/32 - (33*a^3)/4) - tan(c/2 + (d*x)/2)^14*
(12*a^2*b - 4*b^3) - tan(c/2 + (d*x)/2)^2*((12*a^2*b)/7 - (4*b^3)/7) - tan(
c/2 + (d*x)/2)^6*(36*a^2*b - 12*b^3) + tan(c/2 + (d*x)/2)^8*(48*a^2*b + 12*
b^3) + tan(c/2 + (d*x)/2)^12*(36*a^2*b + (20*b^3)/3) - tan(c/2 + (d*x)/2)^1
0*(60*a^2*b - 20*b^3) + tan(c/2 + (d*x)/2)^4*((132*a^2*b)/7 + (12*b^3)/7) -
 (4*b^3)/63 + 6*a^2*b*tan(c/2 + (d*x)/2)^16)/(d*(9*tan(c/2 + (d*x)/2)^2 - 3
6*tan(c/2 + (d*x)/2)^4 + 84*tan(c/2 + (d*x)/2)^6 - 126*tan(c/2 + (d*x)/2)^8
 + 126*tan(c/2 + (d*x)/2)^10 - 84*tan(c/2 + (d*x)/2)^12 + 36*tan(c/2 + (d*x
)/2)^14 - 9*tan(c/2 + (d*x)/2)^16 + tan(c/2 + (d*x)/2)^18 - 1))
```

3.73 $\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx$

Optimal. Leaf size=213

$$\frac{a^3\tan(c+dx)}{d}+\frac{3a^2b\tan^2(c+dx)}{2d}+\frac{a(a^2+b^2)\tan^3(c+dx)}{d}+\frac{b(9a^2+b^2)\tan^4(c+dx)}{4d}+\frac{3a(a^2+3b^2)\tan^5(c+\cdots}{5d}$$

```
[Out] a^3*tan(d*x+c)/d+3/2*a^2*b*tan(d*x+c)^2/d+a*(a^2+b^2)*tan(d*x+c)^3/d+1/4*b*
(9*a^2+b^2)*tan(d*x+c)^4/d+3/5*a*(a^2+3*b^2)*tan(d*x+c)^5/d+1/2*b*(3*a^2+b^
2)*tan(d*x+c)^6/d+1/7*a*(a^2+9*b^2)*tan(d*x+c)^7/d+3/8*b*(a^2+b^2)*tan(d*x+
c)^8/d+1/3*a*b^2*tan(d*x+c)^9/d+1/10*b^3*tan(d*x+c)^10/d
```

Rubi [A]

time = 0.12, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3167, 962}

$$\frac{a^3\tan(c+dx)}{d}+\frac{3b(a^2+b^2)\tan^8(c+dx)}{8d}+\frac{a(a^2+9b^2)\tan^7(c+dx)}{7d}+\frac{b(3a^2+b^2)\tan^6(c+dx)}{2d}+\frac{3a(a^2+3b^2)\tan^5(c+dx)}{5d}+\frac{b(9a^2+b^2)\tan^4(c+dx)}{4d}+\frac{a(a^2+b^2)\tan^3(c+dx)}{d}+\frac{3a^2b\tan^2(c+dx)}{2d}+\frac{ab^2\tan^9(c+dx)}{3d}+\frac{b^3\tan^{10}(c+dx)}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^11*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (a^3*Tan[c + d*x])/d + (3*a^2*b*Tan[c + d*x]^2)/(2*d) + (a*(a^2 + b^2)*Tan[
c + d*x]^3)/d + (b*(9*a^2 + b^2)*Tan[c + d*x]^4)/(4*d) + (3*a*(a^2 + 3*b^2)
*Tan[c + d*x]^5)/(5*d) + (b*(3*a^2 + b^2)*Tan[c + d*x]^6)/(2*d) + (a*(a^2 +
 9*b^2)*Tan[c + d*x]^7)/(7*d) + (3*b*(a^2 + b^2)*Tan[c + d*x]^8)/(8*d) + (a
*b^2*Tan[c + d*x]^9)/(3*d) + (b^3*Tan[c + d*x]^10)/(10*d)
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^3\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^3(1+x^2)^3}{x^{11}}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{b^3}{x^{11}} + \frac{3ab^2}{x^{10}} + \frac{3b(a^2+b^2)}{x^9} + \frac{a^3+9ab^2}{x^8} + \frac{3(3a^2b+b^3}{x^7}\right.\right.}{}$$

$$= \frac{a^3\tan(c+dx)}{d} + \frac{3a^2b\tan^2(c+dx)}{2d} + \frac{a(a^2+b^2)\tan^3}{d}$$

Mathematica [A]
time = 2.13, size = 177, normalized size = 0.83

$$\frac{\frac{1}{4}(a^2+b^2)^3(a+b\tan(c+dx))^4 - \frac{6}{5}a(a^2+b^2)^2(a+b\tan(c+dx))^5 + \frac{1}{2}(a^2+b^2)(5a^2+b^2)(a+b\tan(c+dx))^6 - \frac{4}{7}a(5a^2+3b^2)(a+b\tan(c+dx))^7 + \frac{3}{8}(5a^2+b^2)(a+b\tan(c+dx))^8 - \frac{2}{3}a(a+b\tan(c+dx))^9 + \frac{1}{10}(a+b\tan(c+dx))^{10}}{b^7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^11*(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (((a^2 + b^2)^3*(a + b*Tan[c + d*x])^4)/4 - (6*a*(a^2 + b^2)^2*(a + b*Tan[c
 + d*x])^5)/5 + ((a^2 + b^2)*(5*a^2 + b^2)*(a + b*Tan[c + d*x])^6)/2 - (4*a
*(5*a^2 + 3*b^2)*(a + b*Tan[c + d*x])^7)/7 + (3*(5*a^2 + b^2)*(a + b*Tan[c
+ d*x])^8)/8 - (2*a*(a + b*Tan[c + d*x])^9)/3 + (a + b*Tan[c + d*x])^10/10)
/(b^7*d)
```

Maple [A]
time = 0.35, size = 219, normalized size = 1.03

method	result
derivativedivides	$\frac{-a^3\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)+\frac{3a^2b}{8\cos(dx+c)^8}+3ab^2\left(\frac{\sin^3(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^3(dx+c\right.}{21\cos(dx+c)}\right.}{d}$
default	$\frac{-a^3\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)+\frac{3a^2b}{8\cos(dx+c)^8}+3ab^2\left(\frac{\sin^3(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^3(dx+c\right.}{21\cos(dx+c)}\right.}{d}$
risch	$-\frac{32(126ia\,b^2e^{10i(dx+c)}+45ia\,b^2e^{4i(dx+c)}-315a^2b\,e^{12i(dx+c)}+105b^3e^{12i(dx+c)}-525ia^3e^{8i(dx+c)}-360ia^3e^{6i(dx+c)}-63}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^3*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan
(d*x+c)+3/8*a^2*b/cos(d*x+c)^8+3*a*b^2*(1/9*sin(d*x+c)^3/cos(d*x+c)^9+2/21*
sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(d*x+c)
^3/cos(d*x+c)^3)+b^3*(1/10*sin(d*x+c)^4/cos(d*x+c)^10+3/40*sin(d*x+c)^4/cos
(d*x+c)^8+1/20*sin(d*x+c)^4/cos(d*x+c)^6+1/40*sin(d*x+c)^4/cos(d*x+c)^4))
```

Maxima [A]
time = 0.29, size = 184, normalized size = 0.86

$$\frac{24\left(5\tan\left(dx+c\right)^{7}+21\tan\left(dx+c\right)^{5}+35\tan\left(dx+c\right)^{3}+35\tan\left(dx+c\right)\right)a^{3}+8\left(35\tan\left(dx+c\right)^{9}+135\tan\left(dx+c\right)^{7}+189\tan\left(dx+c\right)^{5}+105\tan\left(dx+c\right)^{3}\right)ab^{2}-\frac{21\left(5\sin(dx+c)^{2}-1\right)b^{3}}{\sin(dx+c)^{10}-5\sin(dx+c)^{8}+10\sin(dx+c)^{6}-10\sin(dx+c)^{4}+5\sin(dx+c)^{2}-1}+\frac{315a^{2}b}{\left(\sin(dx+c)^{2}-1\right)^{4}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima"
)
```

```
[Out] 1/840*(24*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*ta
n(d*x + c))*a^3 + 8*(35*tan(d*x + c)^9 + 135*tan(d*x + c)^7 + 189*tan(d*x +
 c)^5 + 105*tan(d*x + c)^3)*a*b^2 - 21*(5*sin(d*x + c)^2 - 1)*b^3/(sin(d*x
+ c)^10 - 5*sin(d*x + c)^8 + 10*sin(d*x + c)^6 - 10*sin(d*x + c)^4 + 5*sin(
d*x + c)^2 - 1) + 315*a^2*b/(sin(d*x + c)^2 - 1)^4)/d
```

Fricas [A]
time = 3.15, size = 150, normalized size = 0.70

$$\frac{84\,b^{3}+105\left(3\,a^{2}b-b^{3}\right)\cos\left(dx+c\right)^{2}+8\left(16\left(3\,a^{3}-ab^{2}\right)\cos\left(dx+c\right)^{9}+8\left(3\,a^{3}-ab^{2}\right)\cos\left(dx+c\right)^{7}+6\left(3\,a^{3}-ab^{2}\right)\cos\left(dx+c\right)^{5}+35\,ab^{2}\cos\left(dx+c\right)+5\left(3\,a^{3}-ab^{2}\right)\cos\left(dx+c\right)^{3}\right)\sin\left(dx+c\right)}{840\,d\cos\left(dx+c\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas"
)
```

```
[Out] 1/840*(84*b^3 + 105*(3*a^2*b - b^3)*cos(d*x + c)^2 + 8*(16*(3*a^3 - a*b^2)*
cos(d*x + c)^9 + 8*(3*a^3 - a*b^2)*cos(d*x + c)^7 + 6*(3*a^3 - a*b^2)*cos(d
*x + c)^5 + 35*a*b^2*cos(d*x + c) + 5*(3*a^3 - a*b^2)*cos(d*x + c)^3)*sin(d
*x + c))/(d*cos(d*x + c)^10)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**11*(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.51, size = 220, normalized size = 1.03

$$\frac{84\,b^{3}\tan\left(dx+c\right)^{10}+280\,ab^{2}\tan\left(dx+c\right)^{9}+315\,a^{2}b\tan\left(dx+c\right)^{8}+315\,b^{3}\tan\left(dx+c\right)^{8}+120\,a^{3}\tan\left(dx+c\right)^{7}+1080\,ab^{2}\tan\left(dx+c\right)^{7}+1260\,a^{2}b\tan\left(dx+c\right)^{6}+420\,b^{3}\tan\left(dx+c\right)^{6}+504\,a^{3}\tan\left(dx+c\right)^{5}+1512\,ab^{2}\tan\left(dx+c\right)^{5}+1890\,a^{2}b\tan\left(dx+c\right)^{4}+210\,b^{3}\tan\left(dx+c\right)^{4}+840\,a^{3}\tan\left(dx+c\right)^{3}+840\,ab^{2}\tan\left(dx+c\right)^{3}+1260\,a^{2}b\tan\left(dx+c\right)^{2}+840\,a^{3}\tan\left(dx+c\right)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] 1/840*(84*b^3*tan(d*x + c)^10 + 280*a*b^2*tan(d*x + c)^9 + 315*a^2*b*tan(d*x + c)^8 + 315*b^3*tan(d*x + c)^8 + 120*a^3*tan(d*x + c)^7 + 1080*a*b^2*tan(d*x + c)^7 + 1260*a^2*b*tan(d*x + c)^6 + 420*b^3*tan(d*x + c)^6 + 504*a^3*tan(d*x + c)^5 + 1512*a*b^2*tan(d*x + c)^5 + 1890*a^2*b*tan(d*x + c)^4 + 210*b^3*tan(d*x + c)^4 + 840*a^3*tan(d*x + c)^3 + 840*a*b^2*tan(d*x + c)^3 + 1260*a^2*b*tan(d*x + c)^2 + 840*a^3*tan(d*x + c))/d

Mupad [B]
time = 1.58, size = 189, normalized size = 0.89

$$\frac{\cos\left(c+d\,x\right)^3\,\left(\frac{a^3\,\sin(c+d\,x)}{7}-\frac{a\,b^2\,\sin(c+d\,x)}{21}\right)+\cos\left(c+d\,x\right)^5\,\left(\frac{6\,a^3\,\sin(c+d\,x)}{35}-\frac{2\,a\,b^2\,\sin(c+d\,x)}{35}\right)+\cos\left(c+d\,x\right)^7\,\left(\frac{8\,a^3\,\sin(c+d\,x)}{35}-\frac{8\,a\,b^2\,\sin(c+d\,x)}{105}\right)+\cos\left(c+d\,x\right)^9\,\left(\frac{16\,a^3\,\sin(c+d\,x)}{35}-\frac{16\,a\,b^2\,\sin(c+d\,x)}{105}\right)+\cos\left(c+d\,x\right)^2\,\left(\frac{3\,a^2\,b}{8}-\frac{b^3}{8}\right)+\frac{b^3}{10}+\frac{a\,b^2\,\cos(c+d\,x)\,\sin(c+d\,x)}{3}}{d\,\cos\left(c+d\,x\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(c + d*x) + b*sin(c + d*x))^3/cos(c + d*x)^11,x)

[Out] (cos(c + d*x)^3*((a^3*sin(c + d*x))/7 - (a*b^2*sin(c + d*x))/21) + cos(c + d*x)^5*((6*a^3*sin(c + d*x))/35 - (2*a*b^2*sin(c + d*x))/35) + cos(c + d*x)^7*((8*a^3*sin(c + d*x))/35 - (8*a*b^2*sin(c + d*x))/105) + cos(c + d*x)^9*((16*a^3*sin(c + d*x))/35 - (16*a*b^2*sin(c + d*x))/105) + cos(c + d*x)^2*((3*a^2*b)/8 - b^3/8) + b^3/10 + (a*b^2*cos(c + d*x)*sin(c + d*x))/3)/(d*cos(c + d*x)^10)

3.74 $\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=279

$$-\frac{4ab^3\cos^7(c+dx)}{7d}-\frac{4a^3b\cos^9(c+dx)}{9d}+\frac{4ab^3\cos^9(c+dx)}{9d}+\frac{a^4\sin(c+dx)}{d}-\frac{4a^4\sin^3(c+dx)}{3d}+\frac{2a^2b^2\sin^3(c+$$

```
[Out] -4/7*a*b^3*cos(d*x+c)^7/d-4/9*a^3*b*cos(d*x+c)^9/d+4/9*a*b^3*cos(d*x+c)^9/d
+a^4*sin(d*x+c)/d-4/3*a^4*sin(d*x+c)^3/d+2*a^2*b^2*sin(d*x+c)^3/d+6/5*a^4*s
in(d*x+c)^5/d-18/5*a^2*b^2*sin(d*x+c)^5/d+1/5*b^4*sin(d*x+c)^5/d-4/7*a^4*si
n(d*x+c)^7/d+18/7*a^2*b^2*sin(d*x+c)^7/d-2/7*b^4*sin(d*x+c)^7/d+1/9*a^4*sin
(d*x+c)^9/d-2/3*a^2*b^2*sin(d*x+c)^9/d+1/9*b^4*sin(d*x+c)^9/d
```

Rubi [A]

time = 0.18, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3169, 2713, 2645, 30, 2644, 276, 14}

$$\frac{a^4\sin^9(c+dx)}{9d}-\frac{4a^4\sin^7(c+dx)}{7d}+\frac{6a^4\sin^5(c+dx)}{5d}-\frac{4a^4\sin^3(c+dx)}{3d}+\frac{a^4\sin(c+dx)}{d}-\frac{4a^3b\cos^9(c+dx)}{9d}-\frac{2a^2b^2\sin^9(c+dx)}{3d}+\frac{18a^2b^2\sin^7(c+dx)}{7d}-\frac{18a^2b^2\sin^5(c+dx)}{5d}+\frac{2a^2b^2\sin^3(c+dx)}{d}+\frac{4ab^3\cos^9(c+dx)}{9d}-\frac{4ab^3\cos^7(c+dx)}{7d}+\frac{b^4\sin^9(c+dx)}{9d}-\frac{2b^4\sin^7(c+dx)}{7d}+\frac{b^4\sin^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (-4*a*b^3*Cos[c + d*x]^7)/(7*d) - (4*a^3*b*Cos[c + d*x]^9)/(9*d) + (4*a*b^3
*Cos[c + d*x]^9)/(9*d) + (a^4*Sin[c + d*x])/d - (4*a^4*Sin[c + d*x]^3)/(3*d
) + (2*a^2*b^2*Sin[c + d*x]^3)/d + (6*a^4*Sin[c + d*x]^5)/(5*d) - (18*a^2*b
^2*Sin[c + d*x]^5)/(5*d) + (b^4*Sin[c + d*x]^5)/(5*d) - (4*a^4*Sin[c + d*x]
^7)/(7*d) + (18*a^2*b^2*Sin[c + d*x]^7)/(7*d) - (2*b^4*Sin[c + d*x]^7)/(7*d
) + (a^4*Sin[c + d*x]^9)/(9*d) - (2*a^2*b^2*Sin[c + d*x]^9)/(3*d) + (b^4*Si
n[c + d*x]^9)/(9*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx &= \int \left(a^4\cos^9(c+dx)+4a^3b\cos^8(c+dx)\sin(c+dx)+6\right.\\ &= a^4\int \cos^9(c+dx)\,dx+\left(4a^3b\right)\int \cos^8(c+dx)\sin(c+\\ &= -\frac{a^4\text{Subst}\left(\int \left(1-4x^2+6x^4-4x^6+x^8\right)\,dx,x,-\sin(c\right.}{d}\\ &= -\frac{4a^3b\cos^9(c+dx)}{9d}+\frac{a^4\sin(c+dx)}{d}-\frac{4a^4\sin^3(c+dx}{3d}\\ &= -\frac{4ab^3\cos^7(c+dx)}{7d}-\frac{4a^3b\cos^9(c+dx)}{9d}+\frac{4ab^3\cos^9(c}{9d}\end{aligned}$$

Mathematica [A]
time = 0.74, size = 237, normalized size = 0.85

$$\frac{-2520ab(7a^2+3b^2)\cos(c+dx)-1680ab(7a^2+2b^2)\cos(3(c+dx))-5040a^3b\cos(5(c+dx))-180ab(7a^2-3b^2)\cos(7(c+dx))-140ab(a^2-b^2)\cos(9(c+dx))+1890(21a^4+14a^2b^2+b^4)\sin(c+dx)+420(21a^4-b^4)\sin(3(c+dx))+252(9a^4-12a^2b^2-b^4)\sin(5(c+dx))+45(9a^4-30a^2b^2+b^4)\sin(7(c+dx))+35(a^4-6a^2b^2+b^4)\sin(9(c+dx))}{80640d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (-2520*a*b*(7*a^2 + 3*b^2)*Cos[c + d*x] - 1680*a*b*(7*a^2 + 2*b^2)*Cos[3*(c
 + d*x)] - 5040*a^3*b*Cos[5*(c + d*x)] - 180*a*b*(7*a^2 - 3*b^2)*Cos[7*(c +
 d*x)] - 140*a*b*(a^2 - b^2)*Cos[9*(c + d*x)] + 1890*(21*a^4 + 14*a^2*b^2 +
 b^4)*Sin[c + d*x] + 420*(21*a^4 - b^4)*Sin[3*(c + d*x)] + 252*(9*a^4 - 12*
a^2*b^2 - b^4)*Sin[5*(c + d*x)] + 45*(9*a^4 - 30*a^2*b^2 + b^4)*Sin[7*(c +
d*x)] + 35*(a^4 - 6*a^2*b^2 + b^4)*Sin[9*(c + d*x)])/(80640*d)
```

Maple [A]
time = 0.42, size = 236, normalized size = 0.85 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^4*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*cos(d*x+c)^6+1/105
*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+4*a*b^3*(-1/9*sin(d*x+c)^2
*cos(d*x+c)^7-2/63*cos(d*x+c)^7)+6*a^2*b^2*(-1/9*sin(d*x+c)*cos(d*x+c)^8+1/
63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))-4/9*a^
3*b*cos(d*x+c)^9+1/9*a^4*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*
x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.27, size = 186, normalized size = 0.67

$$-\frac{140\,a^3b\cos\left(dx+c\right)^9-\left(35\,\sin\left(dx+c\right)^9-180\,\sin\left(dx+c\right)^7+378\,\sin\left(dx+c\right)^5-420\,\sin\left(dx+c\right)^3+315\,\sin\left(dx+c\right)\right)a^4+6\left(35\,\sin\left(dx+c\right)^9-135\,\sin\left(dx+c\right)^7+189\,\sin\left(dx+c\right)^5-105\,\sin\left(dx+c\right)^3\right)a^2b^2-20\left(7\,\cos\left(dx+c\right)^9-9\,\cos\left(dx+c\right)^7\right)ab^3-\left(35\,\sin\left(dx+c\right)^9-90\,\sin\left(dx+c\right)^7+63\,\sin\left(dx+c\right)^5\right)b^4}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/315*(140*a^3*b*cos(d*x + c)^9 - (35*sin(d*x + c)^9 - 180*sin(d*x + c)^7
+ 378*sin(d*x + c)^5 - 420*sin(d*x + c)^3 + 315*sin(d*x + c))*a^4 + 6*(35*s
in(d*x + c)^9 - 135*sin(d*x + c)^7 + 189*sin(d*x + c)^5 - 105*sin(d*x + c)^
3)*a^2*b^2 - 20*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a*b^3 - (35*sin(d*x +
 c)^9 - 90*sin(d*x + c)^7 + 63*sin(d*x + c)^5)*b^4)/d
```

Fricas [A]
time = 3.30, size = 177, normalized size = 0.63

$$-\frac{180\,ab^3\cos\left(dx+c\right)^7+140\left(a^3b-ab^3\right)\cos\left(dx+c\right)^9-\left(35\left(a^4-6\,a^2b^2+b^4\right)\cos\left(dx+c\right)^8+10\left(4\,a^4+3\,a^2b^2-5\,b^4\right)\cos\left(dx+c\right)^6+3\left(16\,a^4+12\,a^2b^2+b^4\right)\cos\left(dx+c\right)^4+128\,a^4+96\,a^2b^2+8\,b^4+4\left(16\,a^4+12\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/315*(180*a*b^3*cos(d*x + c)^7 + 140*(a^3*b - a*b^3)*cos(d*x + c)^9 - (35
*(a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^8 + 10*(4*a^4 + 3*a^2*b^2 - 5*b^4)*co
s(d*x + c)^6 + 3*(16*a^4 + 12*a^2*b^2 + b^4)*cos(d*x + c)^4 + 128*a^4 + 96*
a^2*b^2 + 8*b^4 + 4*(16*a^4 + 12*a^2*b^2 + b^4)*cos(d*x + c)^2)*sin(d*x + c
))/d
```

Sympy [A]
time = 1.33, size = 367, normalized size = 1.32

$$\begin{cases} \frac{128a^4\sin^9(c+dx)}{315d} + \frac{64a^4\sin^7(c+dx)\cos^2(c+dx)}{35d} + \frac{16a^4\sin^5(c+dx)\cos^4(c+dx)}{5d} + \frac{8a^4\sin^3(c+dx)\cos^6(c+dx)}{3d} + \frac{a^4\sin(c+dx)\cos^8(c+dx)}{d} - \frac{4a^3b\cos^9(c+dx)}{9d} + \frac{32a^2b^2\sin^9(c+dx)}{105d} + \frac{48a^2b^2\sin^7(c+dx)\cos^2(c+dx)}{35d} + \frac{12a^2b^2\sin^5(c+dx)\cos^4(c+dx)}{5d} + \frac{2a^2b^2\sin^3(c+dx)\cos^6(c+dx)}{d} - \frac{4ab^3\sin^2(c+dx)\cos^7(c+dx)}{7d} - \frac{8ab^3\cos^9(c+dx)}{63d} + \frac{8b^4\sin^9(c+dx)}{315d} + \frac{4b^4\sin^7(c+dx)\cos^2(c+dx)}{35d} + \frac{b^4\sin^5(c+dx)\cos^4(c+dx)}{5d} & \text{for } d \neq 0 \\ x(a\cos(c)+b\sin(c))^4\cos^5(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((128*a**4*sin(c + d*x)**9/(315*d) + 64*a**4*sin(c + d*x)**7*cos(c
 + d*x)**2/(35*d) + 16*a**4*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + 8*a**4*
sin(c + d*x)**3*cos(c + d*x)**6/(3*d) + a**4*sin(c + d*x)*cos(c + d*x)**8/d
 - 4*a**3*b*cos(c + d*x)**9/(9*d) + 32*a**2*b**2*sin(c + d*x)**9/(105*d) +
48*a**2*b**2*sin(c + d*x)**7*cos(c + d*x)**2/(35*d) + 12*a**2*b**2*sin(c +
d*x)**5*cos(c + d*x)**4/(5*d) + 2*a**2*b**2*sin(c + d*x)**3*cos(c + d*x)**6
/d - 4*a*b**3*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) - 8*a*b**3*cos(c + d*x)
**9/(63*d) + 8*b**4*sin(c + d*x)**9/(315*d) + 4*b**4*sin(c + d*x)**7*cos(c
+ d*x)**2/(35*d) + b**4*sin(c + d*x)**5*cos(c + d*x)**4/(5*d), Ne(d, 0)), (
x*(a*cos(c) + b*sin(c))**4*cos(c)**5, True))
```

Giac [A]
time = 0.62, size = 269, normalized size = 0.96

$$-\frac{a^3b\cos(5dx+5c)}{16d} - \frac{(a^3b-ab^3)\cos(9dx+9c)}{576d} - \frac{(7a^3b-3ab^3)\cos(7dx+7c)}{448d} - \frac{(7a^3b+2ab^3)\cos(3dx+3c)}{48d} - \frac{(7a^3b+3ab^3)\cos(dx+c)}{32d} + \frac{(a^4-6a^2b^2+b^4)\sin(9dx+9c)}{2304d} + \frac{(9a^4-30a^2b^2+b^4)\sin(7dx+7c)}{1792d} + \frac{(9a^4-12a^2b^2-b^4)\sin(5dx+5c)}{320d} + \frac{(21a^4-b^4)\sin(3dx+3c)}{192d} + \frac{3(21a^4+14a^2b^2+b^4)\sin(dx+c)}{128d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/16*a^3*b*cos(5*d*x + 5*c)/d - 1/576*(a^3*b - a*b^3)*cos(9*d*x + 9*c)/d -
 1/448*(7*a^3*b - 3*a*b^3)*cos(7*d*x + 7*c)/d - 1/48*(7*a^3*b + 2*a*b^3)*co
s(3*d*x + 3*c)/d - 1/32*(7*a^3*b + 3*a*b^3)*cos(d*x + c)/d + 1/2304*(a^4 -
6*a^2*b^2 + b^4)*sin(9*d*x + 9*c)/d + 1/1792*(9*a^4 - 30*a^2*b^2 + b^4)*sin
(7*d*x + 7*c)/d + 1/320*(9*a^4 - 12*a^2*b^2 - b^4)*sin(5*d*x + 5*c)/d + 1/1
92*(21*a^4 - b^4)*sin(3*d*x + 3*c)/d + 3/128*(21*a^4 + 14*a^2*b^2 + b^4)*si
n(d*x + c)/d
```

Mupad [B]
time = 2.00, size = 334, normalized size = 1.20

$$-\frac{\frac{b^4\sin(3c+3dx)}{192} - \frac{3b^4\sin(c+dx)}{128} - \frac{7a^4\sin(3c+3dx)}{64} - \frac{9a^4\sin(5c+5dx)}{320} - \frac{9a^4\sin(7c+7dx)}{1792} - \frac{a^4\sin(9c+9dx)}{2304} - \frac{63a^4\sin(c+dx)}{128} + \frac{b^4\sin(5c+5dx)}{320} - \frac{b^4\sin(7c+7dx)}{1792} - \frac{b^4\sin(9c+9dx)}{2304} + \frac{ab^3\cos(3c+3dx)}{24} + \frac{7a^3b\cos(3c+3dx)}{48} + \frac{a^3b\cos(5c+5dx)}{16} - \frac{3ab^3\cos(7c+7dx)}{448} + \frac{a^3b\cos(7c+7dx)}{64} - \frac{ab^3\cos(9c+9dx)}{576} + \frac{a^3b\cos(9c+9dx)}{576} - \frac{21a^2b^2\sin(c+dx)}{64} + \frac{3a^2b^2\sin(5c+5dx)}{80} + \frac{15a^2b^2\sin(7c+7dx)}{896} + \frac{a^2b^2\sin(9c+9dx)}{384} + \frac{3ab^3\cos(c+dx)}{32} + \frac{7a^3b\cos(c+dx)}{32}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

```
[Out] -((b^4*sin(3*c + 3*d*x))/192 - (3*b^4*sin(c + d*x))/128 - (7*a^4*sin(3*c +
3*d*x))/64 - (9*a^4*sin(5*c + 5*d*x))/320 - (9*a^4*sin(7*c + 7*d*x))/1792 -
 (a^4*sin(9*c + 9*d*x))/2304 - (63*a^4*sin(c + d*x))/128 + (b^4*sin(5*c + 5
*d*x))/320 - (b^4*sin(7*c + 7*d*x))/1792 - (b^4*sin(9*c + 9*d*x))/2304 + (a
*b^3*cos(3*c + 3*d*x))/24 + (7*a^3*b*cos(3*c + 3*d*x))/48 + (a^3*b*cos(5*c
+ 5*d*x))/16 - (3*a*b^3*cos(7*c + 7*d*x))/448 + (a^3*b*cos(7*c + 7*d*x))/64
 - (a*b^3*cos(9*c + 9*d*x))/576 + (a^3*b*cos(9*c + 9*d*x))/576 - (21*a^2*b^
2*sin(c + d*x))/64 + (3*a^2*b^2*sin(5*c + 5*d*x))/80 + (15*a^2*b^2*sin(7*c
+ 7*d*x))/896 + (a^2*b^2*sin(9*c + 9*d*x))/384 + (3*a*b^3*cos(c + d*x))/32
+ (7*a^3*b*cos(c + d*x))/32)/d
```

3.75 $\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=381

$$\frac{35a^4x}{128}+\frac{15}{64}a^2b^2x+\frac{3b^4x}{128}-\frac{2ab^3\cos^6(c+dx)}{3d}-\frac{a^3b\cos^8(c+dx)}{2d}+\frac{ab^3\cos^8(c+dx)}{2d}+\frac{35a^4\cos(c+dx)\sin(c+d$$

```
[Out] 35/128*a^4*x+15/64*a^2*b^2*x+3/128*b^4*x-2/3*a*b^3*cos(d*x+c)^6/d-1/2*a^3*b
*cos(d*x+c)^8/d+1/2*a*b^3*cos(d*x+c)^8/d+35/128*a^4*cos(d*x+c)*sin(d*x+c)/d
+15/64*a^2*b^2*cos(d*x+c)*sin(d*x+c)/d+3/128*b^4*cos(d*x+c)*sin(d*x+c)/d+35
/192*a^4*cos(d*x+c)^3*sin(d*x+c)/d+5/32*a^2*b^2*cos(d*x+c)^3*sin(d*x+c)/d+1
/64*b^4*cos(d*x+c)^3*sin(d*x+c)/d+7/48*a^4*cos(d*x+c)^5*sin(d*x+c)/d+1/8*a^
2*b^2*cos(d*x+c)^5*sin(d*x+c)/d-1/16*b^4*cos(d*x+c)^5*sin(d*x+c)/d+1/8*a^4*
cos(d*x+c)^7*sin(d*x+c)/d-3/4*a^2*b^2*cos(d*x+c)^7*sin(d*x+c)/d-1/8*b^4*cos
(d*x+c)^5*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.27, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3169, 2715, 8, 2645, 30, 2648, 14}

$$\frac{a^4\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{7a^4\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{35a^4\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{35a^4\sin(c+dx)\cos(c+dx)}{128d}+\frac{35a^4x}{128}-\frac{a^3b\cos^8(c+dx)}{2d}-\frac{3a^2b^2\sin(c+dx)\cos^7(c+dx)}{4d}+\frac{a^2b^2\sin(c+dx)\cos^5(c+dx)}{8d}+\frac{5a^2b^2\sin(c+dx)\cos^3(c+dx)}{32d}+\frac{15a^2b^2\sin(c+dx)\cos(c+dx)}{64d}+\frac{15}{64}a^2b^2x+\frac{ab^3\cos^8(c+dx)}{2d}-\frac{2ab^3\cos^6(c+dx)}{3d}-\frac{b^4\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{b^4\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{b^4\sin(c+dx)\cos^3(c+dx)}{64d}+\frac{3b^4\sin(c+dx)\cos(c+dx)}{128d}+\frac{3b^4x}{128}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (35*a^4*x)/128 + (15*a^2*b^2*x)/64 + (3*b^4*x)/128 - (2*a*b^3*Cos[c + d*x]^
6)/(3*d) - (a^3*b*Cos[c + d*x]^8)/(2*d) + (a*b^3*Cos[c + d*x]^8)/(2*d) + (3
5*a^4*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (15*a^2*b^2*Cos[c + d*x]*Sin[c +
 d*x])/(64*d) + (3*b^4*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (35*a^4*Cos[c +
 d*x]^3*Sin[c + d*x])/(192*d) + (5*a^2*b^2*Cos[c + d*x]^3*Sin[c + d*x])/(32
*d) + (b^4*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) + (7*a^4*Cos[c + d*x]^5*Sin[
c + d*x])/(48*d) + (a^2*b^2*Cos[c + d*x]^5*Sin[c + d*x])/(8*d) - (b^4*Cos[c
 + d*x]^5*Sin[c + d*x])/(16*d) + (a^4*Cos[c + d*x]^7*Sin[c + d*x])/(8*d) -
(3*a^2*b^2*Cos[c + d*x]^7*Sin[c + d*x])/(4*d) - (b^4*Cos[c + d*x]^5*Sin[c +
 d*x]^3)/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx &= \int \left(a^4\cos^8(c+dx)+4a^3b\cos^7(c+dx)\sin(c+dx)+6\right. \\
&= a^4\int \cos^8(c+dx)\,dx+\left(4a^3b\right)\int \cos^7(c+dx)\sin(c+ \\
&= \frac{a^4\cos^7(c+dx)\sin(c+dx)}{8d}-\frac{3a^2b^2\cos^7(c+dx)\sin(c}{4d} \\
&= -\frac{a^3b\cos^8(c+dx)}{2d}+\frac{7a^4\cos^5(c+dx)\sin(c+dx)}{48d}+\frac{a^2}{} \\
&= -\frac{2ab^3\cos^6(c+dx)}{3d}-\frac{a^3b\cos^8(c+dx)}{2d}+\frac{ab^3\cos^8(c+}{2d} \\
&= -\frac{2ab^3\cos^6(c+dx)}{3d}-\frac{a^3b\cos^8(c+dx)}{2d}+\frac{ab^3\cos^8(c+}{2d} \\
&= \frac{35a^4x}{128}+\frac{15}{64}a^2b^2x+\frac{3b^4x}{128}-\frac{2ab^3\cos^6(c+dx)}{3d}-\frac{a^3b\cos}{}
\end{aligned}
$$

Mathematica [A]
time = 0.65, size = 222, normalized size = 0.58

$$
\frac{24(35a^4+30a^2b^2+3b^4)(c+dx)-96ab(7a^2+3b^2)\cos(2(c+dx))-48ab(7a^2+b^2)\cos(4(c+dx))-32ab(3a^2-b^2)\cos(6(c+dx))-12ab(a^2-b^2)\cos(8(c+dx))+96a^2(7a^2+3b^2)\sin(2(c+dx))+24(7a^4-6a^2b^2-b^4)\sin(4(c+dx))+32a^2(a^2-3b^2)\sin(6(c+dx))+3(a^4-6a^2b^2+b^4)\sin(8(c+dx))}{3072d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (24*(35*a^4 + 30*a^2*b^2 + 3*b^4)*(c + d*x) - 96*a*b*(7*a^2 + 3*b^2)*Cos[2*
(c + d*x)] - 48*a*b*(7*a^2 + b^2)*Cos[4*(c + d*x)] - 32*a*b*(3*a^2 - b^2)*C
os[6*(c + d*x)] - 12*a*b*(a^2 - b^2)*Cos[8*(c + d*x)] + 96*a^2*(7*a^2 + 3*b
^2)*Sin[2*(c + d*x)] + 24*(7*a^4 - 6*a^2*b^2 - b^4)*Sin[4*(c + d*x)] + 32*a
^2*(a^2 - 3*b^2)*Sin[6*(c + d*x)] + 3*(a^4 - 6*a^2*b^2 + b^4)*Sin[8*(c + d*
x)])/(3072*d)
```

Maple [A]
time = 0.36, size = 250, normalized size = 0.66 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^4*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos(d*x+c)^5+1/64*
(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+4*a*b^3*(-1/8*s
in(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+6*a^2*b^2*(-1/8*sin(d*x+c)*cos(
d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/
128*d*x+5/128*c)-1/2*a^3*b*cos(d*x+c)^8+a^4*(1/8*(cos(d*x+c)^7+7/6*cos(d*x+
c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/128*d*x+35/128*c))
```

Maxima [A]
time = 0.28, size = 199, normalized size = 0.52

$$-\frac{1536\,a^3b\cos\left(dx+c\right)^8+\left(128\,\sin\left(2\,dx+2\,c\right)^3-840\,dx-840\,c-3\,\sin\left(8\,dx+8\,c\right)-168\,\sin\left(4\,dx+4\,c\right)-768\,\sin\left(2\,dx+2\,c\right)\right)a^4-6\left(64\,\sin\left(2\,dx+2\,c\right)^3+120\,dx+120\,c-3\,\sin\left(8\,dx+8\,c\right)-24\,\sin\left(4\,dx+4\,c\right)\right)a^2b^2-512\left(3\,\sin\left(dx+c\right)^8-8\,\sin\left(dx+c\right)^6+6\,\sin\left(dx+c\right)^4\right)ab^3-3\left(24\,dx+24\,c+\sin\left(8\,dx+8\,c\right)-8\,\sin\left(4\,dx+4\,c\right)\right)b^4}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3072*(1536*a^3*b*cos(d*x + c)^8 + (128*sin(2*d*x + 2*c)^3 - 840*d*x - 84
0*c - 3*sin(8*d*x + 8*c) - 168*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*a^4
 - 6*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*sin
(4*d*x + 4*c))*a^2*b^2 - 512*(3*sin(d*x + c)^8 - 8*sin(d*x + c)^6 + 6*sin(d
*x + c)^4)*a*b^3 - 3*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x + 4*c)
)*b^4)/d
```

Fricas [A]
time = 2.88, size = 184, normalized size = 0.48

$$-\frac{256\,ab^3\cos\left(dx+c\right)^6+192\left(a^3b-ab^3\right)\cos\left(dx+c\right)^8-3\left(35\,a^4+30\,a^2b^2+3\,b^4\right)dx-\left(48\left(a^4-6\,a^2b^2+b^4\right)\cos\left(dx+c\right)^7+8\left(7\,a^4+6\,a^2b^2-9\,b^4\right)\cos\left(dx+c\right)^5+2\left(35\,a^4+30\,a^2b^2+3\,b^4\right)\cos\left(dx+c\right)^3+3\left(35\,a^4+30\,a^2b^2+3\,b^4\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/384*(256*a*b^3*cos(d*x + c)^6 + 192*(a^3*b - a*b^3)*cos(d*x + c)^8 - 3*(
35*a^4 + 30*a^2*b^2 + 3*b^4)*d*x - (48*(a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)
^7 + 8*(7*a^4 + 6*a^2*b^2 - 9*b^4)*cos(d*x + c)^5 + 2*(35*a^4 + 30*a^2*b^2
+ 3*b^4)*cos(d*x + c)^3 + 3*(35*a^4 + 30*a^2*b^2 + 3*b^4)*cos(d*x + c))*sin
(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 736 vs. $2(367) = 734$.
time = 0.97, size = 736, normalized size = 1.93

$$\begin{cases} \frac{35a^4x\sin^8(c+dx)}{128}+\frac{35a^4x\sin^6(c+dx)\cos^2(c+dx)}{32}+\frac{105a^4x\sin^4(c+dx)\cos^4(c+dx)}{64}+\frac{35a^4x\sin^2(c+dx)\cos^6(c+dx)}{32}+\frac{35a^4x\cos^8(c+dx)}{128}+\frac{35a^4\sin^7(c+dx)\cos(c+dx)}{128d}+\frac{385a^4\sin^5(c+dx)\cos^3(c+dx)}{384d}+\frac{511a^4\sin^3(c+dx)\cos^5(c+dx)}{384d}+\ldots & [illegible] \\ x(a\cos(c)+b\sin(c))^4\cos^4(c) & [illegible] \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((35*a**4*x*sin(c + d*x)**8/128 + 35*a**4*x*sin(c + d*x)**6*cos(c
+ d*x)**2/32 + 105*a**4*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 35*a**4*x*si
n(c + d*x)**2*cos(c + d*x)**6/32 + 35*a**4*x*cos(c + d*x)**8/128 + 35*a**4*
sin(c + d*x)**7*cos(c + d*x)/(128*d) + 385*a**4*sin(c + d*x)**5*cos(c + d*x
)**3/(384*d) + 511*a**4*sin(c + d*x)**3*cos(c + d*x)**5/(384*d) + 93*a**4*s
in(c + d*x)*cos(c + d*x)**7/(128*d) - a**3*b*cos(c + d*x)**8/(2*d) + 15*a**
2*b**2*x*sin(c + d*x)**8/64 + 15*a**2*b**2*x*sin(c + d*x)**6*cos(c + d*x)**
```

```
2/16 + 45*a**2*b**2*x*sin(c + d*x)**4*cos(c + d*x)**4/32 + 15*a**2*b**2*x*s
in(c + d*x)**2*cos(c + d*x)**6/16 + 15*a**2*b**2*x*cos(c + d*x)**8/64 + 15*
a**2*b**2*sin(c + d*x)**7*cos(c + d*x)/(64*d) + 55*a**2*b**2*sin(c + d*x)**
5*cos(c + d*x)**3/(64*d) + 73*a**2*b**2*sin(c + d*x)**3*cos(c + d*x)**5/(64
*d) - 15*a**2*b**2*sin(c + d*x)*cos(c + d*x)**7/(64*d) - 2*a*b**3*sin(c + d
*x)**2*cos(c + d*x)**6/(3*d) - a*b**3*cos(c + d*x)**8/(6*d) + 3*b**4*x*sin(
c + d*x)**8/128 + 3*b**4*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 9*b**4*x*si
n(c + d*x)**4*cos(c + d*x)**4/64 + 3*b**4*x*sin(c + d*x)**2*cos(c + d*x)**6
/32 + 3*b**4*x*cos(c + d*x)**8/128 + 3*b**4*sin(c + d*x)**7*cos(c + d*x)/(1
28*d) + 11*b**4*sin(c + d*x)**5*cos(c + d*x)**3/(128*d) - 11*b**4*sin(c + d
*x)**3*cos(c + d*x)**5/(128*d) - 3*b**4*sin(c + d*x)*cos(c + d*x)**7/(128*d
), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**4*cos(c)**4, True))
```

Giac [A]
time = 0.58, size = 245, normalized size = 0.64

$$\frac{1}{128}\left(35\,a^4+30\,a^2b^2+3\,b^4\right)x-\frac{\left(a^3b-ab^3\right)\cos\left(8\,dx+8\,c\right)}{256\,d}-\frac{\left(3\,a^3b-ab^3\right)\cos\left(6\,dx+6\,c\right)}{96\,d}-\frac{\left(7\,a^3b+ab^3\right)\cos\left(4\,dx+4\,c\right)}{64\,d}-\frac{\left(7\,a^3b+3\,ab^3\right)\cos\left(2\,dx+2\,c\right)}{32\,d}+\frac{\left(a^4-6\,a^2b^2+b^4\right)\sin\left(8\,dx+8\,c\right)}{1024\,d}+\frac{\left(a^4-3\,a^2b^2\right)\sin\left(6\,dx+6\,c\right)}{96\,d}+\frac{\left(7\,a^4-6\,a^2b^2-b^4\right)\sin\left(4\,dx+4\,c\right)}{128\,d}+\frac{\left(7\,a^4+3\,a^2b^2\right)\sin\left(2\,dx+2\,c\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/128*(35*a^4 + 30*a^2*b^2 + 3*b^4)*x - 1/256*(a^3*b - a*b^3)*cos(8*d*x + 8
*c)/d - 1/96*(3*a^3*b - a*b^3)*cos(6*d*x + 6*c)/d - 1/64*(7*a^3*b + a*b^3)*
cos(4*d*x + 4*c)/d - 1/32*(7*a^3*b + 3*a*b^3)*cos(2*d*x + 2*c)/d + 1/1024*(
a^4 - 6*a^2*b^2 + b^4)*sin(8*d*x + 8*c)/d + 1/96*(a^4 - 3*a^2*b^2)*sin(6*d*
x + 6*c)/d + 1/128*(7*a^4 - 6*a^2*b^2 - b^4)*sin(4*d*x + 4*c)/d + 1/32*(7*a
^4 + 3*a^2*b^2)*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 1.69, size = 343, normalized size = 0.90

$$\frac{35\,a^4\,x}{128}+\frac{3\,b^4\,x}{128}+\frac{15\,a^2\,b^2\,x}{64}-\frac{2\,a\,b^3\cos\left(c+d\,x\right)^6}{3\,d}+\frac{a\,b^3\cos\left(c+d\,x\right)^8}{2\,d}-\frac{a^3\,b\cos\left(c+d\,x\right)^8}{2\,d}+\frac{35\,a^4\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{192\,d}+\frac{7\,a^4\cos\left(c+d\,x\right)^5\,\sin\left(c+d\,x\right)}{48\,d}+\frac{a^4\cos\left(c+d\,x\right)^7\,\sin\left(c+d\,x\right)}{8\,d}+\frac{b^4\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{64\,d}-\frac{3\,b^4\cos\left(c+d\,x\right)^5\,\sin\left(c+d\,x\right)}{16\,d}+\frac{b^4\cos\left(c+d\,x\right)^7\,\sin\left(c+d\,x\right)}{8\,d}+\frac{35\,a^4\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{128\,d}+\frac{3\,b^4\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{128\,d}+\frac{15\,a^2\,b^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{64\,d}+\frac{5\,a^2\,b^2\cos\left(c+d\,x\right)^3\,\sin\left(c+d\,x\right)}{32\,d}+\frac{a^2\,b^2\cos\left(c+d\,x\right)^5\,\sin\left(c+d\,x\right)}{8\,d}-\frac{3\,a^2\,b^2\cos\left(c+d\,x\right)^7\,\sin\left(c+d\,x\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

```
[Out] (35*a^4*x)/128 + (3*b^4*x)/128 + (15*a^2*b^2*x)/64 - (2*a*b^3*cos(c + d*x)^
6)/(3*d) + (a*b^3*cos(c + d*x)^8)/(2*d) - (a^3*b*cos(c + d*x)^8)/(2*d) + (3
5*a^4*cos(c + d*x)^3*sin(c + d*x))/(192*d) + (7*a^4*cos(c + d*x)^5*sin(c +
d*x))/(48*d) + (a^4*cos(c + d*x)^7*sin(c + d*x))/(8*d) + (b^4*cos(c + d*x)^
3*sin(c + d*x))/(64*d) - (3*b^4*cos(c + d*x)^5*sin(c + d*x))/(16*d) + (b^4*
cos(c + d*x)^7*sin(c + d*x))/(8*d) + (35*a^4*cos(c + d*x)*sin(c + d*x))/(12
8*d) + (3*b^4*cos(c + d*x)*sin(c + d*x))/(128*d) + (15*a^2*b^2*cos(c + d*x)
*sin(c + d*x))/(64*d) + (5*a^2*b^2*cos(c + d*x)^3*sin(c + d*x))/(32*d) + (a
^2*b^2*cos(c + d*x)^5*sin(c + d*x))/(8*d) - (3*a^2*b^2*cos(c + d*x)^7*sin(c
 + d*x))/(4*d)
```

3.76 $\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=220

$$-\frac{4ab^3\cos^5(c+dx)}{5d}-\frac{4a^3b\cos^7(c+dx)}{7d}+\frac{4ab^3\cos^7(c+dx)}{7d}+\frac{a^4\sin(c+dx)}{d}-\frac{a^4\sin^3(c+dx)}{d}+\frac{2a^2b^2\sin^3(c+}{d}$$

```
[Out] -4/5*a*b^3*cos(d*x+c)^5/d-4/7*a^3*b*cos(d*x+c)^7/d+4/7*a*b^3*cos(d*x+c)^7/d
+a^4*sin(d*x+c)/d-a^4*sin(d*x+c)^3/d+2*a^2*b^2*sin(d*x+c)^3/d+3/5*a^4*sin(d
*x+c)^5/d-12/5*a^2*b^2*sin(d*x+c)^5/d+1/5*b^4*sin(d*x+c)^5/d-1/7*a^4*sin(d*
x+c)^7/d+6/7*a^2*b^2*sin(d*x+c)^7/d-1/7*b^4*sin(d*x+c)^7/d
```

Rubi [A]
time = 0.16, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3169, 2713, 2645, 30, 2644, 276, 14}

$$-\frac{a^4\sin^7(c+dx)}{7d}+\frac{3a^4\sin^5(c+dx)}{5d}-\frac{a^4\sin^3(c+dx)}{d}+\frac{a^4\sin(c+dx)}{d}-\frac{4a^3b\cos^7(c+dx)}{7d}+\frac{6a^2b^2\sin^7(c+dx)}{7d}-\frac{12a^2b^2\sin^5(c+dx)}{5d}+\frac{2a^2b^2\sin^3(c+dx)}{d}+\frac{4ab^3\cos^7(c+dx)}{7d}-\frac{4ab^3\cos^5(c+dx)}{5d}-\frac{b^4\sin^7(c+dx)}{7d}+\frac{b^4\sin^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (-4*a*b^3*Cos[c + d*x]^5)/(5*d) - (4*a^3*b*Cos[c + d*x]^7)/(7*d) + (4*a*b^3
*Cos[c + d*x]^7)/(7*d) + (a^4*Sin[c + d*x])/d - (a^4*Sin[c + d*x]^3)/d + (2
*a^2*b^2*Sin[c + d*x]^3)/d + (3*a^4*Sin[c + d*x]^5)/(5*d) - (12*a^2*b^2*Sin
[c + d*x]^5)/(5*d) + (b^4*Sin[c + d*x]^5)/(5*d) - (a^4*Sin[c + d*x]^7)/(7*d
) + (6*a^2*b^2*Sin[c + d*x]^7)/(7*d) - (b^4*Sin[c + d*x]^7)/(7*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx &= \int \left(a^4\cos^7(c+dx)+4a^3b\cos^6(c+dx)\sin(c+dx)+6\right.\\ &= a^4\int \cos^7(c+dx)\,dx+\left(4a^3b\right)\int \cos^6(c+dx)\sin(c+\\ &= -\frac{a^4\mathrm{Subst}\left(\int \left(1-3x^2+3x^4-x^6\right)\,dx,x,-\sin(c+dx)\right.}{d}\\ &= -\frac{4a^3b\cos^7(c+dx)}{7d}+\frac{a^4\sin(c+dx)}{d}-\frac{a^4\sin^3(c+dx)}{d}\\ &= -\frac{4ab^3\cos^5(c+dx)}{5d}-\frac{4a^3b\cos^7(c+dx)}{7d}+\frac{4ab^3\cos^7(c}{7d}\end{aligned}$$

Mathematica [A]
time = 0.54, size = 204, normalized size = 0.93

$$\frac{-140ab(5a^2+3b^2)\cos(c+dx)-140ab(3a^2+b^2)\cos(3(c+dx))-28ab(5a^2-b^2)\cos(5(c+dx))-20ab(a^2-b^2)\cos(7(c+dx))+35(35a^4+30a^2b^2+3b^4)\sin(c+dx)+35(7a^4-2a^2b^2-b^4)\sin(3(c+dx))+7(7a^4-18a^2b^2-b^4)\sin(5(c+dx))+5(a^4-6a^2b^2+b^4)\sin(7(c+dx))}{2240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (-140*a*b*(5*a^2 + 3*b^2)*Cos[c + d*x] - 140*a*b*(3*a^2 + b^2)*Cos[3*(c + d
*x)] - 28*a*b*(5*a^2 - b^2)*Cos[5*(c + d*x)] - 20*a*b*(a^2 - b^2)*Cos[7*(c
+ d*x)] + 35*(35*a^4 + 30*a^2*b^2 + 3*b^4)*Sin[c + d*x] + 35*(7*a^4 - 2*a^2
*b^2 - b^4)*Sin[3*(c + d*x)] + 7*(7*a^4 - 18*a^2*b^2 - b^4)*Sin[5*(c + d*x)
] + 5*(a^4 - 6*a^2*b^2 + b^4)*Sin[7*(c + d*x)])/(2240*d)
```

Maple [A]
time = 0.29, size = 206, normalized size = 0.94

method	result
derivativedivides	$\frac{b^4\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^4(dx+c)\right)}{7}-\frac{3\sin(dx+c)\left(\cos^4(dx+c)\right)}{35}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{35}\right)+4a\,b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}\right.}{}$
default	$\frac{b^4\left(-\frac{\left(\sin^3(dx+c)\right)\left(\cos^4(dx+c)\right)}{7}-\frac{3\sin(dx+c)\left(\cos^4(dx+c)\right)}{35}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{35}\right)+4a\,b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^5(dx+c)\right)}{7}\right.}{}$
risch	$-\frac{5a^3b\cos(dx+c)}{16d}-\frac{3a\,b^3\cos(dx+c)}{16d}+\frac{35a^4\sin(dx+c)}{64d}+\frac{15a^2b^2\sin(dx+c)}{32d}+\frac{3b^4\sin(dx+c)}{64d}-\frac{a^3b\cos(7dx+7c)}{112d}$
norman	$\frac{-\frac{40a^3b+16a\,b^3}{35d}+\frac{2a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^4\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{16a\,b^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5d}-\frac{16a\,b^3\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{32a\,b^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^4*(-1/7*sin(d*x+c)^3*cos(d*x+c)^4-3/35*sin(d*x+c)*cos(d*x+c)^4+1/35*
(2+cos(d*x+c)^2)*sin(d*x+c))+4*a*b^3*(-1/7*sin(d*x+c)^2*cos(d*x+c)^5-2/35*c
os(d*x+c)^5)+6*a^2*b^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/35*(8/3+cos(d*x+c)^4
+4/3*cos(d*x+c)^2)*sin(d*x+c))-4/7*a^3*b*cos(d*x+c)^7+1/7*a^4*(16/5+cos(d*x
+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 154, normalized size = 0.70

$$-\frac{20\,a^3b\cos\left(dx+c\right)^7+\left(5\,\sin\left(dx+c\right)^7-21\,\sin\left(dx+c\right)^5+35\,\sin\left(dx+c\right)^3-35\,\sin\left(dx+c\right)\right)a^4-2\left(15\,\sin\left(dx+c\right)^7-42\,\sin\left(dx+c\right)^5+35\,\sin\left(dx+c\right)^3\right)a^2b^2-4\left(5\,\cos\left(dx+c\right)^7-7\,\cos\left(dx+c\right)^5\right)ab^3+\left(5\,\sin\left(dx+c\right)^7-7\,\sin\left(dx+c\right)^5\right)b^4}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/35*(20*a^3*b*cos(d*x + c)^7 + (5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35
*sin(d*x + c)^3 - 35*sin(d*x + c))*a^4 - 2*(15*sin(d*x + c)^7 - 42*sin(d*x
```

```
+ c)^5 + 35*sin(d*x + c)^3)*a^2*b^2 - 4*(5*cos(d*x + c)^7 - 7*cos(d*x + c)^
5)*a*b^3 + (5*sin(d*x + c)^7 - 7*sin(d*x + c)^5)*b^4)/d
```

Fricas [A]
time = 2.47, size = 149, normalized size = 0.68

$$-\frac{28\,ab^3\cos\left(dx+c\right)^5+20\,\left(a^3b-ab^3\right)\cos\left(dx+c\right)^7-\left(5\,\left(a^4-6\,a^2b^2+b^4\right)\cos\left(dx+c\right)^6+2\,\left(3\,a^4+3\,a^2b^2-4\,b^4\right)\cos\left(dx+c\right)^4+16\,a^4+16\,a^2b^2+2\,b^4+\left(8\,a^4+8\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/35*(28*a*b^3*cos(d*x + c)^5 + 20*(a^3*b - a*b^3)*cos(d*x + c)^7 - (5*(a^
4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^6 + 2*(3*a^4 + 3*a^2*b^2 - 4*b^4)*cos(d*x
 + c)^4 + 16*a^4 + 16*a^2*b^2 + 2*b^4 + (8*a^4 + 8*a^2*b^2 + b^4)*cos(d*x +
 c)^2)*sin(d*x + c))/d
```

Sympy [A]
time = 0.65, size = 286, normalized size = 1.30

$$\begin{cases} \frac{16a^4\sin^7\left(c+dx\right)}{35d}+\frac{8a^4\sin^5\left(c+dx\right)\cos^2\left(c+dx\right)}{5d}+\frac{2a^4\sin^3\left(c+dx\right)\cos^4\left(c+dx\right)}{d}+\frac{a^4\sin\left(c+dx\right)\cos^6\left(c+dx\right)}{d}-\frac{4a^3b\cos^7\left(c+dx\right)}{7d}+\frac{16a^2b^2\sin^7\left(c+dx\right)}{35d}+\frac{8a^2b^2\sin^5\left(c+dx\right)\cos^2\left(c+dx\right)}{5d}+\frac{2a^2b^2\sin^3\left(c+dx\right)\cos^4\left(c+dx\right)}{d}-\frac{4ab^3\sin^2\left(c+dx\right)\cos^5\left(c+dx\right)}{5d}-\frac{8ab^3\cos^7\left(c+dx\right)}{35d}+\frac{2b^4\sin^7\left(c+dx\right)}{35d}+\frac{b^4\sin^5\left(c+dx\right)\cos^2\left(c+dx\right)}{5d} & \text{for}\ d\neq 0 \\ x\left(a\cos\left(c\right)+b\sin\left(c\right)\right)^4\cos^3\left(c\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((16*a**4*sin(c + d*x)**7/(35*d) + 8*a**4*sin(c + d*x)**5*cos(c +
d*x)**2/(5*d) + 2*a**4*sin(c + d*x)**3*cos(c + d*x)**4/d + a**4*sin(c + d*x
)*cos(c + d*x)**6/d - 4*a**3*b*cos(c + d*x)**7/(7*d) + 16*a**2*b**2*sin(c +
 d*x)**7/(35*d) + 8*a**2*b**2*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 2*a**
2*b**2*sin(c + d*x)**3*cos(c + d*x)**4/d - 4*a*b**3*sin(c + d*x)**2*cos(c +
 d*x)**5/(5*d) - 8*a*b**3*cos(c + d*x)**7/(35*d) + 2*b**4*sin(c + d*x)**7/(
35*d) + b**4*sin(c + d*x)**5*cos(c + d*x)**2/(5*d), Ne(d, 0)), (x*(a*cos(c)
 + b*sin(c))**4*cos(c)**3, True))
```

Giac [A]
time = 0.58, size = 229, normalized size = 1.04

$$-\frac{\left(a^3b-ab^3\right)\cos\left(7\,dx+7\,c\right)}{112\,d}-\frac{\left(5\,a^3b-ab^3\right)\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{\left(3\,a^3b+ab^3\right)\cos\left(3\,dx+3\,c\right)}{16\,d}-\frac{\left(5\,a^3b+3\,ab^3\right)\cos\left(dx+c\right)}{16\,d}+\frac{\left(a^4-6\,a^2b^2+b^4\right)\sin\left(7\,dx+7\,c\right)}{448\,d}+\frac{\left(7\,a^4-18\,a^2b^2-b^4\right)\sin\left(5\,dx+5\,c\right)}{320\,d}+\frac{\left(7\,a^4-2\,a^2b^2-b^4\right)\sin\left(3\,dx+3\,c\right)}{64\,d}+\frac{\left(35\,a^4+30\,a^2b^2+3\,b^4\right)\sin\left(dx+c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/112*(a^3*b - a*b^3)*cos(7*d*x + 7*c)/d - 1/80*(5*a^3*b - a*b^3)*cos(5*d*
x + 5*c)/d - 1/16*(3*a^3*b + a*b^3)*cos(3*d*x + 3*c)/d - 1/16*(5*a^3*b + 3*
a*b^3)*cos(d*x + c)/d + 1/448*(a^4 - 6*a^2*b^2 + b^4)*sin(7*d*x + 7*c)/d +
1/320*(7*a^4 - 18*a^2*b^2 - b^4)*sin(5*d*x + 5*c)/d + 1/64*(7*a^4 - 2*a^2*b
```

```
^2 - b^4)*sin(3*d*x + 3*c)/d + 1/64*(35*a^4 + 30*a^2*b^2 + 3*b^4)*sin(d*x +
 c)/d
```

Mupad [B]
time = 1.28, size = 291, normalized size = 1.32

$$-\frac{\frac{b^4\sin(3c+3dx)}{64}-\frac{3b^4\sin(c+dx)}{64}-\frac{7a^4\sin(3c+3dx)}{64}-\frac{7a^4\sin(5c+5dx)}{320}-\frac{a^4\sin(7c+7dx)}{448}-\frac{35a^4\sin(c+dx)}{64}+\frac{b^4\sin(5c+5dx)}{320}-\frac{b^4\sin(7c+7dx)}{448}+\frac{ab^3\cos(3c+3dx)}{16}+\frac{3a^3b\cos(3c+3dx)}{16}-\frac{ab^3\cos(5c+5dx)}{80}+\frac{a^3b\cos(5c+5dx)}{16}-\frac{ab^3\cos(7c+7dx)}{112}+\frac{a^3b\cos(7c+7dx)}{112}-\frac{15a^2b^2\sin(c+dx)}{32}+\frac{a^2b^2\sin(3c+3dx)}{32}+\frac{9a^2b^2\sin(5c+5dx)}{160}+\frac{3a^2b^2\sin(7c+7dx)}{224}+\frac{3ab^3\cos(c+dx)}{16}+\frac{5a^3b\cos(c+dx)}{16}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

```
[Out] -((b^4*sin(3*c + 3*d*x))/64 - (3*b^4*sin(c + d*x))/64 - (7*a^4*sin(3*c + 3*
d*x))/64 - (7*a^4*sin(5*c + 5*d*x))/320 - (a^4*sin(7*c + 7*d*x))/448 - (35*
a^4*sin(c + d*x))/64 + (b^4*sin(5*c + 5*d*x))/320 - (b^4*sin(7*c + 7*d*x))/
448 + (a*b^3*cos(3*c + 3*d*x))/16 + (3*a^3*b*cos(3*c + 3*d*x))/16 - (a*b^3*
cos(5*c + 5*d*x))/80 + (a^3*b*cos(5*c + 5*d*x))/16 - (a*b^3*cos(7*c + 7*d*x
))/112 + (a^3*b*cos(7*c + 7*d*x))/112 - (15*a^2*b^2*sin(c + d*x))/32 + (a^2
*b^2*sin(3*c + 3*d*x))/32 + (9*a^2*b^2*sin(5*c + 5*d*x))/160 + (3*a^2*b^2*s
in(7*c + 7*d*x))/224 + (3*a*b^3*cos(c + d*x))/16 + (5*a^3*b*cos(c + d*x))/1
6)/d
```

3.77 $\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=301

$$\frac{5a^4x}{16}+\frac{3}{8}a^2b^2x+\frac{b^4x}{16}-\frac{2a^3b\cos^6(c+dx)}{3d}+\frac{5a^4\cos(c+dx)\sin(c+dx)}{16d}+\frac{3a^2b^2\cos(c+dx)\sin(c+dx)}{8d}+\frac{b^4\cos$$

```
[Out] 5/16*a^4*x+3/8*a^2*b^2*x+1/16*b^4*x-2/3*a^3*b*cos(d*x+c)^6/d+5/16*a^4*cos(d
*x+c)*sin(d*x+c)/d+3/8*a^2*b^2*cos(d*x+c)*sin(d*x+c)/d+1/16*b^4*cos(d*x+c)*
sin(d*x+c)/d+5/24*a^4*cos(d*x+c)^3*sin(d*x+c)/d+1/4*a^2*b^2*cos(d*x+c)^3*si
n(d*x+c)/d-1/8*b^4*cos(d*x+c)^3*sin(d*x+c)/d+1/6*a^4*cos(d*x+c)^5*sin(d*x+c
)/d-a^2*b^2*cos(d*x+c)^5*sin(d*x+c)/d-1/6*b^4*cos(d*x+c)^3*sin(d*x+c)^3/d+a
*b^3*sin(d*x+c)^4/d-2/3*a*b^3*sin(d*x+c)^6/d
```

Rubi [A]

time = 0.22, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3169, 2715, 8, 2645, 30, 2648, 2644, 14}

$$\frac{a^4\sin(c+dx)\cos^5(c+dx)}{6d}+\frac{5a^4\sin(c+dx)\cos^3(c+dx)}{24d}+\frac{5a^4\sin(c+dx)\cos(c+dx)}{16d}+\frac{5a^4x}{16}-\frac{2a^3b\cos^6(c+dx)}{3d}-\frac{a^2b^2\sin(c+dx)\cos^5(c+dx)}{d}+\frac{a^2b^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a^2b^2\sin(c+dx)\cos(c+dx)}{8d}+\frac{3}{8}a^2b^2x-\frac{2ab^3\sin^6(c+dx)}{3d}+\frac{ab^3\sin^4(c+dx)}{d}-\frac{b^4\sin^3(c+dx)\cos^3(c+dx)}{6d}-\frac{b^4\sin(c+dx)\cos^3(c+dx)}{8d}+\frac{b^4\sin(c+dx)\cos(c+dx)}{16d}+\frac{b^4x}{16}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (5*a^4*x)/16 + (3*a^2*b^2*x)/8 + (b^4*x)/16 - (2*a^3*b*Cos[c + d*x]^6)/(3*d
) + (5*a^4*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (3*a^2*b^2*Cos[c + d*x]*Sin[
c + d*x])/(8*d) + (b^4*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (5*a^4*Cos[c + d
*x]^3*Sin[c + d*x])/(24*d) + (a^2*b^2*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) -
(b^4*Cos[c + d*x]^3*Sin[c + d*x])/(8*d) + (a^4*Cos[c + d*x]^5*Sin[c + d*x])
/(6*d) - (a^2*b^2*Cos[c + d*x]^5*Sin[c + d*x])/d - (b^4*Cos[c + d*x]^3*Sin[
c + d*x]^3)/(6*d) + (a*b^3*Sin[c + d*x]^4)/d - (2*a*b^3*Sin[c + d*x]^6)/(3*
d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int \left(a^4\cos^6(c+dx)+4a^3b\cos^5(c+dx)\sin(c+dx)+6\right.$$
$$= a^4\int \cos^6(c+dx)\,dx+\left(4a^3b\right)\int \cos^5(c+dx)\sin(c+$$
$$= \frac{a^4\cos^5(c+dx)\sin(c+dx)}{6d}-\frac{a^2b^2\cos^5(c+dx)\sin(c+}{d}$$
$$= -\frac{2a^3b\cos^6(c+dx)}{3d}+\frac{5a^4\cos^3(c+dx)\sin(c+dx)}{24d}+$$
$$= -\frac{2a^3b\cos^6(c+dx)}{3d}+\frac{5a^4\cos(c+dx)\sin(c+dx)}{16d}+\frac{3}{}$$
$$= \frac{5a^4x}{16}+\frac{3}{8}a^2b^2x+\frac{b^4x}{16}-\frac{2a^3b\cos^6(c+dx)}{3d}+\frac{5a^4\cos(c}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.46, size = 178, normalized size = 0.59

$$\frac{12(a-ib)(a+ib)\left(5a^2+b^2\right)(c+dx)-12ab\left(5a^2+3b^2\right)\cos(2(c+dx))-24a^3b\cos(4(c+dx))-4ab\left(a^2-b^2\right)\cos(6(c+dx))+3\left(15a^4+6a^2b^2-b^4\right)\sin(2(c+dx))+3\left(3a^4-6a^2b^2-b^4\right)\sin(4(c+dx))+\left(a^4-6a^2b^2+b^4\right)\sin(6(c+dx))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (12*(a - I*b)*(a + I*b)*(5*a^2 + b^2)*(c + d*x) - 12*a*b*(5*a^2 + 3*b^2)*Co
s[2*(c + d*x)] - 24*a^3*b*Cos[4*(c + d*x)] - 4*a*b*(a^2 - b^2)*Cos[6*(c + d
*x)] + 3*(15*a^4 + 6*a^2*b^2 - b^4)*Sin[2*(c + d*x)] + 3*(3*a^4 - 6*a^2*b^2
 - b^4)*Sin[4*(c + d*x)] + (a^4 - 6*a^2*b^2 + b^4)*Sin[6*(c + d*x)])/(192*d
)
```

Maple [A]
time = 0.25, size = 219, normalized size = 0.73 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^4*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*sin(d*x+c)*cos(d*x+c)^3+1/16*c
os(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+4*a*b^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)
^4-1/12*cos(d*x+c)^4)+6*a^2*b^2*(-1/6*sin(d*x+c)*cos(d*x+c)^5+1/24*(cos(d*x
+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16*c)-2/3*a^3*b*cos(d*x+c)^6+a^
4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+
5/16*c))
```

Maxima [A]
time = 0.28, size = 170, normalized size = 0.56

$$-\frac{128\,a^3b\cos\left(dx+c\right)^6+\left(4\,\sin\left(2\,dx+2\,c\right)^3-60\,dx-60\,c-9\,\sin\left(4\,dx+4\,c\right)-48\,\sin\left(2\,dx+2\,c\right)\right)a^4-6\left(4\,\sin\left(2\,dx+2\,c\right)^3+12\,dx+12\,c-3\,\sin\left(4\,dx+4\,c\right)\right)a^2b^2+64\left(2\,\sin\left(dx+c\right)^6-3\,\sin\left(dx+c\right)^4\right)ab^3+\left(4\,\sin\left(2\,dx+2\,c\right)^3-12\,dx-12\,c+3\,\sin\left(4\,dx+4\,c\right)\right)b^4}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/192*(128*a^3*b*cos(d*x + c)^6 + (4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c -
9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^4 - 6*(4*sin(2*d*x + 2*c)^3 + 1
2*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^2*b^2 + 64*(2*sin(d*x + c)^6 - 3*sin(d
*x + c)^4)*a*b^3 + (4*sin(2*d*x + 2*c)^3 - 12*d*x - 12*c + 3*sin(4*d*x + 4*
c))*b^4)/d
```

Fricas [A]
time = 2.26, size = 151, normalized size = 0.50

$$-\frac{48\,ab^{3}\cos\left(dx+c\right)^{4}+32\,(a^{3}b-ab^{3})\cos\left(dx+c\right)^{6}-3\,(5\,a^{4}+6\,a^{2}b^{2}+b^{4})dx-\left(8\,(a^{4}-6\,a^{2}b^{2}+b^{4})\cos\left(dx+c\right)^{5}+2\,(5\,a^{4}+6\,a^{2}b^{2}-7\,b^{4})\cos\left(dx+c\right)^{3}+3\,(5\,a^{4}+6\,a^{2}b^{2}+b^{4})\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/48*(48*a*b^3*cos(d*x + c)^4 + 32*(a^3*b - a*b^3)*cos(d*x + c)^6 - 3*(5*a
^4 + 6*a^2*b^2 + b^4)*d*x - (8*(a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^5 + 2*(
5*a^4 + 6*a^2*b^2 - 7*b^4)*cos(d*x + c)^3 + 3*(5*a^4 + 6*a^2*b^2 + b^4)*cos
(d*x + c))*sin(d*x + c))/d
```

Sympy [A]
time = 0.48, size = 563, normalized size = 1.87

$$\begin{cases} \frac{5a^{4}x\sin^{6}(c+dx)}{16}+\frac{15a^{4}x\sin^{4}(c+dx)\cos^{2}(c+dx)}{16}+\frac{15a^{4}x\sin^{2}(c+dx)\cos^{4}(c+dx)}{16}+\frac{5a^{4}x\cos^{6}(c+dx)}{16}+\frac{5a^{4}\sin^{5}(c+dx)\cos(c+dx)}{16d}+\frac{5a^{4}\sin^{3}(c+dx)\cos^{3}(c+dx)}{6d}+\frac{11a^{4}\sin(c+dx)\cos^{5}(c+dx)}{16d}-\frac{2a^{3}b\cos^{6}(c+dx)}{3d}+\frac{3a^{2}b^{2}x\sin^{6}(c+dx)}{8}+\frac{9a^{2}b^{2}x\sin^{4}(c+dx)\cos^{2}(c+dx)}{8}+\frac{9a^{2}b^{2}x\sin^{2}(c+dx)\cos^{4}(c+dx)}{8}+\frac{3a^{2}b^{2}x\cos^{6}(c+dx)}{8}+\frac{3a^{2}b^{2}\sin^{5}(c+dx)\cos(c+dx)}{8d}+\frac{a^{2}b^{2}\sin^{3}(c+dx)\cos^{3}(c+dx)}{d}-\frac{3a^{2}b^{2}\sin(c+dx)\cos^{5}(c+dx)}{8d}-\frac{ab^{3}\sin^{2}(c+dx)\cos^{4}(c+dx)}{d}-\frac{ab^{3}\cos^{6}(c+dx)}{3d}+\frac{b^{4}x\sin^{6}(c+dx)}{16}+\frac{3b^{4}x\sin^{4}(c+dx)\cos^{2}(c+dx)}{16}+\frac{3b^{4}x\sin^{2}(c+dx)\cos^{4}(c+dx)}{16}+\frac{b^{4}x\cos^{6}(c+dx)}{16}+\frac{b^{4}\sin^{5}(c+dx)\cos(c+dx)}{16d}-\frac{b^{4}\sin^{3}(c+dx)\cos^{3}(c+dx)}{6d}-\frac{b^{4}\sin(c+dx)\cos^{5}(c+dx)}{16d} & \text{for}\ d\neq 0 \\ x\left(a\cos(c)+b\sin(c)\right)^{4}\cos^{2}(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((5*a**4*x*sin(c + d*x)**6/16 + 15*a**4*x*sin(c + d*x)**4*cos(c +
d*x)**2/16 + 15*a**4*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a**4*x*cos(c
+ d*x)**6/16 + 5*a**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a**4*sin(c +
d*x)**3*cos(c + d*x)**3/(6*d) + 11*a**4*sin(c + d*x)*cos(c + d*x)**5/(16*d)
 - 2*a**3*b*cos(c + d*x)**6/(3*d) + 3*a**2*b**2*x*sin(c + d*x)**6/8 + 9*a**
2*b**2*x*sin(c + d*x)**4*cos(c + d*x)**2/8 + 9*a**2*b**2*x*sin(c + d*x)**2*
cos(c + d*x)**4/8 + 3*a**2*b**2*x*cos(c + d*x)**6/8 + 3*a**2*b**2*sin(c + d
*x)**5*cos(c + d*x)/(8*d) + a**2*b**2*sin(c + d*x)**3*cos(c + d*x)**3/d - 3
*a**2*b**2*sin(c + d*x)*cos(c + d*x)**5/(8*d) - a*b**3*sin(c + d*x)**2*cos(
c + d*x)**4/d - a*b**3*cos(c + d*x)**6/(3*d) + b**4*x*sin(c + d*x)**6/16 +
3*b**4*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 3*b**4*x*sin(c + d*x)**2*cos(
c + d*x)**4/16 + b**4*x*cos(c + d*x)**6/16 + b**4*sin(c + d*x)**5*cos(c + d
*x)/(16*d) - b**4*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) - b**4*sin(c + d*x)
*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**4*cos(c)**2,
True))
```

Giac [A]
time = 0.52, size = 187, normalized size = 0.62

$$-\frac{a^{3} b \cos\left(4 \, d x + 4 \, c\right)}{8 \, d} + \frac{1}{16} \, {\left(5 \, a^{4} + 6 \, a^{2} b^{2} + b^{4}\right)} x - \frac{{\left(a^{3} b - a b^{3}\right)} \cos\left(6 \, d x + 6 \, c\right)}{48 \, d} - \frac{{\left(5 \, a^{3} b + 3 \, a b^{3}\right)} \cos\left(2 \, d x + 2 \, c\right)}{16 \, d} + \frac{{\left(a^{4} - 6 \, a^{2} b^{2} + b^{4}\right)} \sin\left(6 \, d x + 6 \, c\right)}{192 \, d} + \frac{{\left(3 \, a^{4} - 6 \, a^{2} b^{2} - b^{4}\right)} \sin\left(4 \, d x + 4 \, c\right)}{64 \, d} + \frac{{\left(15 \, a^{4} + 6 \, a^{2} b^{2} - b^{4}\right)} \sin\left(2 \, d x + 2 \, c\right)}{64 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/8*a^3*b*cos(4*d*x + 4*c)/d + 1/16*(5*a^4 + 6*a^2*b^2 + b^4)*x - 1/48*(a^
3*b - a*b^3)*cos(6*d*x + 6*c)/d - 1/16*(5*a^3*b + 3*a*b^3)*cos(2*d*x + 2*c)
/d + 1/192*(a^4 - 6*a^2*b^2 + b^4)*sin(6*d*x + 6*c)/d + 1/64*(3*a^4 - 6*a^2
*b^2 - b^4)*sin(4*d*x + 4*c)/d + 1/64*(15*a^4 + 6*a^2*b^2 - b^4)*sin(2*d*x
+ 2*c)/d
```

Mupad [B]
time = 2.31, size = 471, normalized size = 1.56

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\,\left(-\frac{11\,a^4}{8}+\frac{3\,a^2\,b^2}{4}+\frac{b^4}{8}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(\frac{15\,a^4}{4}-\frac{39\,a^2\,b^2}{2}+\frac{19\,b^4}{4}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(\frac{15\,a^4}{4}-\frac{39\,a^2\,b^2}{2}+\frac{19\,b^4}{4}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{5\,a^4}{24}-\frac{47\,a^2\,b^2}{4}+\frac{17\,b^4}{24}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,\left(\frac{5\,a^4}{24}-\frac{47\,a^2\,b^2}{4}+\frac{17\,b^4}{24}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(-\frac{11\,a^4}{8}+\frac{3\,a^2\,b^2}{4}+\frac{b^4}{8}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(\frac{32\,a\,b^3}{3}-\frac{80\,a^3\,b}{3}\right)+8\,a^3\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+16\,a\,b^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+16\,a\,b^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+8\,a^3\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+20\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)\,\left(5\,a^4+6\,a^2\,b^2+b^4\right)}{8\,d}+\frac{\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(5\,a^2+b^2\right)\,\left(a^2+b^2\right)}{8\,\left(\frac{5\,a^4}{8}+\frac{3\,a^2\,b^2}{4}+\frac{b^4}{8}\right)}\right)\,\left(5\,a^2+b^2\right)\,\left(a^2+b^2\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^11*(b^4/8 - (11*a^4)/8 + (3*a^2*b^2)/4) + tan(c/2 + (d*
x)/2)^5*((15*a^4)/4 + (19*b^4)/4 - (39*a^2*b^2)/2) - tan(c/2 + (d*x)/2)^7*(
(15*a^4)/4 + (19*b^4)/4 - (39*a^2*b^2)/2) - tan(c/2 + (d*x)/2)^3*((5*a^4)/2
4 + (17*b^4)/24 - (47*a^2*b^2)/4) + tan(c/2 + (d*x)/2)^9*((5*a^4)/24 + (17*
b^4)/24 - (47*a^2*b^2)/4) - tan(c/2 + (d*x)/2)*(b^4/8 - (11*a^4)/8 + (3*a^2
*b^2)/4) - tan(c/2 + (d*x)/2)^6*((32*a*b^3)/3 - (80*a^3*b)/3) + 8*a^3*b*tan
(c/2 + (d*x)/2)^2 + 16*a*b^3*tan(c/2 + (d*x)/2)^4 + 16*a*b^3*tan(c/2 + (d*x
)/2)^8 + 8*a^3*b*tan(c/2 + (d*x)/2)^10)/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan
(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*t
an(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) - ((atan(tan(c/2 + (d*x)
/2)) - (d*x)/2)*(5*a^4 + b^4 + 6*a^2*b^2))/(8*d) + (atan((tan(c/2 + (d*x)/2
)*(5*a^2 + b^2)*(a^2 + b^2))/(8*((5*a^4)/8 + b^4/8 + (3*a^2*b^2)/4)))*(5*a^
2 + b^2)*(a^2 + b^2))/(8*d)
```

3.78 $\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=165

$$-\frac{4ab^3\cos^3(c+dx)}{3d}-\frac{4a^3b\cos^5(c+dx)}{5d}+\frac{4ab^3\cos^5(c+dx)}{5d}+\frac{a^4\sin(c+dx)}{d}-\frac{2a^4\sin^3(c+dx)}{3d}+\frac{2a^2b^2\sin^3(c+}{d}$$

```
[Out] -4/3*a*b^3*cos(d*x+c)^3/d-4/5*a^3*b*cos(d*x+c)^5/d+4/5*a*b^3*cos(d*x+c)^5/d
+a^4*sin(d*x+c)/d-2/3*a^4*sin(d*x+c)^3/d+2*a^2*b^2*sin(d*x+c)^3/d+1/5*a^4*s
in(d*x+c)^5/d-6/5*a^2*b^2*sin(d*x+c)^5/d+1/5*b^4*sin(d*x+c)^5/d
```

Rubi [A]
time = 0.13, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3169, 2713, 2645, 30, 2644, 14}

$$\frac{a^4\sin^5(c+dx)}{5d}-\frac{2a^4\sin^3(c+dx)}{3d}+\frac{a^4\sin(c+dx)}{d}-\frac{4a^3b\cos^5(c+dx)}{5d}-\frac{6a^2b^2\sin^5(c+dx)}{5d}+\frac{2a^2b^2\sin^3(c+dx)}{d}+\frac{4ab^3\cos^5(c+dx)}{5d}-\frac{4ab^3\cos^3(c+dx)}{3d}+\frac{b^4\sin^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (-4*a*b^3*Cos[c + d*x]^3)/(3*d) - (4*a^3*b*Cos[c + d*x]^5)/(5*d) + (4*a*b^3
*Cos[c + d*x]^5)/(5*d) + (a^4*Sin[c + d*x])/d - (2*a^4*Sin[c + d*x]^3)/(3*d
) + (2*a^2*b^2*Sin[c + d*x]^3)/d + (a^4*Sin[c + d*x]^5)/(5*d) - (6*a^2*b^2*
Sin[c + d*x]^5)/(5*d) + (b^4*Sin[c + d*x]^5)/(5*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx &= \int \left(a^4\cos^5(c+dx)+4a^3b\cos^4(c+dx)\sin(c+dx)+6a\right.\\
&= a^4\int \cos^5(c+dx)\,dx+\left(4a^3b\right)\int \cos^4(c+dx)\sin(c+d\\
&= -\frac{a^4\mathrm{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,-\sin(c+dx)\right)}{d}-\frac{(4a}{}\\
&= -\frac{4a^3b\cos^5(c+dx)}{5d}+\frac{a^4\sin(c+dx)}{d}-\frac{2a^4\sin^3(c+dx)}{3d}\\
&= -\frac{4ab^3\cos^3(c+dx)}{3d}-\frac{4a^3b\cos^5(c+dx)}{5d}+\frac{4ab^3\cos^5(c-}{5d}
\end{aligned}$$

Mathematica [A]
time = 0.40, size = 146, normalized size = 0.88

$$\frac{-120ab(a^2+b^2)\cos(c+dx)-20ab(3a^2+b^2)\cos(3(c+dx))-12ab(a^2-b^2)\cos(5(c+dx))+30(5a^4+6a^2b^2+b^4)\sin(c+dx)+5(5a^4-6a^2b^2-3b^4)\sin(3(c+dx))+3(a^4-6a^2b^2+b^4)\sin(5(c+dx))}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (-120*a*b*(a^2 + b^2)*Cos[c + d*x] - 20*a*b*(3*a^2 + b^2)*Cos[3*(c + d*x)]
- 12*a*b*(a^2 - b^2)*Cos[5*(c + d*x)] + 30*(5*a^4 + 6*a^2*b^2 + b^4)*Sin[c
+ d*x] + 5*(5*a^4 - 6*a^2*b^2 - 3*b^4)*Sin[3*(c + d*x)] + 3*(a^4 - 6*a^2*b^
2 + b^4)*Sin[5*(c + d*x)])/(240*d)
```

Maple [A]
time = 0.20, size = 142, normalized size = 0.86

method	result
derivativedivides	$\frac{\frac{b^4\left(\sin^5(dx+c)\right)}{5}+4a\,b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+6a^2b^2\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin}{15}\right.}{d}$
default	$\frac{\frac{b^4\left(\sin^5(dx+c)\right)}{5}+4a\,b^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+6a^2b^2\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin}{15}\right.}{d}$
norman	$\frac{-\frac{24a^3b+16a\,b^3}{15d}+\frac{2a^4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^4\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{16a\,b^3\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{16a\,b^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{8a^3b\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan\right.}$
risch	$-\frac{a^3b\cos(dx+c)}{2d}-\frac{a\,b^3\cos(dx+c)}{2d}+\frac{5a^4\sin(dx+c)}{8d}+\frac{3a^2b^2\sin(dx+c)}{4d}+\frac{b^4\sin(dx+c)}{8d}-\frac{a^3b\cos(5dx+5c)}{20d}+a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*b^4*sin(d*x+c)^5+4*a*b^3*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(
d*x+c)^3)+6*a^2*b^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin
(d*x+c))-4/5*a^3*b*cos(d*x+c)^5+1/5*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)
*sin(d*x+c))
```

Maxima [A]
time = 0.27, size = 123, normalized size = 0.75

$$-\frac{12\,a^3b\cos\left(dx+c\right)^5-3\,b^4\sin\left(dx+c\right)^5-\left(3\,\sin\left(dx+c\right)^5-10\,\sin\left(dx+c\right)^3+15\,\sin\left(dx+c\right)\right)a^4+6\left(3\,\sin\left(dx+c\right)^5-5\,\sin\left(dx+c\right)^3\right)a^2b^2-4\left(3\,\cos\left(dx+c\right)^5-5\,\cos\left(dx+c\right)^3\right)ab^3}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/15*(12*a^3*b*cos(d*x + c)^5 - 3*b^4*sin(d*x + c)^5 - (3*sin(d*x + c)^5 -
 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^4 + 6*(3*sin(d*x + c)^5 - 5*sin(d*x
 + c)^3)*a^2*b^2 - 4*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a*b^3)/d
```

Fricas [A]
time = 2.40, size = 123, normalized size = 0.75

$$-\frac{20\,ab^3\cos\left(dx+c\right)^3+12\left(a^3b-ab^3\right)\cos\left(dx+c\right)^5-\left(3\left(a^4-6\,a^2b^2+b^4\right)\cos\left(dx+c\right)^4+8\,a^4+12\,a^2b^2+3\,b^4+2\left(2\,a^4+3\,a^2b^2-3\,b^4\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/15*(20*a*b^3*cos(d*x + c)^3 + 12*(a^3*b - a*b^3)*cos(d*x + c)^5 - (3*(a^
4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^4 + 8*a^4 + 12*a^2*b^2 + 3*b^4 + 2*(2*a^4
 + 3*a^2*b^2 - 3*b^4)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [A]

time = 0.29, size = 206, normalized size = 1.25

$$\begin{cases} \frac{8a^4\sin^5(c+dx)}{15d} + \frac{4a^4\sin^3(c+dx)\cos^2(c+dx)}{3d} + \frac{a^4\sin(c+dx)\cos^4(c+dx)}{d} - \frac{4a^3b\cos^5(c+dx)}{5d} + \frac{4a^2b^2\sin^5(c+dx)}{5d} + \frac{2a^2b^2\sin^3(c+dx)\cos^2(c+dx)}{d} - \frac{4ab^3\sin^2(c+dx)\cos^3(c+dx)}{3d} - \frac{8ab^3\cos^5(c+dx)}{15d} + \frac{b^4\sin^5(c+dx)}{5d} & \text{for } d \neq 0 \\ x(a\cos(c) + b\sin(c))^4\cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((8*a**4*sin(c + d*x)**5/(15*d) + 4*a**4*sin(c + d*x)**3*cos(c + d
*x)**2/(3*d) + a**4*sin(c + d*x)*cos(c + d*x)**4/d - 4*a**3*b*cos(c + d*x)*
*5/(5*d) + 4*a**2*b**2*sin(c + d*x)**5/(5*d) + 2*a**2*b**2*sin(c + d*x)**3*
cos(c + d*x)**2/d - 4*a*b**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 8*a*b*
*3*cos(c + d*x)**5/(15*d) + b**4*sin(c + d*x)**5/(5*d), Ne(d, 0)), (x*(a*co
s(c) + b*sin(c))**4*cos(c), True))
```

Giac [A]

time = 0.51, size = 165, normalized size = 1.00

$$-\frac{(a^3b - ab^3)\cos(5\,dx + 5\,c)}{20\,d} - \frac{(3\,a^3b + ab^3)\cos(3\,dx + 3\,c)}{12\,d} - \frac{(a^3b + ab^3)\cos(dx + c)}{2\,d} + \frac{(a^4 - 6\,a^2b^2 + b^4)\sin(5\,dx + 5\,c)}{80\,d} + \frac{(5\,a^4 - 6\,a^2b^2 - 3\,b^4)\sin(3\,dx + 3\,c)}{48\,d} + \frac{(5\,a^4 + 6\,a^2b^2 + b^4)\sin(dx + c)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/20*(a^3*b - a*b^3)*cos(5*d*x + 5*c)/d - 1/12*(3*a^3*b + a*b^3)*cos(3*d*x
 + 3*c)/d - 1/2*(a^3*b + a*b^3)*cos(d*x + c)/d + 1/80*(a^4 - 6*a^2*b^2 + b^
4)*sin(5*d*x + 5*c)/d + 1/48*(5*a^4 - 6*a^2*b^2 - 3*b^4)*sin(3*d*x + 3*c)/d
 + 1/8*(5*a^4 + 6*a^2*b^2 + b^4)*sin(d*x + c)/d
```

Mupad [B]

time = 0.83, size = 204, normalized size = 1.24

$$\frac{2\left(\frac{3\sin(c+d\,x)\,a^4\cos(c+d\,x)^4}{2} + 2\sin(c+d\,x)\,a^4\cos(c+d\,x)^2 + 4\sin(c+d\,x)\,a^4 - 6\,a^3\,b\cos(c+d\,x)^5 - 9\sin(c+d\,x)\,a^2\,b^2\cos(c+d\,x)^4 + 3\sin(c+d\,x)\,a^2\,b^2\cos(c+d\,x)^2 + 6\sin(c+d\,x)\,a^2\,b^2 + 6\,a\,b^3\cos(c+d\,x)^5 - 10\,a\,b^3\cos(c+d\,x)^3 + \frac{3\sin(c+d\,x)\,b^4\cos(c+d\,x)^4}{2} - 3\sin(c+d\,x)\,b^4\cos(c+d\,x)^2 + \frac{3\sin(c+d\,x)\,b^4}{2}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

```
[Out] (2*(4*a^4*sin(c + d*x) + (3*b^4*sin(c + d*x))/2 - 10*a*b^3*cos(c + d*x)^3 +
 6*a*b^3*cos(c + d*x)^5 - 6*a^3*b*cos(c + d*x)^5 + 2*a^4*cos(c + d*x)^2*sin
(c + d*x) + (3*a^4*cos(c + d*x)^4*sin(c + d*x))/2 + 6*a^2*b^2*sin(c + d*x)
- 3*b^4*cos(c + d*x)^2*sin(c + d*x) + (3*b^4*cos(c + d*x)^4*sin(c + d*x))/2
 + 3*a^2*b^2*cos(c + d*x)^2*sin(c + d*x) - 9*a^2*b^2*cos(c + d*x)^4*sin(c +
 d*x)))/(15*d)
```

3.79 $\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=108

$$\frac{3}{8}\left(a^2+b^2\right)^2 x-\frac{3(a^2+b^2)\left(b\cos(c+dx)-a\sin(c+dx)\right)(a\cos(c+dx)+b\sin(c+dx))}{8d}-\frac{(b\cos(c+dx)-a\sin(c+dx)}{}$$

```
[Out] 3/8*(a^2+b^2)^2*x-3/8*(a^2+b^2)*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b
*sin(d*x+c))/d-1/4*(b*cos(d*x+c)-a*sin(d*x+c))*(a*cos(d*x+c)+b*sin(d*x+c))^
3/d
```

Rubi [A]
time = 0.03, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3152, 8}

$$-\frac{3(a^2+b^2)\left(b\cos(c+dx)-a\sin(c+dx)\right)(a\cos(c+dx)+b\sin(c+dx))}{8d}+\frac{3}{8}x\left(a^2+b^2\right)^2-\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (3*(a^2 + b^2)^2*x)/8 - (3*(a^2 + b^2)*(b*Cos[c + d*x] - a*Sin[c + d*x])*(a
*Cos[c + d*x] + b*Sin[c + d*x]))/(8*d) - ((b*Cos[c + d*x] - a*Sin[c + d*x])
*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3152

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(-(b*Cos[c + d*x] - a*Sin[c + d*x]))*((a*Cos[c + d*x] + b*
Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[(n - 1)*((a^2 + b^2)/n), Int[(a*Cos
[c + d*x] + b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{a, b, c, d}, x] && Ne
Q[a^2 + b^2, 0] &&  !IntegerQ[(n - 1)/2] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}\int (a\cos(c+dx)+b\sin(c+dx))^4\,dx &= -\frac{(b\cos(c+dx)-a\sin(c+dx))(a\cos(c+dx)+b\sin(c+dx))^3}{4d}+\frac{1}{4}\\ &= -\frac{3(a^2+b^2)\left(b\cos(c+dx)-a\sin(c+dx)\right)(a\cos(c+dx)+b\sin(c+}{8d}\\ &= \frac{3}{8}\left(a^2+b^2\right)^2 x-\frac{3(a^2+b^2)\left(b\cos(c+dx)-a\sin(c+dx)\right)(a\cos(c+}{8d}\end{aligned}$$

Mathematica [A]
time = 0.41, size = 107, normalized size = 0.99

$$\frac{12(a^2+b^2)^2(c+dx)-16ab(a^2+b^2)\cos(2(c+dx))-4ab(a^2-b^2)\cos(4(c+dx))+8(a^4-b^4)\sin(2(c+dx))+(a^4-6a^2b^2+b^4)\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (12*(a^2 + b^2)^2*(c + d*x) - 16*a*b*(a^2 + b^2)*Cos[2*(c + d*x)] - 4*a*b*(
a^2 - b^2)*Cos[4*(c + d*x)] + 8*(a^4 - b^4)*Sin[2*(c + d*x)] + (a^4 - 6*a^2
*b^2 + b^4)*Sin[4*(c + d*x)])/(32*d)
```

Maple [A]
time = 0.18, size = 153, normalized size = 1.42

method	result
derivativedivides	$\frac{b^4\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a\,b^3\left(\sin^4(dx+c)\right)+6a^2b^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\,\mathrm{si}}{8}\right.}{d}$
default	$\frac{b^4\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a\,b^3\left(\sin^4(dx+c)\right)+6a^2b^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\,\mathrm{si}}{8}\right.}{d}$
risch	$\frac{3a^4x}{8}+\frac{3a^2b^2x}{4}+\frac{3b^4x}{8}-\frac{a^3b\cos(4dx+4c)}{8d}+\frac{a\,b^3\cos(4dx+4c)}{8d}+\frac{\sin(4dx+4c)a^4}{32d}-\frac{3\sin(4dx+4c)a^2b^2}{16d}+\mathrm{sin}$
norman	$\frac{(\frac{3}{8}a^4+\frac{3}{4}a^2b^2+\frac{3}{8}b^4)x+(\frac{3}{2}a^4+3a^2b^2+\frac{3}{2}b^4)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{3}{2}a^4+3a^2b^2+\frac{3}{2}b^4)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{3}{8}a^4+\frac{3}{4}a^2b^2+\frac{3}{8}b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^4*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+a*b^
3*sin(d*x+c)^4+6*a^2*b^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d
*x+c)+1/8*d*x+1/8*c)-a^3*b*cos(d*x+c)^4+a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+
c))*sin(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.27, size = 136, normalized size = 1.26

$$-\frac{a^3b\cos(dx+c)^4}{d}+\frac{ab^3\sin(dx+c)^4}{d}+\frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))a^4}{32\,d}+\frac{3\,(4\,dx+4\,c-\sin(4\,dx+4\,c))a^2b^2}{16\,d}+\frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)-8\,\sin(2\,dx+2\,c))b^4}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -a^3*b*cos(d*x + c)^4/d + a*b^3*sin(d*x + c)^4/d + 1/32*(12*d*x + 12*c + si
n(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^4/d + 3/16*(4*d*x + 4*c - sin(4*d*x
```

```
+ 4*c))*a^2*b^2/d + 1/32*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x +
2*c))*b^4/d
```

Fricas [A]
time = 3.33, size = 121, normalized size = 1.12

$$-\frac{16\,ab^3\cos\left(dx+c\right)^2+8\left(a^3b-ab^3\right)\cos\left(dx+c\right)^4-3\left(a^4+2\,a^2b^2+b^4\right)dx-\left(2\left(a^4-6\,a^2b^2+b^4\right)\cos\left(dx+c\right)^3+\left(3\,a^4+6\,a^2b^2-5\,b^4\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/8*(16*a*b^3*cos(d*x + c)^2 + 8*(a^3*b - a*b^3)*cos(d*x + c)^4 - 3*(a^4 +
 2*a^2*b^2 + b^4)*d*x - (2*(a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^3 + (3*a^4
+ 6*a^2*b^2 - 5*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(102) = 204$.
time = 0.21, size = 381, normalized size = 3.53

$$\begin{cases} \frac{3a^4x\sin^4(c+dx)}{8}+\frac{3a^4x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3a^4x\cos^4(c+dx)}{8}+\frac{3a^4\sin^3(c+dx)\cos(c+dx)}{8d}+\frac{5a^4\sin(c+dx)\cos^3(c+dx)}{8d}-\frac{a^3b\cos^4(c+dx)}{d}+\frac{3a^2b^2x\sin^4(c+dx)}{4}+\frac{3a^2b^2x\sin^2(c+dx)\cos^2(c+dx)}{2}+\frac{3a^2b^2x\cos^4(c+dx)}{4}+\frac{3a^2b^2\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{3a^2b^2\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{ab^3\sin^4(c+dx)}{d}+\frac{3b^4x\sin^4(c+dx)}{8}+\frac{3b^4x\sin^2(c+dx)\cos^2(c+dx)}{4}+\frac{3b^4x\cos^4(c+dx)}{8}-\frac{5b^4\sin^3(c+dx)\cos(c+dx)}{8d}-\frac{3b^4\sin(c+dx)\cos^3(c+dx)}{8d} & \text{for}\: d\neq 0 \\ x(a\cos(c)+b\sin(c))^4 & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Piecewise((3*a**4*x*sin(c + d*x)**4/8 + 3*a**4*x*sin(c + d*x)**2*cos(c + d*
x)**2/4 + 3*a**4*x*cos(c + d*x)**4/8 + 3*a**4*sin(c + d*x)**3*cos(c + d*x)/
(8*d) + 5*a**4*sin(c + d*x)*cos(c + d*x)**3/(8*d) - a**3*b*cos(c + d*x)**4/
d + 3*a**2*b**2*x*sin(c + d*x)**4/4 + 3*a**2*b**2*x*sin(c + d*x)**2*cos(c +
 d*x)**2/2 + 3*a**2*b**2*x*cos(c + d*x)**4/4 + 3*a**2*b**2*sin(c + d*x)**3*
cos(c + d*x)/(4*d) - 3*a**2*b**2*sin(c + d*x)*cos(c + d*x)**3/(4*d) + a*b**
3*sin(c + d*x)**4/d + 3*b**4*x*sin(c + d*x)**4/8 + 3*b**4*x*sin(c + d*x)**2
*cos(c + d*x)**2/4 + 3*b**4*x*cos(c + d*x)**4/8 - 5*b**4*sin(c + d*x)**3*co
s(c + d*x)/(8*d) - 3*b**4*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x
*(a*cos(c) + b*sin(c))**4, True))
```

Giac [A]
time = 0.45, size = 122, normalized size = 1.13

$$\frac{3}{8}\left(a^4+2\,a^2b^2+b^4\right)x-\frac{\left(a^3b-ab^3\right)\cos\left(4\,dx+4\,c\right)}{8\,d}-\frac{\left(a^3b+ab^3\right)\cos\left(2\,dx+2\,c\right)}{2\,d}+\frac{\left(a^4-6\,a^2b^2+b^4\right)\sin\left(4\,dx+4\,c\right)}{32\,d}+\frac{\left(a^4-b^4\right)\sin\left(2\,dx+2\,c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 3/8*(a^4 + 2*a^2*b^2 + b^4)*x - 1/8*(a^3*b - a*b^3)*cos(4*d*x + 4*c)/d - 1/
2*(a^3*b + a*b^3)*cos(2*d*x + 2*c)/d + 1/32*(a^4 - 6*a^2*b^2 + b^4)*sin(4*d
*x + 4*c)/d + 1/4*(a^4 - b^4)*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 1.96, size = 320, normalized size = 2.96

$$\frac{3\,\mathrm{atan}\left(\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,{\left(a^2+b^2\right)}^2}{4\,\left(\frac{3\,a^4}{4}+\frac{3\,a^2\,b^2}{2}+\frac{3\,b^4}{4}\right)}\right)\,{\left(a^2+b^2\right)}^2}{4\,d}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(-\frac{5\,a^4}{4}+\frac{3\,a^2\,b^2}{2}+\frac{3\,b^4}{4}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{3\,a^4}{4}-\frac{21\,a^2\,b^2}{2}+\frac{11\,b^4}{4}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(\frac{3\,a^4}{4}-\frac{21\,a^2\,b^2}{2}+\frac{11\,b^4}{4}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(-\frac{5\,a^4}{4}+\frac{3\,a^2\,b^2}{2}+\frac{3\,b^4}{4}\right)+8\,a^3\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+16\,a\,b^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+8\,a^3\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{3\,\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)\,{\left(a^2+b^2\right)}^2}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

```
[Out] (3*atan((3*tan(c/2 + (d*x)/2)*(a^2 + b^2)^2)/(4*((3*a^4)/4 + (3*b^4)/4 + (3
*a^2*b^2)/2)))*(a^2 + b^2)^2)/(4*d) + (tan(c/2 + (d*x)/2)^7*((3*b^4)/4 - (5
*a^4)/4 + (3*a^2*b^2)/2) - tan(c/2 + (d*x)/2)^3*((3*a^4)/4 + (11*b^4)/4 - (
21*a^2*b^2)/2) + tan(c/2 + (d*x)/2)^5*((3*a^4)/4 + (11*b^4)/4 - (21*a^2*b^2
)/2) - tan(c/2 + (d*x)/2)*((3*b^4)/4 - (5*a^4)/4 + (3*a^2*b^2)/2) + 8*a^3*b
*tan(c/2 + (d*x)/2)^2 + 16*a*b^3*tan(c/2 + (d*x)/2)^4 + 8*a^3*b*tan(c/2 + (
d*x)/2)^6)/(d*(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2
+ (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) - (3*(atan(tan(c/2 + (d*x)/2)) -
(d*x)/2)*(a^2 + b^2)^2)/(4*d)
```

3.80 $\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=150

$$\frac{b^4\tanh^{-1}(\sin(c+dx))}{d}-\frac{4ab^3\cos(c+dx)}{d}-\frac{4a^3b\cos^3(c+dx)}{3d}+\frac{4ab^3\cos^3(c+dx)}{3d}+\frac{a^4\sin(c+dx)}{d}-\frac{b^4\sin(c+}{d}$$

```
[Out] b^4*arctanh(sin(d*x+c))/d-4*a*b^3*cos(d*x+c)/d-4/3*a^3*b*cos(d*x+c)^3/d+4/3
*a*b^3*cos(d*x+c)^3/d+a^4*sin(d*x+c)/d-b^4*sin(d*x+c)/d-1/3*a^4*sin(d*x+c)^
3/d+2*a^2*b^2*sin(d*x+c)^3/d-1/3*b^4*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.11, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3169, 2713, 2645, 30, 2644, 2672, 308, 212}

$$-\frac{a^4\sin^3(c+dx)}{3d}+\frac{a^4\sin(c+dx)}{d}-\frac{4a^3b\cos^3(c+dx)}{3d}+\frac{2a^2b^2\sin^3(c+dx)}{d}+\frac{4ab^3\cos^3(c+dx)}{3d}-\frac{4ab^3\cos(c+dx)}{d}-\frac{b^4\sin^3(c+dx)}{3d}-\frac{b^4\sin(c+dx)}{d}+\frac{b^4\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (b^4*ArcTanh[Sin[c + d*x]])/d - (4*a*b^3*Cos[c + d*x])/d - (4*a^3*b*Cos[c +
 d*x]^3)/(3*d) + (4*a*b^3*Cos[c + d*x]^3)/(3*d) + (a^4*Sin[c + d*x])/d - (b
^4*Sin[c + d*x])/d - (a^4*Sin[c + d*x]^3)/(3*d) + (2*a^2*b^2*Sin[c + d*x]^3
)/d - (b^4*Sin[c + d*x]^3)/(3*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
```

```
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int \left(a^4\cos^3(c+dx)+4a^3b\cos^2(c+dx)\sin(c+dx)+6a^2\right.$$

$$= a^4\int \cos^3(c+dx)\,dx+\left(4a^3b\right)\int \cos^2(c+dx)\sin(c+dx)$$

$$= -\frac{a^4\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(c+dx)\right)}{d}-\frac{\left(4a^3b\right)\text{Subs}}{}$$

$$= -\frac{4ab^3\cos(c+dx)}{d}-\frac{4a^3b\cos^3(c+dx)}{3d}+\frac{4ab^3\cos^3(c+d}{3d}$$

$$= -\frac{4ab^3\cos(c+dx)}{d}-\frac{4a^3b\cos^3(c+dx)}{3d}+\frac{4ab^3\cos^3(c+d}{3d}$$

$$= \frac{b^4\tanh^{-1}(\sin(c+dx))}{d}-\frac{4ab^3\cos(c+dx)}{d}-\frac{4a^3b\cos^3(c}{3d}$$

Mathematica [A]

time = 1.03, size = 181, normalized size = 1.21

$$\frac{-12ab(a^2+3b^2)\cos(c+dx)+(-4a^3b+4ab^3)\cos(3(c+dx))-12b^4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12b^4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+9a^4\sin(c+dx)+18a^2b^2\sin(c+dx)-15b^4\sin(c+dx)+a^4\sin(3(c+dx))-6a^2b^2\sin(3(c+dx))+b^4\sin(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (-12*a*b*(a^2 + 3*b^2)*Cos[c + d*x] + (-4*a^3*b + 4*a*b^3)*Cos[3*(c + d*x)]
 - 12*b^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*b^4*Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]] + 9*a^4*Sin[c + d*x] + 18*a^2*b^2*Sin[c + d*x] -
15*b^4*Sin[c + d*x] + a^4*Sin[3*(c + d*x)] - 6*a^2*b^2*Sin[3*(c + d*x)] + b
^4*Sin[3*(c + d*x)])/(12*d)
```

Maple [A]

time = 0.28, size = 116, normalized size = 0.77

method	result
derivativedivides	$\frac{\frac{a^4\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}-\frac{4a^3\left(\cos^3(dx+c)\right)b}{3}+2a^2b^2\left(\sin^3(dx+c)\right)-\frac{4a\,b^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+b^4\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\right.}{d}$
default	$\frac{\frac{a^4\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}-\frac{4a^3\left(\cos^3(dx+c)\right)b}{3}+2a^2b^2\left(\sin^3(dx+c)\right)-\frac{4a\,b^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+b^4\left(-\frac{\left(\sin^3(dx+c)\right)}{3}-\right.}{d}$
norman	$\frac{-\frac{8a^3b+16a\,b^3}{3d}+\frac{2\left(a^4-b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2\left(a^4-b^4\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{8a^3b\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{2\left(5a^4+24a^2b^2-13b^4\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{3d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$
risch	$-\frac{e^{i(dx+c)}a^3b}{2d}-\frac{3\,e^{i(dx+c)}a\,b^3}{2d}+\frac{3i e^{-i(dx+c)}a^4}{8d}-\frac{5i e^{-i(dx+c)}b^4}{8d}-\frac{3i e^{i(dx+c)}a^2b^2}{4d}-\frac{e^{-i(dx+c)}a^3b}{2d}-\frac{3\,e^{-i(dx-}}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*a^4*(2+cos(d*x+c)^2)*sin(d*x+c)-4/3*a^3*cos(d*x+c)^3*b+2*a^2*b^2*s
in(d*x+c)^3-4/3*a*b^3*(2+sin(d*x+c)^2)*cos(d*x+c)+b^4*(-1/3*sin(d*x+c)^3-si
n(d*x+c)+ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.27, size = 126, normalized size = 0.84

$$-\frac{8\,a^{3}b\cos\left(dx+c\right)^{3}-12\,a^{2}b^{2}\sin\left(dx+c\right)^{3}+2\,\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)a^{4}-8\,\left(\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)\right)ab^{3}+\left(2\,\sin\left(dx+c\right)^{3}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)+6\,\sin\left(dx+c\right)\right)b^{4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/6*(8*a^3*b*cos(d*x + c)^3 - 12*a^2*b^2*sin(d*x + c)^3 + 2*(sin(d*x + c)^
3 - 3*sin(d*x + c))*a^4 - 8*(cos(d*x + c)^3 - 3*cos(d*x + c))*a*b^3 + (2*si
n(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d*
x + c))*b^4)/d
```

Fricas [A]
time = 3.77, size = 121, normalized size = 0.81

$$-\frac{24\,ab^{3}\cos\left(dx+c\right)-3\,b^{4}\log\left(\sin\left(dx+c\right)+1\right)+3\,b^{4}\log\left(-\sin\left(dx+c\right)+1\right)+8\,\left(a^{3}b-ab^{3}\right)\cos\left(dx+c\right)^{3}-2\,\left(2\,a^{4}+6\,a^{2}b^{2}-4\,b^{4}+\left(a^{4}-6\,a^{2}b^{2}+b^{4}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/6*(24*a*b^3*cos(d*x + c) - 3*b^4*log(sin(d*x + c) + 1) + 3*b^4*log(-sin(
d*x + c) + 1) + 8*(a^3*b - a*b^3)*cos(d*x + c)^3 - 2*(2*a^4 + 6*a^2*b^2 - 4
*b^4 + (a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)^{4}\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Integral((a*cos(c + d*x) + b*sin(c + d*x))**4*sec(c + d*x), x)
```

Giac [A]
time = 0.51, size = 217, normalized size = 1.45

$$\frac{3\,b^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,b^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-12\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+2\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-10\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,a^3b-8\,ab^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*b^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*b^4*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + 2*(3*a^4*tan(1/2*d*x + 1/2*c)^5 - 3*b^4*tan(1/2*d*x + 1/2*c
)^5 - 12*a^3*b*tan(1/2*d*x + 1/2*c)^4 + 2*a^4*tan(1/2*d*x + 1/2*c)^3 + 24*a
^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 10*b^4*tan(1/2*d*x + 1/2*c)^3 - 24*a*b^3*ta
n(1/2*d*x + 1/2*c)^2 + 3*a^4*tan(1/2*d*x + 1/2*c) - 3*b^4*tan(1/2*d*x + 1/2
*c) - 4*a^3*b - 8*a*b^3)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B]
time = 2.75, size = 190, normalized size = 1.27

$$\frac{2\,b^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{\frac{16\,a\,b^3}{3}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(2\,a^4-2\,b^4\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{4\,a^4}{3}+16\,a^2\,b^2-\frac{20\,b^4}{3}\right)+\frac{8\,a^3\,b}{3}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^4-2\,b^4\right)+16\,a\,b^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+8\,a^3\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x),x)
```

```
[Out] (2*b^4*atanh(tan(c/2 + (d*x)/2)))/d - ((16*a*b^3)/3 - tan(c/2 + (d*x)/2)^5*
(2*a^4 - 2*b^4) - tan(c/2 + (d*x)/2)^3*((4*a^4)/3 - (20*b^4)/3 + 16*a^2*b^2
) + (8*a^3*b)/3 - tan(c/2 + (d*x)/2)*(2*a^4 - 2*b^4) + 16*a*b^3*tan(c/2 + (
d*x)/2)^2 + 8*a^3*b*tan(c/2 + (d*x)/2)^4)/(d*(3*tan(c/2 + (d*x)/2)^2 + 3*ta
n(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 + 1))
```

3.81 $\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=119

$$\frac{1}{2}\left(a^4+6a^2b^2-3b^4\right)x-\frac{4ab^3\log(\sin(c+dx))}{d}+\frac{4ab^3\log(\tan(c+dx))}{d}+\frac{(4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\,\mathrm{co}}{2d}$$

```
[Out] 1/2*(a^4+6*a^2*b^2-3*b^4)*x-4*a*b^3*ln(sin(d*x+c))/d+4*a*b^3*ln(tan(d*x+c))
/d+1/2*(4*a*b*(a^2-b^2)+(a^4-6*a^2*b^2+b^4)*cot(d*x+c))*sin(d*x+c)^2/d+b^4*
tan(d*x+c)/d
```

Rubi [A]
time = 0.13, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3167, 1819, 1816, 649, 209, 266}

$$\frac{\sin^2(c+dx)\left(4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\cot(c+dx)\right)}{2d}+\frac{1}{2}x\left(a^4+6a^2b^2-3b^4\right)-\frac{4ab^3\log(\sin(c+dx))}{d}+\frac{4ab^3\log(\tan(c+dx))}{d}+\frac{b^4\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] ((a^4 + 6*a^2*b^2 - 3*b^4)*x)/2 - (4*a*b^3*Log[Sin[c + d*x]])/d + (4*a*b^3*
Log[Tan[c + d*x]])/d + ((4*a*b*(a^2 - b^2) + (a^4 - 6*a^2*b^2 + b^4)*Cot[c
+ d*x])*Sin[c + d*x]^2)/(2*d) + (b^4*Tan[c + d*x])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^4}{x^2(1+x^2)^2}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= \frac{(4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\cot(c+dx))\sin^2(c+ \cdots}{2d}$$

$$= \frac{(4ab(a^2-b^2)+(a^4-6a^2b^2+b^4)\cot(c+dx))\sin^2(c+ \cdots}{2d}$$

$$= \frac{4ab^3\log(\tan(c+dx))}{d} + \frac{(4ab(a^2-b^2)+(a^4-6a^2b^2+ \cdots}{2d}$$

$$= \frac{4ab^3\log(\tan(c+dx))}{d} + \frac{(4ab(a^2-b^2)+(a^4-6a^2b^2+ \cdots}{2d}$$

$$= \frac{1}{2}\left(a^4+6a^2b^2-3b^4\right)x - \frac{4ab^3\log(\sin(c+dx))}{d} + \frac{4ab^3\log \cdots}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 477 vs. 2(119) = 238.

time = 6.29, size = 477, normalized size = 4.01

$$\frac{b^3\left(\frac{\cos^2(c+dx)(a+b\tan(c+dx))^5(b^2+ab\tan(c+dx))}{2b^4(a^2+b^2)} - \frac{(-5a^2+3b^2)\left(\frac{1}{2}\left(4a(a-b)(a+b)+\frac{a^4-6a^2b^2+b^4}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+\frac{1}{2}\left(4a(a-b)(a+b)-\frac{a^4-6a^2b^2+b^4}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)+b(6a^2-b^2)\tan(c+dx)+2ab^2\tan^2(c+dx)+\frac{1}{3}b^3\tan^3(c+dx)\right)+4a\left(\frac{1}{2}\left(5a^4-10a^2b^2+b^4+\frac{a^5-10a^3b^2+5ab^4}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+\frac{1}{2}\left(5a^4-10a^2b^2+b^4-\frac{a^5-10a^3b^2+5ab^4}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)+5ab(2a^2-b^2)\tan(c+dx)+\frac{1}{2}b^2(10a^2-b^2)\tan^2(c+dx)+\frac{5}{3}ab^3\tan^3(c+dx)+\frac{1}{4}b^4\tan^4(c+dx)\right)}{2b^2(a^2+b^2)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (b^3*((Cos[c + d*x]^2*(a + b*Tan[c + d*x])^5*(b^2 + a*b*Tan[c + d*x]))/(2*b
^4*(a^2 + b^2)) - ((-5*a^2 + 3*b^2)*(((4*a*(a - b)*(a + b) + (a^4 - 6*a^2*b
^2 + b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/2 + ((4*a*(a - b)*(
a + b) - (a^4 - 6*a^2*b^2 + b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x
]])/2 + b*(6*a^2 - b^2)*Tan[c + d*x] + 2*a*b^2*Tan[c + d*x]^2 + (b^3*Tan[c
+ d*x]^3)/3) + 4*a*(((5*a^4 - 10*a^2*b^2 + b^4 + (a^5 - 10*a^3*b^2 + 5*a*b^
4)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/2 + ((5*a^4 - 10*a^2*b^2 +
 b^4 - (a^5 - 10*a^3*b^2 + 5*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c +
d*x]])/2 + 5*a*b*(2*a^2 - b^2)*Tan[c + d*x] + (b^2*(10*a^2 - b^2)*Tan[c + d
*x]^2)/2 + (5*a*b^3*Tan[c + d*x]^3)/3 + (b^4*Tan[c + d*x]^4)/4))/(2*b^2*(a^
2 + b^2))))/d
```

Maple [A]

time = 0.30, size = 155, normalized size = 1.30

method	result
derivativedivides	$\frac{a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-2a^3b\left(\cos^2(dx+c)\right)+6a^2b^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4ab^3\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln\right.}{d}$
default	$\frac{a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)-2a^3b\left(\cos^2(dx+c)\right)+6a^2b^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4ab^3\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln\right.}{d}$
risch	$\frac{i e^{-2i(dx+c)}b^4}{8d}+\frac{a^4x}{2}+3a^2b^2x-\frac{3b^4x}{2}-\frac{e^{2i(dx+c)}a^3b}{2d}+\frac{e^{2i(dx+c)}a\,b^3}{2d}-\frac{i e^{2i(dx+c)}b^4}{8d}+\frac{2ib^4}{d\left(e^{2i(dx+c)}+1\right)}$
norman	$\frac{\left(-\frac{1}{2}a^4-3a^2b^2+\frac{3}{2}b^4\right)x+\left(-\frac{3}{2}a^4-9a^2b^2+\frac{9}{2}b^4\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{1}{2}a^4+3a^2b^2-\frac{3}{2}b^4\right)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{3}{2}a^4+9a^2b^2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-2*a^3*b*cos(d*x+c)^2+6*a
^2*b^2*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4*a*b^3*(-1/2*sin(d*x+c)^
2-ln(cos(d*x+c)))+b^4*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*sin(d*x+c)
)*cos(d*x+c)-3/2*d*x-3/2*c))
```

Maxima [A]

time = 0.50, size = 135, normalized size = 1.13

$$\frac{8\,a^3b\sin\left(dx+c\right)^2+\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^4+6\left(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right)\right)a^2b^2-8\left(\sin\left(dx+c\right)^2+\log\left(\sin\left(dx+c\right)^2-1\right)\right)ab^3-2\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan\left(dx+c\right)\right)b^4}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/4*(8*a^3*b*sin(d*x + c)^2 + (2*d*x + 2*c + sin(2*d*x + 2*c))*a^4 + 6*(2*d
*x + 2*c - sin(2*d*x + 2*c))*a^2*b^2 - 8*(sin(d*x + c)^2 + log(sin(d*x + c)
^2 - 1))*a*b^3 - 2*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan
(d*x + c))*b^4)/d
```

Fricas [A]
time = 2.65, size = 136, normalized size = 1.14

$$-\frac{8\,ab^{3}\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)+4\,\left(a^{3}b-ab^{3}\right)\cos\left(dx+c\right)^{3}-\left(2\,a^{3}b-2\,ab^{3}+\left(a^{4}+6\,a^{2}b^{2}-3\,b^{4}\right)dx\right)\cos\left(dx+c\right)-\left(2\,b^{4}+\left(a^{4}-6\,a^{2}b^{2}+b^{4}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/2*(8*a*b^3*cos(d*x + c)*log(-cos(d*x + c)) + 4*(a^3*b - a*b^3)*cos(d*x +
 c)^3 - (2*a^3*b - 2*a*b^3 + (a^4 + 6*a^2*b^2 - 3*b^4)*d*x)*cos(d*x + c) -
(2*b^4 + (a^4 - 6*a^2*b^2 + b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x +
 c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.51, size = 128, normalized size = 1.08

$$\frac{4\,ab^{3}\log\left(\tan\left(dx+c\right)^{2}+1\right)+2\,b^{4}\tan\left(dx+c\right)+\left(a^{4}+6\,a^{2}b^{2}-3\,b^{4}\right)\left(dx+c\right)-\frac{4\,ab^{3}\tan\left(dx+c\right)^{2}-a^{4}\tan\left(dx+c\right)+6\,a^{2}b^{2}\tan\left(dx+c\right)-b^{4}\tan\left(dx+c\right)+4\,a^{3}b}{\tan\left(dx+c\right)^{2}+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2*(4*a*b^3*log(tan(d*x + c)^2 + 1) + 2*b^4*tan(d*x + c) + (a^4 + 6*a^2*b^
2 - 3*b^4)*(d*x + c) - (4*a*b^3*tan(d*x + c)^2 - a^4*tan(d*x + c) + 6*a^2*b
^2*tan(d*x + c) - b^4*tan(d*x + c) + 4*a^3*b)/(tan(d*x + c)^2 + 1))/d
```

Mupad [B]
time = 1.23, size = 255, normalized size = 2.14

$$\frac{a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-3\,b^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+4\,a\,b^3\,\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)+6\,a^2\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-4\,a\,b^3\,\ln\left(\frac{\cos\left(c+d\,x\right)}{\cos\left(c+d\,x\right)+1}\right)}{d}+\frac{\frac{a^4\,\sin\left(c+d\,x\right)}{8}+\frac{9\,b^4\,\sin\left(c+d\,x\right)}{8}+\frac{a^4\,\sin\left(3\,c+3\,d\,x\right)}{8}+\frac{b^4\,\sin\left(3\,c+3\,d\,x\right)}{8}+\frac{a\,b^3\,\cos\left(3\,c+3\,d\,x\right)}{2}-\frac{a^3\,b\,\cos\left(3\,c+3\,d\,x\right)}{2}-\frac{3\,a^2\,b^2\,\sin\left(c+d\,x\right)}{4}-\frac{3\,a^2\,b^2\,\sin\left(3\,c+3\,d\,x\right)}{4}}{d\,\cos\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^2,x)
```

```
[Out] (a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - 3*b^4*atan(sin(c/2 + (d*
x)/2)/cos(c/2 + (d*x)/2)) + 4*a*b^3*log(1/cos(c/2 + (d*x)/2)^2) + 6*a^2*b^2
*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - 4*a*b^3*log(cos(c + d*x)/(co
s(c + d*x) + 1)))/d + ((a^4*sin(c + d*x))/8 + (9*b^4*sin(c + d*x))/8 + (a^4
*sin(3*c + 3*d*x))/8 + (b^4*sin(3*c + 3*d*x))/8 + (a*b^3*cos(3*c + 3*d*x))/
2 - (a^3*b*cos(3*c + 3*d*x))/2 - (3*a^2*b^2*sin(c + d*x))/4 - (3*a^2*b^2*si
n(3*c + 3*d*x))/4)/(d*cos(c + d*x))
```

3.82 $\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=151

$$\frac{6a^2b^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{3b^4\tanh^{-1}(\sin(c+dx))}{2d}-\frac{4a^3b\cos(c+dx)}{d}+\frac{4ab^3\cos(c+dx)}{d}+\frac{4ab^3\sec(c+dx)}{d}+$$

```
[Out] 6*a^2*b^2*arctanh(sin(d*x+c))/d-3/2*b^4*arctanh(sin(d*x+c))/d-4*a^3*b*cos(d
*x+c)/d+4*a*b^3*cos(d*x+c)/d+4*a*b^3*sec(d*x+c)/d+a^4*sin(d*x+c)/d-6*a^2*b^
2*sin(d*x+c)/d+3/2*b^4*sin(d*x+c)/d+1/2*b^4*sin(d*x+c)*tan(d*x+c)^2/d
```

Rubi [A]
time = 0.11, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {3169, 2717, 2718, 2672, 327, 212, 2670, 14, 294}

$$\frac{a^4\sin(c+dx)}{d}-\frac{4a^3b\cos(c+dx)}{d}-\frac{6a^2b^2\sin(c+dx)}{d}+\frac{6a^2b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{4ab^3\cos(c+dx)}{d}+\frac{4ab^3\sec(c+dx)}{d}+\frac{3b^4\sin(c+dx)}{2d}+\frac{b^4\sin(c+dx)\tan^2(c+dx)}{2d}-\frac{3b^4\tanh^{-1}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (6*a^2*b^2*ArcTanh[Sin[c + d*x]])/d - (3*b^4*ArcTanh[Sin[c + d*x]])/(2*d) -
 (4*a^3*b*Cos[c + d*x])/d + (4*a*b^3*Cos[c + d*x])/d + (4*a*b^3*Sec[c + d*x
])/d + (a^4*Sin[c + d*x])/d - (6*a^2*b^2*Sin[c + d*x])/d + (3*b^4*Sin[c + d
*x])/(2*d) + (b^4*Sin[c + d*x]*Tan[c + d*x]^2)/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int \left(a^4\cos(c+dx)+4a^3b\sin(c+dx)+6a^2b^2\sin(c+dx\right.$$

$$= a^4\int \cos(c+dx)\,dx + \left(4a^3b\right)\int \sin(c+dx)\,dx + \left(6a^2b^2\right.$$

$$= -\frac{4a^3b\cos(c+dx)}{d} + \frac{a^4\sin(c+dx)}{d} + \frac{(6a^2b^2)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{4a^3b\cos(c+dx)}{d} + \frac{a^4\sin(c+dx)}{d} - \frac{6a^2b^2\sin(c+dx)}{d}$$

$$= \frac{6a^2b^2\tanh^{-1}(\sin(c+dx))}{d} - \frac{4a^3b\cos(c+dx)}{d} + \frac{4ab^3\,\mathrm{co}}{}$$

$$= \frac{6a^2b^2\tanh^{-1}(\sin(c+dx))}{d} - \frac{3b^4\tanh^{-1}(\sin(c+dx))}{2d} -$$

Mathematica [A]
time = 2.39, size = 268, normalized size = 1.77

$$\frac{16ab^3-16ab(a^2-b^2)\cos(c+dx)-24a^2b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+6b^4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+24a^2b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-6b^4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{b^4}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+32ab^3\sec(c+dx)\sin^2\left(\frac{1}{2}(c+dx)\right)-\frac{b^4}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+4a^4\sin(c+dx)-24a^2b^2\sin(c+dx)+4b^4\sin(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (16*a*b^3 - 16*a*b*(a^2 - b^2)*Cos[c + d*x] - 24*a^2*b^2*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]] + 6*b^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2
4*a^2*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 6*b^4*Log[Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]] + b^4/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + 32
*a*b^3*Sec[c + d*x]*Sin[(c + d*x)/2]^2 - b^4/(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^2 + 4*a^4*Sin[c + d*x] - 24*a^2*b^2*Sin[c + d*x] + 4*b^4*Sin[c + d*
x])/(4*d)
```

Maple [A]
time = 0.36, size = 157, normalized size = 1.04

method	result
derivativedivides	$\frac{\sin(dx+c)a^4-4a^3b\cos(dx+c)+6a^2b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+4a\,b^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos\right.}{d}$
default	$\frac{\sin(dx+c)a^4-4a^3b\cos(dx+c)+6a^2b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+4a\,b^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos\right.}{d}$
risch	$-\frac{2\,e^{i(dx+c)}a^3b}{d}+\frac{2\,e^{i(dx+c)}a\,b^3}{d}-\frac{ie^{i(dx+c)}a^4}{2d}+\frac{3ie^{i(dx+c)}a^2b^2}{d}-\frac{ie^{i(dx+c)}b^4}{2d}-\frac{2\,e^{-i(dx+c)}a^3b}{d}+\frac{2\,e^{-i(dx+c)}}{d}$

norman	$\frac{\frac{8a^3b+16a\,b^3}{d}+\frac{16a^3b\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(2a^4-12a^2b^2+3b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{\left(2a^4-12a^2b^2+3b^4\right)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\left(2a^4-12a^2b^2}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(sin(d*x+c)*a^4-4*a^3*b*cos(d*x+c)+6*a^2*b^2*(-sin(d*x+c)+ln(sec(d*x+c)
+tan(d*x+c)))+4*a*b^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))
+b^4*(1/2*sin(d*x+c)^5/cos(d*x+c)^2+1/2*sin(d*x+c)^3+3/2*sin(d*x+c)-3/2*ln(
sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.28, size = 142, normalized size = 0.94

$$-\frac{b^4\left(\frac{2\,\sin\left(dx+c\right)}{\sin\left(dx+c\right)^2-1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)-4\,\sin\left(dx+c\right)\right)-16\,ab^3\left(\frac{1}{\cos\left(dx+c\right)}+\cos\left(dx+c\right)\right)-12\,a^2b^2\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)-2\,\sin\left(dx+c\right)\right)+16\,a^3b\cos\left(dx+c\right)-4\,a^4\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/4*(b^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) + 1) -
3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) - 16*a*b^3*(1/cos(d*x + c) + cos(
d*x + c)) - 12*a^2*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*s
in(d*x + c)) + 16*a^3*b*cos(d*x + c) - 4*a^4*sin(d*x + c))/d
```

Fricas [A]
time = 2.66, size = 153, normalized size = 1.01

$$\frac{16\,ab^3\cos\left(dx+c\right)-16\,\left(a^3b-ab^3\right)\cos\left(dx+c\right)^3+3\,\left(4\,a^2b^2-b^4\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-3\,\left(4\,a^2b^2-b^4\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(b^4+2\,\left(a^4-6\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/4*(16*a*b^3*cos(d*x + c) - 16*(a^3*b - a*b^3)*cos(d*x + c)^3 + 3*(4*a^2*b
^2 - b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - 3*(4*a^2*b^2 - b^4)*cos(d*
x + c)^2*log(-sin(d*x + c) + 1) + 2*(b^4 + 2*(a^4 - 6*a^2*b^2 + b^4)*cos(d*
x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.55, size = 206, normalized size = 1.36

$$\frac{3\,(4\,a^2b^2-b^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(4\,a^2b^2-b^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{4\,\left(a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-6\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,a^3b+4\,ab^3\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+\frac{2\left(b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-8\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,ab^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2*(3*(4*a^2*b^2 - b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*a^2*b^2
- b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*(a^4*tan(1/2*d*x + 1/2*c) - 6
*a^2*b^2*tan(1/2*d*x + 1/2*c) + b^4*tan(1/2*d*x + 1/2*c) - 4*a^3*b + 4*a*b^
3)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(b^4*tan(1/2*d*x + 1/2*c)^3 - 8*a*b^3*t
an(1/2*d*x + 1/2*c)^2 + b^4*tan(1/2*d*x + 1/2*c) + 8*a*b^3)/(tan(1/2*d*x +
1/2*c)^2 - 1)^2)/d
```

Mupad [B]
time = 2.96, size = 221, normalized size = 1.46

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(4\,a^4-24\,a^2\,b^2+2\,b^4)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(2\,a^4-12\,a^2\,b^2+3\,b^4)-16\,a\,b^3+8\,a^3\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(2\,a^4-12\,a^2\,b^2+3\,b^4)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(16\,a\,b^3-16\,a^3\,b)+8\,a^3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}-\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,(3\,b^4-12\,a^2\,b^2)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(4*a^4 + 2*b^4 - 24*a^2*b^2) - tan(c/2 + (d*x)/2)^5*(
2*a^4 + 3*b^4 - 12*a^2*b^2) - 16*a*b^3 + 8*a^3*b - tan(c/2 + (d*x)/2)*(2*a^
4 + 3*b^4 - 12*a^2*b^2) + tan(c/2 + (d*x)/2)^2*(16*a*b^3 - 16*a^3*b) + 8*a^
3*b*tan(c/2 + (d*x)/2)^4)/(d*(tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 -
 tan(c/2 + (d*x)/2)^6 - 1)) - (atanh(tan(c/2 + (d*x)/2))*(3*b^4 - 12*a^2*b^
2))/d
```

3.83 $\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=103

$$\left(a^4-6a^2b^2+b^4\right)x-\frac{4ab(a^2-b^2)\log(\cos(c+dx))}{d}+\frac{b^2(3a^2-b^2)\tan(c+dx)}{d}+\frac{ab(a+b\tan(c+dx))^2}{d}+\frac{b(a$$

```
[Out] (a^4-6*a^2*b^2+b^4)*x-4*a*b*(a^2-b^2)*ln(cos(d*x+c))/d+b^2*(3*a^2-b^2)*tan(
d*x+c)/d+a*b*(a+b*tan(d*x+c))^2/d+1/3*b*(a+b*tan(d*x+c))^3/d
```

Rubi [A]
time = 0.11, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3165, 3563, 3609, 3606, 3556}

$$\frac{b^2(3a^2-b^2)\tan(c+dx)}{d}-\frac{4ab(a^2-b^2)\log(\cos(c+dx))}{d}+x\left(a^4-6a^2b^2+b^4\right)+\frac{b(a+b\tan(c+dx))^3}{3d}+\frac{ab(a+b\tan(c+dx))^2}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (a^4 - 6*a^2*b^2 + b^4)*x - (4*a*b*(a^2 - b^2)*Log[Cos[c + d*x]])/d + (b^2*
(3*a^2 - b^2)*Tan[c + d*x])/d + (a*b*(a + b*Tan[c + d*x])^2)/d + (b*(a + b*
Tan[c + d*x])^3)/(3*d)
```

Rule 3165

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
```

```
f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx &= \int (a+b\tan(c+dx))^4\,dx\\ &= \frac{b(a+b\tan(c+dx))^3}{3d}+\int (a+b\tan(c+dx))^2\left(a^2-b^2\right.\\ &= \frac{ab(a+b\tan(c+dx))^2}{d}+\frac{b(a+b\tan(c+dx))^3}{3d}+\int (a-\\ &= \left(a^4-6a^2b^2+b^4\right)x+\frac{b^2(3a^2-b^2)\tan(c+dx)}{d}+\frac{ab(a+}{}\\ &= \left(a^4-6a^2b^2+b^4\right)x-\frac{4ab(a^2-b^2)\log(\cos(c+dx))}{d}+\frac{b}{}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.45, size = 105, normalized size = 1.02

$$\frac{-3i(a+ib)^4\log(i-\tan(c+dx))+3i(a-ib)^4\log(i+\tan(c+dx))-6b^2(-6a^2+b^2)\tan(c+dx)+12ab^3\tan^2(c+dx)+2b^4\tan^3(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
[Out] ((-3*I)*(a + I*b)^4*Log[I - Tan[c + d*x]] + (3*I)*(a - I*b)^4*Log[I + Tan[c
 + d*x]] - 6*b^2*(-6*a^2 + b^2)*Tan[c + d*x] + 12*a*b^3*Tan[c + d*x]^2 + 2*
b^4*Tan[c + d*x]^3)/(6*d)
```

Maple [A]
time = 0.36, size = 101, normalized size = 0.98

method	result

derivativedivides	$\frac{a^4(dx+c)-4a^3b\ln(\cos(dx+c))+6a^2b^2(\tan(dx+c)-dx-c)+4a\,b^3\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)+b^4\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\right.}{d}$
default	$\frac{a^4(dx+c)-4a^3b\ln(\cos(dx+c))+6a^2b^2(\tan(dx+c)-dx-c)+4a\,b^3\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)+b^4\left(\frac{\left(\tan^3(dx+c)\right)}{3}-\right.}{d}$
risch	$4ia^3bx-4ixa\,b^3+a^4x-6a^2b^2x+b^4x+\frac{8ia^3bc}{d}-\frac{8ia\,b^3c}{d}-\frac{4ib^2\left(-9a^2e^{4i(dx+c)}+3b^2e^{4i(dx+c)}+6ia\right.}{}$
norman	$\frac{\frac{8a\,b^3}{d}+\left(-a^4+6a^2b^2-b^4\right)x+\frac{40a\,b^3\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\left(-3a^4+18a^2b^2-3b^4\right)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-3a^4+18a^2b^2-3b^4\right)x\left(\tan\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(d*x+c)-4*a^3*b*ln(cos(d*x+c))+6*a^2*b^2*(tan(d*x+c)-d*x-c)+4*a*b^
3*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))+b^4*(1/3*tan(d*x+c)^3-tan(d*x+c)+d*x+c)
)
```

Maxima [A]
time = 0.50, size = 116, normalized size = 1.13

$$\frac{3\,(dx+c)a^4-18\,(dx+c-\tan\,(dx+c))a^2b^2+\left(\tan\,(dx+c)^3+3\,dx+3\,c-3\,\tan\,(dx+c)\right)b^4-6\,ab^3\left(\frac{1}{\sin(dx+c)^2-1}-\log\left(\sin\,(dx+c)^2-1\right)\right)-6\,a^3b\log\left(-\sin\,(dx+c)^2+1\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(d*x + c)*a^4 - 18*(d*x + c - tan(d*x + c))*a^2*b^2 + (tan(d*x + c)^
3 + 3*d*x + 3*c - 3*tan(d*x + c))*b^4 - 6*a*b^3*(1/(sin(d*x + c)^2 - 1) - l
og(sin(d*x + c)^2 - 1)) - 6*a^3*b*log(-sin(d*x + c)^2 + 1))/d
```

Fricas [A]
time = 2.35, size = 119, normalized size = 1.16

$$\frac{3\,(a^4-6\,a^2b^2+b^4)dx\cos\,(dx+c)^3+6\,ab^3\cos\,(dx+c)-12\,(a^3b-ab^3)\cos\,(dx+c)^3\log\,(-\cos\,(dx+c))+\left(b^4+2\,(9\,a^2b^2-2\,b^4)\cos\,(dx+c)^2\right)\sin\,(dx+c)}{3\,d\cos\,(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*d*x*cos(d*x + c)^3 + 6*a*b^3*cos(d*x + c) -
12*(a^3*b - a*b^3)*cos(d*x + c)^3*log(-cos(d*x + c)) + (b^4 + 2*(9*a^2*b^2
- 2*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.52, size = 104, normalized size = 1.01

$$\frac{b^4\tan\left(dx+c\right)^3+6\,ab^3\tan\left(dx+c\right)^2+18\,a^2b^2\tan\left(dx+c\right)-3\,b^4\tan\left(dx+c\right)+3\,(a^4-6\,a^2b^2+b^4)(dx+c)+6\,(a^3b-ab^3)\log\left(\tan\left(dx+c\right)^2+1\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(b^4*tan(d*x + c)^3 + 6*a*b^3*tan(d*x + c)^2 + 18*a^2*b^2*tan(d*x + c)
- 3*b^4*tan(d*x + c) + 3*(a^4 - 6*a^2*b^2 + b^4)*(d*x + c) + 6*(a^3*b - a*b
^3)*log(tan(d*x + c)^2 + 1))/d
```

Mupad [B]
time = 1.79, size = 546, normalized size = 5.30

$$\frac{\frac{3\,a^4\cos(c+d\,x)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}-\frac{b^4\sin(3\,c+3\,d\,x)}{3}+\frac{3\,b^4\cos(c+d\,x)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}-\frac{a\,b^3\cos(3\,c+3\,d\,x)}{2}+\frac{3\,a^2\,b^2\sin(c+d\,x)}{2}+\frac{a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos(3\,c+3\,d\,x)}{2}+\frac{b^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos(3\,c+3\,d\,x)}{2}+\frac{3\,a^2\,b^2\sin(3\,c+3\,d\,x)}{2}+\frac{a\,b^3\cos(c+d\,x)}{2}+3\,a\,b^3\ln\left(-\frac{\cos(c+d\,x)}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)\cos\left(c+d\,x\right)-3\,a^3\,b\ln\left(-\frac{\cos(c+d\,x)}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)\cos\left(c+d\,x\right)-3\,a^2\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos\left(3\,c+3\,d\,x\right)-3\,a\,b^3\cos\left(c+d\,x\right)\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)+3\,a^3\,b\cos\left(c+d\,x\right)\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)+a\,b^3\ln\left(-\frac{\cos(c+d\,x)}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)\cos\left(3\,c+3\,d\,x\right)-a^3\,b\ln\left(-\frac{\cos(c+d\,x)}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)\cos\left(3\,c+3\,d\,x\right)-a\,b^3\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)\cos\left(3\,c+3\,d\,x\right)+a^3\,b\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)\cos\left(3\,c+3\,d\,x\right)-9\,a^2\,b^2\cos\left(c+d\,x\right)\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d\left(\frac{3\cos(c+d\,x)}{4}+\frac{\cos(3\,c+3\,d\,x)}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^4,x)
```

```
[Out] ((3*a^4*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 - (b^4*
sin(3*c + 3*d*x))/3 + (3*b^4*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 +
 (d*x)/2)))/2 - (a*b^3*cos(3*c + 3*d*x))/2 + (3*a^2*b^2*sin(c + d*x))/2 + (
a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/2 + (b^4*
atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/2 + (3*a^2*b^
2*sin(3*c + 3*d*x))/2 + (a*b^3*cos(c + d*x))/2 + 3*a*b^3*log(-cos(c + d*x)/
cos(c/2 + (d*x)/2)^2)*cos(c + d*x) - 3*a^3*b*log(-cos(c + d*x)/cos(c/2 + (d
*x)/2)^2)*cos(c + d*x) - 3*a^2*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/
2))*cos(3*c + 3*d*x) - 3*a*b^3*cos(c + d*x)*log(1/cos(c/2 + (d*x)/2)^2) + 3
*a^3*b*cos(c + d*x)*log(1/cos(c/2 + (d*x)/2)^2) + a*b^3*log(-cos(c + d*x)/c
os(c/2 + (d*x)/2)^2)*cos(3*c + 3*d*x) - a^3*b*log(-cos(c + d*x)/cos(c/2 + (
d*x)/2)^2)*cos(3*c + 3*d*x) - a*b^3*log(1/cos(c/2 + (d*x)/2)^2)*cos(3*c + 3
*d*x) + a^3*b*log(1/cos(c/2 + (d*x)/2)^2)*cos(3*c + 3*d*x) - 9*a^2*b^2*cos(
c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(d*((3*cos(c + d*x))/
4 + cos(3*c + 3*d*x)/4))
```

3.84 $\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=168

$$\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}-\frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{3b^4\tanh^{-1}(\sin(c+dx))}{8d}+\frac{4a^3b\sec(c+dx)}{d}-\frac{4ab^3\sec(c}{d}$$

```
[Out] a^4*arctanh(sin(d*x+c))/d-3*a^2*b^2*arctanh(sin(d*x+c))/d+3/8*b^4*arctanh(s
in(d*x+c))/d+4*a^3*b*sec(d*x+c)/d-4*a*b^3*sec(d*x+c)/d+4/3*a*b^3*sec(d*x+c)
^3/d+3*a^2*b^2*sec(d*x+c)*tan(d*x+c)/d-3/8*b^4*sec(d*x+c)*tan(d*x+c)/d+1/4*
b^4*sec(d*x+c)*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.13, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3169, 3855, 2686, 8, 2691}

$$\frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{4a^3b\sec(c+dx)}{d}-\frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{3a^2b^2\tan(c+dx)\sec(c+dx)}{d}+\frac{4ab^3\sec^3(c+dx)}{3d}-\frac{4ab^3\sec(c+dx)}{d}+\frac{3b^4\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b^4\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{3b^4\tan(c+dx)\sec(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*ArcTanh[Sin[c + d*x]])/d - (3*a^2*b^2*ArcTanh[Sin[c + d*x]])/d + (3*b^
4*ArcTanh[Sin[c + d*x]])/(8*d) + (4*a^3*b*Sec[c + d*x])/d - (4*a*b^3*Sec[c
+ d*x])/d + (4*a*b^3*Sec[c + d*x]^3)/(3*d) + (3*a^2*b^2*Sec[c + d*x]*Tan[c
+ d*x])/d - (3*b^4*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b^4*Sec[c + d*x]*Tan
[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx &= \int \left(a^4\sec(c+dx)+4a^3b\sec(c+dx)\tan(c+dx)+6a^2b\right.\\
&= a^4\int \sec(c+dx)\,dx+\left(4a^3b\right)\int \sec(c+dx)\tan(c+dx)\\
&= \frac{a^4\tanh^{-1}(\sin(c+dx))}{d}+\frac{3a^2b^2\sec(c+dx)\tan(c+dx)}{d}\\
&= \frac{a^4\tanh^{-1}(\sin(c+dx))}{d}-\frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{d}+\\
&= \frac{a^4\tanh^{-1}(\sin(c+dx))}{d}-\frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{d}+
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 936 vs. 2(168) = 336.
time = 6.28, size = 936, normalized size = 5.57

$$\frac{2ab(6a^2-5b^2)\cos^4(c+dx)(a+b\tan(c+dx))^4}{3d(a\cos(c+dx)+b\sin(c+dx))^4}+\frac{(-8a^4+24a^2b^2-3b^4)\cos^4(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{8d(a\cos(c+dx)+b\sin(c+dx))^4}+\frac{(8a^4-24a^2b^2+3b^4)\cos^4(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{8d(a\cos(c+dx)+b\sin(c+dx))^4}+\frac{b^4\cos^4(c+dx)(a+b\tan(c+dx))^4}{16d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^4(a\cos(c+dx)+b\sin(c+dx))^4}+\frac{(72a^2b^2+16ab^3-15b^4)\cos^4(c+dx)(a+b\tan(c+dx))^4}{48d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^4}+\frac{2ab^3\cos^4(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(a\cos(c+dx)+b\sin(c+dx))^4}-\frac{b^4\cos^4(c+dx)(a+b\tan(c+dx))^4}{16d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^4(a\cos(c+dx)+b\sin(c+dx))^4}-\frac{2ab^3\cos^4(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^4}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3(a\cos(c+dx)+b\sin(c+dx))^4}+\frac{(-72a^2b^2+16ab^3+15b^4)\cos^4(c+dx)(a+b\tan(c+dx))^4}{48d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2(a\cos(c+dx)+b\sin(c+dx))^4}+\frac{2\cos^4(c+dx)\left(6a^3b\sin\left(\frac{1}{2}(c+dx)\right)-5ab^3\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^4}-\frac{2\cos^4(c+dx)\left(6a^3b\sin\left(\frac{1}{2}(c+dx)\right)-5ab^3\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^4}{3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (2*a*b*(6*a^2 - 5*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(3*d*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^4) + ((-8*a^4 + 24*a^2*b^2 - 3*b^4)*Cos[c + d*x]^
4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(8*d*(a*
Cos[c + d*x] + b*Sin[c + d*x])^4) + ((8*a^4 - 24*a^2*b^2 + 3*b^4)*Cos[c + d
*x]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(8*d
*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (b^4*Cos[c + d*x]^4*(a + b*Tan[c +
d*x])^4)/(16*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4*(a*Cos[c + d*x] + b*
Sin[c + d*x])^4) + ((72*a^2*b^2 + 16*a*b^3 - 15*b^4)*Cos[c + d*x]^4*(a + b*
```

```
Tan[c + d*x])^4)/(48*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d
*x] + b*Sin[c + d*x])^4) + (2*a*b^3*Cos[c + d*x]^4*Sin[(c + d*x)/2]*(a + b*
Tan[c + d*x])^4)/(3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(a*Cos[c + d*
x] + b*Sin[c + d*x])^4) - (b^4*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(16*d
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^
4) - (2*a*b^3*Cos[c + d*x]^4*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^4)/(3*d*
(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4
) + ((-72*a^2*b^2 + 16*a*b^3 + 15*b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^
4)/(48*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c
+ d*x])^4) + (2*Cos[c + d*x]^4*(6*a^3*b*Sin[(c + d*x)/2] - 5*a*b^3*Sin[(c +
 d*x)/2])*(a + b*Tan[c + d*x])^4)/(3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
)*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (2*Cos[c + d*x]^4*(6*a^3*b*Sin[(c
+ d*x)/2] - 5*a*b^3*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(3*d*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
```

Maple [A]
time = 0.40, size = 225, normalized size = 1.34 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*ln(sec(d*x+c)+tan(d*x+c))+4*a^3*b/cos(d*x+c)+6*a^2*b^2*(1/2*sin(d*
x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))+4*a*b^3*(
1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+c)
^2)*cos(d*x+c))+b^4*(1/4*sin(d*x+c)^5/cos(d*x+c)^4-1/8*sin(d*x+c)^5/cos(d*x
+c)^2-1/8*sin(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.28, size = 192, normalized size = 1.14

$$\frac{3\,b^4\left(\frac{2\left(5\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-72\,a^2b^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)+24\,a^4\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{192\,a^3b}{\cos(dx+c)}-\frac{64\left(3\,\cos(dx+c)^2-1\right)ab^3}{\cos(dx+c)^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/48*(3*b^4*(2*(5*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(
d*x + c)^2 + 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)) - 72*a
^2*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(s
in(d*x + c) - 1)) + 24*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ 192*a^3*b/cos(d*x + c) - 64*(3*cos(d*x + c)^2 - 1)*a*b^3/cos(d*x + c)^3)/
d
```

Fricas [A]
time = 2.53, size = 163, normalized size = 0.97

$$\frac{3\left(8\,a^4-24\,a^2b^2+3\,b^4\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(8\,a^4-24\,a^2b^2+3\,b^4\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+64\,ab^3\cos\left(dx+c\right)+192\left(a^3b-ab^3\right)\cos\left(dx+c\right)^3+6\left(2\,b^4+\left(24\,a^2b^2-5\,b^4\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) -
 3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 64*
a*b^3*cos(d*x + c) + 192*(a^3*b - a*b^3)*cos(d*x + c)^3 + 6*(2*b^4 + (24*a^
2*b^2 - 5*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. 2(160) = 320.
time = 0.55, size = 325, normalized size = 1.93

$$\frac{3\left(8a^4-24a^2b^2+3b^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3\left(8a^4-24a^2b^2+3b^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(72a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-9b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-96a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-72a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+33b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+288a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-192ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-72a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+33b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-288a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+256ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+72a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-9b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+96a^3b-64ab^3\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^4}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/24*(3*(8*a^4 - 24*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3
*(8*a^4 - 24*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(72*a^
2*b^2*tan(1/2*d*x + 1/2*c)^7 - 9*b^4*tan(1/2*d*x + 1/2*c)^7 - 96*a^3*b*tan(
1/2*d*x + 1/2*c)^6 - 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 33*b^4*tan(1/2*d*x
 + 1/2*c)^5 + 288*a^3*b*tan(1/2*d*x + 1/2*c)^4 - 192*a*b^3*tan(1/2*d*x + 1/
2*c)^4 - 72*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 33*b^4*tan(1/2*d*x + 1/2*c)^3
- 288*a^3*b*tan(1/2*d*x + 1/2*c)^2 + 256*a*b^3*tan(1/2*d*x + 1/2*c)^2 + 72*
a^2*b^2*tan(1/2*d*x + 1/2*c) - 9*b^4*tan(1/2*d*x + 1/2*c) + 96*a^3*b - 64*a
*b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B]
time = 4.21, size = 278, normalized size = 1.65

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a^4-6\,a^2\,b^2+\frac{3\,b^4}{4}\right)}{d}-\frac{\frac{16\,a\,b^3}{3}-8\,a^3\,b+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{3\,b^4}{4}-6\,a^2\,b^2\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(\frac{3\,b^4}{4}-6\,a^2\,b^2\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{11\,b^4}{4}-6\,a^2\,b^2\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(\frac{11\,b^4}{4}-6\,a^2\,b^2\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(16\,a\,b^3-24\,a^3\,b\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{64\,a\,b^3}{3}-24\,a^3\,b\right)+8\,a^3\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^5,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(2*a^4 + (3*b^4)/4 - 6*a^2*b^2))/d - ((16*a*b^3)
/3 - 8*a^3*b + tan(c/2 + (d*x)/2)*((3*b^4)/4 - 6*a^2*b^2) + tan(c/2 + (d*x)
/2)^7*((3*b^4)/4 - 6*a^2*b^2) - tan(c/2 + (d*x)/2)^3*((11*b^4)/4 - 6*a^2*b^
2) - tan(c/2 + (d*x)/2)^5*((11*b^4)/4 - 6*a^2*b^2) + tan(c/2 + (d*x)/2)^4*(
16*a*b^3 - 24*a^3*b) - tan(c/2 + (d*x)/2)^2*((64*a*b^3)/3 - 24*a^3*b) + 8*a
^3*b*tan(c/2 + (d*x)/2)^6)/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2
)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.85 $\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=30

$$\frac{(b+a\cot(c+dx))^5\tan^5(c+dx)}{5bd}$$

```
[Out] 1/5*(b+a*cot(d*x+c))^5*tan(d*x+c)^5/b/d
```

Rubi [A]
time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3167, 37}

$$\frac{\tan^5(c+dx)(a\cot(c+dx)+b)^5}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] ((b + a*Cot[c + d*x])^5*Tan[c + d*x]^5)/(5*b*d)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx &= -\frac{\text{Subst}\left(\int \frac{(b+ax)^4}{x^6}\,dx,x,\cot(c+dx)\right)}{d}\\ &= \frac{(b+a\cot(c+dx))^5\tan^5(c+dx)}{5bd}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 131 vs. 2(30) = 60.
time = 1.43, size = 131, normalized size = 4.37

$$\frac{(a+b\tan(c+dx))^4\left(10ab(a^2-b^2)\cos^2(c+dx)+(5a^4-10a^2b^2+b^4)\cos^3(c+dx)\sin(c+dx)+b^2((5a^2-b^2)\sin(2(c+dx))+b(5a+b\tan(c+dx)))\right)}{5d(a\cos(c+dx)+b\sin(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] ((a + b*Tan[c + d*x])^4*(10*a*b*(a^2 - b^2)*Cos[c + d*x]^2 + (5*a^4 - 10*a^
2*b^2 + b^4)*Cos[c + d*x]^3*Sin[c + d*x] + b^2*((5*a^2 - b^2)*Sin[2*(c + d*
x)] + b*(5*a + b*Tan[c + d*x]))))/(5*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(28) = 56.
time = 0.30, size = 96, normalized size = 3.20

method	result
derivativedivides	$\frac{a^4\tan(dx+c)+\frac{2a^3b}{\cos(dx+c)^2}+\frac{2a^2b^2\left(\sin^3(dx+c)\right)}{\cos(dx+c)^3}+\frac{a\,b^3\left(\sin^4(dx+c)\right)}{\cos(dx+c)^4}+\frac{b^4\left(\sin^5(dx+c)\right)}{5\cos(dx+c)^5}}{d}$
default	$\frac{a^4\tan(dx+c)+\frac{2a^3b}{\cos(dx+c)^2}+\frac{2a^2b^2\left(\sin^3(dx+c)\right)}{\cos(dx+c)^3}+\frac{a\,b^3\left(\sin^4(dx+c)\right)}{\cos(dx+c)^4}+\frac{b^4\left(\sin^5(dx+c)\right)}{5\cos(dx+c)^5}}{d}$
risch	$2i\left(-60ia^3b\,\mathrm{e}^{4i(dx+c)}+20ia\,b^3\mathrm{e}^{8i(dx+c)}+5a^4\mathrm{e}^{8i(dx+c)}-30a^2b^2\mathrm{e}^{8i(dx+c)}+5b^4\mathrm{e}^{8i(dx+c)}+20ia\,b^3\mathrm{e}^{6i(dx+c)}-60ia^3b\,\mathrm{e}^{6i(dx\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*tan(d*x+c)+2*a^3*b/cos(d*x+c)^2+2*a^2*b^2*sin(d*x+c)^3/cos(d*x+c)^
3+a*b^3*sin(d*x+c)^4/cos(d*x+c)^4+1/5*b^4*sin(d*x+c)^5/cos(d*x+c)^5)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(28) = 56.
time = 0.28, size = 103, normalized size = 3.43

$$\frac{b^4\tan(dx+c)^5+10\,a^2b^2\tan(dx+c)^3+5\,a^4\tan(dx+c)+\frac{5\left(2\,\sin(dx+c)^2-1\right)ab^3}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-\frac{10\,a^3b}{\sin(dx+c)^2-1}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/5*(b^4*tan(d*x + c)^5 + 10*a^2*b^2*tan(d*x + c)^3 + 5*a^4*tan(d*x + c) +
5*(2*sin(d*x + c)^2 - 1)*a*b^3/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 10
*a^3*b/(sin(d*x + c)^2 - 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(28) = 56$.
time = 2.40, size = 109, normalized size = 3.63

$$\frac{5\,ab^3\cos\left(dx+c\right)+10\left(a^3b-ab^3\right)\cos\left(dx+c\right)^3+\left(\left(5\,a^4-10\,a^2b^2+b^4\right)\cos\left(dx+c\right)^4+b^4+2\left(5\,a^2b^2-b^4\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{5\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/5*(5*a*b^3*cos(d*x + c) + 10*(a^3*b - a*b^3)*cos(d*x + c)^3 + ((5*a^4 - 1
0*a^2*b^2 + b^4)*cos(d*x + c)^4 + b^4 + 2*(5*a^2*b^2 - b^4)*cos(d*x + c)^2)
*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(28) = 56$.
time = 0.54, size = 73, normalized size = 2.43

$$\frac{b^4\tan\left(dx+c\right)^5+5\,ab^3\tan\left(dx+c\right)^4+10\,a^2b^2\tan\left(dx+c\right)^3+10\,a^3b\tan\left(dx+c\right)^2+5\,a^4\tan\left(dx+c\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/5*(b^4*tan(d*x + c)^5 + 5*a*b^3*tan(d*x + c)^4 + 10*a^2*b^2*tan(d*x + c)^
3 + 10*a^3*b*tan(d*x + c)^2 + 5*a^4*tan(d*x + c))/d
```

Mupad [B]
time = 0.80, size = 139, normalized size = 4.63

$$\frac{\frac{b^4\,\sin(c+d\,x)}{5}-\cos\left(c+d\,x\right)^3\left(2\,a\,b^3-2\,a^3\,b\right)-\cos\left(c+d\,x\right)^2\left(\frac{2\,b^4\,\sin(c+d\,x)}{5}-2\,a^2\,b^2\,\sin\left(c+d\,x\right)\right)+\cos\left(c+d\,x\right)^4\left(\sin\left(c+d\,x\right)\,a^4-2\,\sin\left(c+d\,x\right)\,a^2\,b^2+\frac{\sin(c+d\,x)\,b^4}{5}\right)+a\,b^3\,\cos\left(c+d\,x\right)}{d\cos\left(c+d\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^6,x)
```

```
[Out] ((b^4*sin(c + d*x))/5 - cos(c + d*x)^3*(2*a*b^3 - 2*a^3*b) - cos(c + d*x)^2
*((2*b^4*sin(c + d*x))/5 - 2*a^2*b^2*sin(c + d*x)) + cos(c + d*x)^4*(a^4*si
n(c + d*x) + (b^4*sin(c + d*x))/5 - 2*a^2*b^2*sin(c + d*x)) + a*b^3*cos(c +
 d*x))/(d*cos(c + d*x)^5)
```

3.86 $\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=258

$$\frac{a^4\tanh^{-1}(\sin(c+dx))}{2d}-\frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{b^4\tanh^{-1}(\sin(c+dx))}{16d}+\frac{4a^3b\sec^3(c+dx)}{3d}-\frac{4ab^3\sec^3(}{3d}$$

```
[Out] 1/2*a^4*arctanh(sin(d*x+c))/d-3/4*a^2*b^2*arctanh(sin(d*x+c))/d+1/16*b^4*ar
ctanh(sin(d*x+c))/d+4/3*a^3*b*sec(d*x+c)^3/d-4/3*a*b^3*sec(d*x+c)^3/d+4/5*a
*b^3*sec(d*x+c)^5/d+1/2*a^4*sec(d*x+c)*tan(d*x+c)/d-3/4*a^2*b^2*sec(d*x+c)*
tan(d*x+c)/d+1/16*b^4*sec(d*x+c)*tan(d*x+c)/d+3/2*a^2*b^2*sec(d*x+c)^3*tan(
d*x+c)/d-1/8*b^4*sec(d*x+c)^3*tan(d*x+c)/d+1/6*b^4*sec(d*x+c)^3*tan(d*x+c)^
3/d
```

Rubi [A]
time = 0.21, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3169, 3853, 3855, 2686, 30, 2691, 14}

$$\frac{a^4\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^4\tan(c+dx)\sec(c+dx)}{2d}+\frac{4a^3b\sec^3(c+dx)}{3d}-\frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{3a^2b^2\tan(c+dx)\sec^3(c+dx)}{2d}-\frac{3a^2b^2\tan(c+dx)\sec(c+dx)}{4d}+\frac{4ab^3\sec^5(c+dx)}{5d}-\frac{4ab^3\sec^3(c+dx)}{3d}+\frac{b^4\tanh^{-1}(\sin(c+dx))}{16d}+\frac{b^4\tan^3(c+dx)\sec^3(c+dx)}{6d}-\frac{b^4\tan(c+dx)\sec^3(c+dx)}{8d}+\frac{b^4\tan(c+dx)\sec(c+dx)}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*ArcTanh[Sin[c + d*x]])/(2*d) - (3*a^2*b^2*ArcTanh[Sin[c + d*x]])/(4*d)
 + (b^4*ArcTanh[Sin[c + d*x]])/(16*d) + (4*a^3*b*Sec[c + d*x]^3)/(3*d) - (4
*a*b^3*Sec[c + d*x]^3)/(3*d) + (4*a*b^3*Sec[c + d*x]^5)/(5*d) + (a^4*Sec[c
+ d*x]*Tan[c + d*x])/(2*d) - (3*a^2*b^2*Sec[c + d*x]*Tan[c + d*x])/(4*d) +
(b^4*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (3*a^2*b^2*Sec[c + d*x]^3*Tan[c +
d*x])/(2*d) - (b^4*Sec[c + d*x]^3*Tan[c + d*x])/(8*d) + (b^4*Sec[c + d*x]^3
*Tan[c + d*x]^3)/(6*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
```

```
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int \left(a^4\sec^3(c+dx)+4a^3b\sec^3(c+dx)\tan(c+dx)+6\right.$$
$$= a^4\int \sec^3(c+dx)\,dx+\left(4a^3b\right)\int \sec^3(c+dx)\tan(c+$$
$$= \frac{a^4\sec(c+dx)\tan(c+dx)}{2d}+\frac{3a^2b^2\sec^3(c+dx)\tan(c-}{2d}$$
$$= \frac{a^4\tanh^{-1}(\sin(c+dx))}{2d}+\frac{4a^3b\sec^3(c+dx)}{3d}+\frac{a^4\sec(c}{}$$
$$= \frac{a^4\tanh^{-1}(\sin(c+dx))}{2d}-\frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{4d}+$$
$$= \frac{a^4\tanh^{-1}(\sin(c+dx))}{2d}-\frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{4d}+$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1342 vs. $2(258) = 516$.
time = 6.30, size = 1342, normalized size = 5.20

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (a*b*(20*a^2 - 11*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(30*d*(a*Cos[
c + d*x] + b*Sin[c + d*x])^4) + ((-8*a^4 + 12*a^2*b^2 - b^4)*Cos[c + d*x]^4
*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(16*d*(a*
Cos[c + d*x] + b*Sin[c + d*x])^4) + ((8*a^4 - 12*a^2*b^2 + b^4)*Cos[c + d*x
]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(16*d*
(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (b^4*Cos[c + d*x]^4*(a + b*Tan[c + d
*x])^4)/(48*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^6*(a*Cos[c + d*x] + b*S
in[c + d*x])^4) + ((30*a^2*b^2 + 8*a*b^3 - 5*b^4)*Cos[c + d*x]^4*(a + b*Tan
[c + d*x])^4)/(80*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4*(a*Cos[c + d*x]
 + b*Sin[c + d*x])^4) + ((120*a^4 + 160*a^3*b - 180*a^2*b^2 - 88*a*b^3 + 15
*b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(480*d*(Cos[(c + d*x)/2] - Sin
[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (a*b^3*Cos[c + d*x]
^4*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^4)/(5*d*(Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2])^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (b^4*Cos[c + d*x]^4*(a
 + b*Tan[c + d*x])^4)/(48*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(a*Cos[
c + d*x] + b*Sin[c + d*x])^4) - (a*b^3*Cos[c + d*x]^4*Sin[(c + d*x)/2]*(a +
 b*Tan[c + d*x])^4)/(5*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*(a*Cos[c +
 d*x] + b*Sin[c + d*x])^4) + ((-30*a^2*b^2 + 8*a*b^3 + 5*b^4)*Cos[c + d*x]^
4*(a + b*Tan[c + d*x])^4)/(80*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(a*
```

```
Cos[c + d*x] + b*Sin[c + d*x])^4) + ((-120*a^4 + 160*a^3*b + 180*a^2*b^2 -
88*a*b^3 - 15*b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(480*d*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c
 + d*x]^4*(20*a^3*b*Sin[(c + d*x)/2] - 11*a*b^3*Sin[(c + d*x)/2])*(a + b*Ta
n[c + d*x])^4)/(30*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(a*Cos[c + d*x
] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(20*a^3*b*Sin[(c + d*x)/2] - 11*a*
b^3*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(30*d*(Cos[(c + d*x)/2] - Sin
[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(-20*
a^3*b*Sin[(c + d*x)/2] + 11*a*b^3*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)
/(30*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3*(a*Cos[c + d*x] + b*Sin[c +
d*x])^4) + (Cos[c + d*x]^4*(-20*a^3*b*Sin[(c + d*x)/2] + 11*a*b^3*Sin[(c +
d*x)/2])*(a + b*Tan[c + d*x])^4)/(30*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]
)*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
```

Maple [A]
time = 0.38, size = 296, normalized size = 1.15

method	result
derivativedivides	$\frac{a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{4a^3b}{3\cos(dx+c)^3}+6a^2b^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx\cdots}{}}{}$
default	$\frac{a^4\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{4a^3b}{3\cos(dx+c)^3}+6a^2b^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\frac{\ln(\sec(dx\cdots}{}}{}$
risch	$-\frac{i(15b^4e^{11i(dx+c)}+235b^4e^{3i(dx+c)}+120a^4e^{11i(dx+c)}-120a^4e^{i(dx+c)}-390b^4e^{5i(dx+c)}-360a^4e^{3i(dx+c)}+360a^4e^{9i(dx+c)}\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+4/3*a^3*
b/cos(d*x+c)^3+6*a^2*b^2*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/co
s(d*x+c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+4*a*b^3*(1/5*sin(d
*x+c)^4/cos(d*x+c)^5+1/15*sin(d*x+c)^4/cos(d*x+c)^3-1/15*sin(d*x+c)^4/cos(d
*x+c)-1/15*(2+sin(d*x+c)^2)*cos(d*x+c))+b^4*(1/6*sin(d*x+c)^5/cos(d*x+c)^6+
1/24*sin(d*x+c)^5/cos(d*x+c)^4-1/48*sin(d*x+c)^5/cos(d*x+c)^2-1/48*sin(d*x+
c)^3-1/16*sin(d*x+c)+1/16*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.27, size = 251, normalized size = 0.97

$$-\frac{5\,b^4\left(\frac{2\left(3\,\sin(dx+c)^5+8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-180\,a^2b^2\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+120\,a^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-\frac{640\,a^3b}{\cos(dx+c)^3}+\frac{128\left(5\,\cos(dx+c)^2-3\right)ab^3}{\cos(dx+c)^5}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/480*(5*b^4*(2*(3*sin(d*x + c)^5 + 8*sin(d*x + c)^3 - 3*sin(d*x + c))/(si
n(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x + c
) + 1) + 3*log(sin(d*x + c) - 1)) - 180*a^2*b^2*(2*(sin(d*x + c)^3 + sin(d*
x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)) + 120*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(
sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 640*a^3*b/cos(d*x + c)^3 + 128
*(5*cos(d*x + c)^2 - 3)*a*b^3/cos(d*x + c)^5)/d
```

Fricas [A]
time = 2.06, size = 187, normalized size = 0.72

$$\frac{15\,(8\,a^4-12\,a^2b^2+b^4)\cos(dx+c)^6\log(\sin(dx+c)+1)-15\,(8\,a^4-12\,a^2b^2+b^4)\cos(dx+c)^6\log(-\sin(dx+c)+1)+384\,ab^3\cos(dx+c)+640\,(a^3b-ab^3)\cos(dx+c)^3+10\,(3\,(8\,a^4-12\,a^2b^2+b^4)\cos(dx+c)^4+8\,b^4+2\,(36\,a^2b^2-7\,b^4)\cos(dx+c)^2)\sin(dx+c)}{480\,d\cos(dx+c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/480*(15*(8*a^4 - 12*a^2*b^2 + b^4)*cos(d*x + c)^6*log(sin(d*x + c) + 1) -
 15*(8*a^4 - 12*a^2*b^2 + b^4)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 384*
a*b^3*cos(d*x + c) + 640*(a^3*b - a*b^3)*cos(d*x + c)^3 + 10*(3*(8*a^4 - 12
*a^2*b^2 + b^4)*cos(d*x + c)^4 + 8*b^4 + 2*(36*a^2*b^2 - 7*b^4)*cos(d*x + c
)^2)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 536 vs. 2(234) = 468.
time = 0.59, size = 536, normalized size = 2.08

$$\frac{15\,(8\,a^4-12\,a^2b^2+b^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,(8\,a^4-12\,a^2b^2+b^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\,\left(120\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+180\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-15\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-960\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{10}-360\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+900\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+85\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+2880\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-1920\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}+240\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-1080\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+570\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-3200\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+1280\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{6}+240\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-1080\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+570\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+1920\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}-360\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+900\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+85\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-960\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+768\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+120\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+180\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-15\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+320\,a^3b-128\,ab^3\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^6}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/240*(15*(8*a^4 - 12*a^2*b^2 + b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 1
5*(8*a^4 - 12*a^2*b^2 + b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(120*a^
4*tan(1/2*d*x + 1/2*c)^11 + 180*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 - 15*b^4*ta
n(1/2*d*x + 1/2*c)^11 - 960*a^3*b*tan(1/2*d*x + 1/2*c)^10 - 360*a^4*tan(1/2
*d*x + 1/2*c)^9 + 900*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 85*b^4*tan(1/2*d*x +
```

```
 1/2*c)^9 + 2880*a^3*b*tan(1/2*d*x + 1/2*c)^8 - 1920*a*b^3*tan(1/2*d*x + 1/
2*c)^8 + 240*a^4*tan(1/2*d*x + 1/2*c)^7 - 1080*a^2*b^2*tan(1/2*d*x + 1/2*c)
^7 + 570*b^4*tan(1/2*d*x + 1/2*c)^7 - 3200*a^3*b*tan(1/2*d*x + 1/2*c)^6 + 1
280*a*b^3*tan(1/2*d*x + 1/2*c)^6 + 240*a^4*tan(1/2*d*x + 1/2*c)^5 - 1080*a^
2*b^2*tan(1/2*d*x + 1/2*c)^5 + 570*b^4*tan(1/2*d*x + 1/2*c)^5 + 1920*a^3*b*
tan(1/2*d*x + 1/2*c)^4 - 360*a^4*tan(1/2*d*x + 1/2*c)^3 + 900*a^2*b^2*tan(1
/2*d*x + 1/2*c)^3 + 85*b^4*tan(1/2*d*x + 1/2*c)^3 - 960*a^3*b*tan(1/2*d*x +
 1/2*c)^2 + 768*a*b^3*tan(1/2*d*x + 1/2*c)^2 + 120*a^4*tan(1/2*d*x + 1/2*c)
 + 180*a^2*b^2*tan(1/2*d*x + 1/2*c) - 15*b^4*tan(1/2*d*x + 1/2*c) + 320*a^3
*b - 128*a*b^3)/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

Mupad [B]
time = 4.27, size = 419, normalized size = 1.62

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^4-\frac{3\,a^2\,b^2}{2}+\frac{b^4}{8}\right)}{d}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(2\,a^4-9\,a^2\,b^2+\frac{19\,b^4}{4}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(2\,a^4-9\,a^2\,b^2+\frac{19\,b^4}{4}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(-3\,a^4+\frac{15\,a^2\,b^2}{2}+\frac{17\,b^4}{24}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9\,\left(-3\,a^4+\frac{15\,a^2\,b^2}{2}+\frac{17\,b^4}{24}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^4+\frac{3\,a^2\,b^2}{2}-\frac{b^4}{8}\right)-\frac{16\,a\,b^3}{15}+\frac{8\,a^3\,b}{3}+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}\,\left(a^4+\frac{3\,a^2\,b^2}{2}-\frac{b^4}{8}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{32\,a\,b^3}{5}-8\,a^3\,b\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(16\,a\,b^3-24\,a^3\,b\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(\frac{32\,a\,b^3}{3}-\frac{80\,a^3\,b}{3}\right)+16\,a^3\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-8\,a^3\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}-6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-20\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^7,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(a^4 + b^4/8 - (3*a^2*b^2)/2))/d + (tan(c/2 + (d
*x)/2)^5*(2*a^4 + (19*b^4)/4 - 9*a^2*b^2) + tan(c/2 + (d*x)/2)^7*(2*a^4 + (
19*b^4)/4 - 9*a^2*b^2) + tan(c/2 + (d*x)/2)^3*((17*b^4)/24 - 3*a^4 + (15*a^
2*b^2)/2) + tan(c/2 + (d*x)/2)^9*((17*b^4)/24 - 3*a^4 + (15*a^2*b^2)/2) + t
an(c/2 + (d*x)/2)*(a^4 - b^4/8 + (3*a^2*b^2)/2) - (16*a*b^3)/15 + (8*a^3*b)
/3 + tan(c/2 + (d*x)/2)^11*(a^4 - b^4/8 + (3*a^2*b^2)/2) + tan(c/2 + (d*x)/
2)^2*((32*a*b^3)/5 - 8*a^3*b) - tan(c/2 + (d*x)/2)^8*(16*a*b^3 - 24*a^3*b)
+ tan(c/2 + (d*x)/2)^6*((32*a*b^3)/3 - (80*a^3*b)/3) + 16*a^3*b*tan(c/2 + (
d*x)/2)^4 - 8*a^3*b*tan(c/2 + (d*x)/2)^10)/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*
tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 -
6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.87 $\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=143

$$\frac{a^4\tan(c+dx)}{d}+\frac{2a^3b\tan^2(c+dx)}{d}+\frac{a^2(a^2+6b^2)\tan^3(c+dx)}{3d}+\frac{ab(a^2+b^2)\tan^4(c+dx)}{d}+\frac{b^2(6a^2+b^2)\tan^5}{5d}$$

```
[Out] a^4*tan(d*x+c)/d+2*a^3*b*tan(d*x+c)^2/d+1/3*a^2*(a^2+6*b^2)*tan(d*x+c)^3/d+
a*b*(a^2+b^2)*tan(d*x+c)^4/d+1/5*b^2*(6*a^2+b^2)*tan(d*x+c)^5/d+2/3*a*b^3*t
an(d*x+c)^6/d+1/7*b^4*tan(d*x+c)^7/d
```

Rubi [A]
time = 0.09, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3167, 908}

$$\frac{a^4\tan(c+dx)}{d}+\frac{2a^3b\tan^2(c+dx)}{d}+\frac{b^2(6a^2+b^2)\tan^5(c+dx)}{5d}+\frac{ab(a^2+b^2)\tan^4(c+dx)}{d}+\frac{a^2(a^2+6b^2)\tan^3(c+dx)}{3d}+\frac{2ab^3\tan^6(c+dx)}{3d}+\frac{b^4\tan^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Tan[c + d*x])/d + (2*a^3*b*Tan[c + d*x]^2)/d + (a^2*(a^2 + 6*b^2)*Tan[
c + d*x]^3)/(3*d) + (a*b*(a^2 + b^2)*Tan[c + d*x]^4)/d + (b^2*(6*a^2 + b^2)
*Tan[c + d*x]^5)/(5*d) + (2*a*b^3*Tan[c + d*x]^6)/(3*d) + (b^4*Tan[c + d*x]
^7)/(7*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^4(1+x^2)}{x^8}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{b^4}{x^8}+\frac{4ab^3}{x^7}+\frac{6a^2b^2+b^4}{x^6}+\frac{4ab(a^2+b^2)}{x^5}+\frac{a^4+6a^2b^2}{x^4}\right.\right.}{d}$$

$$= \frac{a^4\tan(c+dx)}{d}+\frac{2a^3b\tan^2(c+dx)}{d}+\frac{a^2(a^2+6b^2)\tan^3}{3d}$$

Mathematica [A]

time = 0.58, size = 54, normalized size = 0.38

$$\frac{(a+b\tan(c+dx))^5\left(a^2+21b^2-5ab\tan(c+dx)+15b^2\tan^2(c+dx)\right)}{105b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] ((a + b*Tan[c + d*x])^5*(a^2 + 21*b^2 - 5*a*b*Tan[c + d*x] + 15*b^2*Tan[c +
 d*x]^2))/(105*b^3*d)
```

Maple [A]

time = 0.33, size = 171, normalized size = 1.20

method	result
derivativedivides	$\frac{-a^4\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{a^3b}{\cos(dx+c)^4}+6a^2b^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+4a\,b^3\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx}{12\cos(dx}\right.}{d}$
default	$\frac{-a^4\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{a^3b}{\cos(dx+c)^4}+6a^2b^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+4a\,b^3\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\sin^4(dx}{12\cos(dx}\right.}{d}$
risch	$\frac{4i(35a^4+3b^4-42a^2b^2-420ia^3b\,e^{10i(dx+c)}+420ia\,b^3e^{10i(dx+c)}+210b^4e^{6i(dx+c)}+21b^4e^{2i(dx+c)}+105b^4e^{10i(dx+c)}+105a^4e}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^4*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a^3*b/cos(d*x+c)^4+6*a^2*b^2*(
1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+4*a*b^3*(1/6*
sin(d*x+c)^4/cos(d*x+c)^6+1/12*sin(d*x+c)^4/cos(d*x+c)^4)+b^4*(1/7*sin(d*x+
c)^5/cos(d*x+c)^7+2/35*sin(d*x+c)^5/cos(d*x+c)^5))
```

Maxima [A]

time = 0.27, size = 151, normalized size = 1.06

$$\frac{35\left(\tan(dx+c)^3+3\tan(dx+c)\right)a^4+42\left(3\tan(dx+c)^5+5\tan(dx+c)^3\right)a^2b^2+3\left(5\tan(dx+c)^7+7\tan(dx+c)^5\right)b^4-\frac{35\left(3\sin(dx+c)^2-1\right)ab^3}{\sin(dx+c)^6-3\sin(dx+c)^4+3\sin(dx+c)^2-1}+\frac{105a^3b}{\left(\sin(dx+c)^2-1\right)^2}}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/105*(35*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^4 + 42*(3*tan(d*x + c)^5 + 5*
tan(d*x + c)^3)*a^2*b^2 + 3*(5*tan(d*x + c)^7 + 7*tan(d*x + c)^5)*b^4 - 35*
(3*sin(d*x + c)^2 - 1)*a*b^3/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x
 + c)^2 - 1) + 105*a^3*b/(sin(d*x + c)^2 - 1)^2)/d
```

Fricas [**A**]
time = 1.91, size = 142, normalized size = 0.99

$$\frac{70\,ab^3\cos\left(dx+c\right)+105\,\left(a^3b-ab^3\right)\cos\left(dx+c\right)^3+\left(2\,\left(35\,a^4-42\,a^2b^2+3\,b^4\right)\cos\left(dx+c\right)^6+\left(35\,a^4-42\,a^2b^2+3\,b^4\right)\cos\left(dx+c\right)^4+15\,b^4+6\,\left(21\,a^2b^2-4\,b^4\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{105\,d\cos\left(dx+c\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(70*a*b^3*cos(d*x + c) + 105*(a^3*b - a*b^3)*cos(d*x + c)^3 + (2*(35*
a^4 - 42*a^2*b^2 + 3*b^4)*cos(d*x + c)^6 + (35*a^4 - 42*a^2*b^2 + 3*b^4)*co
s(d*x + c)^4 + 15*b^4 + 6*(21*a^2*b^2 - 4*b^4)*cos(d*x + c)^2)*sin(d*x + c)
)/(d*cos(d*x + c)^7)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [**A**]
time = 0.55, size = 144, normalized size = 1.01

$$\frac{15\,b^4\tan\left(dx+c\right)^7+70\,ab^3\tan\left(dx+c\right)^6+126\,a^2b^2\tan\left(dx+c\right)^5+21\,b^4\tan\left(dx+c\right)^5+105\,a^3b\tan\left(dx+c\right)^4+105\,ab^3\tan\left(dx+c\right)^4+35\,a^4\tan\left(dx+c\right)^3+210\,a^2b^2\tan\left(dx+c\right)^3+210\,a^3b\tan\left(dx+c\right)^2+105\,a^4\tan\left(dx+c\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/105*(15*b^4*tan(d*x + c)^7 + 70*a*b^3*tan(d*x + c)^6 + 126*a^2*b^2*tan(d*
x + c)^5 + 21*b^4*tan(d*x + c)^5 + 105*a^3*b*tan(d*x + c)^4 + 105*a*b^3*tan
(d*x + c)^4 + 35*a^4*tan(d*x + c)^3 + 210*a^2*b^2*tan(d*x + c)^3 + 210*a^3*
b*tan(d*x + c)^2 + 105*a^4*tan(d*x + c))/d
```

Mupad [**B**]
time = 1.10, size = 186, normalized size = 1.30

$$\frac{\frac{b^4\,\sin\left(c+d\,x\right)}{7}-\cos\left(c+d\,x\right)^3\,\left(a\,b^3-a^3\,b\right)-\cos\left(c+d\,x\right)^2\,\left(\frac{8\,b^4\,\sin\left(c+d\,x\right)}{35}-\frac{6\,a^2\,b^2\,\sin\left(c+d\,x\right)}{5}\right)+\cos\left(c+d\,x\right)^4\,\left(\frac{\sin\left(c+d\,x\right)\,a^4}{3}-\frac{2\,\sin\left(c+d\,x\right)\,a^2\,b^2}{5}+\frac{\sin\left(c+d\,x\right)\,b^4}{35}\right)+\cos\left(c+d\,x\right)^6\,\left(\frac{2\,\sin\left(c+d\,x\right)\,a^4}{3}-\frac{4\,\sin\left(c+d\,x\right)\,a^2\,b^2}{5}+\frac{2\,\sin\left(c+d\,x\right)\,b^4}{35}\right)+\frac{2\,a\,b^3\,\cos\left(c+d\,x\right)}{3}}{d\,\cos\left(c+d\,x\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^8,x)
```

```
[Out] ((b^4*sin(c + d*x))/7 - cos(c + d*x)^3*(a*b^3 - a^3*b) - cos(c + d*x)^2*((8
*b^4*sin(c + d*x))/35 - (6*a^2*b^2*sin(c + d*x))/5) + cos(c + d*x)^4*((a^4*
sin(c + d*x))/3 + (b^4*sin(c + d*x))/35 - (2*a^2*b^2*sin(c + d*x))/5) + cos
(c + d*x)^6*((2*a^4*sin(c + d*x))/3 + (2*b^4*sin(c + d*x))/35 - (4*a^2*b^2*
sin(c + d*x))/5) + (2*a*b^3*cos(c + d*x))/3)/(d*cos(c + d*x)^7)
```

3.88 $\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=330

$$\frac{3a^4\tanh^{-1}(\sin(c+dx))}{8d}-\frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3b^4\tanh^{-1}(\sin(c+dx))}{128d}+\frac{4a^3b\sec^5(c+dx)}{5d}-\frac{4ab^3\sec$$

```
[Out] 3/8*a^4*arctanh(sin(d*x+c))/d-3/8*a^2*b^2*arctanh(sin(d*x+c))/d+3/128*b^4*a
rctanh(sin(d*x+c))/d+4/5*a^3*b*sec(d*x+c)^5/d-4/5*a*b^3*sec(d*x+c)^5/d+4/7*
a*b^3*sec(d*x+c)^7/d+3/8*a^4*sec(d*x+c)*tan(d*x+c)/d-3/8*a^2*b^2*sec(d*x+c)
*tan(d*x+c)/d+3/128*b^4*sec(d*x+c)*tan(d*x+c)/d+1/4*a^4*sec(d*x+c)^3*tan(d*
x+c)/d-1/4*a^2*b^2*sec(d*x+c)^3*tan(d*x+c)/d+1/64*b^4*sec(d*x+c)^3*tan(d*x+
c)/d+a^2*b^2*sec(d*x+c)^5*tan(d*x+c)/d-1/16*b^4*sec(d*x+c)^5*tan(d*x+c)/d+1
/8*b^4*sec(d*x+c)^5*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.24, antiderivative size = 330, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3169, 3853, 3855, 2686, 30, 2691, 14}

$$\frac{3a^4\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^4\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a^4\tan(c+dx)\sec(c+dx)}{8d}+\frac{4a^3b\sec^5(c+dx)}{5d}-\frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2b^2\tan(c+dx)\sec^5(c+dx)}{d}-\frac{a^2b^2\tan(c+dx)\sec^3(c+dx)}{4d}-\frac{3a^2b^2\tan(c+dx)\sec(c+dx)}{8d}+\frac{4ab^3\sec^7(c+dx)}{7d}-\frac{4ab^3\sec^5(c+dx)}{5d}+\frac{3b^4\tanh^{-1}(\sin(c+dx))}{128d}+\frac{b^4\tan^3(c+dx)\sec^5(c+dx)}{8d}-\frac{b^4\tan(c+dx)\sec^5(c+dx)}{16d}+\frac{b^4\tan(c+dx)\sec^3(c+dx)}{64d}+\frac{3b^4\tan(c+dx)\sec(c+dx)}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (3*a^4*ArcTanh[Sin[c + d*x]])/(8*d) - (3*a^2*b^2*ArcTanh[Sin[c + d*x]])/(8*
d) + (3*b^4*ArcTanh[Sin[c + d*x]])/(128*d) + (4*a^3*b*Sec[c + d*x]^5)/(5*d)
 - (4*a*b^3*Sec[c + d*x]^5)/(5*d) + (4*a*b^3*Sec[c + d*x]^7)/(7*d) + (3*a^4
*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (3*a^2*b^2*Sec[c + d*x]*Tan[c + d*x])/(
8*d) + (3*b^4*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (a^4*Sec[c + d*x]^3*Tan[
c + d*x])/(4*d) - (a^2*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d) + (b^4*Sec[c
+ d*x]^3*Tan[c + d*x])/(64*d) + (a^2*b^2*Sec[c + d*x]^5*Tan[c + d*x])/d - (
b^4*Sec[c + d*x]^5*Tan[c + d*x])/(16*d) + (b^4*Sec[c + d*x]^5*Tan[c + d*x]^
3)/(8*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int \left(a^4\sec^5(c+dx)+4a^3b\sec^5(c+dx)\tan(c+dx)+6\right.$$
$$= a^4\int \sec^5(c+dx)\,dx + \left(4a^3b\right)\int \sec^5(c+dx)\tan(c+$$
$$= \frac{a^4\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{a^2b^2\sec^5(c+dx)\tan(c-}{d}$$
$$= \frac{4a^3b\sec^5(c+dx)}{5d} + \frac{3a^4\sec(c+dx)\tan(c+dx)}{8d} + \frac{a^4\,\mathrm{s}}{}$$
$$= \frac{3a^4\tanh^{-1}(\sin(c+dx))}{8d} + \frac{4a^3b\sec^5(c+dx)}{5d} - \frac{4ab^3\,\mathrm{se}}{}$$
$$= \frac{3a^4\tanh^{-1}(\sin(c+dx))}{8d} - \frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{8d}$$
$$= \frac{3a^4\tanh^{-1}(\sin(c+dx))}{8d} - \frac{3a^2b^2\tanh^{-1}(\sin(c+dx))}{8d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1732 vs. $2(330) = 660$.
time = 6.42, size = 1732, normalized size = 5.25

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (a*b*(42*a^2 - 17*b^2)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(140*d*(a*Cos
[c + d*x] + b*Sin[c + d*x])^4) - (3*(16*a^4 - 16*a^2*b^2 + b^4)*Cos[c + d*x
]^4*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)/(128*d
*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (3*(16*a^4 - 16*a^2*b^2 + b^4)*Cos[
c + d*x]^4*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^4)
/(128*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (b^4*Cos[c + d*x]^4*(a + b*T
an[c + d*x])^4)/(128*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^8*(a*Cos[c + d
*x] + b*Sin[c + d*x])^4) + ((56*a^2*b^2 + 16*a*b^3 - 7*b^4)*Cos[c + d*x]^4*
(a + b*Tan[c + d*x])^4)/(448*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^6*(a*C
os[c + d*x] + b*Sin[c + d*x])^4) + ((560*a^4 + 896*a^3*b - 256*a*b^3 - 35*b
^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(8960*d*(Cos[(c + d*x)/2] - Sin[
(c + d*x)/2])^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((1680*a^4 + 1344*a^
3*b - 1680*a^2*b^2 - 544*a*b^3 + 105*b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x
])^4)/(8960*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*S
in[c + d*x])^4) + (a*b^3*Cos[c + d*x]^4*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x
])^4)/(14*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^7*(a*Cos[c + d*x] + b*Sin
```

```
[c + d*x])^4) - (b^4*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(128*d*(Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2])^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) - (a*b
^3*Cos[c + d*x]^4*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^4)/(14*d*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + ((-56*
a^2*b^2 + 16*a*b^3 + 7*b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(448*d*(
Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^4)
 + ((-560*a^4 + 896*a^3*b - 256*a*b^3 + 35*b^4)*Cos[c + d*x]^4*(a + b*Tan[c
 + d*x])^4)/(8960*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(a*Cos[c + d*x]
 + b*Sin[c + d*x])^4) + ((-1680*a^4 + 1344*a^3*b + 1680*a^2*b^2 - 544*a*b^3
 - 105*b^4)*Cos[c + d*x]^4*(a + b*Tan[c + d*x])^4)/(8960*d*(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x
]^4*(42*a^3*b*Sin[(c + d*x)/2] - 17*a*b^3*Sin[(c + d*x)/2])*(a + b*Tan[c +
d*x])^4)/(140*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(a*Cos[c + d*x] + b
*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(42*a^3*b*Sin[(c + d*x)/2] - 17*a*b^3*S
in[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(140*d*(Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(7*a^3*b*
Sin[(c + d*x)/2] - 2*a*b^3*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(35*d*
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^4
) + (Cos[c + d*x]^4*(-7*a^3*b*Sin[(c + d*x)/2] + 2*a*b^3*Sin[(c + d*x)/2])*
(a + b*Tan[c + d*x])^4)/(35*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*(a*Co
s[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c + d*x]^4*(-42*a^3*b*Sin[(c + d*x)/
2] + 17*a*b^3*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^4)/(140*d*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^4) + (Cos[c +
 d*x]^4*(-42*a^3*b*Sin[(c + d*x)/2] + 17*a*b^3*Sin[(c + d*x)/2])*(a + b*Tan
[c + d*x])^4)/(140*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x]
+ b*Sin[c + d*x])^4)
```

Maple [A]
time = 0.42, size = 363, normalized size = 1.10 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+
tan(d*x+c)))+4/5*a^3*b/cos(d*x+c)^5+6*a^2*b^2*(1/6*sin(d*x+c)^3/cos(d*x+c)^
6+1/8*sin(d*x+c)^3/cos(d*x+c)^4+1/16*sin(d*x+c)^3/cos(d*x+c)^2+1/16*sin(d*x
+c)-1/16*ln(sec(d*x+c)+tan(d*x+c)))+4*a*b^3*(1/7*sin(d*x+c)^4/cos(d*x+c)^7+
3/35*sin(d*x+c)^4/cos(d*x+c)^5+1/35*sin(d*x+c)^4/cos(d*x+c)^3-1/35*sin(d*x+
c)^4/cos(d*x+c)-1/35*(2+sin(d*x+c)^2)*cos(d*x+c))+b^4*(1/8*sin(d*x+c)^5/cos
(d*x+c)^8+1/16*sin(d*x+c)^5/cos(d*x+c)^6+1/64*sin(d*x+c)^5/cos(d*x+c)^4-1/1
28*sin(d*x+c)^5/cos(d*x+c)^2-1/128*sin(d*x+c)^3-3/128*sin(d*x+c)+3/128*ln(s
ec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.28, size = 322, normalized size = 0.98

$$-\frac{35\,b^4\left(\frac{2\left(3\,\sin(dx+c)^7-11\,\sin(dx+c)^5-11\,\sin(dx+c)^3+3\,\sin(dx+c)\right)}{\sin(dx+c)^8-4\,\sin(dx+c)^6+6\,\sin(dx+c)^4-4\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-560\,a^2b^2\left(\frac{2\left(3\,\sin(dx+c)^5-8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)+560\,a^4\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-\frac{7168\,a^3b}{\cos(dx+c)^5}+\frac{1024\left(7\,\cos(dx+c)^2-5\right)ab^3}{\cos(dx+c)^7}}{8960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/8960*(35*b^4*(2*(3*sin(d*x + c)^7 - 11*sin(d*x + c)^5 - 11*sin(d*x + c)^
3 + 3*sin(d*x + c))/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^4 -
 4*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
 - 560*a^2*b^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x + c))/(s
in(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*x +
c) + 1) + 3*log(sin(d*x + c) - 1)) + 560*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d
*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1)
+ 3*log(sin(d*x + c) - 1)) - 7168*a^3*b/cos(d*x + c)^5 + 1024*(7*cos(d*x +
c)^2 - 5)*a*b^3/cos(d*x + c)^7)/d
```

Fricas [**A**]
time = 2.20, size = 214, normalized size = 0.65

$$\frac{105 \left(16 a^4-16 a^2 b^2+b^4\right) \cos \left(dx+c\right)^8 \log \left(\sin \left(dx+c\right)+1\right)-105 \left(16 a^4-16 a^2 b^2+b^4\right) \cos \left(dx+c\right)^8 \log \left(-\sin \left(dx+c\right)+1\right)+5120 a b^3 \cos \left(dx+c\right)+7168 \left(a^3 b-a b^3\right) \cos \left(dx+c\right)^3+70 \left(3 \left(16 a^4-16 a^2 b^2+b^4\right) \cos \left(dx+c\right)^6+2 \left(16 a^4-16 a^2 b^2+b^4\right) \cos \left(dx+c\right)^4+16 b^4+8 \left(16 a^2 b^2-3 b^4\right) \cos \left(dx+c\right)^2\right) \sin \left(dx+c\right)}{8960 d \cos \left(dx+c\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/8960*(105*(16*a^4 - 16*a^2*b^2 + b^4)*cos(d*x + c)^8*log(sin(d*x + c) + 1
) - 105*(16*a^4 - 16*a^2*b^2 + b^4)*cos(d*x + c)^8*log(-sin(d*x + c) + 1) +
 5120*a*b^3*cos(d*x + c) + 7168*(a^3*b - a*b^3)*cos(d*x + c)^3 + 70*(3*(16*
a^4 - 16*a^2*b^2 + b^4)*cos(d*x + c)^6 + 2*(16*a^4 - 16*a^2*b^2 + b^4)*cos(
d*x + c)^4 + 16*b^4 + 8*(16*a^2*b^2 - 3*b^4)*cos(d*x + c)^2)*sin(d*x + c))/
(d*cos(d*x + c)^8)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 706 vs. 2(302) = 604.
time = 0.60, size = 706, normalized size = 2.14

$$\frac{105 \left(16 a^4-16 a^2 b^2+b^4\right) \log \left(\left|\tan \left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right|\right)-105 \left(16 a^4-16 a^2 b^2+b^4\right) \log \left(\left|\tan \left(\frac{1}{2} dx+\frac{1}{2} c\right)-1\right|\right)+\text{[illegible]}}{4480 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/4480*(105*(16*a^4 - 16*a^2*b^2 + b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1))
- 105*(16*a^4 - 16*a^2*b^2 + b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(2
800*a^4*tan(1/2*d*x + 1/2*c)^15 + 1680*a^2*b^2*tan(1/2*d*x + 1/2*c)^15 - 10
5*b^4*tan(1/2*d*x + 1/2*c)^15 - 17920*a^3*b*tan(1/2*d*x + 1/2*c)^14 - 9520*
a^4*tan(1/2*d*x + 1/2*c)^13 + 22960*a^2*b^2*tan(1/2*d*x + 1/2*c)^13 + 805*b
^4*tan(1/2*d*x + 1/2*c)^13 + 53760*a^3*b*tan(1/2*d*x + 1/2*c)^12 - 35840*a*
b^3*tan(1/2*d*x + 1/2*c)^12 + 11760*a^4*tan(1/2*d*x + 1/2*c)^11 - 7280*a^2*
b^2*tan(1/2*d*x + 1/2*c)^11 + 11655*b^4*tan(1/2*d*x + 1/2*c)^11 - 89600*a^3
*b*tan(1/2*d*x + 1/2*c)^10 - 5040*a^4*tan(1/2*d*x + 1/2*c)^9 - 17360*a^2*b^
2*tan(1/2*d*x + 1/2*c)^9 + 23485*b^4*tan(1/2*d*x + 1/2*c)^9 + 125440*a^3*b*
tan(1/2*d*x + 1/2*c)^8 - 35840*a*b^3*tan(1/2*d*x + 1/2*c)^8 - 5040*a^4*tan(
1/2*d*x + 1/2*c)^7 - 17360*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 23485*b^4*tan(1
/2*d*x + 1/2*c)^7 - 111104*a^3*b*tan(1/2*d*x + 1/2*c)^6 + 57344*a*b^3*tan(1
/2*d*x + 1/2*c)^6 + 11760*a^4*tan(1/2*d*x + 1/2*c)^5 - 7280*a^2*b^2*tan(1/2
*d*x + 1/2*c)^5 + 11655*b^4*tan(1/2*d*x + 1/2*c)^5 + 46592*a^3*b*tan(1/2*d*
x + 1/2*c)^4 + 7168*a*b^3*tan(1/2*d*x + 1/2*c)^4 - 9520*a^4*tan(1/2*d*x + 1
/2*c)^3 + 22960*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 805*b^4*tan(1/2*d*x + 1/2*
c)^3 - 10752*a^3*b*tan(1/2*d*x + 1/2*c)^2 + 8192*a*b^3*tan(1/2*d*x + 1/2*c)
^2 + 2800*a^4*tan(1/2*d*x + 1/2*c) + 1680*a^2*b^2*tan(1/2*d*x + 1/2*c) - 10
5*b^4*tan(1/2*d*x + 1/2*c) + 3584*a^3*b - 1024*a*b^3)/(tan(1/2*d*x + 1/2*c)
^2 - 1)^8)/d
```

<u>Mupad</u> [B]
time = 4.40, size = 566, normalized size = 1.72

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}\left(\frac{5a^4}{4}+\frac{3a^2b^2}{4}-\frac{3b^4}{64}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\left(-\frac{17a^4}{4}+\frac{41a^2b^2}{4}+\frac{23b^4}{64}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\left(-\frac{17a^4}{4}+\frac{41a^2b^2}{4}+\frac{23b^4}{64}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\left(\frac{21a^4}{4}-\frac{13a^2b^2}{4}+\frac{333b^4}{64}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\left(\frac{21a^4}{4}-\frac{13a^2b^2}{4}+\frac{333b^4}{64}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\left(\frac{9a^4}{4}+\frac{31a^2b^2}{4}-\frac{671b^4}{64}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\left(\frac{9a^4}{4}+\frac{31a^2b^2}{4}-\frac{671b^4}{64}\right)-\frac{16ab^3}{35}+\frac{8a^3b}{5}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{5a^4}{4}+\frac{3a^2b^2}{4}-\frac{3b^4}{64}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\left(16ab^3-24a^3b\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}\left(16ab^3-56a^3b\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}\left(\frac{104a^3b}{5}+\frac{16ab^3}{5}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}\left(\frac{128ab^3}{35}-\frac{24a^3b}{5}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\left(\frac{128ab^3}{5}-\frac{248a^3b}{5}\right)-40a^3b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-8a^3b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}-8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}+28\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-56\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+70\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}-56\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+28\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}-8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+1\right)}+\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(\frac{3a^4}{4}-\frac{3a^2b^2}{4}+\frac{3b^4}{64}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^9,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^15*((5*a^4)/4 - (3*b^4)/64 + (3*a^2*b^2)/4) + tan(c/2 +
 (d*x)/2)^3*((23*b^4)/64 - (17*a^4)/4 + (41*a^2*b^2)/4) + tan(c/2 + (d*x)/2
)^13*((23*b^4)/64 - (17*a^4)/4 + (41*a^2*b^2)/4) + tan(c/2 + (d*x)/2)^5*((2
1*a^4)/4 + (333*b^4)/64 - (13*a^2*b^2)/4) + tan(c/2 + (d*x)/2)^11*((21*a^4)
/4 + (333*b^4)/64 - (13*a^2*b^2)/4) - tan(c/2 + (d*x)/2)^7*((9*a^4)/4 - (67
1*b^4)/64 + (31*a^2*b^2)/4) - tan(c/2 + (d*x)/2)^9*((9*a^4)/4 - (671*b^4)/6
4 + (31*a^2*b^2)/4) - (16*a*b^3)/35 + (8*a^3*b)/5 + tan(c/2 + (d*x)/2)*((5*
a^4)/4 - (3*b^4)/64 + (3*a^2*b^2)/4) - tan(c/2 + (d*x)/2)^12*(16*a*b^3 - 24
*a^3*b) - tan(c/2 + (d*x)/2)^8*(16*a*b^3 - 56*a^3*b) + tan(c/2 + (d*x)/2)^4
*((16*a*b^3)/5 + (104*a^3*b)/5) + tan(c/2 + (d*x)/2)^2*((128*a*b^3)/35 - (2
4*a^3*b)/5) + tan(c/2 + (d*x)/2)^6*((128*a*b^3)/5 - (248*a^3*b)/5) - 40*a^3
*b*tan(c/2 + (d*x)/2)^10 - 8*a^3*b*tan(c/2 + (d*x)/2)^14)/(d*(28*tan(c/2 +
(d*x)/2)^4 - 8*tan(c/2 + (d*x)/2)^2 - 56*tan(c/2 + (d*x)/2)^6 + 70*tan(c/2
+ (d*x)/2)^8 - 56*tan(c/2 + (d*x)/2)^10 + 28*tan(c/2 + (d*x)/2)^12 - 8*tan(
```

```
c/2 + (d*x)/2)^14 + tan(c/2 + (d*x)/2)^16 + 1)) + (atanh(tan(c/2 + (d*x)/2)
)*((3*a^4)/4 + (3*b^4)/64 - (3*a^2*b^2)/4))/d
```

3.89 $\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=201

$$\frac{a^4\tan(c+dx)}{d}+\frac{2a^3b\tan^2(c+dx)}{d}+\frac{2a^2(a^2+3b^2)\tan^3(c+dx)}{3d}+\frac{ab(2a^2+b^2)\tan^4(c+dx)}{d}+\frac{(a^4+12a^2b^2+\cdots}{5\cdots}$$

```
[Out] a^4*tan(d*x+c)/d+2*a^3*b*tan(d*x+c)^2/d+2/3*a^2*(a^2+3*b^2)*tan(d*x+c)^3/d+
a*b*(2*a^2+b^2)*tan(d*x+c)^4/d+1/5*(a^4+12*a^2*b^2+b^4)*tan(d*x+c)^5/d+2/3*
a*b*(a^2+2*b^2)*tan(d*x+c)^6/d+2/7*b^2*(3*a^2+b^2)*tan(d*x+c)^7/d+1/2*a*b^3
*tan(d*x+c)^8/d+1/9*b^4*tan(d*x+c)^9/d
```

Rubi [A]
time = 0.12, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3167, 962}

$$\frac{a^4\tan(c+dx)}{d}+\frac{2a^3b\tan^2(c+dx)}{d}+\frac{2b^2(3a^2+b^2)\tan^7(c+dx)}{7d}+\frac{2ab(a^2+2b^2)\tan^6(c+dx)}{3d}+\frac{ab(2a^2+b^2)\tan^4(c+dx)}{d}+\frac{2a^2(a^2+3b^2)\tan^3(c+dx)}{3d}+\frac{(a^4+12a^2b^2+b^4)\tan^5(c+dx)}{5d}+\frac{ab^3\tan^8(c+dx)}{2d}+\frac{b^4\tan^9(c+dx)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Tan[c + d*x])/d + (2*a^3*b*Tan[c + d*x]^2)/d + (2*a^2*(a^2 + 3*b^2)*Ta
n[c + d*x]^3)/(3*d) + (a*b*(2*a^2 + b^2)*Tan[c + d*x]^4)/d + ((a^4 + 12*a^2
*b^2 + b^4)*Tan[c + d*x]^5)/(5*d) + (2*a*b*(a^2 + 2*b^2)*Tan[c + d*x]^6)/(3
*d) + (2*b^2*(3*a^2 + b^2)*Tan[c + d*x]^7)/(7*d) + (a*b^3*Tan[c + d*x]^8)/(
2*d) + (b^4*Tan[c + d*x]^9)/(9*d)
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^4(1+x^2)^2}{x^{10}}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{b^4}{x^{10}} + \frac{4ab^3}{x^9} + \frac{2(3a^2b^2+b^4)}{x^8} + \frac{4ab(a^2+2b^2)}{x^7} + \frac{a^4+\cdots}{}\right.\right.}{}$$

$$= \frac{a^4\tan(c+dx)}{d} + \frac{2a^3b\tan^2(c+dx)}{d} + \frac{2a^2(a^2+3b^2)\,\mathrm{t}\cdots}{3d}$$

Mathematica [A]
time = 0.85, size = 115, normalized size = 0.57

$$\frac{\frac{1}{5}(a^2+b^2)^2(a+b\tan(c+dx))^5 - \frac{2}{3}a(a^2+b^2)(a+b\tan(c+dx))^6 + \frac{2}{7}(3a^2+b^2)(a+b\tan(c+dx))^7 - \frac{1}{2}a(a+b\tan(c+dx))^8 + \frac{1}{9}(a+b\tan(c+dx))^9}{b^5 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (((a^2 + b^2)^2*(a + b*Tan[c + d*x])^5)/5 - (2*a*(a^2 + b^2)*(a + b*Tan[c +
 d*x])^6)/3 + (2*(3*a^2 + b^2)*(a + b*Tan[c + d*x])^7)/7 - (a*(a + b*Tan[c
+ d*x])^8)/2 + (a + b*Tan[c + d*x])^9/9)/(b^5*d)
```

Maple [A]
time = 0.35, size = 236, normalized size = 1.17

method	result
derivativedivides	$\frac{-a^4\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{2a^3b}{3\cos(dx+c)^6}+6a^2b^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c\cdots\right.}{105\cos(dx+\cdots}\right.}{d}$
default	$\frac{-a^4\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{2a^3b}{3\cos(dx+c)^6}+6a^2b^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx+c\cdots\right.}{105\cos(dx+\cdots}\right.}{d}$
risch	$\frac{16i(21a^4+b^4-18a^2b^2-2520ia^3b\,e^{10i(dx+c)}-126b^4e^{6i(dx+c)}+9b^4e^{2i(dx+c)}-315b^4e^{10i(dx+c)}+945a^4e^{10i(dx+c)}+1701a^4\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^4*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+2/3*a^3*b/c
os(d*x+c)^6+6*a^2*b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos(
d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+4*a*b^3*(1/8*sin(d*x+c)^4/cos(d*x
+c)^8+1/12*sin(d*x+c)^4/cos(d*x+c)^6+1/24*sin(d*x+c)^4/cos(d*x+c)^4)+b^4*(1
/9*sin(d*x+c)^5/cos(d*x+c)^9+4/63*sin(d*x+c)^5/cos(d*x+c)^7+8/315*sin(d*x+c
)^5/cos(d*x+c)^5))
```

Maxima [A]
time = 0.29, size = 193, normalized size = 0.96

$$\frac{42\left(3\tan\left(dx+c\right)^{5}+10\tan\left(dx+c\right)^{3}+15\tan\left(dx+c\right)\right)a^{4}+36\left(15\tan\left(dx+c\right)^{7}+42\tan\left(dx+c\right)^{5}+35\tan\left(dx+c\right)^{3}\right)a^{2}b^{2}+2\left(35\tan\left(dx+c\right)^{9}+90\tan\left(dx+c\right)^{7}+63\tan\left(dx+c\right)^{5}\right)b^{4}+\frac{105\left(4\sin(dx+c)^{2}-1\right)ab^{3}}{\sin(dx+c)^{8}-4\sin(dx+c)^{6}+6\sin(dx+c)^{4}-4\sin(dx+c)^{2}+1}-\frac{420a^{3}b}{\left(\sin(dx+c)^{2}-1\right)^{3}}}{630d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima"
)
```

```
[Out] 1/630*(42*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^4 + 36
*(15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*a^2*b^2 + 2*(3
5*tan(d*x + c)^9 + 90*tan(d*x + c)^7 + 63*tan(d*x + c)^5)*b^4 + 105*(4*sin(
d*x + c)^2 - 1)*a*b^3/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^4
 - 4*sin(d*x + c)^2 + 1) - 420*a^3*b/(sin(d*x + c)^2 - 1)^3)/d
```

Fricas [A]
time = 2.09, size = 167, normalized size = 0.83

$$\frac{315\,ab^{3}\cos\left(dx+c\right)+420\left(a^{3}b-ab^{3}\right)\cos\left(dx+c\right)^{3}+2\left(8\left(21\,a^{4}-18\,a^{2}b^{2}+b^{4}\right)\cos\left(dx+c\right)^{8}+4\left(21\,a^{4}-18\,a^{2}b^{2}+b^{4}\right)\cos\left(dx+c\right)^{6}+3\left(21\,a^{4}-18\,a^{2}b^{2}+b^{4}\right)\cos\left(dx+c\right)^{4}+35\,b^{4}+10\left(27\,a^{2}b^{2}-5\,b^{4}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{630\,d\cos\left(dx+c\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas"
)
```

```
[Out] 1/630*(315*a*b^3*cos(d*x + c) + 420*(a^3*b - a*b^3)*cos(d*x + c)^3 + 2*(8*(
21*a^4 - 18*a^2*b^2 + b^4)*cos(d*x + c)^8 + 4*(21*a^4 - 18*a^2*b^2 + b^4)*c
os(d*x + c)^6 + 3*(21*a^4 - 18*a^2*b^2 + b^4)*cos(d*x + c)^4 + 35*b^4 + 10*
(27*a^2*b^2 - 5*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^9)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.58, size = 214, normalized size = 1.06

$$\frac{70\,b^{4}\tan\left(dx+c\right)^{9}+315\,ab^{3}\tan\left(dx+c\right)^{8}+540\,a^{2}b^{2}\tan\left(dx+c\right)^{7}+180\,b^{4}\tan\left(dx+c\right)^{7}+420\,a^{3}b\tan\left(dx+c\right)^{6}+840\,ab^{3}\tan\left(dx+c\right)^{6}+126\,a^{4}\tan\left(dx+c\right)^{5}+1512\,a^{2}b^{2}\tan\left(dx+c\right)^{5}+126\,b^{4}\tan\left(dx+c\right)^{5}+1260\,a^{3}b\tan\left(dx+c\right)^{4}+630\,ab^{3}\tan\left(dx+c\right)^{4}+420\,a^{4}\tan\left(dx+c\right)^{3}+1260\,a^{2}b^{2}\tan\left(dx+c\right)^{3}+1260\,a^{3}b\tan\left(dx+c\right)^{2}+630\,a^{4}\tan\left(dx+c\right)}{630\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/630*(70*b^4*tan(d*x + c)^9 + 315*a*b^3*tan(d*x + c)^8 + 540*a^2*b^2*tan(d
*x + c)^7 + 180*b^4*tan(d*x + c)^7 + 420*a^3*b*tan(d*x + c)^6 + 840*a*b^3*t
an(d*x + c)^6 + 126*a^4*tan(d*x + c)^5 + 1512*a^2*b^2*tan(d*x + c)^5 + 126*
b^4*tan(d*x + c)^5 + 1260*a^3*b*tan(d*x + c)^4 + 630*a*b^3*tan(d*x + c)^4 +
 420*a^4*tan(d*x + c)^3 + 1260*a^2*b^2*tan(d*x + c)^3 + 1260*a^3*b*tan(d*x
+ c)^2 + 630*a^4*tan(d*x + c))/d
```

Mupad [B]
time = 4.28, size = 447, normalized size = 2.22

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(\frac{152\,a^4}{5}-\frac{96\,a^2\,b^2}{5}+\frac{32\,b^4}{5}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}\,\left(\frac{152\,a^4}{5}-\frac{96\,a^2\,b^2}{5}+\frac{32\,b^4}{5}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(-\frac{288\,a^4}{5}+\frac{1488\,a^2\,b^2}{35}+\frac{384\,b^4}{35}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\,\left(-\frac{288\,a^4}{5}+\frac{1488\,a^2\,b^2}{35}+\frac{384\,b^4}{35}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,\left(\frac{1076\,a^4}{15}-\frac{2752\,a^2\,b^2}{35}+\frac{6976\,b^4}{315}\right)+2\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{32\,a^4}{3}-16\,a^2\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}\,\left(\frac{32\,a^4}{3}-16\,a^2\,b^2\right)+2\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(16\,a\,b^3-24\,a^3\,b\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}\,\left(16\,a\,b^3-24\,a^3\,b\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(32\,a\,b^3-88\,a^3\,b\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\,\left(32\,a\,b^3-88\,a^3\,b\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(\frac{152\,a^3\,b}{3}+\frac{16\,a\,b^3}{3}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}\,\left(\frac{152\,a^3\,b}{3}+\frac{16\,a\,b^3}{3}\right)+8\,a^3\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-8\,a^3\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^10,x)
```

```
[Out] -(tan(c/2 + (d*x)/2)^5*((152*a^4)/5 + (32*b^4)/5 - (96*a^2*b^2)/5) + tan(c/
2 + (d*x)/2)^13*((152*a^4)/5 + (32*b^4)/5 - (96*a^2*b^2)/5) + tan(c/2 + (d*
x)/2)^7*((384*b^4)/35 - (288*a^4)/5 + (1488*a^2*b^2)/35) + tan(c/2 + (d*x)/
2)^11*((384*b^4)/35 - (288*a^4)/5 + (1488*a^2*b^2)/35) + tan(c/2 + (d*x)/2)
^9*((1076*a^4)/15 + (6976*b^4)/315 - (2752*a^2*b^2)/35) + 2*a^4*tan(c/2 + (
d*x)/2)^17 - tan(c/2 + (d*x)/2)^3*((32*a^4)/3 - 16*a^2*b^2) - tan(c/2 + (d*
x)/2)^15*((32*a^4)/3 - 16*a^2*b^2) + 2*a^4*tan(c/2 + (d*x)/2) + tan(c/2 + (
d*x)/2)^4*(16*a*b^3 - 24*a^3*b) - tan(c/2 + (d*x)/2)^14*(16*a*b^3 - 24*a^3*
b) + tan(c/2 + (d*x)/2)^8*(32*a*b^3 - 88*a^3*b) - tan(c/2 + (d*x)/2)^10*(32
*a*b^3 - 88*a^3*b) + tan(c/2 + (d*x)/2)^6*((16*a*b^3)/3 + (152*a^3*b)/3) -
tan(c/2 + (d*x)/2)^12*((16*a*b^3)/3 + (152*a^3*b)/3) + 8*a^3*b*tan(c/2 + (d
*x)/2)^2 - 8*a^3*b*tan(c/2 + (d*x)/2)^16)/(d*(tan(c/2 + (d*x)/2)^2 - 1)^9)
```

3.90 $\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=408

$$\frac{5a^4\tanh^{-1}(\sin(c+dx))}{16d}-\frac{15a^2b^2\tanh^{-1}(\sin(c+dx))}{64d}+\frac{3b^4\tanh^{-1}(\sin(c+dx))}{256d}+\frac{4a^3b\sec^7(c+dx)}{7d}-4ab^3\sec$$

```
[Out] 5/16*a^4*arctanh(sin(d*x+c))/d-15/64*a^2*b^2*arctanh(sin(d*x+c))/d+3/256*b^
4*arctanh(sin(d*x+c))/d+4/7*a^3*b*sec(d*x+c)^7/d-4/7*a*b^3*sec(d*x+c)^7/d+4
/9*a*b^3*sec(d*x+c)^9/d+5/16*a^4*sec(d*x+c)*tan(d*x+c)/d-15/64*a^2*b^2*sec(
d*x+c)*tan(d*x+c)/d+3/256*b^4*sec(d*x+c)*tan(d*x+c)/d+5/24*a^4*sec(d*x+c)^3
*tan(d*x+c)/d-5/32*a^2*b^2*sec(d*x+c)^3*tan(d*x+c)/d+1/128*b^4*sec(d*x+c)^3
*tan(d*x+c)/d+1/6*a^4*sec(d*x+c)^5*tan(d*x+c)/d-1/8*a^2*b^2*sec(d*x+c)^5*ta
n(d*x+c)/d+1/160*b^4*sec(d*x+c)^5*tan(d*x+c)/d+3/4*a^2*b^2*sec(d*x+c)^7*tan
(d*x+c)/d-3/80*b^4*sec(d*x+c)^7*tan(d*x+c)/d+1/10*b^4*sec(d*x+c)^7*tan(d*x+
c)^3/d
```

Rubi [A]
time = 0.28, antiderivative size = 408, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3169, 3853, 3855, 2686, 30, 2691, 14}

$$\frac{5a^4\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4\tan(c+dx)\sec^5(c+dx)}{6d}+\frac{5a^4\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{5a^4\tan(c+dx)\sec(c+dx)}{16d}+\frac{4a^3b\sec^7(c+dx)}{7d}-\frac{15a^2b^2\tanh^{-1}(\sin(c+dx))}{64d}+\frac{3a^2b^2\tan(c+dx)\sec^7(c+dx)}{4d}-\frac{a^2b^2\tan(c+dx)\sec^5(c+dx)}{8d}-\frac{5a^2b^2\tan(c+dx)\sec^3(c+dx)}{32d}-\frac{15a^2b^2\tan(c+dx)\sec(c+dx)}{64d}+\frac{4ab^3\sec^9(c+dx)}{9d}-\frac{4ab^3\sec^7(c+dx)}{7d}+\frac{3b^4\tanh^{-1}(\sin(c+dx))}{256d}+\frac{b^4\tan^3(c+dx)\sec^7(c+dx)}{10d}-\frac{3b^4\tan(c+dx)\sec^7(c+dx)}{80d}+\frac{b^4\tan(c+dx)\sec^5(c+dx)}{160d}+\frac{b^4\tan(c+dx)\sec^3(c+dx)}{128d}+\frac{3b^4\tan(c+dx)\sec(c+dx)}{256d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^11*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (5*a^4*ArcTanh[Sin[c + d*x]])/(16*d) - (15*a^2*b^2*ArcTanh[Sin[c + d*x]])/(
64*d) + (3*b^4*ArcTanh[Sin[c + d*x]])/(256*d) + (4*a^3*b*Sec[c + d*x]^7)/(7
*d) - (4*a*b^3*Sec[c + d*x]^7)/(7*d) + (4*a*b^3*Sec[c + d*x]^9)/(9*d) + (5*
a^4*Sec[c + d*x]*Tan[c + d*x])/(16*d) - (15*a^2*b^2*Sec[c + d*x]*Tan[c + d*
x])/(64*d) + (3*b^4*Sec[c + d*x]*Tan[c + d*x])/(256*d) + (5*a^4*Sec[c + d*x
]^3*Tan[c + d*x])/(24*d) - (5*a^2*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(32*d) +
 (b^4*Sec[c + d*x]^3*Tan[c + d*x])/(128*d) + (a^4*Sec[c + d*x]^5*Tan[c + d*
x])/(6*d) - (a^2*b^2*Sec[c + d*x]^5*Tan[c + d*x])/(8*d) + (b^4*Sec[c + d*x]
^5*Tan[c + d*x])/(160*d) + (3*a^2*b^2*Sec[c + d*x]^7*Tan[c + d*x])/(4*d) -
(3*b^4*Sec[c + d*x]^7*Tan[c + d*x])/(80*d) + (b^4*Sec[c + d*x]^7*Tan[c + d*
x]^3)/(10*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = \int \left(a^4\sec^7(c+dx)+4a^3b\sec^7(c+dx)\tan(c+dx)+6a\right.$$
$$= a^4\int \sec^7(c+dx)\,dx+\left(4a^3b\right)\int \sec^7(c+dx)\tan(c+d$$
$$= \frac{a^4\sec^5(c+dx)\tan(c+dx)}{6d}+\frac{3a^2b^2\sec^7(c+dx)\tan(c+}{4d}$$
$$= \frac{4a^3b\sec^7(c+dx)}{7d}+\frac{5a^4\sec^3(c+dx)\tan(c+dx)}{24d}+\frac{a^4 s}{}$$
$$= \frac{4a^3b\sec^7(c+dx)}{7d}-\frac{4ab^3\sec^7(c+dx)}{7d}+\frac{4ab^3\sec^9(c+}{9d}$$
$$= \frac{5a^4\tanh^{-1}(\sin(c+dx))}{16d}+\frac{4a^3b\sec^7(c+dx)}{7d}-\frac{4ab^3\sec}{}$$
$$= \frac{5a^4\tanh^{-1}(\sin(c+dx))}{16d}-\frac{15a^2b^2\tanh^{-1}(\sin(c+dx))}{64d}$$
$$= \frac{5a^4\tanh^{-1}(\sin(c+dx))}{16d}-\frac{15a^2b^2\tanh^{-1}(\sin(c+dx))}{64d}$$

Mathematica [A]
time = 1.42, size = 242, normalized size = 0.59

$$\frac{-80640(80a^4-60a^2b^2+3b^4)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+3\sec^{10}(c+dx)\left(983040ab(a^2-b^2)\cos(3(c+dx))+420(1552a^4+1908a^2b^2-505b^4)\sin(3(c+dx))+7(80a^4-60a^2b^2+3b^4)\left(628\sin(5(c+dx))+145\sin(7(c+dx))+15\sin(9(c+dx))\right)\right)+10\sec^9(c+dx)\left(32768ab(27a^2+b^2)+189(592a^4+1604a^2b^2+739b^4)\tan(c+dx)\right)}{20643840d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^11*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (-80640*(80*a^4 - 60*a^2*b^2 + 3*b^4)*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 3*Sec[c + d*x]^10*(98304
0*a*b*(a^2 - b^2)*Cos[3*(c + d*x)] + 420*(1552*a^4 + 1908*a^2*b^2 - 505*b^4
)*Sin[3*(c + d*x)] + 7*(80*a^4 - 60*a^2*b^2 + 3*b^4)*(628*Sin[5*(c + d*x)]
+ 145*Sin[7*(c + d*x)] + 15*Sin[9*(c + d*x)])) + 10*Sec[c + d*x]^9*(32768*a
*b*(27*a^2 + b^2) + 189*(592*a^4 + 1604*a^2*b^2 + 739*b^4)*Tan[c + d*x]))/(
20643840*d)
```

Maple [A]
time = 0.47, size = 427, normalized size = 1.05 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^4*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x+c))*tan(d*x+c)
+5/16*ln(sec(d*x+c)+tan(d*x+c)))+4/7*a^3*b/cos(d*x+c)^7+6*a^2*b^2*(1/8*sin(
```

d*x+c)^3/cos(d*x+c)^8+5/48*sin(d*x+c)^3/cos(d*x+c)^6+5/64*sin(d*x+c)^3/cos(d*x+c)^4+5/128*sin(d*x+c)^3/cos(d*x+c)^2+5/128*sin(d*x+c)-5/128*ln(sec(d*x+c)+tan(d*x+c)))+4*a*b^3*(1/9*sin(d*x+c)^4/cos(d*x+c)^9+5/63*sin(d*x+c)^4/cos(d*x+c)^7+1/21*sin(d*x+c)^4/cos(d*x+c)^5+1/63*sin(d*x+c)^4/cos(d*x+c)^3-1/63*sin(d*x+c)^4/cos(d*x+c)-1/63*(2+sin(d*x+c)^2)*cos(d*x+c))+b^4*(1/10*sin(d*x+c)^5/cos(d*x+c)^10+1/16*sin(d*x+c)^5/cos(d*x+c)^8+1/32*sin(d*x+c)^5/cos(d*x+c)^6+1/128*sin(d*x+c)^5/cos(d*x+c)^4-1/256*sin(d*x+c)^5/cos(d*x+c)^2-1/256*sin(d*x+c)^3-3/256*sin(d*x+c)+3/256*ln(sec(d*x+c)+tan(d*x+c))))

Maxima [A]
time = 0.28, size = 382, normalized size = 0.94

$$-\frac{63\,b^4\left(\frac{2\left(15\sin(dx+c)^9-70\sin(dx+c)^7+128\sin(dx+c)^5+70\sin(dx+c)^3-15\sin(dx+c)\right)}{\sin(dx+c)^{10}-5\sin(dx+c)^8+10\sin(dx+c)^6-10\sin(dx+c)^4+5\sin(dx+c)^2-1}-15\log(\sin(dx+c)+1)+15\log(\sin(dx+c)-1)\right)-1260\,a^2b^2\left(\frac{2\left(15\sin(dx+c)^7-55\sin(dx+c)^5+73\sin(dx+c)^3+15\sin(dx+c)\right)}{\sin(dx+c)^8-4\sin(dx+c)^6+6\sin(dx+c)^4-4\sin(dx+c)^2+1}-15\log(\sin(dx+c)+1)+15\log(\sin(dx+c)-1)\right)+1680\,a^4\left(\frac{2\left(15\sin(dx+c)^5-40\sin(dx+c)^3+33\sin(dx+c)\right)}{\sin(dx+c)^6-3\sin(dx+c)^4+3\sin(dx+c)^2-1}-15\log(\sin(dx+c)+1)+15\log(\sin(dx+c)-1)\right)-\frac{92160\,a^3b}{\cos(dx+c)^7}+\frac{10240\left(9\cos(dx+c)^2-7\right)ab^3}{\cos(dx+c)^9}}{161280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")

[Out] -1/161280*(63*b^4*(2*(15*sin(d*x + c)^9 - 70*sin(d*x + c)^7 + 128*sin(d*x + c)^5 + 70*sin(d*x + c)^3 - 15*sin(d*x + c))/(sin(d*x + c)^10 - 5*sin(d*x + c)^8 + 10*sin(d*x + c)^6 - 10*sin(d*x + c)^4 + 5*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 1260*a^2*b^2*(2*(15*sin(d*x + c)^7 - 55*sin(d*x + c)^5 + 73*sin(d*x + c)^3 + 15*sin(d*x + c))/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^4 - 4*sin(d*x + c)^2 + 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 1680*a^4*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 92160*a^3*b/cos(d*x + c)^7 + 10240*(9*cos(d*x + c)^2 - 7)*a*b^3/cos(d*x + c)^9)/d

Fricas [A]
time = 3.19, size = 251, normalized size = 0.62

$$\frac{315\left(80a^4-60a^2b^2+3b^4\right)\cos(dx+c)^{10}\log(\sin(dx+c)+1)-315\left(80a^4-60a^2b^2+3b^4\right)\cos(dx+c)^{10}\log(-\sin(dx+c)+1)+71680\,ab^3\cos(dx+c)+92160\left(a^3b-ab^3\right)\cos(dx+c)^3+42\left(15\left(80a^4-60a^2b^2+3b^4\right)\cos(dx+c)^8+10\left(80a^4-60a^2b^2+3b^4\right)\cos(dx+c)^6+8\left(80a^4-60a^2b^2+3b^4\right)\cos(dx+c)^4+384b^4+48\left(60a^2b^2-11b^4\right)\cos(dx+c)^2\right)\sin(dx+c)}{161280\,d\cos(dx+c)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")

[Out] 1/161280*(315*(80*a^4 - 60*a^2*b^2 + 3*b^4)*cos(d*x + c)^10*log(sin(d*x + c) + 1) - 315*(80*a^4 - 60*a^2*b^2 + 3*b^4)*cos(d*x + c)^10*log(-sin(d*x + c) + 1) + 71680*a*b^3*cos(d*x + c) + 92160*(a^3*b - a*b^3)*cos(d*x + c)^3 + 42*(15*(80*a^4 - 60*a^2*b^2 + 3*b^4)*cos(d*x + c)^8 + 10*(80*a^4 - 60*a^2*b^2 + 3*b^4)*cos(d*x + c)^6 + 8*(80*a^4 - 60*a^2*b^2 + 3*b^4)*cos(d*x + c)^4 + 384*b^4 + 48*(60*a^2*b^2 - 11*b^4)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^10)

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**11*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 880 vs. 2(372) = 744.
time = 0.61, size = 880, normalized size = 2.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/80640*(315*(80*a^4 - 60*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) - 315*(80*a^4 - 60*a^2*b^2 + 3*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) +
 2*(55440*a^4*tan(1/2*d*x + 1/2*c)^19 + 18900*a^2*b^2*tan(1/2*d*x + 1/2*c)^
19 - 945*b^4*tan(1/2*d*x + 1/2*c)^19 - 322560*a^3*b*tan(1/2*d*x + 1/2*c)^18
 - 213360*a^4*tan(1/2*d*x + 1/2*c)^17 + 462420*a^2*b^2*tan(1/2*d*x + 1/2*c)
^17 + 9135*b^4*tan(1/2*d*x + 1/2*c)^17 + 967680*a^3*b*tan(1/2*d*x + 1/2*c)^
16 - 645120*a*b^3*tan(1/2*d*x + 1/2*c)^16 + 450240*a^4*tan(1/2*d*x + 1/2*c)
^15 + 146160*a^2*b^2*tan(1/2*d*x + 1/2*c)^15 + 218484*b^4*tan(1/2*d*x + 1/2
*c)^15 - 2580480*a^3*b*tan(1/2*d*x + 1/2*c)^14 - 430080*a*b^3*tan(1/2*d*x +
 1/2*c)^14 - 624960*a^4*tan(1/2*d*x + 1/2*c)^13 + 468720*a^2*b^2*tan(1/2*d*
x + 1/2*c)^13 + 653940*b^4*tan(1/2*d*x + 1/2*c)^13 + 5160960*a^3*b*tan(1/2*
d*x + 1/2*c)^12 - 2150400*a*b^3*tan(1/2*d*x + 1/2*c)^12 + 332640*a^4*tan(1/
2*d*x + 1/2*c)^11 - 1096200*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 + 1183770*b^4*t
an(1/2*d*x + 1/2*c)^11 - 5806080*a^3*b*tan(1/2*d*x + 1/2*c)^10 + 1290240*a*
b^3*tan(1/2*d*x + 1/2*c)^10 + 332640*a^4*tan(1/2*d*x + 1/2*c)^9 - 1096200*a
^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 1183770*b^4*tan(1/2*d*x + 1/2*c)^9 + 451584
0*a^3*b*tan(1/2*d*x + 1/2*c)^8 - 624960*a^4*tan(1/2*d*x + 1/2*c)^7 + 468720
*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 + 653940*b^4*tan(1/2*d*x + 1/2*c)^7 - 29491
20*a^3*b*tan(1/2*d*x + 1/2*c)^6 + 1658880*a*b^3*tan(1/2*d*x + 1/2*c)^6 + 45
0240*a^4*tan(1/2*d*x + 1/2*c)^5 + 146160*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 2
18484*b^4*tan(1/2*d*x + 1/2*c)^5 + 1105920*a^3*b*tan(1/2*d*x + 1/2*c)^4 + 1
84320*a*b^3*tan(1/2*d*x + 1/2*c)^4 - 213360*a^4*tan(1/2*d*x + 1/2*c)^3 + 46
2420*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 9135*b^4*tan(1/2*d*x + 1/2*c)^3 - 138
240*a^3*b*tan(1/2*d*x + 1/2*c)^2 + 102400*a*b^3*tan(1/2*d*x + 1/2*c)^2 + 55
440*a^4*tan(1/2*d*x + 1/2*c) + 18900*a^2*b^2*tan(1/2*d*x + 1/2*c) - 945*b^4
*tan(1/2*d*x + 1/2*c) + 46080*a^3*b - 10240*a*b^3)/(tan(1/2*d*x + 1/2*c)^2
- 1)^10)/d
```

<u>Mupad</u> [B]
time = 5.15, size = 703, normalized size = 1.72

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{5\,a^4}{8}-\frac{15\,a^2\,b^2}{32}+\frac{3\,b^4}{128}\right)}{d}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{19}\,\left(\frac{11\,a^4}{8}+\frac{15\,a^2\,b^2}{32}-\frac{3\,b^4}{128}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{7}\,\left(-\frac{31\,a^4}{2}+\frac{93\,a^2\,b^2}{8}+\frac{519\,b^4}{32}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}\,\left(-\frac{31\,a^4}{2}+\frac{93\,a^2\,b^2}{8}+\frac{519\,b^4}{32}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{3}\,\left(-\frac{127\,a^4}{24}+\frac{367\,a^2\,b^2}{32}+\frac{29\,b^4}{128}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{17}\,\left(-\frac{127\,a^4}{24}+\frac{367\,a^2\,b^2}{32}+\frac{29\,b^4}{128}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{5}\,\left(\frac{67\,a^4}{6}+\frac{29\,a^2\,b^2}{8}+\frac{867\,b^4}{160}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{15}\,\left(\frac{67\,a^4}{6}+\frac{29\,a^2\,b^2}{8}+\frac{867\,b^4}{160}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{9}\,\left(\frac{33\,a^4}{4}-\frac{435\,a^2\,b^2}{16}+\frac{1879\,b^4}{64}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}\,\left(\frac{33\,a^4}{4}-\frac{435\,a^2\,b^2}{16}+\frac{1879\,b^4}{64}\right)-\frac{16\,a\,b^3}{63}+\frac{8\,a^3\,b}{7}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{11\,a^4}{8}+\frac{15\,a^2\,b^2}{32}-\frac{3\,b^4}{128}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{16}\,\left(16\,a\,b^3-24\,a^3\,b\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}\,\left(64\,a^3\,b+\frac{32\,a\,b^3}{3}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\,\left(32\,a\,b^3-144\,a^3\,b\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}\,\left(\frac{192\,a^3\,b}{7}+\frac{32\,a\,b^3}{7}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}\,\left(\frac{160\,a\,b^3}{63}-\frac{24\,a^3\,b}{7}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}\,\left(\frac{160\,a\,b^3}{3}-128\,a^3\,b\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}\,\left(\frac{288\,a\,b^3}{7}-\frac{512\,a^3\,b}{7}\right)+112\,a^3\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}-8\,a^3\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{18}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{20}-10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{18}+45\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{16}-120\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}+210\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}-252\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+210\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}-120\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}+45\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}-10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^11,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*((5*a^4)/8 + (3*b^4)/128 - (15*a^2*b^2)/32))/d +
 (tan(c/2 + (d*x)/2)^19*((11*a^4)/8 - (3*b^4)/128 + (15*a^2*b^2)/32) + tan(
c/2 + (d*x)/2)^7*((519*b^4)/32 - (31*a^4)/2 + (93*a^2*b^2)/8) + tan(c/2 + (
d*x)/2)^13*((519*b^4)/32 - (31*a^4)/2 + (93*a^2*b^2)/8) + tan(c/2 + (d*x)/2
)^3*((29*b^4)/128 - (127*a^4)/24 + (367*a^2*b^2)/32) + tan(c/2 + (d*x)/2)^1
7*((29*b^4)/128 - (127*a^4)/24 + (367*a^2*b^2)/32) + tan(c/2 + (d*x)/2)^5*(
(67*a^4)/6 + (867*b^4)/160 + (29*a^2*b^2)/8) + tan(c/2 + (d*x)/2)^15*((67*a
^4)/6 + (867*b^4)/160 + (29*a^2*b^2)/8) + tan(c/2 + (d*x)/2)^9*((33*a^4)/4
+ (1879*b^4)/64 - (435*a^2*b^2)/16) + tan(c/2 + (d*x)/2)^11*((33*a^4)/4 + (
1879*b^4)/64 - (435*a^2*b^2)/16) - (16*a*b^3)/63 + (8*a^3*b)/7 + tan(c/2 +
(d*x)/2)*((11*a^4)/8 - (3*b^4)/128 + (15*a^2*b^2)/32) - tan(c/2 + (d*x)/2)^
16*(16*a*b^3 - 24*a^3*b) - tan(c/2 + (d*x)/2)^14*((32*a*b^3)/3 + 64*a^3*b)
+ tan(c/2 + (d*x)/2)^10*(32*a*b^3 - 144*a^3*b) + tan(c/2 + (d*x)/2)^4*((32*
a*b^3)/7 + (192*a^3*b)/7) + tan(c/2 + (d*x)/2)^2*((160*a*b^3)/63 - (24*a^3*
b)/7) - tan(c/2 + (d*x)/2)^12*((160*a*b^3)/3 - 128*a^3*b) + tan(c/2 + (d*x)
/2)^6*((288*a*b^3)/7 - (512*a^3*b)/7) + 112*a^3*b*tan(c/2 + (d*x)/2)^8 - 8*
a^3*b*tan(c/2 + (d*x)/2)^18)/(d*(45*tan(c/2 + (d*x)/2)^4 - 10*tan(c/2 + (d*
x)/2)^2 - 120*tan(c/2 + (d*x)/2)^6 + 210*tan(c/2 + (d*x)/2)^8 - 252*tan(c/2
 + (d*x)/2)^10 + 210*tan(c/2 + (d*x)/2)^12 - 120*tan(c/2 + (d*x)/2)^14 + 45
*tan(c/2 + (d*x)/2)^16 - 10*tan(c/2 + (d*x)/2)^18 + tan(c/2 + (d*x)/2)^20 +
 1))
```

3.91 $\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx$

Optimal. Leaf size=254

$$\frac{a^4\tan(c+dx)}{d}+\frac{2a^3b\tan^2(c+dx)}{d}+\frac{a^2(a^2+2b^2)\tan^3(c+dx)}{d}+\frac{ab(3a^2+b^2)\tan^4(c+dx)}{d}+\frac{(3a^4+18a^2b^2+\cdots}{5d}$$

```
[Out] a^4*tan(d*x+c)/d+2*a^3*b*tan(d*x+c)^2/d+a^2*(a^2+2*b^2)*tan(d*x+c)^3/d+a*b*
(3*a^2+b^2)*tan(d*x+c)^4/d+1/5*(3*a^4+18*a^2*b^2+b^4)*tan(d*x+c)^5/d+2*a*b*
(a^2+b^2)*tan(d*x+c)^6/d+1/7*(a^4+18*a^2*b^2+3*b^4)*tan(d*x+c)^7/d+1/2*a*b*
(a^2+3*b^2)*tan(d*x+c)^8/d+1/3*b^2*(2*a^2+b^2)*tan(d*x+c)^9/d+2/5*a*b^3*tan
(d*x+c)^10/d+1/11*b^4*tan(d*x+c)^11/d
```

Rubi [A]

time = 0.16, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3167, 962}

$$\frac{a^4\tan(c+dx)}{d}+\frac{2a^3b\tan^2(c+dx)}{d}+\frac{b^2(2a^2+b^2)\tan^9(c+dx)}{3d}+\frac{ab(a^2+3b^2)\tan^8(c+dx)}{2d}+\frac{2ab(a^2+b^2)\tan^6(c+dx)}{d}+\frac{ab(3a^2+b^2)\tan^4(c+dx)}{d}+\frac{a^2(a^2+2b^2)\tan^3(c+dx)}{d}+\frac{(a^4+18a^2b^2+3b^4)\tan^7(c+dx)}{7d}+\frac{(3a^4+18a^2b^2+b^4)\tan^5(c+dx)}{5d}+\frac{2ab^3\tan^{10}(c+dx)}{5d}+\frac{b^4\tan^{11}(c+dx)}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^12*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (a^4*Tan[c + d*x])/d + (2*a^3*b*Tan[c + d*x]^2)/d + (a^2*(a^2 + 2*b^2)*Tan[
c + d*x]^3)/d + (a*b*(3*a^2 + b^2)*Tan[c + d*x]^4)/d + ((3*a^4 + 18*a^2*b^2
 + b^4)*Tan[c + d*x]^5)/(5*d) + (2*a*b*(a^2 + b^2)*Tan[c + d*x]^6)/d + ((a^
4 + 18*a^2*b^2 + 3*b^4)*Tan[c + d*x]^7)/(7*d) + (a*b*(a^2 + 3*b^2)*Tan[c +
d*x]^8)/(2*d) + (b^2*(2*a^2 + b^2)*Tan[c + d*x]^9)/(3*d) + (2*a*b^3*Tan[c +
 d*x]^10)/(5*d) + (b^4*Tan[c + d*x]^11)/(11*d)
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^4\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^4(1+x^2)^3}{x^{12}}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{b^4}{x^{12}} + \frac{4ab^3}{x^{11}} + \frac{3(2a^2b^2+b^4)}{x^{10}} + \frac{4ab(a^2+3b^2)}{x^9} + \frac{a^4+}{}\right.\right.}{}$$

$$= \frac{a^4\tan(c+dx)}{d} + \frac{2a^3b\tan^2(c+dx)}{d} + \frac{a^2(a^2+2b^2)\tan}{d}$$

Mathematica [A]
time = 1.78, size = 175, normalized size = 0.69

$$\frac{\frac{1}{5}(a^2+b^2)^3(a+b\tan(c+dx))^5 - a(a^2+b^2)^2(a+b\tan(c+dx))^6 + \frac{3}{7}(a^2+b^2)(5a^2+b^2)(a+b\tan(c+dx))^7 - \frac{1}{2}a(5a^2+3b^2)(a+b\tan(c+dx))^8 + \frac{1}{3}(5a^2+b^2)(a+b\tan(c+dx))^9 - \frac{3}{5}a(a+b\tan(c+dx))^{10} + \frac{1}{11}(a+b\tan(c+dx))^{11}}{b^7d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^12*(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (((a^2 + b^2)^3*(a + b*Tan[c + d*x])^5)/5 - a*(a^2 + b^2)^2*(a + b*Tan[c +
d*x])^6 + (3*(a^2 + b^2)*(5*a^2 + b^2)*(a + b*Tan[c + d*x])^7)/7 - (a*(5*a^
2 + 3*b^2)*(a + b*Tan[c + d*x])^8)/2 + ((5*a^2 + b^2)*(a + b*Tan[c + d*x])^
9)/3 - (3*a*(a + b*Tan[c + d*x])^10)/5 + (a + b*Tan[c + d*x])^11/11)/(b^7*d
)
```

Maple [A]
time = 0.37, size = 300, normalized size = 1.18

method	result
derivativedivides	$\frac{-a^4\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)+\frac{a^3b}{2\cos(dx+c)^8}+6a^2b^2\left(\frac{\sin^3(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^3(dx+c\right.}{21\cos(dx+c}\right.}{}$
default	$\frac{-a^4\left(-\frac{16}{35}-\frac{\left(\sec^6(dx+c)\right)}{7}-\frac{6\left(\sec^4(dx+c)\right)}{35}-\frac{8\left(\sec^2(dx+c)\right)}{35}\right)\tan(dx+c)+\frac{a^3b}{2\cos(dx+c)^8}+6a^2b^2\left(\frac{\sin^3(dx+c)}{9\cos(dx+c)^9}+\frac{2\left(\sin^3(dx+c\right.}{21\cos(dx+c}\right.}{}$
risch	$\frac{32i\left(33a^4+b^4-22a^2b^2-13860ia^3b\,e^{10i(dx+c)}-924iab^3e^{10i(dx+c)}-6930a^2b^2e^{14i(dx+c)}+165b^4e^{6i(dx+c)}+11b^4e^{2i(dx+c)}-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^4*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan
(d*x+c)+1/2*a^3*b/cos(d*x+c)^8+6*a^2*b^2*(1/9*sin(d*x+c)^3/cos(d*x+c)^9+2/2
1*sin(d*x+c)^3/cos(d*x+c)^7+8/105*sin(d*x+c)^3/cos(d*x+c)^5+16/315*sin(d*x+
```

```
c)^3/cos(d*x+c)^3)+4*a*b^3*(1/10*sin(d*x+c)^4/cos(d*x+c)^10+3/40*sin(d*x+c)
^4/cos(d*x+c)^8+1/20*sin(d*x+c)^4/cos(d*x+c)^6+1/40*sin(d*x+c)^4/cos(d*x+c)
^4)+b^4*(1/11*sin(d*x+c)^5/cos(d*x+c)^11+2/33*sin(d*x+c)^5/cos(d*x+c)^9+8/2
31*sin(d*x+c)^5/cos(d*x+c)^7+16/1155*sin(d*x+c)^5/cos(d*x+c)^5))
```

Maxima [**A**]
time = 0.29, size = 233, normalized size = 0.92

$$\frac{66\left(5\,\tan\left(dx+c\right)^{7}+21\,\tan\left(dx+c\right)^{5}+35\,\tan\left(dx+c\right)^{3}+35\,\tan\left(dx+c\right)\right)a^{4}+44\left(35\,\tan\left(dx+c\right)^{9}+135\,\tan\left(dx+c\right)^{7}+189\,\tan\left(dx+c\right)^{5}+105\,\tan\left(dx+c\right)^{3}\right)a^{2}b^{2}+2\left(105\,\tan\left(dx+c\right)^{11}+385\,\tan\left(dx+c\right)^{9}+495\,\tan\left(dx+c\right)^{7}+231\,\tan\left(dx+c\right)^{5}\right)b^{4}-\frac{231\left(5\,\sin\left(dx+c\right)^{2}-1\right)ab^{3}}{\sin\left(dx+c\right)^{10}-5\,\sin\left(dx+c\right)^{8}+10\,\sin\left(dx+c\right)^{6}-10\,\sin\left(dx+c\right)^{4}+5\,\sin\left(dx+c\right)^{2}-1}+\frac{1155\,a^{3}b}{\left(\sin\left(dx+c\right)^{2}-1\right)^{4}}}{2310\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima"
)
```

```
[Out] 1/2310*(66*(5*tan(d*x + c)^7 + 21*tan(d*x + c)^5 + 35*tan(d*x + c)^3 + 35*t
an(d*x + c))*a^4 + 44*(35*tan(d*x + c)^9 + 135*tan(d*x + c)^7 + 189*tan(d*x
 + c)^5 + 105*tan(d*x + c)^3)*a^2*b^2 + 2*(105*tan(d*x + c)^11 + 385*tan(d*
x + c)^9 + 495*tan(d*x + c)^7 + 231*tan(d*x + c)^5)*b^4 - 231*(5*sin(d*x +
c)^2 - 1)*a*b^3/(sin(d*x + c)^10 - 5*sin(d*x + c)^8 + 10*sin(d*x + c)^6 - 1
0*sin(d*x + c)^4 + 5*sin(d*x + c)^2 - 1) + 1155*a^3*b/(sin(d*x + c)^2 - 1)^
4)/d
```

Fricas [**A**]
time = 3.27, size = 194, normalized size = 0.76

$$\frac{924\,ab^{3}\cos\left(dx+c\right)+1155\left(a^{3}b-ab^{3}\right)\cos\left(dx+c\right)^{3}+2\left(16\left(33\,a^{4}-22\,a^{2}b^{2}+b^{4}\right)\cos\left(dx+c\right)^{10}+8\left(33\,a^{4}-22\,a^{2}b^{2}+b^{4}\right)\cos\left(dx+c\right)^{8}+6\left(33\,a^{4}-22\,a^{2}b^{2}+b^{4}\right)\cos\left(dx+c\right)^{6}+5\left(33\,a^{4}-22\,a^{2}b^{2}+b^{4}\right)\cos\left(dx+c\right)^{4}+105\,b^{4}+70\left(11\,a^{2}b^{2}-2\,b^{4}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{2310\,d\cos\left(dx+c\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas"
)
```

```
[Out] 1/2310*(924*a*b^3*cos(d*x + c) + 1155*(a^3*b - a*b^3)*cos(d*x + c)^3 + 2*(1
6*(33*a^4 - 22*a^2*b^2 + b^4)*cos(d*x + c)^10 + 8*(33*a^4 - 22*a^2*b^2 + b^
4)*cos(d*x + c)^8 + 6*(33*a^4 - 22*a^2*b^2 + b^4)*cos(d*x + c)^6 + 5*(33*a^
4 - 22*a^2*b^2 + b^4)*cos(d*x + c)^4 + 105*b^4 + 70*(11*a^2*b^2 - 2*b^4)*co
s(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^11)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**12*(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.59, size = 284, normalized size = 1.12

$$\frac{210 b^4 \tan(dx+c)^{11} + 924 a b^3 \tan(dx+c)^{10} + 1540 a^2 b^2 \tan(dx+c)^9 + 770 b^4 \tan(dx+c)^9 + 1155 a^3 b \tan(dx+c)^8 + 3465 a b^3 \tan(dx+c)^8 + 330 a^4 \tan(dx+c)^7 + 5940 a^2 b^2 \tan(dx+c)^7 + 990 b^4 \tan(dx+c)^7 + 4620 a^3 b \tan(dx+c)^6 + 4620 a b^3 \tan(dx+c)^6 + 1386 a^4 \tan(dx+c)^5 + 8316 a^2 b^2 \tan(dx+c)^5 + 462 b^4 \tan(dx+c)^5 + 6930 a^3 b \tan(dx+c)^4 + 2310 a b^3 \tan(dx+c)^4 + 2310 a^4 \tan(dx+c)^3 + 4620 a^2 b^2 \tan(dx+c)^3 + 4620 a^3 b \tan(dx+c)^2 + 2310 a^4 \tan(dx+c)}{2310 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/2310*(210*b^4*tan(d*x + c)^11 + 924*a*b^3*tan(d*x + c)^10 + 1540*a^2*b^2*
tan(d*x + c)^9 + 770*b^4*tan(d*x + c)^9 + 1155*a^3*b*tan(d*x + c)^8 + 3465*
a*b^3*tan(d*x + c)^8 + 330*a^4*tan(d*x + c)^7 + 5940*a^2*b^2*tan(d*x + c)^7
 + 990*b^4*tan(d*x + c)^7 + 4620*a^3*b*tan(d*x + c)^6 + 4620*a*b^3*tan(d*x
+ c)^6 + 1386*a^4*tan(d*x + c)^5 + 8316*a^2*b^2*tan(d*x + c)^5 + 462*b^4*ta
n(d*x + c)^5 + 6930*a^3*b*tan(d*x + c)^4 + 2310*a*b^3*tan(d*x + c)^4 + 2310
*a^4*tan(d*x + c)^3 + 4620*a^2*b^2*tan(d*x + c)^3 + 4620*a^3*b*tan(d*x + c)
^2 + 2310*a^4*tan(d*x + c))/d
```

Mupad [B]
time = 4.82, size = 560, normalized size = 2.20

$$-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(\frac{226a^4}{5}+\frac{32b^4}{5}-\frac{64a^2b^2}{5}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{17}\left(\frac{226a^4}{5}+\frac{32b^4}{5}-\frac{64a^2b^2}{5}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9\left(\frac{1308a^4}{7}+\frac{992b^4}{21}-\frac{3008a^2b^2}{21}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\left(\frac{1308a^4}{7}+\frac{992b^4}{21}-\frac{3008a^2b^2}{21}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(\frac{576b^4}{35}-\frac{3952a^4}{35}+\frac{3008a^2b^2}{35}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}\left(\frac{576b^4}{35}-\frac{3952a^4}{35}+\frac{3008a^2b^2}{35}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\left(\frac{10624b^4}{231}-\frac{1528a^4}{7}+\frac{2272a^2b^2}{21}\right)+2a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{21}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(12a^4-16a^2b^2\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{19}\left(12a^4-16a^2b^2\right)+2a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(16ab^3-24a^3b\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{18}\left(16ab^3-24a^3b\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(16ab^3+80a^3b\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}\left(16ab^3+80a^3b\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\left(80ab^3-176a^3b\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\left(80ab^3-176a^3b\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\left(\frac{112ab^3}{5}-224a^3b\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\left(\frac{112ab^3}{5}-224a^3b\right)+8a^3b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-8a^3b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{20}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^4/cos(c + d*x)^12,x)
```

```
[Out] -(tan(c/2 + (d*x)/2)^5*((226*a^4)/5 + (32*b^4)/5 - (64*a^2*b^2)/5) + tan(c/
2 + (d*x)/2)^17*((226*a^4)/5 + (32*b^4)/5 - (64*a^2*b^2)/5) + tan(c/2 + (d*
x)/2)^9*((1308*a^4)/7 + (992*b^4)/21 - (3008*a^2*b^2)/21) + tan(c/2 + (d*x)
/2)^13*((1308*a^4)/7 + (992*b^4)/21 - (3008*a^2*b^2)/21) + tan(c/2 + (d*x)/
2)^7*((576*b^4)/35 - (3952*a^4)/35 + (3008*a^2*b^2)/35) + tan(c/2 + (d*x)/2
)^15*((576*b^4)/35 - (3952*a^4)/35 + (3008*a^2*b^2)/35) + tan(c/2 + (d*x)/2
)^11*((10624*b^4)/231 - (1528*a^4)/7 + (2272*a^2*b^2)/21) + 2*a^4*tan(c/2 +
 (d*x)/2)^21 - tan(c/2 + (d*x)/2)^3*(12*a^4 - 16*a^2*b^2) - tan(c/2 + (d*x)
/2)^19*(12*a^4 - 16*a^2*b^2) + 2*a^4*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2
)^4*(16*a*b^3 - 24*a^3*b) - tan(c/2 + (d*x)/2)^18*(16*a*b^3 - 24*a^3*b) + t
an(c/2 + (d*x)/2)^6*(16*a*b^3 + 80*a^3*b) - tan(c/2 + (d*x)/2)^16*(16*a*b^3
 + 80*a^3*b) + tan(c/2 + (d*x)/2)^8*(80*a*b^3 - 176*a^3*b) - tan(c/2 + (d*x
)/2)^14*(80*a*b^3 - 176*a^3*b) - tan(c/2 + (d*x)/2)^10*((112*a*b^3)/5 - 224
*a^3*b) + tan(c/2 + (d*x)/2)^12*((112*a*b^3)/5 - 224*a^3*b) + 8*a^3*b*tan(c
/2 + (d*x)/2)^2 - 8*a^3*b*tan(c/2 + (d*x)/2)^20)/(d*(tan(c/2 + (d*x)/2)^2 -
 1)^11)
```

3.92 $\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=515

$$\frac{63a^5x}{256}+\frac{35}{128}a^3b^2x+\frac{15}{256}ab^4x-\frac{5a^2b^3\cos^8(c+dx)}{4d}-\frac{a^4b\cos^{10}(c+dx)}{2d}+\frac{a^2b^3\cos^{10}(c+dx)}{d}+\frac{63a^5\cos(c+dx)\,\mathrm{si}}{256d}$$

```
[Out] 63/256*a^5*x+35/128*a^3*b^2*x+15/256*a*b^4*x-5/4*a^2*b^3*cos(d*x+c)^8/d-1/2
*a^4*b*cos(d*x+c)^10/d+a^2*b^3*cos(d*x+c)^10/d+63/256*a^5*cos(d*x+c)*sin(d*
x+c)/d+35/128*a^3*b^2*cos(d*x+c)*sin(d*x+c)/d+15/256*a*b^4*cos(d*x+c)*sin(d
*x+c)/d+21/128*a^5*cos(d*x+c)^3*sin(d*x+c)/d+35/192*a^3*b^2*cos(d*x+c)^3*si
n(d*x+c)/d+5/128*a*b^4*cos(d*x+c)^3*sin(d*x+c)/d+21/160*a^5*cos(d*x+c)^5*si
n(d*x+c)/d+7/48*a^3*b^2*cos(d*x+c)^5*sin(d*x+c)/d+1/32*a*b^4*cos(d*x+c)^5*s
in(d*x+c)/d+9/80*a^5*cos(d*x+c)^7*sin(d*x+c)/d+1/8*a^3*b^2*cos(d*x+c)^7*sin
(d*x+c)/d-3/16*a*b^4*cos(d*x+c)^7*sin(d*x+c)/d+1/10*a^5*cos(d*x+c)^9*sin(d*
x+c)/d-a^3*b^2*cos(d*x+c)^9*sin(d*x+c)/d-1/2*a*b^4*cos(d*x+c)^7*sin(d*x+c)^
3/d+1/6*b^5*sin(d*x+c)^6/d-1/4*b^5*sin(d*x+c)^8/d+1/10*b^5*sin(d*x+c)^10/d
```

Rubi [A]
time = 0.35, antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3169, 2715, 8, 2645, 30, 2648, 14, 2644, 272, 45}

$$\frac{a^5\sin(c+dx)\cos^9(c+dx)}{10d}+\frac{9a^5\sin(c+dx)\cos^7(c+dx)}{80d}+\frac{21a^5\sin(c+dx)\cos^5(c+dx)}{160d}+\frac{21a^5\sin(c+dx)\cos^3(c+dx)}{128d}+\frac{63a^5\sin(c+dx)\cos(c+dx)}{256d}+\frac{63a^5x}{256}-\frac{a^4b\cos^{10}(c+dx)}{2d}-\frac{a^3b^2\sin(c+dx)\cos^9(c+dx)}{d}+\frac{a^3b^2\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{7a^3b^2\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{35a^3b^2\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{35a^3b^2\sin(c+dx)\cos(c+dx)}{128d}+\frac{35}{128}a^3b^2x+\frac{a^2b^3\cos^{10}(c+dx)}{d}-\frac{5a^2b^3\cos^8(c+dx)}{4d}-\frac{ab^4\sin^3(c+dx)\cos^7(c+dx)}{2d}-\frac{3ab^4\sin(c+dx)\cos^7(c+dx)}{16d}+\frac{ab^4\sin(c+dx)\cos^5(c+dx)}{32d}+\frac{5ab^4\sin(c+dx)\cos^3(c+dx)}{128d}+\frac{15ab^4\sin(c+dx)\cos(c+dx)}{256d}+\frac{15}{256}ab^4x+\frac{b^5\sin^{10}(c+dx)}{10d}-\frac{b^5\sin^8(c+dx)}{4d}+\frac{b^5\sin^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (63*a^5*x)/256 + (35*a^3*b^2*x)/128 + (15*a*b^4*x)/256 - (5*a^2*b^3*Cos[c +
 d*x]^8)/(4*d) - (a^4*b*Cos[c + d*x]^10)/(2*d) + (a^2*b^3*Cos[c + d*x]^10)/
d + (63*a^5*Cos[c + d*x]*Sin[c + d*x])/(256*d) + (35*a^3*b^2*Cos[c + d*x]*S
in[c + d*x])/(128*d) + (15*a*b^4*Cos[c + d*x]*Sin[c + d*x])/(256*d) + (21*a
^5*Cos[c + d*x]^3*Sin[c + d*x])/(128*d) + (35*a^3*b^2*Cos[c + d*x]^3*Sin[c
+ d*x])/(192*d) + (5*a*b^4*Cos[c + d*x]^3*Sin[c + d*x])/(128*d) + (21*a^5*C
os[c + d*x]^5*Sin[c + d*x])/(160*d) + (7*a^3*b^2*Cos[c + d*x]^5*Sin[c + d*x
])/(48*d) + (a*b^4*Cos[c + d*x]^5*Sin[c + d*x])/(32*d) + (9*a^5*Cos[c + d*x
]^7*Sin[c + d*x])/(80*d) + (a^3*b^2*Cos[c + d*x]^7*Sin[c + d*x])/(8*d) - (3
*a*b^4*Cos[c + d*x]^7*Sin[c + d*x])/(16*d) + (a^5*Cos[c + d*x]^9*Sin[c + d*
x])/(10*d) - (a^3*b^2*Cos[c + d*x]^9*Sin[c + d*x])/d - (a*b^4*Cos[c + d*x]^
7*Sin[c + d*x]^3)/(2*d) + (b^5*Sin[c + d*x]^6)/(6*d) - (b^5*Sin[c + d*x]^8)
/(4*d) + (b^5*Sin[c + d*x]^10)/(10*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int \left(a^5\cos^{10}(c+dx)+5a^4b\cos^9(c+dx)\sin(c+dx)+10\right.$$

$$= a^5\int \cos^{10}(c+dx)\,dx+\left(5a^4b\right)\int \cos^9(c+dx)\sin(c+d$$

$$= \frac{a^5\cos^9(c+dx)\sin(c+dx)}{10d}-\frac{a^3b^2\cos^9(c+dx)\sin(c+}{d}$$

$$= -\frac{a^4b\cos^{10}(c+dx)}{2d}+\frac{9a^5\cos^7(c+dx)\sin(c+dx)}{80d}+\frac{a^3}{}$$

$$= -\frac{5a^2b^3\cos^8(c+dx)}{4d}-\frac{a^4b\cos^{10}(c+dx)}{2d}+\frac{a^2b^3\cos^{10}(c}{d}$$

$$= -\frac{5a^2b^3\cos^8(c+dx)}{4d}-\frac{a^4b\cos^{10}(c+dx)}{2d}+\frac{a^2b^3\cos^{10}(c}{d}$$

$$= -\frac{5a^2b^3\cos^8(c+dx)}{4d}-\frac{a^4b\cos^{10}(c+dx)}{2d}+\frac{a^2b^3\cos^{10}(c}{d}$$

$$= \frac{63a^5x}{256}+\frac{35}{128}a^3b^2x+\frac{15}{256}ab^4x-\frac{5a^2b^3\cos^8(c+dx)}{4d}-\frac{a}{}$$

Mathematica [A]
time = 1.30, size = 307, normalized size = 0.60

$$\frac{120a(63a^4+70a^2b^2+15b^4)(c+dx)-300b(21a^4+14a^2b^2+b^4)\cos(2(c+dx))-1200a^2b(3a^2+b^2)\cos(4(c+dx))+50b(-27a^4+6a^2b^2+b^4)\cos(6(c+dx))-300a^2b(a^2-b^2)\cos(8(c+dx))-6b(5a^4-10a^2b^2+b^4)\cos(10(c+dx))+300a(21a^4+14a^2b^2+b^4)\sin(2(c+dx))+600a(3a^4-2a^2b^2-b^4)\sin(4(c+dx))+50a(9a^4-26a^2b^2-3b^4)\sin(6(c+dx))+75a(a^4-6a^2b^2+b^4)\sin(8(c+dx))+6a(a^4-10a^2b^2+5b^4)\sin(10(c+dx))}{30720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (120*a*(63*a^4 + 70*a^2*b^2 + 15*b^4)*(c + d*x) - 300*b*(21*a^4 + 14*a^2*b^
2 + b^4)*Cos[2*(c + d*x)] - 1200*a^2*b*(3*a^2 + b^2)*Cos[4*(c + d*x)] + 50*
b*(-27*a^4 + 6*a^2*b^2 + b^4)*Cos[6*(c + d*x)] - 300*a^2*b*(a^2 - b^2)*Cos[
8*(c + d*x)] - 6*b*(5*a^4 - 10*a^2*b^2 + b^4)*Cos[10*(c + d*x)] + 300*a*(21
*a^4 + 14*a^2*b^2 + b^4)*Sin[2*(c + d*x)] + 600*a*(3*a^4 - 2*a^2*b^2 - b^4)
*Sin[4*(c + d*x)] + 50*a*(9*a^4 - 26*a^2*b^2 - 3*b^4)*Sin[6*(c + d*x)] + 75
*a*(a^4 - 6*a^2*b^2 + b^4)*Sin[8*(c + d*x)] + 6*a*(a^4 - 10*a^2*b^2 + 5*b^4
)*Sin[10*(c + d*x)])/(30720*d)
```

Maple [A]
time = 0.45, size = 335, normalized size = 0.65 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^5*(-1/10*sin(d*x+c)^4*cos(d*x+c)^6-1/20*sin(d*x+c)^2*cos(d*x+c)^6-1/
60*cos(d*x+c)^6)+5*a*b^4*(-1/10*sin(d*x+c)^3*cos(d*x+c)^7-3/80*sin(d*x+c)*c
os(d*x+c)^7+1/160*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c
)+3/256*d*x+3/256*c)+10*b^3*a^2*(-1/10*sin(d*x+c)^2*cos(d*x+c)^8-1/40*cos(d
*x+c)^8)+10*a^3*b^2*(-1/10*sin(d*x+c)*cos(d*x+c)^9+1/80*(cos(d*x+c)^7+7/6*c
os(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+7/256*d*x+7/256
*c)-1/2*b*a^4*cos(d*x+c)^10+a^5*(1/10*(cos(d*x+c)^9+9/8*cos(d*x+c)^7+21/16*
cos(d*x+c)^5+105/64*cos(d*x+c)^3+315/128*cos(d*x+c))*sin(d*x+c)+63/256*d*x+
63/256*c))
```

Maxima [A]
time = 0.30, size = 290, normalized size = 0.56

$$-\frac{15360\,a^{4}b\cos\left(dx+c\right)^{10}-3\,\left(32\,\sin\left(2\,dx+2\,c\right)^{5}-640\,\sin\left(2\,dx+2\,c\right)^{3}+2520\,dx+2520\,c+25\,\sin\left(8\,dx+8\,c\right)+600\,\sin\left(4\,dx+4\,c\right)+2560\,\sin\left(2\,dx+2\,c\right)\right)a^{5}+10\,\left(96\,\sin\left(2\,dx+2\,c\right)^{5}-640\,\sin\left(2\,dx+2\,c\right)^{3}-840\,dx-840\,c+45\,\sin\left(8\,dx+8\,c\right)+120\,\sin\left(4\,dx+4\,c\right)\right)a^{3}b^{2}+7680\,\left(4\,\sin\left(dx+c\right)^{10}-15\,\sin\left(dx+c\right)^{8}+20\,\sin\left(dx+c\right)^{6}-10\,\sin\left(dx+c\right)^{4}\right)a^{2}b^{3}-15\,\left(32\,\sin\left(2\,dx+2\,c\right)^{5}+120\,dx+120\,c+5\,\sin\left(8\,dx+8\,c\right)-40\,\sin\left(4\,dx+4\,c\right)\right)ab^{4}-512\,\left(6\,\sin\left(dx+c\right)^{10}-15\,\sin\left(dx+c\right)^{8}+10\,\sin\left(dx+c\right)^{6}\right)b^{5}}{30720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/30720*(15360*a^4*b*cos(d*x + c)^10 - 3*(32*sin(2*d*x + 2*c)^5 - 640*sin(
2*d*x + 2*c)^3 + 2520*d*x + 2520*c + 25*sin(8*d*x + 8*c) + 600*sin(4*d*x +
4*c) + 2560*sin(2*d*x + 2*c))*a^5 + 10*(96*sin(2*d*x + 2*c)^5 - 640*sin(2*d
*x + 2*c)^3 - 840*d*x - 840*c + 45*sin(8*d*x + 8*c) + 120*sin(4*d*x + 4*c))
*a^3*b^2 + 7680*(4*sin(d*x + c)^10 - 15*sin(d*x + c)^8 + 20*sin(d*x + c)^6
- 10*sin(d*x + c)^4)*a^2*b^3 - 15*(32*sin(2*d*x + 2*c)^5 + 120*d*x + 120*c
+ 5*sin(8*d*x + 8*c) - 40*sin(4*d*x + 4*c))*a*b^4 - 512*(6*sin(d*x + c)^10
- 15*sin(d*x + c)^8 + 10*sin(d*x + c)^6)*b^5)/d
```

Fricas [A]
time = 2.87, size = 250, normalized size = 0.49

$$-\frac{640\,b^{5}\cos\left(dx+c\right)^{6}+384\,\left(5\,a^{4}b-10\,a^{2}b^{3}+b^{5}\right)\cos\left(dx+c\right)^{10}+960\,\left(5\,a^{2}b^{3}-b^{5}\right)\cos\left(dx+c\right)^{8}-15\,\left(63\,a^{5}+70\,a^{3}b^{2}+15\,ab^{4}\right)dx-\left(384\,\left(a^{5}-10\,a^{3}b^{2}+5\,ab^{4}\right)\cos\left(dx+c\right)^{9}+48\,\left(9\,a^{5}+10\,a^{3}b^{2}-55\,ab^{4}\right)\cos\left(dx+c\right)^{7}+8\,\left(63\,a^{5}+70\,a^{3}b^{2}+15\,ab^{4}\right)\cos\left(dx+c\right)^{5}+10\,\left(63\,a^{5}+70\,a^{3}b^{2}+15\,ab^{4}\right)\cos\left(dx+c\right)^{3}+15\,\left(63\,a^{5}+70\,a^{3}b^{2}+15\,ab^{4}\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{3840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/3840*(640*b^5*cos(d*x + c)^6 + 384*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x
+ c)^10 + 960*(5*a^2*b^3 - b^5)*cos(d*x + c)^8 - 15*(63*a^5 + 70*a^3*b^2 +
15*a*b^4)*d*x - (384*(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^9 + 48*(9*a^
5 + 10*a^3*b^2 - 55*a*b^4)*cos(d*x + c)^7 + 8*(63*a^5 + 70*a^3*b^2 + 15*a*b
^4)*cos(d*x + c)^5 + 10*(63*a^5 + 70*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^3 + 1
5*(63*a^5 + 70*a^3*b^2 + 15*a*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A]
time = 2.04, size = 979, normalized size = 1.90

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Piecewise((63*a**5*x*sin(c + d*x)**10/256 + 315*a**5*x*sin(c + d*x)**8*cos(
c + d*x)**2/256 + 315*a**5*x*sin(c + d*x)**6*cos(c + d*x)**4/128 + 315*a**5
*x*sin(c + d*x)**4*cos(c + d*x)**6/128 + 315*a**5*x*sin(c + d*x)**2*cos(c +
 d*x)**8/256 + 63*a**5*x*cos(c + d*x)**10/256 + 63*a**5*sin(c + d*x)**9*cos
(c + d*x)/(256*d) + 147*a**5*sin(c + d*x)**7*cos(c + d*x)**3/(128*d) + 21*a
**5*sin(c + d*x)**5*cos(c + d*x)**5/(10*d) + 237*a**5*sin(c + d*x)**3*cos(c
 + d*x)**7/(128*d) + 193*a**5*sin(c + d*x)*cos(c + d*x)**9/(256*d) - a**4*b
*cos(c + d*x)**10/(2*d) + 35*a**3*b**2*x*sin(c + d*x)**10/128 + 175*a**3*b*
*2*x*sin(c + d*x)**8*cos(c + d*x)**2/128 + 175*a**3*b**2*x*sin(c + d*x)**6*
cos(c + d*x)**4/64 + 175*a**3*b**2*x*sin(c + d*x)**4*cos(c + d*x)**6/64 + 1
75*a**3*b**2*x*sin(c + d*x)**2*cos(c + d*x)**8/128 + 35*a**3*b**2*x*cos(c +
 d*x)**10/128 + 35*a**3*b**2*sin(c + d*x)**9*cos(c + d*x)/(128*d) + 245*a**
3*b**2*sin(c + d*x)**7*cos(c + d*x)**3/(192*d) + 7*a**3*b**2*sin(c + d*x)**
5*cos(c + d*x)**5/(3*d) + 395*a**3*b**2*sin(c + d*x)**3*cos(c + d*x)**7/(19
2*d) - 35*a**3*b**2*sin(c + d*x)*cos(c + d*x)**9/(128*d) - 5*a**2*b**3*sin(
c + d*x)**2*cos(c + d*x)**8/(4*d) - a**2*b**3*cos(c + d*x)**10/(4*d) + 15*a
*b**4*x*sin(c + d*x)**10/256 + 75*a*b**4*x*sin(c + d*x)**8*cos(c + d*x)**2/
256 + 75*a*b**4*x*sin(c + d*x)**6*cos(c + d*x)**4/128 + 75*a*b**4*x*sin(c +
 d*x)**4*cos(c + d*x)**6/128 + 75*a*b**4*x*sin(c + d*x)**2*cos(c + d*x)**8/
256 + 15*a*b**4*x*cos(c + d*x)**10/256 + 15*a*b**4*sin(c + d*x)**9*cos(c +
d*x)/(256*d) + 35*a*b**4*sin(c + d*x)**7*cos(c + d*x)**3/(128*d) + a*b**4*s
in(c + d*x)**5*cos(c + d*x)**5/(2*d) - 35*a*b**4*sin(c + d*x)**3*cos(c + d*
x)**7/(128*d) - 15*a*b**4*sin(c + d*x)*cos(c + d*x)**9/(256*d) - b**5*sin(c
 + d*x)**4*cos(c + d*x)**6/(6*d) - b**5*sin(c + d*x)**2*cos(c + d*x)**8/(12
*d) - b**5*cos(c + d*x)**10/(60*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**5*
cos(c)**5, True))
```

Giac [A]
time = 0.61, size = 342, normalized size = 0.66

$$\frac{1}{256}\left(63\,a^5+70\,a^3b^2+15\,ab^4\right)x-\frac{\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(10\,dx+10\,c\right)}{5120\,d}-\frac{5\left(a^4b-a^2b^3\right)\cos\left(8\,dx+8\,c\right)}{512\,d}-\frac{5\left(27\,a^4b-6\,a^2b^3-b^5\right)\cos\left(6\,dx+6\,c\right)}{3072\,d}-\frac{5\left(3\,a^4b+a^2b^3\right)\cos\left(4\,dx+4\,c\right)}{128\,d}-\frac{5\left(21\,a^4b+14\,a^2b^3+b^5\right)\cos\left(2\,dx+2\,c\right)}{512\,d}+\frac{\left(a^5-10\,a^3b^2+5\,ab^4\right)\sin\left(10\,dx+10\,c\right)}{5120\,d}+\frac{5\left(a^5-6\,a^3b^2+ab^4\right)\sin\left(8\,dx+8\,c\right)}{2048\,d}+\frac{5\left(9\,a^5-26\,a^3b^2-3\,ab^4\right)\sin\left(6\,dx+6\,c\right)}{3072\,d}+\frac{5\left(3\,a^5-2\,a^3b^2-ab^4\right)\sin\left(4\,dx+4\,c\right)}{256\,d}+\frac{5\left(21\,a^5+14\,a^3b^2+ab^4\right)\sin\left(2\,dx+2\,c\right)}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/256*(63*a^5 + 70*a^3*b^2 + 15*a*b^4)*x - 1/5120*(5*a^4*b - 10*a^2*b^3 + b
^5)*cos(10*d*x + 10*c)/d - 5/512*(a^4*b - a^2*b^3)*cos(8*d*x + 8*c)/d - 5/3
072*(27*a^4*b - 6*a^2*b^3 - b^5)*cos(6*d*x + 6*c)/d - 5/128*(3*a^4*b + a^2*
b^3)*cos(4*d*x + 4*c)/d - 5/512*(21*a^4*b + 14*a^2*b^3 + b^5)*cos(2*d*x + 2
*c)/d + 1/5120*(a^5 - 10*a^3*b^2 + 5*a*b^4)*sin(10*d*x + 10*c)/d + 5/2048*(
a^5 - 6*a^3*b^2 + a*b^4)*sin(8*d*x + 8*c)/d + 5/3072*(9*a^5 - 26*a^3*b^2 -
3*a*b^4)*sin(6*d*x + 6*c)/d + 5/256*(3*a^5 - 2*a^3*b^2 - a*b^4)*sin(4*d*x +
 4*c)/d + 5/512*(21*a^5 + 14*a^3*b^2 + a*b^4)*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 2.51, size = 801, normalized size = 1.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(a*cos(c + d*x) + b*sin(c + d*x))^5,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^19*((15*a*b^4)/128 - (193*a^5)/128 + (35*a^3*b^2)/64) -
 tan(c/2 + (d*x)/2)*((15*a*b^4)/128 - (193*a^5)/128 + (35*a^3*b^2)/64) + ta
n(c/2 + (d*x)/2)^3*((159*a^5)/128 - (145*a*b^4)/128 + (4105*a^3*b^2)/192) -
 tan(c/2 + (d*x)/2)^17*((159*a^5)/128 - (145*a*b^4)/128 + (4105*a^3*b^2)/19
2) - tan(c/2 + (d*x)/2)^7*((2595*a*b^4)/32 + (147*a^5)/32 - (2905*a^3*b^2)/
16) + tan(c/2 + (d*x)/2)^13*((2595*a*b^4)/32 + (147*a^5)/32 - (2905*a^3*b^2
)/16) + tan(c/2 + (d*x)/2)^5*((867*a*b^4)/32 + (2847*a^5)/160 - (2891*a^3*b
^2)/48) - tan(c/2 + (d*x)/2)^15*((867*a*b^4)/32 + (2847*a^5)/160 - (2891*a^
3*b^2)/48) + tan(c/2 + (d*x)/2)^9*((9395*a*b^4)/64 + (1827*a^5)/64 - (7945*
a^3*b^2)/32) - tan(c/2 + (d*x)/2)^11*((9395*a*b^4)/64 + (1827*a^5)/64 - (79
45*a^3*b^2)/32) + tan(c/2 + (d*x)/2)^6*(120*a^4*b + (32*b^5)/3 - 80*a^2*b^3
) + tan(c/2 + (d*x)/2)^14*(120*a^4*b + (32*b^5)/3 - 80*a^2*b^3) + tan(c/2 +
 (d*x)/2)^10*(252*a^4*b + (192*b^5)/5 - 224*a^2*b^3) - tan(c/2 + (d*x)/2)^8
*((64*b^5)/3 - 280*a^2*b^3) - tan(c/2 + (d*x)/2)^12*((64*b^5)/3 - 280*a^2*b
^3) + 40*a^2*b^3*tan(c/2 + (d*x)/2)^4 + 40*a^2*b^3*tan(c/2 + (d*x)/2)^16 +
10*a^4*b*tan(c/2 + (d*x)/2)^2 + 10*a^4*b*tan(c/2 + (d*x)/2)^18)/(d*(10*tan(
c/2 + (d*x)/2)^2 + 45*tan(c/2 + (d*x)/2)^4 + 120*tan(c/2 + (d*x)/2)^6 + 210
*tan(c/2 + (d*x)/2)^8 + 252*tan(c/2 + (d*x)/2)^10 + 210*tan(c/2 + (d*x)/2)^
12 + 120*tan(c/2 + (d*x)/2)^14 + 45*tan(c/2 + (d*x)/2)^16 + 10*tan(c/2 + (d
*x)/2)^18 + tan(c/2 + (d*x)/2)^20 + 1)) + (a*atan((a*tan(c/2 + (d*x)/2)*(63
```

```
*a^4 + 15*b^4 + 70*a^2*b^2))/(128*((15*a*b^4)/128 + (63*a^5)/128 + (35*a^3*
b^2)/64)))*(63*a^4 + 15*b^4 + 70*a^2*b^2))/(128*d) - (a*(atan(tan(c/2 + (d*
x)/2)) - (d*x)/2)*(63*a^4 + 15*b^4 + 70*a^2*b^2))/(128*d)
```

3.93 $\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=337

$$-\frac{b^5\cos^5(c+dx)}{5d}-\frac{10a^2b^3\cos^7(c+dx)}{7d}+\frac{2b^5\cos^7(c+dx)}{7d}-\frac{5a^4b\cos^9(c+dx)}{9d}+\frac{10a^2b^3\cos^9(c+dx)}{9d}-\frac{b^5\cos^9(}{9}$$

```
[Out] -1/5*b^5*cos(d*x+c)^5/d-10/7*a^2*b^3*cos(d*x+c)^7/d+2/7*b^5*cos(d*x+c)^7/d-
5/9*a^4*b*cos(d*x+c)^9/d+10/9*a^2*b^3*cos(d*x+c)^9/d-1/9*b^5*cos(d*x+c)^9/d
+a^5*sin(d*x+c)/d-4/3*a^5*sin(d*x+c)^3/d+10/3*a^3*b^2*sin(d*x+c)^3/d+6/5*a^
5*sin(d*x+c)^5/d-6*a^3*b^2*sin(d*x+c)^5/d+a*b^4*sin(d*x+c)^5/d-4/7*a^5*sin(
d*x+c)^7/d+30/7*a^3*b^2*sin(d*x+c)^7/d-10/7*a*b^4*sin(d*x+c)^7/d+1/9*a^5*si
n(d*x+c)^9/d-10/9*a^3*b^2*sin(d*x+c)^9/d+5/9*a*b^4*sin(d*x+c)^9/d
```

Rubi [A]
time = 0.21, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3169, 2713, 2645, 30, 2644, 276, 14}

$$\frac{a^5\sin^9(c+dx)}{9d}-\frac{4a^5\sin^7(c+dx)}{7d}+\frac{6a^5\sin^5(c+dx)}{5d}-\frac{4a^5\sin^3(c+dx)}{3d}+\frac{a^5\sin(c+dx)}{d}-\frac{5a^4b\cos^9(c+dx)}{9d}-\frac{10a^3b^2\sin^9(c+dx)}{9d}+\frac{30a^3b^2\sin^7(c+dx)}{7d}-\frac{6a^3b^2\sin^5(c+dx)}{d}+\frac{10a^3b^2\sin^3(c+dx)}{3d}+\frac{10a^2b^3\cos^9(c+dx)}{9d}-\frac{10a^2b^3\cos^7(c+dx)}{7d}+\frac{5ab^4\sin^9(c+dx)}{9d}-\frac{10ab^4\sin^7(c+dx)}{7d}+\frac{ab^4\sin^5(c+dx)}{d}-\frac{b^5\cos^9(c+dx)}{9d}+\frac{2b^5\cos^7(c+dx)}{7d}-\frac{b^5\cos^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] -1/5*(b^5*Cos[c + d*x]^5)/d - (10*a^2*b^3*Cos[c + d*x]^7)/(7*d) + (2*b^5*Co
s[c + d*x]^7)/(7*d) - (5*a^4*b*Cos[c + d*x]^9)/(9*d) + (10*a^2*b^3*Cos[c +
d*x]^9)/(9*d) - (b^5*Cos[c + d*x]^9)/(9*d) + (a^5*Sin[c + d*x])/d - (4*a^5*
Sin[c + d*x]^3)/(3*d) + (10*a^3*b^2*Sin[c + d*x]^3)/(3*d) + (6*a^5*Sin[c +
d*x]^5)/(5*d) - (6*a^3*b^2*Sin[c + d*x]^5)/d + (a*b^4*Sin[c + d*x]^5)/d - (
4*a^5*Sin[c + d*x]^7)/(7*d) + (30*a^3*b^2*Sin[c + d*x]^7)/(7*d) - (10*a*b^4
*Sin[c + d*x]^7)/(7*d) + (a^5*Sin[c + d*x]^9)/(9*d) - (10*a^3*b^2*Sin[c + d
*x]^9)/(9*d) + (5*a*b^4*Sin[c + d*x]^9)/(9*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int \left(a^5\cos^9(c+dx)+5a^4b\cos^8(c+dx)\sin(c+dx)+10$$

$$= a^5\int \cos^9(c+dx)\,dx+\left(5a^4b\right)\int \cos^8(c+dx)\sin(c+dx$$

$$= -\frac{a^5\text{Subst}\left(\int \left(1-4x^2+6x^4-4x^6+x^8\right)\,dx,x,-\sin(c+}{d}$$

$$= -\frac{5a^4b\cos^9(c+dx)}{9d}+\frac{a^5\sin(c+dx)}{d}-\frac{4a^5\sin^3(c+dx)}{3d}$$

$$= -\frac{b^5\cos^5(c+dx)}{5d}-\frac{10a^2b^3\cos^7(c+dx)}{7d}+\frac{2b^5\cos^7(c+}{7d}$$

Mathematica [A]
time = 1.07, size = 278, normalized size = 0.82

$$\frac{-630b(35a^4+30a^2b^2+3b^4)\cos(c+dx)-420b(35a^4+20a^2b^2+b^4)\cos(3(c+dx))+252b(-25a^4+b^4)\cos(5(c+dx))+45b(-35a^4+30a^2b^2+b^4)\cos(7(c+dx))-35b(5a^4-10a^2b^2+b^4)\cos(9(c+dx))+630a(63a^4+70a^2b^2+15b^4)\sin(c+dx)+420a(21a^4-5b^4)\sin(3(c+dx))+252a(9a^4-20a^2b^2-5b^4)\sin(5(c+dx))+45a(9a^4-50a^2b^2+5b^4)\sin(7(c+dx))+35a(a^4-10a^2b^2+5b^4)\sin(9(c+dx))}{80640d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (-630*b*(35*a^4 + 30*a^2*b^2 + 3*b^4)*Cos[c + d*x] - 420*b*(35*a^4 + 20*a^2
*b^2 + b^4)*Cos[3*(c + d*x)] + 252*b*(-25*a^4 + b^4)*Cos[5*(c + d*x)] + 45*
b*(-35*a^4 + 30*a^2*b^2 + b^4)*Cos[7*(c + d*x)] - 35*b*(5*a^4 - 10*a^2*b^2
+ b^4)*Cos[9*(c + d*x)] + 630*a*(63*a^4 + 70*a^2*b^2 + 15*b^4)*Sin[c + d*x]
 + 420*a*(21*a^4 - 5*b^4)*Sin[3*(c + d*x)] + 252*a*(9*a^4 - 20*a^2*b^2 - 5*
b^4)*Sin[5*(c + d*x)] + 45*a*(9*a^4 - 50*a^2*b^2 + 5*b^4)*Sin[7*(c + d*x)]
+ 35*a*(a^4 - 10*a^2*b^2 + 5*b^4)*Sin[9*(c + d*x)])/(80640*d)
```

Maple [A]
time = 0.42, size = 291, normalized size = 0.86 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^5*(-1/9*sin(d*x+c)^4*cos(d*x+c)^5-4/63*sin(d*x+c)^2*cos(d*x+c)^5-8/3
15*cos(d*x+c)^5)+5*a*b^4*(-1/9*sin(d*x+c)^3*cos(d*x+c)^6-1/21*sin(d*x+c)*co
s(d*x+c)^6+1/105*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))+10*b^3*a^2
*(-1/9*sin(d*x+c)^2*cos(d*x+c)^7-2/63*cos(d*x+c)^7)+10*a^3*b^2*(-1/9*sin(d*
x+c)*cos(d*x+c)^8+1/63*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2
)*sin(d*x+c))-5/9*b*a^4*cos(d*x+c)^9+1/9*a^5*(128/35+cos(d*x+c)^8+8/7*cos(d
*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.30, size = 224, normalized size = 0.66

$$-\frac{175\,a^4b\cos\left(dx+c\right)^9-\left(35\,\sin\left(dx+c\right)^9-180\,\sin\left(dx+c\right)^7+378\,\sin\left(dx+c\right)^5-420\,\sin\left(dx+c\right)^3+315\,\sin\left(dx+c\right)\right)a^5+10\left(35\,\sin\left(dx+c\right)^9-135\,\sin\left(dx+c\right)^7+189\,\sin\left(dx+c\right)^5-105\,\sin\left(dx+c\right)^3\right)a^3b^2-50\left(7\,\cos\left(dx+c\right)^9-9\,\cos\left(dx+c\right)^7\right)a^2b^3-5\left(35\,\sin\left(dx+c\right)^9-90\,\sin\left(dx+c\right)^7+63\,\sin\left(dx+c\right)^5\right)ab^4+\left(35\,\cos\left(dx+c\right)^9-90\,\cos\left(dx+c\right)^7+63\,\cos\left(dx+c\right)^5\right)b^5}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/315*(175*a^4*b*cos(d*x + c)^9 - (35*sin(d*x + c)^9 - 180*sin(d*x + c)^7
+ 378*sin(d*x + c)^5 - 420*sin(d*x + c)^3 + 315*sin(d*x + c))*a^5 + 10*(35*
sin(d*x + c)^9 - 135*sin(d*x + c)^7 + 189*sin(d*x + c)^5 - 105*sin(d*x + c)
^3)*a^3*b^2 - 50*(7*cos(d*x + c)^9 - 9*cos(d*x + c)^7)*a^2*b^3 - 5*(35*sin(
d*x + c)^9 - 90*sin(d*x + c)^7 + 63*sin(d*x + c)^5)*a*b^4 + (35*cos(d*x + c
)^9 - 90*cos(d*x + c)^7 + 63*cos(d*x + c)^5)*b^5)/d
```

Fricas [A]
time = 2.73, size = 217, normalized size = 0.64

$$-\frac{63\,b^5\cos\left(dx+c\right)^5+35\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(dx+c\right)^9+90\left(5\,a^2b^3-b^5\right)\cos\left(dx+c\right)^7-\left(35\left(a^5-10\,a^3b^2+5\,ab^4\right)\cos\left(dx+c\right)^8+10\left(4\,a^5+5\,a^3b^2-25\,ab^4\right)\cos\left(dx+c\right)^6+128\,a^5+160\,a^3b^2+40\,ab^4+3\left(16\,a^5+20\,a^3b^2+5\,ab^4\right)\cos\left(dx+c\right)^4+4\left(16\,a^5+20\,a^3b^2+5\,ab^4\right)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/315*(63*b^5*cos(d*x + c)^5 + 35*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c
)^9 + 90*(5*a^2*b^3 - b^5)*cos(d*x + c)^7 - (35*(a^5 - 10*a^3*b^2 + 5*a*b^4
)*cos(d*x + c)^8 + 10*(4*a^5 + 5*a^3*b^2 - 25*a*b^4)*cos(d*x + c)^6 + 128*a
^5 + 160*a^3*b^2 + 40*a*b^4 + 3*(16*a^5 + 20*a^3*b^2 + 5*a*b^4)*cos(d*x + c
)^4 + 4*(16*a^5 + 20*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [A]
time = 1.44, size = 440, normalized size = 1.31

$$\begin{cases} \frac{128a^5\sin^9(c+dx)}{315d} + \frac{64a^5\sin^7(c+dx)\cos^2(c+dx)}{35d} + \frac{16a^5\sin^5(c+dx)\cos^4(c+dx)}{5d} + \frac{8a^5\sin^3(c+dx)\cos^6(c+dx)}{3d} + \frac{a^5\sin(c+dx)\cos^8(c+dx)}{d} - \frac{5a^4b\cos^9(c+dx)}{9d} + \frac{32a^3b^2\sin^9(c+dx)}{63d} + \frac{16a^3b^2\sin^7(c+dx)\cos^2(c+dx)}{7d} + \frac{4a^3b^2\sin^5(c+dx)\cos^4(c+dx)}{d} + \frac{10a^3b^2\sin^3(c+dx)\cos^6(c+dx)}{3d} - \frac{10a^2b^3\sin^2(c+dx)\cos^7(c+dx)}{7d} - \frac{20a^2b^3\cos^9(c+dx)}{63d} + \frac{8ab^4\sin^9(c+dx)}{63d} + \frac{4ab^4\sin^7(c+dx)\cos^2(c+dx)}{7d} + \frac{ab^4\sin^5(c+dx)\cos^4(c+dx)}{d} - \frac{b^5\sin^4(c+dx)\cos^5(c+dx)}{5d} - \frac{4b^5\sin^2(c+dx)\cos^7(c+dx)}{35d} - \frac{8b^5\cos^9(c+dx)}{315d} & \text{for}\ d \neq 0 \\ x(a\cos(c) + b\sin(c))^5\cos^4(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Piecewise((128*a**5*sin(c + d*x)**9/(315*d) + 64*a**5*sin(c + d*x)**7*cos(c
 + d*x)**2/(35*d) + 16*a**5*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + 8*a**5*
sin(c + d*x)**3*cos(c + d*x)**6/(3*d) + a**5*sin(c + d*x)*cos(c + d*x)**8/d
 - 5*a**4*b*cos(c + d*x)**9/(9*d) + 32*a**3*b**2*sin(c + d*x)**9/(63*d) + 1
6*a**3*b**2*sin(c + d*x)**7*cos(c + d*x)**2/(7*d) + 4*a**3*b**2*sin(c + d*x
)**5*cos(c + d*x)**4/d + 10*a**3*b**2*sin(c + d*x)**3*cos(c + d*x)**6/(3*d)
 - 10*a**2*b**3*sin(c + d*x)**2*cos(c + d*x)**7/(7*d) - 20*a**2*b**3*cos(c
+ d*x)**9/(63*d) + 8*a*b**4*sin(c + d*x)**9/(63*d) + 4*a*b**4*sin(c + d*x)*
*7*cos(c + d*x)**2/(7*d) + a*b**4*sin(c + d*x)**5*cos(c + d*x)**4/d - b**5*
sin(c + d*x)**4*cos(c + d*x)**5/(5*d) - 4*b**5*sin(c + d*x)**2*cos(c + d*x)
**7/(35*d) - 8*b**5*cos(c + d*x)**9/(315*d), Ne(d, 0)), (x*(a*cos(c) + b*si
n(c))**5*cos(c)**4, True))
```

Giac [A]
time = 0.65, size = 313, normalized size = 0.93

$$-\frac{(5a^4b - 10a^2b^3 + b^5)\cos(9dx + 9c)}{2304d} - \frac{(35a^4b - 30a^2b^3 - b^5)\cos(7dx + 7c)}{1792d} - \frac{(25a^4b - b^5)\cos(5dx + 5c)}{320d} - \frac{(35a^4b + 20a^2b^3 + b^5)\cos(3dx + 3c)}{192d} - \frac{(35a^4b + 30a^2b^3 + 3b^5)\cos(dx + c)}{128d} + \frac{(a^5 - 10a^3b^2 + 5ab^4)\sin(9dx + 9c)}{2304d} + \frac{(9a^5 - 50a^3b^2 + 5ab^4)\sin(7dx + 7c)}{1792d} + \frac{(9a^5 - 20a^3b^2 - 5ab^4)\sin(5dx + 5c)}{320d} + \frac{(21a^5 - 5ab^4)\sin(3dx + 3c)}{192d} + \frac{(63a^5 + 70a^3b^2 + 15ab^4)\sin(dx + c)}{128d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/2304*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(9*d*x + 9*c)/d - 1/1792*(35*a^4*b
- 30*a^2*b^3 - b^5)*cos(7*d*x + 7*c)/d - 1/320*(25*a^4*b - b^5)*cos(5*d*x +
 5*c)/d - 1/192*(35*a^4*b + 20*a^2*b^3 + b^5)*cos(3*d*x + 3*c)/d - 1/128*(3
5*a^4*b + 30*a^2*b^3 + 3*b^5)*cos(d*x + c)/d + 1/2304*(a^5 - 10*a^3*b^2 + 5
*a*b^4)*sin(9*d*x + 9*c)/d + 1/1792*(9*a^5 - 50*a^3*b^2 + 5*a*b^4)*sin(7*d*
x + 7*c)/d + 1/320*(9*a^5 - 20*a^3*b^2 - 5*a*b^4)*sin(5*d*x + 5*c)/d + 1/19
2*(21*a^5 - 5*a*b^4)*sin(3*d*x + 3*c)/d + 1/128*(63*a^5 + 70*a^3*b^2 + 15*a
*b^4)*sin(d*x + c)/d
```

Mupad [B]
time = 4.34, size = 495, normalized size = 1.47

$$\frac{2\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(\frac{152\,a^5}{5}-32\,a^3\,b^2+32\,a\,b^4\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}\left(\frac{152\,a^5}{5}-32\,a^3\,b^2+32\,a\,b^4\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\left(\frac{1136\,a^5}{35}+\frac{1264\,a^3\,b^2}{7}-\frac{384\,a\,b^4}{7}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\left(\frac{1136\,a^5}{35}+\frac{1264\,a^3\,b^2}{7}-\frac{384\,a\,b^4}{7}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\left(\frac{21316\,a^5}{315}-\frac{5696\,a^3\,b^2}{63}+\frac{6976\,a\,b^4}{63}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(40\,a^4\,b-\frac{120\,a^2\,b^3}{7}+\frac{64\,b^5}{35}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\left(140\,a^4\,b-120\,a^2\,b^3+\frac{112\,b^5}{5}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}\left(\frac{280\,a^4\,b}{3}-\frac{200\,a^2\,b^3}{3}+\frac{32\,b^5}{3}\right)-\frac{10\,a^4\,b}{9}-\frac{16\,b^5}{315}-\frac{40\,a^2\,b^3}{63}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(\frac{16\,a^5}{3}+\frac{80\,a^3\,b^2}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}\left(\frac{16\,a^5}{3}+\frac{80\,a^3\,b^2}{3}\right)+2\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{40\,a^2\,b^3}{7}+\frac{16\,b^5}{35}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{32\,b^5}{5}-120\,a^2\,b^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\left(16\,b^5-200\,a^2\,b^3\right)-40\,a^2\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-10\,a^4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(a*cos(c + d*x) + b*sin(c + d*x))^5,x)
```

```
[Out] (2*a^5*tan(c/2 + (d*x)/2)^17 + tan(c/2 + (d*x)/2)^5*(32*a*b^4 + (152*a^5)/5
 - 32*a^3*b^2) + tan(c/2 + (d*x)/2)^13*(32*a*b^4 + (152*a^5)/5 - 32*a^3*b^2
) + tan(c/2 + (d*x)/2)^7*((1136*a^5)/35 - (384*a*b^4)/7 + (1264*a^3*b^2)/7)
 + tan(c/2 + (d*x)/2)^11*((1136*a^5)/35 - (384*a*b^4)/7 + (1264*a^3*b^2)/7)
 + tan(c/2 + (d*x)/2)^9*((6976*a*b^4)/63 + (21316*a^5)/315 - (5696*a^3*b^2)
/63) - tan(c/2 + (d*x)/2)^4*(40*a^4*b + (64*b^5)/35 - (120*a^2*b^3)/7) - ta
n(c/2 + (d*x)/2)^8*(140*a^4*b + (112*b^5)/5 - 120*a^2*b^3) - tan(c/2 + (d*x
)/2)^12*((280*a^4*b)/3 + (32*b^5)/3 - (200*a^2*b^3)/3) - (10*a^4*b)/9 - (16
*b^5)/315 - (40*a^2*b^3)/63 + tan(c/2 + (d*x)/2)^3*((16*a^5)/3 + (80*a^3*b^
2)/3) + tan(c/2 + (d*x)/2)^15*((16*a^5)/3 + (80*a^3*b^2)/3) + 2*a^5*tan(c/2
 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*((16*b^5)/35 + (40*a^2*b^3)/7) + tan(c/2
 + (d*x)/2)^6*((32*b^5)/5 - 120*a^2*b^3) + tan(c/2 + (d*x)/2)^10*(16*b^5 -
200*a^2*b^3) - 40*a^2*b^3*tan(c/2 + (d*x)/2)^14 - 10*a^4*b*tan(c/2 + (d*x)/
2)^16)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^9)
```

3.94 $\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=426

$$\frac{35a^5x}{128}+\frac{25}{64}a^3b^2x+\frac{15}{128}ab^4x-\frac{5a^2b^3\cos^6(c+dx)}{3d}-\frac{5a^4b\cos^8(c+dx)}{8d}+\frac{5a^2b^3\cos^8(c+dx)}{4d}+\frac{35a^5\cos(c+dx)\sin}{128d}$$

```
[Out] 35/128*a^5*x+25/64*a^3*b^2*x+15/128*a*b^4*x-5/3*a^2*b^3*cos(d*x+c)^6/d-5/8*
a^4*b*cos(d*x+c)^8/d+5/4*a^2*b^3*cos(d*x+c)^8/d+35/128*a^5*cos(d*x+c)*sin(d
*x+c)/d+25/64*a^3*b^2*cos(d*x+c)*sin(d*x+c)/d+15/128*a*b^4*cos(d*x+c)*sin(d
*x+c)/d+35/192*a^5*cos(d*x+c)^3*sin(d*x+c)/d+25/96*a^3*b^2*cos(d*x+c)^3*sin
(d*x+c)/d+5/64*a*b^4*cos(d*x+c)^3*sin(d*x+c)/d+7/48*a^5*cos(d*x+c)^5*sin(d*
x+c)/d+5/24*a^3*b^2*cos(d*x+c)^5*sin(d*x+c)/d-5/16*a*b^4*cos(d*x+c)^5*sin(d
*x+c)/d+1/8*a^5*cos(d*x+c)^7*sin(d*x+c)/d-5/4*a^3*b^2*cos(d*x+c)^7*sin(d*x+
c)/d-5/8*a*b^4*cos(d*x+c)^5*sin(d*x+c)^3/d+1/6*b^5*sin(d*x+c)^6/d-1/8*b^5*s
in(d*x+c)^8/d
```

Rubi [A]
time = 0.29, antiderivative size = 426, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3169, 2715, 8, 2645, 30, 2648, 14, 2644}

$$\frac{a^5\sin(c+dx)\cos^7(c+dx)}{8d}+\frac{7a^5\sin(c+dx)\cos^5(c+dx)}{48d}+\frac{35a^5\sin(c+dx)\cos^3(c+dx)}{192d}+\frac{35a^5\sin(c+dx)\cos(c+dx)}{128d}+\frac{35a^5x}{128}-\frac{5a^4b\cos^8(c+dx)}{8d}-\frac{5a^3b^2\sin(c+dx)\cos^7(c+dx)}{4d}+\frac{5a^3b^2\sin(c+dx)\cos^5(c+dx)}{24d}+\frac{25a^3b^2\sin(c+dx)\cos^3(c+dx)}{96d}+\frac{25a^3b^2\sin(c+dx)\cos(c+dx)}{64d}+\frac{25}{64}a^3b^2x+\frac{5a^2b^3\cos^8(c+dx)}{4d}-\frac{5a^2b^3\cos^6(c+dx)}{3d}-\frac{5ab^4\sin^3(c+dx)\cos^5(c+dx)}{8d}-\frac{5ab^4\sin(c+dx)\cos^5(c+dx)}{16d}+\frac{5ab^4\sin(c+dx)\cos^3(c+dx)}{64d}+\frac{15ab^4\sin(c+dx)\cos(c+dx)}{128d}+\frac{15}{128}ab^4x-\frac{b^5\sin^8(c+dx)}{8d}+\frac{b^5\sin^6(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (35*a^5*x)/128 + (25*a^3*b^2*x)/64 + (15*a*b^4*x)/128 - (5*a^2*b^3*Cos[c +
d*x]^6)/(3*d) - (5*a^4*b*Cos[c + d*x]^8)/(8*d) + (5*a^2*b^3*Cos[c + d*x]^8)
/(4*d) + (35*a^5*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (25*a^3*b^2*Cos[c + d
*x]*Sin[c + d*x])/(64*d) + (15*a*b^4*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (
35*a^5*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + (25*a^3*b^2*Cos[c + d*x]^3*Si
n[c + d*x])/(96*d) + (5*a*b^4*Cos[c + d*x]^3*Sin[c + d*x])/(64*d) + (7*a^5*
Cos[c + d*x]^5*Sin[c + d*x])/(48*d) + (5*a^3*b^2*Cos[c + d*x]^5*Sin[c + d*x
])/(24*d) - (5*a*b^4*Cos[c + d*x]^5*Sin[c + d*x])/(16*d) + (a^5*Cos[c + d*x
]^7*Sin[c + d*x])/(8*d) - (5*a^3*b^2*Cos[c + d*x]^7*Sin[c + d*x])/(4*d) - (
5*a*b^4*Cos[c + d*x]^5*Sin[c + d*x]^3)/(8*d) + (b^5*Sin[c + d*x]^6)/(6*d) -
 (b^5*Sin[c + d*x]^8)/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int \left(a^5\cos^8(c+dx)+5a^4b\cos^7(c+dx)\sin(c+dx)+10\right.$$
$$= a^5\int \cos^8(c+dx)\,dx + \left(5a^4b\right)\int \cos^7(c+dx)\sin(c+dx$$
$$= \frac{a^5\cos^7(c+dx)\sin(c+dx)}{8d} - \frac{5a^3b^2\cos^7(c+dx)\sin(c+}{4d}$$
$$= -\frac{5a^4b\cos^8(c+dx)}{8d} + \frac{7a^5\cos^5(c+dx)\sin(c+dx)}{48d} + \frac{5a}{}$$
$$= -\frac{5a^2b^3\cos^6(c+dx)}{3d} - \frac{5a^4b\cos^8(c+dx)}{8d} + \frac{5a^2b^3\cos^8(c}{4d}$$
$$= -\frac{5a^2b^3\cos^6(c+dx)}{3d} - \frac{5a^4b\cos^8(c+dx)}{8d} + \frac{5a^2b^3\cos^8(c}{4d}$$
$$= \frac{35a^5x}{128} + \frac{25}{64}a^3b^2x + \frac{15}{128}ab^4x - \frac{5a^2b^3\cos^6(c+dx)}{3d} - \frac{5a}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.91, size = 259, normalized size = 0.61

$$\frac{120a(a-ib)(a+ib)\left(7a^2+3b^2\right)(c+dx)-24b\left(35a^4+30a^2b^2+3b^4\right)\cos(2(c+dx))+12b\left(-35a^4-10a^2b^2+b^4\right)\cos(4(c+dx))+8b\left(-15a^4+10a^2b^2+b^4\right)\cos(6(c+dx))-3b\left(5a^4-10a^2b^2+b^4\right)\cos(8(c+dx))+96a^3\left(7a^2+5b^2\right)\sin(2(c+dx))+24a\left(7a^4-10a^2b^2-5b^4\right)\sin(4(c+dx))+32a^3\left(a^2-5b^2\right)\sin(6(c+dx))+3a\left(a^4-10a^2b^2+5b^4\right)\sin(8(c+dx))}{3072d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (120*a*(a - I*b)*(a + I*b)*(7*a^2 + 3*b^2)*(c + d*x) - 24*b*(35*a^4 + 30*a^
2*b^2 + 3*b^4)*Cos[2*(c + d*x)] + 12*b*(-35*a^4 - 10*a^2*b^2 + b^4)*Cos[4*(
c + d*x)] + 8*b*(-15*a^4 + 10*a^2*b^2 + b^4)*Cos[6*(c + d*x)] - 3*b*(5*a^4
- 10*a^2*b^2 + b^4)*Cos[8*(c + d*x)] + 96*a^3*(7*a^2 + 5*b^2)*Sin[2*(c + d*
x)] + 24*a*(7*a^4 - 10*a^2*b^2 - 5*b^4)*Sin[4*(c + d*x)] + 32*a^3*(a^2 - 5*
b^2)*Sin[6*(c + d*x)] + 3*a*(a^4 - 10*a^2*b^2 + 5*b^4)*Sin[8*(c + d*x)])/(3
072*d)
```

Maple [A]
time = 0.38, size = 305, normalized size = 0.72 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^5*(-1/8*sin(d*x+c)^4*cos(d*x+c)^4-1/12*sin(d*x+c)^2*cos(d*x+c)^4-1/2
4*cos(d*x+c)^4)+5*a*b^4*(-1/8*sin(d*x+c)^3*cos(d*x+c)^5-1/16*sin(d*x+c)*cos
(d*x+c)^5+1/64*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/128*d*x+3/128*c)+
10*b^3*a^2*(-1/8*sin(d*x+c)^2*cos(d*x+c)^6-1/24*cos(d*x+c)^6)+10*a^3*b^2*(-
1/8*sin(d*x+c)*cos(d*x+c)^7+1/48*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*
```

```
x+c))*sin(d*x+c)+5/128*d*x+5/128*c)-5/8*b*a^4*cos(d*x+c)^8+a^5*(1/8*(cos(d*
x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/
128*d*x+35/128*c))
```

Maxima [A]
time = 0.30, size = 228, normalized size = 0.54

$$-\frac{1920\,a^{4}b\cos\left(dx+c\right)^{8}+\left(128\,\sin\left(2\,dx+2\,c\right)^{3}-840\,dx-840\,c-3\,\sin\left(8\,dx+8\,c\right)-168\,\sin\left(4\,dx+4\,c\right)-768\,\sin\left(2\,dx+2\,c\right)\right)a^{5}-10\,\left(64\,\sin\left(2\,dx+2\,c\right)^{3}+120\,dx+120\,c-3\,\sin\left(8\,dx+8\,c\right)-24\,\sin\left(4\,dx+4\,c\right)\right)a^{3}b^{2}-1280\,\left(3\,\sin\left(dx+c\right)^{8}-8\,\sin\left(dx+c\right)^{6}+6\,\sin\left(dx+c\right)^{4}\right)a^{2}b^{3}-15\,\left(24\,dx+24\,c+\sin\left(8\,dx+8\,c\right)-8\,\sin\left(4\,dx+4\,c\right)\right)ab^{4}+128\,\left(3\,\sin\left(dx+c\right)^{8}-4\,\sin\left(dx+c\right)^{6}\right)b^{5}}{3072\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/3072*(1920*a^4*b*cos(d*x + c)^8 + (128*sin(2*d*x + 2*c)^3 - 840*d*x - 84
0*c - 3*sin(8*d*x + 8*c) - 168*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*a^5
 - 10*(64*sin(2*d*x + 2*c)^3 + 120*d*x + 120*c - 3*sin(8*d*x + 8*c) - 24*si
n(4*d*x + 4*c))*a^3*b^2 - 1280*(3*sin(d*x + c)^8 - 8*sin(d*x + c)^6 + 6*sin
(d*x + c)^4)*a^2*b^3 - 15*(24*d*x + 24*c + sin(8*d*x + 8*c) - 8*sin(4*d*x +
 4*c))*a*b^4 + 128*(3*sin(d*x + c)^8 - 4*sin(d*x + c)^6)*b^5)/d
```

Fricas [A]
time = 2.93, size = 220, normalized size = 0.52

$$-\frac{96\,b^{5}\cos\left(dx+c\right)^{4}+48\,\left(5\,a^{4}b-10\,a^{2}b^{3}+b^{5}\right)\cos\left(dx+c\right)^{8}+128\,\left(5\,a^{2}b^{3}-b^{5}\right)\cos\left(dx+c\right)^{6}-15\,\left(7\,a^{5}+10\,a^{3}b^{2}+3\,ab^{4}\right)dx-\left(48\,\left(a^{5}-10\,a^{3}b^{2}+5\,ab^{4}\right)\cos\left(dx+c\right)^{7}+8\,\left(7\,a^{5}+10\,a^{3}b^{2}-45\,ab^{4}\right)\cos\left(dx+c\right)^{5}+10\,\left(7\,a^{5}+10\,a^{3}b^{2}+3\,ab^{4}\right)\cos\left(dx+c\right)^{3}+15\,\left(7\,a^{5}+10\,a^{3}b^{2}+3\,ab^{4}\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/384*(96*b^5*cos(d*x + c)^4 + 48*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c
)^8 + 128*(5*a^2*b^3 - b^5)*cos(d*x + c)^6 - 15*(7*a^5 + 10*a^3*b^2 + 3*a*b
^4)*d*x - (48*(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^7 + 8*(7*a^5 + 10*a
^3*b^2 - 45*a*b^4)*cos(d*x + c)^5 + 10*(7*a^5 + 10*a^3*b^2 + 3*a*b^4)*cos(d
*x + c)^3 + 15*(7*a^5 + 10*a^3*b^2 + 3*a*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [A]
time = 1.09, size = 821, normalized size = 1.93

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Piecewise((35*a**5*x*sin(c + d*x)**8/128 + 35*a**5*x*sin(c + d*x)**6*cos(c
+ d*x)**2/32 + 105*a**5*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 35*a**5*x*si
n(c + d*x)**2*cos(c + d*x)**6/32 + 35*a**5*x*cos(c + d*x)**8/128 + 35*a**5*
sin(c + d*x)**7*cos(c + d*x)/(128*d) + 385*a**5*sin(c + d*x)**5*cos(c + d*x
)**3/(384*d) + 511*a**5*sin(c + d*x)**3*cos(c + d*x)**5/(384*d) + 93*a**5*s
```

```
in(c + d*x)*cos(c + d*x)**7/(128*d) - 5*a**4*b*cos(c + d*x)**8/(8*d) + 25*a
**3*b**2*x*sin(c + d*x)**8/64 + 25*a**3*b**2*x*sin(c + d*x)**6*cos(c + d*x)
**2/16 + 75*a**3*b**2*x*sin(c + d*x)**4*cos(c + d*x)**4/32 + 25*a**3*b**2*x
*sin(c + d*x)**2*cos(c + d*x)**6/16 + 25*a**3*b**2*x*cos(c + d*x)**8/64 + 2
5*a**3*b**2*sin(c + d*x)**7*cos(c + d*x)/(64*d) + 275*a**3*b**2*sin(c + d*x
)**5*cos(c + d*x)**3/(192*d) + 365*a**3*b**2*sin(c + d*x)**3*cos(c + d*x)**
5/(192*d) - 25*a**3*b**2*sin(c + d*x)*cos(c + d*x)**7/(64*d) - 5*a**2*b**3*
sin(c + d*x)**2*cos(c + d*x)**6/(3*d) - 5*a**2*b**3*cos(c + d*x)**8/(12*d)
+ 15*a*b**4*x*sin(c + d*x)**8/128 + 15*a*b**4*x*sin(c + d*x)**6*cos(c + d*x
)**2/32 + 45*a*b**4*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 15*a*b**4*x*sin(
c + d*x)**2*cos(c + d*x)**6/32 + 15*a*b**4*x*cos(c + d*x)**8/128 + 15*a*b**
4*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 55*a*b**4*sin(c + d*x)**5*cos(c +
d*x)**3/(128*d) - 55*a*b**4*sin(c + d*x)**3*cos(c + d*x)**5/(128*d) - 15*a*
b**4*sin(c + d*x)*cos(c + d*x)**7/(128*d) - b**5*sin(c + d*x)**4*cos(c + d*
x)**4/(4*d) - b**5*sin(c + d*x)**2*cos(c + d*x)**6/(6*d) - b**5*cos(c + d*x
)**8/(24*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**5*cos(c)**3, True))
```

Giac [A]
time = 0.70, size = 278, normalized size = 0.65

$$\frac{5}{128}\left(7\,a^5+10\,a^3b^2+3\,ab^4\right)x-\frac{\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(8\,dx+8\,c\right)}{1024\,d}-\frac{\left(15\,a^4b-10\,a^2b^3-b^5\right)\cos\left(6\,dx+6\,c\right)}{384\,d}-\frac{\left(35\,a^4b+10\,a^2b^3-b^5\right)\cos\left(4\,dx+4\,c\right)}{256\,d}-\frac{\left(35\,a^4b+30\,a^2b^3+3\,b^5\right)\cos\left(2\,dx+2\,c\right)}{128\,d}+\frac{\left(a^5-10\,a^3b^2+5\,ab^4\right)\sin\left(8\,dx+8\,c\right)}{1024\,d}+\frac{\left(a^5-5\,a^3b^2\right)\sin\left(6\,dx+6\,c\right)}{96\,d}+\frac{\left(7\,a^5-10\,a^3b^2-5\,ab^4\right)\sin\left(4\,dx+4\,c\right)}{128\,d}+\frac{\left(7\,a^5+5\,a^3b^2\right)\sin\left(2\,dx+2\,c\right)}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 5/128*(7*a^5 + 10*a^3*b^2 + 3*a*b^4)*x - 1/1024*(5*a^4*b - 10*a^2*b^3 + b^5
)*cos(8*d*x + 8*c)/d - 1/384*(15*a^4*b - 10*a^2*b^3 - b^5)*cos(6*d*x + 6*c)
/d - 1/256*(35*a^4*b + 10*a^2*b^3 - b^5)*cos(4*d*x + 4*c)/d - 1/128*(35*a^4
*b + 30*a^2*b^3 + 3*b^5)*cos(2*d*x + 2*c)/d + 1/1024*(a^5 - 10*a^3*b^2 + 5*
a*b^4)*sin(8*d*x + 8*c)/d + 1/96*(a^5 - 5*a^3*b^2)*sin(6*d*x + 6*c)/d + 1/1
28*(7*a^5 - 10*a^3*b^2 - 5*a*b^4)*sin(4*d*x + 4*c)/d + 1/32*(7*a^5 + 5*a^3*
b^2)*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 2.48, size = 650, normalized size = 1.53

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}\left(-\frac{93\,a^5}{64}+\frac{25\,a^3b^2}{32}+\frac{15\,a\,b^4}{64}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(-\frac{93\,a^5}{64}+\frac{25\,a^3b^2}{32}+\frac{15\,a\,b^4}{64}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}\left(\frac{91\,a^5}{192}+\frac{1985\,a^3b^2}{96}-\frac{115\,a\,b^4}{64}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}\left(\frac{91\,a^5}{192}+\frac{1985\,a^3b^2}{96}-\frac{115\,a\,b^4}{64}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}\left(\frac{1799\,a^5}{192}-\frac{4475\,a^3b^2}{96}+\frac{1665\,a\,b^4}{64}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\left(\frac{1799\,a^5}{192}-\frac{4475\,a^3b^2}{96}+\frac{1665\,a\,b^4}{64}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}\left(\frac{1085\,a^5}{192}-\frac{8825\,a^3b^2}{96}+\frac{3355\,a\,b^4}{64}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}\left(\frac{1085\,a^5}{192}-\frac{8825\,a^3b^2}{96}+\frac{3355\,a\,b^4}{64}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}\left(70\,a^4b-\frac{160\,a^2b^3}{3}+\frac{32\,b^5}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\left(70\,a^4b-\frac{160\,a^2b^3}{3}+\frac{32\,b^5}{3}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}\left(\frac{32\,b^5}{3}-\frac{400\,a^2b^3}{3}\right)+40\,a^2b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+40\,a^2b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+10\,a^4b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+10\,a^4b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+28\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+56\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+70\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+56\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+28\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}-\frac{5\,a\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\frac{d\,x}{2}\right)\left(7\,a^4+10\,a^2b^2+3\,b^4\right)}{64\,d}+\frac{5\,a\,\mathrm{atan}\left(\frac{5\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(7\,a^2+3\,b^2\right)\left(a^2+b^2\right)}{64\left(\frac{35\,a^5}{64}+\frac{25\,a^3b^2}{32}+\frac{15\,a\,b^4}{64}\right)}\right)\left(7\,a^2+3\,b^2\right)\left(a^2+b^2\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a*cos(c + d*x) + b*sin(c + d*x))^5,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^15*((15*a*b^4)/64 - (93*a^5)/64 + (25*a^3*b^2)/32) - ta
n(c/2 + (d*x)/2)*((15*a*b^4)/64 - (93*a^5)/64 + (25*a^3*b^2)/32) + tan(c/2
+ (d*x)/2)^3*((91*a^5)/192 - (115*a*b^4)/64 + (1985*a^3*b^2)/96) - tan(c/2
+ (d*x)/2)^13*((91*a^5)/192 - (115*a*b^4)/64 + (1985*a^3*b^2)/96) + tan(c/2
 + (d*x)/2)^5*((1665*a*b^4)/64 + (1799*a^5)/192 - (4475*a^3*b^2)/96) - tan(
```

```
c/2 + (d*x)/2)^11*((1665*a*b^4)/64 + (1799*a^5)/192 - (4475*a^3*b^2)/96) -
tan(c/2 + (d*x)/2)^7*((3355*a*b^4)/64 + (1085*a^5)/192 - (8825*a^3*b^2)/96)
 + tan(c/2 + (d*x)/2)^9*((3355*a*b^4)/64 + (1085*a^5)/192 - (8825*a^3*b^2)/
96) + tan(c/2 + (d*x)/2)^6*(70*a^4*b + (32*b^5)/3 - (160*a^2*b^3)/3) + tan(
c/2 + (d*x)/2)^10*(70*a^4*b + (32*b^5)/3 - (160*a^2*b^3)/3) - tan(c/2 + (d*
x)/2)^8*((32*b^5)/3 - (400*a^2*b^3)/3) + 40*a^2*b^3*tan(c/2 + (d*x)/2)^4 +
40*a^2*b^3*tan(c/2 + (d*x)/2)^12 + 10*a^4*b*tan(c/2 + (d*x)/2)^2 + 10*a^4*b
*tan(c/2 + (d*x)/2)^14)/(d*(8*tan(c/2 + (d*x)/2)^2 + 28*tan(c/2 + (d*x)/2)^
4 + 56*tan(c/2 + (d*x)/2)^6 + 70*tan(c/2 + (d*x)/2)^8 + 56*tan(c/2 + (d*x)/
2)^10 + 28*tan(c/2 + (d*x)/2)^12 + 8*tan(c/2 + (d*x)/2)^14 + tan(c/2 + (d*x
)/2)^16 + 1)) - (5*a*(atan(tan(c/2 + (d*x)/2)) - (d*x)/2)*(7*a^4 + 3*b^4 +
10*a^2*b^2))/(64*d) + (5*a*atan((5*a*tan(c/2 + (d*x)/2)*(7*a^2 + 3*b^2)*(a^
2 + b^2))/(64*((15*a*b^4)/64 + (35*a^5)/64 + (25*a^3*b^2)/32)))*(7*a^2 + 3*
b^2)*(a^2 + b^2))/(64*d)
```

3.95 $\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=275

$$-\frac{b^5\cos^3(c+dx)}{3d}-\frac{2a^2b^3\cos^5(c+dx)}{d}+\frac{2b^5\cos^5(c+dx)}{5d}-\frac{5a^4b\cos^7(c+dx)}{7d}+\frac{10a^2b^3\cos^7(c+dx)}{7d}-\frac{b^5\cos^7(c\cdots}{7d}$$

```
[Out] -1/3*b^5*cos(d*x+c)^3/d-2*a^2*b^3*cos(d*x+c)^5/d+2/5*b^5*cos(d*x+c)^5/d-5/7
*a^4*b*cos(d*x+c)^7/d+10/7*a^2*b^3*cos(d*x+c)^7/d-1/7*b^5*cos(d*x+c)^7/d+a^
5*sin(d*x+c)/d-a^5*sin(d*x+c)^3/d+10/3*a^3*b^2*sin(d*x+c)^3/d+3/5*a^5*sin(d
*x+c)^5/d-4*a^3*b^2*sin(d*x+c)^5/d+a*b^4*sin(d*x+c)^5/d-1/7*a^5*sin(d*x+c)^
7/d+10/7*a^3*b^2*sin(d*x+c)^7/d-5/7*a*b^4*sin(d*x+c)^7/d
```

Rubi [A]
time = 0.20, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3169, 2713, 2645, 30, 2644, 276, 14}

$$-\frac{a^5\sin^7(c+dx)}{7d}+\frac{3a^5\sin^5(c+dx)}{5d}-\frac{a^5\sin^3(c+dx)}{d}+\frac{a^5\sin(c+dx)}{d}-\frac{5a^4b\cos^7(c+dx)}{7d}+\frac{10a^3b^2\sin^7(c+dx)}{7d}-\frac{4a^3b^2\sin^5(c+dx)}{d}+\frac{10a^3b^2\sin^3(c+dx)}{3d}+\frac{10a^2b^3\cos^7(c+dx)}{7d}-\frac{2a^2b^3\cos^5(c+dx)}{d}-\frac{5ab^4\sin^7(c+dx)}{7d}+\frac{ab^4\sin^5(c+dx)}{d}-\frac{b^5\cos^7(c+dx)}{7d}+\frac{2b^5\cos^5(c+dx)}{5d}-\frac{b^5\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] -1/3*(b^5*Cos[c + d*x]^3)/d - (2*a^2*b^3*Cos[c + d*x]^5)/d + (2*b^5*Cos[c +
 d*x]^5)/(5*d) - (5*a^4*b*Cos[c + d*x]^7)/(7*d) + (10*a^2*b^3*Cos[c + d*x]^
7)/(7*d) - (b^5*Cos[c + d*x]^7)/(7*d) + (a^5*Sin[c + d*x])/d - (a^5*Sin[c +
 d*x]^3)/d + (10*a^3*b^2*Sin[c + d*x]^3)/(3*d) + (3*a^5*Sin[c + d*x]^5)/(5*
d) - (4*a^3*b^2*Sin[c + d*x]^5)/d + (a*b^4*Sin[c + d*x]^5)/d - (a^5*Sin[c +
 d*x]^7)/(7*d) + (10*a^3*b^2*Sin[c + d*x]^7)/(7*d) - (5*a*b^4*Sin[c + d*x]^
7)/(7*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \cos^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int \left(a^5\cos^7(c+dx)+5a^4b\cos^6(c+dx)\sin(c+dx)+1\right.$$

$$= a^5\int \cos^7(c+dx)\,dx+\left(5a^4b\right)\int \cos^6(c+dx)\sin(c+$$

$$= -\frac{a^5\mathrm{Subst}\left(\int \left(1-3x^2+3x^4-x^6\right)\,dx,x,-\sin(c+dx)\right)}{d}$$

$$= -\frac{5a^4b\cos^7(c+dx)}{7d}+\frac{a^5\sin(c+dx)}{d}-\frac{a^5\sin^3(c+dx)}{d}$$

$$= -\frac{b^5\cos^3(c+dx)}{3d}-\frac{2a^2b^3\cos^5(c+dx)}{d}+\frac{2b^5\cos^5(c+}{5d}$$

Mathematica [A]
time = 0.80, size = 236, normalized size = 0.86

$$\frac{-525b(5a^4+6a^2b^2+b^4)\cos(c+dx)-35b(45a^4+30a^2b^2+b^4)\cos(3(c+dx))+21b(-25a^4+10a^2b^2+3b^4)\cos(5(c+dx))-15b(5a^4-10a^2b^2+b^4)\cos(7(c+dx))+525a(7a^4+10a^2b^2+3b^4)\sin(c+dx)+35a(21a^4-10a^2b^2-15b^4)\sin(3(c+dx))+21a(7a^4-30a^2b^2-5b^4)\sin(5(c+dx))+15a(a^4-10a^2b^2+5b^4)\sin(7(c+dx))}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (-525*b*(5*a^4 + 6*a^2*b^2 + b^4)*Cos[c + d*x] - 35*b*(45*a^4 + 30*a^2*b^2
+ b^4)*Cos[3*(c + d*x)] + 21*b*(-25*a^4 + 10*a^2*b^2 + 3*b^4)*Cos[5*(c + d*
x)] - 15*b*(5*a^4 - 10*a^2*b^2 + b^4)*Cos[7*(c + d*x)] + 525*a*(7*a^4 + 10*
a^2*b^2 + 3*b^4)*Sin[c + d*x] + 35*a*(21*a^4 - 10*a^2*b^2 - 15*b^4)*Sin[3*(
c + d*x)] + 21*a*(7*a^4 - 30*a^2*b^2 - 5*b^4)*Sin[5*(c + d*x)] + 15*a*(a^4
- 10*a^2*b^2 + 5*b^4)*Sin[7*(c + d*x)])/(6720*d)
```

Maple [A]
time = 0.30, size = 261, normalized size = 0.95 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^5*(-1/7*sin(d*x+c)^4*cos(d*x+c)^3-4/35*sin(d*x+c)^2*cos(d*x+c)^3-8/1
05*cos(d*x+c)^3)+5*a*b^4*(-1/7*sin(d*x+c)^3*cos(d*x+c)^4-3/35*sin(d*x+c)*co
s(d*x+c)^4+1/35*(2+cos(d*x+c)^2)*sin(d*x+c))+10*b^3*a^2*(-1/7*sin(d*x+c)^2*
cos(d*x+c)^5-2/35*cos(d*x+c)^5)+10*a^3*b^2*(-1/7*sin(d*x+c)*cos(d*x+c)^6+1/
35*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))-5/7*b*a^4*cos(d*x+c)^7+1
/7*a^5*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 194, normalized size = 0.71

$$-\frac{75a^4b\cos(dx+c)^7+3\left(5\sin(dx+c)^7-21\sin(dx+c)^5+35\sin(dx+c)^3-35\sin(dx+c)\right)a^5-10\left(15\sin(dx+c)^7-42\sin(dx+c)^5+35\sin(dx+c)^3\right)a^3b^2-30\left(5\cos(dx+c)^7-7\cos(dx+c)^5\right)a^2b^3+15\left(5\sin(dx+c)^7-7\sin(dx+c)^5\right)ab^4+\left(15\cos(dx+c)^7-42\cos(dx+c)^5+35\cos(dx+c)^3\right)b^5}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/105*(75*a^4*b*cos(d*x + c)^7 + 3*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 +
 35*sin(d*x + c)^3 - 35*sin(d*x + c))*a^5 - 10*(15*sin(d*x + c)^7 - 42*sin(
d*x + c)^5 + 35*sin(d*x + c)^3)*a^3*b^2 - 30*(5*cos(d*x + c)^7 - 7*cos(d*x
+ c)^5)*a^2*b^3 + 15*(5*sin(d*x + c)^7 - 7*sin(d*x + c)^5)*a*b^4 + (15*cos(
d*x + c)^7 - 42*cos(d*x + c)^5 + 35*cos(d*x + c)^3)*b^5)/d
```

Fricas [A]
time = 2.92, size = 186, normalized size = 0.68

$$-\frac{35\,b^5\cos(dx+c)^3+15\,(5\,a^4b-10\,a^2b^3+b^5)\cos(dx+c)^7+42\,(5\,a^2b^3-b^5)\cos(dx+c)^5-\left(15\,(a^5-10\,a^3b^2+5\,ab^4)\cos(dx+c)^6+48\,a^5+80\,a^3b^2+30\,ab^4+6\,(3\,a^5+5\,a^3b^2-20\,ab^4)\cos(dx+c)^4+(24\,a^5+40\,a^3b^2+15\,ab^4)\cos(dx+c)^2\right)\sin(dx+c)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/105*(35*b^5*cos(d*x + c)^3 + 15*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c
)^7 + 42*(5*a^2*b^3 - b^5)*cos(d*x + c)^5 - (15*(a^5 - 10*a^3*b^2 + 5*a*b^4
)*cos(d*x + c)^6 + 48*a^5 + 80*a^3*b^2 + 30*a*b^4 + 6*(3*a^5 + 5*a^3*b^2 -
20*a*b^4)*cos(d*x + c)^4 + (24*a^5 + 40*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^2)
*sin(d*x + c))/d
```

Sympy [A]
time = 0.65, size = 357, normalized size = 1.30

$$\begin{cases} \frac{16a^5\sin^7(c+dx)}{35d} + \frac{8a^5\sin^5(c+dx)\cos^2(c+dx)}{5d} + \frac{2a^5\sin^3(c+dx)\cos^4(c+dx)}{d} + \frac{a^5\sin(c+dx)\cos^6(c+dx)}{d} - \frac{5a^4b\cos^7(c+dx)}{7d} + \frac{16a^3b^2\sin^7(c+dx)}{21d} + \frac{8a^3b^2\sin^5(c+dx)\cos^2(c+dx)}{3d} + \frac{10a^3b^2\sin^3(c+dx)\cos^4(c+dx)}{3d} - \frac{2a^2b^3\sin^2(c+dx)\cos^5(c+dx)}{d} - \frac{4a^2b^3\cos^7(c+dx)}{7d} + \frac{2ab^4\sin^7(c+dx)}{7d} + \frac{ab^4\sin^5(c+dx)\cos^2(c+dx)}{d} - \frac{b^5\sin^4(c+dx)\cos^3(c+dx)}{3d} - \frac{4b^5\sin^2(c+dx)\cos^5(c+dx)}{15d} - \frac{8b^5\cos^7(c+dx)}{105d} & \text{for}\ d \neq 0 \\ x(a\cos(c)+b\sin(c))^5\cos^2(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Piecewise((16*a**5*sin(c + d*x)**7/(35*d) + 8*a**5*sin(c + d*x)**5*cos(c +
d*x)**2/(5*d) + 2*a**5*sin(c + d*x)**3*cos(c + d*x)**4/d + a**5*sin(c + d*x
)*cos(c + d*x)**6/d - 5*a**4*b*cos(c + d*x)**7/(7*d) + 16*a**3*b**2*sin(c +
 d*x)**7/(21*d) + 8*a**3*b**2*sin(c + d*x)**5*cos(c + d*x)**2/(3*d) + 10*a*
*3*b**2*sin(c + d*x)**3*cos(c + d*x)**4/(3*d) - 2*a**2*b**3*sin(c + d*x)**2
*cos(c + d*x)**5/d - 4*a**2*b**3*cos(c + d*x)**7/(7*d) + 2*a*b**4*sin(c + d
*x)**7/(7*d) + a*b**4*sin(c + d*x)**5*cos(c + d*x)**2/d - b**5*sin(c + d*x)
**4*cos(c + d*x)**3/(3*d) - 4*b**5*sin(c + d*x)**2*cos(c + d*x)**5/(15*d) -
 8*b**5*cos(c + d*x)**7/(105*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**5*cos
(c)**2, True))
```

Giac [A]
time = 0.65, size = 259, normalized size = 0.94

$$-\frac{(5a^4b-10a^2b^3+b^5)\cos(7dx+7c)}{448d} - \frac{(25a^4b-10a^2b^3-3b^5)\cos(5dx+5c)}{320d} - \frac{(45a^4b+30a^2b^3+b^5)\cos(3dx+3c)}{192d} - \frac{5(5a^4b+6a^2b^3+b^5)\cos(dx+c)}{64d} + \frac{(a^5-10a^3b^2+5ab^4)\sin(7dx+7c)}{448d} + \frac{(7a^5-30a^3b^2-5ab^4)\sin(5dx+5c)}{320d} + \frac{(21a^5-10a^3b^2-15ab^4)\sin(3dx+3c)}{192d} + \frac{5(7a^5+10a^3b^2+3ab^4)\sin(dx+c)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/448*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(7*d*x + 7*c)/d - 1/320*(25*a^4*b -
10*a^2*b^3 - 3*b^5)*cos(5*d*x + 5*c)/d - 1/192*(45*a^4*b + 30*a^2*b^3 + b^5
)*cos(3*d*x + 3*c)/d - 5/64*(5*a^4*b + 6*a^2*b^3 + b^5)*cos(d*x + c)/d + 1/
448*(a^5 - 10*a^3*b^2 + 5*a*b^4)*sin(7*d*x + 7*c)/d + 1/320*(7*a^5 - 30*a^3
*b^2 - 5*a*b^4)*sin(5*d*x + 5*c)/d + 1/192*(21*a^5 - 10*a^3*b^2 - 15*a*b^4)
*sin(3*d*x + 3*c)/d + 5/64*(7*a^5 + 10*a^3*b^2 + 3*a*b^4)*sin(d*x + c)/d
```

Mupad [B]
time = 4.42, size = 372, normalized size = 1.35

$$\frac{2a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13} + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(\frac{86a^5}{5} - \frac{64a^3b^2}{3} + 32ab^4\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9\left(\frac{86a^5}{5} - \frac{64a^3b^2}{3} + 32ab^4\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(\frac{424a^5}{35} + \frac{608a^3b^2}{7} - \frac{192ab^4}{7}\right) - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(30a^4b - 16a^2b^3 + \frac{16b^5}{5}\right) - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\left(50a^4b - 40a^2b^3 + \frac{32b^5}{3}\right) - \frac{10a^4b}{7} - \frac{16b^5}{105} - \frac{8a^2b^3}{7} + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(4a^5 + \frac{80a^3b^2}{3}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\left(4a^5 + \frac{80a^3b^2}{3}\right) + 2a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right) - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(8a^2b^3 + \frac{16b^5}{15}\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(\frac{16b^5}{3} - 80a^2b^3\right) - 40a^2b^3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10} - 10a^4b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a*cos(c + d*x) + b*sin(c + d*x))^5,x)
```

```
[Out] (2*a^5*tan(c/2 + (d*x)/2)^13 + tan(c/2 + (d*x)/2)^5*(32*a*b^4 + (86*a^5)/5
- (64*a^3*b^2)/3) + tan(c/2 + (d*x)/2)^9*(32*a*b^4 + (86*a^5)/5 - (64*a^3*b
^2)/3) + tan(c/2 + (d*x)/2)^7*((424*a^5)/35 - (192*a*b^4)/7 + (608*a^3*b^2)
/7) - tan(c/2 + (d*x)/2)^4*(30*a^4*b + (16*b^5)/5 - 16*a^2*b^3) - tan(c/2 +
 (d*x)/2)^8*(50*a^4*b + (32*b^5)/3 - 40*a^2*b^3) - (10*a^4*b)/7 - (16*b^5)/
105 - (8*a^2*b^3)/7 + tan(c/2 + (d*x)/2)^3*(4*a^5 + (80*a^3*b^2)/3) + tan(c
/2 + (d*x)/2)^11*(4*a^5 + (80*a^3*b^2)/3) + 2*a^5*tan(c/2 + (d*x)/2) - tan(
c/2 + (d*x)/2)^2*((16*b^5)/15 + 8*a^2*b^3) + tan(c/2 + (d*x)/2)^6*((16*b^5)
/3 - 80*a^2*b^3) - 40*a^2*b^3*tan(c/2 + (d*x)/2)^10 - 10*a^4*b*tan(c/2 + (d
*x)/2)^12)/(d*(tan(c/2 + (d*x)/2)^2 + 1)^7)
```

3.96 $\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=126

$$\frac{5}{16}a\left(a^2+b^2\right)^2 x+\frac{5a(a^2+b^2)\left(b+a\cot(c+dx)\right)(a-b\cot(c+dx))\sin^2(c+dx)}{16d}+\frac{5a(b+a\cot(c+dx))^3(a$$

```
[Out] 5/16*a*(a^2+b^2)^2*x+5/16*a*(a^2+b^2)*(b+a*cot(d*x+c))*(a-b*cot(d*x+c))*sin
(d*x+c)^2/d+5/24*a*(b+a*cot(d*x+c))^3*(a-b*cot(d*x+c))*sin(d*x+c)^4/d+1/6*(
b+a*cot(d*x+c))^5*sin(d*x+c)^6/d
```

Rubi [A]
time = 0.06, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3167, 819, 737, 209}

$$\frac{5a(a^2+b^2)\sin^2(c+dx)(a\cot(c+dx)+b)(a-b\cot(c+dx))}{16d}+\frac{5}{16}ax\left(a^2+b^2\right)^2+\frac{\sin^6(c+dx)(a\cot(c+dx)+b)^5}{6d}+\frac{5a\sin^4(c+dx)(a\cot(c+dx)+b)^3(a-b\cot(c+dx))}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (5*a*(a^2 + b^2)^2*x)/16 + (5*a*(a^2 + b^2)*(b + a*Cot[c + d*x])*(a - b*Cot
[c + d*x])*Sin[c + d*x]^2)/(16*d) + (5*a*(b + a*Cot[c + d*x])^3*(a - b*Cot[
c + d*x])*Sin[c + d*x]^4)/(24*d) + ((b + a*Cot[c + d*x])^5*Sin[c + d*x]^6)/
(6*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 737

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(d + e*x)^(m - 1)*(a*e - c*d*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1))), x] +
 Dist[(2*p + 3)*((c*d^2 + a*e^2)/(2*a*c*(p + 1))), Int[(d + e*x)^(m - 2)*(a
 + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0
] && EqQ[m + 2*p + 2, 0] && LtQ[p, -1]
```

Rule 819

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(d + e*x)^m*(a + c*x^2)^(p + 1)*((a*g - c*f*x)/(2*a*c
*(p + 1))), x] - Dist[m*((c*d*f + a*e*g)/(2*a*c*(p + 1))), Int[(d + e*x)^(m
 - 1)*(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
```

```
d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \cos(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{\text{Subst}\left(\int \frac{x(b+ax)^5}{(1+x^2)^4}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= \frac{(b+a\cot(c+dx))^5\sin^6(c+dx)}{6d} - \frac{(5a)\text{Subst}\left(\int \frac{(b+ax)^4}{(1+x^2)^3}\right.}{6d}$$

$$= \frac{5a(b+a\cot(c+dx))^3(a-b\cot(c+dx))\sin^4(c+dx)}{24d} +$$

$$= \frac{5a(a^2+b^2)(b+a\cot(c+dx))(a-b\cot(c+dx))\sin^2(c}{16d}$$

$$= \frac{5}{16}a\left(a^2+b^2\right)^2 x + \frac{5a(a^2+b^2)(b+a\cot(c+dx))(a-b}{16d}$$

Mathematica [A]
time = 0.66, size = 188, normalized size = 1.49

$$\frac{60a(a^2+b^2)^2(c+dx) - 15b(5a^4+6a^2b^2+b^4)\cos(2(c+dx)) + 6b(-5a^4+b^4)\cos(4(c+dx)) - b(5a^4-10a^2b^2+b^4)\cos(6(c+dx)) + 15a(3a^4+2a^2b^2-b^4)\sin(2(c+dx)) + 3a(3a^4-10a^2b^2-5b^4)\sin(4(c+dx)) + a(a^4-10a^2b^2+5b^4)\sin(6(c+dx))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (60*a*(a^2 + b^2)^2*(c + d*x) - 15*b*(5*a^4 + 6*a^2*b^2 + b^4)*Cos[2*(c + d
*x)] + 6*b*(-5*a^4 + b^4)*Cos[4*(c + d*x)] - b*(5*a^4 - 10*a^2*b^2 + b^4)*C
os[6*(c + d*x)] + 15*a*(3*a^4 + 2*a^2*b^2 - b^4)*Sin[2*(c + d*x)] + 3*a*(3*
a^4 - 10*a^2*b^2 - 5*b^4)*Sin[4*(c + d*x)] + a*(a^4 - 10*a^2*b^2 + 5*b^4)*S
in[6*(c + d*x)])/(192*d)
```

Maple [A]
time = 0.26, size = 236, normalized size = 1.87 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/6*b^5*sin(d*x+c)^6+5*a*b^4*(-1/6*sin(d*x+c)^3*cos(d*x+c)^3-1/8*sin(d
*x+c)*cos(d*x+c)^3+1/16*cos(d*x+c)*sin(d*x+c)+1/16*d*x+1/16*c)+10*b^3*a^2*(
-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+10*a^3*b^2*(-1/6*sin(d*x+
c)*cos(d*x+c)^5+1/24*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+1/16*d*x+1/16
*c)-5/6*b*a^4*cos(d*x+c)^6+a^5*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos
(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A]
time = 0.29, size = 187, normalized size = 1.48

$$-\frac{160\,a^{4}b\cos\left(dx+c\right)^{6}-32\,b^{5}\sin\left(dx+c\right)^{6}+\left(4\,\sin\left(2\,dx+2\,c\right)^{3}-60\,dx-60\,c-9\,\sin\left(4\,dx+4\,c\right)-48\,\sin\left(2\,dx+2\,c\right)\right)a^{5}-10\left(4\,\sin\left(2\,dx+2\,c\right)^{3}+12\,dx+12\,c-3\,\sin\left(4\,dx+4\,c\right)\right)a^{3}b^{2}+160\left(2\,\sin\left(dx+c\right)^{6}-3\,\sin\left(dx+c\right)^{4}\right)a^{2}b^{3}+5\left(4\,\sin\left(2\,dx+2\,c\right)^{3}-12\,dx-12\,c+3\,\sin\left(4\,dx+4\,c\right)\right)ab^{4}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/192*(160*a^4*b*cos(d*x + c)^6 - 32*b^5*sin(d*x + c)^6 + (4*sin(2*d*x + 2
*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^5 - 10*
(4*sin(2*d*x + 2*c)^3 + 12*d*x + 12*c - 3*sin(4*d*x + 4*c))*a^3*b^2 + 160*(
2*sin(d*x + c)^6 - 3*sin(d*x + c)^4)*a^2*b^3 + 5*(4*sin(2*d*x + 2*c)^3 - 12
*d*x - 12*c + 3*sin(4*d*x + 4*c))*a*b^4)/d
```

Fricas [A]
time = 2.38, size = 182, normalized size = 1.44

$$-\frac{24\,b^{5}\cos\left(dx+c\right)^{2}+8\left(5\,a^{4}b-10\,a^{2}b^{3}+b^{5}\right)\cos\left(dx+c\right)^{6}+24\left(5\,a^{2}b^{3}-b^{5}\right)\cos\left(dx+c\right)^{4}-15\left(a^{5}+2\,a^{3}b^{2}+ab^{4}\right)dx-\left(8\left(a^{5}-10\,a^{3}b^{2}+5\,ab^{4}\right)\cos\left(dx+c\right)^{5}+10\left(a^{5}+2\,a^{3}b^{2}-7\,ab^{4}\right)\cos\left(dx+c\right)^{3}+15\left(a^{5}+2\,a^{3}b^{2}+ab^{4}\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/48*(24*b^5*cos(d*x + c)^2 + 8*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^
6 + 24*(5*a^2*b^3 - b^5)*cos(d*x + c)^4 - 15*(a^5 + 2*a^3*b^2 + a*b^4)*d*x
- (8*(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^5 + 10*(a^5 + 2*a^3*b^2 - 7*
a*b^4)*cos(d*x + c)^3 + 15*(a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c))*sin(d*x
+ c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(117) = 234$.
time = 0.49, size = 609, normalized size = 4.83

$$\begin{cases} \frac{5a^{5}x\sin^{6}{\left(c+dx\right)}}{16}+\frac{15a^{5}x\sin^{4}{\left(c+dx\right)}\cos^{2}{\left(c+dx\right)}}{16}+\frac{15a^{5}x\sin^{2}{\left(c+dx\right)}\cos^{4}{\left(c+dx\right)}}{16}+\frac{5a^{5}x\cos^{6}{\left(c+dx\right)}}{16}+\frac{5a^{5}\sin^{5}{\left(c+dx\right)}\cos{\left(c+dx\right)}}{16d}+\frac{5a^{5}\sin^{3}{\left(c+dx\right)}\cos^{3}{\left(c+dx\right)}}{6d}+\frac{11a^{5}\sin{\left(c+dx\right)}\cos^{5}{\left(c+dx\right)}}{16d}-\frac{5a^{4}b\cos^{6}{\left(c+dx\right)}}{6d}+\frac{5a^{3}b^{2}x\sin^{6}{\left(c+dx\right)}}{8}+\frac{15a^{3}b^{2}x\sin^{4}{\left(c+dx\right)}\cos^{2}{\left(c+dx\right)}}{8}+\frac{15a^{3}b^{2}x\sin^{2}{\left(c+dx\right)}\cos^{4}{\left(c+dx\right)}}{8}+\frac{5a^{3}b^{2}x\cos^{6}{\left(c+dx\right)}}{8}+\frac{5a^{3}b^{2}\sin^{5}{\left(c+dx\right)}\cos{\left(c+dx\right)}}{8d}+\frac{5a^{3}b^{2}\sin^{3}{\left(c+dx\right)}\cos^{3}{\left(c+dx\right)}}{3d}-\frac{5a^{3}b^{2}\sin{\left(c+dx\right)}\cos^{5}{\left(c+dx\right)}}{8d}-\frac{5a^{2}b^{3}\sin^{2}{\left(c+dx\right)}\cos^{4}{\left(c+dx\right)}}{2d}-\frac{5a^{2}b^{3}\cos^{6}{\left(c+dx\right)}}{6d}+\frac{5ab^{4}x\sin^{6}{\left(c+dx\right)}}{16}+\frac{15ab^{4}x\sin^{4}{\left(c+dx\right)}\cos^{2}{\left(c+dx\right)}}{16}+\frac{15ab^{4}x\sin^{2}{\left(c+dx\right)}\cos^{4}{\left(c+dx\right)}}{16}+\frac{5ab^{4}x\cos^{6}{\left(c+dx\right)}}{16}+\frac{5ab^{4}\sin^{5}{\left(c+dx\right)}\cos{\left(c+dx\right)}}{16d}-\frac{5ab^{4}\sin^{3}{\left(c+dx\right)}\cos^{3}{\left(c+dx\right)}}{6d}-\frac{5ab^{4}\sin{\left(c+dx\right)}\cos^{5}{\left(c+dx\right)}}{16d}+\frac{b^{5}\sin^{6}{\left(c+dx\right)}}{6d} & \text{for}\: d \neq 0 \\ x\left(a\cos{\left(c\right)}+b\sin{\left(c\right)}\right)^{5}\cos{\left(c\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Piecewise((5*a**5*x*sin(c + d*x)**6/16 + 15*a**5*x*sin(c + d*x)**4*cos(c +
d*x)**2/16 + 15*a**5*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a**5*x*cos(c
+ d*x)**6/16 + 5*a**5*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*a**5*sin(c +
d*x)**3*cos(c + d*x)**3/(6*d) + 11*a**5*sin(c + d*x)*cos(c + d*x)**5/(16*d)
 - 5*a**4*b*cos(c + d*x)**6/(6*d) + 5*a**3*b**2*x*sin(c + d*x)**6/8 + 15*a*
*3*b**2*x*sin(c + d*x)**4*cos(c + d*x)**2/8 + 15*a**3*b**2*x*sin(c + d*x)**
2*cos(c + d*x)**4/8 + 5*a**3*b**2*x*cos(c + d*x)**6/8 + 5*a**3*b**2*sin(c +
 d*x)**5*cos(c + d*x)/(8*d) + 5*a**3*b**2*sin(c + d*x)**3*cos(c + d*x)**3/(
3*d) - 5*a**3*b**2*sin(c + d*x)*cos(c + d*x)**5/(8*d) - 5*a**2*b**3*sin(c +
 d*x)**2*cos(c + d*x)**4/(2*d) - 5*a**2*b**3*cos(c + d*x)**6/(6*d) + 5*a*b*
*4*x*sin(c + d*x)**6/16 + 15*a*b**4*x*sin(c + d*x)**4*cos(c + d*x)**2/16 +
15*a*b**4*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*a*b**4*x*cos(c + d*x)**6
/16 + 5*a*b**4*sin(c + d*x)**5*cos(c + d*x)/(16*d) - 5*a*b**4*sin(c + d*x)*
*3*cos(c + d*x)**3/(6*d) - 5*a*b**4*sin(c + d*x)*cos(c + d*x)**5/(16*d) + b
**5*sin(c + d*x)**6/(6*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**5*cos(c), T
rue))
```

Giac [A]
time = 0.59, size = 211, normalized size = 1.67

$$\frac{5}{16}\left(a^{5}+2a^{3}b^{2}+ab^{4}\right)x-\frac{\left(5a^{4}b-10a^{2}b^{3}+b^{5}\right)\cos\left(6dx+6c\right)}{192d}-\frac{\left(5a^{4}b-b^{5}\right)\cos\left(4dx+4c\right)}{32d}-\frac{5\left(5a^{4}b+6a^{2}b^{3}+b^{5}\right)\cos\left(2dx+2c\right)}{64d}+\frac{\left(a^{5}-10a^{3}b^{2}+5ab^{4}\right)\sin\left(6dx+6c\right)}{192d}+\frac{\left(3a^{5}-10a^{3}b^{2}-5ab^{4}\right)\sin\left(4dx+4c\right)}{64d}+\frac{5\left(3a^{5}+2a^{3}b^{2}-ab^{4}\right)\sin\left(2dx+2c\right)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 5/16*(a^5 + 2*a^3*b^2 + a*b^4)*x - 1/192*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(6
*d*x + 6*c)/d - 1/32*(5*a^4*b - b^5)*cos(4*d*x + 4*c)/d - 5/64*(5*a^4*b + 6
*a^2*b^3 + b^5)*cos(2*d*x + 2*c)/d + 1/192*(a^5 - 10*a^3*b^2 + 5*a*b^4)*sin
(6*d*x + 6*c)/d + 1/64*(3*a^5 - 10*a^3*b^2 - 5*a*b^4)*sin(4*d*x + 4*c)/d +
5/64*(3*a^5 + 2*a^3*b^2 - a*b^4)*sin(2*d*x + 2*c)/d
```

Mupad [B]
time = 2.32, size = 472, normalized size = 3.75

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\left(-\frac{11a^{5}}{8}+\frac{5a^{3}b^{2}}{4}+\frac{5ab^{4}}{8}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-\frac{11a^{5}}{8}+\frac{5a^{3}b^{2}}{4}+\frac{5ab^{4}}{8}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}\left(\frac{15a^{5}}{4}-\frac{65a^{3}b^{2}}{2}+\frac{95ab^{4}}{4}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}\left(\frac{15a^{5}}{4}-\frac{65a^{3}b^{2}}{2}+\frac{95ab^{4}}{4}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}\left(\frac{5a^{5}}{24}-\frac{235a^{3}b^{2}}{12}+\frac{85ab^{4}}{24}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}\left(\frac{5a^{5}}{24}-\frac{235a^{3}b^{2}}{12}+\frac{85ab^{4}}{24}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}\left(\frac{100a^{4}b}{3}-\frac{80a^{2}b^{3}}{3}+\frac{32b^{5}}{3}\right)+40a^{2}b^{3}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+40a^{2}b^{3}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+10a^{4}b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+10a^{4}b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}+20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}+15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+1\right)}+\frac{5a\,\mathrm{atan}\left(\frac{5a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(a^{2}+b^{2}\right)^{2}}{8\left(\frac{5a^{5}}{8}+\frac{5a^{3}b^{2}}{4}+\frac{5ab^{4}}{8}\right)}\right)\left(a^{2}+b^{2}\right)^{2}}{8d}-\frac{5a\left(\mathrm{atan}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)-\frac{dx}{2}\right)\left(a^{2}+b^{2}\right)^{2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a*cos(c + d*x) + b*sin(c + d*x))^5,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^11*((5*a*b^4)/8 - (11*a^5)/8 + (5*a^3*b^2)/4) - tan(c/2
 + (d*x)/2)*((5*a*b^4)/8 - (11*a^5)/8 + (5*a^3*b^2)/4) + tan(c/2 + (d*x)/2)
^5*((95*a*b^4)/4 + (15*a^5)/4 - (65*a^3*b^2)/2) - tan(c/2 + (d*x)/2)^7*((95
*a*b^4)/4 + (15*a^5)/4 - (65*a^3*b^2)/2) - tan(c/2 + (d*x)/2)^3*((85*a*b^4)
/24 + (5*a^5)/24 - (235*a^3*b^2)/12) + tan(c/2 + (d*x)/2)^9*((85*a*b^4)/24
+ (5*a^5)/24 - (235*a^3*b^2)/12) + tan(c/2 + (d*x)/2)^6*((100*a^4*b)/3 + (3
2*b^5)/3 - (80*a^2*b^3)/3) + 40*a^2*b^3*tan(c/2 + (d*x)/2)^4 + 40*a^2*b^3*t
```

```
an(c/2 + (d*x)/2)^8 + 10*a^4*b*tan(c/2 + (d*x)/2)^2 + 10*a^4*b*tan(c/2 + (d
*x)/2)^10)/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/
2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/
2 + (d*x)/2)^12 + 1)) + (5*a*atan((5*a*tan(c/2 + (d*x)/2)*(a^2 + b^2)^2)/(8
*((5*a*b^4)/8 + (5*a^5)/8 + (5*a^3*b^2)/4)))*(a^2 + b^2)^2)/(8*d) - (5*a*(a
tan(tan(c/2 + (d*x)/2)) - (d*x)/2)*(a^2 + b^2)^2)/(8*d)
```

3.97 $\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=94

$$-\frac{(a^2+b^2)^2\,(b\cos(c+dx)-a\sin(c+dx))}{d}+\frac{2(a^2+b^2)\,(b\cos(c+dx)-a\sin(c+dx))^3}{3d}-\frac{(b\cos(c+dx)-a\,\mathrm{s}}{5d}$$

```
[Out] -(a^2+b^2)^2*(b*cos(d*x+c)-a*sin(d*x+c))/d+2/3*(a^2+b^2)*(b*cos(d*x+c)-a*si
n(d*x+c))^3/d-1/5*(b*cos(d*x+c)-a*sin(d*x+c))^5/d
```

Rubi [A]
time = 0.03, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3151, 200}

$$\frac{2(a^2+b^2)\,(b\cos(c+dx)-a\sin(c+dx))^3}{3d}-\frac{(a^2+b^2)^2\,(b\cos(c+dx)-a\sin(c+dx))}{d}-\frac{(b\cos(c+dx)-a\sin(c+dx))^5}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] -(((a^2 + b^2)^2*(b*Cos[c + d*x] - a*Sin[c + d*x]))/d) + (2*(a^2 + b^2)*(b*
Cos[c + d*x] - a*Sin[c + d*x])^3)/(3*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])
^5/(5*d)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3151

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Dist[-d^(-1), Subst[Int[(a^2 + b^2 - x^2)^((n - 1)/2), x], x, b
*Cos[c + d*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
b^2, 0] && IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a\cos(c+dx)+b\sin(c+dx))^5\,dx &= -\frac{\mathrm{Subst}\left(\int \left(a^2+b^2-x^2\right)^2\,dx, x, b\cos(c+dx)-a\sin(c+dx)\right)}{d}\\ &= -\frac{\mathrm{Subst}\left(\int \left(a^4\left(1+\frac{2a^2b^2+b^4}{a^4}\right)-2a^2\left(1+\frac{b^2}{a^2}\right)x^2+x^4\right)\,dx, x, b\cos(c\right.}{d}\\ &= -\frac{(a^2+b^2)^2\,(b\cos(c+dx)-a\sin(c+dx))}{d}+\frac{2(a^2+b^2)\,(b\cos(c+dx}{3d}\end{aligned}$$

Mathematica [A]

time = 0.50, size = 156, normalized size = 1.66

$$\frac{-150b(a^2+b^2)^2\cos(c+dx)+25b(-3a^4-2a^2b^2+b^4)\cos(3(c+dx))-3b(5a^4-10a^2b^2+b^4)\cos(5(c+dx))+150a(a^2+b^2)^2\sin(c+dx)+25a(a^4-2a^2b^2-3b^4)\sin(3(c+dx))+3a(a^4-10a^2b^2+5b^4)\sin(5(c+dx))}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (-150*b*(a^2 + b^2)^2*Cos[c + d*x] + 25*b*(-3*a^4 - 2*a^2*b^2 + b^4)*Cos[3*
(c + d*x)] - 3*b*(5*a^4 - 10*a^2*b^2 + b^4)*Cos[5*(c + d*x)] + 150*a*(a^2 +
 b^2)^2*Sin[c + d*x] + 25*a*(a^4 - 2*a^2*b^2 - 3*b^4)*Sin[3*(c + d*x)] + 3*
a*(a^4 - 10*a^2*b^2 + 5*b^4)*Sin[5*(c + d*x)])/(240*d)
```

Maple [A]

time = 0.24, size = 175, normalized size = 1.86

method	result
derivativedivides	$-\frac{b^5\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}+a\,b^4\left(\sin^5(dx+c)\right)+10b^3a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right.$
default	$-\frac{b^5\left(\frac{8}{3}+\sin^4(dx+c)+\frac{4\left(\sin^2(dx+c)\right)}{3}\right)\cos(dx+c)}{5}+a\,b^4\left(\sin^5(dx+c)\right)+10b^3a^2\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right.$
norman	$-\frac{30b\,a^4+40b^3a^2+16b^5}{15d}+\frac{2a^5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2a^5\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{40b^3a^2\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{10b\,a^4\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{(40b^3a^2+1}{}$
risch	$-\frac{5a^4b\cos(dx+c)}{8d}-\frac{5a^2b^3\cos(dx+c)}{4d}-\frac{5b^5\cos(dx+c)}{8d}+\frac{5a^5\sin(dx+c)}{8d}+\frac{5a^3b^2\sin(dx+c)}{4d}+\frac{5a\,b^4\sin(dx+c)}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5*b^5*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)+a*b^4*sin(d*x+
c)^5+10*b^3*a^2*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+10*a^3*b
^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))-b*a^4*co
s(d*x+c)^5+1/5*a^5*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]

time = 0.27, size = 172, normalized size = 1.83

$$-\frac{a^4b\cos(dx+c)^5}{d}+\frac{ab^4\sin(dx+c)^5}{d}+\frac{(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c))a^5}{15\,d}-\frac{2\,(3\sin(dx+c)^5-5\sin(dx+c)^3)a^3b^2}{3\,d}+\frac{2\,(3\cos(dx+c)^5-5\cos(dx+c)^3)a^2b^3}{3\,d}-\frac{(3\cos(dx+c)^5-10\cos(dx+c)^3+15\cos(dx+c))b^5}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -a^4*b*cos(d*x + c)^5/d + a*b^4*sin(d*x + c)^5/d + 1/15*(3*sin(d*x + c)^5 -
 10*sin(d*x + c)^3 + 15*sin(d*x + c))*a^5/d - 2/3*(3*sin(d*x + c)^5 - 5*sin
(d*x + c)^3)*a^3*b^2/d + 2/3*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^2*b^3/
d - 1/15*(3*cos(d*x + c)^5 - 10*cos(d*x + c)^3 + 15*cos(d*x + c))*b^5/d
```

Fricas [A]
time = 2.25, size = 155, normalized size = 1.65

$$-\frac{15\,b^{5}\cos\left(dx+c\right)+3\,\left(5\,a^{4}b-10\,a^{2}b^{3}+b^{5}\right)\cos\left(dx+c\right)^{5}+10\,\left(5\,a^{2}b^{3}-b^{5}\right)\cos\left(dx+c\right)^{3}-\left(8\,a^{5}+20\,a^{3}b^{2}+15\,ab^{4}+3\,\left(a^{5}-10\,a^{3}b^{2}+5\,ab^{4}\right)\cos\left(dx+c\right)^{4}+2\,\left(2\,a^{5}+5\,a^{3}b^{2}-15\,ab^{4}\right)\cos\left(dx+c\right)^{2}\right)\sin\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/15*(15*b^5*cos(d*x + c) + 3*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^5
+ 10*(5*a^2*b^3 - b^5)*cos(d*x + c)^3 - (8*a^5 + 20*a^3*b^2 + 15*a*b^4 + 3*
(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^4 + 2*(2*a^5 + 5*a^3*b^2 - 15*a*b
^4)*cos(d*x + c)^2)*sin(d*x + c))/d
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(82) = 164.
time = 0.34, size = 267, normalized size = 2.84

$$\begin{cases} \frac{8a^{5}\sin^{5}{\left(c+dx\right)}}{15d}+\frac{4a^{5}\sin^{3}{\left(c+dx\right)}\cos^{2}{\left(c+dx\right)}}{3d}+\frac{a^{5}\sin{\left(c+dx\right)}\cos^{4}{\left(c+dx\right)}}{d}-\frac{a^{4}b\cos^{5}{\left(c+dx\right)}}{d}+\frac{4a^{3}b^{2}\sin^{5}{\left(c+dx\right)}}{3d}+\frac{10a^{3}b^{2}\sin^{3}{\left(c+dx\right)}\cos^{2}{\left(c+dx\right)}}{3d}-\frac{10a^{2}b^{3}\sin^{2}{\left(c+dx\right)}\cos^{3}{\left(c+dx\right)}}{3d}-\frac{4a^{2}b^{3}\cos^{5}{\left(c+dx\right)}}{3d}+\frac{ab^{4}\sin^{5}{\left(c+dx\right)}}{d}-\frac{b^{5}\sin^{4}{\left(c+dx\right)}\cos{\left(c+dx\right)}}{d}-\frac{4b^{5}\sin^{2}{\left(c+dx\right)}\cos^{3}{\left(c+dx\right)}}{3d}-\frac{8b^{5}\cos^{5}{\left(c+dx\right)}}{15d} & \text{for}\: d\neq 0 \\ x\left(a\cos{\left(c\right)}+b\sin{\left(c\right)}\right)^{5} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Piecewise((8*a**5*sin(c + d*x)**5/(15*d) + 4*a**5*sin(c + d*x)**3*cos(c + d
*x)**2/(3*d) + a**5*sin(c + d*x)*cos(c + d*x)**4/d - a**4*b*cos(c + d*x)**5
/d + 4*a**3*b**2*sin(c + d*x)**5/(3*d) + 10*a**3*b**2*sin(c + d*x)**3*cos(c
 + d*x)**2/(3*d) - 10*a**2*b**3*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 4*a
**2*b**3*cos(c + d*x)**5/(3*d) + a*b**4*sin(c + d*x)**5/d - b**5*sin(c + d*
x)**4*cos(c + d*x)/d - 4*b**5*sin(c + d*x)**2*cos(c + d*x)**3/(3*d) - 8*b**
5*cos(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a*cos(c) + b*sin(c))**5, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(90) = 180.
time = 0.47, size = 187, normalized size = 1.99

$$-\frac{\left(5\,a^{4}b-10\,a^{2}b^{3}+b^{5}\right)\cos\left(5\,dx+5\,c\right)}{80\,d}-\frac{5\,\left(3\,a^{4}b+2\,a^{2}b^{3}-b^{5}\right)\cos\left(3\,dx+3\,c\right)}{48\,d}-\frac{5\,\left(a^{4}b+2\,a^{2}b^{3}+b^{5}\right)\cos\left(dx+c\right)}{8\,d}+\frac{\left(a^{5}-10\,a^{3}b^{2}+5\,ab^{4}\right)\sin\left(5\,dx+5\,c\right)}{80\,d}+\frac{5\,\left(a^{5}-2\,a^{3}b^{2}-3\,ab^{4}\right)\sin\left(3\,dx+3\,c\right)}{48\,d}+\frac{5\,\left(a^{5}+2\,a^{3}b^{2}+ab^{4}\right)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] -1/80*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(5*d*x + 5*c)/d - 5/48*(3*a^4*b + 2*a
^2*b^3 - b^5)*cos(3*d*x + 3*c)/d - 5/8*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x +
```

```
c)/d + 1/80*(a^5 - 10*a^3*b^2 + 5*a*b^4)*sin(5*d*x + 5*c)/d + 5/48*(a^5 - 2
*a^3*b^2 - 3*a*b^4)*sin(3*d*x + 3*c)/d + 5/8*(a^5 + 2*a^3*b^2 + a*b^4)*sin(
d*x + c)/d
```

Mupad [B]
time = 0.94, size = 248, normalized size = 2.64

$$\frac{2\left(\frac{3\sin(c+dx)\,a^5\cos(c+dx)^4}{2}+2\sin\left(c+dx\right)\,a^5\cos\left(c+dx\right)^2+4\sin\left(c+dx\right)\,a^5-\frac{15\,a^4\,b\cos(c+dx)^5}{2}-15\sin\left(c+dx\right)\,a^3\,b^2\cos\left(c+dx\right)^4+5\sin\left(c+dx\right)\,a^3\,b^2\cos\left(c+dx\right)^2+10\sin\left(c+dx\right)\,a^3\,b^2+15\,a^2\,b^3\cos\left(c+dx\right)^5-25\,a^2\,b^3\cos\left(c+dx\right)^3+\frac{15\sin(c+dx)\,a\,b^4\cos(c+dx)^4}{2}-15\sin\left(c+dx\right)\,a\,b^4\cos\left(c+dx\right)^2+\frac{15\sin(c+dx)\,a\,b^4}{2}-\frac{3\,b^5\cos(c+dx)^5}{2}+5\,b^5\cos\left(c+dx\right)^3-\frac{15\,b^5\cos(c+dx)}{2}\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5,x)
```

```
[Out] (2*(4*a^5*sin(c + d*x) - (15*b^5*cos(c + d*x))/2 + 5*b^5*cos(c + d*x)^3 - (
3*b^5*cos(c + d*x)^5)/2 - (15*a^4*b*cos(c + d*x)^5)/2 + 2*a^5*cos(c + d*x)^
2*sin(c + d*x) + (3*a^5*cos(c + d*x)^4*sin(c + d*x))/2 + 10*a^3*b^2*sin(c +
 d*x) - 25*a^2*b^3*cos(c + d*x)^3 + 15*a^2*b^3*cos(c + d*x)^5 + (15*a*b^4*s
in(c + d*x))/2 + 5*a^3*b^2*cos(c + d*x)^2*sin(c + d*x) - 15*a^3*b^2*cos(c +
 d*x)^4*sin(c + d*x) - 15*a*b^4*cos(c + d*x)^2*sin(c + d*x) + (15*a*b^4*cos
(c + d*x)^4*sin(c + d*x))/2))/(15*d)
```

3.98 $\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=170

$$\frac{1}{8}a\left(3a^4+10a^2b^2+15b^4\right)x-\frac{b^5\log(\sin(c+dx))}{d}+\frac{b^5\log(\tan(c+dx))}{d}+\frac{\left(4b(5a^4-b^4)+5a(a^2-3b^2)\left(a^2+b^2\right)\right.}{8d}$$

```
[Out] 1/8*a*(3*a^4+10*a^2*b^2+15*b^4)*x-b^5*ln(sin(d*x+c))/d+b^5*ln(tan(d*x+c))/d
+1/8*(4*b*(5*a^4-b^4)+5*a*(a^2-3*b^2)*(a^2+b^2)*cot(d*x+c))*sin(d*x+c)^2/d-
1/4*(b*(5*a^4-10*a^2*b^2+b^4)+a*(a^4-10*a^2*b^2+5*b^4)*cot(d*x+c))*sin(d*x+
c)^4/d
```

Rubi [A]
time = 0.15, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3167, 1819, 815, 649, 209, 266}

$$-\frac{\sin^4(c+dx)\left(a(a^4-10a^2b^2+5b^4)\cot(c+dx)+b(5a^4-10a^2b^2+b^4)\right)}{4d}+\frac{\sin^2(c+dx)\left(4b(5a^4-b^4)+5a(a^2-3b^2)\left(a^2+b^2\right)\cot(c+dx)\right)}{8d}+\frac{1}{8}ax\left(3a^4+10a^2b^2+15b^4\right)-\frac{b^5\log(\sin(c+dx))}{d}+\frac{b^5\log(\tan(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (a*(3*a^4 + 10*a^2*b^2 + 15*b^4)*x)/8 - (b^5*Log[Sin[c + d*x]])/d + (b^5*Lo
g[Tan[c + d*x]])/d + ((4*b*(5*a^4 - b^4) + 5*a*(a^2 - 3*b^2)*(a^2 + b^2)*Co
t[c + d*x])*Sin[c + d*x]^2)/(8*d) - ((b*(5*a^4 - 10*a^2*b^2 + b^4) + a*(a^4
 - 10*a^2*b^2 + 5*b^4)*Cot[c + d*x])*Sin[c + d*x]^4)/(4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \sec(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^5}{x(1+x^2)^3}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{(b(5a^4-10a^2b^2+b^4)+a(a^4-10a^2b^2+5b^4)\cot(c+}{4d}$$

$$= \frac{(4b(5a^4-b^4)+5a(a^2-3b^2)(a^2+b^2)\cot(c+dx))\sin^2(}{8d}$$

$$= \frac{(4b(5a^4-b^4)+5a(a^2-3b^2)(a^2+b^2)\cot(c+dx))\sin^2(}{8d}$$

$$= \frac{b^5\log(\tan(c+dx))}{d} + \frac{(4b(5a^4-b^4)+5a(a^2-3b^2)(a^2}{8d}$$

$$= \frac{b^5\log(\tan(c+dx))}{d} + \frac{(4b(5a^4-b^4)+5a(a^2-3b^2)(a^2}{8d}$$

$$= \frac{1}{8}a\left(3a^4+10a^2b^2+15b^4\right)x - \frac{b^5\log(\sin(c+dx))}{d} + \frac{b^5\,\text{lo}}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 711 vs. 2(170) = 340.

time = 6.48, size = 711, normalized size = 4.18

$$\frac{b^5\left(\frac{\cos^4(c+dx)(a+b\tan(c+dx))^6(b^2+ab\tan(c+dx))}{4b^6(a^2+b^2)}-\frac{\cos^2(c+dx)(a+b\tan(c+dx))^6\left(-3a^2b^2+b^2(-3a^2+2b^2)+b(3ab^2+a(-3a^2+2b^2))\tan(c+dx)\right)}{2b^4(a^2+b^2)}-\frac{(3a^4-29a^2b^2+8b^4+5a^2(3a^2-5b^2))\left(\frac{1}{2}\left(5a^4-10a^2b^2+b^4+\frac{a^5-10a^3b^2+5ab^4}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+\frac{1}{2}\left(5a^4-10a^2b^2+b^4-\frac{a^5-10a^3b^2+5ab^4}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)+5ab(2a^2-b^2)\tan(c+dx)+\frac{1}{2}b^2(10a^2-b^2)\tan^2(c+dx)+\frac{5}{3}ab^3\tan^3(c+dx)+\frac{1}{4}b^4\tan^4(c+dx)\right)-\frac{5a(3a^2-5b^2)\left(\frac{1}{2}\left(6a^5-20a^3b^2+6ab^4+\frac{a^6-15a^4b^2+15a^2b^4-b^6}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+\frac{1}{2}\left(6a^5-20a^3b^2+6ab^4-\frac{a^6-15a^4b^2+15a^2b^4-b^6}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)+b(15a^4-15a^2b^2+b^4)\tan(c+dx)+ab^2(10a^2-3b^2)\tan^2(c+dx)+\frac{1}{3}b^3(15a^2-b^2)\tan^3(c+dx)+\frac{3}{2}ab^4\tan^4(c+dx)+\frac{1}{5}b^5\tan^5(c+dx)\right)}{2b^2(a^2+b^2)}}{4b^2(a^2+b^2)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (b^5*((Cos[c + d*x]^4*(a + b*Tan[c + d*x])^6*(b^2 + a*b*Tan[c + d*x]))/(4*b
^6*(a^2 + b^2)) - ((Cos[c + d*x]^2*(a + b*Tan[c + d*x])^6*(-3*a^2*b^2 + b^2
*(-3*a^2 + 2*b^2) + b*(3*a*b^2 + a*(-3*a^2 + 2*b^2))*Tan[c + d*x]))/(2*b^4*
(a^2 + b^2)) - ((3*a^4 - 29*a^2*b^2 + 8*b^4 + 5*a^2*(3*a^2 - 5*b^2))*(((5*a
^4 - 10*a^2*b^2 + b^4 + (a^5 - 10*a^3*b^2 + 5*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-
b^2] - b*Tan[c + d*x]])/2 + ((5*a^4 - 10*a^2*b^2 + b^4 - (a^5 - 10*a^3*b^2
+ 5*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/2 + 5*a*b*(2*a^2 -
 b^2)*Tan[c + d*x] + (b^2*(10*a^2 - b^2)*Tan[c + d*x]^2)/2 + (5*a*b^3*Tan[c
 + d*x]^3)/3 + (b^4*Tan[c + d*x]^4)/4) - 5*a*(3*a^2 - 5*b^2)*(((6*a^5 - 20*
a^3*b^2 + 6*a*b^4 + (a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)/Sqrt[-b^2])*Log[S
qrt[-b^2] - b*Tan[c + d*x]])/2 + ((6*a^5 - 20*a^3*b^2 + 6*a*b^4 - (a^6 - 15
*a^4*b^2 + 15*a^2*b^4 - b^6)/Sqrt[-b^2])*Log[Sqrt[-b^2] + b*Tan[c + d*x]])/
2 + b*(15*a^4 - 15*a^2*b^2 + b^4)*Tan[c + d*x] + a*b^2*(10*a^2 - 3*b^2)*Tan
[c + d*x]^2 + (b^3*(15*a^2 - b^2)*Tan[c + d*x]^3)/3 + (3*a*b^4*Tan[c + d*x]
^4)/2 + (b^5*Tan[c + d*x]^5)/5))/(2*b^2*(a^2 + b^2)))/(4*b^2*(a^2 + b^2))))
/d
```

Maple [A]
time = 0.32, size = 192, normalized size = 1.13

method	result
derivativedivides	$\frac{a^5\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{5a^4\left(\cos^4(dx+c)\right)b}{4}+10a^3b^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(\ldots}{8}\right.}{}$
default	$\frac{a^5\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)-\frac{5a^4\left(\cos^4(dx+c)\right)b}{4}+10a^3b^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(\ldots}{8}\right.}{}$
risch	$ix\,b^5+\frac{3a^5x}{8}+\frac{5a^3b^2x}{4}+\frac{15a\,b^4x}{8}-\frac{5\,\mathrm{e}^{2i(dx+c)}a^4b}{16d}-\frac{5\,\mathrm{e}^{2i(dx+c)}a^2b^3}{8d}+\frac{3\,\mathrm{e}^{2i(dx+c)}b^5}{16d}+\frac{2ib^5c}{d}-\frac{i\mathrm{e}^{2i(dx+c)}\ldots}{8d}$
norman	$\frac{\left(\frac{3}{8}a^5+\frac{5}{4}a^3b^2+\frac{15}{8}a\,b^4\right)x+\left(\frac{3}{8}a^5+\frac{5}{4}a^3b^2+\frac{15}{8}a\,b^4\right)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{15}{4}a^5+\frac{25}{2}a^3b^2+\frac{75}{4}a\,b^4\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{15}{4}a^5+\ldots\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^5*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)-5/4*a
^4*cos(d*x+c)^4*b+10*a^3*b^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*s
```

```
in(d*x+c)+1/8*d*x+1/8*c)+5/2*b^3*a^2*sin(d*x+c)^4+5*a*b^4*(-1/4*(sin(d*x+c)
^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+b^5*(-1/4*sin(d*x+c)^4-1/2*sin
(d*x+c)^2-ln(cos(d*x+c))))
```

Maxima [A]
time = 0.28, size = 170, normalized size = 1.00

$$\frac{80\,a^2b^3\sin\left(dx+c\right)^4-40\,\left(\sin\left(dx+c\right)^2-1\right)^2a^4b+\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)a^5+10\,\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)a^3b^2+5\,\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)-8\,\sin\left(2\,dx+2\,c\right)\right)ab^4-8\,\left(\sin\left(dx+c\right)^4+2\,\sin\left(dx+c\right)^2+2\,\log\left(\sin\left(dx+c\right)^2-1\right)\right)b^5}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/32*(80*a^2*b^3*sin(d*x + c)^4 - 40*(sin(d*x + c)^2 - 1)^2*a^4*b + (12*d*x
 + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*a^5 + 10*(4*d*x + 4*c - si
n(4*d*x + 4*c))*a^3*b^2 + 5*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x
 + 2*c))*a*b^4 - 8*(sin(d*x + c)^4 + 2*sin(d*x + c)^2 + 2*log(sin(d*x + c)^
2 - 1))*b^5)/d
```

Fricas [A]
time = 3.04, size = 160, normalized size = 0.94

$$-\frac{8\,b^5\log\left(-\cos\left(dx+c\right)\right)+2\,\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(dx+c\right)^4-\left(3\,a^5+10\,a^3b^2+15\,ab^4\right)dx+8\,\left(5\,a^2b^3-b^5\right)\cos\left(dx+c\right)^2-\left(2\,\left(a^5-10\,a^3b^2+5\,ab^4\right)\cos\left(dx+c\right)^3+\left(3\,a^5+10\,a^3b^2-25\,ab^4\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] -1/8*(8*b^5*log(-cos(d*x + c)) + 2*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c
)^4 - (3*a^5 + 10*a^3*b^2 + 15*a*b^4)*d*x + 8*(5*a^2*b^3 - b^5)*cos(d*x + c
)^2 - (2*(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^3 + (3*a^5 + 10*a^3*b^2
- 25*a*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.60, size = 199, normalized size = 1.17

$$\frac{4\,b^5\log\left(\tan\left(dx+c\right)^2+1\right)+\left(3\,a^5+10\,a^3b^2+15\,ab^4\right)\left(dx+c\right)-\frac{6\,b^5\tan(dx+c)^4-3\,a^5\tan(dx+c)^3-10\,a^3b^2\tan(dx+c)^3+25\,ab^4\tan(dx+c)^3+40\,a^2b^3\tan(dx+c)^2+4\,b^5\tan(dx+c)^2-5\,a^5\tan(dx+c)+10\,a^3b^2\tan(dx+c)+15\,ab^4\tan(dx+c)+10\,a^4b+20\,a^2b^3}{\left(\tan(dx+c)^2+1\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/8*(4*b^5*log(tan(d*x + c)^2 + 1) + (3*a^5 + 10*a^3*b^2 + 15*a*b^4)*(d*x +
 c) - (6*b^5*tan(d*x + c)^4 - 3*a^5*tan(d*x + c)^3 - 10*a^3*b^2*tan(d*x + c
)^3 + 25*a*b^4*tan(d*x + c)^3 + 40*a^2*b^3*tan(d*x + c)^2 + 4*b^5*tan(d*x +
 c)^2 - 5*a^5*tan(d*x + c) + 10*a^3*b^2*tan(d*x + c) + 15*a*b^4*tan(d*x + c
) + 10*a^4*b + 20*a^2*b^3)/(tan(d*x + c)^2 + 1)^2)/d
```

Mupad [B]
time = 2.65, size = 297, normalized size = 1.75

$$\frac{4\,b^5\,\ln\left(\frac{1}{{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}\right)-4\,b^5\,\ln\left(\frac{\cos(c+d\,x)}{\cos(c+d\,x)+1}\right)+3\,a^5\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+\frac{3\,b^5\,\cos(2\,c+2\,d\,x)}{2}-\frac{b^5\,\cos(4\,c+4\,d\,x)}{8}+a^5\,\sin\left(2\,c+2\,d\,x\right)+\frac{a^5\,\sin(4\,c+4\,d\,x)}{8}+15\,a\,b^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-\frac{5\,a^4\,b\,\cos(2\,c+2\,d\,x)}{2}-\frac{5\,a^4\,b\,\cos(4\,c+4\,d\,x)}{8}-5\,a\,b^4\,\sin\left(2\,c+2\,d\,x\right)+\frac{5\,a\,b^4\,\sin(4\,c+4\,d\,x)}{8}+10\,a^3\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-5\,a^2\,b^3\,\cos\left(2\,c+2\,d\,x\right)+\frac{5\,a^2\,b^3\,\cos(4\,c+4\,d\,x)}{4}-\frac{5\,a^3\,b^2\,\sin(4\,c+4\,d\,x)}{4}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x),x)
```

```
[Out] (4*b^5*log(1/cos(c/2 + (d*x)/2)^2) - 4*b^5*log(cos(c + d*x)/(cos(c + d*x) +
 1)) + 3*a^5*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + (3*b^5*cos(2*c +
 2*d*x))/2 - (b^5*cos(4*c + 4*d*x))/8 + a^5*sin(2*c + 2*d*x) + (a^5*sin(4*c
 + 4*d*x))/8 + 15*a*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - (5*a^
4*b*cos(2*c + 2*d*x))/2 - (5*a^4*b*cos(4*c + 4*d*x))/8 - 5*a*b^4*sin(2*c +
2*d*x) + (5*a*b^4*sin(4*c + 4*d*x))/8 + 10*a^3*b^2*atan(sin(c/2 + (d*x)/2)/
cos(c/2 + (d*x)/2)) - 5*a^2*b^3*cos(2*c + 2*d*x) + (5*a^2*b^3*cos(4*c + 4*d
*x))/4 - (5*a^3*b^2*sin(4*c + 4*d*x))/4)/(4*d)
```

3.99 $\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=205

$$\frac{5ab^4\tanh^{-1}(\sin(c+dx))}{d}-\frac{10a^2b^3\cos(c+dx)}{d}+\frac{2b^5\cos(c+dx)}{d}-\frac{5a^4b\cos^3(c+dx)}{3d}+\frac{10a^2b^3\cos^3(c+dx)}{3d}-$$

```
[Out] 5*a*b^4*arctanh(sin(d*x+c))/d-10*a^2*b^3*cos(d*x+c)/d+2*b^5*cos(d*x+c)/d-5/
3*a^4*b*cos(d*x+c)^3/d+10/3*a^2*b^3*cos(d*x+c)^3/d-1/3*b^5*cos(d*x+c)^3/d+b
^5*sec(d*x+c)/d+a^5*sin(d*x+c)/d-5*a*b^4*sin(d*x+c)/d-1/3*a^5*sin(d*x+c)^3/
d+10/3*a^3*b^2*sin(d*x+c)^3/d-5/3*a*b^4*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.16, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3169, 2713, 2645, 30, 2644, 2672, 308, 212, 2670, 276}

$$-\frac{a^5\sin^3(c+dx)}{3d}+\frac{a^5\sin(c+dx)}{d}-\frac{5a^4b\cos^3(c+dx)}{3d}+\frac{10a^3b^2\sin^3(c+dx)}{3d}+\frac{10a^2b^3\cos^3(c+dx)}{3d}-\frac{10a^2b^3\cos(c+dx)}{d}-\frac{5ab^4\sin^3(c+dx)}{3d}-\frac{5ab^4\sin(c+dx)}{d}+\frac{5ab^4\tanh^{-1}(\sin(c+dx))}{d}-\frac{b^5\cos^3(c+dx)}{3d}+\frac{2b^5\cos(c+dx)}{d}+\frac{b^5\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (5*a*b^4*ArcTanh[Sin[c + d*x]])/d - (10*a^2*b^3*Cos[c + d*x])/d + (2*b^5*Co
s[c + d*x])/d - (5*a^4*b*Cos[c + d*x]^3)/(3*d) + (10*a^2*b^3*Cos[c + d*x]^3
)/(3*d) - (b^5*Cos[c + d*x]^3)/(3*d) + (b^5*Sec[c + d*x])/d + (a^5*Sin[c +
d*x])/d - (5*a*b^4*Sin[c + d*x])/d - (a^5*Sin[c + d*x]^3)/(3*d) + (10*a^3*b
^2*Sin[c + d*x]^3)/(3*d) - (5*a*b^4*Sin[c + d*x]^3)/(3*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 308

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \sec^2(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int \left(a^5\cos^3(c+dx)+5a^4b\cos^2(c+dx)\sin(c+dx)+1\right.$$
$$= a^5\int \cos^3(c+dx)\,dx + \left(5a^4b\right)\int \cos^2(c+dx)\sin(c+$$
$$= -\frac{a^5\text{Subst}\left(\int \left(1-x^2\right)\,dx, x, -\sin(c+dx)\right)}{d} - \frac{\left(5a^4b\right)\text{Su}}{}$$
$$= -\frac{10a^2b^3\cos(c+dx)}{d} - \frac{5a^4b\cos^3(c+dx)}{3d} + \frac{10a^2b^3\cos}{3}$$
$$= -\frac{10a^2b^3\cos(c+dx)}{d} + \frac{2b^5\cos(c+dx)}{d} - \frac{5a^4b\cos^3(c+}{3d}$$
$$= \frac{5ab^4\tanh^{-1}(\sin(c+dx))}{d} - \frac{10a^2b^3\cos(c+dx)}{d} + \frac{2b^5\,c}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 632 vs. 2(205) = 410.
time = 6.32, size = 632, normalized size = 3.08

$$\frac{b^5\cos^5(c+dx)(a+b\tan(c+dx))^5}{d(a\cos(c+dx)+b\sin(c+dx))^5} - \frac{b(5a^4+30a^2b^2-7b^4)\cos^6(c+dx)(a+b\tan(c+dx))^5}{4d(a\cos(c+dx)+b\sin(c+dx))^5} - \frac{b(5a^4-10a^2b^2+b^4)\cos^5(c+dx)\cos(3(c+dx))(a+b\tan(c+dx))^5}{12d(a\cos(c+dx)+b\sin(c+dx))^5} - \frac{5ab^4\cos^5(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^5}{d(a\cos(c+dx)+b\sin(c+dx))^5} + \frac{5ab^4\cos^5(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^5}{d(a\cos(c+dx)+b\sin(c+dx))^5} + \frac{b^5\cos^5(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^5}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^5} - \frac{b^5\cos^5(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)(a+b\tan(c+dx))^5}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^5} + \frac{a(3a^4+10a^2b^2-25b^4)\cos^5(c+dx)\sin(c+dx)(a+b\tan(c+dx))^5}{4d(a\cos(c+dx)+b\sin(c+dx))^5} + \frac{a(a^4-10a^2b^2+5b^4)\cos^5(c+dx)\sin(3(c+dx))(a+b\tan(c+dx))^5}{12d(a\cos(c+dx)+b\sin(c+dx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (b^5*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(d*(a*Cos[c + d*x] + b*Sin[c +
d*x])^5) - (b*(5*a^4 + 30*a^2*b^2 - 7*b^4)*Cos[c + d*x]^6*(a + b*Tan[c + d*
x])^5)/(4*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) - (b*(5*a^4 - 10*a^2*b^2 +
 b^4)*Cos[c + d*x]^5*Cos[3*(c + d*x)]*(a + b*Tan[c + d*x])^5)/(12*d*(a*Cos[
c + d*x] + b*Sin[c + d*x])^5) - (5*a*b^4*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^5)/(d*(a*Cos[c + d*x] + b*Sin[c
+ d*x])^5) + (5*a*b^4*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]]*(a + b*Tan[c + d*x])^5)/(d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (b^5*C
os[c + d*x]^5*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^5)/(d*(Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) - (b^5*Cos[c + d*
x]^5*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^5)/(d*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (a*(3*a^4 + 10*a^2*b^2 -
 25*b^4)*Cos[c + d*x]^5*Sin[c + d*x]*(a + b*Tan[c + d*x])^5)/(4*d*(a*Cos[c
+ d*x] + b*Sin[c + d*x])^5) + (a*(a^4 - 10*a^2*b^2 + 5*b^4)*Cos[c + d*x]^5*
Sin[3*(c + d*x)]*(a + b*Tan[c + d*x])^5)/(12*d*(a*Cos[c + d*x] + b*Sin[c +
d*x])^5)
```

Maple [**A**]
time = 0.37, size = 169, normalized size = 0.82

method	result
derivativedivides	$\frac{\frac{a^5\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}-\frac{5b\,a^4\left(\cos^3(dx+c)\right)}{3}+\frac{10a^3b^2\left(\sin^3(dx+c)\right)}{3}-\frac{10b^3a^2\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+5a\,b^4\left(-\frac{\left(\sin^3(dx+\right.}{3}\right.}{d}$
default	$\frac{\frac{a^5\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}-\frac{5b\,a^4\left(\cos^3(dx+c)\right)}{3}+\frac{10a^3b^2\left(\sin^3(dx+c)\right)}{3}-\frac{10b^3a^2\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+5a\,b^4\left(-\frac{\left(\sin^3(dx+\right.}{3}\right.}{d}$
risch	$-\frac{5\,e^{i(dx+c)}a^4b}{8d}-\frac{15\,e^{i(dx+c)}a^2b^3}{4d}+\frac{7\,e^{i(dx+c)}b^5}{8d}-\frac{3ie^{i(dx+c)}a^5}{8d}+\frac{25ie^{i(dx+c)}a\,b^4}{8d}-\frac{5ie^{i(dx+c)}a^3b^2}{4d}-\frac{5\,e^{-i(d}}{8}$
norman	$\frac{10b\,a^4+40b^3a^2-16b^5}{3d}-\frac{10b\,a^4\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{5\left(2b\,a^4+8b^3a^2\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{5\left(4b\,a^4+16b^3a^2-16b^5\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2\left(5b\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*a^5*(2+cos(d*x+c)^2)*sin(d*x+c)-5/3*b*a^4*cos(d*x+c)^3+10/3*a^3*b^
2*sin(d*x+c)^3-10/3*b^3*a^2*(2+sin(d*x+c)^2)*cos(d*x+c)+5*a*b^4*(-1/3*sin(d
*x+c)^3-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+b^5*(sin(d*x+c)^6/cos(d*x+c)+
(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.28, size = 162, normalized size = 0.79

$$-\frac{10\,a^4b\cos\left(dx+c\right)^3-20\,a^3b^2\sin\left(dx+c\right)^3+2\left(\sin\left(dx+c\right)^3-3\sin\left(dx+c\right)\right)a^5-20\left(\cos\left(dx+c\right)^3-3\cos\left(dx+c\right)\right)a^2b^3+5\left(2\sin\left(dx+c\right)^3-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)+6\sin\left(dx+c\right)\right)ab^4+2\left(\cos\left(dx+c\right)^3-\frac{3}{\cos(dx+c)}-6\cos\left(dx+c\right)\right)b^5}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/6*(10*a^4*b*cos(d*x + c)^3 - 20*a^3*b^2*sin(d*x + c)^3 + 2*(sin(d*x + c)
^3 - 3*sin(d*x + c))*a^5 - 20*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^2*b^3 + 5
*(2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*
sin(d*x + c))*a*b^4 + 2*(cos(d*x + c)^3 - 3/cos(d*x + c) - 6*cos(d*x + c))*
b^5)/d
```

Fricas [A]
time = 2.40, size = 177, normalized size = 0.86

$$\frac{15\,ab^4\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-15\,ab^4\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+6\,b^5-2\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(dx+c\right)^4-12\left(5\,a^2b^3-b^5\right)\cos\left(dx+c\right)^2+2\left(\left(a^5-10\,a^3b^2+5\,ab^4\right)\cos\left(dx+c\right)^3+2\left(a^5+5\,a^3b^2-10\,ab^4\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/6*(15*a*b^4*cos(d*x + c)*log(sin(d*x + c) + 1) - 15*a*b^4*cos(d*x + c)*lo
g(-sin(d*x + c) + 1) + 6*b^5 - 2*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^
```

```
4 - 12*(5*a^2*b^3 - b^5)*cos(d*x + c)^2 + 2*((a^5 - 10*a^3*b^2 + 5*a*b^4)*c
os(d*x + c)^3 + 2*(a^5 + 5*a^3*b^2 - 10*a*b^4)*cos(d*x + c))*sin(d*x + c))/
(d*cos(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.61, size = 283, normalized size = 1.38

$$\frac{15\,ab^{4}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,ab^{4}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{6\,b^{5}}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}+\frac{2\left(3\,a^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-15\,ab^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-15\,a^{4}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+3\,b^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+2\,a^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+40\,a^{3}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-50\,ab^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-60\,a^{2}b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+12\,b^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+3\,a^{5}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-15\,ab^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\,a^{4}b-20\,a^{2}b^{3}+5\,b^{5}\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/3*(15*a*b^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*a*b^4*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 6*b^5/(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(3*a^5*tan(1/2
*d*x + 1/2*c)^5 - 15*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 15*a^4*b*tan(1/2*d*x +
1/2*c)^4 + 3*b^5*tan(1/2*d*x + 1/2*c)^4 + 2*a^5*tan(1/2*d*x + 1/2*c)^3 + 40
*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 50*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 60*a^2*
b^3*tan(1/2*d*x + 1/2*c)^2 + 12*b^5*tan(1/2*d*x + 1/2*c)^2 + 3*a^5*tan(1/2*
d*x + 1/2*c) - 15*a*b^4*tan(1/2*d*x + 1/2*c) - 5*a^4*b - 20*a^2*b^3 + 5*b^5
)/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B]
time = 3.98, size = 277, normalized size = 1.35

$$\frac{10\,a\,b^{4}\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(10\,a\,b^{4}-2\,a^{5}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\,\left(10\,a^{4}\,b-40\,a^{2}\,b^{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}\,\left(\frac{2\,a^{5}}{3}-\frac{80\,a^{3}\,b^{2}}{3}+\frac{70\,a\,b^{4}}{3}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}\,\left(\frac{2\,a^{5}}{3}-\frac{80\,a^{3}\,b^{2}}{3}+\frac{70\,a\,b^{4}}{3}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\,\left(\frac{10\,a^{4}\,b}{3}-\frac{80\,a^{2}\,b^{3}}{3}+\frac{32\,b^{5}}{3}\right)+\frac{10\,a^{4}\,b}{3}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}\,\left(10\,a\,b^{4}-2\,a^{5}\right)-\frac{16\,b^{5}}{3}+\frac{40\,a^{2}\,b^{3}}{3}-10\,a^{4}\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^2,x)
```

```
[Out] (10*a*b^4*atanh(tan(c/2 + (d*x)/2)))/d - (tan(c/2 + (d*x)/2)*(10*a*b^4 - 2*
a^5) + tan(c/2 + (d*x)/2)^4*(10*a^4*b - 40*a^2*b^3) + tan(c/2 + (d*x)/2)^3*
((70*a*b^4)/3 + (2*a^5)/3 - (80*a^3*b^2)/3) - tan(c/2 + (d*x)/2)^5*((70*a*b
^4)/3 + (2*a^5)/3 - (80*a^3*b^2)/3) - tan(c/2 + (d*x)/2)^2*((10*a^4*b)/3 +
(32*b^5)/3 - (80*a^2*b^3)/3) + (10*a^4*b)/3 - tan(c/2 + (d*x)/2)^7*(10*a*b^
4 - 2*a^5) - (16*b^5)/3 + (40*a^2*b^3)/3 - 10*a^4*b*tan(c/2 + (d*x)/2)^6)/(
d*(2*tan(c/2 + (d*x)/2)^2 - 2*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^8 +
 1))
```

3.100 $\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=169

$$\frac{1}{2}a\left(a^4+10a^2b^2-15b^4\right)x-\frac{2b^3(5a^2-b^2)\log(\sin(c+dx))}{d}+\frac{2b^3(5a^2-b^2)\log(\tan(c+dx))}{d}+\frac{(b(5a^4-10a^2b^2}{}$$

```
[Out] 1/2*a*(a^4+10*a^2*b^2-15*b^4)*x-2*b^3*(5*a^2-b^2)*ln(sin(d*x+c))/d+2*b^3*(5
*a^2-b^2)*ln(tan(d*x+c))/d+1/2*(b*(5*a^4-10*a^2*b^2+b^4)+a*(a^4-10*a^2*b^2+
5*b^4)*cot(d*x+c))*sin(d*x+c)^2/d+5*a*b^4*tan(d*x+c)/d+1/2*b^5*tan(d*x+c)^2
/d
```

Rubi [A]
time = 0.17, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3167, 1819, 1816, 649, 209, 266}

$$-\frac{2b^3(5a^2-b^2)\log(\sin(c+dx))}{d}+\frac{2b^3(5a^2-b^2)\log(\tan(c+dx))}{d}+\frac{\sin^2(c+dx)\left(a(a^4-10a^2b^2+5b^4)\cot(c+dx)+b(5a^4-10a^2b^2+b^4)\right)}{2d}+\frac{1}{2}ax\left(a^4+10a^2b^2-15b^4\right)+\frac{5ab^4\tan(c+dx)}{d}+\frac{b^5\tan^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (a*(a^4 + 10*a^2*b^2 - 15*b^4)*x)/2 - (2*b^3*(5*a^2 - b^2)*Log[Sin[c + d*x]
])/d + (2*b^3*(5*a^2 - b^2)*Log[Tan[c + d*x]])/d + ((b*(5*a^4 - 10*a^2*b^2
+ b^4) + a*(a^4 - 10*a^2*b^2 + 5*b^4)*Cot[c + d*x])*Sin[c + d*x]^2)/(2*d) +
 (5*a*b^4*Tan[c + d*x])/d + (b^5*Tan[c + d*x]^2)/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
```

```
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^5}{x^3(1+x^2)^2}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= \frac{(b(5a^4-10a^2b^2+b^4)+a(a^4-10a^2b^2+5b^4)\cot(c+d}{2d}$$

$$= \frac{(b(5a^4-10a^2b^2+b^4)+a(a^4-10a^2b^2+5b^4)\cot(c+d}{2d}$$

$$= \frac{2b^3(5a^2-b^2)\log(\tan(c+dx))}{d} + \frac{(b(5a^4-10a^2b^2+b^4)}{}$$

$$= \frac{2b^3(5a^2-b^2)\log(\tan(c+dx))}{d} + \frac{(b(5a^4-10a^2b^2+b^4)}{}$$

$$= \frac{1}{2}a\left(a^4+10a^2b^2-15b^4\right)x - \frac{2b^3(5a^2-b^2)\log(\sin(c+}{d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 571 vs. 2(169) = 338.

time = 6.38, size = 571, normalized size = 3.38

$$\frac{b^3\left(\frac{\cos^2(c+dx)(a+b\tan(c+dx))^6(b^2+ab\tan(c+dx))}{2b^4(a^2+b^2)} - \frac{(-6a^2+4b^2)\left(\frac{1}{2}\left(5a^4-10a^2b^2+b^4+\frac{a^5-10a^3b^2+5ab^4}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+\frac{1}{2}\left(5a^4-10a^2b^2+b^4-\frac{a^5-10a^3b^2+5ab^4}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)+5ab(2a^2-b^2)\tan(c+dx)+\frac{1}{2}b^2(10a^2-b^2)\tan^2(c+dx)+\frac{5}{3}ab^3\tan^3(c+dx)+\frac{1}{4}b^4\tan^4(c+dx)\right)+5a\left(\frac{1}{2}\left(6a^5-20a^3b^2+6ab^4+\frac{a^6-15a^4b^2+15a^2b^4-b^6}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}-b\tan(c+dx)\right)+\frac{1}{2}\left(6a^5-20a^3b^2+6ab^4-\frac{a^6-15a^4b^2+15a^2b^4-b^6}{\sqrt{-b^2}}\right)\log\left(\sqrt{-b^2}+b\tan(c+dx)\right)+b(15a^4-15a^2b^2+b^4)\tan(c+dx)+ab^2(10a^2-3b^2)\tan^2(c+dx)+\frac{1}{3}b^3(15a^2-b^2)\tan^3(c+dx)+\frac{3}{2}ab^4\tan^4(c+dx)+\frac{1}{5}b^5\tan^5(c+dx)\right)}{2b^2(a^2+b^2)}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (b^3*((Cos[c + d*x]^2*(a + b*Tan[c + d*x])^6*(b^2 + a*b*Tan[c + d*x]))/(2*b
^4*(a^2 + b^2)) - ((-6*a^2 + 4*b^2)*(((5*a^4 - 10*a^2*b^2 + b^4 + (a^5 - 10
*a^3*b^2 + 5*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/2 + ((5*a
^4 - 10*a^2*b^2 + b^4 - (a^5 - 10*a^3*b^2 + 5*a*b^4)/Sqrt[-b^2])*Log[Sqrt[-
b^2] + b*Tan[c + d*x]])/2 + 5*a*b*(2*a^2 - b^2)*Tan[c + d*x] + (b^2*(10*a^2
 - b^2)*Tan[c + d*x]^2)/2 + (5*a*b^3*Tan[c + d*x]^3)/3 + (b^4*Tan[c + d*x]^
4)/4) + 5*a*(((6*a^5 - 20*a^3*b^2 + 6*a*b^4 + (a^6 - 15*a^4*b^2 + 15*a^2*b^
4 - b^6)/Sqrt[-b^2])*Log[Sqrt[-b^2] - b*Tan[c + d*x]])/2 + ((6*a^5 - 20*a^3
*b^2 + 6*a*b^4 - (a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)/Sqrt[-b^2])*Log[Sqrt
[-b^2] + b*Tan[c + d*x]])/2 + b*(15*a^4 - 15*a^2*b^2 + b^4)*Tan[c + d*x] +
a*b^2*(10*a^2 - 3*b^2)*Tan[c + d*x]^2 + (b^3*(15*a^2 - b^2)*Tan[c + d*x]^3)
/3 + (3*a*b^4*Tan[c + d*x]^4)/2 + (b^5*Tan[c + d*x]^5)/5))/(2*b^2*(a^2 + b^
2))))/d
```

Maple [A]
time = 0.40, size = 209, normalized size = 1.24 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^5*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)-5/2*b*a^4*cos(d*x+c)^2+1
0*a^3*b^2*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+10*b^3*a^2*(-1/2*sin(d
*x+c)^2-ln(cos(d*x+c)))+5*a*b^4*(sin(d*x+c)^5/cos(d*x+c)+(sin(d*x+c)^3+3/2*
sin(d*x+c))*cos(d*x+c)-3/2*d*x-3/2*c)+b^5*(1/2*sin(d*x+c)^6/cos(d*x+c)^2+1/
2*sin(d*x+c)^4+sin(d*x+c)^2+2*ln(cos(d*x+c))))
```

Maxima [A]
time = 0.50, size = 179, normalized size = 1.06

$$\frac{10\,a^4b\sin\left(dx+c\right)^2+\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)a^5+10\left(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right)\right)a^3b^2-20\left(\sin\left(dx+c\right)^2+\log\left(\sin\left(dx+c\right)^2-1\right)\right)a^2b^3-10\left(3\,dx+3\,c-\frac{\tan(dx+c)}{\tan(dx+c)^2+1}-2\,\tan\left(dx+c\right)\right)ab^4+2\left(\sin\left(dx+c\right)^2-\frac{1}{\sin(dx+c)^2-1}+2\,\log\left(\sin\left(dx+c\right)^2-1\right)\right)b^5}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/4*(10*a^4*b*sin(d*x + c)^2 + (2*d*x + 2*c + sin(2*d*x + 2*c))*a^5 + 10*(2
*d*x + 2*c - sin(2*d*x + 2*c))*a^3*b^2 - 20*(sin(d*x + c)^2 + log(sin(d*x +
 c)^2 - 1))*a^2*b^3 - 10*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) -
 2*tan(d*x + c))*a*b^4 + 2*(sin(d*x + c)^2 - 1/(sin(d*x + c)^2 - 1) + 2*log
(sin(d*x + c)^2 - 1))*b^5)/d
```

Fricas [A]
time = 2.89, size = 177, normalized size = 1.05

$$\frac{2\,b^5-2\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(dx+c\right)^4-8\left(5\,a^2b^3-b^5\right)\cos\left(dx+c\right)^2\log\left(-\cos\left(dx+c\right)\right)+\left(5\,a^4b-10\,a^2b^3+b^5+2\left(a^5+10\,a^3b^2-15\,ab^4\right)dx\right)\cos\left(dx+c\right)^2+2\left(10\,ab^4\cos\left(dx+c\right)+\left(a^5-10\,a^3b^2+5\,ab^4\right)\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^5 - 2*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 - 8*(5*a^2*b^3 -
 b^5)*cos(d*x + c)^2*log(-cos(d*x + c)) + (5*a^4*b - 10*a^2*b^3 + b^5 + 2*(
a^5 + 10*a^3*b^2 - 15*a*b^4)*d*x)*cos(d*x + c)^2 + 2*(10*a*b^4*cos(d*x + c)
 + (a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x +
c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.65, size = 173, normalized size = 1.02

$$\frac{b^5\tan\left(dx+c\right)^2+10\,ab^4\tan\left(dx+c\right)+(a^5+10\,a^3b^2-15\,ab^4)(dx+c)+2\left(5\,a^2b^3-b^5\right)\log\left(\tan\left(dx+c\right)^2+1\right)-\frac{10\,a^2b^3\tan(dx+c)^2-2\,b^5\tan(dx+c)^2-a^5\tan(dx+c)+10\,a^3b^2\tan(dx+c)-5\,ab^4\tan(dx+c)+5\,a^4b-b^5}{\tan(dx+c)^2+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/2*(b^5*tan(d*x + c)^2 + 10*a*b^4*tan(d*x + c) + (a^5 + 10*a^3*b^2 - 15*a*
b^4)*(d*x + c) + 2*(5*a^2*b^3 - b^5)*log(tan(d*x + c)^2 + 1) - (10*a^2*b^3*
tan(d*x + c)^2 - 2*b^5*tan(d*x + c)^2 - a^5*tan(d*x + c) + 10*a^3*b^2*tan(d
*x + c) - 5*a*b^4*tan(d*x + c) + 5*a^4*b - b^5)/(tan(d*x + c)^2 + 1))/d
```

Mupad [**B**]
time = 2.53, size = 354, normalized size = 2.09

$$\frac{2\left(b^5\,\ln\left(\frac{\cos(c+d\,x)}{\cos(c+d\,x)+1}\right)-b^5\,\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)+\frac{a^5\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}-5\,a^2\,b^3\,\ln\left(\frac{\cos(c+d\,x)}{\cos(c+d\,x)+1}\right)-\frac{15\,a\,b^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}+5\,a^2\,b^3\,\ln\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)+5\,a^3\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\right)}{d}+\frac{\frac{5\,a^4\,b}{16}+\frac{9\,b^5}{16}-\frac{5\,a^2\,b^3}{8}-\frac{b^5\,\cos(4\,c+4\,d\,x)}{16}+\frac{a^5\,\sin(2\,c+2\,d\,x)}{8}+\frac{a^5\,\sin(4\,c+4\,d\,x)}{16}-\frac{5\,a^4\,b\,\cos(4\,c+4\,d\,x)}{16}+\frac{25\,a\,b^4\,\sin(2\,c+2\,d\,x)}{8}+\frac{5\,a\,b^4\,\sin(4\,c+4\,d\,x)}{16}+\frac{5\,a^2\,b^3\,\cos(4\,c+4\,d\,x)}{8}-\frac{5\,a^3\,b^2\,\sin(2\,c+2\,d\,x)}{4}-\frac{5\,a^3\,b^2\,\sin(4\,c+4\,d\,x)}{8}}{d\,\left(\frac{\cos(2\,c+2\,d\,x)}{2}+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^3,x)
```

```
[Out] (2*(b^5*log(cos(c + d*x)/(cos(c + d*x) + 1)) - b^5*log(1/cos(c/2 + (d*x)/2)
^2) + (a^5*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 - 5*a^2*b^3*log(c
os(c + d*x)/(cos(c + d*x) + 1)) - (15*a*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2
 + (d*x)/2)))/2 + 5*a^2*b^3*log(1/cos(c/2 + (d*x)/2)^2) + 5*a^3*b^2*atan(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))))/d + ((5*a^4*b)/16 + (9*b^5)/16 - (5*
```

```
a^2*b^3)/8 - (b^5*cos(4*c + 4*d*x))/16 + (a^5*sin(2*c + 2*d*x))/8 + (a^5*si
n(4*c + 4*d*x))/16 - (5*a^4*b*cos(4*c + 4*d*x))/16 + (25*a*b^4*sin(2*c + 2*
d*x))/8 + (5*a*b^4*sin(4*c + 4*d*x))/16 + (5*a^2*b^3*cos(4*c + 4*d*x))/8 -
(5*a^3*b^2*sin(2*c + 2*d*x))/4 - (5*a^3*b^2*sin(4*c + 4*d*x))/8)/(d*(cos(2*
c + 2*d*x)/2 + 1/2))
```

3.101 $\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=204

$$\frac{10a^3b^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{15ab^4\tanh^{-1}(\sin(c+dx))}{2d}-\frac{5a^4b\cos(c+dx)}{d}+\frac{10a^2b^3\cos(c+dx)}{d}-\frac{b^5\cos(c+}{d}$$

```
[Out] 10*a^3*b^2*arctanh(sin(d*x+c))/d-15/2*a*b^4*arctanh(sin(d*x+c))/d-5*a^4*b*c
os(d*x+c)/d+10*a^2*b^3*cos(d*x+c)/d-b^5*cos(d*x+c)/d+10*a^2*b^3*sec(d*x+c)/
d-2*b^5*sec(d*x+c)/d+1/3*b^5*sec(d*x+c)^3/d+a^5*sin(d*x+c)/d-10*a^3*b^2*sin
(d*x+c)/d+15/2*a*b^4*sin(d*x+c)/d+5/2*a*b^4*sin(d*x+c)*tan(d*x+c)^2/d
```

Rubi [A]
time = 0.14, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 10, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3169, 2717, 2718, 2672, 327, 212, 2670, 14, 294, 276}

$$\frac{a^5\sin(c+dx)}{d}-\frac{5a^4b\cos(c+dx)}{d}-\frac{10a^3b^2\sin(c+dx)}{d}+\frac{10a^3b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{10a^2b^3\cos(c+dx)}{d}+\frac{10a^2b^3\sec(c+dx)}{d}+\frac{15ab^4\sin(c+dx)}{2d}+\frac{5ab^4\sin(c+dx)\tan^2(c+dx)}{2d}-\frac{15ab^4\tanh^{-1}(\sin(c+dx))}{2d}-\frac{b^5\cos(c+dx)}{d}+\frac{b^5\sec^3(c+dx)}{3d}-\frac{2b^5\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (10*a^3*b^2*ArcTanh[Sin[c + d*x]])/d - (15*a*b^4*ArcTanh[Sin[c + d*x]])/(2*
d) - (5*a^4*b*Cos[c + d*x])/d + (10*a^2*b^3*Cos[c + d*x])/d - (b^5*Cos[c +
d*x])/d + (10*a^2*b^3*Sec[c + d*x])/d - (2*b^5*Sec[c + d*x])/d + (b^5*Sec[c
 + d*x]^3)/(3*d) + (a^5*Sin[c + d*x])/d - (10*a^3*b^2*Sin[c + d*x])/d + (15
*a*b^4*Sin[c + d*x])/(2*d) + (5*a*b^4*Sin[c + d*x]*Tan[c + d*x]^2)/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rubi steps

$$\int \sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int \left(a^5\cos(c+dx)+5a^4b\sin(c+dx)+10a^3b^2\sin(c+\right.$$
$$= a^5\int\cos(c+dx)\,dx+\left(5a^4b\right)\int\sin(c+dx)\,dx+\left(10a^3\right.$$
$$= -\frac{5a^4b\cos(c+dx)}{d}+\frac{a^5\sin(c+dx)}{d}+\frac{(10a^3b^2)\,\mathrm{Subst}\Big(}{}$$
$$= -\frac{5a^4b\cos(c+dx)}{d}+\frac{a^5\sin(c+dx)}{d}-\frac{10a^3b^2\sin(c+d}{d}$$
$$= \frac{10a^3b^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{5a^4b\cos(c+dx)}{d}+\frac{10a^2}{}$$
$$= \frac{10a^3b^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{15ab^4\tanh^{-1}(\sin(c+dx}{2d}$$

Mathematica [A]
time = 6.00, size = 397, normalized size = 1.95

$$\frac{120a^2b^3-22b^5-12b(5a^4-10a^2b^2+b^4)\cos(c+dx)-30ab^2(4a^2-3b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+30ab^2(4a^2-3b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{b^4(15a+b)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{2b^5\sin\left(\frac{1}{2}(c+dx)\right)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{2b^3(60a^2-11b^2)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}-\frac{2b^5\sin\left(\frac{1}{2}(c+dx)\right)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{b^4(-15a+b)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{2b^3(-60a^2+11b^2)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}+12a(a^4-10a^2b^2+5b^4)\sin(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (120*a^2*b^3 - 22*b^5 - 12*b*(5*a^4 - 10*a^2*b^2 + b^4)*Cos[c + d*x] - 30*a
*b^2*(4*a^2 - 3*b^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 30*a*b^2*(4
*a^2 - 3*b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (b^4*(15*a + b))/(
Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (2*b^5*Sin[(c + d*x)/2])/(Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2])^3 + (2*b^3*(60*a^2 - 11*b^2)*Sin[(c + d*x)/2])
/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (2*b^5*Sin[(c + d*x)/2])/(Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2])^3 + (b^4*(-15*a + b))/(Cos[(c + d*x)/2] + Sin[
(c + d*x)/2])^2 + (2*b^3*(-60*a^2 + 11*b^2)*Sin[(c + d*x)/2])/(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]) + 12*a*(a^4 - 10*a^2*b^2 + 5*b^4)*Sin[c + d*x])/(1
2*d)
```

Maple [A]
time = 0.46, size = 230, normalized size = 1.13 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(sin(d*x+c)*a^5-5*b*a^4*cos(d*x+c)+10*a^3*b^2*(-sin(d*x+c)+ln(sec(d*x+c
)+tan(d*x+c)))+10*b^3*a^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x
+c))+5*a*b^4*(1/2*sin(d*x+c)^5/cos(d*x+c)^2+1/2*sin(d*x+c)^3+3/2*sin(d*x+c)
```

```
-3/2*ln(sec(d*x+c)+tan(d*x+c)))+b^5*(1/3*sin(d*x+c)^6/cos(d*x+c)^3-sin(d*x+
c)^6/cos(d*x+c)-(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.28, size = 181, normalized size = 0.89

$$-\frac{15\,ab^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)-4\,\sin\left(dx+c\right)\right)-120\,a^2b^3\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)+4\,b^5\left(\frac{6\,\cos(dx+c)^2-1}{\cos(dx+c)^3}+3\,\cos\left(dx+c\right)\right)-60\,a^3b^2(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)-2\,\sin\left(dx+c\right))+60\,a^4b\cos\left(dx+c\right)-12\,a^5\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/12*(15*a*b^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x + c) +
 1) - 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)) - 120*a^2*b^3*(1/cos(d*x +
c) + cos(d*x + c)) + 4*b^5*((6*cos(d*x + c)^2 - 1)/cos(d*x + c)^3 + 3*cos(d
*x + c)) - 60*a^3*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*si
n(d*x + c)) + 60*a^4*b*cos(d*x + c) - 12*a^5*sin(d*x + c))/d
```

Fricas [A]
time = 4.77, size = 190, normalized size = 0.93

$$\frac{4\,b^5-12\,(5\,a^4b-10\,a^2b^3+b^5)\cos\left(dx+c\right)^4+15\,(4\,a^3b^2-3\,ab^4)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-15\,(4\,a^3b^2-3\,ab^4)\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+24\,(5\,a^2b^3-b^5)\cos\left(dx+c\right)^2+6\left(5\,ab^4\cos\left(dx+c\right)+2\,(a^5-10\,a^3b^2+5\,ab^4)\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/12*(4*b^5 - 12*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 + 15*(4*a^3*b^
2 - 3*a*b^4)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 15*(4*a^3*b^2 - 3*a*b^4
)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 24*(5*a^2*b^3 - b^5)*cos(d*x + c)
^2 + 6*(5*a*b^4*cos(d*x + c) + 2*(a^5 - 10*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^
3)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.64, size = 281, normalized size = 1.38

$$\frac{15\,(4\,a^3b^2-3\,ab^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,(4\,a^3b^2-3\,ab^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{12\,(a^5\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)-10\,a^3b^2\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)+5\,ab^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)-5\,a^4b+10\,a^2b^3-b^5)}{\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2+1}+\frac{2\left(15\,ab^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^5-60\,a^2b^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^4+6\,b^5\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^4+120\,a^2b^3\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2-24\,b^5\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2-15\,ab^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)-60\,a^2b^3+10\,b^5\right)}{\left(\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/6*(15*(4*a^3*b^2 - 3*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*a^
3*b^2 - 3*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 12*(a^5*tan(1/2*d*x +
 1/2*c) - 10*a^3*b^2*tan(1/2*d*x + 1/2*c) + 5*a*b^4*tan(1/2*d*x + 1/2*c) -
5*a^4*b + 10*a^2*b^3 - b^5)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(15*a*b^4*tan(
1/2*d*x + 1/2*c)^5 - 60*a^2*b^3*tan(1/2*d*x + 1/2*c)^4 + 6*b^5*tan(1/2*d*x
+ 1/2*c)^4 + 120*a^2*b^3*tan(1/2*d*x + 1/2*c)^2 - 24*b^5*tan(1/2*d*x + 1/2*
c)^2 - 15*a*b^4*tan(1/2*d*x + 1/2*c) - 60*a^2*b^3 + 10*b^5)/(tan(1/2*d*x +
1/2*c)^2 - 1)^3)/d
```

Mupad [B]
time = 4.05, size = 302, normalized size = 1.48

$$-\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(15\,a\,b^4-20\,a^3\,b^2\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^5-20\,a^3\,b^2+15\,a\,b^4\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(30\,a^4\,b-40\,a^2\,b^3\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(2\,a^5-20\,a^3\,b^2+15\,a\,b^4\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(6\,a^5-60\,a^3\,b^2+25\,a\,b^4\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(6\,a^5-60\,a^3\,b^2+25\,a\,b^4\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(30\,a^4\,b-80\,a^2\,b^3+\frac{32\,b^5}{3}\right)-10\,a^4\,b-\frac{16\,b^5}{3}+40\,a^2\,b^3+10\,a^4\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^4,x)
```

```
[Out] - (atanh(tan(c/2 + (d*x)/2))*(15*a*b^4 - 20*a^3*b^2))/d - (tan(c/2 + (d*x)/
2)*(15*a*b^4 + 2*a^5 - 20*a^3*b^2) - tan(c/2 + (d*x)/2)^4*(30*a^4*b - 40*a^
2*b^3) - tan(c/2 + (d*x)/2)^7*(15*a*b^4 + 2*a^5 - 20*a^3*b^2) - tan(c/2 + (
d*x)/2)^3*(25*a*b^4 + 6*a^5 - 60*a^3*b^2) + tan(c/2 + (d*x)/2)^5*(25*a*b^4
+ 6*a^5 - 60*a^3*b^2) + tan(c/2 + (d*x)/2)^2*(30*a^4*b + (32*b^5)/3 - 80*a^
2*b^3) - 10*a^4*b - (16*b^5)/3 + 40*a^2*b^3 + 10*a^4*b*tan(c/2 + (d*x)/2)^6
)/(d*(2*tan(c/2 + (d*x)/2)^2 - 2*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^
8 - 1))
```

3.102 $\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=147

$$a\left(a^4-10a^2b^2+5b^4\right)x-\frac{b(5a^4-10a^2b^2+b^4)\log(\cos(c+dx))}{d}+\frac{4ab^2(a^2-b^2)\tan(c+dx)}{d}+\frac{b(3a^2-b^2)\,(a+}{2d}$$

```
[Out] a*(a^4-10*a^2*b^2+5*b^4)*x-b*(5*a^4-10*a^2*b^2+b^4)*ln(cos(d*x+c))/d+4*a*b^
2*(a^2-b^2)*tan(d*x+c)/d+1/2*b*(3*a^2-b^2)*(a+b*tan(d*x+c))^2/d+2/3*a*b*(a+
b*tan(d*x+c))^3/d+1/4*b*(a+b*tan(d*x+c))^4/d
```

Rubi [A]
time = 0.16, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3165, 3563, 3609, 3606, 3556}

$$\frac{b(3a^2-b^2)\,(a+b\tan(c+dx))^2}{2d}+\frac{4ab^2(a^2-b^2)\tan(c+dx)}{d}-\frac{b(5a^4-10a^2b^2+b^4)\log(\cos(c+dx))}{d}+ax\left(a^4-10a^2b^2+5b^4\right)+\frac{b(a+b\tan(c+dx))^4}{4d}+\frac{2ab(a+b\tan(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] a*(a^4 - 10*a^2*b^2 + 5*b^4)*x - (b*(5*a^4 - 10*a^2*b^2 + b^4)*Log[Cos[c +
d*x]])/d + (4*a*b^2*(a^2 - b^2)*Tan[c + d*x])/d + (b*(3*a^2 - b^2)*(a + b*T
an[c + d*x])^2)/(2*d) + (2*a*b*(a + b*Tan[c + d*x])^3)/(3*d) + (b*(a + b*Ta
n[c + d*x])^4)/(4*d)
```

Rule 3165

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3563

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n - 1)/(d*(n - 1))), x] + Int[(a^2 - b^2 + 2*a*b*Tan[c + d
*x])*(a + b*Tan[c + d*x])^(n - 2), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2
+ b^2, 0] && GtQ[n, 1]
```

Rule 3606

```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*tan[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(a*c - b*d)*x, x] + (Dist[b*c + a*d, Int[Tan[e +
 f*x], x], x] + Simp[b*d*(Tan[e + f*x]/f), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 3609

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[d*((a + b*Tan[e + f*x])^m/(f*m)), x] + Int
[(a + b*Tan[e + f*x])^(m - 1)*Simp[a*c - b*d + (b*c + a*d)*Tan[e + f*x], x]
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2,
 0] && GtQ[m, 0]
```

Rubi steps

$$\int \sec^5(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int (a+b\tan(c+dx))^5\,dx$$
$$= \frac{b(a+b\tan(c+dx))^4}{4d} + \int (a+b\tan(c+dx))^3\left(a^2-b\right.$$
$$= \frac{2ab(a+b\tan(c+dx))^3}{3d} + \frac{b(a+b\tan(c+dx))^4}{4d} + \int($$
$$= \frac{b(3a^2-b^2)(a+b\tan(c+dx))^2}{2d} + \frac{2ab(a+b\tan(c+d}{3d}$$
$$= a\left(a^4-10a^2b^2+5b^4\right)x + \frac{4ab^2(a^2-b^2)\tan(c+dx)}{d} +$$
$$= a\left(a^4-10a^2b^2+5b^4\right)x - \frac{b(5a^4-10a^2b^2+b^4)\log(\cos(}{d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.79, size = 126, normalized size = 0.86

$$\frac{6(-ia+b)^5\log(i-\tan(c+dx))+6(ia+b)^5\log(i+\tan(c+dx))+60ab^2(2a^2-b^2)\tan(c+dx)-6b^3(-10a^2+b^2)\tan^2(c+dx)+20ab^4\tan^3(c+dx)+3b^5\tan^4(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (6*((-I)*a + b)^5*Log[I - Tan[c + d*x]] + 6*(I*a + b)^5*Log[I + Tan[c + d*x
]] + 60*a*b^2*(2*a^2 - b^2)*Tan[c + d*x] - 6*b^3*(-10*a^2 + b^2)*Tan[c + d*
x]^2 + 20*a*b^4*Tan[c + d*x]^3 + 3*b^5*Tan[c + d*x]^4)/(12*d)
```

Maple [A]
time = 0.33, size = 139, normalized size = 0.95

method	result
derivativedivides	$\frac{a^5(dx+c)-5b\,a^4\ln(\cos(dx+c))+10a^3b^2(\tan(dx+c)-dx-c)+10b^3a^2\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)+5a\,b^4\left(\frac{\left(\tan^3(dx+c\right.}{3}\right.}{d}$
default	$\frac{a^5(dx+c)-5b\,a^4\ln(\cos(dx+c))+10a^3b^2(\tan(dx+c)-dx-c)+10b^3a^2\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)+5a\,b^4\left(\frac{\left(\tan^3(dx+c\right.}{3}\right.}{d}$
risch	$\frac{10ib\,a^4c}{d}-\frac{20ib^3a^2c}{d}+\frac{2ib^5c}{d}+a^5x-10a^3b^2x+5a\,b^4x+ix\,b^5+5ia^4bx-10ix\,a^2b^3-\frac{4b^2(-15}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^5*(d*x+c)-5*b*a^4*ln(cos(d*x+c))+10*a^3*b^2*(tan(d*x+c)-d*x-c)+10*b^
3*a^2*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))+5*a*b^4*(1/3*tan(d*x+c)^3-tan(d*x+c
)+d*x+c)+b^5*(1/4*tan(d*x+c)^4-1/2*tan(d*x+c)^2-ln(cos(d*x+c))))
```

Maxima [A]

time = 0.48, size = 174, normalized size = 1.18

$$\frac{12\,(dx+c)a^5-120\,(dx+c-\tan(dx+c))a^3b^2+20\,\left(\tan(dx+c)^3+3\,dx+3\,c-3\,\tan(dx+c)\right)ab^4+3\,b^5\left(\frac{4\,\sin(dx+c)^2-3}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-2\,\log\left(\sin(dx+c)^2-1\right)\right)-60\,a^2b^3\left(\frac{1}{\sin(dx+c)^2-1}-\log\left(\sin(dx+c)^2-1\right)\right)-30\,a^4b\log\left(-\sin(dx+c)^2+1\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/12*(12*(d*x + c)*a^5 - 120*(d*x + c - tan(d*x + c))*a^3*b^2 + 20*(tan(d*x
 + c)^3 + 3*d*x + 3*c - 3*tan(d*x + c))*a*b^4 + 3*b^5*((4*sin(d*x + c)^2 -
3)/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 2*log(sin(d*x + c)^2 - 1)) - 6
0*a^2*b^3*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1)) - 30*a^4*b*log
(-sin(d*x + c)^2 + 1))/d
```

Fricas [A]

time = 2.85, size = 155, normalized size = 1.05

$$\frac{12\,(a^5-10\,a^3b^2+5\,ab^4)dx\cos(dx+c)^4-12\,(5\,a^4b-10\,a^2b^3+b^5)\cos(dx+c)^4\log\left(-\cos(dx+c)\right)+3\,b^5+12\,(5\,a^2b^3-b^5)\cos(dx+c)^2+20\,\left(ab^4\cos(dx+c)+2\,(3\,a^3b^2-2\,ab^4)\cos(dx+c)^3\right)\sin(dx+c)}{12\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/12*(12*(a^5 - 10*a^3*b^2 + 5*a*b^4)*d*x*cos(d*x + c)^4 - 12*(5*a^4*b - 10
*a^2*b^3 + b^5)*cos(d*x + c)^4*log(-cos(d*x + c)) + 3*b^5 + 12*(5*a^2*b^3 -
 b^5)*cos(d*x + c)^2 + 20*(a*b^4*cos(d*x + c) + 2*(3*a^3*b^2 - 2*a*b^4)*cos
(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.65, size = 144, normalized size = 0.98

$$\frac{3\,b^5\tan\left(dx+c\right)^4+20\,ab^4\tan\left(dx+c\right)^3+60\,a^2b^3\tan\left(dx+c\right)^2-6\,b^5\tan\left(dx+c\right)^2+120\,a^3b^2\tan\left(dx+c\right)-60\,ab^4\tan\left(dx+c\right)+12\,(a^5-10\,a^3b^2+5\,ab^4)(dx+c)+6\,(5\,a^4b-10\,a^2b^3+b^5)\log\left(\tan\left(dx+c\right)^2+1\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/12*(3*b^5*tan(d*x + c)^4 + 20*a*b^4*tan(d*x + c)^3 + 60*a^2*b^3*tan(d*x +
 c)^2 - 6*b^5*tan(d*x + c)^2 + 120*a^3*b^2*tan(d*x + c) - 60*a*b^4*tan(d*x
+ c) + 12*(a^5 - 10*a^3*b^2 + 5*a*b^4)*(d*x + c) + 6*(5*a^4*b - 10*a^2*b^3
+ b^5)*log(tan(d*x + c)^2 + 1))/d
```

Mupad [B]
time = 3.45, size = 971, normalized size = 6.61

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^5,x)
```

```
[Out] ((3*b^5*log(1/cos(c/2 + (d*x)/2)^2))/8 - (3*b^5*log(-cos(c + d*x)/cos(c/2 +
 (d*x)/2)^2))/8 + b^5/32 + (3*a^5*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
)))/4 + (5*a^2*b^3)/8 - (b^5*cos(2*c + 2*d*x))/8 + (3*b^5*cos(4*c + 4*d*x))
/32 + (15*a^4*b*log(1/cos(c/2 + (d*x)/2)^2))/8 + (15*a^2*b^3*log(-cos(c + d
*x)/cos(c/2 + (d*x)/2)^2))/4 - (b^5*log(-cos(c + d*x)/cos(c/2 + (d*x)/2)^2)
*cos(2*c + 2*d*x))/2 - (b^5*log(-cos(c + d*x)/cos(c/2 + (d*x)/2)^2)*cos(4*c
 + 4*d*x))/8 + (15*a*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 - (
5*a*b^4*sin(2*c + 2*d*x))/6 - (5*a*b^4*sin(4*c + 4*d*x))/6 - (15*a^2*b^3*lo
g(1/cos(c/2 + (d*x)/2)^2))/4 + (b^5*log(1/cos(c/2 + (d*x)/2)^2)*cos(2*c + 2
*d*x))/2 + (b^5*log(1/cos(c/2 + (d*x)/2)^2)*cos(4*c + 4*d*x))/8 + a^5*atan(
sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x) + (a^5*atan(sin(c/2
 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(4*c + 4*d*x))/4 - (15*a^3*b^2*atan(sin(
c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 - (5*a^2*b^3*cos(4*c + 4*d*x))/8 + (5
*a^3*b^2*sin(2*c + 2*d*x))/2 + (5*a^3*b^2*sin(4*c + 4*d*x))/4 - (15*a^4*b*l
og(-cos(c + d*x)/cos(c/2 + (d*x)/2)^2))/8 - 5*a^2*b^3*log(1/cos(c/2 + (d*x)
/2)^2)*cos(2*c + 2*d*x) - (5*a^2*b^3*log(1/cos(c/2 + (d*x)/2)^2)*cos(4*c +
4*d*x))/4 - 10*a^3*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c
+ 2*d*x) - (5*a^3*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(4*c +
 4*d*x))/2 - (5*a^4*b*log(-cos(c + d*x)/cos(c/2 + (d*x)/2)^2)*cos(2*c + 2*d
```

```
*x))/2 - (5*a^4*b*log(-cos(c + d*x)/cos(c/2 + (d*x)/2)^2)*cos(4*c + 4*d*x))
/8 + (5*a^4*b*log(1/cos(c/2 + (d*x)/2)^2)*cos(2*c + 2*d*x))/2 + (5*a^4*b*lo
g(1/cos(c/2 + (d*x)/2)^2)*cos(4*c + 4*d*x))/8 + 5*a^2*b^3*log(-cos(c + d*x)
/cos(c/2 + (d*x)/2)^2)*cos(2*c + 2*d*x) + (5*a^2*b^3*log(-cos(c + d*x)/cos(
c/2 + (d*x)/2)^2)*cos(4*c + 4*d*x))/4 + 5*a*b^4*atan(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2))*cos(2*c + 2*d*x) + (5*a*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/
2 + (d*x)/2))*cos(4*c + 4*d*x))/4)/(d*(cos(2*c + 2*d*x)/2 + cos(4*c + 4*d*x
)/8 + 3/8))
```

3.103 $\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=224

$$\frac{a^5\tanh^{-1}(\sin(c+dx))}{d}-\frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{15ab^4\tanh^{-1}(\sin(c+dx))}{8d}+\frac{5a^4b\sec(c+dx)}{d}-\frac{10a^2b^3\sec(c+dx)}{d}$$

```
[Out] a^5*arctanh(sin(d*x+c))/d-5*a^3*b^2*arctanh(sin(d*x+c))/d+15/8*a*b^4*arctan
h(sin(d*x+c))/d+5*a^4*b*sec(d*x+c)/d-10*a^2*b^3*sec(d*x+c)/d+b^5*sec(d*x+c)
/d+10/3*a^2*b^3*sec(d*x+c)^3/d-2/3*b^5*sec(d*x+c)^3/d+1/5*b^5*sec(d*x+c)^5/
d+5*a^3*b^2*sec(d*x+c)*tan(d*x+c)/d-15/8*a*b^4*sec(d*x+c)*tan(d*x+c)/d+5/4*
a*b^4*sec(d*x+c)*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.16, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3169, 3855, 2686, 8, 2691, 200}

$$\frac{a^5\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^4b\sec(c+dx)}{d}-\frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^3b^2\tan(c+dx)\sec(c+dx)}{d}+\frac{10a^2b^3\sec^3(c+dx)}{3d}-\frac{10a^2b^3\sec(c+dx)}{d}+\frac{15ab^4\tanh^{-1}(\sin(c+dx))}{8d}+\frac{5ab^4\tan^3(c+dx)\sec(c+dx)}{4d}-\frac{15ab^4\tan(c+dx)\sec(c+dx)}{8d}+\frac{b^5\sec^5(c+dx)}{5d}-\frac{2b^5\sec^3(c+dx)}{3d}+\frac{b^5\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (a^5*ArcTanh[Sin[c + d*x]])/d - (5*a^3*b^2*ArcTanh[Sin[c + d*x]])/d + (15*a
*b^4*ArcTanh[Sin[c + d*x]])/(8*d) + (5*a^4*b*Sec[c + d*x])/d - (10*a^2*b^3*
Sec[c + d*x])/d + (b^5*Sec[c + d*x])/d + (10*a^2*b^3*Sec[c + d*x]^3)/(3*d)
- (2*b^5*Sec[c + d*x]^3)/(3*d) + (b^5*Sec[c + d*x]^5)/(5*d) + (5*a^3*b^2*Se
c[c + d*x]*Tan[c + d*x])/d - (15*a*b^4*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (
5*a*b^4*Sec[c + d*x]*Tan[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^6(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int \left(a^5\sec(c+dx)+5a^4b\sec(c+dx)\tan(c+dx)+10a^3\right.$$

$$= a^5\int \sec(c+dx)\,dx + \left(5a^4b\right)\int \sec(c+dx)\tan(c+dx)$$

$$= \frac{a^5\tanh^{-1}(\sin(c+dx))}{d} + \frac{5a^3b^2\sec(c+dx)\tan(c+dx)}{d}$$

$$= \frac{a^5\tanh^{-1}(\sin(c+dx))}{d} - \frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{5}{}$$

$$= \frac{a^5\tanh^{-1}(\sin(c+dx))}{d} - \frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{1}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1219 vs. $2(224) = 448$.
time = 6.36, size = 1219, normalized size = 5.44

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (b*(600*a^4 - 1000*a^2*b^2 + 89*b^4)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)
/(120*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((-8*a^5 + 40*a^3*b^2 - 15*a
```

```
*b^4)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[c
+ d*x])^5)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((8*a^5 - 40*a^3*b^2
 + 15*a*b^4)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a + b
*Tan[c + d*x])^5)/(8*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((25*a*b^4 +
2*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(80*d*(Cos[(c + d*x)/2] - Sin
[(c + d*x)/2])^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((600*a^3*b^2 + 200
*a^2*b^3 - 375*a*b^4 - 31*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(240*
d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])
^5) + (b^5*Cos[c + d*x]^5*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^5)/(20*d*(C
os[(c + d*x)/2] - Sin[(c + d*x)/2])^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^5)
- (b^5*Cos[c + d*x]^5*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^5)/(20*d*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((
-25*a*b^4 + 2*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(80*d*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2])^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((-600*
a^3*b^2 + 200*a^2*b^3 + 375*a*b^4 - 31*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d
*x])^5)/(240*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*
Sin[c + d*x])^5) + (Cos[c + d*x]^5*(-600*a^4*b*Sin[(c + d*x)/2] + 1000*a^2*
b^3*Sin[(c + d*x)/2] - 89*b^5*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^5)/(12
0*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])
^5) + (Cos[c + d*x]^5*(200*a^2*b^3*Sin[(c + d*x)/2] - 31*b^5*Sin[(c + d*x)/
2])*(a + b*Tan[c + d*x])^5)/(120*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*
(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (Cos[c + d*x]^5*(-200*a^2*b^3*Sin[(c
 + d*x)/2] + 31*b^5*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^5)/(120*d*(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (C
os[c + d*x]^5*(600*a^4*b*Sin[(c + d*x)/2] - 1000*a^2*b^3*Sin[(c + d*x)/2] +
 89*b^5*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^5)/(120*d*(Cos[(c + d*x)/2]
- Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^5)
```

Maple [A]

time = 0.38, size = 316, normalized size = 1.41

method	result
derivativedivides	$a^5\ln(\sec(dx+c)+\tan(dx+c))+\frac{5b\,a^4}{\cos(dx+c)}+10a^3b^2\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+10b^3a^2\left(\frac{\sin^4(}{3\cos(d}\right.$
default	$a^5\ln(\sec(dx+c)+\tan(dx+c))+\frac{5b\,a^4}{\cos(dx+c)}+10a^3b^2\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+10b^3a^2\left(\frac{\sin^4(}{3\cos(d}\right.$
risch	$b(150ia\,b^3e^{7i(dx+c)}-600ia^3b\,e^{9i(dx+c)}+600a^4e^{9i(dx+c)}-1200a^2b^2e^{9i(dx+c)}+120b^4e^{9i(dx+c)}+1200ia^3b\,e^{3i(dx+c)}-375$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^5*ln(sec(d*x+c)+tan(d*x+c))+5*b*a^4/cos(d*x+c)+10*a^3*b^2*(1/2*sin(d
*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))+10*b^3*a
^2*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*
x+c)^2)*cos(d*x+c))+5*a*b^4*(1/4*sin(d*x+c)^5/cos(d*x+c)^4-1/8*sin(d*x+c)^5
/cos(d*x+c)^2-1/8*sin(d*x+c)^3-3/8*sin(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))
)+b^5*(1/5*sin(d*x+c)^6/cos(d*x+c)^5-1/15*sin(d*x+c)^6/cos(d*x+c)^3+1/5*sin
(d*x+c)^6/cos(d*x+c)+1/5*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.28, size = 230, normalized size = 1.03

$$\frac{75\,ab^4\left(\frac{2\left(5\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}+3\,\log\left(\sin\left(dx+c\right)+1\right)-3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-600\,a^3b^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)+120\,a^5(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+\frac{1200\,a^4b}{\cos(dx+c)}-\frac{800\left(3\,\cos(dx+c)^2-1\right)a^2b^3}{\cos(dx+c)^3}+\frac{16\left(15\,\cos(dx+c)^4-10\,\cos(dx+c)^2+3\right)b^5}{\cos(dx+c)^5}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/240*(75*a*b^4*(2*(5*sin(d*x + c)^3 - 3*sin(d*x + c))/(sin(d*x + c)^4 - 2*
sin(d*x + c)^2 + 1) + 3*log(sin(d*x + c) + 1) - 3*log(sin(d*x + c) - 1)) -
600*a^3*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) -
log(sin(d*x + c) - 1)) + 120*a^5*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
- 1)) + 1200*a^4*b/cos(d*x + c) - 800*(3*cos(d*x + c)^2 - 1)*a^2*b^3/cos(d*
x + c)^3 + 16*(15*cos(d*x + c)^4 - 10*cos(d*x + c)^2 + 3)*b^5/cos(d*x + c)^
5)/d
```

Fricas [A]
time = 3.18, size = 196, normalized size = 0.88

$$\frac{15\left(8\,a^5-40\,a^3b^2+15\,ab^4\right)\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\left(8\,a^5-40\,a^3b^2+15\,ab^4\right)\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+48\,b^5+240\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(dx+c\right)^4+160\left(5\,a^2b^3-b^5\right)\cos\left(dx+c\right)^2+150\left(2\,ab^4\cos\left(dx+c\right)+\left(8\,a^3b^2-5\,ab^4\right)\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)}{240\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/240*(15*(8*a^5 - 40*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5*log(sin(d*x + c) +
 1) - 15*(8*a^5 - 40*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^5*log(-sin(d*x + c) +
 1) + 48*b^5 + 240*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 + 160*(5*a^2
*b^3 - b^5)*cos(d*x + c)^2 + 150*(2*a*b^4*cos(d*x + c) + (8*a^3*b^2 - 5*a*b
^4)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [A]
time = 0.67, size = 410, normalized size = 1.83

$$\frac{15\,(8\,a^5-40\,a^3b^2+15\,ab^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,(8\,a^5-40\,a^3b^2+15\,ab^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(600\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-225\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-600\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-1200\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1050\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+2400\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-2400\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-3600\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+5600\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-640\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+1200\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-1050\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2400\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-4000\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+320\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-600\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+225\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-600\,a^4b+800\,a^2b^3-64\,b^5\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/120*(15*(8*a^5 - 40*a^3*b^2 + 15*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)
) - 15*(8*a^5 - 40*a^3*b^2 + 15*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) +
 2*(600*a^3*b^2*tan(1/2*d*x + 1/2*c)^9 - 225*a*b^4*tan(1/2*d*x + 1/2*c)^9 -
 600*a^4*b*tan(1/2*d*x + 1/2*c)^8 - 1200*a^3*b^2*tan(1/2*d*x + 1/2*c)^7 + 1
050*a*b^4*tan(1/2*d*x + 1/2*c)^7 + 2400*a^4*b*tan(1/2*d*x + 1/2*c)^6 - 2400
*a^2*b^3*tan(1/2*d*x + 1/2*c)^6 - 3600*a^4*b*tan(1/2*d*x + 1/2*c)^4 + 5600*
a^2*b^3*tan(1/2*d*x + 1/2*c)^4 - 640*b^5*tan(1/2*d*x + 1/2*c)^4 + 1200*a^3*
b^2*tan(1/2*d*x + 1/2*c)^3 - 1050*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 2400*a^4*b
*tan(1/2*d*x + 1/2*c)^2 - 4000*a^2*b^3*tan(1/2*d*x + 1/2*c)^2 + 320*b^5*tan
(1/2*d*x + 1/2*c)^2 - 600*a^3*b^2*tan(1/2*d*x + 1/2*c) + 225*a*b^4*tan(1/2*
d*x + 1/2*c) - 600*a^4*b + 800*a^2*b^3 - 64*b^5)/(tan(1/2*d*x + 1/2*c)^2 -
1)^5)/d
```

Mupad [B]
time = 4.26, size = 345, normalized size = 1.54

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a^5-10\,a^3\,b^2+\frac{15\,a\,b^4}{4}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,\left(\frac{15\,a\,b^4}{4}-10\,a^3\,b^2\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{35\,a\,b^4}{2}-20\,a^3\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(\frac{35\,a\,b^4}{2}-20\,a^3\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(40\,a^4\,b-40\,a^2\,b^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(40\,a^4\,b-\frac{200\,a^2\,b^3}{3}+\frac{16\,b^5}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(60\,a^4\,b-\frac{280\,a^2\,b^3}{3}+\frac{32\,b^5}{3}\right)+10\,a^4\,b+\frac{16\,b^5}{15}-\frac{40\,a^2\,b^3}{3}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{15\,a\,b^4}{4}-10\,a^3\,b^2\right)+10\,a^4\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^6,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*((15*a*b^4)/4 + 2*a^5 - 10*a^3*b^2))/d - (tan(c/
2 + (d*x)/2)^9*((15*a*b^4)/4 - 10*a^3*b^2) + tan(c/2 + (d*x)/2)^3*((35*a*b^
4)/2 - 20*a^3*b^2) - tan(c/2 + (d*x)/2)^7*((35*a*b^4)/2 - 20*a^3*b^2) - tan
(c/2 + (d*x)/2)^6*(40*a^4*b - 40*a^2*b^3) - tan(c/2 + (d*x)/2)^2*(40*a^4*b
+ (16*b^5)/3 - (200*a^2*b^3)/3) + tan(c/2 + (d*x)/2)^4*(60*a^4*b + (32*b^5)
/3 - (280*a^2*b^3)/3) + 10*a^4*b + (16*b^5)/15 - (40*a^2*b^3)/3 - tan(c/2 +
 (d*x)/2)*((15*a*b^4)/4 - 10*a^3*b^2) + 10*a^4*b*tan(c/2 + (d*x)/2)^8)/(d*(
5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6
- 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.104 $\int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=30

$$\frac{(b+a\cot(c+dx))^6\tan^6(c+dx)}{6bd}$$

```
[Out] 1/6*(b+a*cot(d*x+c))^6*tan(d*x+c)^6/b/d
```

Rubi [A]
time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3167, 37}

$$\frac{\tan^6(c+dx)(a\cot(c+dx)+b)^6}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] ((b + a*Cot[c + d*x])^6*Tan[c + d*x]^6)/(6*b*d)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\begin{aligned} \int \sec^7(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx &= -\frac{\text{Subst}\left(\int \frac{(b+ax)^5}{x^7}\,dx, x, \cot(c+dx)\right)}{d} \\ &= \frac{(b+a\cot(c+dx))^6\tan^6(c+dx)}{6bd} \end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 370 vs. $2(30) = 60$.
time = 6.24, size = 370, normalized size = 12.33

$$-\frac{b^3(-5a^2+b^2)\cos(c+dx)(a+b\tan(c+dx))^5}{2d(a\cos(c+dx)+b\sin(c+dx))^5}+\frac{b(5a^4-10a^2b^2+b^4)\cos^3(c+dx)(a+b\tan(c+dx))^5}{2d(a\cos(c+dx)+b\sin(c+dx))^5}+\frac{b^5\sec(c+dx)(a+b\tan(c+dx))^5}{6d(a\cos(c+dx)+b\sin(c+dx))^5}+\frac{ab^4\sin(c+dx)(a+b\tan(c+dx))^5}{d(a\cos(c+dx)+b\sin(c+dx))^5}+\frac{2\cos^2(c+dx)\left(5a^3b^2\sin(c+dx)-3ab^4\sin(c+dx)\right)(a+b\tan(c+dx))^5}{3d(a\cos(c+dx)+b\sin(c+dx))^5}+\frac{\cos^4(c+dx)\left(3a^5\sin(c+dx)-10a^3b^2\sin(c+dx)+3ab^4\sin(c+dx)\right)(a+b\tan(c+dx))^5}{3d(a\cos(c+dx)+b\sin(c+dx))^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] -1/2*(b^3*(-5*a^2 + b^2)*Cos[c + d*x]*(a + b*Tan[c + d*x])^5)/(d*(a*Cos[c +
 d*x] + b*Sin[c + d*x])^5) + (b*(5*a^4 - 10*a^2*b^2 + b^4)*Cos[c + d*x]^3*(
a + b*Tan[c + d*x])^5)/(2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (b^5*Sec
[c + d*x]*(a + b*Tan[c + d*x])^5)/(6*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5)
 + (a*b^4*Sin[c + d*x]*(a + b*Tan[c + d*x])^5)/(d*(a*Cos[c + d*x] + b*Sin[c
 + d*x])^5) + (2*Cos[c + d*x]^2*(5*a^3*b^2*Sin[c + d*x] - 3*a*b^4*Sin[c + d
*x])*(a + b*Tan[c + d*x])^5)/(3*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (C
os[c + d*x]^4*(3*a^5*Sin[c + d*x] - 10*a^3*b^2*Sin[c + d*x] + 3*a*b^4*Sin[c
 + d*x])*(a + b*Tan[c + d*x])^5)/(3*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 119 vs. $2(28) = 56$.
time = 0.37, size = 120, normalized size = 4.00

method	result
derivativedivides	$\frac{a^5\tan(dx+c)+\frac{5ba^4}{2\cos(dx+c)^2}+\frac{10a^3b^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+\frac{5b^3a^2\left(\sin^4(dx+c)\right)}{2\cos(dx+c)^4}+\frac{a\,b^4\left(\sin^5(dx+c)\right)}{\cos(dx+c)^5}+\frac{b^5\left(\sin^6(dx+c)\right)}{6\cos(dx+c)^6}}{d}$
default	$\frac{a^5\tan(dx+c)+\frac{5ba^4}{2\cos(dx+c)^2}+\frac{10a^3b^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+\frac{5b^3a^2\left(\sin^4(dx+c)\right)}{2\cos(dx+c)^4}+\frac{a\,b^4\left(\sin^5(dx+c)\right)}{\cos(dx+c)^5}+\frac{b^5\left(\sin^6(dx+c)\right)}{6\cos(dx+c)^6}}{d}$
risch	$-20ia^3b^2e^{10i(dx+c)}+10ia\,b^4e^{10i(dx+c)}-60ia^3b^2e^{8i(dx+c)}+10ia\,b^4e^{8i(dx+c)}-\frac{200ia^3b^2e^{6i(dx+c)}}{3}+20ia\,b^4e^{6i(dx+c)}-40i$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^5*tan(d*x+c)+5/2*b*a^4/cos(d*x+c)^2+10/3*a^3*b^2*sin(d*x+c)^3/cos(d*
x+c)^3+5/2*b^3*a^2*sin(d*x+c)^4/cos(d*x+c)^4+a*b^4*sin(d*x+c)^5/cos(d*x+c)^
5+1/6*b^5*sin(d*x+c)^6/cos(d*x+c)^6)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(28) = 56$.
time = 0.29, size = 166, normalized size = 5.53

$$\frac{6\,ab^4\tan\left(dx+c\right)^5+20\,a^3b^2\tan\left(dx+c\right)^3+6\,a^5\tan\left(dx+c\right)+\frac{15\left(2\,\sin(dx+c)^2-1\right)a^2b^3}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-\frac{\left(3\,\sin(dx+c)^4-3\,\sin(dx+c)^2+1\right)b^5}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-\frac{15\,a^4b}{\sin(dx+c)^2-1}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/6*(6*a*b^4*tan(d*x + c)^5 + 20*a^3*b^2*tan(d*x + c)^3 + 6*a^5*tan(d*x + c
) + 15*(2*sin(d*x + c)^2 - 1)*a^2*b^3/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 +
1) - (3*sin(d*x + c)^4 - 3*sin(d*x + c)^2 + 1)*b^5/(sin(d*x + c)^6 - 3*sin(
d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*a^4*b/(sin(d*x + c)^2 - 1))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(28) = 56$.
time = 2.17, size = 144, normalized size = 4.80

$$\frac{b^5 + 3\left(5 a^4 b - 10 a^2 b^3 + b^5\right)\cos\left(dx+c\right)^4 + 3\left(5 a^2 b^3 - b^5\right)\cos\left(dx+c\right)^2 + 2\left(3 a b^4 \cos\left(dx+c\right) + \left(3 a^5 - 10 a^3 b^2 + 3 a b^4\right)\cos\left(dx+c\right)^5 + 2\left(5 a^3 b^2 - 3 a b^4\right)\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)}{6 d \cos\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/6*(b^5 + 3*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 + 3*(5*a^2*b^3 - b
^5)*cos(d*x + c)^2 + 2*(3*a*b^4*cos(d*x + c) + (3*a^5 - 10*a^3*b^2 + 3*a*b^
4)*cos(d*x + c)^5 + 2*(5*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(
d*cos(d*x + c)^6)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(28) = 56$.
time = 0.68, size = 89, normalized size = 2.97

$$\frac{b^5 \tan\left(dx+c\right)^6 + 6 a b^4 \tan\left(dx+c\right)^5 + 15 a^2 b^3 \tan\left(dx+c\right)^4 + 20 a^3 b^2 \tan\left(dx+c\right)^3 + 15 a^4 b \tan\left(dx+c\right)^2 + 6 a^5 \tan\left(dx+c\right)}{6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/6*(b^5*tan(d*x + c)^6 + 6*a*b^4*tan(d*x + c)^5 + 15*a^2*b^3*tan(d*x + c)^
4 + 20*a^3*b^2*tan(d*x + c)^3 + 15*a^4*b*tan(d*x + c)^2 + 6*a^5*tan(d*x + c
))/d
```

Mupad [**B**]
time = 0.98, size = 169, normalized size = 5.63

$$\frac{\cos\left(c+d\,x\right)^4\,\left(\frac{5\,a^4\,b}{2}-5\,a^2\,b^3+\frac{b^5}{2}\right)+\cos\left(c+d\,x\right)^5\,\left(\sin\left(c+d\,x\right)\,a^5-\frac{10\,\sin\left(c+d\,x\right)\,a^3\,b^2}{3}+\sin\left(c+d\,x\right)\,a\,b^4\right)-\cos\left(c+d\,x\right)^2\,\left(\frac{b^5}{2}-\frac{5\,a^2\,b^3}{2}\right)+\frac{b^5}{6}+\cos\left(c+d\,x\right)^3\,\left(\frac{10\,a^3\,b^2\,\sin\left(c+d\,x\right)}{3}-2\,a\,b^4\,\sin\left(c+d\,x\right)\right)+a\,b^4\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^7,x)
```

```
[Out] (cos(c + d*x)^4*((5*a^4*b)/2 + b^5/2 - 5*a^2*b^3) + cos(c + d*x)^5*(a^5*sin
(c + d*x) - (10*a^3*b^2*sin(c + d*x))/3 + a*b^4*sin(c + d*x)) - cos(c + d*x
)^2*(b^5/2 - (5*a^2*b^3)/2) + b^5/6 + cos(c + d*x)^3*((10*a^3*b^2*sin(c + d
*x))/3 - 2*a*b^4*sin(c + d*x)) + a*b^4*cos(c + d*x)*sin(c + d*x))/(d*cos(c
+ d*x)^6)
```

3.105 $\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=318

$$\frac{a^5\tanh^{-1}(\sin(c+dx))}{2d}-\frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{5ab^4\tanh^{-1}(\sin(c+dx))}{16d}+\frac{5a^4b\sec^3(c+dx)}{3d}-\frac{10a^2b^3\text{se}}{3}$$

```
[Out] 1/2*a^5*arctanh(sin(d*x+c))/d-5/4*a^3*b^2*arctanh(sin(d*x+c))/d+5/16*a*b^4*
arctanh(sin(d*x+c))/d+5/3*a^4*b*sec(d*x+c)^3/d-10/3*a^2*b^3*sec(d*x+c)^3/d+
1/3*b^5*sec(d*x+c)^3/d+2*a^2*b^3*sec(d*x+c)^5/d-2/5*b^5*sec(d*x+c)^5/d+1/7*
b^5*sec(d*x+c)^7/d+1/2*a^5*sec(d*x+c)*tan(d*x+c)/d-5/4*a^3*b^2*sec(d*x+c)*t
an(d*x+c)/d+5/16*a*b^4*sec(d*x+c)*tan(d*x+c)/d+5/2*a^3*b^2*sec(d*x+c)^3*tan
(d*x+c)/d-5/8*a*b^4*sec(d*x+c)^3*tan(d*x+c)/d+5/6*a*b^4*sec(d*x+c)^3*tan(d*
x+c)^3/d
```

Rubi [A]
time = 0.24, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3169, 3853, 3855, 2686, 30, 2691, 14, 276}

$$\frac{a^5\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^5\tan(c+dx)\sec(c+dx)}{2d}+\frac{5a^4b\sec^3(c+dx)}{3d}-\frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{4d}+\frac{5a^3b^2\tan(c+dx)\sec^3(c+dx)}{2d}-\frac{5a^3b^2\tan(c+dx)\sec(c+dx)}{4d}+\frac{2a^2b^3\sec^5(c+dx)}{d}-\frac{10a^2b^3\sec^3(c+dx)}{3d}+\frac{5ab^4\tanh^{-1}(\sin(c+dx))}{16d}+\frac{5ab^4\tan^3(c+dx)\sec^3(c+dx)}{6d}-\frac{5ab^4\tan(c+dx)\sec^3(c+dx)}{8d}+\frac{5ab^4\tan(c+dx)\sec(c+dx)}{16d}+\frac{b^5\sec^7(c+dx)}{7d}-\frac{2b^5\sec^5(c+dx)}{5d}+\frac{b^5\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (a^5*ArcTanh[Sin[c + d*x]])/(2*d) - (5*a^3*b^2*ArcTanh[Sin[c + d*x]])/(4*d)
 + (5*a*b^4*ArcTanh[Sin[c + d*x]])/(16*d) + (5*a^4*b*Sec[c + d*x]^3)/(3*d)
- (10*a^2*b^3*Sec[c + d*x]^3)/(3*d) + (b^5*Sec[c + d*x]^3)/(3*d) + (2*a^2*b
^3*Sec[c + d*x]^5)/d - (2*b^5*Sec[c + d*x]^5)/(5*d) + (b^5*Sec[c + d*x]^7)/
(7*d) + (a^5*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (5*a^3*b^2*Sec[c + d*x]*Tan
[c + d*x])/(4*d) + (5*a*b^4*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (5*a^3*b^2*
Sec[c + d*x]^3*Tan[c + d*x])/(2*d) - (5*a*b^4*Sec[c + d*x]^3*Tan[c + d*x])/
(8*d) + (5*a*b^4*Sec[c + d*x]^3*Tan[c + d*x]^3)/(6*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^8(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int \left(a^5\sec^3(c+dx)+5a^4b\sec^3(c+dx)\tan(c+dx)+10\right.$$
$$= a^5\int \sec^3(c+dx)\,dx + \left(5a^4b\right)\int \sec^3(c+dx)\tan(c+dx$$
$$= \frac{a^5\sec(c+dx)\tan(c+dx)}{2d} + \frac{5a^3b^2\sec^3(c+dx)\tan(c+}{2d}$$
$$= \frac{a^5\tanh^{-1}(\sin(c+dx))}{2d} + \frac{5a^4b\sec^3(c+dx)}{3d} + \frac{a^5\sec(c+}{}$$
$$= \frac{a^5\tanh^{-1}(\sin(c+dx))}{2d} - \frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{4d} + $$
$$= \frac{a^5\tanh^{-1}(\sin(c+dx))}{2d} - \frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{4d} + $$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1677 vs. $2(318) = 636$.

time = 6.42, size = 1677, normalized size = 5.27

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^8*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (b*(1400*a^4 - 1540*a^2*b^2 + 103*b^4)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^
5)/(1680*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((-8*a^5 + 20*a^3*b^2 - 5
*a*b^4)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a + b*Tan[
c + d*x])^5)/(16*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((8*a^5 - 20*a^3*
b^2 + 5*a*b^4)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a +
 b*Tan[c + d*x])^5)/(16*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((35*a*b^4
 + 3*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(336*d*(Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2])^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((350*a^3*b^2 +
 140*a^2*b^3 - 175*a*b^4 - 18*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(
560*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4*(a*Cos[c + d*x] + b*Sin[c + d
*x])^5) + ((840*a^5 + 1400*a^4*b - 2100*a^3*b^2 - 1540*a^2*b^3 + 525*a*b^4
+ 103*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(3360*d*(Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (b^5*Cos[c +
d*x]^5*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^5)/(56*d*(Cos[(c + d*x)/2] - S
in[(c + d*x)/2])^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) - (b^5*Cos[c + d*x]
^5*Sin[(c + d*x)/2]*(a + b*Tan[c + d*x])^5)/(56*d*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])^7*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((-35*a*b^4 + 3*b^5)*
Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(336*d*(Cos[(c + d*x)/2] + Sin[(c +
```

```
d*x)/2])^6*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + ((-350*a^3*b^2 + 140*a^2*
b^3 + 175*a*b^4 - 18*b^5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(560*d*(Co
s[(c + d*x)/2] + Sin[(c + d*x)/2])^4*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) +
 ((-840*a^5 + 1400*a^4*b + 2100*a^3*b^2 - 1540*a^2*b^3 - 525*a*b^4 + 103*b^
5)*Cos[c + d*x]^5*(a + b*Tan[c + d*x])^5)/(3360*d*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (Cos[c + d*x]^5*(-140
0*a^4*b*Sin[(c + d*x)/2] + 1540*a^2*b^3*Sin[(c + d*x)/2] - 103*b^5*Sin[(c +
 d*x)/2])*(a + b*Tan[c + d*x])^5)/(1680*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)
/2])^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (Cos[c + d*x]^5*(-1400*a^4*b*
Sin[(c + d*x)/2] + 1540*a^2*b^3*Sin[(c + d*x)/2] - 103*b^5*Sin[(c + d*x)/2]
)*(a + b*Tan[c + d*x])^5)/(1680*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])*(a*
Cos[c + d*x] + b*Sin[c + d*x])^5) + (Cos[c + d*x]^5*(70*a^2*b^3*Sin[(c + d*
x)/2] - 9*b^5*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^5)/(140*d*(Cos[(c + d*
x)/2] - Sin[(c + d*x)/2])^5*(a*Cos[c + d*x] + b*Sin[c + d*x])^5) + (Cos[c +
 d*x]^5*(-70*a^2*b^3*Sin[(c + d*x)/2] + 9*b^5*Sin[(c + d*x)/2])*(a + b*Tan[
c + d*x])^5)/(140*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5*(a*Cos[c + d*x]
 + b*Sin[c + d*x])^5) + (Cos[c + d*x]^5*(1400*a^4*b*Sin[(c + d*x)/2] - 1540
*a^2*b^3*Sin[(c + d*x)/2] + 103*b^5*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^
5)/(1680*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3*(a*Cos[c + d*x] + b*Sin[
c + d*x])^5) + (Cos[c + d*x]^5*(1400*a^4*b*Sin[(c + d*x)/2] - 1540*a^2*b^3*
Sin[(c + d*x)/2] + 103*b^5*Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^5)/(1680*
d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a*Cos[c + d*x] + b*Sin[c + d*x])^5
)
```

Maple [A]
time = 0.45, size = 405, normalized size = 1.27 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^5*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+5/3*b*a^
4/cos(d*x+c)^3+10*a^3*b^2*(1/4*sin(d*x+c)^3/cos(d*x+c)^4+1/8*sin(d*x+c)^3/c
os(d*x+c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x+c)))+10*b^3*a^2*(1/5*s
in(d*x+c)^4/cos(d*x+c)^5+1/15*sin(d*x+c)^4/cos(d*x+c)^3-1/15*sin(d*x+c)^4/c
os(d*x+c)-1/15*(2+sin(d*x+c)^2)*cos(d*x+c))+5*a*b^4*(1/6*sin(d*x+c)^5/cos(d
*x+c)^6+1/24*sin(d*x+c)^5/cos(d*x+c)^4-1/48*sin(d*x+c)^5/cos(d*x+c)^2-1/48*
sin(d*x+c)^3-1/16*sin(d*x+c)+1/16*ln(sec(d*x+c)+tan(d*x+c)))+b^5*(1/7*sin(d
*x+c)^6/cos(d*x+c)^7+1/35*sin(d*x+c)^6/cos(d*x+c)^5-1/105*sin(d*x+c)^6/cos(
d*x+c)^3+1/35*sin(d*x+c)^6/cos(d*x+c)+1/35*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)
^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.28, size = 289, normalized size = 0.91

$$-\frac{175\,ab^4\left(\frac{2\left(3\,\sin(dx+c)^5+8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-2100\,a^3b^2\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+840\,a^5\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-\frac{5600\,a^4b}{\cos(dx+c)^3}+\frac{2240\left(5\,\cos(dx+c)^2-3\right)a^2b^3}{\cos(dx+c)^5}-\frac{32\left(35\,\cos(dx+c)^4-42\,\cos(dx+c)^2+15\right)b^5}{\cos(dx+c)^7}}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] -1/3360*(175*a*b^4*(2*(3*sin(d*x + c)^5 + 8*sin(d*x + c)^3 - 3*sin(d*x + c)
)/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(d*
x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 2100*a^3*b^2*(2*(sin(d*x + c)^3 +
sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - log(sin(d*x + c) +
1) + log(sin(d*x + c) - 1)) + 840*a^5*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1)
- log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 5600*a^4*b/cos(d*x + c)^
3 + 2240*(5*cos(d*x + c)^2 - 3)*a^2*b^3/cos(d*x + c)^5 - 32*(35*cos(d*x + c
)^4 - 42*cos(d*x + c)^2 + 15)*b^5/cos(d*x + c)^7)/d
```

Fricas [**A**]
time = 2.93, size = 227, normalized size = 0.71

$$\frac{105\,(8\,a^5-20\,a^3b^2+5\,ab^4)\cos(dx+c)^7\log(\sin(dx+c)+1)-105\,(8\,a^5-20\,a^3b^2+5\,ab^4)\cos(dx+c)^7\log(-\sin(dx+c)+1)+480\,b^5+1120\,(5\,a^4b-10\,a^2b^3+b^5)\cos(dx+c)^4+1344\,(5\,a^2b^3-b^5)\cos(dx+c)^2+70\,(40\,ab^4\cos(dx+c)+3\,(8\,a^5-20\,a^3b^2+5\,ab^4)\cos(dx+c)^5+10\,(12\,a^3b^2-7\,ab^4)\cos(dx+c)^3)\sin(dx+c)}{3360\,d\cos(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/3360*(105*(8*a^5 - 20*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^7*log(sin(d*x + c)
+ 1) - 105*(8*a^5 - 20*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^7*log(-sin(d*x + c)
+ 1) + 480*b^5 + 1120*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 + 1344*(5
*a^2*b^3 - b^5)*cos(d*x + c)^2 + 70*(40*a*b^4*cos(d*x + c) + 3*(8*a^5 - 20*
a^3*b^2 + 5*a*b^4)*cos(d*x + c)^5 + 10*(12*a^3*b^2 - 7*a*b^4)*cos(d*x + c)^
3)*sin(d*x + c))/(d*cos(d*x + c)^7)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**8*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 680 vs. 2(290) = 580.
time = 0.68, size = 680, normalized size = 2.14

$$\frac{105\,(8\,a^5-20\,a^3b^2+5\,ab^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\,(8\,a^5-20\,a^3b^2+5\,ab^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{[illegible]}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^7}}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^8*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/1680*(105*(8*a^5 - 20*a^3*b^2 + 5*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) - 105*(8*a^5 - 20*a^3*b^2 + 5*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1))
```

```
+ 2*(840*a^5*tan(1/2*d*x + 1/2*c)^13 + 2100*a^3*b^2*tan(1/2*d*x + 1/2*c)^13
 - 525*a*b^4*tan(1/2*d*x + 1/2*c)^13 - 8400*a^4*b*tan(1/2*d*x + 1/2*c)^12 -
 3360*a^5*tan(1/2*d*x + 1/2*c)^11 + 8400*a^3*b^2*tan(1/2*d*x + 1/2*c)^11 +
3500*a*b^4*tan(1/2*d*x + 1/2*c)^11 + 33600*a^4*b*tan(1/2*d*x + 1/2*c)^10 -
33600*a^2*b^3*tan(1/2*d*x + 1/2*c)^10 + 4200*a^5*tan(1/2*d*x + 1/2*c)^9 - 2
3100*a^3*b^2*tan(1/2*d*x + 1/2*c)^9 + 16975*a*b^4*tan(1/2*d*x + 1/2*c)^9 -
53200*a^4*b*tan(1/2*d*x + 1/2*c)^8 + 56000*a^2*b^3*tan(1/2*d*x + 1/2*c)^8 -
 8960*b^5*tan(1/2*d*x + 1/2*c)^8 + 44800*a^4*b*tan(1/2*d*x + 1/2*c)^6 - 224
00*a^2*b^3*tan(1/2*d*x + 1/2*c)^6 - 4480*b^5*tan(1/2*d*x + 1/2*c)^6 - 4200*
a^5*tan(1/2*d*x + 1/2*c)^5 + 23100*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 16975*a
*b^4*tan(1/2*d*x + 1/2*c)^5 - 25200*a^4*b*tan(1/2*d*x + 1/2*c)^4 + 13440*a^
2*b^3*tan(1/2*d*x + 1/2*c)^4 - 2688*b^5*tan(1/2*d*x + 1/2*c)^4 + 3360*a^5*t
an(1/2*d*x + 1/2*c)^3 - 8400*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 3500*a*b^4*ta
n(1/2*d*x + 1/2*c)^3 + 11200*a^4*b*tan(1/2*d*x + 1/2*c)^2 - 15680*a^2*b^3*t
an(1/2*d*x + 1/2*c)^2 + 896*b^5*tan(1/2*d*x + 1/2*c)^2 - 840*a^5*tan(1/2*d*
x + 1/2*c) - 2100*a^3*b^2*tan(1/2*d*x + 1/2*c) + 525*a*b^4*tan(1/2*d*x + 1/
2*c) - 2800*a^4*b + 2240*a^2*b^3 - 128*b^5)/(tan(1/2*d*x + 1/2*c)^2 - 1)^7)
/d
```

Mupad [B]

time = 4.21, size = 514, normalized size = 1.62

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^5-\frac{5\,a^3\,b^2}{2}+\frac{5\,a\,b^4}{8}\right)}{d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(-4\,a^5+10\,a^3\,b^2+\frac{25\,a\,b^4}{6}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\,\left(40\,a^4\,b-40\,a^2\,b^3\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}\,\left(a^5+\frac{5\,a^3\,b^2}{2}-\frac{5\,a\,b^4}{8}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}\,\left(-4\,a^5+10\,a^3\,b^2+\frac{25\,a\,b^4}{6}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(5\,a^5-\frac{55\,a^3\,b^2}{2}+\frac{485\,a\,b^4}{24}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9\,\left(5\,a^5-\frac{55\,a^3\,b^2}{2}+\frac{485\,a\,b^4}{24}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(30\,a^4\,b-16\,a^2\,b^3+\frac{16\,b^5}{5}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(\frac{40\,a^4\,b}{3}-\frac{56\,a^2\,b^3}{3}+\frac{16\,b^5}{15}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(-\frac{160\,a^4\,b}{3}+\frac{80\,a^2\,b^3}{3}+\frac{16\,b^5}{3}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(\frac{190\,a^4\,b}{3}-\frac{200\,a^2\,b^3}{3}+\frac{32\,b^5}{3}\right)+\frac{10\,a^4\,b}{3}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^5+\frac{5\,a^3\,b^2}{2}-\frac{5\,a\,b^4}{8}\right)+\frac{16\,b^5}{105}-\frac{8\,a^2\,b^3}{3}+10\,a^4\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}-7\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+21\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-35\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+35\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-21\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+7\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^8,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*((5*a*b^4)/8 + a^5 - (5*a^3*b^2)/2))/d - (tan(c/
2 + (d*x)/2)^3*((25*a*b^4)/6 - 4*a^5 + 10*a^3*b^2) - tan(c/2 + (d*x)/2)^10*
(40*a^4*b - 40*a^2*b^3) - tan(c/2 + (d*x)/2)^13*(a^5 - (5*a*b^4)/8 + (5*a^3
*b^2)/2) - tan(c/2 + (d*x)/2)^11*((25*a*b^4)/6 - 4*a^5 + 10*a^3*b^2) + tan(
c/2 + (d*x)/2)^5*((485*a*b^4)/24 + 5*a^5 - (55*a^3*b^2)/2) - tan(c/2 + (d*x
)/2)^9*((485*a*b^4)/24 + 5*a^5 - (55*a^3*b^2)/2) + tan(c/2 + (d*x)/2)^4*(30
*a^4*b + (16*b^5)/5 - 16*a^2*b^3) - tan(c/2 + (d*x)/2)^2*((40*a^4*b)/3 + (1
6*b^5)/15 - (56*a^2*b^3)/3) + tan(c/2 + (d*x)/2)^6*((16*b^5)/3 - (160*a^4*b
)/3 + (80*a^2*b^3)/3) + tan(c/2 + (d*x)/2)^8*((190*a^4*b)/3 + (32*b^5)/3 -
(200*a^2*b^3)/3) + (10*a^4*b)/3 + tan(c/2 + (d*x)/2)*(a^5 - (5*a*b^4)/8 + (
5*a^3*b^2)/2) + (16*b^5)/105 - (8*a^2*b^3)/3 + 10*a^4*b*tan(c/2 + (d*x)/2)^
12)/(d*(7*tan(c/2 + (d*x)/2)^2 - 21*tan(c/2 + (d*x)/2)^4 + 35*tan(c/2 + (d*
x)/2)^6 - 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 - 7*tan(c/2 +
(d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 - 1))
```

3.106 $\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=177

$$\frac{a^5\tan(c+dx)}{d}+\frac{5a^4b\tan^2(c+dx)}{2d}+\frac{a^3(a^2+10b^2)\tan^3(c+dx)}{3d}+\frac{5a^2b(a^2+2b^2)\tan^4(c+dx)}{4d}+\frac{ab^2(2a^2+b^2)}{d}$$

```
[Out] a^5*tan(d*x+c)/d+5/2*a^4*b*tan(d*x+c)^2/d+1/3*a^3*(a^2+10*b^2)*tan(d*x+c)^3
/d+5/4*a^2*b*(a^2+2*b^2)*tan(d*x+c)^4/d+a*b^2*(2*a^2+b^2)*tan(d*x+c)^5/d+1/
6*b^3*(10*a^2+b^2)*tan(d*x+c)^6/d+5/7*a*b^4*tan(d*x+c)^7/d+1/8*b^5*tan(d*x+
c)^8/d
```

Rubi [A]
time = 0.11, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3167, 908}

$$\frac{a^5\tan(c+dx)}{d}+\frac{5a^4b\tan^2(c+dx)}{2d}+\frac{ab^2(2a^2+b^2)\tan^5(c+dx)}{d}+\frac{5a^2b(a^2+2b^2)\tan^4(c+dx)}{4d}+\frac{b^3(10a^2+b^2)\tan^6(c+dx)}{6d}+\frac{a^3(a^2+10b^2)\tan^3(c+dx)}{3d}+\frac{5ab^4\tan^7(c+dx)}{7d}+\frac{b^5\tan^8(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^9*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (a^5*Tan[c + d*x])/d + (5*a^4*b*Tan[c + d*x]^2)/(2*d) + (a^3*(a^2 + 10*b^2)
*Tan[c + d*x]^3)/(3*d) + (5*a^2*b*(a^2 + 2*b^2)*Tan[c + d*x]^4)/(4*d) + (a*
b^2*(2*a^2 + b^2)*Tan[c + d*x]^5)/d + (b^3*(10*a^2 + b^2)*Tan[c + d*x]^6)/(
6*d) + (5*a*b^4*Tan[c + d*x]^7)/(7*d) + (b^5*Tan[c + d*x]^8)/(8*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \sec^9(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^5(1+x^2)}{x^9}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{b^5}{x^9}+\frac{5ab^4}{x^8}+\frac{10a^2b^3+b^5}{x^7}+\frac{5ab^2(2a^2+b^2)}{x^6}+\frac{5a^2b(a}{x}\right.\right.}{d}$$
$$= \frac{a^5\tan(c+dx)}{d}+\frac{5a^4b\tan^2(c+dx)}{2d}+\frac{a^3(a^2+10b^2)\,\text{ta}}{3d}$$

Mathematica [A]
time = 0.48, size = 54, normalized size = 0.31

$$\frac{(a+b\tan(c+dx))^6\left(a^2+28b^2-6ab\tan(c+dx)+21b^2\tan^2(c+dx)\right)}{168b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^9*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] ((a + b*Tan[c + d*x])^6*(a^2 + 28*b^2 - 6*a*b*Tan[c + d*x] + 21*b^2*Tan[c +
 d*x]^2))/(168*b^3*d)
```

Maple [A]
time = 0.39, size = 217, normalized size = 1.23

method	result
derivativedivides	$\frac{-a^5\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{5b\,a^4}{4\cos(dx+c)^4}+10a^3b^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+10b^3a^2\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\text{s}}{12}\right.}{d}$
default	$\frac{-a^5\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+\frac{5b\,a^4}{4\cos(dx+c)^4}+10a^3b^2\left(\frac{\sin^3(dx+c)}{5\cos(dx+c)^5}+\frac{2\left(\sin^3(dx+c)\right)}{15\cos(dx+c)^3}\right)+10b^3a^2\left(\frac{\sin^4(dx+c)}{6\cos(dx+c)^6}+\frac{\text{s}}{12}\right.}{d}$
risch	$-\frac{320ia^3b^2e^{10i(dx+c)}}{3}-\frac{280ia^3b^2e^{8i(dx+c)}}{3}+20ia\,b^4e^{8i(dx+c)}-\frac{128ia^3b^2e^{6i(dx+c)}}{3}+32ia\,b^4e^{6i(dx+c)}-\frac{104ia^3b^2e^{4i(dx+c)}}{3}-4$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^5*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+5/4*b*a^4/cos(d*x+c)^4+10*a^3*
b^2*(1/5*sin(d*x+c)^3/cos(d*x+c)^5+2/15*sin(d*x+c)^3/cos(d*x+c)^3)+10*b^3*a
^2*(1/6*sin(d*x+c)^4/cos(d*x+c)^6+1/12*sin(d*x+c)^4/cos(d*x+c)^4)+5*a*b^4*(
1/7*sin(d*x+c)^5/cos(d*x+c)^7+2/35*sin(d*x+c)^5/cos(d*x+c)^5)+b^5*(1/8*sin(
d*x+c)^6/cos(d*x+c)^8+1/24*sin(d*x+c)^6/cos(d*x+c)^6))
```

Maxima [A]
time = 0.27, size = 223, normalized size = 1.26

$$\frac{56 \left(\tan\left(dx+c\right)^{3}+3 \tan\left(dx+c\right)\right) a^{5}+112 \left(3 \tan\left(dx+c\right)^{5}+5 \tan\left(dx+c\right)^{3}\right) a^{3} b^{2}+24 \left(5 \tan\left(dx+c\right)^{7}+7 \tan\left(dx+c\right)^{5}\right) a b^{4}-\frac{140 \left(3 \sin(dx+c)^{2}-1\right) a^{2} b^{3}}{\sin(dx+c)^{6}-3 \sin(dx+c)^{4}+3 \sin(dx+c)^{2}-1}+\frac{7 \left(6 \sin(dx+c)^{4}-4 \sin(dx+c)^{2}+1\right) b^{5}}{\sin(dx+c)^{8}-4 \sin(dx+c)^{6}+6 \sin(dx+c)^{4}-4 \sin(dx+c)^{2}+1}+\frac{210 a^{4} b}{\left(\sin(dx+c)^{2}-1\right)^{2}}}{168 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima")
```

```
[Out] 1/168*(56*(tan(d*x + c)^3 + 3*tan(d*x + c))*a^5 + 112*(3*tan(d*x + c)^5 + 5
*tan(d*x + c)^3)*a^3*b^2 + 24*(5*tan(d*x + c)^7 + 7*tan(d*x + c)^5)*a*b^4 -
 140*(3*sin(d*x + c)^2 - 1)*a^2*b^3/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*
sin(d*x + c)^2 - 1) + 7*(6*sin(d*x + c)^4 - 4*sin(d*x + c)^2 + 1)*b^5/(sin(
d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^4 - 4*sin(d*x + c)^2 + 1) +
210*a^4*b/(sin(d*x + c)^2 - 1)^2)/d
```

Fricas [A]
time = 2.28, size = 176, normalized size = 0.99

$$\frac{21 b^{5}+42 \left(5 a^{4} b-10 a^{2} b^{3}+b^{5}\right) \cos\left(dx+c\right)^{4}+56 \left(5 a^{2} b^{3}-b^{5}\right) \cos\left(dx+c\right)^{2}+8 \left(2 \left(7 a^{5}-14 a^{3} b^{2}+3 a b^{4}\right) \cos\left(dx+c\right)^{7}+15 a b^{4} \cos\left(dx+c\right)+\left(7 a^{5}-14 a^{3} b^{2}+3 a b^{4}\right) \cos\left(dx+c\right)^{5}+6 \left(7 a^{3} b^{2}-4 a b^{4}\right) \cos\left(dx+c\right)^{3}\right) \sin\left(dx+c\right)}{168 d \cos\left(dx+c\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas")
```

```
[Out] 1/168*(21*b^5 + 42*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 + 56*(5*a^2*
b^3 - b^5)*cos(d*x + c)^2 + 8*(2*(7*a^5 - 14*a^3*b^2 + 3*a*b^4)*cos(d*x + c
)^7 + 15*a*b^4*cos(d*x + c) + (7*a^5 - 14*a^3*b^2 + 3*a*b^4)*cos(d*x + c)^5
 + 6*(7*a^3*b^2 - 4*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^8)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**9*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.73, size = 176, normalized size = 0.99

$$\frac{21 b^{5} \tan\left(dx+c\right)^{8}+120 a b^{4} \tan\left(dx+c\right)^{7}+280 a^{2} b^{3} \tan\left(dx+c\right)^{6}+28 b^{5} \tan\left(dx+c\right)^{6}+336 a^{3} b^{2} \tan\left(dx+c\right)^{5}+168 a b^{4} \tan\left(dx+c\right)^{5}+210 a^{4} b \tan\left(dx+c\right)^{4}+420 a^{2} b^{3} \tan\left(dx+c\right)^{4}+56 a^{5} \tan\left(dx+c\right)^{3}+560 a^{3} b^{2} \tan\left(dx+c\right)^{3}+420 a^{4} b \tan\left(dx+c\right)^{2}+168 a^{5} \tan\left(dx+c\right)}{168 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^9*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/168*(21*b^5*tan(d*x + c)^8 + 120*a*b^4*tan(d*x + c)^7 + 280*a^2*b^3*tan(d
*x + c)^6 + 28*b^5*tan(d*x + c)^6 + 336*a^3*b^2*tan(d*x + c)^5 + 168*a*b^4*
tan(d*x + c)^5 + 210*a^4*b*tan(d*x + c)^4 + 420*a^2*b^3*tan(d*x + c)^4 + 56
*a^5*tan(d*x + c)^3 + 560*a^3*b^2*tan(d*x + c)^3 + 420*a^4*b*tan(d*x + c)^2
 + 168*a^5*tan(d*x + c))/d
```

Mupad [B]

time = 4.27, size = 419, normalized size = 2.37

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(\frac{86a^5}{3}-\frac{208a^3b^2}{3}+32ab^4\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(40a^4b-40a^2b^3\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\left(40a^4b-40a^2b^3\right)-2a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}\left(\frac{86a^5}{3}-\frac{208a^3b^2}{3}+32ab^4\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\left(\frac{130a^5}{3}-\frac{224a^3b^2}{3}+\frac{32ab^4}{7}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9\left(\frac{130a^5}{3}-\frac{224a^3b^2}{3}+\frac{32ab^4}{7}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8\left(-80a^4b+\frac{80a^2b^3}{3}+\frac{32b^5}{3}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(70a^4b-\frac{160a^2b^3}{3}+\frac{32b^5}{3}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}\left(70a^4b-\frac{160a^2b^3}{3}+\frac{32b^5}{3}\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{34a^5}{3}-\frac{80a^3b^2}{3}\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}\left(\frac{34a^5}{3}-\frac{80a^3b^2}{3}\right)+2a^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+10a^4b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+10a^4b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^9,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*(32*a*b^4 + (86*a^5)/3 - (208*a^3*b^2)/3) - tan(c/2 +
 (d*x)/2)^4*(40*a^4*b - 40*a^2*b^3) - tan(c/2 + (d*x)/2)^12*(40*a^4*b - 40*
a^2*b^3) - 2*a^5*tan(c/2 + (d*x)/2)^15 - tan(c/2 + (d*x)/2)^11*(32*a*b^4 +
(86*a^5)/3 - (208*a^3*b^2)/3) - tan(c/2 + (d*x)/2)^7*((32*a*b^4)/7 + (130*a
^5)/3 - (224*a^3*b^2)/3) + tan(c/2 + (d*x)/2)^9*((32*a*b^4)/7 + (130*a^5)/3
 - (224*a^3*b^2)/3) + tan(c/2 + (d*x)/2)^8*((32*b^5)/3 - 80*a^4*b + (80*a^2
*b^3)/3) + tan(c/2 + (d*x)/2)^6*(70*a^4*b + (32*b^5)/3 - (160*a^2*b^3)/3) +
 tan(c/2 + (d*x)/2)^10*(70*a^4*b + (32*b^5)/3 - (160*a^2*b^3)/3) - tan(c/2
+ (d*x)/2)^3*((34*a^5)/3 - (80*a^3*b^2)/3) + tan(c/2 + (d*x)/2)^13*((34*a^5
)/3 - (80*a^3*b^2)/3) + 2*a^5*tan(c/2 + (d*x)/2) + 10*a^4*b*tan(c/2 + (d*x)
/2)^2 + 10*a^4*b*tan(c/2 + (d*x)/2)^14)/(d*(tan(c/2 + (d*x)/2)^2 - 1)^8)
```

3.107 $\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=391

$$\frac{3a^5\tanh^{-1}(\sin(c+dx))}{8d}-\frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{15ab^4\tanh^{-1}(\sin(c+dx))}{128d}+\frac{a^4b\sec^5(c+dx)}{d}-\frac{2a^2b^3\operatorname{se}}{}$$

```
[Out] 3/8*a^5*arctanh(sin(d*x+c))/d-5/8*a^3*b^2*arctanh(sin(d*x+c))/d+15/128*a*b^
4*arctanh(sin(d*x+c))/d+a^4*b*sec(d*x+c)^5/d-2*a^2*b^3*sec(d*x+c)^5/d+1/5*b
^5*sec(d*x+c)^5/d+10/7*a^2*b^3*sec(d*x+c)^7/d-2/7*b^5*sec(d*x+c)^7/d+1/9*b^
5*sec(d*x+c)^9/d+3/8*a^5*sec(d*x+c)*tan(d*x+c)/d-5/8*a^3*b^2*sec(d*x+c)*tan
(d*x+c)/d+15/128*a*b^4*sec(d*x+c)*tan(d*x+c)/d+1/4*a^5*sec(d*x+c)^3*tan(d*x
+c)/d-5/12*a^3*b^2*sec(d*x+c)^3*tan(d*x+c)/d+5/64*a*b^4*sec(d*x+c)^3*tan(d*
x+c)/d+5/3*a^3*b^2*sec(d*x+c)^5*tan(d*x+c)/d-5/16*a*b^4*sec(d*x+c)^5*tan(d*
x+c)/d+5/8*a*b^4*sec(d*x+c)^5*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.27, antiderivative size = 391, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3169, 3853, 3855, 2686, 30, 2691, 14, 276}

$$\frac{3a^5\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^5\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a^5\tan(c+dx)\sec(c+dx)}{8d}+\frac{a^4b\sec^5(c+dx)}{d}-\frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{8d}+\frac{5a^3b^2\tan(c+dx)\sec^5(c+dx)}{3d}-\frac{5a^3b^2\tan(c+dx)\sec^3(c+dx)}{12d}-\frac{5a^3b^2\tan(c+dx)\sec(c+dx)}{8d}+\frac{10a^2b^3\sec^7(c+dx)}{7d}-\frac{2a^2b^3\sec^5(c+dx)}{d}+\frac{15ab^4\tanh^{-1}(\sin(c+dx))}{128d}+\frac{5ab^4\tan^3(c+dx)\sec^5(c+dx)}{8d}-\frac{5ab^4\tan(c+dx)\sec^5(c+dx)}{16d}+\frac{5ab^4\tan(c+dx)\sec^3(c+dx)}{64d}+\frac{15ab^4\tan(c+dx)\sec(c+dx)}{128d}+\frac{b^5\sec^9(c+dx)}{9d}-\frac{2b^5\sec^7(c+dx)}{7d}+\frac{b^5\sec^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^10*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (3*a^5*ArcTanh[Sin[c + d*x]])/(8*d) - (5*a^3*b^2*ArcTanh[Sin[c + d*x]])/(8*
d) + (15*a*b^4*ArcTanh[Sin[c + d*x]])/(128*d) + (a^4*b*Sec[c + d*x]^5)/d -
(2*a^2*b^3*Sec[c + d*x]^5)/d + (b^5*Sec[c + d*x]^5)/(5*d) + (10*a^2*b^3*Sec
[c + d*x]^7)/(7*d) - (2*b^5*Sec[c + d*x]^7)/(7*d) + (b^5*Sec[c + d*x]^9)/(9
*d) + (3*a^5*Sec[c + d*x]*Tan[c + d*x])/(8*d) - (5*a^3*b^2*Sec[c + d*x]*Tan
[c + d*x])/(8*d) + (15*a*b^4*Sec[c + d*x]*Tan[c + d*x])/(128*d) + (a^5*Sec[
c + d*x]^3*Tan[c + d*x])/(4*d) - (5*a^3*b^2*Sec[c + d*x]^3*Tan[c + d*x])/(1
2*d) + (5*a*b^4*Sec[c + d*x]^3*Tan[c + d*x])/(64*d) + (5*a^3*b^2*Sec[c + d*
x]^5*Tan[c + d*x])/(3*d) - (5*a*b^4*Sec[c + d*x]^5*Tan[c + d*x])/(16*d) + (
5*a*b^4*Sec[c + d*x]^5*Tan[c + d*x]^3)/(8*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{10}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int \left(a^5\sec^5(c+dx)+5a^4b\sec^5(c+dx)\tan(c+dx)+1\right.$$
$$= a^5\int \sec^5(c+dx)\,dx + \left(5a^4b\right)\int \sec^5(c+dx)\tan(c+d$$
$$= \frac{a^5\sec^3(c+dx)\tan(c+dx)}{4d} + \frac{5a^3b^2\sec^5(c+dx)\tan(c}{3d}$$
$$= \frac{a^4b\sec^5(c+dx)}{d} + \frac{3a^5\sec(c+dx)\tan(c+dx)}{8d} + \frac{a^5\sec}{}$$
$$= \frac{3a^5\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^4b\sec^5(c+dx)}{d} - \frac{2a^2b^3\sec}{}$$
$$= \frac{3a^5\tanh^{-1}(\sin(c+dx))}{8d} - \frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{8d} +$$
$$= \frac{3a^5\tanh^{-1}(\sin(c+dx))}{8d} - \frac{5a^3b^2\tanh^{-1}(\sin(c+dx))}{8d} +$$

Mathematica [A]
time = 2.46, size = 331, normalized size = 0.85

$$\frac{-40320a(48a^4-80a^2b^2+15b^4)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sec^9(c+dx)\left(1935360a^4b-184320a^2b^3+223232b^5+73728(35a^4b-20a^2b^3-3b^5)\cos(2(c+dx))+129024(5a^4b-10a^2b^3+b^5)\cos(4(c+dx))+372960a^5\sin(4(c+dx))+453600a^3b^2\sin(4(c+dx))-488250ab^4\sin(4(c+dx))+131040a^5\sin(6(c+dx))-218400a^3b^2\sin(6(c+dx))+40950ab^4\sin(6(c+dx))+15120a^5\sin(8(c+dx))-25200a^3b^2\sin(8(c+dx))+4725ab^4\sin(8(c+dx))\right)+1260a(656a^4+2320a^2b^2+845b^4)\sec^7(c+dx)\tan(c+dx)}{5160960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^10*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (-40320*a*(48*a^4 - 80*a^2*b^2 + 15*b^4)*(Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c + d*x]^9*(19353
60*a^4*b - 184320*a^2*b^3 + 223232*b^5 + 73728*(35*a^4*b - 20*a^2*b^3 - 3*b
^5)*Cos[2*(c + d*x)] + 129024*(5*a^4*b - 10*a^2*b^3 + b^5)*Cos[4*(c + d*x)]
 + 372960*a^5*Sin[4*(c + d*x)] + 453600*a^3*b^2*Sin[4*(c + d*x)] - 488250*a
*b^4*Sin[4*(c + d*x)] + 131040*a^5*Sin[6*(c + d*x)] - 218400*a^3*b^2*Sin[6*
(c + d*x)] + 40950*a*b^4*Sin[6*(c + d*x)] + 15120*a^5*Sin[8*(c + d*x)] - 25
200*a^3*b^2*Sin[8*(c + d*x)] + 4725*a*b^4*Sin[8*(c + d*x)]) + 1260*a*(656*a
^4 + 2320*a^2*b^2 + 845*b^4)*Sec[c + d*x]^7*Tan[c + d*x])/(5160960*d)
```

Maple [A]
time = 0.50, size = 489, normalized size = 1.25 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^5*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+
tan(d*x+c)))+b*a^4/cos(d*x+c)^5+10*a^3*b^2*(1/6*sin(d*x+c)^3/cos(d*x+c)^6+1
```

```
/8*sin(d*x+c)^3/cos(d*x+c)^4+1/16*sin(d*x+c)^3/cos(d*x+c)^2+1/16*sin(d*x+c)
-1/16*ln(sec(d*x+c)+tan(d*x+c)))+10*b^3*a^2*(1/7*sin(d*x+c)^4/cos(d*x+c)^7+
3/35*sin(d*x+c)^4/cos(d*x+c)^5+1/35*sin(d*x+c)^4/cos(d*x+c)^3-1/35*sin(d*x+
c)^4/cos(d*x+c)-1/35*(2+sin(d*x+c)^2)*cos(d*x+c))+5*a*b^4*(1/8*sin(d*x+c)^5
/cos(d*x+c)^8+1/16*sin(d*x+c)^5/cos(d*x+c)^6+1/64*sin(d*x+c)^5/cos(d*x+c)^4
-1/128*sin(d*x+c)^5/cos(d*x+c)^2-1/128*sin(d*x+c)^3-3/128*sin(d*x+c)+3/128*
ln(sec(d*x+c)+tan(d*x+c)))+b^5*(1/9*sin(d*x+c)^6/cos(d*x+c)^9+1/21*sin(d*x+
c)^6/cos(d*x+c)^7+1/105*sin(d*x+c)^6/cos(d*x+c)^5-1/315*sin(d*x+c)^6/cos(d*
x+c)^3+1/105*sin(d*x+c)^6/cos(d*x+c)+1/105*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)
^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.28, size = 360, normalized size = 0.92

$$-\frac{1575\,ab^4\left(\frac{2\left(3\,\sin(dx+c)^7-11\,\sin(dx+c)^5-11\,\sin(dx+c)^3+3\,\sin(dx+c)\right)}{\sin(dx+c)^8-4\,\sin(dx+c)^6+6\,\sin(dx+c)^4-4\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-8400\,a^3b^2\left(\frac{2\left(3\,\sin(dx+c)^5-8\,\sin(dx+c)^3-3\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)+5040\,a^5\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-\frac{80640\,a^4b}{\cos(dx+c)^5}+\frac{23040\left(7\,\cos(dx+c)^2-5\right)a^2b^3}{\cos(dx+c)^7}-\frac{256\left(63\,\cos(dx+c)^4-90\,\cos(dx+c)^2+35\right)b^5}{\cos(dx+c)^9}}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima"
)
```

```
[Out] -1/80640*(1575*a*b^4*(2*(3*sin(d*x + c)^7 - 11*sin(d*x + c)^5 - 11*sin(d*x
 + c)^3 + 3*sin(d*x + c))/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c
)^4 - 4*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
 - 1)) - 8400*a^3*b^2*(2*(3*sin(d*x + c)^5 - 8*sin(d*x + c)^3 - 3*sin(d*x +
c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 3*log(sin(
d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) + 5040*a^5*(2*(3*sin(d*x + c)^3 -
5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c
) + 1) + 3*log(sin(d*x + c) - 1)) - 80640*a^4*b/cos(d*x + c)^5 + 23040*(7*c
os(d*x + c)^2 - 5)*a^2*b^3/cos(d*x + c)^7 - 256*(63*cos(d*x + c)^4 - 90*cos
(d*x + c)^2 + 35)*b^5/cos(d*x + c)^9)/d
```

Fricas [A]
time = 2.42, size = 257, normalized size = 0.66

$$\frac{315\,(48\,a^5-80\,a^3b^2+15\,ab^4)\cos\left(dx+c\right)^9\log\left(\sin\left(dx+c\right)+1\right)-315\,(48\,a^5-80\,a^3b^2+15\,ab^4)\cos\left(dx+c\right)^9\log\left(-\sin\left(dx+c\right)+1\right)+8960\,b^5+16128\,(5\,a^4b-10\,a^2b^3+b^5)\cos\left(dx+c\right)^4+23040\,(5\,a^2b^3-b^5)\cos\left(dx+c\right)^2+210\left(3\,(48\,a^5-80\,a^3b^2+15\,ab^4)\cos\left(dx+c\right)^7+240\,ab^4\cos\left(dx+c\right)+2\,(48\,a^5-80\,a^3b^2+15\,ab^4)\cos\left(dx+c\right)^5+40\,(16\,a^3b^2-9\,ab^4)\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)}{80640\,d\cos\left(dx+c\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas"
)
```

```
[Out] 1/80640*(315*(48*a^5 - 80*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^9*log(sin(d*x +
c) + 1) - 315*(48*a^5 - 80*a^3*b^2 + 15*a*b^4)*cos(d*x + c)^9*log(-sin(d*x
 + c) + 1) + 8960*b^5 + 16128*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 +
23040*(5*a^2*b^3 - b^5)*cos(d*x + c)^2 + 210*(3*(48*a^5 - 80*a^3*b^2 + 15*a
*b^4)*cos(d*x + c)^7 + 240*a*b^4*cos(d*x + c) + 2*(48*a^5 - 80*a^3*b^2 + 15
```

```
*a*b^4)*cos(d*x + c)^5 + 40*(16*a^3*b^2 - 9*a*b^4)*cos(d*x + c)^3)*sin(d*x
+ c))/(d*cos(d*x + c)^9)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**10*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 888 vs. 2(359) = 718.
time = 0.76, size = 888, normalized size = 2.27

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^10*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
[Out] 1/40320*(315*(48*a^5 - 80*a^3*b^2 + 15*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c)
+ 1)) - 315*(48*a^5 - 80*a^3*b^2 + 15*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) -
 1)) + 2*(25200*a^5*tan(1/2*d*x + 1/2*c)^17 + 25200*a^3*b^2*tan(1/2*d*x + 1
/2*c)^17 - 4725*a*b^4*tan(1/2*d*x + 1/2*c)^17 - 201600*a^4*b*tan(1/2*d*x +
1/2*c)^16 - 110880*a^5*tan(1/2*d*x + 1/2*c)^15 + 319200*a^3*b^2*tan(1/2*d*x
 + 1/2*c)^15 + 40950*a*b^4*tan(1/2*d*x + 1/2*c)^15 + 806400*a^4*b*tan(1/2*d
*x + 1/2*c)^14 - 806400*a^2*b^3*tan(1/2*d*x + 1/2*c)^14 + 191520*a^5*tan(1/
2*d*x + 1/2*c)^13 - 453600*a^3*b^2*tan(1/2*d*x + 1/2*c)^13 + 488250*a*b^4*t
an(1/2*d*x + 1/2*c)^13 - 1612800*a^4*b*tan(1/2*d*x + 1/2*c)^12 + 806400*a^2
*b^3*tan(1/2*d*x + 1/2*c)^12 - 215040*b^5*tan(1/2*d*x + 1/2*c)^12 - 151200*
a^5*tan(1/2*d*x + 1/2*c)^11 - 151200*a^3*b^2*tan(1/2*d*x + 1/2*c)^11 + 5323
50*a*b^4*tan(1/2*d*x + 1/2*c)^11 + 2419200*a^4*b*tan(1/2*d*x + 1/2*c)^10 -
806400*a^2*b^3*tan(1/2*d*x + 1/2*c)^10 - 322560*b^5*tan(1/2*d*x + 1/2*c)^10
 - 2661120*a^4*b*tan(1/2*d*x + 1/2*c)^8 + 2096640*a^2*b^3*tan(1/2*d*x + 1/2
*c)^8 - 451584*b^5*tan(1/2*d*x + 1/2*c)^8 + 151200*a^5*tan(1/2*d*x + 1/2*c)
^7 + 151200*a^3*b^2*tan(1/2*d*x + 1/2*c)^7 - 532350*a*b^4*tan(1/2*d*x + 1/2
*c)^7 + 1774080*a^4*b*tan(1/2*d*x + 1/2*c)^6 - 1128960*a^2*b^3*tan(1/2*d*x
+ 1/2*c)^6 - 129024*b^5*tan(1/2*d*x + 1/2*c)^6 - 191520*a^5*tan(1/2*d*x + 1
/2*c)^5 + 453600*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 488250*a*b^4*tan(1/2*d*x
+ 1/2*c)^5 - 645120*a^4*b*tan(1/2*d*x + 1/2*c)^4 + 23040*a^2*b^3*tan(1/2*d*
x + 1/2*c)^4 - 36864*b^5*tan(1/2*d*x + 1/2*c)^4 + 110880*a^5*tan(1/2*d*x +
1/2*c)^3 - 319200*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 40950*a*b^4*tan(1/2*d*x
+ 1/2*c)^3 + 161280*a^4*b*tan(1/2*d*x + 1/2*c)^2 - 207360*a^2*b^3*tan(1/2*d
*x + 1/2*c)^2 + 9216*b^5*tan(1/2*d*x + 1/2*c)^2 - 25200*a^5*tan(1/2*d*x + 1
```

```
/2*c) - 25200*a^3*b^2*tan(1/2*d*x + 1/2*c) + 4725*a*b^4*tan(1/2*d*x + 1/2*c
) - 40320*a^4*b + 23040*a^2*b^3 - 1024*b^5)/(tan(1/2*d*x + 1/2*c)^2 - 1)^9)
/d
```

Mupad [B]
time = 4.71, size = 675, normalized size = 1.73

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{3\,a^5}{4}-\frac{5\,a^3\,b^2}{4}+\frac{15\,a\,b^4}{64}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{5\,a^5}{4}+\frac{5\,a^3\,b^2}{4}-\frac{15\,a\,b^4}{64}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}\,\left(40\,a^4\,b-40\,a^2\,b^3\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{17}\,\left(\frac{5\,a^5}{4}+\frac{5\,a^3\,b^2}{4}-\frac{15\,a\,b^4}{64}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(-\frac{11\,a^5}{2}+\frac{95\,a^3\,b^2}{6}+\frac{65\,a\,b^4}{32}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{15}\,\left(-\frac{11\,a^5}{2}+\frac{95\,a^3\,b^2}{6}+\frac{65\,a\,b^4}{32}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(\frac{19\,a^5}{2}-\frac{45\,a^3\,b^2}{2}+\frac{775\,a\,b^4}{32}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{13}\,\left(\frac{19\,a^5}{2}-\frac{45\,a^3\,b^2}{2}+\frac{775\,a\,b^4}{32}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(\frac{15\,a^5}{2}+\frac{15\,a^3\,b^2}{2}-\frac{845\,a\,b^4}{32}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{11}\,\left(\frac{15\,a^5}{2}+\frac{15\,a^3\,b^2}{2}-\frac{845\,a\,b^4}{32}\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(8\,a^4\,b-\frac{72\,a^2\,b^3}{7}+\frac{16\,b^5}{35}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(32\,a^4\,b-\frac{8\,a^2\,b^3}{7}+\frac{64\,b^5}{35}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}\,\left(80\,a^4\,b-40\,a^2\,b^3+\frac{32\,b^5}{3}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}\,\left(-120\,a^4\,b+40\,a^2\,b^3+16\,b^5\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(-88\,a^4\,b+56\,a^2\,b^3+\frac{32\,b^5}{5}\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,\left(132\,a^4\,b-104\,a^2\,b^3+\frac{112\,b^5}{5}\right)+2\,a^4\,b+\frac{16\,b^5}{315}-\frac{8\,a^2\,b^3}{7}+10\,a^4\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{16}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{18}-9\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{16}+36\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}-84\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+126\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-126\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+84\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-36\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+9\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^10,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*((15*a*b^4)/64 + (3*a^5)/4 - (5*a^3*b^2)/4))/d -
 (tan(c/2 + (d*x)/2)*((5*a^5)/4 - (15*a*b^4)/64 + (5*a^3*b^2)/4) - tan(c/2
+ (d*x)/2)^14*(40*a^4*b - 40*a^2*b^3) - tan(c/2 + (d*x)/2)^17*((5*a^5)/4 -
(15*a*b^4)/64 + (5*a^3*b^2)/4) + tan(c/2 + (d*x)/2)^3*((65*a*b^4)/32 - (11*
a^5)/2 + (95*a^3*b^2)/6) - tan(c/2 + (d*x)/2)^15*((65*a*b^4)/32 - (11*a^5)/
2 + (95*a^3*b^2)/6) + tan(c/2 + (d*x)/2)^5*((775*a*b^4)/32 + (19*a^5)/2 - (
45*a^3*b^2)/2) - tan(c/2 + (d*x)/2)^13*((775*a*b^4)/32 + (19*a^5)/2 - (45*a
^3*b^2)/2) - tan(c/2 + (d*x)/2)^7*((15*a^5)/2 - (845*a*b^4)/32 + (15*a^3*b^
2)/2) + tan(c/2 + (d*x)/2)^11*((15*a^5)/2 - (845*a*b^4)/32 + (15*a^3*b^2)/2
) - tan(c/2 + (d*x)/2)^2*(8*a^4*b + (16*b^5)/35 - (72*a^2*b^3)/7) + tan(c/2
 + (d*x)/2)^4*(32*a^4*b + (64*b^5)/35 - (8*a^2*b^3)/7) + tan(c/2 + (d*x)/2)
^12*(80*a^4*b + (32*b^5)/3 - 40*a^2*b^3) + tan(c/2 + (d*x)/2)^10*(16*b^5 -
120*a^4*b + 40*a^2*b^3) + tan(c/2 + (d*x)/2)^6*((32*b^5)/5 - 88*a^4*b + 56*
a^2*b^3) + tan(c/2 + (d*x)/2)^8*(132*a^4*b + (112*b^5)/5 - 104*a^2*b^3) + 2
*a^4*b + (16*b^5)/315 - (8*a^2*b^3)/7 + 10*a^4*b*tan(c/2 + (d*x)/2)^16)/(d*
(9*tan(c/2 + (d*x)/2)^2 - 36*tan(c/2 + (d*x)/2)^4 + 84*tan(c/2 + (d*x)/2)^6
 - 126*tan(c/2 + (d*x)/2)^8 + 126*tan(c/2 + (d*x)/2)^10 - 84*tan(c/2 + (d*x
)/2)^12 + 36*tan(c/2 + (d*x)/2)^14 - 9*tan(c/2 + (d*x)/2)^16 + tan(c/2 + (d
*x)/2)^18 - 1))
```

3.108 $\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=242

$$\frac{a^5\tan(c+dx)}{d}+\frac{5a^4b\tan^2(c+dx)}{2d}+\frac{2a^3(a^2+5b^2)\tan^3(c+dx)}{3d}+\frac{5a^2b(a^2+b^2)\tan^4(c+dx)}{2d}+\frac{a(a^4+20a^2b^2+\cdots}{\cdots}$$

```
[Out] a^5*tan(d*x+c)/d+5/2*a^4*b*tan(d*x+c)^2/d+2/3*a^3*(a^2+5*b^2)*tan(d*x+c)^3/
d+5/2*a^2*b*(a^2+b^2)*tan(d*x+c)^4/d+1/5*a*(a^4+20*a^2*b^2+5*b^4)*tan(d*x+c
)^5/d+1/6*b*(5*a^4+20*a^2*b^2+b^4)*tan(d*x+c)^6/d+10/7*a*b^2*(a^2+b^2)*tan(
d*x+c)^7/d+1/4*b^3*(5*a^2+b^2)*tan(d*x+c)^8/d+5/9*a*b^4*tan(d*x+c)^9/d+1/10
*b^5*tan(d*x+c)^10/d
```

Rubi [A]
time = 0.15, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3167, 962}

$$\frac{a^5\tan(c+dx)}{d}+\frac{5a^4b\tan^2(c+dx)}{2d}+\frac{10ab^2(a^2+b^2)\tan^7(c+dx)}{7d}+\frac{5a^2b(a^2+b^2)\tan^4(c+dx)}{2d}+\frac{b^3(5a^2+b^2)\tan^8(c+dx)}{4d}+\frac{b(5a^4+20a^2b^2+b^4)\tan^6(c+dx)}{6d}+\frac{a(a^4+20a^2b^2+5b^4)\tan^5(c+dx)}{5d}+\frac{2a^3(a^2+5b^2)\tan^3(c+dx)}{3d}+\frac{5ab^4\tan^9(c+dx)}{9d}+\frac{b^5\tan^{10}(c+dx)}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^11*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (a^5*Tan[c + d*x])/d + (5*a^4*b*Tan[c + d*x]^2)/(2*d) + (2*a^3*(a^2 + 5*b^2
)*Tan[c + d*x]^3)/(3*d) + (5*a^2*b*(a^2 + b^2)*Tan[c + d*x]^4)/(2*d) + (a*(
a^4 + 20*a^2*b^2 + 5*b^4)*Tan[c + d*x]^5)/(5*d) + (b*(5*a^4 + 20*a^2*b^2 +
b^4)*Tan[c + d*x]^6)/(6*d) + (10*a*b^2*(a^2 + b^2)*Tan[c + d*x]^7)/(7*d) +
(b^3*(5*a^2 + b^2)*Tan[c + d*x]^8)/(4*d) + (5*a*b^4*Tan[c + d*x]^9)/(9*d) +
 (b^5*Tan[c + d*x]^10)/(10*d)
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \sec^{11}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = -\frac{\text{Subst}\left(\int \frac{(b+ax)^5(1+x^2)^2}{x^{11}}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{b^5}{x^{11}} + \frac{5ab^4}{x^{10}} + \frac{2(5a^2b^3+b^5)}{x^9} + \frac{10ab^2(a^2+b^2)}{x^8} + \frac{5a}{}\right.\right.}{}$$

$$= \frac{a^5\tan(c+dx)}{d} + \frac{5a^4b\tan^2(c+dx)}{2d} + \frac{2a^3(a^2+5b^2)\,\mathrm{t}}{3d}$$

Mathematica [A]
time = 1.23, size = 115, normalized size = 0.48

$$\frac{\frac{1}{6}(a^2+b^2)^2(a+b\tan(c+dx))^6 - \frac{4}{7}a(a^2+b^2)(a+b\tan(c+dx))^7 + \frac{1}{4}(3a^2+b^2)(a+b\tan(c+dx))^8 - \frac{4}{9}a(a+b\tan(c+dx))^9 + \frac{1}{10}(a+b\tan(c+dx))^{10}}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^11*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (((a^2 + b^2)^2*(a + b*Tan[c + d*x])^6)/6 - (4*a*(a^2 + b^2)*(a + b*Tan[c +
 d*x])^7)/7 + ((3*a^2 + b^2)*(a + b*Tan[c + d*x])^8)/4 - (4*a*(a + b*Tan[c
+ d*x])^9)/9 + (a + b*Tan[c + d*x])^10/10)/(b^5*d)
```

Maple [A]
time = 0.42, size = 299, normalized size = 1.24

method	result
derivativedivides	$\frac{-a^5\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{5b\,a^4}{6\cos(dx+c)^6}+10a^3b^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx-\right.}{105\cos(dx-}\right.}{}$
default	$\frac{-a^5\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right)}{15}\right)\tan(dx+c)+\frac{5b\,a^4}{6\cos(dx+c)^6}+10a^3b^2\left(\frac{\sin^3(dx+c)}{7\cos(dx+c)^7}+\frac{4\left(\sin^3(dx+c)\right)}{35\cos(dx+c)^5}+\frac{8\left(\sin^3(dx-\right.}{105\cos(dx-}\right.}{}$
risch	$\frac{-\frac{320a^2b^3e^{14i(dx+c)}}{3}+\frac{160a^4b\,e^{14i(dx+c)}}{3}+\frac{32ia^5e^{14i(dx+c)}}{3}+\frac{160ia\,b^4e^{14i(dx+c)}}{3}-\frac{320ia^3b^2e^{14i(dx+c)}}{3}-192ia^3b^2e^{10i(dx+c)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^5*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+5/6*b*a^4/c
os(d*x+c)^6+10*a^3*b^2*(1/7*sin(d*x+c)^3/cos(d*x+c)^7+4/35*sin(d*x+c)^3/cos
(d*x+c)^5+8/105*sin(d*x+c)^3/cos(d*x+c)^3)+10*b^3*a^2*(1/8*sin(d*x+c)^4/cos
(d*x+c)^8+1/12*sin(d*x+c)^4/cos(d*x+c)^6+1/24*sin(d*x+c)^4/cos(d*x+c)^4)+5*
a*b^4*(1/9*sin(d*x+c)^5/cos(d*x+c)^9+4/63*sin(d*x+c)^5/cos(d*x+c)^7+8/315*s
```

```
in(d*x+c)^5/cos(d*x+c)^5)+b^5*(1/10*sin(d*x+c)^6/cos(d*x+c)^10+1/20*sin(d*x
+c)^6/cos(d*x+c)^8+1/60*sin(d*x+c)^6/cos(d*x+c)^6))
```

Maxima [A]
time = 0.30, size = 275, normalized size = 1.14

$$\frac{84\,\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)a^{5}+120\,\left(15\,\tan\left(dx+c\right)^{7}+42\,\tan\left(dx+c\right)^{5}+35\,\tan\left(dx+c\right)^{3}\right)a^{3}b^{2}+20\,\left(35\,\tan\left(dx+c\right)^{9}+90\,\tan\left(dx+c\right)^{7}+63\,\tan\left(dx+c\right)^{5}\right)ab^{4}+\frac{525\,\left(4\,\sin\left(dx+c\right)^{2}-1\right)a^{2}b^{3}}{\sin\left(dx+c\right)^{8}-4\,\sin\left(dx+c\right)^{6}+6\,\sin\left(dx+c\right)^{4}-4\,\sin\left(dx+c\right)^{2}+1}-\frac{21\,\left(10\,\sin\left(dx+c\right)^{4}-5\,\sin\left(dx+c\right)^{2}+1\right)b^{5}}{\sin\left(dx+c\right)^{10}-5\,\sin\left(dx+c\right)^{8}+10\,\sin\left(dx+c\right)^{6}-10\,\sin\left(dx+c\right)^{4}+5\,\sin\left(dx+c\right)^{2}-1}-\frac{1050\,a^{4}b}{\left(\sin\left(dx+c\right)^{2}-1\right)^{3}}}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima"
)
```

```
[Out] 1/1260*(84*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*a^5 + 1
20*(15*tan(d*x + c)^7 + 42*tan(d*x + c)^5 + 35*tan(d*x + c)^3)*a^3*b^2 + 20
*(35*tan(d*x + c)^9 + 90*tan(d*x + c)^7 + 63*tan(d*x + c)^5)*a*b^4 + 525*(4
*sin(d*x + c)^2 - 1)*a^2*b^3/(sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x
 + c)^4 - 4*sin(d*x + c)^2 + 1) - 21*(10*sin(d*x + c)^4 - 5*sin(d*x + c)^2
+ 1)*b^5/(sin(d*x + c)^10 - 5*sin(d*x + c)^8 + 10*sin(d*x + c)^6 - 10*sin(d
*x + c)^4 + 5*sin(d*x + c)^2 - 1) - 1050*a^4*b/(sin(d*x + c)^2 - 1)^3)/d
```

Fricas [A]
time = 3.01, size = 207, normalized size = 0.86

$$\frac{126\,b^{5}+210\,\left(5\,a^{4}b-10\,a^{2}b^{3}+b^{5}\right)\cos\left(dx+c\right)^{4}+315\,\left(5\,a^{2}b^{3}-b^{5}\right)\cos\left(dx+c\right)^{2}+4\,\left(8\,\left(21\,a^{5}-30\,a^{3}b^{2}+5\,ab^{4}\right)\cos\left(dx+c\right)^{9}+4\,\left(21\,a^{5}-30\,a^{3}b^{2}+5\,ab^{4}\right)\cos\left(dx+c\right)^{7}+175\,ab^{4}\cos\left(dx+c\right)+3\,\left(21\,a^{5}-30\,a^{3}b^{2}+5\,ab^{4}\right)\cos\left(dx+c\right)^{5}+50\,\left(9\,a^{3}b^{2}-5\,ab^{4}\right)\cos\left(dx+c\right)^{3}\right)\sin\left(dx+c\right)}{1260\,d\cos\left(dx+c\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas"
)
```

```
[Out] 1/1260*(126*b^5 + 210*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^4 + 315*(5*
a^2*b^3 - b^5)*cos(d*x + c)^2 + 4*(8*(21*a^5 - 30*a^3*b^2 + 5*a*b^4)*cos(d*
x + c)^9 + 4*(21*a^5 - 30*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^7 + 175*a*b^4*cos
(d*x + c) + 3*(21*a^5 - 30*a^3*b^2 + 5*a*b^4)*cos(d*x + c)^5 + 50*(9*a^3*b^
2 - 5*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x + c)^10)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**11*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.72, size = 262, normalized size = 1.08

$$\frac{126\,b^{5}\tan\left(dx+c\right)^{10}+700\,ab^{4}\tan\left(dx+c\right)^{9}+1575\,a^{2}b^{3}\tan\left(dx+c\right)^{8}+315\,b^{5}\tan\left(dx+c\right)^{8}+1800\,a^{3}b^{2}\tan\left(dx+c\right)^{7}+1800\,ab^{4}\tan\left(dx+c\right)^{7}+1050\,a^{4}b\tan\left(dx+c\right)^{6}+4200\,a^{2}b^{3}\tan\left(dx+c\right)^{6}+210\,b^{5}\tan\left(dx+c\right)^{6}+252\,a^{5}\tan\left(dx+c\right)^{5}+5040\,a^{3}b^{2}\tan\left(dx+c\right)^{5}+1260\,ab^{4}\tan\left(dx+c\right)^{5}+3150\,a^{4}b\tan\left(dx+c\right)^{4}+3150\,a^{2}b^{3}\tan\left(dx+c\right)^{4}+840\,a^{5}\tan\left(dx+c\right)^{3}+4200\,a^{3}b^{2}\tan\left(dx+c\right)^{3}+3150\,a^{4}b\tan\left(dx+c\right)^{2}+1260\,a^{5}\tan\left(dx+c\right)}{1260\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^11*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/1260*(126*b^5*tan(d*x + c)^10 + 700*a*b^4*tan(d*x + c)^9 + 1575*a^2*b^3*t
an(d*x + c)^8 + 315*b^5*tan(d*x + c)^8 + 1800*a^3*b^2*tan(d*x + c)^7 + 1800
*a*b^4*tan(d*x + c)^7 + 1050*a^4*b*tan(d*x + c)^6 + 4200*a^2*b^3*tan(d*x +
c)^6 + 210*b^5*tan(d*x + c)^6 + 252*a^5*tan(d*x + c)^5 + 5040*a^3*b^2*tan(d
*x + c)^5 + 1260*a*b^4*tan(d*x + c)^5 + 3150*a^4*b*tan(d*x + c)^4 + 3150*a^
2*b^3*tan(d*x + c)^4 + 840*a^5*tan(d*x + c)^3 + 4200*a^3*b^2*tan(d*x + c)^3
 + 3150*a^4*b*tan(d*x + c)^2 + 1260*a^5*tan(d*x + c))/d
```

Mupad [B]
time = 4.44, size = 548, normalized size = 2.26

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(\frac{616\,a^5}{15}-\frac{176\,a^3\,b^2}{3}+32\,a\,b^4\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(40\,a^4\,b-40\,a^2\,b^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}\left(40\,a^4\,b-40\,a^2\,b^3\right)-2\,a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{19}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}\left(\frac{616\,a^5}{15}-\frac{176\,a^3\,b^2}{3}+32\,a\,b^4\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\left(-88\,a^5+\frac{720\,a^3\,b^2}{7}+\frac{160\,a\,b^4}{7}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}\left(-88\,a^5+\frac{720\,a^3\,b^2}{7}+\frac{160\,a\,b^4}{7}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\left(\frac{388\,a^5}{3}-\frac{4240\,a^3\,b^2}{21}+\frac{3520\,a\,b^4}{63}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\left(\frac{388\,a^5}{3}-\frac{4240\,a^3\,b^2}{21}+\frac{3520\,a\,b^4}{63}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(\frac{280\,a^4\,b}{3}-\frac{80\,a^2\,b^3}{3}+\frac{32\,b^5}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}\left(\frac{280\,a^4\,b}{3}-\frac{80\,a^2\,b^3}{3}+\frac{32\,b^5}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\left(220\,a^4\,b-160\,a^2\,b^3+\frac{192\,b^5}{5}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\left(-\frac{520\,a^4\,b}{3}+\frac{200\,a^2\,b^3}{3}+\frac{64\,b^5}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}\left(-\frac{520\,a^4\,b}{3}+\frac{200\,a^2\,b^3}{3}+\frac{64\,b^5}{3}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(\frac{38\,a^5}{3}-\frac{80\,a^3\,b^2}{3}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}\left(\frac{38\,a^5}{3}-\frac{80\,a^3\,b^2}{3}\right)+2\,a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+10\,a^4\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+10\,a^4\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^11,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*(32*a*b^4 + (616*a^5)/15 - (176*a^3*b^2)/3) - tan(c/2
 + (d*x)/2)^4*(40*a^4*b - 40*a^2*b^3) - tan(c/2 + (d*x)/2)^16*(40*a^4*b - 4
0*a^2*b^3) - 2*a^5*tan(c/2 + (d*x)/2)^19 - tan(c/2 + (d*x)/2)^15*(32*a*b^4
+ (616*a^5)/15 - (176*a^3*b^2)/3) + tan(c/2 + (d*x)/2)^7*((160*a*b^4)/7 - 8
8*a^5 + (720*a^3*b^2)/7) - tan(c/2 + (d*x)/2)^13*((160*a*b^4)/7 - 88*a^5 +
(720*a^3*b^2)/7) + tan(c/2 + (d*x)/2)^9*((3520*a*b^4)/63 + (388*a^5)/3 - (4
240*a^3*b^2)/21) - tan(c/2 + (d*x)/2)^11*((3520*a*b^4)/63 + (388*a^5)/3 - (
4240*a^3*b^2)/21) + tan(c/2 + (d*x)/2)^6*((280*a^4*b)/3 + (32*b^5)/3 - (80*
a^2*b^3)/3) + tan(c/2 + (d*x)/2)^14*((280*a^4*b)/3 + (32*b^5)/3 - (80*a^2*b
^3)/3) + tan(c/2 + (d*x)/2)^10*(220*a^4*b + (192*b^5)/5 - 160*a^2*b^3) + ta
n(c/2 + (d*x)/2)^8*((64*b^5)/3 - (520*a^4*b)/3 + (200*a^2*b^3)/3) + tan(c/2
 + (d*x)/2)^12*((64*b^5)/3 - (520*a^4*b)/3 + (200*a^2*b^3)/3) - tan(c/2 + (
d*x)/2)^3*((38*a^5)/3 - (80*a^3*b^2)/3) + tan(c/2 + (d*x)/2)^17*((38*a^5)/3
 - (80*a^3*b^2)/3) + 2*a^5*tan(c/2 + (d*x)/2) + 10*a^4*b*tan(c/2 + (d*x)/2)
^2 + 10*a^4*b*tan(c/2 + (d*x)/2)^18)/(d*(tan(c/2 + (d*x)/2)^2 - 1)^10)
```

3.109 $\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx$

Optimal. Leaf size=472

$$\frac{5a^5\tanh^{-1}(\sin(c+dx))}{16d}-\frac{25a^3b^2\tanh^{-1}(\sin(c+dx))}{64d}+\frac{15ab^4\tanh^{-1}(\sin(c+dx))}{256d}+\frac{5a^4b\sec^7(c+dx)}{7d}-\frac{10a^2b}{}$$

```
[Out] 5/16*a^5*arctanh(sin(d*x+c))/d-25/64*a^3*b^2*arctanh(sin(d*x+c))/d+15/256*a
*b^4*arctanh(sin(d*x+c))/d+5/7*a^4*b*sec(d*x+c)^7/d-10/7*a^2*b^3*sec(d*x+c)
^7/d+1/7*b^5*sec(d*x+c)^7/d+10/9*a^2*b^3*sec(d*x+c)^9/d-2/9*b^5*sec(d*x+c)^
9/d+1/11*b^5*sec(d*x+c)^11/d+5/16*a^5*sec(d*x+c)*tan(d*x+c)/d-25/64*a^3*b^2
*sec(d*x+c)*tan(d*x+c)/d+15/256*a*b^4*sec(d*x+c)*tan(d*x+c)/d+5/24*a^5*sec(
d*x+c)^3*tan(d*x+c)/d-25/96*a^3*b^2*sec(d*x+c)^3*tan(d*x+c)/d+5/128*a*b^4*s
ec(d*x+c)^3*tan(d*x+c)/d+1/6*a^5*sec(d*x+c)^5*tan(d*x+c)/d-5/24*a^3*b^2*sec
(d*x+c)^5*tan(d*x+c)/d+1/32*a*b^4*sec(d*x+c)^5*tan(d*x+c)/d+5/4*a^3*b^2*sec
(d*x+c)^7*tan(d*x+c)/d-3/16*a*b^4*sec(d*x+c)^7*tan(d*x+c)/d+1/2*a*b^4*sec(d
*x+c)^7*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.31, antiderivative size = 472, normalized size of antiderivative = 1.00, number of steps used = 25, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3169, 3853, 3855, 2686, 30, 2691, 14, 276}

$$\frac{5a^5\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^5\tan(c+dx)\sec^5(c+dx)}{6d}+\frac{5a^5\tan(c+dx)\sec^3(c+dx)}{24d}+\frac{5a^5\tan(c+dx)\sec(c+dx)}{16d}+\frac{5a^4b\sec^7(c+dx)}{7d}-\frac{25a^3b^2\tanh^{-1}(\sin(c+dx))}{64d}+\frac{5a^3b^2\tan(c+dx)\sec^7(c+dx)}{4d}-\frac{5a^3b^2\tan(c+dx)\sec^5(c+dx)}{24d}-\frac{25a^3b^2\tan(c+dx)\sec^3(c+dx)}{96d}-\frac{25a^3b^2\tan(c+dx)\sec(c+dx)}{64d}+\frac{10a^2b^3\sec^9(c+dx)}{9d}-\frac{10a^2b^3\sec^7(c+dx)}{7d}+\frac{15ab^4\tanh^{-1}(\sin(c+dx))}{256d}+\frac{ab^4\tan^3(c+dx)\sec^7(c+dx)}{2d}-\frac{3ab^4\tan(c+dx)\sec^7(c+dx)}{16d}+\frac{ab^4\tan(c+dx)\sec^5(c+dx)}{32d}+\frac{5ab^4\tan(c+dx)\sec^3(c+dx)}{128d}+\frac{15ab^4\tan(c+dx)\sec(c+dx)}{256d}+\frac{b^5\sec^{11}(c+dx)}{11d}-\frac{2b^5\sec^9(c+dx)}{9d}+\frac{b^5\sec^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^12*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (5*a^5*ArcTanh[Sin[c + d*x]])/(16*d) - (25*a^3*b^2*ArcTanh[Sin[c + d*x]])/(
64*d) + (15*a*b^4*ArcTanh[Sin[c + d*x]])/(256*d) + (5*a^4*b*Sec[c + d*x]^7)
/(7*d) - (10*a^2*b^3*Sec[c + d*x]^7)/(7*d) + (b^5*Sec[c + d*x]^7)/(7*d) + (
10*a^2*b^3*Sec[c + d*x]^9)/(9*d) - (2*b^5*Sec[c + d*x]^9)/(9*d) + (b^5*Sec[
c + d*x]^11)/(11*d) + (5*a^5*Sec[c + d*x]*Tan[c + d*x])/(16*d) - (25*a^3*b^
2*Sec[c + d*x]*Tan[c + d*x])/(64*d) + (15*a*b^4*Sec[c + d*x]*Tan[c + d*x])/
(256*d) + (5*a^5*Sec[c + d*x]^3*Tan[c + d*x])/(24*d) - (25*a^3*b^2*Sec[c +
d*x]^3*Tan[c + d*x])/(96*d) + (5*a*b^4*Sec[c + d*x]^3*Tan[c + d*x])/(128*d)
 + (a^5*Sec[c + d*x]^5*Tan[c + d*x])/(6*d) - (5*a^3*b^2*Sec[c + d*x]^5*Tan[
c + d*x])/(24*d) + (a*b^4*Sec[c + d*x]^5*Tan[c + d*x])/(32*d) + (5*a^3*b^2*
Sec[c + d*x]^7*Tan[c + d*x])/(4*d) - (3*a*b^4*Sec[c + d*x]^7*Tan[c + d*x])/
(16*d) + (a*b^4*Sec[c + d*x]^7*Tan[c + d*x]^3)/(2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sec^{12}(c+dx)(a\cos(c+dx)+b\sin(c+dx))^5\,dx = \int \left(a^5\sec^7(c+dx)+5a^4b\sec^7(c+dx)\tan(c+dx)+1\ldots\right.$$

$$= a^5\int \sec^7(c+dx)\,dx + \left(5a^4b\right)\int \sec^7(c+dx)\tan(c+\ldots$$

$$= \frac{a^5\sec^5(c+dx)\tan(c+dx)}{6d} + \frac{5a^3b^2\sec^7(c+dx)\tan(c\ldots}{4d}$$

$$= \frac{5a^4b\sec^7(c+dx)}{7d} + \frac{5a^5\sec^3(c+dx)\tan(c+dx)}{24d} + \frac{a^5\ldots}{}$$

$$= \frac{5a^4b\sec^7(c+dx)}{7d} - \frac{10a^2b^3\sec^7(c+dx)}{7d} + \frac{b^5\sec^7(c+\ldots}{7d}$$

$$= \frac{5a^5\tanh^{-1}(\sin(c+dx))}{16d} + \frac{5a^4b\sec^7(c+dx)}{7d} - \frac{10a^2b^3\ldots}{}$$

$$= \frac{5a^5\tanh^{-1}(\sin(c+dx))}{16d} - \frac{25a^3b^2\tanh^{-1}(\sin(c+dx))}{64d}$$

$$= \frac{5a^5\tanh^{-1}(\sin(c+dx))}{16d} - \frac{25a^3b^2\tanh^{-1}(\sin(c+dx))}{64d}$$

Mathematica [A]
time = 1.93, size = 374, normalized size = 0.79

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^12*(a*Cos[c + d*x] + b*Sin[c + d*x])^5,x]
```

```
[Out] (-1774080*a*(16*a^4 - 20*a^2*b^2 + 3*b^4)*(Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c + d*x]^11*(243
30240*a^4*b + 1802240*a^2*b^3 + 3031040*b^5 + 3604480*(9*a^4*b - 4*a^2*b^3
- b^5)*Cos[2*(c + d*x)] + 1622016*(5*a^4*b - 10*a^2*b^3 + b^5)*Cos[4*(c + d
*x)] + 6623232*a^5*Sin[4*(c + d*x)] + 5913600*a^3*b^2*Sin[4*(c + d*x)] - 65
64096*a*b^4*Sin[4*(c + d*x)] + 2857008*a^5*Sin[6*(c + d*x)] - 3571260*a^3*b
^2*Sin[6*(c + d*x)] + 535689*a*b^4*Sin[6*(c + d*x)] + 591360*a^5*Sin[8*(c +
 d*x)] - 739200*a^3*b^2*Sin[8*(c + d*x)] + 110880*a*b^4*Sin[8*(c + d*x)] +
55440*a^5*Sin[10*(c + d*x)] - 69300*a^3*b^2*Sin[10*(c + d*x)] + 10395*a*b^4
*Sin[10*(c + d*x)]) + 13860*a*(976*a^4 + 2876*a^2*b^2 + 1207*b^4)*Sec[c + d
*x]^9*Tan[c + d*x])/(90832896*d)
```

Maple [A]
time = 0.56, size = 572, normalized size = 1.21 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^5*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x+c))*tan(d*x+c)
+5/16*ln(sec(d*x+c)+tan(d*x+c)))+5/7*b*a^4/cos(d*x+c)^7+10*a^3*b^2*(1/8*sin
(d*x+c)^3/cos(d*x+c)^8+5/48*sin(d*x+c)^3/cos(d*x+c)^6+5/64*sin(d*x+c)^3/cos
(d*x+c)^4+5/128*sin(d*x+c)^3/cos(d*x+c)^2+5/128*sin(d*x+c)-5/128*ln(sec(d*x
+c)+tan(d*x+c)))+10*b^3*a^2*(1/9*sin(d*x+c)^4/cos(d*x+c)^9+5/63*sin(d*x+c)^
4/cos(d*x+c)^7+1/21*sin(d*x+c)^4/cos(d*x+c)^5+1/63*sin(d*x+c)^4/cos(d*x+c)^
3-1/63*sin(d*x+c)^4/cos(d*x+c)-1/63*(2+sin(d*x+c)^2)*cos(d*x+c))+5*a*b^4*(1
/10*sin(d*x+c)^5/cos(d*x+c)^10+1/16*sin(d*x+c)^5/cos(d*x+c)^8+1/32*sin(d*x+
c)^5/cos(d*x+c)^6+1/128*sin(d*x+c)^5/cos(d*x+c)^4-1/256*sin(d*x+c)^5/cos(d*
x+c)^2-1/256*sin(d*x+c)^3-3/256*sin(d*x+c)+3/256*ln(sec(d*x+c)+tan(d*x+c)))
+b^5*(1/11*sin(d*x+c)^6/cos(d*x+c)^11+5/99*sin(d*x+c)^6/cos(d*x+c)^9+5/231*
sin(d*x+c)^6/cos(d*x+c)^7+1/231*sin(d*x+c)^6/cos(d*x+c)^5-1/693*sin(d*x+c)^
6/cos(d*x+c)^3+1/231*sin(d*x+c)^6/cos(d*x+c)+1/231*(8/3+sin(d*x+c)^4+4/3*si
n(d*x+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.27, size = 420, normalized size = 0.89

$$-\frac{693\,ab^4\left(\frac{2\left(15\,\sin(dx+c)^9-70\,\sin(dx+c)^7+128\,\sin(dx+c)^5+70\,\sin(dx+c)^3-15\,\sin(dx+c)\right)}{\sin(dx+c)^{10}-5\,\sin(dx+c)^8+10\,\sin(dx+c)^6-10\,\sin(dx+c)^4+5\,\sin(dx+c)^2-1}-15\,\log\left(\sin\left(dx+c\right)+1\right)+15\,\log\left(\sin\left(dx+c\right)-1\right)\right)-4620\,a^3b^2\left(\frac{2\left(15\,\sin(dx+c)^7-55\,\sin(dx+c)^5+73\,\sin(dx+c)^3+15\,\sin(dx+c)\right)}{\sin(dx+c)^8-4\,\sin(dx+c)^6+6\,\sin(dx+c)^4-4\,\sin(dx+c)^2+1}-15\,\log\left(\sin\left(dx+c\right)+1\right)+15\,\log\left(\sin\left(dx+c\right)-1\right)\right)+3696\,a^5\left(\frac{2\left(15\,\sin(dx+c)^5-40\,\sin(dx+c)^3+33\,\sin(dx+c)\right)}{\sin(dx+c)^6-3\,\sin(dx+c)^4+3\,\sin(dx+c)^2-1}-15\,\log\left(\sin\left(dx+c\right)+1\right)+15\,\log\left(\sin\left(dx+c\right)-1\right)\right)-\frac{253440\,a^4b}{\cos(dx+c)^7}+\frac{56320\left(9\,\cos(dx+c)^2-7\right)a^2b^3}{\cos(dx+c)^9}-\frac{512\left(99\,\cos(dx+c)^4-154\,\cos(dx+c)^2+63\right)b^5}{\cos(dx+c)^{11}}}{354816\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="maxima"
)
```

```
[Out] -1/354816*(693*a*b^4*(2*(15*sin(d*x + c)^9 - 70*sin(d*x + c)^7 + 128*sin(d*
x + c)^5 + 70*sin(d*x + c)^3 - 15*sin(d*x + c))/(sin(d*x + c)^10 - 5*sin(d*
x + c)^8 + 10*sin(d*x + c)^6 - 10*sin(d*x + c)^4 + 5*sin(d*x + c)^2 - 1) -
15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 4620*a^3*b^2*(2*(15*
sin(d*x + c)^7 - 55*sin(d*x + c)^5 + 73*sin(d*x + c)^3 + 15*sin(d*x + c))/(
sin(d*x + c)^8 - 4*sin(d*x + c)^6 + 6*sin(d*x + c)^4 - 4*sin(d*x + c)^2 + 1
) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 3696*a^5*(2*(15*
sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x + c))/(sin(d*x + c)^6 - 3*s
in(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(s
in(d*x + c) - 1)) - 253440*a^4*b/cos(d*x + c)^7 + 56320*(9*cos(d*x + c)^2 -
 7)*a^2*b^3/cos(d*x + c)^9 - 512*(99*cos(d*x + c)^4 - 154*cos(d*x + c)^2 +
63)*b^5/cos(d*x + c)^11)/d
```

Fricas [A]
time = 2.90, size = 287, normalized size = 0.61

$$\frac{3465\left(16\,a^5-20\,a^3b^2+3\,ab^4\right)\cos\left(dx+c\right)^{11}\log\left(\sin\left(dx+c\right)+1\right)-3465\left(16\,a^5-20\,a^3b^2+3\,ab^4\right)\cos\left(dx+c\right)^{11}\log\left(-\sin\left(dx+c\right)+1\right)+32256\,b^5+50688\left(5\,a^4b-10\,a^2b^3+b^5\right)\cos\left(dx+c\right)^4+78848\left(5\,a^2b^3-b^5\right)\cos\left(dx+c\right)^2+462\left(15\left(16\,a^5-20\,a^3b^2+3\,ab^4\right)\cos\left(dx+c\right)^9+10\left(16\,a^5-20\,a^3b^2+3\,ab^4\right)\cos\left(dx+c\right)^7+384\,ab^4\cos\left(dx+c\right)+8\left(16\,a^5-20\,a^3b^2+3\,ab^4\right)\cos\left(dx+c\right)^5+48\left(20\,a^3b^2-11\,ab^4\right)\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)}{354816\,d\cos\left(dx+c\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="fricas"
)
```

```
[Out] 1/354816*(3465*(16*a^5 - 20*a^3*b^2 + 3*a*b^4)*cos(d*x + c)^11*log(sin(d*x
+ c) + 1) - 3465*(16*a^5 - 20*a^3*b^2 + 3*a*b^4)*cos(d*x + c)^11*log(-sin(d
*x + c) + 1) + 32256*b^5 + 50688*(5*a^4*b - 10*a^2*b^3 + b^5)*cos(d*x + c)^
4 + 78848*(5*a^2*b^3 - b^5)*cos(d*x + c)^2 + 462*(15*(16*a^5 - 20*a^3*b^2 +
 3*a*b^4)*cos(d*x + c)^9 + 10*(16*a^5 - 20*a^3*b^2 + 3*a*b^4)*cos(d*x + c)^
7 + 384*a*b^4*cos(d*x + c) + 8*(16*a^5 - 20*a^3*b^2 + 3*a*b^4)*cos(d*x + c)
^5 + 48*(20*a^3*b^2 - 11*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(d*cos(d*x +
c)^11)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**12*(a*cos(d*x+c)+b*sin(d*x+c))**5,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1096 vs. 2(430) = 860.
time = 0.74, size = 1096, normalized size = 2.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^12*(a*cos(d*x+c)+b*sin(d*x+c))^5,x, algorithm="giac")
```

```
[Out] 1/177408*(3465*(16*a^5 - 20*a^3*b^2 + 3*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c)
 + 1)) - 3465*(16*a^5 - 20*a^3*b^2 + 3*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + 2*(121968*a^5*tan(1/2*d*x + 1/2*c)^21 + 69300*a^3*b^2*tan(1/2*d*x +
 1/2*c)^21 - 10395*a*b^4*tan(1/2*d*x + 1/2*c)^21 - 887040*a^4*b*tan(1/2*d*x
 + 1/2*c)^20 - 591360*a^5*tan(1/2*d*x + 1/2*c)^19 + 1626240*a^3*b^2*tan(1/2
*d*x + 1/2*c)^19 + 110880*a*b^4*tan(1/2*d*x + 1/2*c)^19 + 3548160*a^4*b*tan
(1/2*d*x + 1/2*c)^18 - 3548160*a^2*b^3*tan(1/2*d*x + 1/2*c)^18 + 1459920*a^
5*tan(1/2*d*x + 1/2*c)^17 - 1159620*a^3*b^2*tan(1/2*d*x + 1/2*c)^17 + 23028
39*a*b^4*tan(1/2*d*x + 1/2*c)^17 - 9757440*a^4*b*tan(1/2*d*x + 1/2*c)^16 +
1182720*a^2*b^3*tan(1/2*d*x + 1/2*c)^16 - 946176*b^5*tan(1/2*d*x + 1/2*c)^1
6 - 2365440*a^5*tan(1/2*d*x + 1/2*c)^15 + 1182720*a^3*b^2*tan(1/2*d*x + 1/2
*c)^15 + 4790016*a*b^4*tan(1/2*d*x + 1/2*c)^15 + 21288960*a^4*b*tan(1/2*d*x
 + 1/2*c)^14 - 9461760*a^2*b^3*tan(1/2*d*x + 1/2*c)^14 - 2365440*b^5*tan(1/
2*d*x + 1/2*c)^14 + 2106720*a^5*tan(1/2*d*x + 1/2*c)^13 - 5738040*a^3*b^2*t
an(1/2*d*x + 1/2*c)^13 + 5828130*a*b^4*tan(1/2*d*x + 1/2*c)^13 - 30159360*a
^4*b*tan(1/2*d*x + 1/2*c)^12 + 18923520*a^2*b^3*tan(1/2*d*x + 1/2*c)^12 - 5
203968*b^5*tan(1/2*d*x + 1/2*c)^12 + 28385280*a^4*b*tan(1/2*d*x + 1/2*c)^10
 - 7096320*a^2*b^3*tan(1/2*d*x + 1/2*c)^10 - 4257792*b^5*tan(1/2*d*x + 1/2*
```

```
c)^10 - 2106720*a^5*tan(1/2*d*x + 1/2*c)^9 + 5738040*a^3*b^2*tan(1/2*d*x +
1/2*c)^9 - 5828130*a*b^4*tan(1/2*d*x + 1/2*c)^9 - 20528640*a^4*b*tan(1/2*d*
x + 1/2*c)^8 + 9123840*a^2*b^3*tan(1/2*d*x + 1/2*c)^8 - 3041280*b^5*tan(1/2
*d*x + 1/2*c)^8 + 2365440*a^5*tan(1/2*d*x + 1/2*c)^7 - 1182720*a^3*b^2*tan(
1/2*d*x + 1/2*c)^7 - 4790016*a*b^4*tan(1/2*d*x + 1/2*c)^7 + 11151360*a^4*b*
tan(1/2*d*x + 1/2*c)^6 - 8110080*a^2*b^3*tan(1/2*d*x + 1/2*c)^6 - 608256*b^
5*tan(1/2*d*x + 1/2*c)^6 - 1459920*a^5*tan(1/2*d*x + 1/2*c)^5 + 1159620*a^3
*b^2*tan(1/2*d*x + 1/2*c)^5 - 2302839*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 342144
0*a^4*b*tan(1/2*d*x + 1/2*c)^4 - 450560*a^2*b^3*tan(1/2*d*x + 1/2*c)^4 - 11
2640*b^5*tan(1/2*d*x + 1/2*c)^4 + 591360*a^5*tan(1/2*d*x + 1/2*c)^3 - 16262
40*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 110880*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 5
06880*a^4*b*tan(1/2*d*x + 1/2*c)^2 - 619520*a^2*b^3*tan(1/2*d*x + 1/2*c)^2
+ 22528*b^5*tan(1/2*d*x + 1/2*c)^2 - 121968*a^5*tan(1/2*d*x + 1/2*c) - 6930
0*a^3*b^2*tan(1/2*d*x + 1/2*c) + 10395*a*b^4*tan(1/2*d*x + 1/2*c) - 126720*
a^4*b + 56320*a^2*b^3 - 2048*b^5)/(tan(1/2*d*x + 1/2*c)^2 - 1)^11)/d
```

Mupad [B]
time = 5.92, size = 831, normalized size = 1.76

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + b*sin(c + d*x))^5/cos(c + d*x)^12,x)
```

```
[Out] (5*a*atanh(tan(c/2 + (d*x)/2))*(16*a^4 + 3*b^4 - 20*a^2*b^2))/(128*d) - (ta
n(c/2 + (d*x)/2)*((11*a^5)/8 - (15*a*b^4)/128 + (25*a^3*b^2)/32) - tan(c/2
+ (d*x)/2)^18*(40*a^4*b - 40*a^2*b^3) + tan(c/2 + (d*x)/2)^3*((5*a*b^4)/4 -
 (20*a^5)/3 + (55*a^3*b^2)/3) - tan(c/2 + (d*x)/2)^19*((5*a*b^4)/4 - (20*a^
5)/3 + (55*a^3*b^2)/3) + tan(c/2 + (d*x)/2)^7*(54*a*b^4 - (80*a^5)/3 + (40*
a^3*b^2)/3) - tan(c/2 + (d*x)/2)^15*(54*a*b^4 - (80*a^5)/3 + (40*a^3*b^2)/3
) - tan(c/2 + (d*x)/2)^21*((11*a^5)/8 - (15*a*b^4)/128 + (25*a^3*b^2)/32) +
 tan(c/2 + (d*x)/2)^5*((3323*a*b^4)/128 + (395*a^5)/24 - (1255*a^3*b^2)/96)
 - tan(c/2 + (d*x)/2)^17*((3323*a*b^4)/128 + (395*a^5)/24 - (1255*a^3*b^2)/
96) + tan(c/2 + (d*x)/2)^9*((4205*a*b^4)/64 + (95*a^5)/4 - (1035*a^3*b^2)/1
6) - tan(c/2 + (d*x)/2)^13*((4205*a*b^4)/64 + (95*a^5)/4 - (1035*a^3*b^2)/1
6) + tan(c/2 + (d*x)/2)^16*(110*a^4*b + (32*b^5)/3 - (40*a^2*b^3)/3) + tan(
c/2 + (d*x)/2)^10*(48*b^5 - 320*a^4*b + 80*a^2*b^3) - tan(c/2 + (d*x)/2)^2*
((40*a^4*b)/7 + (16*b^5)/63 - (440*a^2*b^3)/63) + tan(c/2 + (d*x)/2)^14*((8
0*b^5)/3 - 240*a^4*b + (320*a^2*b^3)/3) + tan(c/2 + (d*x)/2)^4*((270*a^4*b)
/7 + (80*b^5)/63 + (320*a^2*b^3)/63) + tan(c/2 + (d*x)/2)^12*(340*a^4*b + (
176*b^5)/3 - (640*a^2*b^3)/3) + tan(c/2 + (d*x)/2)^6*((48*b^5)/7 - (880*a^4
*b)/7 + (640*a^2*b^3)/7) + tan(c/2 + (d*x)/2)^8*((1620*a^4*b)/7 + (240*b^5)
/7 - (720*a^2*b^3)/7) + (10*a^4*b)/7 + (16*b^5)/693 - (40*a^2*b^3)/63 + 10*
a^4*b*tan(c/2 + (d*x)/2)^20)/(d*(11*tan(c/2 + (d*x)/2)^2 - 55*tan(c/2 + (d*
x)/2)^4 + 165*tan(c/2 + (d*x)/2)^6 - 330*tan(c/2 + (d*x)/2)^8 + 462*tan(c/2
```

```
 + (d*x)/2)^10 - 462*tan(c/2 + (d*x)/2)^12 + 330*tan(c/2 + (d*x)/2)^14 - 16
5*tan(c/2 + (d*x)/2)^16 + 55*tan(c/2 + (d*x)/2)^18 - 11*tan(c/2 + (d*x)/2)^
20 + tan(c/2 + (d*x)/2)^22 - 1))
```

3.110 $\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=227

$$\frac{ab^4x}{\left(a^2+b^2\right)^3}+\frac{ab^2x}{2\left(a^2+b^2\right)^2}+\frac{3ax}{8\left(a^2+b^2\right)}+\frac{b^3\cos^2(c+dx)}{2\left(a^2+b^2\right)^2d}+\frac{b\cos^4(c+dx)}{4\left(a^2+b^2\right)d}+\frac{b^5\log(a\cos(c+dx)+b\sin(c+dx)}{\left(a^2+b^2\right)^3d}$$

```
[Out] a*b^4*x/(a^2+b^2)^3+1/2*a*b^2*x/(a^2+b^2)^2+3/8*a*x/(a^2+b^2)+1/2*b^3*cos(d
*x+c)^2/(a^2+b^2)^2/d+1/4*b*cos(d*x+c)^4/(a^2+b^2)/d+b^5*ln(a*cos(d*x+c)+b*
sin(d*x+c))/(a^2+b^2)^3/d+1/2*a*b^2*cos(d*x+c)*sin(d*x+c)/(a^2+b^2)^2/d+3/8
*a*cos(d*x+c)*sin(d*x+c)/(a^2+b^2)/d+1/4*a*cos(d*x+c)^3*sin(d*x+c)/(a^2+b^2
)/d
```

Rubi [A]

time = 0.16, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3179, 2715, 8, 3177, 3212}

$$\frac{b\cos^4(c+dx)}{4d\left(a^2+b^2\right)}+\frac{a\sin(c+dx)\cos^3(c+dx)}{4d\left(a^2+b^2\right)}+\frac{ab^2\sin(c+dx)\cos(c+dx)}{2d\left(a^2+b^2\right)^2}+\frac{3a\sin(c+dx)\cos(c+dx)}{8d\left(a^2+b^2\right)}+\frac{ab^2x}{2\left(a^2+b^2\right)^2}+\frac{3ax}{8\left(a^2+b^2\right)}+\frac{b^5\log(a\cos(c+dx)+b\sin(c+dx))}{d\left(a^2+b^2\right)^3}+\frac{ab^4x}{\left(a^2+b^2\right)^3}+\frac{b^3\cos^2(c+dx)}{2d\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (a*b^4*x)/(a^2 + b^2)^3 + (a*b^2*x)/(2*(a^2 + b^2)^2) + (3*a*x)/(8*(a^2 + b
^2)) + (b^3*Cos[c + d*x]^2)/(2*(a^2 + b^2)^2*d) + (b*Cos[c + d*x]^4)/(4*(a^
2 + b^2)*d) + (b^5*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^3*d)
+ (a*b^2*Cos[c + d*x]*Sin[c + d*x])/(2*(a^2 + b^2)^2*d) + (3*a*Cos[c + d*x]
*Sin[c + d*x])/(8*(a^2 + b^2)*d) + (a*Cos[c + d*x]^3*Sin[c + d*x])/(4*(a^2
+ b^2)*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3177

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
```

```
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3179

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{b\cos^4(c+dx)}{4\left(a^2+b^2\right)d} + \frac{a\int\cos^4(c+dx)\,dx}{a^2+b^2} + \frac{b^2\int\frac{\cos^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{b^3\cos^2(c+dx)}{2\left(a^2+b^2\right)^2 d} + \frac{b\cos^4(c+dx)}{4\left(a^2+b^2\right)d} + \frac{a\cos^3(c+dx)\sin(c+dx)}{4\left(a^2+b^2\right)d} + \frac{\left(ab^2\right) \ldots}{\ldots}$$

$$= \frac{ab^4x}{\left(a^2+b^2\right)^3} + \frac{b^3\cos^2(c+dx)}{2\left(a^2+b^2\right)^2 d} + \frac{b\cos^4(c+dx)}{4\left(a^2+b^2\right)d} + \frac{ab^2\cos(c+dx)\sin(c+\ldots}{2\left(a^2+b^2\right)^2 d}$$

$$= \frac{ab^4x}{\left(a^2+b^2\right)^3} + \frac{ab^2x}{2\left(a^2+b^2\right)^2} + \frac{3ax}{8\left(a^2+b^2\right)} + \frac{b^3\cos^2(c+dx)}{2\left(a^2+b^2\right)^2 d} + \frac{b\cos^4(c+\ldots}{4\left(a^2+b^2\ldots}$$

Mathematica [A]
time = 0.46, size = 218, normalized size = 0.96

$$\frac{12a^5c+40a^3b^2c+60ab^4c+12a^5dx+40a^3b^2dx+60ab^4dx+4b\left(a^4+4a^2b^2+3b^4\right)\cos(2(c+dx))+b\left(a^2+b^2\right)^2\cos(4(c+dx))+32b^5\log(a\cos(c+dx)+b\sin(c+dx))+8a^5\sin(2(c+dx))+24a^3b^2\sin(2(c+dx))+16ab^4\sin(2(c+dx))+a^5\sin(4(c+dx))+2a^3b^2\sin(4(c+dx))+ab^4\sin(4(c+dx))}{32\left(a^2+b^2\right)^3 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
[Out] (12*a^5*c + 40*a^3*b^2*c + 60*a*b^4*c + 12*a^5*d*x + 40*a^3*b^2*d*x + 60*a*
b^4*d*x + 4*b*(a^4 + 4*a^2*b^2 + 3*b^4)*Cos[2*(c + d*x)] + b*(a^2 + b^2)^2*
```

```
Cos[4*(c + d*x)] + 32*b^5*Log[a*Cos[c + d*x] + b*Sin[c + d*x]] + 8*a^5*Sin[
2*(c + d*x)] + 24*a^3*b^2*Sin[2*(c + d*x)] + 16*a*b^4*Sin[2*(c + d*x)] + a^
5*Sin[4*(c + d*x)] + 2*a^3*b^2*Sin[4*(c + d*x)] + a*b^4*Sin[4*(c + d*x)])/(
32*(a^2 + b^2)^3*d)
```

Maple [A]
time = 0.35, size = 197, normalized size = 0.87 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^5/(a^2+b^2)^3*ln(a+b*tan(d*x+c))+1/(a^2+b^2)^3*(((3/8*a^5+5/4*a^3*b^
2+7/8*a*b^4)*tan(d*x+c)^3+(1/2*b^3*a^2+1/2*b^5)*tan(d*x+c)^2+(7/4*a^3*b^2+9
/8*a*b^4+5/8*a^5)*tan(d*x+c)+1/4*b*a^4+b^3*a^2+3/4*b^5)/(tan(d*x+c)^2+1)^2-
1/2*b^5*ln(tan(d*x+c)^2+1)+1/8*(3*a^5+10*a^3*b^2+15*a*b^4)*arctan(tan(d*x+c
))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 564 vs. 2(213) = 426.
time = 0.49, size = 564, normalized size = 2.48

$$\frac{\frac{4\,b^5\log\left(-a-\frac{2\,b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{4\,b^5\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(3\,a^5+10\,a^3b^2+15\,ab^4\right)\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{\frac{16\,b^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{\left(5\,a^3+9\,ab^2\right)\sin(dx+c)}{\cos(dx+c)+1}+\frac{8\left(a^2b+2\,b^3\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\left(3\,a^3-ab^2\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{\left(3\,a^3-ab^2\right)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{8\left(a^2b+2\,b^3\right)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{\left(5\,a^3+9\,ab^2\right)\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4+2\,a^2b^2+b^4+\frac{4\left(a^4+2\,a^2b^2+b^4\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\left(a^4+2\,a^2b^2+b^4\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\left(a^4+2\,a^2b^2+b^4\right)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{\left(a^4+2\,a^2b^2+b^4\right)\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*b^5*log(-a - 2*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 4*b^5*log(sin(d
*x + c)^2/(cos(d*x + c) + 1)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (
3*a^5 + 10*a^3*b^2 + 15*a*b^4)*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (16*b^3*sin(d*x + c)^4/(cos(d*x + c) + 1)
^4 - (5*a^3 + 9*a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) + 8*(a^2*b + 2*b^3)*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + (3*a^3 - a*b^2)*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 - (3*a^3 - a*b^2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 8*(a
^2*b + 2*b^3)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + (5*a^3 + 9*a*b^2)*sin(d
*x + c)^7/(cos(d*x + c) + 1)^7)/(a^4 + 2*a^2*b^2 + b^4 + 4*(a^4 + 2*a^2*b^2
 + b^4)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*(a^4 + 2*a^2*b^2 + b^4)*sin
(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^6
/(cos(d*x + c) + 1)^6 + (a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^8/(cos(d*x + c
) + 1)^8))/d
```

Fricas [A]
time = 3.49, size = 208, normalized size = 0.92

$$\frac{4\,b^5\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)+2\left(a^4b+2\,a^2b^3+b^5\right)\cos\left(dx+c\right)^4+\left(3\,a^5+10\,a^3b^2+15\,ab^4\right)dx+4\left(a^2b^3+b^5\right)\cos\left(dx+c\right)^2+\left(2\left(a^5+2\,a^3b^2+ab^4\right)\cos\left(dx+c\right)^3+\left(3\,a^5+10\,a^3b^2+7\,ab^4\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{8\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*b^5*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
 + b^2) + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^4 + (3*a^5 + 10*a^3*b^2
+ 15*a*b^4)*d*x + 4*(a^2*b^3 + b^5)*cos(d*x + c)^2 + (2*(a^5 + 2*a^3*b^2 +
a*b^4)*cos(d*x + c)^3 + (3*a^5 + 10*a^3*b^2 + 7*a*b^4)*cos(d*x + c))*sin(d*
x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.44, size = 322, normalized size = 1.42

$$\frac{\frac{8\,b^{6}\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^{6}b+3\,a^{4}b^{3}+3\,a^{2}b^{5}+b^{7}}-\frac{4\,b^{5}\log\left(\tan\left(dx+c\right)^{2}+1\right)}{a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}}+\frac{\left(3\,a^{5}+10\,a^{3}b^{2}+15\,ab^{4}\right)\left(dx+c\right)}{a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}}+\frac{6\,b^{5}\tan\left(dx+c\right)^{4}+3\,a^{5}\tan\left(dx+c\right)^{3}+10\,a^{3}b^{2}\tan\left(dx+c\right)^{3}+7\,ab^{4}\tan\left(dx+c\right)^{3}+4\,a^{2}b^{3}\tan\left(dx+c\right)^{2}+16\,b^{5}\tan\left(dx+c\right)^{2}+5\,a^{5}\tan\left(dx+c\right)+14\,a^{3}b^{2}\tan\left(dx+c\right)+9\,ab^{4}\tan\left(dx+c\right)+2\,a^{4}b+8\,a^{2}b^{3}+12\,b^{5}}{\left(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}\right)\left(\tan\left(dx+c\right)^{2}+1\right)^{2}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(8*b^6*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^
7) - 4*b^5*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (3
*a^5 + 10*a^3*b^2 + 15*a*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
 + (6*b^5*tan(d*x + c)^4 + 3*a^5*tan(d*x + c)^3 + 10*a^3*b^2*tan(d*x + c)^3
 + 7*a*b^4*tan(d*x + c)^3 + 4*a^2*b^3*tan(d*x + c)^2 + 16*b^5*tan(d*x + c)^
2 + 5*a^5*tan(d*x + c) + 14*a^3*b^2*tan(d*x + c) + 9*a*b^4*tan(d*x + c) + 2
*a^4*b + 8*a^2*b^3 + 12*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(tan(d*x
+ c)^2 + 1)^2))/d
```

Mupad [**B**]
time = 11.16, size = 2500, normalized size = 11.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

```
[Out] (b^5*log(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^2))/(d*(a^6 + b^
6 + 3*a^2*b^4 + 3*a^4*b^2)) - (64*b^5*log(1/(cos(c + d*x) + 1)))/(d*(64*a^6
 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)) - ((4*b^3*tan(c/2 + (d*x)/2)^4)/(a^
```

4 + b^4 + 2*a^2*b^2) - (tan(c/2 + (d*x)/2)*(9*a*b^2 + 5*a^3))/(4*(a^4 + b^4 + 2*a^2*b^2)) - (tan(c/2 + (d*x)/2)^3*(a*b^2 - 3*a^3))/(4*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c/2 + (d*x)/2)^5*(a*b^2 - 3*a^3))/(4*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c/2 + (d*x)/2)^7*(9*a*b^2 + 5*a^3))/(4*(a^4 + b^4 + 2*a^2*b^2)) + (2*b*tan(c/2 + (d*x)/2)^2*(a^2 + 2*b^2))/(a^4 + b^4 + 2*a^2*b^2) + (2*b*tan(c/2 + (d*x)/2)^6*(a^2 + 2*b^2))/(a^4 + b^4 + 2*a^2*b^2))/(d*(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) - (a*atan((tan(c/2 + (d*x)/2)*((((64*b^5*((a*((64*a*b^15 + 48*a^15*b + 624*a^3*b^13 + 2016*a^5*b^11 + 3152*a^7*b^9 + 2688*a^9*b^7 + 1296*a^11*b^5 + 352*a^13*b^3)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (32*b^5*(192*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(3*a^4 + 15*b^4 + 10*a^2*b^2))/(8*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (4*a*b^5*(3*a^4 + 15*b^4 + 10*a^2*b^2)*(192*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2) - (a*((9*a^15 - 192*a*b^14 - 222*a^3*b^12 + 475*a^5*b^10 + 1089*a^7*b^8 + 894*a^9*b^6 + 388*a^11*b^4 + 87*a^13*b^2)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (64*b^5*((64*a*b^15 + 48*a^15*b + 624*a^3*b^13 + 2016*a^5*b^11 + 3152*a^7*b^9 + 2688*a^9*b^7 + 1296*a^11*b^5 + 352*a^13*b^3)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (32*b^5*(192*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2))*(3*a^4 + 15*b^4 + 10*a^2*b^2))/(8*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (a^3*(3*a^4 + 15*b^4 + 10*a^2*b^2)^3*(192*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/(1024*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^3*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(9*a^12 + 256*b^12 - 1441*a^2*b^10 - 715*a^4*b^8 - 82*a^6*b^6 + 130*a^8*b^4 + 51*a^10*b^2))/(9*a^12 + 256*b^12 + 481*a^2*b^10 + 525*a^4*b^8 + 490*a^6*b^6 + 250*a^8*b^4 + 69*a^10*b^2)^2 - (2*a*b*((64*a*b^13 + 210*a^3*b^11 + 181*a^5*b^9 + 60*a^7*b^7 + 9*a^9*b^5)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) + (64*b^5*((9*a^15 - 192*a*b^14 - 222*a^3*b^12 + 475*a^5*b^10 + 1089*a^7*b^8 + 894*a^9*b^6 + 388*a^11*b^4 + 87*a^13*b^2)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (64*b^5*((64*a*b^15 + 48*a^15*b + 624*a^3*b^13 + 2016*a^5*b^11 + 3152*a^7*b^9 + 2688*a^9*b^7 + 1296*a^11*b^5 + 352*a^13*b^3)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (32*b^5*(192*a*b^16 + 1344*a^3*b^14 + 4032*

```
a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^4 + 1
92*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^12 + b^12 +
 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(64*a^6
 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)))/(64*a^6 + 64*b^6 + 192*a^2*b^4 + 1
92*a^4*b^2) + (a*((a*((64*a*b^15 + 48*a^15*b + 624*a^3*b^13 + 2016*a^5*b^11
 + 3152*a^7*b^9 + 2688*a^9*b^7 + 1296*a^11*b^5 + 352*a^13*b^3)/(2*(a^12 + b
^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (3
2*b^5*(192*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^
9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 +
192*a^2*b^4 + 192*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*
b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(3*a^4 + 15*b^4 + 10*a^2*b^2))/(8*(a^6 + b
^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (4*a*b^5*(3*a^4 + 15*b^4 + 10*a^2*b^2)*(192*
a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 403
2*a^11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^2*b^4
 + 192*a^4*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^
10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(3*a^4 + 15*b^4 +
 10*a^2*b^2))/(8*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (a^2*b^5*(3*a^4 + 1
5*b^4 + 10*a^2*b^2)^2*(192*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^
7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6 + 1344*a^...
```

3.111 $\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=166

$$-\frac{b^4\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}\,d}+\frac{b^3\cos(c+dx)}{(a^2+b^2)^2\,d}+\frac{b\cos^3(c+dx)}{3\,(a^2+b^2)\,d}+\frac{ab^2\sin(c+dx)}{(a^2+b^2)^2\,d}+\frac{a\sin(c+dx)}{(a^2+b^2)\,d}-\frac{a\sin^3(}{3\,(a^2}$$

```
[Out] -b^4*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)/d
+b^3*cos(d*x+c)/(a^2+b^2)^2/d+1/3*b*cos(d*x+c)^3/(a^2+b^2)/d+a*b^2*sin(d*x+
c)/(a^2+b^2)^2/d+a*sin(d*x+c)/(a^2+b^2)/d-1/3*a*sin(d*x+c)^3/(a^2+b^2)/d
```

Rubi [A]

time = 0.14, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3179, 2713, 2717, 3153, 212}

$$-\frac{a\sin^3(c+dx)}{3d\,(a^2+b^2)}+\frac{ab^2\sin(c+dx)}{d\,(a^2+b^2)^2}+\frac{a\sin(c+dx)}{d\,(a^2+b^2)}+\frac{b\cos^3(c+dx)}{3d\,(a^2+b^2)}-\frac{b^4\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\,(a^2+b^2)^{5/2}}+\frac{b^3\cos(c+dx)}{d\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] -((b^4*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 +
b^2)^(5/2)*d)) + (b^3*Cos[c + d*x])/((a^2 + b^2)^2*d) + (b*Cos[c + d*x]^3)/
(3*(a^2 + b^2)*d) + (a*b^2*Sin[c + d*x])/((a^2 + b^2)^2*d) + (a*Sin[c + d*x
])/((a^2 + b^2)*d) - (a*Sin[c + d*x]^3)/(3*(a^2 + b^2)*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3179

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx &= \frac{b\cos^3(c+dx)}{3\left(a^2+b^2\right)d}+\frac{a\int \cos^3(c+dx)\,dx}{a^2+b^2}+\frac{b^2\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a^2+b^2}\\ &= \frac{b^3\cos(c+dx)}{\left(a^2+b^2\right)^2 d}+\frac{b\cos^3(c+dx)}{3\left(a^2+b^2\right)d}+\frac{\left(ab^2\right)\int \cos(c+dx)\,dx}{\left(a^2+b^2\right)^2}+\frac{b^4\int \frac{1}{a\cos(c+}}{(a^2}\\ &= \frac{b^3\cos(c+dx)}{\left(a^2+b^2\right)^2 d}+\frac{b\cos^3(c+dx)}{3\left(a^2+b^2\right)d}+\frac{ab^2\sin(c+dx)}{\left(a^2+b^2\right)^2 d}+\frac{a\sin(c+dx)}{\left(a^2+b^2\right)d}-\frac{a}{3}\\ &= -\frac{b^4\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}d}+\frac{b^3\cos(c+dx)}{\left(a^2+b^2\right)^2 d}+\frac{b\cos^3(c+dx)}{3\left(a^2+b^2\right)d}+\end{aligned}$$

Mathematica [A]
time = 1.08, size = 137, normalized size = 0.83

$$\frac{24b^4\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)+\sqrt{a^2+b^2}\left(3b\left(a^2+5b^2\right)\cos(c+dx)+b\left(a^2+b^2\right)\cos(3(c+dx))+2a\left(5a^2+11b^2+\left(a^2+b^2\right)\cos(2(c+dx))\right)\sin(c+dx)\right)}{12\left(a^2+b^2\right)^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (24*b^4*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 + b^2
]*(3*b*(a^2 + 5*b^2)*Cos[c + d*x] + b*(a^2 + b^2)*Cos[3*(c + d*x)] + 2*a*(5
*a^2 + 11*b^2 + (a^2 + b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/(12*(a^2 + b^2
)^(5/2)*d)
```

Maple [A]
time = 0.40, size = 221, normalized size = 1.33

method	result
derivativedivides	$\frac{\frac{2b^4 \operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^4+2a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(\left(-a^3-2a\,b^2\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-a^2b-2b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{2}{3}a^3-\frac{8}{3}a\,b^2\right)\left(\tan^3\left(\frac{dx}{2}+\right.\right.\right.}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan^2\left(\frac{dx}{2}+\right.\right.}}{d}$
default	$\frac{\frac{2b^4 \operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^4+2a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(\left(-a^3-2a\,b^2\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-a^2b-2b^3\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{2}{3}a^3-\frac{8}{3}a\,b^2\right)\left(\tan^3\left(\frac{dx}{2}+\right.\right.\right.}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan^2\left(\frac{dx}{2}+\right.\right.}}{d}$
risch	$-\frac{5\,\mathrm{e}^{i(dx+c)}b}{8(-2iab+a^2-b^2)d}-\frac{3i\mathrm{e}^{i(dx+c)}a}{8(-2iab+a^2-b^2)d}-\frac{5\,\mathrm{e}^{-i(dx+c)}b}{8(ib+a)^2d}+\frac{3i\mathrm{e}^{-i(dx+c)}a}{8(ib+a)^2d}+\frac{b^4\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^5+2ia^3b^2+ia\,b^4-}{\left(a^2+b^2\right.}\right.}{\left(a^2+b^2\right)^{\frac{5}{2}}d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b^4/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x
+1/2*c)-2*b)/(a^2+b^2)^(1/2))-2/(a^4+2*a^2*b^2+b^4)*((-a^3-2*a*b^2)*tan(1/2
*d*x+1/2*c)^5+(-a^2*b-2*b^3)*tan(1/2*d*x+1/2*c)^4+(-2/3*a^3-8/3*a*b^2)*tan(
1/2*d*x+1/2*c)^3-2*b^3*tan(1/2*d*x+1/2*c)^2+(-a^3-2*a*b^2)*tan(1/2*d*x+1/2*
c)-1/3*a^2*b-4/3*b^3)/(1+tan(1/2*d*x+1/2*c)^2)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 379 vs. 2(158) = 316.
time = 0.48, size = 379, normalized size = 2.28

$$-\frac{\frac{3\,b^4\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(a^2b+4\,b^3+\frac{6\,b^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\left(a^3+2\,ab^2\right)\sin(dx+c)}{\cos(dx+c)+1}+\frac{2\left(a^3+4\,ab^2\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\left(a^2b+2\,b^3\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{3\left(a^3+2\,ab^2\right)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^4+2\,a^2b^2+b^4+\frac{3\left(a^4+2\,a^2b^2+b^4\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3\left(a^4+2\,a^2b^2+b^4\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\left(a^4+2\,a^2b^2+b^4\right)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/3*(3*b^4*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(
b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2
 + b^4)*sqrt(a^2 + b^2)) - 2*(a^2*b + 4*b^3 + 6*b^3*sin(d*x + c)^2/(cos(d*x
 + c) + 1)^2 + 3*(a^3 + 2*a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) + 2*(a^3 +
 4*a*b^2)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*(a^2*b + 2*b^3)*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 + 3*(a^3 + 2*a*b^2)*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5)/(a^4 + 2*a^2*b^2 + b^4 + 3*(a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 + (a^4 + 2*a^2*b^2 + b^4)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6))/d
```

Fricas [A]
time = 3.56, size = 262, normalized size = 1.58

$$\frac{3\sqrt{a^2+b^2}\,b^4\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2+2\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)+2\,(a^4b+2\,a^2b^3+b^5)\cos(dx+c)^3+6\,(a^2b^3+b^5)\cos(dx+c)+2\,(2\,a^5+7\,a^3b^2+5\,ab^4+(a^5+2\,a^3b^2+ab^4)\cos(dx+c)^2)\sin(dx+c)}{6\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(a^2 + b^2)*b^4*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b
^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*si
n(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
+ b^2)) + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^3 + 6*(a^2*b^3 + b^5)*co
s(d*x + c) + 2*(2*a^5 + 7*a^3*b^2 + 5*a*b^4 + (a^5 + 2*a^3*b^2 + a*b^4)*cos
(d*x + c)^2)*sin(d*x + c))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.50, size = 286, normalized size = 1.72

$$-\frac{\frac{3\,b^4\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}}-\frac{2\left(3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+6\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+2\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+8\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^2b+4\,b^3\right)}{(a^4+2\,a^2b^2+b^4)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3*(3*b^4*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs
(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b
^4)*sqrt(a^2 + b^2)) - 2*(3*a^3*tan(1/2*d*x + 1/2*c)^5 + 6*a*b^2*tan(1/2*d*
x + 1/2*c)^5 + 3*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 6*b^3*tan(1/2*d*x + 1/2*c)^
4 + 2*a^3*tan(1/2*d*x + 1/2*c)^3 + 8*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*b^3*t
an(1/2*d*x + 1/2*c)^2 + 3*a^3*tan(1/2*d*x + 1/2*c) + 6*a*b^2*tan(1/2*d*x +
1/2*c) + a^2*b + 4*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(tan(1/2*d*x + 1/2*c)^2 +
1)^3))/d
```

Mupad [B]
time = 3.32, size = 342, normalized size = 2.06

$$\frac{\frac{\frac{2\,a^2\,b}{3}+\frac{8\,b^3}{3}}{a^4+2\,a^2\,b^2+b^4}+\frac{4\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^4+2\,a^2\,b^2+b^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(2\,a^3+4\,a\,b^2)}{a^4+2\,a^2\,b^2+b^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{4\,a^3}{3}+\frac{16\,a\,b^2}{3}\right)}{a^4+2\,a^2\,b^2+b^4}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^3+2\,a\,b^2)}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,(a^2+2\,b^2)}{a^4+2\,a^2\,b^2+b^4}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}-\frac{2\,b^4\,\mathrm{atanh}\left(\frac{a^4\,b+b^5+2\,a^2\,b^3-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^4+2\,a^2\,b^2+b^4)}{(a^2+b^2)^{5/2}}\right)}{d\,(a^2+b^2)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

```
[Out] (((2*a^2*b)/3 + (8*b^3)/3)/(a^4 + b^4 + 2*a^2*b^2) + (4*b^3*tan(c/2 + (d*x)
/2)^2)/(a^4 + b^4 + 2*a^2*b^2) + (tan(c/2 + (d*x)/2)^5*(4*a*b^2 + 2*a^3))/(
a^4 + b^4 + 2*a^2*b^2) + (tan(c/2 + (d*x)/2)^3*((16*a*b^2)/3 + (4*a^3)/3))/
(a^4 + b^4 + 2*a^2*b^2) + (2*tan(c/2 + (d*x)/2)*(2*a*b^2 + a^3))/(a^4 + b^4
 + 2*a^2*b^2) + (2*b*tan(c/2 + (d*x)/2)^4*(a^2 + 2*b^2))/(a^4 + b^4 + 2*a^2
*b^2))/(d*(3*tan(c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x
)/2)^6 + 1)) - (2*b^4*atanh((a^4*b + b^5 + 2*a^2*b^3 - a*tan(c/2 + (d*x)/2)
*(a^4 + b^4 + 2*a^2*b^2))/(a^2 + b^2)^(5/2)))/(d*(a^2 + b^2)^(5/2))
```

3.112 $\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=119

$$\frac{ab^2x}{(a^2+b^2)^2}+\frac{ax}{2(a^2+b^2)}+\frac{b\cos^2(c+dx)}{2(a^2+b^2)d}+\frac{b^3\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^2 d}+\frac{a\cos(c+dx)\sin(c+dx)}{2(a^2+b^2)d}$$

```
[Out] a*b^2*x/(a^2+b^2)^2+1/2*a*x/(a^2+b^2)+1/2*b*cos(d*x+c)^2/(a^2+b^2)/d+b^3*ln
(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^2/d+1/2*a*cos(d*x+c)*sin(d*x+c)/(a^2+
b^2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3179, 2715, 8, 3177, 3212}

$$\frac{b\cos^2(c+dx)}{2d(a^2+b^2)}+\frac{a\sin(c+dx)\cos(c+dx)}{2d(a^2+b^2)}+\frac{ab^2x}{(a^2+b^2)^2}+\frac{ax}{2(a^2+b^2)}+\frac{b^3\log(a\cos(c+dx)+b\sin(c+dx))}{d(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (a*b^2*x)/(a^2 + b^2)^2 + (a*x)/(2*(a^2 + b^2)) + (b*Cos[c + d*x]^2)/(2*(a^
2 + b^2)*d) + (b^3*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^2*d)
+ (a*Cos[c + d*x]*Sin[c + d*x])/(2*(a^2 + b^2)*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3177

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3179

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{b\cos^2(c+dx)}{2\left(a^2+b^2\right)d}+\frac{a\int\cos^2(c+dx)\,dx}{a^2+b^2}+\frac{b^2\int\frac{\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a^2+b^2}$$

$$= \frac{ab^2x}{\left(a^2+b^2\right)^2}+\frac{b\cos^2(c+dx)}{2\left(a^2+b^2\right)d}+\frac{a\cos(c+dx)\sin(c+dx)}{2\left(a^2+b^2\right)d}+\frac{b^3\int\frac{b\cos(c+\ldots}{a\cos(c+\ldots}}{(a\ldots}$$

$$= \frac{ab^2x}{\left(a^2+b^2\right)^2}+\frac{ax}{2\left(a^2+b^2\right)}+\frac{b\cos^2(c+dx)}{2\left(a^2+b^2\right)d}+\frac{b^3\log(a\cos(c+dx)+b\,\mathrm{s}\ldots}{\left(a^2+b^2\right)^2d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.25, size = 143, normalized size = 1.20

$$\frac{2a^3c+6ab^2c+4ib^3c+2a^3dx+6ab^2dx+4ib^3dx-4ib^3\mathrm{ArcTan}(\tan(c+dx))+b(a^2+b^2)\cos(2(c+dx))+2b^3\log\left((a\cos(c+dx)+b\sin(c+dx))^2\right)+a^3\sin(2(c+dx))+ab^2\sin(2(c+dx))}{4\left(a^2+b^2\right)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (2*a^3*c + 6*a*b^2*c + (4*I)*b^3*c + 2*a^3*d*x + 6*a*b^2*d*x + (4*I)*b^3*d*
x - (4*I)*b^3*ArcTan[Tan[c + d*x]] + b*(a^2 + b^2)*Cos[2*(c + d*x)] + 2*b^3
*Log[(a*Cos[c + d*x] + b*Sin[c + d*x])^2] + a^3*Sin[2*(c + d*x)] + a*b^2*Si
n[2*(c + d*x)])/(4*(a^2 + b^2)^2*d)
```

Maple [A]
time = 0.26, size = 120, normalized size = 1.01

method	result
derivativedivides	$\frac{\frac{b^3 \ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^2}+\frac{\frac{\left(\frac{1}{2}a^3+\frac{1}{2}a\,b^2\right)\tan(dx+c)+\frac{a^2b}{2}+\frac{b^3}{2}}{\tan^2(dx+c)+1}-\frac{b^3\ln\left(\tan^2(dx+c)+1\right)}{2}+\frac{\left(a^3+3a\,b^2\right)\arctan(\tan(dx+c))}{2}}{\left(a^2+b^2\right)^2}}{d}$
default	$\frac{\frac{b^3 \ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^2}+\frac{\frac{\left(\frac{1}{2}a^3+\frac{1}{2}a\,b^2\right)\tan(dx+c)+\frac{a^2b}{2}+\frac{b^3}{2}}{\tan^2(dx+c)+1}-\frac{b^3\ln\left(\tan^2(dx+c)+1\right)}{2}+\frac{\left(a^3+3a\,b^2\right)\arctan(\tan(dx+c))}{2}}{\left(a^2+b^2\right)^2}}{d}$
risch	$\frac{2ixb}{4iab-2a^2+2b^2}-\frac{xa}{4iab-2a^2+2b^2}-\frac{i\mathrm{e}^{2i(dx+c)}}{8(-ib+a)d}+\frac{i\mathrm{e}^{-2i(dx+c)}}{8(ib+a)d}-\frac{2ib^3x}{a^4+2a^2b^2+b^4}-\frac{2ib^3c}{d(a^4+2a^2b^2+b^4)}+\frac{b^3\ln\left(\mathrm{e}^{2i(\right.}}{d(a^4+}$
norman	$\frac{\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(a^2+b^2\right)}-\frac{2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(a^2+b^2\right)}-\frac{2b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(a^2+b^2\right)}-\frac{a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(a^2+b^2\right)}+\frac{a\left(a^2+3b^2\right)x}{2a^4+4a^2b^2+2b^4}+\frac{3\left(a^2+3b^2\right)ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^4+2a^2b^2+b^4\right)}+\frac{3}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b^3/(a^2+b^2)^2*ln(a+b*tan(d*x+c))+1/(a^2+b^2)^2*(((1/2*a^3+1/2*a*b^2)
*tan(d*x+c)+1/2*a^2*b+1/2*b^3)/(tan(d*x+c)^2+1)-1/2*b^3*ln(tan(d*x+c)^2+1)+
1/2*(a^3+3*a*b^2)*arctan(tan(d*x+c))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(113) = 226$.

time = 0.48, size = 284, normalized size = 2.39

$$\frac{\frac{b^3\log\left(-a-\frac{2\,b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^4+2\,a^2b^2+b^4}-\frac{b^3\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{(a^3+3\,ab^2)\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+2\,a^2b^2+b^4}+\frac{\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\,b\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2+b^2+\frac{2\left(a^2+b^2\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\left(a^2+b^2\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b^3*log(-a - 2*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2)/(a^4 + 2*a^2*b^2 + b^4) - b^3*log(sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 + 3*a*b^2)*arctan(sin(d*x +
c)/(cos(d*x + c) + 1))/(a^4 + 2*a^2*b^2 + b^4) + (a*sin(d*x + c)/(cos(d*x +
 c) + 1) - 2*b*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - a*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3)/(a^2 + b^2 + 2*(a^2 + b^2)*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 + (a^2 + b^2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4))/d
```

Fricas [A]

time = 2.88, size = 119, normalized size = 1.00

$$\frac{b^3\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)+\left(a^3+3\,ab^2\right)dx+\left(a^2b+b^3\right)\cos\left(dx+c\right)^2+\left(a^3+ab^2\right)\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\left(a^4+2\,a^2b^2+b^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(b^3*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 +
 b^2) + (a^3 + 3*a*b^2)*d*x + (a^2*b + b^3)*cos(d*x + c)^2 + (a^3 + a*b^2)*
cos(d*x + c)*sin(d*x + c))/((a^4 + 2*a^2*b^2 + b^4)*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.47, size = 182, normalized size = 1.53

$$\frac{\frac{2\,b^4\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^4b+2\,a^2b^3+b^5}-\frac{b^3\log\left(\tan\left(dx+c\right)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{\left(a^3+3\,ab^2\right)\left(dx+c\right)}{a^4+2\,a^2b^2+b^4}+\frac{b^3\tan\left(dx+c\right)^2+a^3\tan\left(dx+c\right)+ab^2\tan\left(dx+c\right)+a^2b+2\,b^3}{\left(a^4+2\,a^2b^2+b^4\right)\left(\tan\left(dx+c\right)^2+1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^4*log(abs(b*tan(d*x + c) + a))/(a^4*b + 2*a^2*b^3 + b^5) - b^3*log
(tan(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^3 + 3*a*b^2)*(d*x + c)/(a
^4 + 2*a^2*b^2 + b^4) + (b^3*tan(d*x + c)^2 + a^3*tan(d*x + c) + a*b^2*tan(
d*x + c) + a^2*b + 2*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(tan(d*x + c)^2 + 1)))/d
```

Mupad [**B**]
time = 6.16, size = 2500, normalized size = 21.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

```
[Out] (b^3*log(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^2))/(d*(a^4 + b^
4 + 2*a^2*b^2)) - (4*b^3*log(1/(cos(c + d*x) + 1)))/(d*(4*a^4 + 4*b^4 + 8*a
^2*b^2)) - ((a*tan(c/2 + (d*x)/2)^3)/(a^2 + b^2) + (2*b*tan(c/2 + (d*x)/2)^
2)/(a^2 + b^2) - (a*tan(c/2 + (d*x)/2))/(a^2 + b^2))/(d*(2*tan(c/2 + (d*x)/
2)^2 + tan(c/2 + (d*x)/2)^4 + 1)) - (a*atan((tan(c/2 + (d*x)/2)*((((4*b^3*(
(a*((8*(4*a*b^9 + 4*a^9*b + 28*a^3*b^7 + 48*a^5*b^5 + 28*a^7*b^3))/(a^6 + b
```

^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*b^3*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a^2 + 3*b^2))/(2*(a^4 + b^4 + 2*a^2*b^2)) - (16*a*b^3*(a^2 + 3*b^2)*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2) - (a*((8*(a^9 - 12*a*b^8 - 6*a^3*b^6 + 13*a^5*b^4 + 8*a^7*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (4*b^3*((8*(4*a*b^9 + 4*a^9*b + 28*a^3*b^7 + 48*a^5*b^5 + 28*a^7*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*b^3*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2))*(a^2 + 3*b^2))/(2*(a^4 + b^4 + 2*a^2*b^2)) + (a^3*(a^2 + 3*b^2)^3*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((a^4 + b^4 + 2*a^2*b^2)^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a^8 + 16*b^8 - 73*a^2*b^6 - 13*a^4*b^4 + 5*a^6*b^2))/(a^8 + 16*b^8 + 25*a^2*b^6 + 15*a^4*b^4 + 7*a^6*b^2)^2 - (2*a*b*(a^6 - 28*b^6 + 17*a^2*b^4 + 10*a^4*b^2)*((8*(4*a*b^7 + 6*a^3*b^5 + a^5*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (4*b^3*((8*(a^9 - 12*a*b^8 - 6*a^3*b^6 + 13*a^5*b^4 + 8*a^7*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (4*b^3*((8*(4*a*b^9 + 4*a^9*b + 28*a^3*b^7 + 48*a^5*b^5 + 28*a^7*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*b^3*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2)))/(4*a^4 + 4*b^4 + 8*a^2*b^2) + (a*(a^2 + 3*b^2)*((a*((8*(4*a*b^9 + 4*a^9*b + 28*a^3*b^7 + 48*a^5*b^5 + 28*a^7*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*b^3*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a^2 + 3*b^2))/(2*(a^4 + b^4 + 2*a^2*b^2)) - (16*a*b^3*(a^2 + 3*b^2)*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(2*(a^4 + b^4 + 2*a^2*b^2)) - (8*a^2*b^3*(a^2 + 3*b^2)^2*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(a^8 + 16*b^8 + 25*a^2*b^6 + 15*a^4*b^4 + 7*a^6*b^2)^2)*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2))/(4*a^4 + 12*a^2*b^2) + (((4*b^3*((a*(a^2 + 3*b^2)*((8*(2*a^10 - 10*a^2*b^8 - 16*a^4*b^6 + 8*a^8*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*b^3*(12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(2*(a^4 + b^4 + 2*a^2*b^2)) - (16*a*b^3*(a^2 + 3*b^2)*(12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2) - (a*(a^2 + 3*b^2)*((8*(a^8*b + a^4*b^5 + 2*a^6*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (4*b^3*((8*(2*a^10 - 10*a^2*b^8 - 16*a^4*b^6 + 8*a^8*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*b^3*(12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2)))/(2*(a^4 + b^4 + 2*a^

```
2*b^2)) + (a^3*(a^2 + 3*b^2)^3*(12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^
6*b^5 + 48*a^8*b^3))/((a^4 + b^4 + 2*a^2*b^2)^3*(a^6 + b^6 + 3*a^2*b^4 + 3*
a^4*b^2)))*(a^8 + 16*b^8 - 73*a^2*b^6 - 13*a^4*b^4 + 5*a^6*b^2)*(a^10 + b^1
0 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2))/((4*a^4 + 12*a^2*b^2)
*(a^8 + 16*b^8 + 25*a^2*b^6 + 15*a^4*b^4 + 7*a^6*b^2)^2) - (2*a*b*(a^6 - 28
*b^6 + 17*a^2*b^4 + 10*a^4*b^2)*((8*(2*a^2*b^6 + a^4*b^4))/(a^6 + b^6 + 3*a
^2*b^4 + 3*a^4*b^2) + (4*b^3*((8*(a^8*b + a^4*b^5 + 2*a^6*b^3))/(a^6 + b^6
+ 3*a^2*b^4 + 3*a^4*b^2) - (4*b^3*((8*(2*a^10 - 10*a^2*b^8 - 16*a^4*b^6 + 8
*a^8*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*b^3*(12*a^10*b + 12*a^
2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)
*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2)))/(4*a^
4 + 4*b^4 + 8*a^2*b^2) + (a*((a*(a^2 + 3*b^2)*((8*(2*a^10 - 10*a^2*b^8 - 16
*a^4*b^6 + 8*a^8*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*b^3*(12*a^
10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4
+ 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(2*(a^4 + b^4 + 2*a^2*b
^2)) - (16*a*b^3*(a^2 + 3*b^2)*(12*a^10*b + 12*...
```

3.113 $\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=91

$$-\frac{b^2\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}+\frac{b\cos(c+dx)}{(a^2+b^2)\,d}+\frac{a\sin(c+dx)}{(a^2+b^2)\,d}$$

```
[Out] -b^2*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d
+b*cos(d*x+c)/(a^2+b^2)/d+a*sin(d*x+c)/(a^2+b^2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3179, 2717, 3153, 212}

$$\frac{a\sin(c+dx)}{d\,(a^2+b^2)}+\frac{b\cos(c+dx)}{d\,(a^2+b^2)}-\frac{b^2\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\,(a^2+b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] -((b^2*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 +
b^2)^(3/2)*d)) + (b*Cos[c + d*x])/((a^2 + b^2)*d) + (a*Sin[c + d*x])/((a^2
+ b^2)*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3179

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx &= \frac{b\cos(c+dx)}{(a^2+b^2)\,d}+\frac{a\int \cos(c+dx)\,dx}{a^2+b^2}+\frac{b^2\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a^2+b^2}\\ &= \frac{b\cos(c+dx)}{(a^2+b^2)\,d}+\frac{a\sin(c+dx)}{(a^2+b^2)\,d}-\frac{b^2\mathrm{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,b\cos(c+dx)\ldots\right)}{(a^2+b^2)\,d}\\ &= -\frac{b^2\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}+\frac{b\cos(c+dx)}{(a^2+b^2)\,d}+\frac{a\sin(c+dx)}{(a^2+b^2)\,d}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 79, normalized size = 0.87

$$\frac{2b^2\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)+\sqrt{a^2+b^2}\,(b\cos(c+dx)+a\sin(c+dx))}{(a^2+b^2)^{3/2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (2*b^2*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + Sqrt[a^2 + b^2]
*(b*Cos[c + d*x] + a*Sin[c + d*x]))/((a^2 + b^2)^(3/2)*d)
```

Maple [A]
time = 0.29, size = 90, normalized size = 0.99

method	result	siz…
derivativedivides	$\frac{\frac{2b^2\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(-a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-b\right)}{\left(a^2+b^2\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{d}$	90

default	$\frac{\frac{2b^2 \operatorname{arctanh}\left(\frac{2a \tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(-a \tan\left(\frac{dx}{2}+\frac{c}{2}\right)-b\right)}{\left(a^2+b^2\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{d}$	90
risch	$-\frac{i e^{i(dx+c)}}{2(-ib+a)d}+\frac{i e^{-i(dx+c)}}{2(ib+a)d}+\frac{b^2 \ln\left(e^{i(dx+c)}+\frac{i a^3+i a b^2-a^2 b-b^3}{\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}} d}-\frac{b^2 \ln\left(e^{i(dx+c)}-\frac{i a^3+i a b^2-a^2 b-b^3}{\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}} d}$	174

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*b^2/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^
2)^(1/2))-2/(a^2+b^2)*(-a*tan(1/2*d*x+1/2*c)-b)/(1+tan(1/2*d*x+1/2*c)^2))
```

Maxima [A]
time = 0.49, size = 142, normalized size = 1.56

$$-\frac{\frac{b^2 \log\left(\frac{b-\frac{a \sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a \sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\left(b+\frac{a \sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2+b^2+\frac{\left(a^2+b^2\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(b^2*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*
sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(
b + a*sin(d*x + c)/(cos(d*x + c) + 1))/(a^2 + b^2 + (a^2 + b^2)*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(87) = 174.
time = 2.54, size = 187, normalized size = 2.05

$$\frac{\sqrt{a^2+b^2}\, b^2 \log\left(-\frac{2ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2a^2-b^2+2\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)+2\left(a^2b+b^3\right)\cos(dx+c)+2\left(a^3+ab^2\right)\sin(dx+c)}{2\left(a^4+2a^2b^2+b^4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + b^2)*b^2*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2
)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(
d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 +
b^2)) + 2*(a^2*b + b^3)*cos(d*x + c) + 2*(a^3 + a*b^2)*sin(d*x + c))/((a^4
+ 2*a^2*b^2 + b^4)*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 117.79, size = 1034, normalized size = 11.36

$$\begin{cases} \frac{\tilde{\infty} x \cos^{2}{\left(c \right)}}{\sin{\left(c \right)}} & \text{for}\: a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{\frac{\log{\left(\tan{\left(\frac{c}{2} + \frac{d x}{2} \right)} \right)} \tan^{2}{\left(\frac{c}{2} + \frac{d x}{2} \right)}}{d \tan^{2}{\left(\frac{c}{2} + \frac{d x}{2} \right)} + d} + \frac{\log{\left(\tan{\left(\frac{c}{2} + \frac{d x}{2} \right)} \right)}}{d \tan^{2}{\left(\frac{c}{2} + \frac{d x}{2} \right)} + d} + \frac{2}{d \tan^{2}{\left(\frac{c}{2} + \frac{d x}{2} \right)} + d}}{b} & \text{for}\: a = 0 \\ - \frac{2 \sin^{2}{\left(c + d x \right)}}{3 i b d \sin{\left(c + d x \right)} + 3 b d \cos{\left(c + d x \right)}} + \frac{2 i \sin{\left(c + d x \right)} \cos{\left(c + d x \right)}}{3 i b d \sin{\left(c + d x \right)} + 3 b d \cos{\left(c + d x \right)}} - \frac{\cos^{2}{\left(c + d x \right)}}{3 i b d \sin{\left(c + d x \right)} + 3 b d \cos{\left(c + d x \right)}} & \text{for}\: a = - i b \\ - \frac{2 \sin^{2}{\left(c + d x \right)}}{- 3 i b d \sin{\left(c + d x \right)} + 3 b d \cos{\left(c + d x \right)}} - \frac{2 i \sin{\left(c + d x \right)} \cos{\left(c + d x \right)}}{- 3 i b d \sin{\left(c + d x \right)} + 3 b d \cos{\left(c + d x \right)}} - \frac{\cos^{2}{\left(c + d x \right)}}{- 3 i b d \sin{\left(c + d x \right)} + 3 b d \cos{\left(c + d x \right)}} & \text{for}\: a = i b \\ \frac{x \cos^{2}{\left(c \right)}}{a \cos{\left(c \right)} + b \sin{\left(c \right)}} & \text{for}\: d = 0 \\ \frac{2 a \sqrt{a^{2} + b^{2}} \tan{\left(\frac{c}{2} + \frac{d x}{2} \right)}}{a^{2} d \sqrt{a^{2} + b^{2}} \tan^{2}{\left(\frac{c}{2} + \frac{d x}{2} \right)} + a^{2} d \sqrt{a^{2} + b^{2}} + b^{2} d \sqrt{a^{2} + b^{2}} \tan^{2}{\left(\frac{c}{2} + \frac{d x}{2} \right)} + b^{2} d \sqrt{a^{2} + b^{2}}} - \frac{b^{2} \log{\left(\tan{\left(\frac{c}{2} + \frac{d x}{2} \right)} - \frac{b}{a} - \frac{\sqrt{a^{2} + b^{2}}}{a} \right)} \tan^{2}{\left(\frac{c}{2} + \frac{d x}{2} \right)}}{a^{2} d \sqrt{a^{2} + b^{2}} \tan^{2}{\left(\frac{c}{2} + \frac{d x}{2} \right)} + a^{2} d \sqrt{a^{2} + b^{2}} + b^{2} d \sqrt{a^{2} + b^{2}} \tan^{2}{\left(\frac{c}{2} + \frac{d x}{2} \right)} + b^{2} d \sqrt{a^{2} + b^{2}}} - \frac{b^{2} \log{\left(\tan{\left(\frac{c}{2} + \frac{d x}{2} \right)} - \frac{b}{a} - \frac{\sqrt{a^{2} + b^{2}}}{a} \right)}}{a^{2} d \sqrt{a^{2} + b^{2}} \tan^{2}{\left(\frac{c}{2} + \frac{d x}{2} \right)} + a^{2} d \sqrt{a^{2} + b^{2}} + b^{2} d \sqrt{a^{2} + b^{2}} \tan^{2}{\left(\frac{c}{2} + \frac{d x}{2} \right)} + b^{2} d \sqrt{a^{2} + b^{2}}} + \frac{b^{2} \log{\left(\tan{\left(\frac{c}{2} + \frac{d x}{2} \right)} - \frac{b}{a} + \frac{\sqrt{a^{2} + b^{2}}}{a} \right)} \tan^{2}{\left(\frac{c}{2} + \frac{d x}{2} \right)}}{a^{2} d \sqrt{a^{2} + b^{2}} \tan^{2}{\left(\frac{c}{2} + \frac{d x}{2} \right)} + a^{2} d \sqrt{a^{2} + b^{2}} + b^{2} d \sqrt{a^{2} + b^{2}} \tan^{2}{\left(\frac{c}{2} + \frac{d x}{2} \right)} + b^{2} d \sqrt{a^{2} + b^{2}}} + \frac{b^{2} \log{\left(\tan{\left(\frac{c}{2} + \frac{d x}{2} \right)} - \frac{b}{a} + \frac{\sqrt{a^{2} + b^{2}}}{a} \right)}}{a^{2} d \sqrt{a^{2} + b^{2}} \tan^{2}{\left(\frac{c}{2} + \frac{d x}{2} \right)} + a^{2} d \sqrt{a^{2} + b^{2}} + b^{2} d \sqrt{a^{2} + b^{2}} \tan^{2}{\left(\frac{c}{2} + \frac{d x}{2} \right)} + b^{2} d \sqrt{a^{2} + b^{2}}} + \frac{2 b \sqrt{a^{2} + b^{2}}}{a^{2} d \sqrt{a^{2} + b^{2}} \tan^{2}{\left(\frac{c}{2} + \frac{d x}{2} \right)} + a^{2} d \sqrt{a^{2} + b^{2}} + b^{2} d \sqrt{a^{2} + b^{2}} \tan^{2}{\left(\frac{c}{2} + \frac{d x}{2} \right)} + b^{2} d \sqrt{a^{2} + b^{2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*cos(c)**2/sin(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((log(t
an(c/2 + d*x/2))*tan(c/2 + d*x/2)**2/(d*tan(c/2 + d*x/2)**2 + d) + log(tan(
c/2 + d*x/2))/(d*tan(c/2 + d*x/2)**2 + d) + 2/(d*tan(c/2 + d*x/2)**2 + d))/
b, Eq(a, 0)), (-2*sin(c + d*x)**2/(3*I*b*d*sin(c + d*x) + 3*b*d*cos(c + d*x
)) + 2*I*sin(c + d*x)*cos(c + d*x)/(3*I*b*d*sin(c + d*x) + 3*b*d*cos(c + d*
x)) - cos(c + d*x)**2/(3*I*b*d*sin(c + d*x) + 3*b*d*cos(c + d*x)), Eq(a, -I
*b)), (-2*sin(c + d*x)**2/(-3*I*b*d*sin(c + d*x) + 3*b*d*cos(c + d*x)) - 2*
I*sin(c + d*x)*cos(c + d*x)/(-3*I*b*d*sin(c + d*x) + 3*b*d*cos(c + d*x)) -
cos(c + d*x)**2/(-3*I*b*d*sin(c + d*x) + 3*b*d*cos(c + d*x)), Eq(a, I*b)),
(x*cos(c)**2/(a*cos(c) + b*sin(c)), Eq(d, 0)), (2*a*sqrt(a**2 + b**2)*tan(c
/2 + d*x/2)/(a**2*d*sqrt(a**2 + b**2)*tan(c/2 + d*x/2)**2 + a**2*d*sqrt(a**
2 + b**2) + b**2*d*sqrt(a**2 + b**2)*tan(c/2 + d*x/2)**2 + b**2*d*sqrt(a**2
 + b**2)) - b**2*log(tan(c/2 + d*x/2) - b/a - sqrt(a**2 + b**2)/a)*tan(c/2
+ d*x/2)**2/(a**2*d*sqrt(a**2 + b**2)*tan(c/2 + d*x/2)**2 + a**2*d*sqrt(a**
2 + b**2) + b**2*d*sqrt(a**2 + b**2)*tan(c/2 + d*x/2)**2 + b**2*d*sqrt(a**2
 + b**2)) - b**2*log(tan(c/2 + d*x/2) - b/a - sqrt(a**2 + b**2)/a)/(a**2*d*
sqrt(a**2 + b**2)*tan(c/2 + d*x/2)**2 + a**2*d*sqrt(a**2 + b**2) + b**2*d*s
qrt(a**2 + b**2)*tan(c/2 + d*x/2)**2 + b**2*d*sqrt(a**2 + b**2)) + b**2*log
(tan(c/2 + d*x/2) - b/a + sqrt(a**2 + b**2)/a)*tan(c/2 + d*x/2)**2/(a**2*d*
sqrt(a**2 + b**2)*tan(c/2 + d*x/2)**2 + a**2*d*sqrt(a**2 + b**2) + b**2*d*s
qrt(a**2 + b**2)*tan(c/2 + d*x/2)**2 + b**2*d*sqrt(a**2 + b**2)) + b**2*log
(tan(c/2 + d*x/2) - b/a + sqrt(a**2 + b**2)/a)/(a**2*d*sqrt(a**2 + b**2)*ta
n(c/2 + d*x/2)**2 + a**2*d*sqrt(a**2 + b**2) + b**2*d*sqrt(a**2 + b**2)*tan
(c/2 + d*x/2)**2 + b**2*d*sqrt(a**2 + b**2)) + 2*b*sqrt(a**2 + b**2)/(a**2*
d*sqrt(a**2 + b**2)*tan(c/2 + d*x/2)**2 + a**2*d*sqrt(a**2 + b**2) + b**2*d
*sqrt(a**2 + b**2)*tan(c/2 + d*x/2)**2 + b**2*d*sqrt(a**2 + b**2)), True))
```

Giac [A]
time = 0.46, size = 118, normalized size = 1.30

$$-\frac{\frac{b^{2} \log\left(\frac{{\left| 2 \, a \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) - 2 \, b - 2 \, \sqrt{a^{2} + b^{2}} \right|}}{{\left| 2 \, a \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) - 2 \, b + 2 \, \sqrt{a^{2} + b^{2}} \right|}}\right)}{{\left(a^{2} + b^{2}\right)}^{\frac{3}{2}}} - \frac{2 \, {\left(a \tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right) + b\right)}}{{\left(a^{2} + b^{2}\right)} {\left(\tan\left(\frac{1}{2} \, d x + \frac{1}{2} \, c\right)^{2} + 1\right)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(b^2*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*t
an(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a*ta
n(1/2*d*x + 1/2*c) + b)/((a^2 + b^2)*(tan(1/2*d*x + 1/2*c)^2 + 1)))/d
```

Mupad [B]
time = 0.63, size = 110, normalized size = 1.21

$$\frac{\frac{2\,b}{a^2+b^2}+\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2+b^2}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}-\frac{2\,b^2\,\mathrm{atanh}\left(\frac{a^2\,b+b^3-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2+b^2\right)}{{\left(a^2+b^2\right)}^{3/2}}\right)}{d\,{\left(a^2+b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

```
[Out] ((2*b)/(a^2 + b^2) + (2*a*tan(c/2 + (d*x)/2))/(a^2 + b^2))/(d*(tan(c/2 + (d
*x)/2)^2 + 1)) - (2*b^2*atanh((a^2*b + b^3 - a*tan(c/2 + (d*x)/2)*(a^2 + b^
2))/(a^2 + b^2)^(3/2)))/(d*(a^2 + b^2)^(3/2))
```

3.114 $\int \frac{\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=45

$$\frac{ax}{a^2+b^2}+\frac{b\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)\,d}$$

```
[Out] a*x/(a^2+b^2)+b*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)/d
```

Rubi [A]
time = 0.05, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3177, 3212}

$$\frac{b\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)}+\frac{ax}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (a*x)/(a^2 + b^2) + (b*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d
)
```

Rule 3177

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx &= \frac{ax}{a^2+b^2}+\frac{b\int \frac{b\cos(c+dx)-a\sin(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{a^2+b^2}\\ &= \frac{ax}{a^2+b^2}+\frac{b\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)\,d}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 41, normalized size = 0.91

$$\frac{a(c+dx)+b\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (a*(c + d*x) + b*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2 + b^2)*d)
```

Maple [A]
time = 0.20, size = 62, normalized size = 1.38

method	result	size
derivativedivides	$\frac{\frac{b\ln(a+b\tan(dx+c))}{a^2+b^2}+\frac{-\frac{b\ln\left(\tan^2(dx+c)+1\right)}{2}+a\arctan(\tan(dx+c))}{a^2+b^2}}{d}$	62
default	$\frac{\frac{b\ln(a+b\tan(dx+c))}{a^2+b^2}+\frac{-\frac{b\ln\left(\tan^2(dx+c)+1\right)}{2}+a\arctan(\tan(dx+c))}{a^2+b^2}}{d}$	62
risch	$-\frac{x}{ib-a}-\frac{2ibx}{a^2+b^2}-\frac{2ibc}{d(a^2+b^2)}+\frac{b\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{d(a^2+b^2)}$	89
norman	$\frac{\frac{ax}{a^2+b^2}+\frac{ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2+b^2}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{b\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{d(a^2+b^2)}-\frac{b\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d(a^2+b^2)}$	127

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(b/(a^2+b^2)*ln(a+b*tan(d*x+c))+1/(a^2+b^2)*(-1/2*b*ln(tan(d*x+c)^2+1)+
a*arctan(tan(d*x+c))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(45) = 90$.
time = 0.48, size = 124, normalized size = 2.76

$$\frac{\frac{2\,a\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2+b^2}+\frac{b\log\left(-a-\frac{2\,b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^2+b^2}-\frac{b\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a^2+b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (2*a*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(a^2 + b^2) + b*log(-a - 2*b*s
in(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^
2 + b^2) - b*log(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)/(a^2 + b^2))/d
```

Fricas [A]
time = 2.17, size = 61, normalized size = 1.36

$$\frac{2\,adx + b\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right) + \left(a^2-b^2\right)\cos\left(dx+c\right)^2 + b^2\right)}{2\,(a^2+b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a*d*x + b*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x
+ c)^2 + b^2))/((a^2 + b^2)*d)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.90, size = 291, normalized size = 6.47

$$\begin{cases} \frac{\tilde{\infty}x\cos\left(c\right)}{\sin\left(c\right)} & \text{for}\: a=0 \wedge b=0 \wedge d=0 \\ \frac{x}{a} & \text{for}\: b=0 \\ -\frac{dx\sin\left(c+dx\right)}{2ibd\sin\left(c+dx\right)+2bd\cos\left(c+dx\right)} + \frac{idx\cos\left(c+dx\right)}{2ibd\sin\left(c+dx\right)+2bd\cos\left(c+dx\right)} + \frac{i\sin\left(c+dx\right)}{2ibd\sin\left(c+dx\right)+2bd\cos\left(c+dx\right)} & \text{for}\: a=-ib \\ -\frac{dx\sin\left(c+dx\right)}{-2ibd\sin\left(c+dx\right)+2bd\cos\left(c+dx\right)} - \frac{idx\cos\left(c+dx\right)}{-2ibd\sin\left(c+dx\right)+2bd\cos\left(c+dx\right)} - \frac{i\sin\left(c+dx\right)}{-2ibd\sin\left(c+dx\right)+2bd\cos\left(c+dx\right)} & \text{for}\: a=ib \\ \frac{x\cos\left(c\right)}{a\cos\left(c\right)+b\sin\left(c\right)} & \text{for}\: d=0 \\ \frac{adx}{a^2d+b^2d} + \frac{b\log\left(\frac{a\cos\left(c+dx\right)}{b}+\sin\left(c+dx\right)\right)}{a^2d+b^2d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Piecewise((zoo*x*cos(c)/sin(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/a, Eq(b
, 0)), (-d*x*sin(c + d*x)/(2*I*b*d*sin(c + d*x) + 2*b*d*cos(c + d*x)) + I*d
*x*cos(c + d*x)/(2*I*b*d*sin(c + d*x) + 2*b*d*cos(c + d*x)) + I*sin(c + d*x
)/(2*I*b*d*sin(c + d*x) + 2*b*d*cos(c + d*x)), Eq(a, -I*b)), (-d*x*sin(c +
d*x)/(-2*I*b*d*sin(c + d*x) + 2*b*d*cos(c + d*x)) - I*d*x*cos(c + d*x)/(-2*
I*b*d*sin(c + d*x) + 2*b*d*cos(c + d*x)) - I*sin(c + d*x)/(-2*I*b*d*sin(c +
 d*x) + 2*b*d*cos(c + d*x)), Eq(a, I*b)), (x*cos(c)/(a*cos(c) + b*sin(c)),
Eq(d, 0)), (a*d*x/(a**2*d + b**2*d) + b*log(a*cos(c + d*x)/b + sin(c + d*x)
)/(a**2*d + b**2*d), True))
```

Giac [A]
time = 0.44, size = 74, normalized size = 1.64

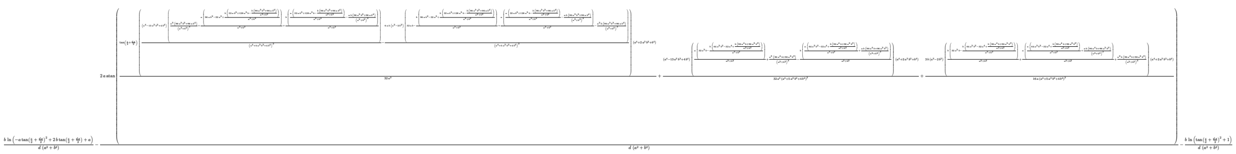
$$\frac{\frac{2\,b^2\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{a^2b+b^3} + \frac{2\,(dx+c)a}{a^2+b^2} - \frac{b\log\left(\tan\left(dx+c\right)^2+1\right)}{a^2+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^2*log(abs(b*tan(d*x + c) + a))/(a^2*b + b^3) + 2*(d*x + c)*a/(a^2
+ b^2) - b*log(tan(d*x + c)^2 + 1)/(a^2 + b^2))/d
```

Mupad [B]
time = 1.16, size = 1069, normalized size = 23.76



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

```
[Out] (b*log(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^2))/(d*(a^2 + b^2)
) - (2*a*atan((tan(c/2 + (d*x)/2)*(((a^4 + 4*b^4 - 13*a^2*b^2)*((a^3*(96*a*
b^4 + 96*a^3*b^2))/(a^2 + b^2)^3 + (a*(96*a*b^2 - 32*a^3 + (b*(32*a*b^3 + 1
28*a^3*b - (b*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)))/(a^2 + b^2)))/(a^2 + b
^2) + (b*((a*(32*a*b^3 + 128*a^3*b - (b*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2
)))/(a^2 + b^2) - (a*b*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)^2))/(a^2 + b^2)
))/(a^4 + 4*b^4 + 5*a^2*b^2)^2 - (6*a*b*(a^2 - 2*b^2)*(32*a*b - (b*(96*a*b^
2 - 32*a^3 + (b*(32*a*b^3 + 128*a^3*b - (b*(96*a*b^4 + 96*a^3*b^2))/(a^2 +
b^2)))/(a^2 + b^2)))/(a^2 + b^2) + (a*((a*(32*a*b^3 + 128*a^3*b - (b*(96*a*
b^4 + 96*a^3*b^2))/(a^2 + b^2)))/(a^2 + b^2) - (a*b*(96*a*b^4 + 96*a^3*b^2)
)/(a^2 + b^2)^2))/(a^2 + b^2) - (a^2*b*(96*a*b^4 + 96*a^3*b^2))/(a^2 + b^2)
^3))/(a^4 + 4*b^4 + 5*a^2*b^2)^2)*(a^4 + b^4 + 2*a^2*b^2))/(32*a^2) + ((a^4
 + 4*b^4 - 13*a^2*b^2)*((a*(32*a^2*b - (b*(64*a^2*b^2 - 32*a^4 + (b*(96*a^4
*b + 96*a^2*b^3))/(a^2 + b^2)))/(a^2 + b^2)))/(a^2 + b^2) + (a^3*(96*a^4*b
+ 96*a^2*b^3))/(a^2 + b^2)^3 - (b*((a*(64*a^2*b^2 - 32*a^4 + (b*(96*a^4*b +
 96*a^2*b^3))/(a^2 + b^2)))/(a^2 + b^2) + (a*b*(96*a^4*b + 96*a^2*b^3))/(a^
2 + b^2)^2))/(a^2 + b^2))*(a^4 + b^4 + 2*a^2*b^2))/(32*a^2*(a^4 + 4*b^4 + 5
*a^2*b^2)^2) + (3*b*(a^2 - 2*b^2)*((b*(32*a^2*b - (b*(64*a^2*b^2 - 32*a^4 +
 (b*(96*a^4*b + 96*a^2*b^3))/(a^2 + b^2)))/(a^2 + b^2)))/(a^2 + b^2) + (a*(
(a*(64*a^2*b^2 - 32*a^4 + (b*(96*a^4*b + 96*a^2*b^3))/(a^2 + b^2)))/(a^2 +
b^2) + (a*b*(96*a^4*b + 96*a^2*b^3))/(a^2 + b^2)^2))/(a^2 + b^2) + (a^2*b*(
96*a^4*b + 96*a^2*b^3))/(a^2 + b^2)^3)*(a^4 + b^4 + 2*a^2*b^2))/(16*a*(a^4
+ 4*b^4 + 5*a^2*b^2)^2)))/(d*(a^2 + b^2)) - (b*log(tan(c/2 + (d*x)/2)^2 + 1
))/(d*(a^2 + b^2))
```

3.115 $\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=47

$$-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$$

```
[Out] -arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/d/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = $\{3153, 212\}$

$$-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-1),x]
```

```
[Out] -(ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]/(Sqrt[a^2 + b^
2]*d))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx, x, b\cos(c+dx)-a\sin(c+dx)\right)}{d}\\ &= -\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 45, normalized size = 0.96

$$\frac{2\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-1),x]
```

```
[Out] (2*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(Sqrt[a^2 + b^2]*d)
```

Maple [A]
time = 0.21, size = 43, normalized size = 0.91

method	result	size
derivativedivides	$\frac{2\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$	43
default	$\frac{2\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{d\sqrt{a^2+b^2}}$	43
risch	$\frac{\ln\left(e^{i(dx+c)}+\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}-\frac{\ln\left(e^{i(dx+c)}-\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2
))
```

Maxima [A]
time = 0.47, size = 80, normalized size = 1.70

$$-\frac{\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d
*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(43) = 86$.
time = 2.70, size = 131, normalized size = 2.79

$$\frac{\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)}{2\,\sqrt{a^2+b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*
a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos
(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2))/(sqrt(a^2 + b^2
)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral(1/(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.43, size = 74, normalized size = 1.57

$$-\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/
2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*d)
```

Mupad [B]
time = 0.47, size = 39, normalized size = 0.83

$$-\frac{2\,\mathrm{atanh}\left(\frac{b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{d\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x)),x)
```

```
[Out] -(2*atanh((b - a*tan(c/2 + (d*x)/2))/(a^2 + b^2)^(1/2)))/(d*(a^2 + b^2)^(1/
2))
```

3.116 $\int \frac{\sec(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=41

$$-\frac{\log(\cos(c+dx))}{bd}+\frac{\log(a\cos(c+dx)+b\sin(c+dx))}{bd}$$

```
[Out] -ln(cos(d*x+c))/b/d+ln(a*cos(d*x+c)+b*sin(d*x+c))/b/d
```

Rubi [A]
time = 0.06, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.115$, Rules used = {3181, 3556, 3212}

$$\frac{\log(a\cos(c+dx)+b\sin(c+dx))}{bd}-\frac{\log(\cos(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/(b*d)) + Log[a*Cos[c + d*x] + b*Sin[c + d*x]]/(b*d)
```

Rule 3181

```
Int[1/(cos[(c_.) + (d_.)*(x_)]*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(
c_.) + (d_.)*(x_)])), x_Symbol] :> Dist[1/b, Int[Tan[c + d*x], x], x] + Dis
t[1/b, Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*
x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\int \frac{b\cos(c+dx)-a\sin(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{b} + \frac{\int \tan(c+dx)\,dx}{b}$$
$$= -\frac{\log(\cos(c+dx))}{bd} + \frac{\log(a\cos(c+dx)+b\sin(c+dx))}{bd}$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 0.44

$$\frac{\log(a+b\tan(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
[Out] Log[a + b*Tan[c + d*x]]/(b*d)
```

Maple [A]
time = 0.28, size = 19, normalized size = 0.46

method	result	size
derivativedivides	$\frac{\ln(a+b\tan(dx+c))}{db}$	19
default	$\frac{\ln(a+b\tan(dx+c))}{db}$	19
risch	$-\frac{\ln(e^{2i(dx+c)}+1)}{bd}+\frac{\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{bd}$	58
norman	$\frac{\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{bd}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{bd}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{bd}$	79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/b*ln(a+b*tan(d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(41) = 82.
time = 0.29, size = 103, normalized size = 2.51

$$\frac{\frac{\log\left(-a-\frac{2\,b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{b}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (log(-a - 2*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2)/b - log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b - log(sin(d*x +
c)/(cos(d*x + c) + 1) - 1)/b)/d
```

Fricas [A]
time = 2.57, size = 59, normalized size = 1.44

$$\frac{\log\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)-\log\left(\cos\left(dx+c\right)^2\right)}{2\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2
) - log(cos(d*x + c)^2))/(b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(c+dx\right)}{a\cos\left(c+dx\right)+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.44, size = 19, normalized size = 0.46

$$\frac{\log\left(\left|b\tan\left(dx+c\right)+a\right|\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] log(abs(b*tan(d*x + c) + a))/(b*d)
```

Mupad [B]
time = 0.72, size = 62, normalized size = 1.51

$$-\frac{2\,\mathrm{atanh}\left(\frac{b\,\left(b\,\cos(c+d\,x)-a\,\sin(c+d\,x)\right)}{2\,\cos(c+d\,x)\,a^2+\sin(c+d\,x)\,a\,b+\cos(c+d\,x)\,b^2}\right)}{b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a*cos(c + d*x) + b*sin(c + d*x))),x)
```

```
[Out] -(2*atanh((b*(b*cos(c + d*x) - a*sin(c + d*x)))/(2*a^2*cos(c + d*x) + b^2*c
os(c + d*x) + a*b*sin(c + d*x))))/(b*d)
```

3.117 $\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=80

$$-\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d}-\frac{\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}+\frac{\sec(c+dx)}{bd}$$

```
[Out] -a*arctanh(sin(d*x+c))/b^2/d+sec(d*x+c)/b/d-arctanh((b*cos(d*x+c)-a*sin(d*x
+c))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/b^2/d
```

Rubi [A]
time = 0.06, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3183, 3855, 3153, 212}

$$-\frac{\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d}-\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{\sec(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Sin[c + d*x]])/(b^2*d)) - (Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[c +
d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^2*d) + Sec[c + d*x]/(b*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3183

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\sec(c+dx)}{bd} - \frac{a\int \sec(c+dx)\,dx}{b^2} + \frac{(a^2+b^2)\int \frac{1}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{b^2}$$

$$= -\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d} + \frac{\sec(c+dx)}{bd} - \frac{(a^2+b^2)\,\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,d\right.}{}$$

$$= -\frac{a\tanh^{-1}(\sin(c+dx))}{b^2d} - \frac{\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d} +$$

Mathematica [A]
time = 0.15, size = 109, normalized size = 1.36

$$\frac{2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)+a\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+b\sec(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (2*Sqrt[a^2 + b^2]*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + a*(
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]]) + b*Sec[c + d*x])/(b^2*d)
```

Maple [A]
time = 0.45, size = 129, normalized size = 1.61

method	result
derivativedivides	$\frac{-\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}-\frac{2\left(-a^2-b^2\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\sqrt{a^2+b}}\right.}{b^2\sqrt{a^2+b^2}}}{d}$
default	$\frac{-\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{1}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}-\frac{2\left(-a^2-b^2\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\sqrt{a^2+b}}\right.}{b^2\sqrt{a^2+b^2}}}{d}$
risch	$\frac{2\,e^{i(dx+c)}}{db\left(e^{2i(dx+c)}+1\right)}+\frac{a\ln\left(e^{i(dx+c)}-i\right)}{b^2d}-\frac{a\ln\left(e^{i(dx+c)}+i\right)}{b^2d}+\frac{\sqrt{a^2+b^2}\,\ln\left(e^{i(dx+c)}+\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{d\,b^2}-\frac{\sqrt{a^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b/(tan(1/2*d*x+1/2*c)-1)+a/b^2*ln(tan(1/2*d*x+1/2*c)-1)+1/b/(tan(1/
2*d*x+1/2*c)+1)-a/b^2*ln(tan(1/2*d*x+1/2*c)+1)-2/b^2*(-a^2-b^2)/(a^2+b^2)^(
1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(76) = 152$.
time = 0.49, size = 163, normalized size = 2.04

$$-\frac{\frac{a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^2}-\frac{a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^2}+\frac{\sqrt{a^2+b^2}\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{b^2}-\frac{2}{b-\frac{b\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(a*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^2 - a*log(sin(d*x + c)/(cos(
d*x + c) + 1) - 1)/b^2 + sqrt(a^2 + b^2)*log((b - a*sin(d*x + c)/(cos(d*x +
 c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a
^2 + b^2)))/b^2 - 2/(b - b*sin(d*x + c)^2/(cos(d*x + c) + 1)^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(76) = 152$.
time = 3.23, size = 191, normalized size = 2.39

$$-\frac{a\cos(dx+c)\log(\sin(dx+c)+1)-a\cos(dx+c)\log(-\sin(dx+c)+1)-\sqrt{a^2+b^2}\cos(dx+c)\log\left(-\frac{2ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2a^2-b^2+2\sqrt{a^2+b^2}(b\cos(dx+c)-a\sin(dx+c))}{2ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)-2b}{2b^2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*cos(d*x + c)*log(sin(d*x + c) + 1) - a*cos(d*x + c)*log(-sin(d*x +
c) + 1) - sqrt(a^2 + b^2)*cos(d*x + c)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c
) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x
 + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos
(d*x + c)^2 + b^2)) - 2*b)/(b^2*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.49, size = 136, normalized size = 1.70

$$-\frac{\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^2}-\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^2}+\frac{\sqrt{a^2+b^2}\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{b^2}+\frac{2}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^2 - a*log(abs(tan(1/2*d*x + 1/2*c)
 - 1))/b^2 + sqrt(a^2 + b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqr
t(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/b^2
+ 2/((tan(1/2*d*x + 1/2*c)^2 - 1)*b))/d
```

Mupad [B]
time = 0.71, size = 310, normalized size = 3.88

$$\frac{2\,\mathrm{atanh}\left(\frac{64\,a^2\,\sqrt{a^2+b^2}}{64\,a^2\,b+\frac{64\,a^4}{b}+128\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+128\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{128\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^2+b^2}}{64\,a^2+\frac{64\,a^4}{b^2}+\frac{128\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b}+128\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{64\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^2+b^2}}{64\,a^4+128\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^3\,b+64\,a^2\,b^2+128\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b^3}\right)\sqrt{a^2+b^2}}{b^2\,d}-\frac{2\,a\,\mathrm{atanh}\left(\frac{64\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^2+\frac{64\,a^4}{b^2}}+\frac{64\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^4+64\,a^2\,b^2}\right)}{b^2\,d}-\frac{2}{b\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a*cos(c + d*x) + b*sin(c + d*x))),x)
```

```
[Out] (2*atanh((64*a^2*(a^2 + b^2)^(1/2))/(64*a^2*b + (64*a^4)/b + 128*a^3*tan(c/
2 + (d*x)/2) + 128*a*b^2*tan(c/2 + (d*x)/2)) + (128*a*tan(c/2 + (d*x)/2)*(a
^2 + b^2)^(1/2))/(64*a^2 + (64*a^4)/b^2 + (128*a^3*tan(c/2 + (d*x)/2))/b +
128*a*b*tan(c/2 + (d*x)/2)) + (64*a^3*tan(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2))
/(64*a^4 + 64*a^2*b^2 + 128*a*b^3*tan(c/2 + (d*x)/2) + 128*a^3*b*tan(c/2 +
(d*x)/2)))*(a^2 + b^2)^(1/2))/(b^2*d) - (2*a*atanh((64*a^2*tan(c/2 + (d*x)/
2))/(64*a^2 + (64*a^4)/b^2) + (64*a^4*tan(c/2 + (d*x)/2))/(64*a^4 + 64*a^2*
b^2)))/(b^2*d) - 2/(b*d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.118 $\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=88

$$-\frac{(a^2+b^2)\log(\cos(c+dx))}{b^3d}+\frac{(a^2+b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{b^3d}+\frac{\sec^2(c+dx)}{2bd}-\frac{a\tan(c+dx)}{b^2d}$$

```
[Out] -(a^2+b^2)*ln(cos(d*x+c))/b^3/d+(a^2+b^2)*ln(a*cos(d*x+c)+b*sin(d*x+c))/b^3
/d+1/2*sec(d*x+c)^2/b/d-a*tan(d*x+c)/b^2/d
```

Rubi [A]
time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3183, 3852, 8, 3181, 3556, 3212}

$$-\frac{(a^2+b^2)\log(\cos(c+dx))}{b^3d}+\frac{(a^2+b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{b^3d}-\frac{a\tan(c+dx)}{b^2d}+\frac{\sec^2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] -(((a^2 + b^2)*Log[Cos[c + d*x]])/(b^3*d)) + ((a^2 + b^2)*Log[a*Cos[c + d*x
] + b*Sin[c + d*x]])/(b^3*d) + Sec[c + d*x]^2/(2*b*d) - (a*Tan[c + d*x])/(b
^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3181

```
Int[1/(cos[(c_.) + (d_.)*(x_)]*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(
c_.) + (d_.)*(x_)])), x_Symbol] :> Dist[1/b, Int[Tan[c + d*x], x], x] + Dis
t[1/b, Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*
x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3183

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
```

```
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\sec^2(c+dx)}{2bd} - \frac{a\int \sec^2(c+dx)\,dx}{b^2} + \frac{(a^2+b^2)\int \frac{\sec(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,d}{b^2}$$

$$= \frac{\sec^2(c+dx)}{2bd} + \frac{(a^2+b^2)\int \frac{b\cos(c+dx)-a\sin(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{b^3} + \frac{(a^2+b^2)\int \tan(c}{b^3}$$

$$= -\frac{(a^2+b^2)\log(\cos(c+dx))}{b^3d} + \frac{(a^2+b^2)\log(a\cos(c+dx)+b\sin(c+}{b^3d}$$

Mathematica [A]
time = 0.32, size = 71, normalized size = 0.81

$$\frac{b^2\sec^2(c+dx) - 2((a^2+b^2)(\log(\cos(c+dx)) - \log(a\cos(c+dx)+b\sin(c+dx))) + ab\tan(c+dx))}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (b^2*Sec[c + d*x]^2 - 2*((a^2 + b^2)*(Log[Cos[c + d*x]] - Log[a*Cos[c + d*x
] + b*Sin[c + d*x]]) + a*b*Tan[c + d*x]))/(2*b^3*d)
```

Maple [A]
time = 0.35, size = 53, normalized size = 0.60

method	result

derivativedivides	$\frac{-\frac{-\frac{b\left(\tan^2(dx+c)\right)}{2}+a\tan(dx+c)}{b^2}+\frac{\left(a^2+b^2\right)\ln(a+b\tan(dx+c))}{b^3}}{d}$
default	$\frac{-\frac{-\frac{b\left(\tan^2(dx+c)\right)}{2}+a\tan(dx+c)}{b^2}+\frac{\left(a^2+b^2\right)\ln(a+b\tan(dx+c))}{b^3}}{d}$
risch	$\frac{-2ia\,e^{2i(dx+c)}+2b\,e^{2i(dx+c)}-2ia}{b^2d\left(e^{2i(dx+c)}+1\right)^2}-\frac{\ln(e^{2i(dx+c)}+1)a^2}{b^3d}-\frac{\ln(e^{2i(dx+c)}+1)}{bd}+\frac{\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)a^2}{b^3d}+\frac{\ln\left(e^{2i(dx+\cdots\right.}}{b\cdots}$
norman	$\frac{-\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2d}+\frac{2a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2d}+\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{bd}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{(a^2+b^2)\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}{b^3d}-\frac{(a^2+\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^2*(-1/2*b*tan(d*x+c)^2+a*tan(d*x+c))+(a^2+b^2)/b^3*ln(a+b*tan(d*x
+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(86) = 172$.

time = 0.27, size = 238, normalized size = 2.70

$$-\frac{\frac{2\left(\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\frac{b\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{b^2-\frac{2b^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{b^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{(a^2+b^2)\log\left(-a-\frac{2b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{b^3}+\frac{(a^2+b^2)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^3}+\frac{(a^2+b^2)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(2*(a*sin(d*x + c)/(cos(d*x + c) + 1) - b*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2 - a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(b^2 - 2*b^2*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + b^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - (a^2 + b^2)
*log(-a - 2*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2)/b^3 + (a^2 + b^2)*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^3
+ (a^2 + b^2)*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/b^3)/d
```

Fricas [A]

time = 2.52, size = 117, normalized size = 1.33

$$\frac{(a^2+b^2)\cos(dx+c)^2\log\left(2ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2\right)-(a^2+b^2)\cos(dx+c)^2\log\left(\cos(dx+c)^2\right)-2ab\cos(dx+c)\sin(dx+c)+b^2}{2b^3d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 + b^2)*cos(d*x + c)^2*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2
- b^2)*cos(d*x + c)^2 + b^2) - (a^2 + b^2)*cos(d*x + c)^2*log(cos(d*x + c)^
2) - 2*a*b*cos(d*x + c)*sin(d*x + c) + b^2)/(b^3*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.45, size = 54, normalized size = 0.61

$$\frac{\frac{b\tan(dx+c)^2-2\,a\tan(dx+c)}{b^2}+\frac{2\,(a^2+b^2)\log(|b\tan(dx+c)+a|)}{b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((b*tan(d*x + c)^2 - 2*a*tan(d*x + c))/b^2 + 2*(a^2 + b^2)*log(abs(b*ta
n(d*x + c) + a))/b^3)/d
```

Mupad [B]
time = 1.50, size = 300, normalized size = 3.41

$$\frac{2\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-2\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b^3\right)}-\frac{a^2\,\mathrm{atan}\left(\frac{b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,1\mathrm{i}-b^2\,1\mathrm{i}+2\mathrm{i}\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{-2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^2+2\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b^2}\right)\,2\mathrm{i}+b^2\,\mathrm{atan}\left(\frac{b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,1\mathrm{i}-b^2\,1\mathrm{i}+2\mathrm{i}\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{-2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,a^2+2\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b^2}\right)\,2\mathrm{i}}{b^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a*cos(c + d*x) + b*sin(c + d*x))),x)
```

```
[Out] (2*b^2*tan(c/2 + (d*x)/2)^2 + 2*a*b*tan(c/2 + (d*x)/2)^3 - 2*a*b*tan(c/2 +
(d*x)/2))/(d*(b^3*tan(c/2 + (d*x)/2)^4 - 2*b^3*tan(c/2 + (d*x)/2)^2 + b^3))
 - (a^2*atan((b^2*tan(c/2 + (d*x)/2)^2*1i - b^2*1i + a*b*tan(c/2 + (d*x)/2)
*2i)/(2*a^2 - b^2*tan(c/2 + (d*x)/2)^2 - 2*a^2*tan(c/2 + (d*x)/2)^2 + b^2 +
 2*a*b*tan(c/2 + (d*x)/2)))*2i + b^2*atan((b^2*tan(c/2 + (d*x)/2)^2*1i - b^
2*1i + a*b*tan(c/2 + (d*x)/2)*2i)/(2*a^2 - b^2*tan(c/2 + (d*x)/2)^2 - 2*a^2
*tan(c/2 + (d*x)/2)^2 + b^2 + 2*a*b*tan(c/2 + (d*x)/2)))*2i)/(b^3*d)
```

3.119 $\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=153

$$-\frac{a\tanh^{-1}(\sin(c+dx))}{2b^2d}-\frac{a(a^2+b^2)\tanh^{-1}(\sin(c+dx))}{b^4d}-\frac{(a^2+b^2)^{3/2}\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4d}+\frac{(a^2+$$

[Out] -1/2*a*arctanh(sin(d*x+c))/b^2/d-a*(a^2+b^2)*arctanh(sin(d*x+c))/b^4/d-(a^2+b^2)^(3/2)*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/b^4/d+(a^2+b^2)*sec(d*x+c)/b^3/d+1/3*sec(d*x+c)^3/b/d-1/2*a*sec(d*x+c)*tan(d*x+c)/b^2/d

Rubi [A]
time = 0.12, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3183, 3853, 3855, 3153, 212}

$$-\frac{a(a^2+b^2)\tanh^{-1}(\sin(c+dx))}{b^4d}-\frac{(a^2+b^2)^{3/2}\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4d}+\frac{(a^2+b^2)\sec(c+dx)}{b^3d}-\frac{a\tanh^{-1}(\sin(c+dx))}{2b^2d}-\frac{a\tan(c+dx)\sec(c+dx)}{2b^2d}+\frac{\sec^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]

[Out] -1/2*(a*ArcTanh[Sin[c + d*x]])/(b^2*d) - (a*(a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(b^4*d) - ((a^2 + b^2)^(3/2)*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^4*d) + ((a^2 + b^2)*Sec[c + d*x])/(b^3*d) + Sec[c + d*x]^3/(3*b*d) - (a*Sec[c + d*x]*Tan[c + d*x])/(2*b^2*d)

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3183

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
```

```
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\sec^3(c+dx)}{3bd} - \frac{a\int \sec^3(c+dx)\,dx}{b^2} + \frac{(a^2+b^2)\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}}{b^2}$$

$$= \frac{(a^2+b^2)\sec(c+dx)}{b^3d} + \frac{\sec^3(c+dx)}{3bd} - \frac{a\sec(c+dx)\tan(c+dx)}{2b^2d} -$$

$$= -\frac{a\tanh^{-1}(\sin(c+dx))}{2b^2d} - \frac{a(a^2+b^2)\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{(a^2+b^2}{}$$

$$= -\frac{a\tanh^{-1}(\sin(c+dx))}{2b^2d} - \frac{a(a^2+b^2)\tanh^{-1}(\sin(c+dx))}{b^4d} - \frac{(a^2+b^2}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 321 vs. 2(153) = 306.
time = 2.20, size = 321, normalized size = 2.10

$$\frac{48(a^2+b^2)^{3/2}\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)+\sec^3(c+dx)\left(12a^2b+20b^3+12b(a^2+b^2)\cos(2(c+dx))+6a^3\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+9ab^2\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+9a(2a^2+3b^2)\cos(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-6a^3\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-9ab^2\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-6ab^2\sin(2(c+dx))\right)}{24b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (48*(a^2 + b^2)^(3/2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] +
Sec[c + d*x]^3*(12*a^2*b + 20*b^3 + 12*b*(a^2 + b^2)*Cos[2*(c + d*x)] + 6*a
^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 9*a*b^2*Cos[
3*(c + d*x)]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 9*a*(2*a^2 + 3*b^2)
*Cos[c + d*x]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)
```

```
/2] + Sin[(c + d*x)/2]]) - 6*a^3*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] - 9*a*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] - 6*a*b^2*Sin[2*(c + d*x)]))/(24*b^4*d)
```

Maple [A]
time = 0.62, size = 269, normalized size = 1.76

method	result
derivativedivides	$\frac{-\frac{1}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{a+b}{2b^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{2a^{2}+ab+3b^{2}}{2b^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\left(2a^{2}+3b^{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^{4}}+\frac{1}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}}{}$
default	$\frac{-\frac{1}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{a+b}{2b^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{2a^{2}+ab+3b^{2}}{2b^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\left(2a^{2}+3b^{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^{4}}+\frac{1}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}}{}$
risch	$\frac{3iab\,e^{5i(dx+c)}+6a^{2}e^{5i(dx+c)}+6b^{2}e^{5i(dx+c)}+12a^{2}e^{3i(dx+c)}+20b^{2}e^{3i(dx+c)}-3iab\,e^{i(dx+c)}+6a^{2}e^{i(dx+c)}+6b^{2}e^{i(dx+c)}}{3d\,b^{3}\left(e^{2i(dx+c)}+1\right)^{3}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3/b/(tan(1/2*d*x+1/2*c)-1)^3-1/2*(a+b)/b^2/(tan(1/2*d*x+1/2*c)-1)^2
-1/2*(2*a^2+a*b+3*b^2)/b^3/(tan(1/2*d*x+1/2*c)-1)+1/2*a*(2*a^2+3*b^2)/b^4*l
n(tan(1/2*d*x+1/2*c)-1)+1/3/b/(tan(1/2*d*x+1/2*c)+1)^3-1/2*(b-a)/b^2/(tan(1
/2*d*x+1/2*c)+1)^2-1/2*(-2*a^2+a*b-3*b^2)/b^3/(tan(1/2*d*x+1/2*c)+1)-1/2*a*
(2*a^2+3*b^2)/b^4*ln(tan(1/2*d*x+1/2*c)+1)-2/b^4*(-a^4-2*a^2*b^2-b^4)/(a^2+
b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 361 vs. 2(143) = 286.
time = 0.50, size = 361, normalized size = 2.36

$$\frac{\frac{2\left(6a^{2}+8b^{2}-\frac{3ab\sin(dx+c)}{\cos(dx+c)+1}+\frac{3ab\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{12\left(a^{2}+b^{2}\right)\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{6\left(a^{2}+2b^{2}\right)\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}\right)}{b^{3}-\frac{3b^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3b^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{b^{3}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}}-\frac{3\left(2a^{3}+3ab^{2}\right)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^{4}}+\frac{3\left(2a^{3}+3ab^{2}\right)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^{4}}-\frac{6\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^{2}+b^{2}}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^{2}+b^{2}}}\right)}{\sqrt{a^{2}+b^{2}}\,b^{4}}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*(6*a^2 + 8*b^2 - 3*a*b*sin(d*x + c)/(cos(d*x + c) + 1) + 3*a*b*sin(d
*x + c)^5/(cos(d*x + c) + 1)^5 - 12*(a^2 + b^2)*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 + 6*(a^2 + 2*b^2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/(b^3 - 3*b^
3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*b^3*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4 - b^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) - 3*(2*a^3 + 3*a*b^2)*log
```

```
(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^4 + 3*(2*a^3 + 3*a*b^2)*log(sin(d*x
 + c)/(cos(d*x + c) + 1) - 1)/b^4 - 6*(a^4 + 2*a^2*b^2 + b^4)*log((b - a*si
n(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d
*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4))/d
```

Fricas [A]
time = 1.24, size = 259, normalized size = 1.69

$$\frac{6\,(a^2+b^2)^{\frac{3}{2}}\cos\left(dx+c\right)^3\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)-3\,(2\,a^3+3\,ab^2)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)+3\,(2\,a^3+3\,ab^2)\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)-6\,ab^2\cos\left(dx+c\right)\sin\left(dx+c\right)+4\,b^3+12\,(a^2b+b^3)\cos\left(dx+c\right)^2}{12\,b^4d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(6*(a^2 + b^2)^(3/2)*cos(d*x + c)^3*log(-(2*a*b*cos(d*x + c)*sin(d*x +
 c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d
*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*c
os(d*x + c)^2 + b^2)) - 3*(2*a^3 + 3*a*b^2)*cos(d*x + c)^3*log(sin(d*x + c)
 + 1) + 3*(2*a^3 + 3*a*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 6*a*b^2
*cos(d*x + c)*sin(d*x + c) + 4*b^3 + 12*(a^2*b + b^3)*cos(d*x + c)^2)/(b^4*
d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4\left(c+dx\right)}{a\cos\left(c+dx\right)+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**4/(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.49, size = 278, normalized size = 1.82

$$-\frac{\frac{3\,(2\,a^3+3\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^4}-\frac{3\,(2\,a^3+3\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^4}+\frac{6\,(a^4+2\,a^2b^2+b^4)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^4}+\frac{2\left(3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-12\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,a^2+8\,b^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3b^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6*(3*(2*a^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 3*(2*a^3
 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 + 6*(a^4 + 2*a^2*b^2 + b
^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan
(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4) + 2*(3*
a*b*tan(1/2*d*x + 1/2*c)^5 + 6*a^2*tan(1/2*d*x + 1/2*c)^4 + 12*b^2*tan(1/2*
```

```
d*x + 1/2*c)^4 - 12*a^2*tan(1/2*d*x + 1/2*c)^2 - 12*b^2*tan(1/2*d*x + 1/2*c
)^2 - 3*a*b*tan(1/2*d*x + 1/2*c) + 6*a^2 + 8*b^2)/((tan(1/2*d*x + 1/2*c)^2
- 1)^3*b^3))/d
```

Mupad [B]
time = 2.21, size = 724, normalized size = 4.73

$$\frac{b^3\left(\cos\left(c+d\,x\right)+\frac{\cos\left(2\,c+2\,d\,x\right)}{2}+\frac{\cos\left(3\,c+3\,d\,x\right)}{3}+\frac{5}{6}\right)-b^2\left(\frac{a\,\sin\left(2\,c+2\,d\,x\right)}{4}+\frac{3\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\cos\left(3\,c+3\,d\,x\right)}{4}+\frac{9\,a\,\cos\left(c+d\,x\right)\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4}\right)+b\left(\frac{3\,a^2\,\cos\left(c+d\,x\right)}{4}+\frac{a^2}{2}+\frac{a^2\,\cos\left(2\,c+2\,d\,x\right)}{2}+\frac{a^2\,\cos\left(3\,c+3\,d\,x\right)}{4}\right)+\frac{\mathrm{atanh}\left(\frac{a^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+2\,b^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+a\,b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^5+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^4\,b+2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3\,b^2+4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^3+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^4+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^5}\right)\,\cos\left(3\,c+3\,d\,x\right)\,\sqrt{{\left(a^2+b^2\right)}^3}}{2}-\frac{3\,a^3\,\cos\left(c+d\,x\right)\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}-\frac{a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\cos\left(3\,c+3\,d\,x\right)}{2}+\frac{3\,\cos\left(c+d\,x\right)\,\mathrm{atanh}\left(\frac{a^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+2\,b^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+a\,b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^5+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^4\,b+2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3\,b^2+4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^3+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^4+2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^5}\right)\,\sqrt{{\left(a^2+b^2\right)}^3}}{2}}{b^4\,d\,\left(\frac{3\,\cos\left(c+d\,x\right)}{4}+\frac{\cos\left(3\,c+3\,d\,x\right)}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a*cos(c + d*x) + b*sin(c + d*x))),x)
```

```
[Out] (b^3*(cos(c + d*x) + cos(2*c + 2*d*x)/2 + cos(3*c + 3*d*x)/3 + 5/6) - b^2*(
(a*sin(2*c + 2*d*x))/4 + (3*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*
cos(3*c + 3*d*x))/4 + (9*a*cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 +
(d*x)/2)))/4) + b*((3*a^2*cos(c + d*x))/4 + a^2/2 + (a^2*cos(2*c + 2*d*x))/
2 + (a^2*cos(3*c + 3*d*x))/4) + (atanh((a^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 +
 3*a^2*b^4 + 3*a^4*b^2)^(1/2) + 2*b^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2
*b^4 + 3*a^4*b^2)^(1/2) + a*b*cos(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3
*a^4*b^2)^(1/2))/(a^5*cos(c/2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*b^4
*cos(c/2 + (d*x)/2) + 2*a^4*b*sin(c/2 + (d*x)/2) + 2*a^3*b^2*cos(c/2 + (d*x
)/2) + 4*a^2*b^3*sin(c/2 + (d*x)/2)))*cos(3*c + 3*d*x)*((a^2 + b^2)^3)^(1/2
))/2 - (3*a^3*cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2
- (a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x))/2 + (
3*cos(c + d*x)*atanh((a^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4
*b^2)^(1/2) + 2*b^2*sin(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^
(1/2) + a*b*cos(c/2 + (d*x)/2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/(
a^5*cos(c/2 + (d*x)/2) + 2*b^5*sin(c/2 + (d*x)/2) + a*b^4*cos(c/2 + (d*x)/2
) + 2*a^4*b*sin(c/2 + (d*x)/2) + 2*a^3*b^2*cos(c/2 + (d*x)/2) + 4*a^2*b^3*s
in(c/2 + (d*x)/2)))*((a^2 + b^2)^3)^(1/2))/2)/(b^4*d*((3*cos(c + d*x))/4 +
cos(3*c + 3*d*x)/4))
```

3.120 $\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=158

$$-\frac{(a^2+b^2)^2\log(\cos(c+dx))}{b^5d}+\frac{(a^2+b^2)^2\log(a\cos(c+dx)+b\sin(c+dx))}{b^5d}+\frac{(a^2+b^2)\sec^2(c+dx)}{2b^3d}+\frac{\sec^4(c$$

```
[Out] -(a^2+b^2)^2*ln(cos(d*x+c))/b^5/d+(a^2+b^2)^2*ln(a*cos(d*x+c)+b*sin(d*x+c))
/b^5/d+1/2*(a^2+b^2)*sec(d*x+c)^2/b^3/d+1/4*sec(d*x+c)^4/b/d-a*tan(d*x+c)/b
^2/d-a*(a^2+b^2)*tan(d*x+c)/b^4/d-1/3*a*tan(d*x+c)^3/b^2/d
```

Rubi [A]
time = 0.16, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3183, 3852, 8, 3181, 3556, 3212}

$$-\frac{(a^2+b^2)^2\log(\cos(c+dx))}{b^5d}+\frac{(a^2+b^2)^2\log(a\cos(c+dx)+b\sin(c+dx))}{b^5d}-\frac{a(a^2+b^2)\tan(c+dx)}{b^4d}+\frac{(a^2+b^2)\sec^2(c+dx)}{2b^3d}-\frac{a\tan^3(c+dx)}{3b^2d}-\frac{a\tan(c+dx)}{b^2d}+\frac{\sec^4(c+dx)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] -(((a^2 + b^2)^2*Log[Cos[c + d*x]])/(b^5*d)) + ((a^2 + b^2)^2*Log[a*Cos[c +
 d*x] + b*Sin[c + d*x]])/(b^5*d) + ((a^2 + b^2)*Sec[c + d*x]^2)/(2*b^3*d) +
 Sec[c + d*x]^4/(4*b*d) - (a*Tan[c + d*x])/(b^2*d) - (a*(a^2 + b^2)*Tan[c +
 d*x])/(b^4*d) - (a*Tan[c + d*x]^3)/(3*b^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3181

```
Int[1/(cos[(c_.) + (d_.)*(x_)]*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(
c_.) + (d_.)*(x_)])), x_Symbol] :> Dist[1/b, Int[Tan[c + d*x], x], x] + Dis
t[1/b, Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*
x]), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3183

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx &= \frac{\sec^4(c+dx)}{4bd}-\frac{a\int \sec^4(c+dx)\,dx}{b^2}+\frac{(a^2+b^2)\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{b^2}\\ &= \frac{(a^2+b^2)\sec^2(c+dx)}{2b^3d}+\frac{\sec^4(c+dx)}{4bd}-\frac{(a(a^2+b^2))\int \sec^2(c+dx)\,dx}{b^4}\\ &= \frac{(a^2+b^2)\sec^2(c+dx)}{2b^3d}+\frac{\sec^4(c+dx)}{4bd}-\frac{a\tan(c+dx)}{b^2d}-\frac{a\tan^3(c+dx)}{3b^2d}\\ &= -\frac{(a^2+b^2)^2\log(\cos(c+dx))}{b^5d}+\frac{(a^2+b^2)^2\log(a\cos(c+dx)+b\sin(c+\end{aligned}$$

Mathematica [A]
time = 1.27, size = 119, normalized size = 0.75

$$\frac{3b^4\sec^4(c+dx)+2b^2\sec^2(c+dx)\left(3(a^2+b^2)-2ab\tan(c+dx)\right)-4\left(3(a^2+b^2)^2\left(\log(\cos(c+dx))-\log(a\cos(c+dx)+b\sin(c+dx))\right)+ab(3a^2+5b^2)\tan(c+dx)\right)}{12b^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (3*b^4*Sec[c + d*x]^4 + 2*b^2*Sec[c + d*x]^2*(3*(a^2 + b^2) - 2*a*b*Tan[c +
 d*x]) - 4*(3*(a^2 + b^2)^2*(Log[Cos[c + d*x]] - Log[a*Cos[c + d*x] + b*Sin
[c + d*x]]) + a*b*(3*a^2 + 5*b^2)*Tan[c + d*x]))/(12*b^5*d)
```

Maple [A]
time = 0.48, size = 106, normalized size = 0.67

method	result
derivativedivides	$\frac{-\frac{-\frac{\left(\tan^4(dx+c)\right)b^3}{4}+\frac{a\left(\tan^3(dx+c)\right)b^2}{3}-\frac{\left(a^2+2b^2\right)\left(\tan^2(dx+c)\right)b}{2}+a\left(a^2+2b^2\right)\tan(dx+c)}{b^4}+\frac{\left(a^4+2a^2b^2+b^4\right)\ln(a+b\tan(dx+c))}{b^5}}{d}$
default	$\frac{-\frac{-\frac{\left(\tan^4(dx+c)\right)b^3}{4}+\frac{a\left(\tan^3(dx+c)\right)b^2}{3}-\frac{\left(a^2+2b^2\right)\left(\tan^2(dx+c)\right)b}{2}+a\left(a^2+2b^2\right)\tan(dx+c)}{b^4}+\frac{\left(a^4+2a^2b^2+b^4\right)\ln(a+b\tan(dx+c))}{b^5}}{d}$
norman	$\frac{-\frac{2\left(2a^2+2b^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3d}+\frac{2\left(a^2+2b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3d}+\frac{2\left(a^2+2b^2\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3d}-\frac{2a\left(a^2+2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^4d}+\frac{2a\left(a^2+2\right.}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}$
risch	$\frac{-2ia^3e^{6i(dx+c)}-2ia\,b^2e^{6i(dx+c)}+2a^2b\,e^{6i(dx+c)}+2b^3e^{6i(dx+c)}-6ia^3e^{4i(dx+c)}-10ia\,b^2e^{4i(dx+c)}+4a^2b\,e^{4i(dx+c)}+8b^3e}{b^4d\left(e^{2i(dx+c)}+1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^4*(-1/4*tan(d*x+c)^4*b^3+1/3*a*tan(d*x+c)^3*b^2-1/2*(a^2+2*b^2)*t
an(d*x+c)^2*b+a*(a^2+2*b^2)*tan(d*x+c))+(a^4+2*a^2*b^2+b^4)/b^5*ln(a+b*tan(
d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 462 vs. 2(152) = 304.
time = 0.28, size = 462, normalized size = 2.92

$$-\frac{\frac{2\left(\frac{3\left(a^3+2ab^2\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\left(a^2b+2b^3\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\left(9a^3+14ab^2\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{6\left(a^2b+b^3\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\left(9a^3+14ab^2\right)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3\left(a^2b+2b^3\right)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{3\left(a^3+2ab^2\right)\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{b^4-\frac{4b^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6b^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4b^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{b^4\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{3\left(a^4+2a^2b^2+b^4\right)\log\left(-a-\frac{2b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{b^5}+\frac{3\left(a^4+2a^2b^2+b^4\right)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^5}+\frac{3\left(a^4+2a^2b^2+b^4\right)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^5}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/3*(2*(3*(a^3 + 2*a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) - 3*(a^2*b + 2*b
^3)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - (9*a^3 + 14*a*b^2)*sin(d*x + c)^3
/(cos(d*x + c) + 1)^3 + 6*(a^2*b + b^3)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
 + (9*a^3 + 14*a*b^2)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*(a^2*b + 2*b^
3)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 3*(a^3 + 2*a*b^2)*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7)/(b^4 - 4*b^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*b
^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*b^4*sin(d*x + c)^6/(cos(d*x + c)
 + 1)^6 + b^4*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 3*(a^4 + 2*a^2*b^2 + b
^4)*log(-a - 2*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2)/b^5 + 3*(a^4 + 2*a^2*b^2 + b^4)*log(sin(d*x + c)/(cos(d*x +
c) + 1) + 1)/b^5 + 3*(a^4 + 2*a^2*b^2 + b^4)*log(sin(d*x + c)/(cos(d*x + c)
 + 1) - 1)/b^5)/d
```

Fricas [A]
time = 1.93, size = 183, normalized size = 1.16

$$\frac{6\,(a^4+2\,a^2b^2+b^4)\cos(dx+c)^4\log\left(2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2\right)-6\,(a^4+2\,a^2b^2+b^4)\cos(dx+c)^4\log\left(\cos(dx+c)^2\right)+3\,b^4+6\,(a^2b^2+b^4)\cos(dx+c)^2-4\,\left(ab^3\cos(dx+c)+(3\,a^3b+5\,ab^3)\cos(dx+c)^3\right)\sin(dx+c)}{12\,b^5d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(6*(a^4 + 2*a^2*b^2 + b^4)*cos(d*x + c)^4*log(2*a*b*cos(d*x + c)*sin(d
*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 6*(a^4 + 2*a^2*b^2 + b^4)*cos
(d*x + c)^4*log(cos(d*x + c)^2) + 3*b^4 + 6*(a^2*b^2 + b^4)*cos(d*x + c)^2
- 4*(a*b^3*cos(d*x + c) + (3*a^3*b + 5*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))
/(b^5*d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**5/(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

Giac [A]
time = 0.47, size = 120, normalized size = 0.76

$$\frac{\frac{3\,b^3\tan(dx+c)^4-4\,ab^2\tan(dx+c)^3+6\,a^2b\tan(dx+c)^2+12\,b^3\tan(dx+c)^2-12\,a^3\tan(dx+c)-24\,ab^2\tan(dx+c)}{b^4}+\frac{12\,(a^4+2\,a^2b^2+b^4)\log(|b\tan(dx+c)+a|)}{b^5}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*((3*b^3*tan(d*x + c)^4 - 4*a*b^2*tan(d*x + c)^3 + 6*a^2*b*tan(d*x + c)
^2 + 12*b^3*tan(d*x + c)^2 - 12*a^3*tan(d*x + c) - 24*a*b^2*tan(d*x + c))/b
^4 + 12*(a^4 + 2*a^2*b^2 + b^4)*log(abs(b*tan(d*x + c) + a))/b^5)/d
```

Mupad [B]
time = 3.68, size = 575, normalized size = 3.64

$$\frac{(6\,a^3b+12\,ab^3)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+(6\,a^2b^2+12\,b^4)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+(-18\,a^3b-28\,ab^3)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+(-12\,a^2b^2-12\,b^4)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+(18\,a^3b+28\,ab^3)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+(6\,a^2b^2+12\,b^4)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+(-6\,a^3b-12\,ab^3)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(3\,b^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-12\,b^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+18\,b^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-12\,b^5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+3\,b^5\right)}-\frac{a^4\,\mathrm{atan}\left(\frac{b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\,1\mathrm{i}-b^2\,1\mathrm{i}+2\mathrm{i}\,ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{-2\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,a^2+2\,ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b^2}\right)2\mathrm{i}+b^4\,\mathrm{atan}\left(\frac{b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\,1\mathrm{i}-b^2\,1\mathrm{i}+2\mathrm{i}\,ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{-2\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,a^2+2\,ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b^2}\right)2\mathrm{i}+a^2b^2\,\mathrm{atan}\left(\frac{b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\,1\mathrm{i}-b^2\,1\mathrm{i}+2\mathrm{i}\,ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{-2\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2\,a^2+2\,ab\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-b^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b^2}\right)4\mathrm{i}}{b^5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a*cos(c + d*x) + b*sin(c + d*x))),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*(12*b^4 + 6*a^2*b^2) - tan(c/2 + (d*x)/2)*(12*a*b^3 +
 6*a^3*b) + tan(c/2 + (d*x)/2)^6*(12*b^4 + 6*a^2*b^2) - tan(c/2 + (d*x)/2)^
4*(12*b^4 + 12*a^2*b^2) + tan(c/2 + (d*x)/2)^7*(12*a*b^3 + 6*a^3*b) + tan(c
/2 + (d*x)/2)^3*(28*a*b^3 + 18*a^3*b) - tan(c/2 + (d*x)/2)^5*(28*a*b^3 + 18
*a^3*b))/(d*(18*b^5*tan(c/2 + (d*x)/2)^4 - 12*b^5*tan(c/2 + (d*x)/2)^2 - 12
*b^5*tan(c/2 + (d*x)/2)^6 + 3*b^5*tan(c/2 + (d*x)/2)^8 + 3*b^5)) - (a^4*ata
n((b^2*tan(c/2 + (d*x)/2)^2*1i - b^2*1i + a*b*tan(c/2 + (d*x)/2)*2i)/(2*a^2
 - b^2*tan(c/2 + (d*x)/2)^2 - 2*a^2*tan(c/2 + (d*x)/2)^2 + b^2 + 2*a*b*tan(
c/2 + (d*x)/2)))*2i + b^4*atan((b^2*tan(c/2 + (d*x)/2)^2*1i - b^2*1i + a*b*
tan(c/2 + (d*x)/2)*2i)/(2*a^2 - b^2*tan(c/2 + (d*x)/2)^2 - 2*a^2*tan(c/2 +
(d*x)/2)^2 + b^2 + 2*a*b*tan(c/2 + (d*x)/2)))*2i + a^2*b^2*atan((b^2*tan(c/
2 + (d*x)/2)^2*1i - b^2*1i + a*b*tan(c/2 + (d*x)/2)*2i)/(2*a^2 - b^2*tan(c/
2 + (d*x)/2)^2 - 2*a^2*tan(c/2 + (d*x)/2)^2 + b^2 + 2*a*b*tan(c/2 + (d*x)/2
)))*4i)/(b^5*d)
```

3.121 $\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx$

Optimal. Leaf size=262

$$-\frac{3a\tanh^{-1}(\sin(c+dx))}{8b^2d}-\frac{a(a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2b^4d}-\frac{a(a^2+b^2)^2\tanh^{-1}(\sin(c+dx))}{b^6d}-\frac{(a^2+b^2)^{5/2}\tan$$

```
[Out] -3/8*a*arctanh(sin(d*x+c))/b^2/d-1/2*a*(a^2+b^2)*arctanh(sin(d*x+c))/b^4/d-
a*(a^2+b^2)^2*arctanh(sin(d*x+c))/b^6/d-(a^2+b^2)^(5/2)*arctanh((b*cos(d*x+
c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/b^6/d+(a^2+b^2)^2*sec(d*x+c)/b^5/d+1/3*(a
^2+b^2)*sec(d*x+c)^3/b^3/d+1/5*sec(d*x+c)^5/b/d-3/8*a*sec(d*x+c)*tan(d*x+c)
/b^2/d-1/2*a*(a^2+b^2)*sec(d*x+c)*tan(d*x+c)/b^4/d-1/4*a*sec(d*x+c)^3*tan(d
*x+c)/b^2/d
```

Rubi [A]
time = 0.20, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3183, 3853, 3855, 3153, 212}

$$-\frac{a(a^2+b^2)^2\tanh^{-1}(\sin(c+dx))}{b^6d}-\frac{(a^2+b^2)^{5/2}\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^6d}+\frac{(a^2+b^2)^2\sec(c+dx)}{b^5d}-\frac{a(a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2b^4d}-\frac{a(a^2+b^2)\tan(c+dx)\sec(c+dx)}{2b^4d}+\frac{(a^2+b^2)\sec^3(c+dx)}{3b^3d}-\frac{3a\tanh^{-1}(\sin(c+dx))}{8b^2d}-\frac{a\tan(c+dx)\sec^3(c+dx)}{4b^2d}-\frac{3a\tan(c+dx)\sec(c+dx)}{8b^2d}+\frac{\sec^5(c+dx)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (-3*a*ArcTanh[Sin[c + d*x]])/(8*b^2*d) - (a*(a^2 + b^2)*ArcTanh[Sin[c + d*x
]])/(2*b^4*d) - (a*(a^2 + b^2)^2*ArcTanh[Sin[c + d*x]])/(b^6*d) - ((a^2 + b
^2)^(5/2)*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^6*
d) + ((a^2 + b^2)^2*Sec[c + d*x])/(b^5*d) + ((a^2 + b^2)*Sec[c + d*x]^3)/(3
*b^3*d) + Sec[c + d*x]^5/(5*b*d) - (3*a*Sec[c + d*x]*Tan[c + d*x])/(8*b^2*d
) - (a*(a^2 + b^2)*Sec[c + d*x]*Tan[c + d*x])/(2*b^4*d) - (a*Sec[c + d*x]^3
*Tan[c + d*x])/(4*b^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3183

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx = \frac{\sec^5(c+dx)}{5bd} - \frac{a\int \sec^5(c+dx)\,dx}{b^2} + \frac{(a^2+b^2)\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,d}{b^2}$$

$$= \frac{(a^2+b^2)\sec^3(c+dx)}{3b^3d} + \frac{\sec^5(c+dx)}{5bd} - \frac{a\sec^3(c+dx)\tan(c+dx)}{4b^2d} -$$

$$= \frac{(a^2+b^2)^2\sec(c+dx)}{b^5d} + \frac{(a^2+b^2)\sec^3(c+dx)}{3b^3d} + \frac{\sec^5(c+dx)}{5bd} - \frac{3a\,\mathrm{s}}{}$$

$$= -\frac{3a\tanh^{-1}(\sin(c+dx))}{8b^2d} - \frac{a(a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{a(a^2+}{}$$

$$= -\frac{3a\tanh^{-1}(\sin(c+dx))}{8b^2d} - \frac{a(a^2+b^2)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{a(a^2+}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 661 vs. 2(262) = 524.
time = 5.42, size = 661, normalized size = 2.52

$$\frac{\sec(c+dx)\left(240a^4b+520a^2b^3+298b^5+480(a^2+b^2)^{5/2}\tanh^{-1}\left(\frac{-b+a\tan(\frac{1}{2}(c+dx))}{\sqrt{a^2+b^2}}\right)+30a(8a^4+20a^2b^2+15b^4)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-30a(8a^4+20a^2b^2+15b^4)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{3b^4(-5a+2b)}{(\cos(\frac{1}{2}(c+dx))-\sin(\frac{1}{2}(c+dx)))^4}+\frac{b^2(-60a^3+20a^2b-105ab^2+29b^3)}{(\cos(\frac{1}{2}(c+dx))-\sin(\frac{1}{2}(c+dx)))^2}+\frac{12b^5\sin(\frac{1}{2}(c+dx))}{(\cos(\frac{1}{2}(c+dx))-\sin(\frac{1}{2}(c+dx)))^5}+\frac{2b^3(20a^2+29b^2)\sin(\frac{1}{2}(c+dx))}{(\cos(\frac{1}{2}(c+dx))-\sin(\frac{1}{2}(c+dx)))^3}+\frac{2b(120a^4+260a^2b^2+149b^4)\sin(\frac{1}{2}(c+dx))}{\cos(\frac{1}{2}(c+dx))-\sin(\frac{1}{2}(c+dx))}-\frac{12b^5\sin(\frac{1}{2}(c+dx))}{(\cos(\frac{1}{2}(c+dx))+\sin(\frac{1}{2}(c+dx)))^5}+\frac{3b^4(5a+2b)}{(\cos(\frac{1}{2}(c+dx))+\sin(\frac{1}{2}(c+dx)))^4}-\frac{2b^3(20a^2+29b^2)\sin(\frac{1}{2}(c+dx))}{(\cos(\frac{1}{2}(c+dx))+\sin(\frac{1}{2}(c+dx)))^3}+\frac{b^2(60a^3+20a^2b+105ab^2+29b^3)}{(\cos(\frac{1}{2}(c+dx))+\sin(\frac{1}{2}(c+dx)))^2}-\frac{2b(120a^4+260a^2b^2+149b^4)\sin(\frac{1}{2}(c+dx))}{\cos(\frac{1}{2}(c+dx))+\sin(\frac{1}{2}(c+dx))}\right)(a\cos(c+dx)+b\sin(c+dx))}{240b^6d(a+b\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a*Cos[c + d*x] + b*Sin[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]*(240*a^4*b + 520*a^2*b^3 + 298*b^5 + 480*(a^2 + b^2)^(5/2)*Ar
cTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]] + 30*a*(8*a^4 + 20*a^2*b^2
 + 15*b^4)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 30*a*(8*a^4 + 20*a^2*
b^2 + 15*b^4)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (3*b^4*(-5*a + 2*b
))/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^4 + (b^2*(-60*a^3 + 20*a^2*b - 105
*a*b^2 + 29*b^3))/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (12*b^5*Sin[(c
+ d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^5 + (2*b^3*(20*a^2 + 29*b^
2)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (2*b*(120*a^
4 + 260*a^2*b^2 + 149*b^4)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]) - (12*b^5*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^5
 + (3*b^4*(5*a + 2*b))/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^4 - (2*b^3*(20
*a^2 + 29*b^2)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 +
(b^2*(60*a^3 + 20*a^2*b + 105*a*b^2 + 29*b^3))/(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])^2 - (2*b*(120*a^4 + 260*a^2*b^2 + 149*b^4)*Sin[(c + d*x)/2])/(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2]))*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(240*
b^6*d*(a + b*Tan[c + d*x]))
```

Maple [A]
time = 0.81, size = 479, normalized size = 1.83 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a*cos(d*x+c)+b*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/5/b/(tan(1/2*d*x+1/2*c)-1)^5-1/4*(a+2*b)/b^2/(tan(1/2*d*x+1/2*c)-1)
^4-1/12*(4*a^2+6*a*b+13*b^2)/b^3/(tan(1/2*d*x+1/2*c)-1)^3-1/8*(4*a^3+4*a^2*
b+11*a*b^2+9*b^3)/b^4/(tan(1/2*d*x+1/2*c)-1)^2-1/8*(8*a^4+4*a^3*b+20*a^2*b^
2+9*a*b^3+15*b^4)/b^5/(tan(1/2*d*x+1/2*c)-1)+1/8*a*(8*a^4+20*a^2*b^2+15*b^4
)/b^6*ln(tan(1/2*d*x+1/2*c)-1)+1/5/b/(tan(1/2*d*x+1/2*c)+1)^5-1/4*(2*b-a)/b
^2/(tan(1/2*d*x+1/2*c)+1)^4-1/12*(-4*a^2+6*a*b-13*b^2)/b^3/(tan(1/2*d*x+1/2
*c)+1)^3-1/8*(-4*a^3+4*a^2*b-11*a*b^2+9*b^3)/b^4/(tan(1/2*d*x+1/2*c)+1)^2-1
/8*(-8*a^4+4*a^3*b-20*a^2*b^2+9*a*b^3-15*b^4)/b^5/(tan(1/2*d*x+1/2*c)+1)-1/
8*a*(8*a^4+20*a^2*b^2+15*b^4)/b^6*ln(tan(1/2*d*x+1/2*c)+1)-2/b^6*(-a^6-3*a^
4*b^2-3*a^2*b^4-b^6)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*
b)/(a^2+b^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 625 vs. 2(244) = 488.
time = 0.49, size = 625, normalized size = 2.39

$$\frac{\frac{2\left(120a^4+280a^2b^2+184b^4-\frac{15\left(4a^3b+9ab^3\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{80\left(6a^4+13a^2b^2+7b^4\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{30\left(4a^3b+5ab^3\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{80\left(9a^4+20a^2b^2+14b^4\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{240\left(2a^4+5a^2b^2+3b^4\right)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{30\left(4a^3b+5ab^3\right)\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{120\left(a^4+3a^2b^2+3b^4\right)\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{15\left(4a^3b+9ab^3\right)\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right)}{b^5-\frac{5b^5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{10b^5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{10b^5\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{5b^5\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{b^5\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{15\left(8a^5+20a^3b^2+15ab^4\right)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^6}+\frac{15\left(8a^5+20a^3b^2+15ab^4\right)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^6}-\frac{120\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b^6}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/120*(2*(120*a^4 + 280*a^2*b^2 + 184*b^4 - 15*(4*a^3*b + 9*a*b^3)*sin(d*x
+ c)/(cos(d*x + c) + 1) - 80*(6*a^4 + 13*a^2*b^2 + 7*b^4)*sin(d*x + c)^2/(c
```

```
os(d*x + c) + 1)^2 + 30*(4*a^3*b + 5*a*b^3)*sin(d*x + c)^3/(cos(d*x + c) +
1)^3 + 80*(9*a^4 + 20*a^2*b^2 + 14*b^4)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
 - 240*(2*a^4 + 5*a^2*b^2 + 3*b^4)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 30
*(4*a^3*b + 5*a*b^3)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 120*(a^4 + 3*a^2
*b^2 + 3*b^4)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 15*(4*a^3*b + 9*a*b^3)*
sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(b^5 - 5*b^5*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 10*b^5*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 10*b^5*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 5*b^5*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - b^5
*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) - 15*(8*a^5 + 20*a^3*b^2 + 15*a*b^4
)*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^6 + 15*(8*a^5 + 20*a^3*b^2 + 1
5*a*b^4)*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/b^6 - 120*(a^6 + 3*a^4*b^
2 + 3*a^2*b^4 + b^6)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2
+ b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^
2 + b^2)*b^6))/d
```

Fricas [A]
time = 2.53, size = 346, normalized size = 1.32

$$\frac{120\left(a^{4}+2a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}+b^{2}}\cos\left(dx+c\right)^{5}\log\left(-\frac{2ab\cos(dx+c)\sin(dx+c)+(a^{2}-b^{2})\cos(dx+c)^{2}-2a^{2}-b^{2}+2\sqrt{a^{2}+b^{2}}\,(b\cos(dx+c)-a\sin(dx+c))}{2ab\cos(dx+c)\sin(dx+c)+(a^{2}-b^{2})\cos(dx+c)^{2}+b^{2}}\right)-15\left(8a^{5}+20a^{3}b^{2}+15ab^{4}\right)\cos\left(dx+c\right)^{5}\log\left(\sin\left(dx+c\right)+1\right)+15\left(8a^{5}+20a^{3}b^{2}+15ab^{4}\right)\cos\left(dx+c\right)^{5}\log\left(-\sin\left(dx+c\right)+1\right)+48b^{5}+240\left(a^{4}b+2a^{2}b^{3}+b^{5}\right)\cos\left(dx+c\right)^{4}+80\left(a^{2}b^{3}+b^{5}\right)\cos\left(dx+c\right)^{2}-30\left(2ab^{4}\cos\left(dx+c\right)+\left(4a^{3}b^{2}+7ab^{4}\right)\cos\left(dx+c\right)^{3}\right)\sin\left(dx+c\right)}{240b^{6}d\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(120*(a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)*cos(d*x + c)^5*log(-(2*a
*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2
*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin
(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 15*(8*a^5 + 20*a^3*b^2 + 1
5*a*b^4)*cos(d*x + c)^5*log(sin(d*x + c) + 1) + 15*(8*a^5 + 20*a^3*b^2 + 15
*a*b^4)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 48*b^5 + 240*(a^4*b + 2*a^2
*b^3 + b^5)*cos(d*x + c)^4 + 80*(a^2*b^3 + b^5)*cos(d*x + c)^2 - 30*(2*a*b^
4*cos(d*x + c) + (4*a^3*b^2 + 7*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(b^6*d
*cos(d*x + c)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{6}\left(c+dx\right)}{a\cos\left(c+dx\right)+b\sin\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a*cos(d*x+c)+b*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**6/(a*cos(c + d*x) + b*sin(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 554 vs. 2(244) = 488.

time = 0.50, size = 554, normalized size = 2.11

$$-\frac{\frac{15\left(8a^5+20a^3b^2+15ab^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^6}-\frac{15\left(8a^5+20a^3b^2+15ab^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^6}+\frac{120\left(a^6+3a^4b^2+3a^2b^4+b^6\right)\log\left(\frac{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^6}+\frac{2\left(60a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+135ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+120a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+360a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+360b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8-120a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-150ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-480a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-1200a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-720b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+720a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+1600a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+1120b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+120a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+150ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-480a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1040a^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-560b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-60a^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-135ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+120a^4+280a^2b^2+184b^4\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^5b^5}}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+b*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/120*(15*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1
))/b^6 - 15*(8*a^5 + 20*a^3*b^2 + 15*a*b^4)*log(abs(tan(1/2*d*x + 1/2*c) -
1))/b^6 + 120*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*log(abs(2*a*tan(1/2*d*x +
 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*s
qrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^6) + 2*(60*a^3*b*tan(1/2*d*x + 1/2*c)^9
 + 135*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*a^4*tan(1/2*d*x + 1/2*c)^8 + 360*
a^2*b^2*tan(1/2*d*x + 1/2*c)^8 + 360*b^4*tan(1/2*d*x + 1/2*c)^8 - 120*a^3*b
*tan(1/2*d*x + 1/2*c)^7 - 150*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 480*a^4*tan(1/
2*d*x + 1/2*c)^6 - 1200*a^2*b^2*tan(1/2*d*x + 1/2*c)^6 - 720*b^4*tan(1/2*d*
x + 1/2*c)^6 + 720*a^4*tan(1/2*d*x + 1/2*c)^4 + 1600*a^2*b^2*tan(1/2*d*x +
1/2*c)^4 + 1120*b^4*tan(1/2*d*x + 1/2*c)^4 + 120*a^3*b*tan(1/2*d*x + 1/2*c)
^3 + 150*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 480*a^4*tan(1/2*d*x + 1/2*c)^2 - 10
40*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 560*b^4*tan(1/2*d*x + 1/2*c)^2 - 60*a^3
*b*tan(1/2*d*x + 1/2*c) - 135*a*b^3*tan(1/2*d*x + 1/2*c) + 120*a^4 + 280*a^
2*b^2 + 184*b^4)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*b^5))/d
```

Mupad [B]
time = 2.84, size = 2500, normalized size = 9.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a*cos(c + d*x) + b*sin(c + d*x))),x)
```

```
[Out] (atan(((((a^2 + b^2)^5)^(1/2)*(((225*a^4*b^13)/2 + 300*a^6*b^11 + 320*a^8*b
^9 + 160*a^10*b^7 + 32*a^12*b^5)/b^14 + (tan(c/2 + (d*x)/2)*(64*a*b^17 + 83
4*a^3*b^15 + 2385*a^5*b^13 + 3160*a^7*b^11 + 2240*a^9*b^9 + 832*a^11*b^7 +
128*a^13*b^5))/(2*b^15) - (((a^2 + b^2)^5)^(1/2)*((28*a^2*b^16 + 44*a^4*b^1
4 + 16*a^6*b^12)/b^14 - (tan(c/2 + (d*x)/2)*(128*a*b^18 + 384*a^3*b^16 + 38
4*a^5*b^14 + 128*a^7*b^12))/(2*b^15) + (((a^2 + b^2)^5)^(1/2)*(32*a^2*b^3 +
 (tan(c/2 + (d*x)/2)*(192*a*b^19 + 128*a^3*b^17))/(2*b^15)))/b^6))/b^6)*1i)
/b^6 + (((a^2 + b^2)^5)^(1/2)*(((225*a^4*b^13)/2 + 300*a^6*b^11 + 320*a^8*b
^9 + 160*a^10*b^7 + 32*a^12*b^5)/b^14 + (tan(c/2 + (d*x)/2)*(64*a*b^17 + 83
4*a^3*b^15 + 2385*a^5*b^13 + 3160*a^7*b^11 + 2240*a^9*b^9 + 832*a^11*b^7 +
128*a^13*b^5))/(2*b^15) - (((a^2 + b^2)^5)^(1/2)*((tan(c/2 + (d*x)/2)*(128*
a*b^18 + 384*a^3*b^16 + 384*a^5*b^14 + 128*a^7*b^12))/(2*b^15) - (28*a^2*b^
16 + 44*a^4*b^14 + 16*a^6*b^12)/b^14 + (((a^2 + b^2)^5)^(1/2)*(32*a^2*b^3 +
 (tan(c/2 + (d*x)/2)*(192*a*b^19 + 128*a^3*b^17))/(2*b^15)))/b^6))/b^6)*1i)
```

/b^6)/((32*a^16 + 120*a^2*b^14 + 655*a^4*b^12 + 1549*a^6*b^10 + 2069*a^8*b^
8 + 1695*a^10*b^6 + 856*a^12*b^4 + 248*a^14*b^2)/b^14 + (((a^2 + b^2)^5)^(1
/2)*(((225*a^4*b^13)/2 + 300*a^6*b^11 + 320*a^8*b^9 + 160*a^10*b^7 + 32*a^1
2*b^5)/b^14 + (tan(c/2 + (d*x)/2)*(64*a*b^17 + 834*a^3*b^15 + 2385*a^5*b^13
 + 3160*a^7*b^11 + 2240*a^9*b^9 + 832*a^11*b^7 + 128*a^13*b^5))/(2*b^15) -
(((a^2 + b^2)^5)^(1/2)*((28*a^2*b^16 + 44*a^4*b^14 + 16*a^6*b^12)/b^14 - (t
an(c/2 + (d*x)/2)*(128*a*b^18 + 384*a^3*b^16 + 384*a^5*b^14 + 128*a^7*b^12)
)/(2*b^15) + (((a^2 + b^2)^5)^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*
a*b^19 + 128*a^3*b^17))/(2*b^15)))/b^6))/b^6))/b^6 - (((a^2 + b^2)^5)^(1/2)
*(((225*a^4*b^13)/2 + 300*a^6*b^11 + 320*a^8*b^9 + 160*a^10*b^7 + 32*a^12*b
^5)/b^14 + (tan(c/2 + (d*x)/2)*(64*a*b^17 + 834*a^3*b^15 + 2385*a^5*b^13 +
3160*a^7*b^11 + 2240*a^9*b^9 + 832*a^11*b^7 + 128*a^13*b^5))/(2*b^15) - (((
a^2 + b^2)^5)^(1/2)*((tan(c/2 + (d*x)/2)*(128*a*b^18 + 384*a^3*b^16 + 384*a
^5*b^14 + 128*a^7*b^12))/(2*b^15) - (28*a^2*b^16 + 44*a^4*b^14 + 16*a^6*b^1
2)/b^14 + (((a^2 + b^2)^5)^(1/2)*(32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b
^19 + 128*a^3*b^17))/(2*b^15)))/b^6))/b^6))/b^6 - (tan(c/2 + (d*x)/2)*(128*
a^17 + 450*a^3*b^14 + 2550*a^5*b^12 + 6230*a^7*b^10 + 8530*a^9*b^8 + 7088*a
^11*b^6 + 3584*a^13*b^4 + 1024*a^15*b^2))/b^15))*((a^2 + b^2)^5)^(1/2)*2i)/
(b^6*d) - ((2*(15*a^4 + 23*b^4 + 35*a^2*b^2))/(15*b^5) + (tan(c/2 + (d*x)/2
)^3*(5*a*b^2 + 4*a^3))/(2*b^4) - (tan(c/2 + (d*x)/2)^7*(5*a*b^2 + 4*a^3))/(
2*b^4) + (tan(c/2 + (d*x)/2)^9*(9*a*b^2 + 4*a^3))/(4*b^4) + (2*tan(c/2 + (d
x)/2)^8(a^4 + 3*b^4 + 3*a^2*b^2))/b^5 - (4*tan(c/2 + (d*x)/2)^6*(2*a^4 +
3*b^4 + 5*a^2*b^2))/b^5 - (4*tan(c/2 + (d*x)/2)^2*(6*a^4 + 7*b^4 + 13*a^2*b
^2))/(3*b^5) + (4*tan(c/2 + (d*x)/2)^4*(9*a^4 + 14*b^4 + 20*a^2*b^2))/(3*b^
5) - (tan(c/2 + (d*x)/2)*(9*a*b^2 + 4*a^3))/(4*b^4))/(d*(5*tan(c/2 + (d*x)/
2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x
)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1)) + (atan(((((15*a*b^4)/8 + a^5 + (5*a^3
b^2)/2)(((225*a^4*b^13)/2 + 300*a^6*b^11 + 320*a^8*b^9 + 160*a^10*b^7 + 3
2*a^12*b^5)/b^14 + (tan(c/2 + (d*x)/2)*(64*a*b^17 + 834*a^3*b^15 + 2385*a^5
*b^13 + 3160*a^7*b^11 + 2240*a^9*b^9 + 832*a^11*b^7 + 128*a^13*b^5))/(2*b^1
5) - (((15*a*b^4)/8 + a^5 + (5*a^3*b^2)/2)*((28*a^2*b^16 + 44*a^4*b^14 + 16
*a^6*b^12)/b^14 - (tan(c/2 + (d*x)/2)*(128*a*b^18 + 384*a^3*b^16 + 384*a^5*
b^14 + 128*a^7*b^12))/(2*b^15) + ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*
b^19 + 128*a^3*b^17))/(2*b^15))*((15*a*b^4)/8 + a^5 + (5*a^3*b^2)/2))/b^6))
/b^6)*1i)/b^6 + (((15*a*b^4)/8 + a^5 + (5*a^3*b^2)/2)*(((225*a^4*b^13)/2 +
300*a^6*b^11 + 320*a^8*b^9 + 160*a^10*b^7 + 32*a^12*b^5)/b^14 + (tan(c/2 +
(d*x)/2)*(64*a*b^17 + 834*a^3*b^15 + 2385*a^5*b^13 + 3160*a^7*b^11 + 2240*a
^9*b^9 + 832*a^11*b^7 + 128*a^13*b^5))/(2*b^15) - (((15*a*b^4)/8 + a^5 + (5
*a^3*b^2)/2)*((tan(c/2 + (d*x)/2)*(128*a*b^18 + 384*a^3*b^16 + 384*a^5*b^14
 + 128*a^7*b^12))/(2*b^15) - (28*a^2*b^16 + 44*a^4*b^14 + 16*a^6*b^12)/b^14
 + ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^19 + 128*a^3*b^17))/(2*b^15)
)*((15*a*b^4)/8 + a^5 + (5*a^3*b^2)/2))/b^6))/b^6)*1i)/b^6)/((32*a^16 + 120
*a^2*b^14 + 655*a^4*b^12 + 1549*a^6*b^10 + 2069*a^8*b^8 + 1695*a^10*b^6 + 8
56*a^12*b^4 + 248*a^14*b^2)/b^14 + (((15*a*b^4)/8 + a^5 + (5*a^3*b^2)/2)*((
(225*a^4*b^13)/2 + 300*a^6*b^11 + 320*a^8*b^9 + 160*a^10*b^7 + 32*a^12*b^5)

```
/b^14 + (tan(c/2 + (d*x)/2)*(64*a*b^17 + 834*a^3*b^15 + 2385*a^5*b^13 + 316
0*a^7*b^11 + 2240*a^9*b^9 + 832*a^11*b^7 + 128*a^13*b^5))/(2*b^15) - (((15*
a*b^4)/8 + a^5 + (5*a^3*b^2)/2)*((28*a^2*b^16 + 44*a^4*b^14 + 16*a^6*b^12)/
b^14 - (tan(c/2 + (d*x)/2)*(128*a*b^18 + 384*a^3*b^16 + 384*a^5*b^14 + 128*
a^7*b^12))/(2*b^15) + ((32*a^2*b^3 + (tan(c/2 + (d*x)/2)*(192*a*b^19 + 128*
a^3*b^17))/(2*b^15))*((15*a*b^4)/8 + a^5 + (5*a^3*b^2)/2))/b^6))/b^6))/b^6
- (((15*a*b^4)/8 + a^5 + (5*a^3*b^2)/2)*(((225*a^4*b^13)/2 + 300*a^6*b^11 +
 320*a^8*b^9 + 160*a^10*b^7 + 32*a^12*b^5)/b^14...
```

3.122 $\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=145

$$\frac{(a^4+6a^2b^2-3b^4)\,x}{2\,(a^2+b^2)^3}+\frac{b^4}{a\,(a^2+b^2)^2\,d(b+a\cot(c+dx))}+\frac{4ab^3\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d}-\frac{(2ab-(a^2$$

```
[Out] 1/2*(a^4+6*a^2*b^2-3*b^4)*x/(a^2+b^2)^3+b^4/a/(a^2+b^2)^2/d/(b+a*cot(d*x+c)
)+4*a*b^3*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^3/d-1/2*(2*a*b-(a^2-b^2)*
cot(d*x+c))*sin(d*x+c)^2/(a^2+b^2)^2/d
```

Rubi [A]
time = 0.23, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3167, 1661, 1643, 649, 209, 266}

$$-\frac{\sin^2(c+dx)\,(2ab-(a^2-b^2)\cot(c+dx))}{2d\,(a^2+b^2)^2}+\frac{b^4}{ad\,(a^2+b^2)^2\,(a\cot(c+dx)+b)}+\frac{4ab^3\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^3}+\frac{x(a^4+6a^2b^2-3b^4)}{2\,(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^4 + 6*a^2*b^2 - 3*b^4)*x)/(2*(a^2 + b^2)^3) + b^4/(a*(a^2 + b^2)^2*d*(b
 + a*Cot[c + d*x])) + (4*a*b^3*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])/((a^2
+ b^2)^3*d) - ((2*a*b - (a^2 - b^2)*Cot[c + d*x])*Sin[c + d*x]^2)/(2*(a^2 +
 b^2)^2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
```

```
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{x^4}{(b+ax)^2(1+x^2)^2}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{(2ab-(a^2-b^2)\cot(c+dx))\sin^2(c+dx)}{2\left(a^2+b^2\right)^2 d} + \frac{\text{Subst}\left(\int \frac{-\frac{b^2\left(a^2-b^2\right)}{\left(a^2+b^2\right)^2} - \cdots}{(b \cdots}\right.}{\cdots}$$

$$= -\frac{(2ab-(a^2-b^2)\cot(c+dx))\sin^2(c+dx)}{2\left(a^2+b^2\right)^2 d} + \frac{\text{Subst}\left(\int \left(-\frac{2b^4}{(a^2+b^2)^2(b \cdots}\right.\right.}{\cdots}$$

$$= \frac{b^4}{a\left(a^2+b^2\right)^2 d(b+a\cot(c+dx))} + \frac{4ab^3\log(b+a\cot(c+dx))}{\left(a^2+b^2\right)^3 d} - \frac{(2a \cdots}{\cdots}$$

$$= \frac{b^4}{a\left(a^2+b^2\right)^2 d(b+a\cot(c+dx))} + \frac{4ab^3\log(b+a\cot(c+dx))}{\left(a^2+b^2\right)^3 d} - \frac{(2a \cdots}{\cdots}$$

$$= \frac{\left(a^4+6a^2b^2-3b^4\right)x}{2\left(a^2+b^2\right)^3} + \frac{b^4}{a\left(a^2+b^2\right)^2 d(b+a\cot(c+dx))} + \frac{4ab^3\log(\cdots}{(\cdots}$$

Mathematica [A]

time = 1.02, size = 149, normalized size = 1.03

$$\frac{2(a^4+6a^2b^2-3b^4)(c+dx)+2ab(a^2+b^2)\cos(2(c+dx))+16ab^3\log(a\cos(c+dx)+b\sin(c+dx))+\frac{4b^4(a^2+b^2)\sin(c+dx)}{a(a\cos(c+dx)+b\sin(c+dx))}+(a^2-b^2)(a^2+b^2)\sin(2(c+dx))}{4(a^2+b^2)^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*(a^4 + 6*a^2*b^2 - 3*b^4)*(c + d*x) + 2*a*b*(a^2 + b^2)*Cos[2*(c + d*x)]
 + 16*a*b^3*Log[a*Cos[c + d*x] + b*Sin[c + d*x]] + (4*b^4*(a^2 + b^2)*Sin[c
 + d*x])/(a*(a*Cos[c + d*x] + b*Sin[c + d*x])) + (a^2 - b^2)*(a^2 + b^2)*Si
n[2*(c + d*x)])/(4*(a^2 + b^2)^3*d)
```

Maple [A]
time = 0.53, size = 154, normalized size = 1.06 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b^3/(a^2+b^2)^2/(a+b*tan(d*x+c))+4*b^3/(a^2+b^2)^3*a*ln(a+b*tan(d*x+c
))+1/(a^2+b^2)^3*(((1/2*a^4-1/2*b^4)*tan(d*x+c)+a^3*b+a*b^3)/(tan(d*x+c)^2+
1)-2*a*b^3*ln(tan(d*x+c)^2+1)+1/2*(a^4+6*a^2*b^2-3*b^4)*arctan(tan(d*x+c)))
)
```

Maxima [A]
time = 0.49, size = 282, normalized size = 1.94

$$\frac{\frac{8ab^3\log(b\tan(dx+c)+a)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{4ab^3\log\left(\tan(dx+c)^2+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{(a^4+6a^2b^2-3b^4)(dx+c)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{2a^2b-2b^3+(a^2b-3b^3)\tan(dx+c)^2+(a^3+ab^2)\tan(dx+c)}{a^5+2a^3b^2+ab^4+(a^4b+2a^2b^3+b^5)\tan(dx+c)^3+(a^5+2a^3b^2+ab^4)\tan(dx+c)^2+(a^4b+2a^2b^3+b^5)\tan(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(8*a*b^3*log(b*tan(d*x + c) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) -
4*a*b^3*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (a^4
+ 6*a^2*b^2 - 3*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + (2*a^2
*b - 2*b^3 + (a^2*b - 3*b^3)*tan(d*x + c)^2 + (a^3 + a*b^2)*tan(d*x + c))/(
a^5 + 2*a^3*b^2 + a*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*tan(d*x + c)^3 + (a^5 +
 2*a^3*b^2 + a*b^4)*tan(d*x + c)^2 + (a^4*b + 2*a^2*b^3 + b^5)*tan(d*x + c)
))/d
```

Fricas [A]
time = 2.24, size = 279, normalized size = 1.92

$$\frac{(a^4b+2a^2b^3+b^5)\cos(dx+c)^3-(a^2b^3+3b^5-(a^5+6a^3b^2-3ab^4)dx)\cos(dx+c)+4(a^2b^3\cos(dx+c)+ab^4\sin(dx+c))\log\left(2ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2\right)-(a^3b^2-ab^4-(a^4b+6a^2b^3-3b^5)dx-(a^5+2a^3b^2+ab^4)\cos(dx+c)^2)\sin(dx+c)}{2((a^7+3a^5b^2+3a^3b^4+ab^6)d\cos(dx+c)+(a^6b+3a^4b^3+3a^2b^5+b^7)d\sin(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)^3 - (a^2*b^3 + 3*b^5 - (a^5 + 6
*a^3*b^2 - 3*a*b^4)*d*x)*cos(d*x + c) + 4*(a^2*b^3*cos(d*x + c) + a*b^4*sin
(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
 + b^2) - (a^3*b^2 - a*b^4 - (a^4*b + 6*a^2*b^3 - 3*b^5)*d*x - (a^5 + 2*a^3
*b^2 + a*b^4)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 +
 a*b^6)*d*cos(d*x + c) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*d*sin(d*x +
c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.44, size = 250, normalized size = 1.72

$$\frac{\frac{8\,ab^4\log\left(|b\tan\left(dx+c\right)+a|\right)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}-\frac{4\,ab^3\log\left(\tan\left(dx+c\right)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{\left(a^4+6\,a^2b^2-3\,b^4\right)\left(dx+c\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{a^2b\tan\left(dx+c\right)^2-3\,b^3\tan\left(dx+c\right)^2+a^3\tan\left(dx+c\right)+ab^2\tan\left(dx+c\right)+2\,a^2b-2\,b^3}{\left(a^4+2\,a^2b^2+b^4\right)\left(b\tan\left(dx+c\right)^3+a\tan\left(dx+c\right)^2+b\tan\left(dx+c\right)+a\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(8*a*b^4*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 +
b^7) - 4*a*b^3*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
+ (a^4 + 6*a^2*b^2 - 3*b^4)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) +
 (a^2*b*tan(d*x + c)^2 - 3*b^3*tan(d*x + c)^2 + a^3*tan(d*x + c) + a*b^2*ta
n(d*x + c) + 2*a^2*b - 2*b^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*tan(d*x + c)^3 +
a*tan(d*x + c)^2 + b*tan(d*x + c) + a)))/d
```

Mupad [B]
time = 11.46, size = 2500, normalized size = 17.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

```
[Out] ((2*b*tan(c/2 + (d*x)/2)^4)/(a^2 + b^2) - (2*b*tan(c/2 + (d*x)/2)^2)/(a^2 +
 b^2) + (tan(c/2 + (d*x)/2)*(a^4 + 2*b^4 - a^2*b^2))/(a*(a^2 + b^2)^2) + (t
an(c/2 + (d*x)/2)^5*(a^4 + 2*b^4 - a^2*b^2))/(a*(a^4 + b^4 + 2*a^2*b^2)) -
```

```
(2*tan(c/2 + (d*x)/2)^3*(a^4 - 2*b^4 + 3*a^2*b^2))/(a*(a^2 + b^2)^2))/(d*(a
 + 2*b*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^2 - a*tan(c/2 + (d*x)/2)^4
 - a*tan(c/2 + (d*x)/2)^6 + 4*b*tan(c/2 + (d*x)/2)^3 + 2*b*tan(c/2 + (d*x)/
2)^5)) - (atan((tan(c/2 + (d*x)/2)*(((((a^4 - 3*b^4 + 6*a^2*b^2)^3*(12*a*b^
16 + 84*a^3*b^14 + 252*a^5*b^12 + 420*a^7*b^10 + 420*a^9*b^8 + 252*a^11*b^6
 + 84*a^13*b^4 + 12*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^3*(a^12
 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))
- (((8*(18*a*b^12 + a^13 - 141*a^3*b^10 - 327*a^5*b^8 - 146*a^7*b^6 + 36*a^
9*b^4 + 15*a^11*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 +
 15*a^8*b^4 + 6*a^10*b^2) - (16*a*b^3*((8*(4*a^14*b + 4*a^2*b^13 + 72*a^4*b
^11 + 252*a^6*b^9 + 368*a^8*b^7 + 252*a^10*b^5 + 72*a^12*b^3))/(a^12 + b^12
 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (128*a
*b^3*(12*a*b^16 + 84*a^3*b^14 + 252*a^5*b^12 + 420*a^7*b^10 + 420*a^9*b^8 +
 252*a^11*b^6 + 84*a^13*b^4 + 12*a^15*b^2))/((4*a^6 + 4*b^6 + 12*a^2*b^4 +
12*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^
4 + 6*a^10*b^2))))/(4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2))*(a^4 - 3*b^4
+ 6*a^2*b^2))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (16*a*b^3*((((8*(4*
a^14*b + 4*a^2*b^13 + 72*a^4*b^11 + 252*a^6*b^9 + 368*a^8*b^7 + 252*a^10*b^
5 + 72*a^12*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*
a^8*b^4 + 6*a^10*b^2) - (128*a*b^3*(12*a*b^16 + 84*a^3*b^14 + 252*a^5*b^12
+ 420*a^7*b^10 + 420*a^9*b^8 + 252*a^11*b^6 + 84*a^13*b^4 + 12*a^15*b^2))/(
(4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^
4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^4 - 3*b^4 + 6*a^2*b^2))/
(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (64*a*b^3*(a^4 - 3*b^4 + 6*a^2*b^
2)*(12*a*b^16 + 84*a^3*b^14 + 252*a^5*b^12 + 420*a^7*b^10 + 420*a^9*b^8 + 2
52*a^11*b^6 + 84*a^13*b^4 + 12*a^15*b^2))/((4*a^6 + 4*b^6 + 12*a^2*b^4 + 12
*a^4*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 1
5*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(4*a^6 + 4*b^6 + 12*a^
2*b^4 + 12*a^4*b^2))*(a^10 - 9*b^10 + 493*a^2*b^8 - 706*a^4*b^6 - 46*a^6*b^
4 + 11*a^8*b^2))/(a^10 + 9*b^10 + 229*a^2*b^8 + 250*a^4*b^6 + 42*a^6*b^4 +
13*a^8*b^2)^2 - (2*a*b*((8*(72*a^2*b^9 + 52*a^4*b^7 + 48*a^6*b^5 + 4*a^8*b^
3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^
10*b^2) + (((((8*(4*a^14*b + 4*a^2*b^13 + 72*a^4*b^11 + 252*a^6*b^9 + 368*a
^8*b^7 + 252*a^10*b^5 + 72*a^12*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^
8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (128*a*b^3*(12*a*b^16 + 84*a^3*
b^14 + 252*a^5*b^12 + 420*a^7*b^10 + 420*a^9*b^8 + 252*a^11*b^6 + 84*a^13*b
^4 + 12*a^15*b^2))/((4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)*(a^12 + b^12
+ 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^4 -
3*b^4 + 6*a^2*b^2))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (64*a*b^3*(a^
4 - 3*b^4 + 6*a^2*b^2)*(12*a*b^16 + 84*a^3*b^14 + 252*a^5*b^12 + 420*a^7*b^
10 + 420*a^9*b^8 + 252*a^11*b^6 + 84*a^13*b^4 + 12*a^15*b^2))/((4*a^6 + 4*b
^6 + 12*a^2*b^4 + 12*a^4*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b
^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^
4 - 3*b^4 + 6*a^2*b^2))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (16*a*b^3
*((8*(18*a*b^12 + a^13 - 141*a^3*b^10 - 327*a^5*b^8 - 146*a^7*b^6 + 36*a^9*
```

b^4 + 15*a^11*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 1
5*a^8*b^4 + 6*a^10*b^2) - (16*a*b^3*((8*(4*a^14*b + 4*a^2*b^13 + 72*a^4*b^1
1 + 252*a^6*b^9 + 368*a^8*b^7 + 252*a^10*b^5 + 72*a^12*b^3))/(a^12 + b^12 +
 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (128*a*b
^3*(12*a*b^16 + 84*a^3*b^14 + 252*a^5*b^12 + 420*a^7*b^10 + 420*a^9*b^8 + 2
52*a^11*b^6 + 84*a^13*b^4 + 12*a^15*b^2))/((4*a^6 + 4*b^6 + 12*a^2*b^4 + 12
*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4
+ 6*a^10*b^2))))/(4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)))/(4*a^6 + 4*b^6
 + 12*a^2*b^4 + 12*a^4*b^2) - (32*a*b^3*(a^4 - 3*b^4 + 6*a^2*b^2)^2*(12*a*b
^16 + 84*a^3*b^14 + 252*a^5*b^12 + 420*a^7*b^10 + 420*a^9*b^8 + 252*a^11*b^
6 + 84*a^13*b^4 + 12*a^15*b^2))/((4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)*
(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^
8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^8 + 57*b^8 - 436*a^2*b^6 + 1
10*a^4*b^4 + 28*a^6*b^2))/(a^10 + 9*b^10 + 229*a^2*b^8 + 250*a^4*b^6 + 42*a
^6*b^4 + 13*a^8*b^2)^2)*(a^16 + b^16 + 8*a^2*b^14 + 28*a^4*b^12 + 56*a^6*b^
10 + 70*a^8*b^8 + 56*a^10*b^6 + 28*a^12*b^4 + 8*a^14*b^2))/(4*a^5 - 12*a*b^
4 + 24*a^3*b^2) + (((((8*(39*a^2*b^11 - a^12*b + 123*a^4*b^9 + 134*a^6*b^7
+ 54*a^8*b^5 + 3*a^10*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6
*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + (16*a*b^3*((8...

3.123 $\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=138

$$-\frac{3ab^2\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}\,d}+\frac{2ab\cos(c+dx)}{(a^2+b^2)^2\,d}+\frac{(a^2-b^2)\sin(c+dx)}{(a^2+b^2)^2\,d}-\frac{b^3}{(a^2+b^2)^2\,d(a\cos(c+dx)+}$$

```
[Out] -3*a*b^2*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/
2)/d+2*a*b*cos(d*x+c)/(a^2+b^2)^2/d+(a^2-b^2)*sin(d*x+c)/(a^2+b^2)^2/d-b^3/
(a^2+b^2)^2/d/(a*cos(d*x+c)+b*sin(d*x+c))
```

Rubi [A]
time = 0.78, antiderivative size = 231, normalized size of antiderivative = 1.67, number of steps used = 11, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {6874, 653, 209, 652, 632, 212}

$$\frac{2\left((a^2-b^2)\tan\left(\frac{1}{2}(c+dx)\right)+2ab\right)}{d\left(a^2+b^2\right)^2\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)}-\frac{2b^2(3a^2+b^2)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{ad\left(a^2+b^2\right)^{5/2}}+\frac{2b^4\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{ad\left(a^2+b^2\right)^{5/2}}-\frac{2b^3\left(a+b\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad\left(a^2+b^2\right)^2\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*b^4*ArcTanh[(b - a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a*(a^2 + b^2)^(5
/2)*d) - (2*b^2*(3*a^2 + b^2)*ArcTanh[(b - a*Tan[(c + d*x)/2])/Sqrt[a^2 + b
^2]])/(a*(a^2 + b^2)^(5/2)*d) + (2*(2*a*b + (a^2 - b^2)*Tan[(c + d*x)/2]))/
((a^2 + b^2)^2*d*(1 + Tan[(c + d*x)/2]^2)) - (2*b^3*(a + b*Tan[(c + d*x)/2]
))/(a*(a^2 + b^2)^2*d*(a + 2*b*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/2]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{2\text{Subst}\left(\int \frac{(1-x^2)^3}{(1+x^2)^2(a+2bx-ax^2)^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{2(a^2-b^2-2abx)}{(a^2+b^2)^2(1+x^2)^2} + \frac{-a^2+b^2}{(a^2+b^2)^2(1+x^2)} - \frac{2b^3x}{a(a^2+b^2)(-a-2bx+ax^2)^2} - \frac{}{a(}\right.\right.}{d}$$

$$= \frac{4\text{Subst}\left(\int \frac{a^2-b^2-2abx}{(1+x^2)^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^2+b^2)^2 d} - \frac{(2(a^2-b^2))\,\text{Subst}\left(\int\right.}{(a}$$

$$= -\frac{(a^2-b^2)\,x}{(a^2+b^2)^2} + \frac{2\left(2ab+(a^2-b^2)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^2+b^2)^2\,d\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)} - \frac{}{a\,(a^2+b^2)^2\,d}$$

$$= -\frac{2b^2(3a^2+b^2)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^{5/2}\,d} + \frac{2\left(2ab+(a^2-b^2)\tan\left(\frac{1}{2}\right.\right.}{(a^2+b^2)^2\,d\left(1+\tan^2\left(\frac{1}{2}\right.\right.}$$

$$= \frac{2b^4\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a\,(a^2+b^2)^{5/2}\,d} - \frac{2b^2(3a^2+b^2)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx\right.}{\sqrt{a^2+b^2}}\right.}{a\,(a^2+b^2)^{5/2}\,d}$$

Mathematica [A]

time = 0.84, size = 130, normalized size = 0.94

$$\frac{\frac{12ab^2\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}+\frac{3b(a^2-b^2)+b(a^2+b^2)\cos(2(c+dx))+a(a^2+b^2)\sin(2(c+dx))}{(a^2+b^2)^2(a\cos(c+dx)+b\sin(c+dx))}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((12*a*b^2*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^
(5/2) + (3*b*(a^2 - b^2) + b*(a^2 + b^2)*Cos[2*(c + d*x)] + a*(a^2 + b^2)*S
in[2*(c + d*x)])/((a^2 + b^2)^2*(a*Cos[c + d*x] + b*Sin[c + d*x])))/(2*d)
```

Maple [A]
time = 0.53, size = 172, normalized size = 1.25

method	result
derivativedivides	$\frac{-\frac{2b^2\left(\frac{-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-b}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\frac{3a\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2}-\frac{2\left(\left(-a^2+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2ab\right)}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{d}$
default	$\frac{-\frac{2b^2\left(\frac{-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}-b}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\frac{3a\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2}-\frac{2\left(\left(-a^2+b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2ab\right)}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{d}$
risch	$-\frac{i e^{i(dx+c)}}{2(-2iab+a^2-b^2)d}+\frac{i e^{-i(dx+c)}}{2(2iab+a^2-b^2)d}-\frac{2ib^3 e^{i(dx+c)}}{(-ia+b)^2 d(ia+b)^2\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)}+\frac{3b^2a\ln\left(e^{i(dx+}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b^2/(a^2+b^2)^2*((-b^2/a*tan(1/2*d*x+1/2*c)-b)/(a*tan(1/2*d*x+1/2*c
)^2-2*b*tan(1/2*d*x+1/2*c)-a)-3*a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*
d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))-2/(a^4+2*a^2*b^2+b^4)*((-a^2+b^2)*tan(1/2
*d*x+1/2*c)-2*a*b)/(1+tan(1/2*d*x+1/2*c)^2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(134) = 268.
time = 0.48, size = 348, normalized size = 2.52

$$-\frac{\frac{3\,ab^2\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}}-\frac{2\left(2\,a^3b-ab^3-\frac{3\,ab^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{(a^4+3\,a^2b^2-b^4)\sin(dx+c)}{\cos(dx+c)+1}-\frac{(a^4-a^2b^2+b^4)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^6+2\,a^4b^2+a^2b^4+\frac{2\,(a^5b+2\,a^3b^3+ab^5)\sin(dx+c)}{\cos(dx+c)+1}+\frac{2\,(a^5b+2\,a^3b^3+ab^5)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{(a^6+2\,a^4b^2+a^2b^4)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(3*a*b^2*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b
- a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 +
 b^4)*sqrt(a^2 + b^2)) - 2*(2*a^3*b - a*b^3 - 3*a*b^3*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 + (a^4 + 3*a^2*b^2 - b^4)*sin(d*x + c)/(cos(d*x + c) + 1) -
(a^4 - a^2*b^2 + b^4)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^6 + 2*a^4*b^2
 + a^2*b^4 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*sin(d*x + c)/(cos(d*x + c) + 1)
+ 2*(a^5*b + 2*a^3*b^3 + a*b^5)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - (a^6
+ 2*a^4*b^2 + a^2*b^4)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(134) = 268$.
time = 2.40, size = 302, normalized size = 2.19

$$\frac{2\,a^4b-2\,a^2b^3-4\,b^5+2\,(a^4b+2\,a^2b^3+b^5)\cos\left(dx+c\right)^2+2\,(a^5+2\,a^3b^2+ab^4)\cos\left(dx+c\right)\sin\left(dx+c\right)+3\,(a^2b^2\cos\left(dx+c\right)+ab^3\sin\left(dx+c\right))\sqrt{a^2+b^2}\,\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)}{2\,((a^7+3\,a^5b^2+3\,a^3b^4+ab^6)d\cos\left(dx+c\right)+(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)d\sin\left(dx+c\right))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^4*b - 2*a^2*b^3 - 4*b^5 + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(d*x + c)
^2 + 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(d*x + c)*sin(d*x + c) + 3*(a^2*b^2*cos
(d*x + c) + a*b^3*sin(d*x + c))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*si
n(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(
b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 -
 b^2)*cos(d*x + c)^2 + b^2)))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*d*cos(
d*x + c) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*d*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(134) = 268$.
time = 0.50, size = 286, normalized size = 2.07

$$-\frac{\frac{3\,ab^2\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}}-\frac{2\left(a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,a^3b+ab^3\right)}{(a^5+2\,a^3b^2+ab^4)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(3*a*b^2*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2
*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4
)*sqrt(a^2 + b^2)) - 2*(a^4*tan(1/2*d*x + 1/2*c)^3 - a^2*b^2*tan(1/2*d*x +
1/2*c)^3 + b^4*tan(1/2*d*x + 1/2*c)^3 + 3*a*b^3*tan(1/2*d*x + 1/2*c)^2 - a^
4*tan(1/2*d*x + 1/2*c) - 3*a^2*b^2*tan(1/2*d*x + 1/2*c) + b^4*tan(1/2*d*x +
 1/2*c) - 2*a^3*b + a*b^3)/((a^5 + 2*a^3*b^2 + a*b^4)*(a*tan(1/2*d*x + 1/2*
c)^4 - 2*b*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1/2*c) - a)))/d
```

Mupad [B]
time = 2.81, size = 286, normalized size = 2.07

$$\frac{\frac{4\,a^2\,b-2\,b^3}{a^4+2\,a^2\,b^2+b^4}-\frac{6\,b^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^4+3\,a^2\,b^2-b^4\right)}{a\,\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(2\,a^4-2\,a^2\,b^2+2\,b^4\right)}{a\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left(-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}-\frac{6\,a\,b^2\,\mathrm{atanh}\left(\frac{a^4\,b+b^5+2\,a^2\,b^3-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}{{\left(a^2+b^2\right)}^{5/2}}\right)}{d\,{\left(a^2+b^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

```
[Out] ((4*a^2*b - 2*b^3)/(a^4 + b^4 + 2*a^2*b^2) - (6*b^3*tan(c/2 + (d*x)/2)^2)/(
a^4 + b^4 + 2*a^2*b^2) + (2*tan(c/2 + (d*x)/2)*(a^4 - b^4 + 3*a^2*b^2))/(a*
(a^4 + b^4 + 2*a^2*b^2)) - (tan(c/2 + (d*x)/2)^3*(2*a^4 + 2*b^4 - 2*a^2*b^2
))/(a*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2
+ (d*x)/2)^4 + 2*b*tan(c/2 + (d*x)/2)^3)) - (6*a*b^2*atanh((a^4*b + b^5 + 2
*a^2*b^3 - a*tan(c/2 + (d*x)/2)*(a^4 + b^4 + 2*a^2*b^2))/(a^2 + b^2)^(5/2))
)/(d*(a^2 + b^2)^(5/2))
```

3.124 $\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=82

$$\frac{(a^2-b^2)\,x}{(a^2+b^2)^2}+\frac{2ab\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^2\,d}-\frac{b}{(a^2+b^2)\,d(a+b\tan(c+dx))}$$

```
[Out] (a^2-b^2)*x/(a^2+b^2)^2+2*a*b*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^2+b^2)^2/d-b
/(a^2+b^2)/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3165, 3564, 3612, 3611}

$$-\frac{b}{d\,(a^2+b^2)\,(a+b\tan(c+dx))}+\frac{2ab\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^2}+\frac{x(a^2-b^2)}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ((a^2 - b^2)*x)/(a^2 + b^2)^2 + (2*a*b*Log[a*Cos[c + d*x] + b*Sin[c + d*x]]
)/((a^2 + b^2)^2*d) - b/((a^2 + b^2)*d*(a + b*Tan[c + d*x]))
```

Rule 3165

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx &= \int \frac{1}{(a+b\tan(c+dx))^2}\,dx \\
&= -\frac{b}{(a^2+b^2)\,d(a+b\tan(c+dx))}+\frac{\int \frac{a-b\tan(c+dx)}{a+b\tan(c+dx)}\,dx}{a^2+b^2} \\
&= \frac{(a^2-b^2)\,x}{(a^2+b^2)^2}-\frac{b}{(a^2+b^2)\,d(a+b\tan(c+dx))}+\frac{(2ab)\int \frac{b-a\tan(c+dx)}{a+b\tan(c+dx)} \\
&= \frac{(a^2-b^2)\,x}{(a^2+b^2)^2}+\frac{2ab\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^2\,d}-\frac{}{(a^2+b^2)\,d(}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.44, size = 192, normalized size = 2.34

$$
\frac{a^2\cos(c+dx)\left((a+ib)^2(c+dx)+ab\log\left((a\cos(c+dx)+b\sin(c+dx))^2\right)\right)+b\left((a+ib)\left(-ib^2+ab(1+ic+idx)+a^2(c+dx)\right)+a^2b\log\left((a\cos(c+dx)+b\sin(c+dx))^2\right)\right)\sin(c+dx)-2ia^2b\,\mathrm{ArcTan}(\tan(c+dx))(a\cos(c+dx)+b\sin(c+dx))}{a\left(a^2+b^2\right)^2d(a\cos(c+dx)+b\sin(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2*Cos[c + d*x]*((a + I*b)^2*(c + d*x) + a*b*Log[(a*Cos[c + d*x] + b*Sin[
c + d*x])^2]) + b*((a + I*b)*((-I)*b^2 + a*b*(1 + I*c + I*d*x) + a^2*(c + d
*x)) + a^2*b*Log[(a*Cos[c + d*x] + b*Sin[c + d*x])^2])*Sin[c + d*x] - (2*I)
*a^2*b*ArcTan[Tan[c + d*x]]*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(a*(a^2 + b^
2)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Maple [A]
time = 0.34, size = 97, normalized size = 1.18

method	result
derivativedivides	$\frac{-\frac{b}{\left(a^2+b^2\right)(a+b\tan(dx+c))}+\frac{2ba\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^2}+\frac{-ab\ln\left(\tan^2(dx+c)+1\right)+\left(a^2-b^2\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^2}}{d}$
default	$\frac{-\frac{b}{\left(a^2+b^2\right)(a+b\tan(dx+c))}+\frac{2ba\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^2}+\frac{-ab\ln\left(\tan^2(dx+c)+1\right)+\left(a^2-b^2\right)\arctan(\tan(dx+c))}{\left(a^2+b^2\right)^2}}{d}$

risch	$-\frac{x}{2iab-a^2+b^2}-\frac{4iabx}{a^4+2a^2b^2+b^4}-\frac{4iabc}{d(a^4+2a^2b^2+b^4)}-\frac{2ib^2}{(-ia+b)d(ia+b)^2(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia)}+\frac{2ab\ln}{d}$
norman	$\frac{\frac{(a^2-b^2)ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a^2+b^2)^2}+\frac{(a^2-b^2)ax\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a^2+b^2)^2}-\frac{(a^2-b^2)ax}{(a^2+b^2)^2}-\frac{2b(a^2-b^2)x\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a^2+b^2)^2}-\frac{4b(a^2-b^2)x\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a^2+b^2)^2}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(a\left(\tan^2\left(\frac{dx}{2}\right.\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b/(a^2+b^2)/(a+b*tan(d*x+c))+2*b*a/(a^2+b^2)^2*ln(a+b*tan(d*x+c))+1/(
a^2+b^2)^2*(-a*b*ln(tan(d*x+c)^2+1)+(a^2-b^2)*arctan(tan(d*x+c))))
```

Maxima [A]
time = 0.47, size = 131, normalized size = 1.60

$$\frac{\frac{2\,ab\log(b\tan(dx+c)+a)}{a^4+2\,a^2b^2+b^4}-\frac{ab\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{(a^2-b^2)(dx+c)}{a^4+2\,a^2b^2+b^4}-\frac{b}{a^3+ab^2+(a^2b+b^3)\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (2*a*b*log(b*tan(d*x + c) + a)/(a^4 + 2*a^2*b^2 + b^4) - a*b*log(tan(d*x +
c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^2 - b^2)*(d*x + c)/(a^4 + 2*a^2*b^2
+ b^4) - b/(a^3 + a*b^2 + (a^2*b + b^3)*tan(d*x + c)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(82) = 164$.
time = 3.24, size = 173, normalized size = 2.11

$$-\frac{(b^3-(a^3-ab^2)dx)\cos(dx+c)-(a^2b\cos(dx+c)+ab^2\sin(dx+c))\log\left(2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2\right)-(ab^2+(a^2b-b^3)dx)\sin(dx+c)}{(a^5+2\,a^3b^2+ab^4)d\cos(dx+c)+(a^4b+2\,a^2b^3+b^5)d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -((b^3 - (a^3 - a*b^2)*d*x)*cos(d*x + c) - (a^2*b*cos(d*x + c) + a*b^2*sin(
d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
+ b^2) - (a*b^2 + (a^2*b - b^3)*d*x)*sin(d*x + c))/((a^5 + 2*a^3*b^2 + a*b^
4)*d*cos(d*x + c) + (a^4*b + 2*a^2*b^3 + b^5)*d*sin(d*x + c))
```

Sympy [C] Result contains complex when optimal does not.
time = 3.06, size = 1583, normalized size = 19.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((zoo*x*cos(c)**2/sin(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/a
**2, Eq(b, 0)), ((-x - cos(c + d*x)/(d*sin(c + d*x)))/b**2, Eq(a, 0)), (2*d
*x*sin(c + d*x)**2/(-8*b**2*d*sin(c + d*x)**2 + 16*I*b**2*d*sin(c + d*x)*co
s(c + d*x) + 8*b**2*d*cos(c + d*x)**2) - 4*I*d*x*sin(c + d*x)*cos(c + d*x)/
(-8*b**2*d*sin(c + d*x)**2 + 16*I*b**2*d*sin(c + d*x)*cos(c + d*x) + 8*b**2
*d*cos(c + d*x)**2) - 2*d*x*cos(c + d*x)**2/(-8*b**2*d*sin(c + d*x)**2 + 16
*I*b**2*d*sin(c + d*x)*cos(c + d*x) + 8*b**2*d*cos(c + d*x)**2) - I*sin(c +
 d*x)**2/(-8*b**2*d*sin(c + d*x)**2 + 16*I*b**2*d*sin(c + d*x)*cos(c + d*x)
 + 8*b**2*d*cos(c + d*x)**2) - 3*I*cos(c + d*x)**2/(-8*b**2*d*sin(c + d*x)*
*2 + 16*I*b**2*d*sin(c + d*x)*cos(c + d*x) + 8*b**2*d*cos(c + d*x)**2), Eq(
a, -I*b)), (2*d*x*sin(c + d*x)**2/(-8*b**2*d*sin(c + d*x)**2 - 16*I*b**2*d*
sin(c + d*x)*cos(c + d*x) + 8*b**2*d*cos(c + d*x)**2) + 4*I*d*x*sin(c + d*x
)*cos(c + d*x)/(-8*b**2*d*sin(c + d*x)**2 - 16*I*b**2*d*sin(c + d*x)*cos(c
+ d*x) + 8*b**2*d*cos(c + d*x)**2) - 2*d*x*cos(c + d*x)**2/(-8*b**2*d*sin(c
 + d*x)**2 - 16*I*b**2*d*sin(c + d*x)*cos(c + d*x) + 8*b**2*d*cos(c + d*x)*
*2) + I*sin(c + d*x)**2/(-8*b**2*d*sin(c + d*x)**2 - 16*I*b**2*d*sin(c + d*
x)*cos(c + d*x) + 8*b**2*d*cos(c + d*x)**2) + 3*I*cos(c + d*x)**2/(-8*b**2*
d*sin(c + d*x)**2 - 16*I*b**2*d*sin(c + d*x)*cos(c + d*x) + 8*b**2*d*cos(c
+ d*x)**2), Eq(a, I*b)), (x*cos(c)**2/(a*cos(c) + b*sin(c))**2, Eq(d, 0)),
(a**4*d*x*cos(c + d*x)/(a**6*d*cos(c + d*x) + a**5*b*d*sin(c + d*x) + 2*a**
4*b**2*d*cos(c + d*x) + 2*a**3*b**3*d*sin(c + d*x) + a**2*b**4*d*cos(c + d*
x) + a*b**5*d*sin(c + d*x)) + a**3*b*d*x*sin(c + d*x)/(a**6*d*cos(c + d*x)
+ a**5*b*d*sin(c + d*x) + 2*a**4*b**2*d*cos(c + d*x) + 2*a**3*b**3*d*sin(c
+ d*x) + a**2*b**4*d*cos(c + d*x) + a*b**5*d*sin(c + d*x)) + 2*a**3*b*log(a
*cos(c + d*x)/b + sin(c + d*x))*cos(c + d*x)/(a**6*d*cos(c + d*x) + a**5*b*
d*sin(c + d*x) + 2*a**4*b**2*d*cos(c + d*x) + 2*a**3*b**3*d*sin(c + d*x) +
a**2*b**4*d*cos(c + d*x) + a*b**5*d*sin(c + d*x)) - a**2*b**2*d*x*cos(c + d
*x)/(a**6*d*cos(c + d*x) + a**5*b*d*sin(c + d*x) + 2*a**4*b**2*d*cos(c + d*
x) + 2*a**3*b**3*d*sin(c + d*x) + a**2*b**4*d*cos(c + d*x) + a*b**5*d*sin(c
 + d*x)) + 2*a**2*b**2*log(a*cos(c + d*x)/b + sin(c + d*x))*sin(c + d*x)/(a
**6*d*cos(c + d*x) + a**5*b*d*sin(c + d*x) + 2*a**4*b**2*d*cos(c + d*x) + 2
*a**3*b**3*d*sin(c + d*x) + a**2*b**4*d*cos(c + d*x) + a*b**5*d*sin(c + d*x
)) + a**2*b**2*sin(c + d*x)/(a**6*d*cos(c + d*x) + a**5*b*d*sin(c + d*x) +
2*a**4*b**2*d*cos(c + d*x) + 2*a**3*b**3*d*sin(c + d*x) + a**2*b**4*d*cos(c
 + d*x) + a*b**5*d*sin(c + d*x)) - a*b**3*d*x*sin(c + d*x)/(a**6*d*cos(c +
d*x) + a**5*b*d*sin(c + d*x) + 2*a**4*b**2*d*cos(c + d*x) + 2*a**3*b**3*d*s
in(c + d*x) + a**2*b**4*d*cos(c + d*x) + a*b**5*d*sin(c + d*x)) + b**4*sin(
c + d*x)/(a**6*d*cos(c + d*x) + a**5*b*d*sin(c + d*x) + 2*a**4*b**2*d*cos(c
 + d*x) + 2*a**3*b**3*d*sin(c + d*x) + a**2*b**4*d*cos(c + d*x) + a*b**5*d*
sin(c + d*x)), True))
```

Giac [**A**]

time = 0.44, size = 159, normalized size = 1.94

$$\frac{\frac{2\,ab^2\log(|b\tan(dx+c)+a|)}{a^4b+2\,a^2b^3+b^5}-\frac{ab\log\left(\tan(dx+c)^2+1\right)}{a^4+2\,a^2b^2+b^4}+\frac{(a^2-b^2)(dx+c)}{a^4+2\,a^2b^2+b^4}-\frac{2\,ab^2\tan(dx+c)+3\,a^2b+b^3}{(a^4+2\,a^2b^2+b^4)(b\tan(dx+c)+a)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*a*b^2*log(abs(b*tan(d*x + c) + a))/(a^4*b + 2*a^2*b^3 + b^5) - a*b*log(t
an(d*x + c)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + (a^2 - b^2)*(d*x + c)/(a^4 + 2
*a^2*b^2 + b^4) - (2*a*b^2*tan(d*x + c) + 3*a^2*b + b^3)/((a^4 + 2*a^2*b^2
+ b^4)*(b*tan(d*x + c) + a)))/d
```

Mupad [B]
time = 4.87, size = 3114, normalized size = 37.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

```
[Out] (2*a*b*log(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^2))/(d*(a^4 +
b^4 + 2*a^2*b^2)) - (2*a*b*log(1/(cos(c + d*x) + 1)))/(d*(a^4 + b^4 + 2*a^2
*b^2)) + (2*atan((tan(c/2 + (d*x)/2)*((((2*a*b*((((32*(6*a^8*b + 6*a^4*b^5
+ 12*a^6*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (64*a*b*(3*a*b^10 + 12
*a^3*b^8 + 18*a^5*b^6 + 12*a^7*b^4 + 3*a^9*b^2))/((a^4 + b^4 + 2*a^2*b^2)*(
a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a + b)*(a - b))/(a^4 + b^4 + 2*a^2*b^
2) - (64*a*b*(a + b)*(a - b)*(3*a*b^10 + 12*a^3*b^8 + 18*a^5*b^6 + 12*a^7*b
^4 + 3*a^9*b^2))/((a^4 + b^4 + 2*a^2*b^2)^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*
b^2))))/(a^4 + b^4 + 2*a^2*b^2) - ((a + b)*((32*(2*a*b^6 + a^7 - 7*a^3*b^4
- 8*a^5*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (2*a*b*((32*(6*a^8*b +
6*a^4*b^5 + 12*a^6*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (64*a*b*(3*a
*b^10 + 12*a^3*b^8 + 18*a^5*b^6 + 12*a^7*b^4 + 3*a^9*b^2))/((a^4 + b^4 + 2*
a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(a^4 + b^4 + 2*a^2*b^2))*(a
 - b))/(a^4 + b^4 + 2*a^2*b^2) + (32*(a + b)^3*(a - b)^3*(3*a*b^10 + 12*a^3
*b^8 + 18*a^5*b^6 + 12*a^7*b^4 + 3*a^9*b^2))/((a^4 + b^4 + 2*a^2*b^2)^3*(a^
6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a^6 - b^6 + 35*a^2*b^4 - 35*a^4*b^2))/(
a^6 + b^6 + 15*a^2*b^4 + 15*a^4*b^2)^2 - (2*a*b*(5*a^4 + 5*b^4 - 26*a^2*b^2
)*((32*(2*a^4*b + 4*a^2*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + ((a + b
)*(a - b)*((((32*(6*a^8*b + 6*a^4*b^5 + 12*a^6*b^3))/(a^6 + b^6 + 3*a^2*b^4
 + 3*a^4*b^2) - (64*a*b*(3*a*b^10 + 12*a^3*b^8 + 18*a^5*b^6 + 12*a^7*b^4 +
3*a^9*b^2))/((a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*
(a + b)*(a - b))/(a^4 + b^4 + 2*a^2*b^2) - (64*a*b*(a + b)*(a - b)*(3*a*b^1
0 + 12*a^3*b^8 + 18*a^5*b^6 + 12*a^7*b^4 + 3*a^9*b^2))/((a^4 + b^4 + 2*a^2*
b^2)^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(a^4 + b^4 + 2*a^2*b^2) + (2*
```

```
a*b*((32*(2*a*b^6 + a^7 - 7*a^3*b^4 - 8*a^5*b^2))/(a^6 + b^6 + 3*a^2*b^4 +
3*a^4*b^2) - (2*a*b*((32*(6*a^8*b + 6*a^4*b^5 + 12*a^6*b^3))/(a^6 + b^6 + 3
*a^2*b^4 + 3*a^4*b^2) - (64*a*b*(3*a*b^10 + 12*a^3*b^8 + 18*a^5*b^6 + 12*a^
7*b^4 + 3*a^9*b^2))/((a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4
*b^2))))/(a^4 + b^4 + 2*a^2*b^2)))/(a^4 + b^4 + 2*a^2*b^2) - (64*a*b*(a + b
)^2*(a - b)^2*(3*a*b^10 + 12*a^3*b^8 + 18*a^5*b^6 + 12*a^7*b^4 + 3*a^9*b^2)
)/((a^4 + b^4 + 2*a^2*b^2)^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(a^6 +
b^6 + 15*a^2*b^4 + 15*a^4*b^2)^2)*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 1
0*a^6*b^4 + 5*a^8*b^2))/(32*a*b^2 - 32*a^3) + ((((a + b)*(a - b)*((32*(3*a^
6*b + 3*a^2*b^5 + 6*a^4*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (2*a*b*
((32*(2*a^3*b^6 - a^9 - a*b^8 + 6*a^5*b^4 + 2*a^7*b^2))/(a^6 + b^6 + 3*a^2*
b^4 + 3*a^4*b^2) + (64*a*b*(3*a^10*b + 3*a^2*b^9 + 12*a^4*b^7 + 18*a^6*b^5
+ 12*a^8*b^3))/((a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)
)))/(a^4 + b^4 + 2*a^2*b^2)))/(a^4 + b^4 + 2*a^2*b^2) - (2*a*b*((((32*(2*a^
3*b^6 - a^9 - a*b^8 + 6*a^5*b^4 + 2*a^7*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^
4*b^2) + (64*a*b*(3*a^10*b + 3*a^2*b^9 + 12*a^4*b^7 + 18*a^6*b^5 + 12*a^8*b
^3))/((a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a + b)
*(a - b))/(a^4 + b^4 + 2*a^2*b^2) + (64*a*b*(a + b)*(a - b)*(3*a^10*b + 3*a
^2*b^9 + 12*a^4*b^7 + 18*a^6*b^5 + 12*a^8*b^3))/((a^4 + b^4 + 2*a^2*b^2)^2*
(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(a^4 + b^4 + 2*a^2*b^2) + (32*(a + b
)^3*(a - b)^3*(3*a^10*b + 3*a^2*b^9 + 12*a^4*b^7 + 18*a^6*b^5 + 12*a^8*b^3)
)/((a^4 + b^4 + 2*a^2*b^2)^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a^6 - b
^6 + 35*a^2*b^4 - 35*a^4*b^2)*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^
6*b^4 + 5*a^8*b^2))/((32*a*b^2 - 32*a^3)*(a^6 + b^6 + 15*a^2*b^4 + 15*a^4*b
^2)^2) + (2*a*b*(5*a^4 + 5*b^4 - 26*a^2*b^2)*((64*a^3*b^2)/(a^6 + b^6 + 3*a
^2*b^4 + 3*a^4*b^2) + (2*a*b*((32*(3*a^6*b + 3*a^2*b^5 + 6*a^4*b^3))/(a^6 +
 b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (2*a*b*((32*(2*a^3*b^6 - a^9 - a*b^8 + 6*a^
5*b^4 + 2*a^7*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (64*a*b*(3*a^10*b
 + 3*a^2*b^9 + 12*a^4*b^7 + 18*a^6*b^5 + 12*a^8*b^3))/((a^4 + b^4 + 2*a^2*b
^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(a^4 + b^4 + 2*a^2*b^2)))/(a^4 +
 b^4 + 2*a^2*b^2) + ((a + b)*(a - b)*((((32*(2*a^3*b^6 - a^9 - a*b^8 + 6*a^
5*b^4 + 2*a^7*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (64*a*b*(3*a^10*b
 + 3*a^2*b^9 + 12*a^4*b^7 + 18*a^6*b^5 + 12*a^8*b^3))/((a^4 + b^4 + 2*a^2*b
^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a + b)*(a - b))/(a^4 + b^4 + 2*a
^2*b^2) + (64*a*b*(a + b)*(a - b)*(3*a^10*b + 3*a^2*b^9 + 12*a^4*b^7 + 18*a
^6*b^5 + 12*a^8*b^3))/((a^4 + b^4 + 2*a^2*b^2)^2*(a^6 + b^6 + 3*a^2*b^4 + 3
*a^4*b^2))))/(a^4 + b^4 + 2*a^2*b^2) + (64*a*b*(a + b)^2*(a - b)^2*(3*a^10*
b + 3*a^2*b^9 + 12*a^4*b^7 + 18*a^6*b^5 + 12*a^8*b^3))/((a^4 + b^4 + 2*a^2*
b^2)^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a^10 + b^10 + 5*a^2*b^8 + 10*
a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2))/((32*a*b^2 - 32*a^3)*(a^6 + b^6 + 15*a^2
*b^4 + 15*a^4*b^2)^2))*(a + b)*(a - b))/(d*(a^4 + b^4 + 2*a^2*b^2)) + (2*b^
2*tan(c/2 + (d*x)/2))/(a*d*(a^2 + b^2)*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(
c/2 + (d*x)/2)^2))
```

3.125 $\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=83

$$-\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d}-\frac{b}{(a^2+b^2)\,d(a\cos(c+dx)+b\sin(c+dx))}$$

```
[Out] -a*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d-b
/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))
```

Rubi [A]
time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3234, 3153, 212}

$$-\frac{b}{d\,(a^2+b^2)\,(a\cos(c+dx)+b\sin(c+dx))}-\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{d\,(a^2+b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] -((a*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 + b^
2)^(3/2)*d)) - b/((a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3234

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
```

```
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{b}{(a^2+b^2)\,d(a\cos(c+dx)+b\sin(c+dx))} + \frac{a\int \frac{1}{a\cos(c+dx)+b\sin(c+\cdots}}{a^2+b^2}$$

$$= -\frac{b}{(a^2+b^2)\,d(a\cos(c+dx)+b\sin(c+dx))} - \frac{a\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,d\cdots\right.}{}$$

$$= -\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}\,d} - \frac{b}{(a^2+b^2)\,d(a\cos(c+dx)+b\,\mathrm{s}\cdots}$$

Mathematica [A]
time = 0.25, size = 79, normalized size = 0.95

$$\frac{\frac{2a\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}} - \frac{b}{(a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
[Out] ((2*a*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2)
 - b/((a^2 + b^2)*(a*Cos[c + d*x] + b*Sin[c + d*x])))/d
```

Maple [A]
time = 0.36, size = 118, normalized size = 1.42

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a^2+b^2)a}-\frac{b}{a^2+b^2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}+\frac{2a\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}}{d}$
default	$\frac{-\frac{2\left(-\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a^2+b^2)a}-\frac{b}{a^2+b^2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}+\frac{2a\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}}{d}$
risch	$-\frac{2ib\,\mathrm{e}^{i(dx+c)}}{(-ia+b)d(ia+b)\left(b\,\mathrm{e}^{2i(dx+c)}+ia\,\mathrm{e}^{2i(dx+c)}-b+ia\right)}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^3+ia\,b^2-a^2b-b^3}{(a^2+b^2)^{\frac{3}{2}}}\right)}{(a^2+b^2)^{\frac{3}{2}}\,d}-\frac{a\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^3+i\cdots}{(a\cdots}\right)}{(a^2+b^2)^{\frac{3}{2}}\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-b^2/(a^2+b^2)/a*tan(1/2*d*x+1/2*c)-b/(a^2+b^2))/(a*tan(1/2*d*x+1/
2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)+2*a/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1
/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(79) = 158$.
time = 0.49, size = 182, normalized size = 2.19

$$-\frac{\frac{a \log\left(\frac{b-\frac{a \sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a \sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}+\frac{2\left(ab+\frac{b^2 \sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+a^2b^2+\frac{2\left(a^3b+ab^3\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{\left(a^4+a^2b^2\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(a*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*si
n(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(a*
b + b^2*sin(d*x + c)/(cos(d*x + c) + 1))/(a^4 + a^2*b^2 + 2*(a^3*b + a*b^3)
*sin(d*x + c)/(cos(d*x + c) + 1) - (a^4 + a^2*b^2)*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(79) = 158$.
time = 3.11, size = 215, normalized size = 2.59

$$-\frac{2\,a^2b+2\,b^3-\left(a^2\cos\left(dx+c\right)+ab\sin\left(dx+c\right)\right)\sqrt{a^2+b^2}\,\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)}{2\left((a^5+2\,a^3b^2+ab^4)d\cos\left(dx+c\right)+(a^4b+2\,a^2b^3+b^5)d\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*b + 2*b^3 - (a^2*cos(d*x + c) + a*b*sin(d*x + c))*sqrt(a^2 + b^
2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a
^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(
d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)))/((a^5 + 2*a^3*b
^2 + a*b^4)*d*cos(d*x + c) + (a^4*b + 2*a^2*b^3 + b^5)*d*sin(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [A]
time = 0.46, size = 138, normalized size = 1.66

$$-\frac{\frac{a\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}}-\frac{2\,\left(b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+ab\right)}{\left(a^3+ab^2\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(a*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan
(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(b^2*ta
n(1/2*d*x + 1/2*c) + a*b)/((a^3 + a*b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*ta
n(1/2*d*x + 1/2*c) - a)))/d
```

Mupad [B]
time = 0.84, size = 136, normalized size = 1.64

$$-\frac{\frac{2\,b}{a^2+b^2}+\frac{2\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,\left(a^2+b^2\right)}}{d\,\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}+\frac{a\,\mathrm{atan}\left(\frac{a^2\,b\,1\mathrm{i}+b^3\,1\mathrm{i}-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2+b^2\right)\,1\mathrm{i}}{\left(a^2+b^2\right)^{3/2}}\right)\,2\mathrm{i}}{d\,\left(a^2+b^2\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

```
[Out] (a*atan((a^2*b*1i + b^3*1i - a*tan(c/2 + (d*x)/2)*(a^2 + b^2)*1i)/(a^2 + b^
2)^(3/2))*2i)/(d*(a^2 + b^2)^(3/2)) - ((2*b)/(a^2 + b^2) + (2*b^2*tan(c/2 +
 (d*x)/2))/(a*(a^2 + b^2)))/(d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d
*x)/2)^2))
```

3.126 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=32

$$\frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

```
[Out] sin(d*x+c)/a/d/(a*cos(d*x+c)+b*sin(d*x+c))
```

Rubi [A]
time = 0.01, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = $\{3154\}$

$$\frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-2),x]
```

```
[Out] Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Rule 3154

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

Mathematica [A]
time = 0.04, size = 32, normalized size = 1.00

$$\frac{\sin(c+dx)}{ad(a\cos(c+dx)+b\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-2),x]
```

```
[Out] Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Maple [A]
time = 0.25, size = 21, normalized size = 0.66

method	result	size
derivativedivides	$-\frac{1}{db(a+b\tan(dx+c))}$	21
default	$-\frac{1}{db(a+b\tan(dx+c))}$	21
risch	$\frac{2i}{d(-ib+a)\left(-ib\,\mathrm{e}^{2i(dx+c)}+a\,\mathrm{e}^{2i(dx+c)}+ib+a\right)}$	47
norman	$\frac{\frac{1}{bd}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{bd}}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d/b/(a+b*tan(d*x+c))
```

Maxima [**A**]
time = 0.28, size = 21, normalized size = 0.66

$$-\frac{1}{(b^2\tan(dx+c)+ab)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/((b^2*tan(d*x + c) + a*b)*d)
```

Fricas [**A**]
time = 2.59, size = 57, normalized size = 1.78

$$-\frac{b\cos(dx+c)-a\sin(dx+c)}{(a^3+ab^2)d\cos(dx+c)+(a^2b+b^3)d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(b*cos(d*x + c) - a*sin(d*x + c))/((a^3 + a*b^2)*d*cos(d*x + c) + (a^2*b +
 b^3)*d*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.40, size = 20, normalized size = 0.62

$$-\frac{1}{(b\tan(dx+c)+a)bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/((b*tan(d*x + c) + a)*b*d)
```

Mupad [B]
time = 0.49, size = 47, normalized size = 1.47

$$\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a\,d\,\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^2,x)
```

```
[Out] (2*tan(c/2 + (d*x)/2))/(a*d*(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)
/2)^2))
```

3.127 $\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=92

$$\frac{\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}$$

```
[Out] arctanh(sin(d*x+c))/b^2/d-1/b/d/(a*cos(d*x+c)+b*sin(d*x+c))+a*arctanh((b*co
s(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/b^2/d/(a^2+b^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3173, 3855, 3153, 212}

$$\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2d\sqrt{a^2+b^2}}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}+\frac{\tanh^{-1}(\sin(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(b^2*d) + (a*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x]
)/Sqrt[a^2 + b^2]])/(b^2*Sqrt[a^2 + b^2]*d) - 1/(b*d*(a*Cos[c + d*x] + b*Si
n[c + d*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3173

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/co
s[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^
(n + 1)/(b*d*(n + 1)), x] + (Dist[1/b^2, Int[(a*Cos[c + d*x] + b*Sin[c + d*
x])^(n + 2)/Cos[c + d*x], x], x] - Dist[a/b^2, Int[(a*Cos[c + d*x] + b*Sin[
c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &
```

```
& LtQ[n, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}+\frac{\int \sec(c+dx)\,dx}{b^2}-\frac{a\int \frac{}{a\cos(c+}}{}$$
$$=\frac{\tanh^{-1}(\sin(c+dx))}{b^2d}-\frac{1}{bd(a\cos(c+dx)+b\sin(c+dx))}+\frac{a\text{Subst}(}{}$$
$$=\frac{\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^2\sqrt{a^2+b^2}\,d}-\frac{}{bd(a\cos(}$$

Mathematica [A]
time = 0.87, size = 120, normalized size = 1.30

$$-\frac{\frac{2a\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{b\sec(c+dx)}{a+b\tan(c+dx)}}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] -(((2*a*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2]
 + Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]] + (b*Sec[c + d*x])/(a + b*Tan[c + d*x]))/(b^2*d))
```

Maple [A]
time = 0.52, size = 135, normalized size = 1.47

method	result
derivativedivides	$\frac{-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}+\frac{2\left(\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+b\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\frac{2a\,\text{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{d}$

default	$\frac{-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}+\frac{\frac{2\left(\frac{b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}+b\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a}-\frac{2a\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}}{b^2}}{d}$
risch	$-\frac{2\,e^{i(dx+c)}}{db\left(-ib\,e^{2i(dx+c)}+a\,e^{2i(dx+c)}+ib+a\right)}+\frac{a\ln\left(e^{i(dx+c)}-\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d\,b^2}-\frac{a\ln\left(e^{i(dx+c)}+\frac{ia-b}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,d\,b^2}-\frac{\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^2*ln(tan(1/2*d*x+1/2*c)-1)+1/b^2*ln(tan(1/2*d*x+1/2*c)+1)+2/b^2*(
(b^2/a*tan(1/2*d*x+1/2*c)+b)/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)
-a)-a/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1
/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(88) = 176$.
time = 0.50, size = 212, normalized size = 2.30

$$-\frac{\frac{2\left(a+\frac{b\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2b+\frac{2\,ab^2\sin(dx+c)}{\cos(dx+c)+1}-\frac{a^2b\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}-\frac{a\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b^2}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^2}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(2*(a + b*sin(d*x + c)/(cos(d*x + c) + 1))/(a^2*b + 2*a*b^2*sin(d*x + c)/(
cos(d*x + c) + 1) - a^2*b*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) - a*log((b -
 a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(
cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) - log(sin(d*x +
 c)/(cos(d*x + c) + 1) + 1)/b^2 + log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/
b^2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(88) = 176$.
time = 2.76, size = 293, normalized size = 3.18

$$-\frac{2\,a^2b+2\,b^3-\left(a^2\cos\left(dx+c\right)+ab\sin\left(dx+c\right)\right)\sqrt{a^2+b^2}\,\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)-\left((a^3+ab^2)\cos\left(dx+c\right)+(a^2b+b^3)\sin\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)+\left((a^3+ab^2)\cos\left(dx+c\right)+(a^2b+b^3)\sin\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)}{2\left((a^3b^2+ab^4)d\cos\left(dx+c\right)+(a^2b^3+b^5)d\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^2*b + 2*b^3 - (a^2*cos(d*x + c) + a*b*sin(d*x + c))*sqrt(a^2 + b^
2)*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^
```

```
2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d
*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - ((a^3 + a*b^2)*
cos(d*x + c) + (a^2*b + b^3)*sin(d*x + c))*log(sin(d*x + c) + 1) + ((a^3 +
a*b^2)*cos(d*x + c) + (a^2*b + b^3)*sin(d*x + c))*log(-sin(d*x + c) + 1))/(
(a^3*b^2 + a*b^4)*d*cos(d*x + c) + (a^2*b^3 + b^5)*d*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)/(a*cos(c + d*x) + b*sin(c + d*x))**2, x)
```

Giac [A]
time = 0.51, size = 166, normalized size = 1.80

$$\frac{\frac{a\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^2}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^2}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^2}+\frac{2\,\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)ab}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (a*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(
1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^2) + log(ab
s(tan(1/2*d*x + 1/2*c) + 1))/b^2 - log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^2 +
 2*(b*tan(1/2*d*x + 1/2*c) + a)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*
x + 1/2*c) - a)*a*b))/d
```

Mupad [B]
time = 1.17, size = 383, normalized size = 4.16

$$-\frac{b^2\sin(c+dx)-\frac{2\left(a^2\cos(c+dx)\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\sqrt{a^2+b^2}+a^3\,\mathrm{atan}\left(\frac{1\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^2+1\mathrm{i}\,\cos\left(\frac{c}{2}+\frac{dx}{2}\right)ab+2\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)b^2}{a\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2+b^2}+2\,b\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2+b^2}}\right)\cos(c+dx)\,1\mathrm{i}\right)}{\sqrt{a^2+b^2}}+\frac{2\,b\left(\frac{a\sqrt{a^2+b^2}}{2}+\frac{a\cos(c+dx)\sqrt{a^2+b^2}}{2}-a\sin(c+dx)\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\sqrt{a^2+b^2}-a^2\,\mathrm{atan}\left(\frac{1\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)a^2+1\mathrm{i}\,\cos\left(\frac{c}{2}+\frac{dx}{2}\right)ab+2\mathrm{i}\,\sin\left(\frac{c}{2}+\frac{dx}{2}\right)b^2}{a\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2+b^2}+2\,b\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2+b^2}}\right)\sin(c+dx)\,1\mathrm{i}\right)}{\sqrt{a^2+b^2}}}{a\,b^2\,d\,(a\cos(c+dx)+b\sin(c+dx))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a*cos(c + d*x) + b*sin(c + d*x))^2),x)
```

```
[Out] -(b^2*sin(c + d*x) - (2*(a^3*atan((a^2*sin(c/2 + (d*x)/2)*1i + b^2*sin(c/2
+ (d*x)/2)*2i + a*b*cos(c/2 + (d*x)/2)*1i)/(a*cos(c/2 + (d*x)/2)*(a^2 + b^2
)^(1/2) + 2*b*sin(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)))*cos(c + d*x)*1i + a^2*
cos(c + d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*(a^2 + b^2)^(1/2)
```

```
))/(a^2 + b^2)^(1/2) + (2*b*((a*(a^2 + b^2)^(1/2))/2 + (a*cos(c + d*x)*(a^2
 + b^2)^(1/2))/2 - a^2*atan((a^2*sin(c/2 + (d*x)/2)*1i + b^2*sin(c/2 + (d*x
)/2)*2i + a*b*cos(c/2 + (d*x)/2)*1i)/(a*cos(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2
) + 2*b*sin(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2)))*sin(c + d*x)*1i - a*sin(c +
d*x)*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*(a^2 + b^2)^(1/2)))/(a^2
+ b^2)^(1/2))/(a*b^2*d*(a*cos(c + d*x) + b*sin(c + d*x)))
```

3.128 $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=75

$$\frac{\frac{1}{a}+\frac{a}{b^2}}{d(b+a\cot(c+dx))}-\frac{2a\log(b+a\cot(c+dx))}{b^3d}-\frac{2a\log(\tan(c+dx))}{b^3d}+\frac{\tan(c+dx)}{b^2d}$$

```
[Out] (1/a+a/b^2)/d/(b+a*cot(d*x+c))-2*a*ln(b+a*cot(d*x+c))/b^3/d-2*a*ln(tan(d*x+c))/b^3/d+tan(d*x+c)/b^2/d
```

Rubi [A]

time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3167, 908}

$$-\frac{2a\log(\tan(c+dx))}{b^3d}-\frac{2a\log(a\cot(c+dx)+b)}{b^3d}+\frac{\frac{a}{b^2}+\frac{1}{a}}{d(a\cot(c+dx)+b)}+\frac{\tan(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^(-1) + a/b^2)/(d*(b + a*Cot[c + d*x])) - (2*a*Log[b + a*Cot[c + d*x]])/(b^3*d) - (2*a*Log[Tan[c + d*x]])/(b^3*d) + Tan[c + d*x]/(b^2*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0]))
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b + a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] && !(GtQ[n, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{1+x^2}{x^2(b+ax)^2}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\operatorname{Subst}\left(\int \left(\frac{1}{b^2x^2}-\frac{2a}{b^3x}+\frac{a^2+b^2}{b^2(b+ax)^2}+\frac{2a^2}{b^3(b+ax)}\right)\,dx, x, \cot(c+dx)\right)}{d}$$
$$= \frac{\frac{1}{a}+\frac{a}{b^2}}{d(b+a\cot(c+dx))} - \frac{2a\log(b+a\cot(c+dx))}{b^3d} - \frac{2a\log(\tan(c+}{b^3d}$$

Mathematica [A]
time = 0.37, size = 121, normalized size = 1.61

$$\frac{2a^3(\log(\cos(c+dx))-\log(a\cos(c+dx)+b\sin(c+dx)))+b(2a^2+b^2+2a^2\log(\cos(c+dx))-2a^2\log(a\cos(c+dx)+b\sin(c+dx)))\tan(c+dx)+ab^2\tan^2(c+dx)}{ab^3d(a+b\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*a^3*(Log[Cos[c + d*x]] - Log[a*Cos[c + d*x] + b*Sin[c + d*x]]) + b*(2*a^
2 + b^2 + 2*a^2*Log[Cos[c + d*x]] - 2*a^2*Log[a*Cos[c + d*x] + b*Sin[c + d*
x]])*Tan[c + d*x] + a*b^2*Tan[c + d*x]^2)/(a*b^3*d*(a + b*Tan[c + d*x]))
```

Maple [A]
time = 0.44, size = 57, normalized size = 0.76

method	result
derivativedivides	$\frac{\frac{\tan(dx+c)}{b^2}-\frac{2a\ln(a+b\tan(dx+c))}{b^3}-\frac{a^2+b^2}{b^3(a+b\tan(dx+c))}}{d}$
default	$\frac{\frac{\tan(dx+c)}{b^2}-\frac{2a\ln(a+b\tan(dx+c))}{b^3}-\frac{a^2+b^2}{b^3(a+b\tan(dx+c))}}{d}$
risch	$-\frac{4i\left(-ia\,e^{2i(dx+c)}+b-ia\right)}{\left(e^{2i(dx+c)}+1\right)\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)b^2d}+\frac{2a\ln\left(e^{2i(dx+c)}+1\right)}{b^3d}-\frac{2a\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{b^3d}$
norman	$\frac{\frac{(4a^2+6b^2)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3d}-\frac{4a^2+2b^2}{2b^3d}-\frac{(4a^2+2b^2)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2b^3d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)}+\frac{2a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^3d}+\frac{2a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^2*tan(d*x+c)-2*a/b^3*ln(a+b*tan(d*x+c))-1/b^3*(a^2+b^2)/(a+b*tan(d
*x+c)))
```

Maxima [A]
time = 0.27, size = 60, normalized size = 0.80

$$-\frac{\frac{a^2+b^2}{b^4\tan(dx+c)+ab^3}+\frac{2\,a\log(b\tan(dx+c)+a)}{b^3}-\frac{\tan(dx+c)}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -((a^2 + b^2)/(b^4*tan(d*x + c) + a*b^3) + 2*a*log(b*tan(d*x + c) + a)/b^3
- tan(d*x + c)/b^2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(75) = 150.
time = 2.70, size = 178, normalized size = 2.37

$$-\frac{2\,b^2\cos(dx+c)^2-2\,ab\cos(dx+c)\sin(dx+c)-b^2+\left(a^2\cos(dx+c)^2+ab\cos(dx+c)\sin(dx+c)\right)\log\left(2\,ab\cos(dx+c)\sin(dx+c)+\left(a^2-b^2\right)\cos(dx+c)^2+b^2\right)-\left(a^2\cos(dx+c)^2+ab\cos(dx+c)\sin(dx+c)\right)\log\left(\cos(dx+c)^2\right)}{ab^3d\cos(dx+c)^2+b^4d\cos(dx+c)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(2*b^2*cos(d*x + c)^2 - 2*a*b*cos(d*x + c)*sin(d*x + c) - b^2 + (a^2*cos(d
*x + c)^2 + a*b*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x +
 c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (a^2*cos(d*x + c)^2 + a*b*cos(d*x
 + c)*sin(d*x + c))*log(cos(d*x + c)^2))/(a*b^3*d*cos(d*x + c)^2 + b^4*d*co
s(d*x + c)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^2(c+dx)}{\left(a\cos(c+dx)+b\sin(c+dx)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*cos(c + d*x) + b*sin(c + d*x))**2, x)
```

Giac [A]
time = 0.46, size = 71, normalized size = 0.95

$$-\frac{\frac{2\,a\log(|b\tan(dx+c)+a|)}{b^3}-\frac{\tan(dx+c)}{b^2}-\frac{2\,ab\tan(dx+c)+a^2-b^2}{(b\tan(dx+c)+a)b^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(2*a*log(abs(b*tan(d*x + c) + a))/b^3 - tan(d*x + c)/b^2 - (2*a*b*tan(d*x
+ c) + a^2 - b^2)/((b*tan(d*x + c) + a)*b^3))/d
```

Mupad [B]
time = 2.36, size = 382, normalized size = 5.09

$$\frac{\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{b}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(2\,a^2+b^2\right)}{a\,b^2}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^2+b^2\right)}{a\,b^2}}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}-\frac{4\,a\,\mathrm{atanh}\left(\frac{64\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{64\,a^3-64\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{128\,a^5}{b^2}-\frac{128\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{b^2}+\frac{128\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b}}-\frac{64\,a^3}{64\,a^3-64\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{128\,a^5}{b^2}-\frac{128\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{b^2}+\frac{128\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b}}+\frac{128\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^3\,b+\frac{128\,a^5}{b}+128\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{128\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{b}-64\,a^3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)}{b^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a*cos(c + d*x) + b*sin(c + d*x))^2),x)
```

```
[Out] ((4*tan(c/2 + (d*x)/2)^2)/b - (2*tan(c/2 + (d*x)/2)^3*(2*a^2 + b^2))/(a*b^2
) + (2*tan(c/2 + (d*x)/2)*(2*a^2 + b^2))/(a*b^2))/(d*(a + 2*b*tan(c/2 + (d*
x)/2) - 2*a*tan(c/2 + (d*x)/2)^2 + a*tan(c/2 + (d*x)/2)^4 - 2*b*tan(c/2 + (
d*x)/2)^3)) - (4*a*atanh((64*a^3*tan(c/2 + (d*x)/2)^2)/(64*a^3 - 64*a^3*tan
(c/2 + (d*x)/2)^2 + (128*a^5)/b^2 - (128*a^5*tan(c/2 + (d*x)/2)^2)/b^2 + (1
28*a^4*tan(c/2 + (d*x)/2))/b) - (64*a^3)/(64*a^3 - 64*a^3*tan(c/2 + (d*x)/2
)^2 + (128*a^5)/b^2 - (128*a^5*tan(c/2 + (d*x)/2)^2)/b^2 + (128*a^4*tan(c/2
 + (d*x)/2))/b) + (128*a^4*tan(c/2 + (d*x)/2))/(64*a^3*b + (128*a^5)/b + 12
8*a^4*tan(c/2 + (d*x)/2) - (128*a^5*tan(c/2 + (d*x)/2)^2)/b - 64*a^3*b*tan(
c/2 + (d*x)/2)^2)))/(b^3*d)
```

3.129 $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=179

$$\frac{2a^2\tanh^{-1}(\sin(c+dx))}{b^4d}+\frac{\tanh^{-1}(\sin(c+dx))}{2b^2d}+\frac{(a^2+b^2)\tanh^{-1}(\sin(c+dx))}{b^4d}+\frac{3a\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b\cos(c+}{\sqrt{}}\right.}{b^4d}$$

```
[Out] 2*a^2*arctanh(sin(d*x+c))/b^4/d+1/2*arctanh(sin(d*x+c))/b^2/d+(a^2+b^2)*arc
tanh(sin(d*x+c))/b^4/d-2*a*sec(d*x+c)/b^3/d+(-a^2-b^2)/b^3/d/(a*cos(d*x+c)+
b*sin(d*x+c))+3*a*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))*(a^2
+b^2)^(1/2)/b^4/d+1/2*sec(d*x+c)*tan(d*x+c)/b^2/d
```

Rubi [A]
time = 0.18, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3185, 3173, 3855, 3153, 212, 3853, 3183}

$$\frac{2a^2\tanh^{-1}(\sin(c+dx))}{b^4d}+\frac{(a^2+b^2)\tanh^{-1}(\sin(c+dx))}{b^4d}+\frac{3a\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4d}-\frac{a^2+b^2}{b^3d(a\cos(c+dx)+b\sin(c+dx))}-\frac{2a\sec(c+dx)}{b^3d}+\frac{\tanh^{-1}(\sin(c+dx))}{2b^2d}+\frac{\tan(c+dx)\sec(c+dx)}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/(b^4*d) + ArcTanh[Sin[c + d*x]]/(2*b^2*d) + (
(a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(b^4*d) + (3*a*Sqrt[a^2 + b^2]*ArcTanh[(
b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^4*d) - (2*a*Sec[c + d
*x])/(b^3*d) - (a^2 + b^2)/(b^3*d*(a*Cos[c + d*x] + b*Sin[c + d*x])) + (Sec
[c + d*x]*Tan[c + d*x])/(2*b^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3173

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/co
s[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^
```

```
(n + 1)/(b*d*(n + 1)), x] + (Dist[1/b^2, Int[(a*Cos[c + d*x] + b*Sin[c + d*
x])^(n + 2)/Cos[c + d*x], x], x] - Dist[a/b^2, Int[(a*Cos[c + d*x] + b*Sin[
c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &
& LtQ[n, -1]
```

Rule 3183

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3185

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2 + b^2)/b^2, Int[Cos[c +
d*x]^(m + 2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^n, x], x] + (Dist[1/b^2, Int
[Cos[c + d*x]^m*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] - Dist[2*
(a/b^2), Int[Cos[c + d*x]^(m + 1)*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 1)
, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && L
tQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = \frac{\int \sec^3(c+dx)\,dx}{b^2} - \frac{(2a)\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{b^2} + \frac{(a^2+b^2)\int \frac{}{(a\cos}}{}$$

$$= -\frac{2a\sec(c+dx)}{b^3d} - \frac{a^2+b^2}{b^3d(a\cos(c+dx)+b\sin(c+dx))} + \frac{\sec(c+dx)}{2b}$$

$$= \frac{2a^2\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{\tanh^{-1}(\sin(c+dx))}{2b^2d} + \frac{(a^2+b^2)\tanh^{-1}(}{b^4d}$$

$$= \frac{2a^2\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{\tanh^{-1}(\sin(c+dx))}{2b^2d} + \frac{(a^2+b^2)\tanh^{-1}(}{b^4d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.13, size = 709, normalized size = 3.96

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] -(((a - I*b)*(a + I*b)*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x]))/(b
^3*d*(a + b*Tan[c + d*x])^2)) - (2*a*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2)/(b^3*d*(a + b*Tan[c + d*x])^2) - (6*a*Sqrt[a^2 + b^2]*ArcTanh
[(Sqrt[a^2 + b^2]*(-(b*Cos[(c + d*x)/2]) + a*Sin[(c + d*x)/2]))/(a^2*Cos[(c
 + d*x)/2] + b^2*Cos[(c + d*x)/2])]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[
c + d*x])^2)/(b^4*d*(a + b*Tan[c + d*x])^2) - (3*(2*a^2 + b^2)*Log[Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x
])^2)/(2*b^4*d*(a + b*Tan[c + d*x])^2) + (3*(2*a^2 + b^2)*Log[Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]]*Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
/(2*b^4*d*(a + b*Tan[c + d*x])^2) + (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2)/(4*b^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2*(a + b*Tan[c
 + d*x])^2) - (2*a*Sec[c + d*x]^2*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[
c + d*x])^2)/(b^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(a + b*Tan[c + d*
x])^2) - (Sec[c + d*x]^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(4*b^2*d*(Cos
[(c + d*x)/2] + Sin[(c + d*x)/2])^2*(a + b*Tan[c + d*x])^2) + (2*a*Sec[c +
d*x]^2*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(b^3*d*(Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2])*(a + b*Tan[c + d*x])^2)
```

Maple [A]
time = 0.68, size = 259, normalized size = 1.45

method	result

derivativedivides	$\frac{\frac{1}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-4a-b}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-6a^2-3b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^4}-\frac{1}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{4a-b}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right.}}{}$
default	$\frac{\frac{1}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-4a-b}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-6a^2-3b^2\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^4}-\frac{1}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{4a-b}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right.}}{}$
risch	$-\frac{-3iab\,e^{5i(dx+c)}+6a^2e^{5i(dx+c)}+3b^2e^{5i(dx+c)}+12a^2e^{3i(dx+c)}+2b^2e^{3i(dx+c)}+3iab\,e^{i(dx+c)}+6a^2e^{i(dx+c)}+3b^2e^{i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)^2\left(-ib\,e^{2i(dx+c)}+a\,e^{2i(dx+c)}+ib+a\right)b^3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/b^2/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(-4*a-b)/b^3/(tan(1/2*d*x+1/2*c)-
1)+1/2/b^4*(-6*a^2-3*b^2)*ln(tan(1/2*d*x+1/2*c)-1)-1/2/b^2/(tan(1/2*d*x+1/2
*c)+1)^2-1/2*(4*a-b)/b^3/(tan(1/2*d*x+1/2*c)+1)+1/2/b^4*(6*a^2+3*b^2)*ln(ta
n(1/2*d*x+1/2*c)+1)+2/b^4*(((a^2+b^2)*b^2/a*tan(1/2*d*x+1/2*c)+b*(a^2+b^2))
/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)-3*(a^2+b^2)^(1/2)*a*arct
anh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(171) = 342$.
time = 0.51, size = 471, normalized size = 2.63

$$-\frac{\frac{2\left(6a^3+2ab^2+\frac{6a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{(9a^2b+2b^3)\sin(dx+c)}{\cos(dx+c)+1}-\frac{6(2a^3+ab^2)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4(3a^2b+b^3)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{(3a^2b+2b^3)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2b^3+\frac{2ab^4\sin(dx+c)}{\cos(dx+c)+1}-\frac{3a^2b^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4ab^4\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3a^2b^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{2ab^4\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a^2b^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{6\sqrt{a^2+b^2}\,a\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{b^4}-\frac{3(2a^2+b^2)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^4}+\frac{3(2a^2+b^2)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^4}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(6*a^3 + 2*a*b^2 + 6*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (9*a
^2*b + 2*b^3)*sin(d*x + c)/(cos(d*x + c) + 1) - 6*(2*a^3 + a*b^2)*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 - 4*(3*a^2*b + b^3)*sin(d*x + c)^3/(cos(d*x + c)
 + 1)^3 + (3*a^2*b + 2*b^3)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2*b^3 +
 2*a*b^4*sin(d*x + c)/(cos(d*x + c) + 1) - 3*a^2*b^3*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 - 4*a*b^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*a^2*b^3*sin
(d*x + c)^4/(cos(d*x + c) + 1)^4 + 2*a*b^4*sin(d*x + c)^5/(cos(d*x + c) + 1
)^5 - a^2*b^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) - 6*sqrt(a^2 + b^2)*a*lo
g((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x
+ c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/b^4 - 3*(2*a^2 + b^2)*log(sin(d
*x + c)/(cos(d*x + c) + 1) + 1)/b^4 + 3*(2*a^2 + b^2)*log(sin(d*x + c)/(cos
(d*x + c) + 1) - 1)/b^4)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(171) = 342$.
time = 3.13, size = 355, normalized size = 1.98

$$-\frac{6\,ab^{2}\cos\left(dx+c\right)\sin\left(dx+c\right)-2\,b^{3}+6\left(2\,a^{2}b+b^{3}\right)\cos\left(dx+c\right)^{2}-6\left(a^{2}\cos\left(dx+c\right)^{3}+ab\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)\right)\sqrt{a^{2}+b^{2}}\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^{2}-b^{2})\cos(dx+c)^{2}-2\,a^{2}-b^{2}-2\sqrt{a^{2}+b^{2}}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^{2}-b^{2})\cos(dx+c)^{2}+b^{2}}\right)-3\left(\left(2\,a^{3}+ab^{2}\right)\cos\left(dx+c\right)^{3}+\left(2\,a^{2}b+b^{3}\right)\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)+3\left(\left(2\,a^{3}+ab^{2}\right)\cos\left(dx+c\right)^{3}+\left(2\,a^{2}b+b^{3}\right)\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)}{4\left(ab^{4}d\cos\left(dx+c\right)^{3}+b^{5}d\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(6*a*b^2*cos(d*x + c)*sin(d*x + c) - 2*b^3 + 6*(2*a^2*b + b^3)*cos(d*x
 + c)^2 - 6*(a^2*cos(d*x + c)^3 + a*b*cos(d*x + c)^2*sin(d*x + c))*sqrt(a^2
 + b^2)*log((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 -
 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*
cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 3*((2*a^3
+ a*b^2)*cos(d*x + c)^3 + (2*a^2*b + b^3)*cos(d*x + c)^2*sin(d*x + c))*log(
sin(d*x + c) + 1) + 3*((2*a^3 + a*b^2)*cos(d*x + c)^3 + (2*a^2*b + b^3)*cos
(d*x + c)^2*sin(d*x + c))*log(-sin(d*x + c) + 1))/(a*b^4*d*cos(d*x + c)^3 +
 b^5*d*cos(d*x + c)^2*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{3}\left(c+dx\right)}{\left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*cos(c + d*x) + b*sin(c + d*x))**2, x)
```

Giac [A]
time = 0.54, size = 280, normalized size = 1.56

$$\frac{\frac{3\,(2\,a^{2}+b^{2})\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^{4}}-\frac{3\,(2\,a^{2}+b^{2})\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^{4}}+\frac{6\,(a^{3}+ab^{2})\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\sqrt{a^{2}+b^{2}}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\sqrt{a^{2}+b^{2}}\right|}\right)}{\sqrt{a^{2}+b^{2}}\,b^{4}}+\frac{2\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+4\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}b^{3}}+\frac{4\,(a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+a^{3}+ab^{2})}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)ab^{3}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*(2*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 3*(2*a^2 + b^
2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 + 6*(a^3 + a*b^2)*log(abs(2*a*tan
(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) -
 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^4) + 2*(b*tan(1/2*d*x + 1/2*c
)^3 + 4*a*tan(1/2*d*x + 1/2*c)^2 + b*tan(1/2*d*x + 1/2*c) - 4*a)/((tan(1/2*
d*x + 1/2*c)^2 - 1)^2*b^3) + 4*(a^2*b*tan(1/2*d*x + 1/2*c) + b^3*tan(1/2*d*
```

x + 1/2*c) + a^3 + a*b^2)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)*a*b^3))/d

Mupad [B]
time = 1.90, size = 585, normalized size = 3.27

$$\frac{\operatorname{atanh}\left(\frac{648\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{216\,a\,b^2+648\,a^3+\frac{432\,a^5}{b^2}}+\frac{432\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{432\,a^5+648\,a^3\,b^2+216\,a\,b^4}+\frac{216\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{216\,a+\frac{648\,a^3}{b^2}+\frac{432\,a^5}{b^4}}\right)\,\left(6\,a^2+3\,b^2\right)}{b^4\,d}-\frac{\frac{2\,\left(3\,a^2+b^2\right)}{b^3}+\frac{6\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{b^3}-\frac{6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a^2+b^2\right)}{b^3}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(9\,a^2+2\,b^2\right)}{a\,b^2}-\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(3\,a^2+b^2\right)}{a\,b^2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(3\,a^2+2\,b^2\right)}{a\,b^2}}{d\,\left(-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\right)}-\frac{6\,a\operatorname{atanh}\left(\frac{432\,a^3\,\sqrt{a^2+b^2}}{432\,a^3\,b+\frac{432\,a^5}{b}+864\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+864\,a^2\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}+\frac{864\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2+b^2}}{432\,a^3+\frac{432\,a^5}{b^2}+864\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\frac{864\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b}}+\frac{432\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2+b^2}}{432\,a^5+864\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^4\,b+432\,a^3\,b^2+864\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^3}\right)\,\sqrt{a^2+b^2}}{b^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a*cos(c + d*x) + b*sin(c + d*x))^2),x)
```

```
[Out] (atanh((648*a^3*tan(c/2 + (d*x)/2))/(216*a*b^2 + 648*a^3 + (432*a^5)/b^2) +
 (432*a^5*tan(c/2 + (d*x)/2))/(216*a*b^4 + 432*a^5 + 648*a^3*b^2) + (216*a*
tan(c/2 + (d*x)/2))/(216*a + (648*a^3)/b^2 + (432*a^5)/b^4))*(6*a^2 + 3*b^2
))/(b^4*d) - ((2*(3*a^2 + b^2))/b^3 + (6*a^2*tan(c/2 + (d*x)/2)^4)/b^3 - (6
*tan(c/2 + (d*x)/2)^2*(2*a^2 + b^2))/b^3 + (tan(c/2 + (d*x)/2)*(9*a^2 + 2*b
^2))/(a*b^2) - (4*tan(c/2 + (d*x)/2)^3*(3*a^2 + b^2))/(a*b^2) + (tan(c/2 +
(d*x)/2)^5*(3*a^2 + 2*b^2))/(a*b^2))/(d*(a + 2*b*tan(c/2 + (d*x)/2) - 3*a*t
an(c/2 + (d*x)/2)^2 + 3*a*tan(c/2 + (d*x)/2)^4 - a*tan(c/2 + (d*x)/2)^6 - 4
*b*tan(c/2 + (d*x)/2)^3 + 2*b*tan(c/2 + (d*x)/2)^5)) - (6*a*atanh((432*a^3*
(a^2 + b^2)^(1/2))/(432*a^3*b + (432*a^5)/b + 864*a^4*tan(c/2 + (d*x)/2) +
864*a^2*b^2*tan(c/2 + (d*x)/2)) + (864*a^2*tan(c/2 + (d*x)/2)*(a^2 + b^2)^(
1/2))/(432*a^3 + (432*a^5)/b^2 + 864*a^2*b*tan(c/2 + (d*x)/2) + (864*a^4*ta
n(c/2 + (d*x)/2))/b) + (432*a^4*tan(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2))/(432*
a^5 + 432*a^3*b^2 + 864*a^4*b*tan(c/2 + (d*x)/2) + 864*a^2*b^3*tan(c/2 + (d
*x)/2)))*(a^2 + b^2)^(1/2))/(b^4*d)
```

3.130 $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx$

Optimal. Leaf size=141

$$\frac{(a^2+b^2)^2}{ab^4d(b+a\cot(c+dx))}-\frac{4a(a^2+b^2)\log(b+a\cot(c+dx))}{b^5d}-\frac{4a(a^2+b^2)\log(\tan(c+dx))}{b^5d}+\frac{(3a^2+2b^2)\tan(}{b^4d}$$

```
[Out] (a^2+b^2)^2/a/b^4/d/(b+a*cot(d*x+c))-4*a*(a^2+b^2)*ln(b+a*cot(d*x+c))/b^5/d
-4*a*(a^2+b^2)*ln(tan(d*x+c))/b^5/d+(3*a^2+2*b^2)*tan(d*x+c)/b^4/d-a*tan(d*
x+c)^2/b^3/d+1/3*tan(d*x+c)^3/b^2/d
```

Rubi [A]
time = 0.10, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3167, 908}

$$-\frac{4a(a^2+b^2)\log(\tan(c+dx))}{b^5d}-\frac{4a(a^2+b^2)\log(a\cot(c+dx)+b)}{b^5d}+\frac{(3a^2+2b^2)\tan(c+dx)}{b^4d}+\frac{(a^2+b^2)^2}{ab^4d(a\cot(c+dx)+b)}-\frac{a\tan^2(c+dx)}{b^3d}+\frac{\tan^3(c+dx)}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (a^2 + b^2)^2/(a*b^4*d*(b + a*Cot[c + d*x])) - (4*a*(a^2 + b^2)*Log[b + a*C
ot[c + d*x]])/(b^5*d) - (4*a*(a^2 + b^2)*Log[Tan[c + d*x]])/(b^5*d) + ((3*a
^2 + 2*b^2)*Tan[c + d*x])/(b^4*d) - (a*Tan[c + d*x]^2)/(b^3*d) + Tan[c + d*
x]^3/(3*b^2*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^4(b+ax)^2}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{b^2x^4} - \frac{2a}{b^3x^3} + \frac{3a^2+2b^2}{b^4x^2} - \frac{4a(a^2+b^2)}{b^5x} + \frac{(a^2+b^2)^2}{b^4(b+ax)^2} + \frac{4a^2(a^2+b^2)}{b^5(b+ax)}\right.\right.}{d}$$

$$= \frac{(a^2+b^2)^2}{ab^4d(b+a\cot(c+dx))} - \frac{4a(a^2+b^2)\log(b+a\cot(c+dx))}{b^5d} - \frac{4a}{}$$

Mathematica [A]

time = 1.63, size = 207, normalized size = 1.47

$$\frac{12a^3(a^2+b^2)(\log(\cos(c+dx))-\log(a\cos(c+dx)+b\sin(c+dx)))+b(12a^4+11a^2b^2+3b^4+12a^2(a^2+b^2)\log(\cos(c+dx))-12a^2(a^2+b^2)\log(a\cos(c+dx)+b\sin(c+dx)))\tan(c+dx)+ab^2(9a^2+5b^2)\tan^2(c+dx)+ab^2\sec^2(c+dx)(-3a^2-2ab\tan(c+dx)+b^2\tan^2(c+dx))}{3ab^5d(a+b\tan(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^2,x]
```

```
[Out] (12*a^3*(a^2 + b^2)*(Log[Cos[c + d*x]] - Log[a*Cos[c + d*x] + b*Sin[c + d*x
]]) + b*(12*a^4 + 11*a^2*b^2 + 3*b^4 + 12*a^2*(a^2 + b^2)*Log[Cos[c + d*x]]
 - 12*a^2*(a^2 + b^2)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])*Tan[c + d*x] +
a*b^2*(9*a^2 + 5*b^2)*Tan[c + d*x]^2 + a*b^2*Sec[c + d*x]^2*(-3*a^2 - 2*a*b
*Tan[c + d*x] + b^2*Tan[c + d*x]^2))/(3*a*b^5*d*(a + b*Tan[c + d*x]))
```

Maple [A]

time = 0.56, size = 114, normalized size = 0.81

method	result
derivativedivides	$\frac{\frac{\frac{b^2\left(\tan^3(dx+c)\right)}{3}-ab\left(\tan^2(dx+c)\right)+3a^2\tan(dx+c)+2b^2\tan(dx+c)}{b^4}-\frac{4a\left(a^2+b^2\right)\ln(a+b\tan(dx+c))}{b^5}-\frac{a^4+2a^2b^2+b^4}{b^5(a+b\tan(dx+c))}}{d}$
default	$\frac{\frac{\frac{b^2\left(\tan^3(dx+c)\right)}{3}-ab\left(\tan^2(dx+c)\right)+3a^2\tan(dx+c)+2b^2\tan(dx+c)}{b^4}-\frac{4a\left(a^2+b^2\right)\ln(a+b\tan(dx+c))}{b^5}-\frac{a^4+2a^2b^2+b^4}{b^5(a+b\tan(dx+c))}}{d}$
risch	$-\frac{8i\left(-5iab^2e^{2i(dx+c)}+6a^2be^{2i(dx+c)}-3ia^3-2iab^2+3a^2b-3iab^2e^{6i(dx+c)}-6iab^2e^{4i(dx+c)}+2b^3+4b^3e^{2i(dx+c)}+3a^2b\right.}{3\left(e^{2i(dx+c)}+1\right)^3\left(be^{2i(dx+c)}+iae^{2i(dx+c)}-b+ia\right)b^4d}$
norman	$\frac{-\frac{2\left(24a^2+16b^2\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3b^3d}+\frac{4\left(2a^2+2b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3d}+\frac{4\left(2a^2+2b^2\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^3d}-\frac{2\left(36a^4+44a^2b^2+9b^4\right)\left(\tan^3\left(\frac{dx}{2}\right.\right.}{3adb^4}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3\left(a\left(\tan^2\left(\frac{dx}{2}+\right.\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^4*(1/3*b^2*tan(d*x+c)^3-a*b*tan(d*x+c)^2+3*a^2*tan(d*x+c)+2*b^2*ta
n(d*x+c))-4*a/b^5*(a^2+b^2)*ln(a+b*tan(d*x+c))-1/b^5*(a^4+2*a^2*b^2+b^4)/(a
+b*tan(d*x+c)))
```

Maxima [A]
time = 0.30, size = 115, normalized size = 0.82

$$-\frac{\frac{3\left(a^4+2a^2b^2+b^4\right)}{b^6\tan(dx+c)+ab^5}-\frac{b^2\tan(dx+c)^3-3ab\tan(dx+c)^2+3\left(3a^2+2b^2\right)\tan(dx+c)}{b^4}+\frac{12\left(a^3+ab^2\right)\log(b\tan(dx+c)+a)}{b^5}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(3*(a^4 + 2*a^2*b^2 + b^4)/(b^6*tan(d*x + c) + a*b^5) - (b^2*tan(d*x +
 c)^3 - 3*a*b*tan(d*x + c)^2 + 3*(3*a^2 + 2*b^2)*tan(d*x + c))/b^4 + 12*(a^
3 + a*b^2)*log(b*tan(d*x + c) + a)/b^5)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 281 vs. 2(139) = 278.
time = 3.50, size = 281, normalized size = 1.99

$$-\frac{4\left(3a^2b^2+2b^4\right)\cos\left(dx+c\right)^4-b^4-2\left(3a^2b^2+2b^4\right)\cos\left(dx+c\right)^2+6\left(\left(a^4+a^2b^2\right)\cos\left(dx+c\right)^4+\left(a^3b+ab^3\right)\cos\left(dx+c\right)^3\sin\left(dx+c\right)\right)\log\left(2ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)-6\left(\left(a^4+a^2b^2\right)\cos\left(dx+c\right)^4+\left(a^3b+ab^3\right)\cos\left(dx+c\right)^3\sin\left(dx+c\right)\right)\log\left(\cos\left(dx+c\right)^2\right)+2\left(ab^3\cos\left(dx+c\right)-2\left(3a^3b+2ab^3\right)\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)}{3\left(ab^5d\cos\left(dx+c\right)^4+b^6d\cos\left(dx+c\right)^3\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(4*(3*a^2*b^2 + 2*b^4)*cos(d*x + c)^4 - b^4 - 2*(3*a^2*b^2 + 2*b^4)*co
s(d*x + c)^2 + 6*((a^4 + a^2*b^2)*cos(d*x + c)^4 + (a^3*b + a*b^3)*cos(d*x
+ c)^3*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(
d*x + c)^2 + b^2) - 6*((a^4 + a^2*b^2)*cos(d*x + c)^4 + (a^3*b + a*b^3)*cos
(d*x + c)^3*sin(d*x + c))*log(cos(d*x + c)^2) + 2*(a*b^3*cos(d*x + c) - 2*(
3*a^3*b + 2*a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(a*b^5*d*cos(d*x + c)^4 +
b^6*d*cos(d*x + c)^3*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^4\left(c+dx\right)}{\left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a*cos(c + d*x) + b*sin(c + d*x))**2, x)
```

Giac [A]
time = 0.45, size = 149, normalized size = 1.06

$$-\frac{\frac{12\,(a^3+ab^2)\log(|b\tan(dx+c)+a|)}{b^5}-\frac{b^4\tan(dx+c)^3-3\,ab^3\tan(dx+c)^2+9\,a^2b^2\tan(dx+c)+6\,b^4\tan(dx+c)}{b^6}-\frac{3\,(4\,a^3b\tan(dx+c)+4\,ab^3\tan(dx+c)+3\,a^4+2\,a^2b^2-b^4)}{(b\tan(dx+c)+a)b^5}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(12*(a^3 + a*b^2)*log(abs(b*tan(d*x + c) + a))/b^5 - (b^4*tan(d*x + c)
^3 - 3*a*b^3*tan(d*x + c)^2 + 9*a^2*b^2*tan(d*x + c) + 6*b^4*tan(d*x + c))/
b^6 - 3*(4*a^3*b*tan(d*x + c) + 4*a*b^3*tan(d*x + c) + 3*a^4 + 2*a^2*b^2 -
b^4)/((b*tan(d*x + c) + a)*b^5))/d
```

Mupad [B]
time = 4.20, size = 1132, normalized size = 8.03

$$\frac{\frac{8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(a^2+b^2)}{b^3}+\frac{8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6(a^2+b^2)}{b^3}-\frac{16\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(3a^2+2b^2)}{3b^3}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7(4a^4+4a^2b^2+b^4)}{ab^4}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(36a^4+44a^2b^2+9b^4)}{3ab^4}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5(36a^4+44a^2b^2+9b^4)}{3ab^4}+\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(4a^4+4a^2b^2+b^4)}{ab^4}}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7-4a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+6a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-6b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-4a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+2b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+a\right)}+\ldots$$

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a*cos(c + d*x) + b*sin(c + d*x))^2),x)
```

```
[Out] ((8*tan(c/2 + (d*x)/2)^2*(a^2 + b^2))/b^3 + (8*tan(c/2 + (d*x)/2)^6*(a^2 +
b^2))/b^3 - (16*tan(c/2 + (d*x)/2)^4*(3*a^2 + 2*b^2))/(3*b^3) - (2*tan(c/2
+ (d*x)/2)^7*(4*a^4 + b^4 + 4*a^2*b^2))/(a*b^4) - (2*tan(c/2 + (d*x)/2)^3*(
36*a^4 + 9*b^4 + 44*a^2*b^2))/(3*a*b^4) + (2*tan(c/2 + (d*x)/2)^5*(36*a^4 +
 9*b^4 + 44*a^2*b^2))/(3*a*b^4) + (2*tan(c/2 + (d*x)/2)*(4*a^4 + b^4 + 4*a^
2*b^2))/(a*b^4))/(d*(a + 2*b*tan(c/2 + (d*x)/2) - 4*a*tan(c/2 + (d*x)/2)^2
+ 6*a*tan(c/2 + (d*x)/2)^4 - 4*a*tan(c/2 + (d*x)/2)^6 + a*tan(c/2 + (d*x)/2
)^8 - 6*b*tan(c/2 + (d*x)/2)^3 + 6*b*tan(c/2 + (d*x)/2)^5 - 2*b*tan(c/2 + (
d*x)/2)^7)) + (a*atan(((a*(a^2 + b^2)*((16*tan(c/2 + (d*x)/2)*(4*a^5 + 4*a^
3*b^2))/b^4 - (4*(8*a^2*b^7 + 8*a^4*b^5))/b^8 + (4*tan(c/2 + (d*x)/2)^2*(8*
a^2*b^7 + 8*a^4*b^5))/b^8 + (4*a*(a^2 + b^2)*((4*(a*b^10 + 4*a^3*b^8))/b^8
- (4*tan(c/2 + (d*x)/2)^2*(3*a*b^10 + 4*a^3*b^8))/b^8 + 16*a^2*b*tan(c/2 +
(d*x)/2)))/b^5)*4i)/b^5 - (a*(a^2 + b^2)*((4*(8*a^2*b^7 + 8*a^4*b^5))/b^8 -
 (16*tan(c/2 + (d*x)/2)*(4*a^5 + 4*a^3*b^2))/b^4 - (4*tan(c/2 + (d*x)/2)^2*
(8*a^2*b^7 + 8*a^4*b^5))/b^8 + (4*a*(a^2 + b^2)*((4*(a*b^10 + 4*a^3*b^8))/b
^8 - (4*tan(c/2 + (d*x)/2)^2*(3*a*b^10 + 4*a^3*b^8))/b^8 + 16*a^2*b*tan(c/2
 + (d*x)/2)))/b^5)*4i)/b^5)/((8*(16*a^7 + 16*a^3*b^4 + 32*a^5*b^2))/b^8 + (
8*tan(c/2 + (d*x)/2)^2*(16*a^7 + 16*a^3*b^4 + 32*a^5*b^2))/b^8 + (4*a*(a^2
+ b^2)*((16*tan(c/2 + (d*x)/2)*(4*a^5 + 4*a^3*b^2))/b^4 - (4*(8*a^2*b^7 + 8
*a^4*b^5))/b^8 + (4*tan(c/2 + (d*x)/2)^2*(8*a^2*b^7 + 8*a^4*b^5))/b^8 + (4*
a*(a^2 + b^2)*((4*(a*b^10 + 4*a^3*b^8))/b^8 - (4*tan(c/2 + (d*x)/2)^2*(3*a*
b^10 + 4*a^3*b^8))/b^8 + 16*a^2*b*tan(c/2 + (d*x)/2)))/b^5))/b^5 + (4*a*(a^
2 + b^2)*((4*(8*a^2*b^7 + 8*a^4*b^5))/b^8 - (16*tan(c/2 + (d*x)/2)*(4*a^5 +
```

```
 4*a^3*b^2))/b^4 - (4*tan(c/2 + (d*x)/2)^2*(8*a^2*b^7 + 8*a^4*b^5))/b^8 + (
4*a*(a^2 + b^2)*((4*(a*b^10 + 4*a^3*b^8))/b^8 - (4*tan(c/2 + (d*x)/2)^2*(3*
a*b^10 + 4*a^3*b^8))/b^8 + 16*a^2*b*tan(c/2 + (d*x)/2)))/b^5))/b^5))*(a^2 +
 b^2)*8i)/(b^5*d)
```

3.131 $\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=216

$$-\frac{3b^2(4a^2-b^2)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}d}+\frac{b(3a^2-b^2)\cos(c+dx)}{\left(a^2+b^2\right)^3 d}+\frac{a(a^2-3b^2)\sin(c+dx)}{\left(a^2+b^2\right)^3 d}+\frac{}{2a\left(a^2+b^2\right)^2 \cdots}$$

```
[Out] -3*b^2*(4*a^2-b^2)*arctanh((b-a*tan(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b
^2)^(7/2)/d+b*(3*a^2-b^2)*cos(d*x+c)/(a^2+b^2)^3/d+a*(a^2-3*b^2)*sin(d*x+c)
/(a^2+b^2)^3/d+1/2*b^4*sin(d*x+c)/a/(a^2+b^2)^2/d/(a*cos(d*x+c)+b*sin(d*x+c
))^2-1/2*b^3*(8*a^2+b^2)/a/(a^2+b^2)^3/d/(a*cos(d*x+c)+b*sin(d*x+c))
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 492 vs. $2(216) = 432$.
time = 1.22, antiderivative size = 492, normalized size of antiderivative = 2.28, number of steps used = 15, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$,
Rules used = {6874, 653, 209, 652, 632, 212, 628}

$$\frac{2\left(a(a^2-3b^2)\tan\left(\frac{1}{2}(c+dx)\right)+b(3a^2-b^2)\right)}{d\left(a^2+b^2\right)^3\left(\tan^2\left(\frac{1}{2}(c+dx)\right)+1\right)}-\frac{3b^4(a^2+2b^2)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2d\left(a^2+b^2\right)^{7/2}}+\frac{4b^4(3a^2+2b^2)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2d\left(a^2+b^2\right)^{7/2}}-\frac{2b^2(6a^4+3a^2b^2+b^4)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2d\left(a^2+b^2\right)^{7/2}}-\frac{3b^4(a^2+2b^2)\left(b-a\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^3d\left(a^2+b^2\right)^3\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{2b^4\left((a^2+2b^2)\tan\left(\frac{1}{2}(c+dx)\right)+ab\right)}{a^3d\left(a^2+b^2\right)^2\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{4b^3\left(2a^4+ab(3a^2+2b^2)\tan\left(\frac{1}{2}(c+dx)\right)-b^4\right)}{a^3d\left(a^2+b^2\right)^3\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*b^4*(a^2 + 2*b^2)*ArcTanh[(b - a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a
^2*(a^2 + b^2)^(7/2)*d) + (4*b^4*(3*a^2 + 2*b^2)*ArcTanh[(b - a*Tan[(c + d*
x)/2])/Sqrt[a^2 + b^2]])/(a^2*(a^2 + b^2)^(7/2)*d) - (2*b^2*(6*a^4 + 3*a^2*
b^2 + b^4)*ArcTanh[(b - a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2*(a^2 + b
^2)^(7/2)*d) + (2*(b*(3*a^2 - b^2) + a*(a^2 - 3*b^2)*Tan[(c + d*x)/2]))/((a
^2 + b^2)^3*d*(1 + Tan[(c + d*x)/2]^2)) + (2*b^4*(a*b + (a^2 + 2*b^2)*Tan[(
c + d*x)/2]))/(a^3*(a^2 + b^2)^2*d*(a + 2*b*Tan[(c + d*x)/2] - a*Tan[(c + d
*x)/2]^2)^2) - (3*b^4*(a^2 + 2*b^2)*(b - a*Tan[(c + d*x)/2]))/(a^3*(a^2 + b
^2)^3*d*(a + 2*b*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/2]^2)) - (4*b^3*(2*a^4
- b^4 + a*b*(3*a^2 + 2*b^2)*Tan[(c + d*x)/2]))/(a^3*(a^2 + b^2)^3*d*(a + 2*
b*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/2]^2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 6874

```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{2\text{Subst}\left(\int \frac{(1-x^2)^4}{(1+x^2)^2(a+2bx-ax^2)^3}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{2(a(a^2-3b^2)-b(3a^2-b^2)x)}{(a^2+b^2)^3(1+x^2)^2} - \frac{a(a^2-3b^2)}{(a^2+b^2)^3(1+x^2)} + \frac{4b^3(-b(a^2+b^2)-a(2}{a^3(a^2+b^2)^2(a+2bx}\right.\right.}{}$$

$$= \frac{4\text{Subst}\left(\int \frac{a(a^2-3b^2)-b(3a^2-b^2)x}{(1+x^2)^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^2+b^2)^3\,d} - \frac{(2a(a^2-3}{}$$

$$= -\frac{a(a^2-3b^2)\,x}{(a^2+b^2)^3} + \frac{2\left(b(3a^2-b^2)+a(a^2-3b^2)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^2+b^2)^3\,d\left(1+\tan^2\left(\frac{1}{2}(c+dx)\right)\right)} +$$

$$= -\frac{2b^2(6a^4+3a^2b^2+b^4)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{7/2}d} + \frac{2\left(b(3a^2-b^2)\right.}{(a^2+b^2)}$$

$$= \frac{4b^4(3a^2+2b^2)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{7/2}d} - \frac{2b^2(6a^4+3a^2b^2+b^4)\,\mathrm{t}}{a^2\,(a^2+}$$

$$= -\frac{3b^4(a^2+2b^2)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{a^2\left(a^2+b^2\right)^{7/2}d} + \frac{4b^4(3a^2+2b^2)\tanh^{-1}}{a^2\,(a^2+b^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.15, size = 211, normalized size = 0.98

$$\frac{-\frac{6b^2(-4a^2+b^2)\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} - \frac{2b(-3a^2+b^2)\cos(c+dx)}{(a^2+b^2)^3} + \frac{2a(a^2-3b^2)\sin(c+dx)}{(a^2+b^2)^3} + \frac{b^4\sin(c+dx)}{a(a-ib)^2(a+ib)^2(a\cos(c+dx)+b\sin(c+dx))^2} - \frac{b^3(8a^2+b^2)}{a(a^2+b^2)^3(a\cos(c+dx)+b\sin(c+dx))}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] ((-6*b^2*(-4*a^2 + b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])
/(a^2 + b^2)^(7/2) - (2*b*(-3*a^2 + b^2)*Cos[c + d*x])/(a^2 + b^2)^3 + (2*a
*(a^2 - 3*b^2)*Sin[c + d*x])/(a^2 + b^2)^3 + (b^4*Sin[c + d*x])/(a*(a - I*b
)^2*(a + I*b)^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - (b^3*(8*a^2 + b^2))/
(a*(a^2 + b^2)^3*(a*Cos[c + d*x] + b*Sin[c + d*x])))/(2*d)
```

Maple [A]
time = 1.02, size = 283, normalized size = 1.31

method	result
derivativedivides	$-\frac{2b^2\left(\frac{-\frac{b^2(9a^2+2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{b(8a^4-15a^2b^2-2b^4)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2}+\frac{b^2(23a^2+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+4a^2b+\frac{b^3}{2}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}-\frac{3\left(4a^2-b^2 \cdots\right.}{\cdots}\right.}{(a^2+b^2)^3} \cdots}{d}$
default	$-\frac{2b^2\left(\frac{-\frac{b^2(9a^2+2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{b(8a^4-15a^2b^2-2b^4)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2}+\frac{b^2(23a^2+2b^2)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}+4a^2b+\frac{b^3}{2}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}-\frac{3\left(4a^2-b^2 \cdots\right.}{\cdots}\right.}{(a^2+b^2)^3} \cdots}{d}$
risch	$-\frac{ie^{i(dx+c)}}{2(-3ia^2b+ib^3+a^3-3ab^2)d}+\frac{ie^{-i(dx+c)}}{2(3ia^2b-ib^3+a^3-3ab^2)d}+\frac{b^3\left(-7iab\,e^{3i(dx+c)}+8a^2e^{3i(dx+c)}+b^2e^{3i(dx+c)}+7iab\,e^{i(\cdots}\right.}{(-ia+b)^3\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b\cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b^2/(a^2+b^2)^3*((-1/2*b^2*(9*a^2+2*b^2)/a*tan(1/2*d*x+1/2*c)^3-1/2
*b*(8*a^4-15*a^2*b^2-2*b^4)/a^2*tan(1/2*d*x+1/2*c)^2+1/2*b^2*(23*a^2+2*b^2)
/a*tan(1/2*d*x+1/2*c)+4*a^2*b+1/2*b^3)/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*
d*x+1/2*c)-a)^2-3/2*(4*a^2-b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*
x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))-2/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*((-a^3+3*a*
b^2)*tan(1/2*d*x+1/2*c)-3*a^2*b+b^3)/(1+tan(1/2*d*x+1/2*c)^2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(209) = 418$.

time = 0.51, size = 658, normalized size = 3.05

$$-\frac{\frac{3\,(4\,a^2b^2-b^4)\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}}-\frac{2\left(6\,a^6b-10\,a^4b^3-a^2b^5+\frac{(2\,a^7+18\,a^5b^2-31\,a^3b^4-2\,ab^6)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\,(2\,a^6b-2\,a^4b^3+12\,a^2b^5+b^7)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2\,(2\,a^7+2\,a^5b^2+15\,a^3b^4)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{(2\,a^6b-30\,a^4b^3+15\,a^2b^5+2\,b^7)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{(2\,a^7-6\,a^5b^2+9\,a^3b^4+2\,ab^6)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^{10}+3\,a^8b^2+3\,a^6b^4+a^4b^6+\frac{4\,(a^9b+3\,a^7b^3+3\,a^5b^5+a^3b^7)\sin(dx+c)}{\cos(dx+c)+1}-\frac{(a^{10}-a^8b^2-9\,a^6b^4-11\,a^4b^6-4\,a^2b^8)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{(a^{10}-a^8b^2-9\,a^6b^4-11\,a^4b^6-4\,a^2b^8)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4\,(a^9b+3\,a^7b^3+3\,a^5b^5+a^3b^7)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{(a^{10}+3\,a^8b^2+3\,a^6b^4+a^4b^6)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(3*(4*a^2*b^2 - b^4)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt
(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 2*(6*a^6*b - 10*a^4*b^
3 - a^2*b^5 + (2*a^7 + 18*a^5*b^2 - 31*a^3*b^4 - 2*a*b^6)*sin(d*x + c)/(cos
(d*x + c) + 1) - 2*(2*a^6*b - 2*a^4*b^3 + 12*a^2*b^5 + b^7)*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 - 2*(2*a^7 + 2*a^5*b^2 + 15*a^3*b^4)*sin(d*x + c)^3/(c
os(d*x + c) + 1)^3 - (2*a^6*b - 30*a^4*b^3 + 15*a^2*b^5 + 2*b^7)*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 + (2*a^7 - 6*a^5*b^2 + 9*a^3*b^4 + 2*a*b^6)*sin(d
```

```
*x + c)^5/(cos(d*x + c) + 1)^5)/(a^10 + 3*a^8*b^2 + 3*a^6*b^4 + a^4*b^6 + 4
*(a^9*b + 3*a^7*b^3 + 3*a^5*b^5 + a^3*b^7)*sin(d*x + c)/(cos(d*x + c) + 1)
- (a^10 - a^8*b^2 - 9*a^6*b^4 - 11*a^4*b^6 - 4*a^2*b^8)*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 - (a^10 - a^8*b^2 - 9*a^6*b^4 - 11*a^4*b^6 - 4*a^2*b^8)*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*(a^9*b + 3*a^7*b^3 + 3*a^5*b^5 + a^3*
b^7)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + (a^10 + 3*a^8*b^2 + 3*a^6*b^4 +
a^4*b^6)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(209) = 418$.
time = 2.74, size = 480, normalized size = 2.22

$$\frac{4\,(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)\cos(dx+c)^3-3\,\left(4\,a^2b^4-b^6+(4\,a^4b^2-5\,a^2b^4+b^6)\cos(dx+c)^2+2\,(4\,a^3b^3-ab^5)\cos(dx+c)\sin(dx+c)\right)\sqrt{a^2+b^2}\,\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)+2\,(4\,a^6b-10\,a^4b^3-17\,a^2b^5-3\,b^7)\cos(dx+c)+2\,\left(2\,a^5b^2-11\,a^3b^4-13\,ab^6+2\,(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)\cos(dx+c)^2\right)\sin(dx+c)}{4\,\left((a^{10}+3\,a^8b^2+2\,a^6b^4-2\,a^4b^6-3\,a^2b^8-b^{10})d\cos(dx+c)^2+2\,(a^9b+4\,a^7b^3+6\,a^5b^5+4\,a^3b^7+ab^9)d\cos(dx+c)\sin(dx+c)+(a^8b^2+4\,a^6b^4+6\,a^4b^6+4\,a^2b^8+b^{10})d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d*x + c)^3 - 3*(4*a^2*b^4
- b^6 + (4*a^4*b^2 - 5*a^2*b^4 + b^6)*cos(d*x + c)^2 + 2*(4*a^3*b^3 - a*b^5
)*cos(d*x + c)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x + c)*sin(d*
x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*co
s(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2
)*cos(d*x + c)^2 + b^2)) + 2*(4*a^6*b - 10*a^4*b^3 - 17*a^2*b^5 - 3*b^7)*co
s(d*x + c) + 2*(2*a^5*b^2 - 11*a^3*b^4 - 13*a*b^6 + 2*(a^7 + 3*a^5*b^2 + 3*
a^3*b^4 + a*b^6)*cos(d*x + c)^2)*sin(d*x + c))/((a^10 + 3*a^8*b^2 + 2*a^6*b
^4 - 2*a^4*b^6 - 3*a^2*b^8 - b^10)*d*cos(d*x + c)^2 + 2*(a^9*b + 4*a^7*b^3
+ 6*a^5*b^5 + 4*a^3*b^7 + a*b^9)*d*cos(d*x + c)*sin(d*x + c) + (a^8*b^2 + 4
*a^6*b^4 + 6*a^4*b^6 + 4*a^2*b^8 + b^10)*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.56, size = 399, normalized size = 1.85

$$-\frac{\frac{3\,(4\,a^2b^2-b^4)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}}-\frac{4\,(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,a^2b-b^3)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)}-\frac{2\,\left(9\,a^3b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,ab^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+8\,a^4b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,a^2b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b^7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-23\,a^3b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,ab^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-8\,a^4b^3-a^2b^5\right)}{(a^8+3\,a^6b^2+3\,a^4b^4+a^2b^6)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(3*(4*a^2*b^2 - b^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a
^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 +
 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 4*(a^3*tan(1/2*d*x + 1/2*c
) - 3*a*b^2*tan(1/2*d*x + 1/2*c) + 3*a^2*b - b^3)/((a^6 + 3*a^4*b^2 + 3*a^2
*b^4 + b^6)*(tan(1/2*d*x + 1/2*c)^2 + 1)) - 2*(9*a^3*b^4*tan(1/2*d*x + 1/2*
c)^3 + 2*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 8*a^4*b^3*tan(1/2*d*x + 1/2*c)^2 -
15*a^2*b^5*tan(1/2*d*x + 1/2*c)^2 - 2*b^7*tan(1/2*d*x + 1/2*c)^2 - 23*a^3*b
^4*tan(1/2*d*x + 1/2*c) - 2*a*b^6*tan(1/2*d*x + 1/2*c) - 8*a^4*b^3 - a^2*b^
5)/((a^8 + 3*a^6*b^2 + 3*a^4*b^4 + a^2*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b
*tan(1/2*d*x + 1/2*c) - a)^2))/d
```

Mupad [B]
time = 4.25, size = 610, normalized size = 2.82

$$-\frac{\frac{-6\,a^4\,b+10\,a^2\,b^3+b^5}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(2\,a^5+2\,a^3\,b^2+15\,a\,b^4\right)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(2\,a^6\,b-30\,a^4\,b^3+15\,a^2\,b^5+2\,b^7\right)}{a^2\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a^6+18\,a^4\,b^2-31\,a^2\,b^4-2\,b^6\right)}{a\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(2\,a^6-6\,a^4\,b^2+9\,a^2\,b^4+2\,b^6\right)}{a\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a^6\,b-2\,a^4\,b^3+12\,a^2\,b^5+b^7\right)}{a^2\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}}{d\,\left(a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+a^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(a^2-4\,b^2\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(a^2-4\,b^2\right)-4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{\mathrm{atan}\left(\frac{-1\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^7+a^6\,b\,1\mathrm{i}-3\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^5\,b^2+a^4\,b^3\,3\mathrm{i}-3\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^3\,b^4+a^2\,b^5\,3\mathrm{i}-1\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a\,b^6+b^7\,1\mathrm{i}}{{\left(a^2+b^2\right)}^{7/2}}\right)\,\left(3\,b^4-12\,a^2\,b^2\right)\,1\mathrm{i}}{d\,{\left(a^2+b^2\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

```
[Out] - ((b^5 - 6*a^4*b + 10*a^2*b^3)/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (2*ta
n(c/2 + (d*x)/2)^3*(15*a*b^4 + 2*a^5 + 2*a^3*b^2))/(a^6 + b^6 + 3*a^2*b^4 +
 3*a^4*b^2) + (tan(c/2 + (d*x)/2)^4*(2*a^6*b + 2*b^7 + 15*a^2*b^5 - 30*a^4*
b^3))/(a^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (tan(c/2 + (d*x)/2)*(2*a^
6 - 2*b^6 - 31*a^2*b^4 + 18*a^4*b^2))/(a*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2
)) - (tan(c/2 + (d*x)/2)^5*(2*a^6 + 2*b^6 + 9*a^2*b^4 - 6*a^4*b^2))/(a*(a^6
 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (2*tan(c/2 + (d*x)/2)^2*(2*a^6*b + b^7 +
 12*a^2*b^5 - 2*a^4*b^3))/(a^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))/(d*(a^
2*tan(c/2 + (d*x)/2)^6 + a^2 - tan(c/2 + (d*x)/2)^2*(a^2 - 4*b^2) - tan(c/2
 + (d*x)/2)^4*(a^2 - 4*b^2) - 4*a*b*tan(c/2 + (d*x)/2)^5 + 4*a*b*tan(c/2 +
(d*x)/2))) - (atan((a^6*b*1i + b^7*1i + a^2*b^5*3i + a^4*b^3*3i - a^7*tan(c
/2 + (d*x)/2)*1i - a*b^6*tan(c/2 + (d*x)/2)*1i - a^3*b^4*tan(c/2 + (d*x)/2)
*3i - a^5*b^2*tan(c/2 + (d*x)/2)*3i)/(a^2 + b^2)^(7/2))*(3*b^4 - 12*a^2*b^2
)*1i)/(d*(a^2 + b^2)^(7/2))
```

3.132 $\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=122

$$\frac{a(a^2-3b^2)\,x}{(a^2+b^2)^3}+\frac{b(3a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^3\,d}-\frac{b}{2\,(a^2+b^2)\,d(a+b\tan(c+dx))^2}-\frac{}{(a^2+b^2)^2}$$

```
[Out] a*(a^2-3*b^2)*x/(a^2+b^2)^3+b*(3*a^2-b^2)*ln(a*cos(d*x+c)+b*sin(d*x+c))/(a^
2+b^2)^3/d-1/2*b/(a^2+b^2)/d/(a+b*tan(d*x+c))^2-2*a*b/(a^2+b^2)^2/d/(a+b*ta
n(d*x+c))
```

Rubi [A]
time = 0.15, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {3165, 3564, 3610, 3612, 3611}

$$-\frac{2ab}{d\,(a^2+b^2)^2\,(a+b\tan(c+dx))}-\frac{b}{2d\,(a^2+b^2)\,(a+b\tan(c+dx))^2}+\frac{b(3a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^3}+\frac{ax(a^2-3b^2)}{(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (a*(a^2 - 3*b^2)*x)/(a^2 + b^2)^3 + (b*(3*a^2 - b^2)*Log[a*Cos[c + d*x] + b
*Sin[c + d*x]])/((a^2 + b^2)^3*d) - b/(2*(a^2 + b^2)*d*(a + b*Tan[c + d*x])
^2) - (2*a*b)/((a^2 + b^2)^2*d*(a + b*Tan[c + d*x]))
```

Rule 3165

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
```

```
 b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \int \frac{1}{(a+b\tan(c+dx))^3}\,dx$$

$$= -\frac{b}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} + \frac{\int \frac{a-b\tan(c+dx)}{(a+b\tan(c+dx))^2}\,dx}{a^2+b^2}$$

$$= -\frac{b}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{2ab}{\left(a^2+b^2\right)^2 d(a+b\tan(c+dx))}$$

$$= \frac{a\left(a^2-3b^2\right)x}{\left(a^2+b^2\right)^3} - \frac{b}{2\left(a^2+b^2\right)d(a+b\tan(c+dx))^2} - \frac{2a}{\left(a^2+b^2\right)^2 d(a+}$$

$$= \frac{a\left(a^2-3b^2\right)x}{\left(a^2+b^2\right)^3} + \frac{b\left(3a^2-b^2\right)\log(a\cos(c+dx)+b\sin(c+dx))}{\left(a^2+b^2\right)^3 d} - \frac{}{2(}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.36, size = 154, normalized size = 1.26

$$\frac{\frac{2a\left(a^2-3b^2\right)(c+dx)}{\left(a^2+b^2\right)^3} - \frac{2b\left(-3a^2+b^2\right)\log(a\cos(c+dx)+b\sin(c+dx))}{\left(a^2+b^2\right)^3} - \frac{b^3}{(a-ib)^2(a+ib)^2(a\cos(c+dx)+b\sin(c+dx))^2} + \frac{6b^2\sin(c+dx)}{\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*a*(a^2 - 3*b^2)*(c + d*x))/(a^2 + b^2)^3 - (2*b*(-3*a^2 + b^2)*Log[a*Co
s[c + d*x] + b*Sin[c + d*x]])/(a^2 + b^2)^3 - b^3/((a - I*b)^2*(a + I*b)^2*
```

```
(a*Cos[c + d*x] + b*Sin[c + d*x])^2) + (6*b^2*Sin[c + d*x])/((a^2 + b^2)^2*
(a*Cos[c + d*x] + b*Sin[c + d*x])))/(2*d)
```

Maple [A]
time = 0.55, size = 140, normalized size = 1.15 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*b/(a^2+b^2)/(a+b*tan(d*x+c))^2+b*(3*a^2-b^2)/(a^2+b^2)^3*ln(a+b*t
an(d*x+c))-2*b*a/(a^2+b^2)^2/(a+b*tan(d*x+c))+1/(a^2+b^2)^3*(1/2*(-3*a^2*b+
b^3)*ln(tan(d*x+c)^2+1)+(a^3-3*a*b^2)*arctan(tan(d*x+c))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(120) = 240$.
time = 0.50, size = 481, normalized size = 3.94

$$\frac{\frac{2\left(a^{3}-3ab^{2}\right)\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}}+\frac{\left(3a^{2}b-b^{3}\right)\log\left(-a-\frac{2b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)}{a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}}-\frac{\left(3a^{2}b-b^{3}\right)\log\left(\frac{\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+1\right)}{a^{6}+3a^{4}b^{2}+3a^{2}b^{4}+b^{6}}+\frac{2\left(\frac{\left(3a^{3}b^{2}+ab^{4}\right)\sin(dx+c)}{\cos(dx+c)+1}+\frac{\left(5a^{2}b^{3}+b^{5}\right)\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{\left(3a^{3}b^{2}+ab^{4}\right)\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{a^{8}+2a^{6}b^{2}+a^{4}b^{4}+\frac{4\left(a^{7}b+2a^{5}b^{3}+a^{3}b^{5}\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\left(a^{8}-3a^{4}b^{4}-2a^{2}b^{6}\right)\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{4\left(a^{7}b+2a^{5}b^{3}+a^{3}b^{5}\right)\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{\left(a^{8}+2a^{6}b^{2}+a^{4}b^{4}\right)\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (2*(a^3 - 3*a*b^2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/(a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6) + (3*a^2*b - b^3)*log(-a - 2*b*sin(d*x + c)/(cos(d*x +
c) + 1) + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^6 + 3*a^4*b^2 + 3*a^2*b
^4 + b^6) - (3*a^2*b - b^3)*log(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)/(a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*((3*a^3*b^2 + a*b^4)*sin(d*x + c)/(co
s(d*x + c) + 1) + (5*a^2*b^3 + b^5)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - (
3*a^3*b^2 + a*b^4)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^8 + 2*a^6*b^2 +
a^4*b^4 + 4*(a^7*b + 2*a^5*b^3 + a^3*b^5)*sin(d*x + c)/(cos(d*x + c) + 1) -
 2*(a^8 - 3*a^4*b^4 - 2*a^2*b^6)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4*(a
^7*b + 2*a^5*b^3 + a^3*b^5)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + (a^8 + 2*
a^6*b^2 + a^4*b^4)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(120) = 240$.
time = 2.83, size = 341, normalized size = 2.80

$$\frac{5a^{2}b^{3}-b^{5}+2\left(a^{3}b^{2}-3ab^{4}\right)dx-2\left(6a^{2}b^{3}-\left(a^{5}-4a^{3}b^{2}+3ab^{4}\right)dx\right)\cos\left(dx+c\right)^{2}+2\left(3a^{3}b^{2}-3ab^{4}+2\left(a^{4}b-3a^{2}b^{3}\right)dx\right)\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(3a^{2}b^{3}-b^{5}+\left(3a^{4}b-4a^{2}b^{3}+b^{5}\right)\cos\left(dx+c\right)^{2}+2\left(3a^{3}b^{2}-ab^{4}\right)\cos\left(dx+c\right)\sin\left(dx+c\right)\right)\log\left(2ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}+b^{2}\right)}{2\left(\left(a^{8}+2a^{6}b^{2}-2a^{2}b^{6}-b^{8}\right)d\cos\left(dx+c\right)^{2}+2\left(a^{7}b+3a^{5}b^{3}+3a^{3}b^{5}+ab^{7}\right)d\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^{6}b^{2}+3a^{4}b^{4}+3a^{2}b^{6}+b^{8}\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(5*a^2*b^3 - b^5 + 2*(a^3*b^2 - 3*a*b^4)*d*x - 2*(6*a^2*b^3 - (a^5 - 4*
a^3*b^2 + 3*a*b^4)*d*x)*cos(d*x + c)^2 + 2*(3*a^3*b^2 - 3*a*b^4 + 2*(a^4*b
- 3*a^2*b^3)*d*x)*cos(d*x + c)*sin(d*x + c) + (3*a^2*b^3 - b^5 + (3*a^4*b -
```

```
 4*a^2*b^3 + b^5)*cos(d*x + c)^2 + 2*(3*a^3*b^2 - a*b^4)*cos(d*x + c)*sin(d
*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 +
 b^2))/((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b^8)*d*cos(d*x + c)^2 + 2*(a^7*b + 3
*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*cos(d*x + c)*sin(d*x + c) + (a^6*b^2 + 3*a^
4*b^4 + 3*a^2*b^6 + b^8)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. 2(120) = 240.
time = 0.52, size = 265, normalized size = 2.17

$$\frac{\frac{2\,(a^3-3\,ab^2)\,(dx+c)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{(3\,a^2b-b^3)\log\left(\tan(dx+c)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{2\,(3\,a^2b^2-b^4)\log(|b\tan(dx+c)+a|)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}-\frac{9\,a^2b^3\tan(dx+c)^2-3\,b^5\tan(dx+c)^2+22\,a^3b^2\tan(dx+c)-2\,ab^4\tan(dx+c)+14\,a^4b+3\,a^2b^3+b^5}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)(b\tan(dx+c)+a)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(a^3 - 3*a*b^2)*(d*x + c)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (3*a
^2*b - b^3)*log(tan(d*x + c)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2
*(3*a^2*b^2 - b^4)*log(abs(b*tan(d*x + c) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*
b^5 + b^7) - (9*a^2*b^3*tan(d*x + c)^2 - 3*b^5*tan(d*x + c)^2 + 22*a^3*b^2*
tan(d*x + c) - 2*a*b^4*tan(d*x + c) + 14*a^4*b + 3*a^2*b^3 + b^5)/((a^6 + 3
*a^4*b^2 + 3*a^2*b^4 + b^6)*(b*tan(d*x + c) + a)^2))/d
```

Mupad [B]
time = 8.54, size = 2500, normalized size = 20.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^2*(b^5 + 5*a^2*b^3))/(a^2*(a^4 + b^4 + 2*a^2*b^2)) +
 (2*b*tan(c/2 + (d*x)/2)*(3*a^2*b + b^3))/(a*(a^4 + b^4 + 2*a^2*b^2)) - (2*
b*tan(c/2 + (d*x)/2)^3*(3*a^2*b + b^3))/(a*(a^4 + b^4 + 2*a^2*b^2)))/(d*(a^
2*tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^2*(2*a^2 - 4*b^2) + a^2 - 4*a*b
```

*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/2 + (d*x)/2))) - (log((((-(a^2*(a^2 - 3*b^2)^2)/(a^2 + b^2)^6)^(1/2) + (3*a^2*b - b^3)/(a^2 + b^2)^3)*(((-(a^2*(a^2 - 3*b^2)^2)/(a^2 + b^2)^6)^(1/2) + (3*a^2*b - b^3)/(a^2 + b^2)^3)*((32*a*b*tan(c/2 + (d*x)/2)*(b^4 - 8*a^4 + 5*a^2*b^2))/(a^2 + b^2)^2 - (32*a^2*(a^4 + 4*b^4 - 7*a^2*b^2))/(a^2 + b^2)^2 + 96*a*b*(a + b*tan(c/2 + (d*x)/2))*((-(a^2*(a^2 - 3*b^2)^2)/(a^2 + b^2)^6)^(1/2) + (3*a^2*b - b^3)/(a^2 + b^2)^3)*(a^2 + b^2)) - (32*a^2*b*(5*a^2 - 3*b^2))/(a^2 + b^2)^3 + (32*a*tan(c/2 + (d*x)/2)*(a^6 - 3*b^6 + 27*a^2*b^4 - 17*a^4*b^2))/(a^2 + b^2)^4) - (64*a^2*b^2*(3*a^4 + b^4 - 4*a^2*b^2))/(a^2 + b^2)^6 + (32*a*b*tan(c/2 + (d*x)/2)*(3*a^6 - b^6 - 3*a^2*b^4 + 17*a^4*b^2))/(a^2 + b^2)^6)*(((-(a^2*(a^2 - 3*b^2)^2)/(a^2 + b^2)^6)^(1/2) - (3*a^2*b - b^3)/(a^2 + b^2)^3)*(((-(a^2*(a^2 - 3*b^2)^2)/(a^2 + b^2)^6)^(1/2) - (3*a^2*b - b^3)/(a^2 + b^2)^3)*((32*a^2*(a^4 + 4*b^4 - 7*a^2*b^2))/(a^2 + b^2)^2 - (32*a*b*tan(c/2 + (d*x)/2)*(b^4 - 8*a^4 + 5*a^2*b^2))/(a^2 + b^2)^2 + 96*a*b*(a + b*tan(c/2 + (d*x)/2))*((-(a^2*(a^2 - 3*b^2)^2)/(a^2 + b^2)^6)^(1/2) - (3*a^2*b - b^3)/(a^2 + b^2)^3)*(a^2 + b^2)) - (32*a^2*b*(5*a^2 - 3*b^2))/(a^2 + b^2)^3 + (32*a*tan(c/2 + (d*x)/2)*(a^6 - 3*b^6 + 27*a^2*b^4 - 17*a^4*b^2))/(a^2 + b^2)^4) + (64*a^2*b^2*(3*a^4 + b^4 - 4*a^2*b^2))/(a^2 + b^2)^6 - (32*a*b*tan(c/2 + (d*x)/2)*(3*a^6 - b^6 - 3*a^2*b^4 + 17*a^4*b^2))/(a^2 + b^2)^6))*(6*a^2*b - 2*b^3))/(2*d*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (log(a + 2*b*tan(c/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^2)*(3*a^2*b - b^3))/(d*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (2*a*atan((tan(c/2 + (d*x)/2)*((((a*(a^2 - 3*b^2)*((32*(3*a*b^10 - a^11 - 21*a^3*b^8 - 34*a^5*b^6 + 6*a^7*b^4 + 15*a^9*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - ((6*a^2*b - 2*b^3)*((32*(a*b^13 - 8*a^13*b + 9*a^3*b^11 + 18*a^5*b^9 + 2*a^7*b^7 - 27*a^9*b^5 - 27*a^11*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + (16*(6*a^2*b - 2*b^3)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - ((6*a^2*b - 2*b^3)*((a*(a^2 - 3*b^2)*((32*(a*b^13 - 8*a^13*b + 9*a^3*b^11 + 18*a^5*b^9 + 2*a^7*b^7 - 27*a^9*b^5 - 27*a^11*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + (16*(6*a^2*b - 2*b^3)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (16*a*(6*a^2*b - 2*b^3)*(a^2 - 3*b^2)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (32*a^3*(a^2 - 3*b^2)^3*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^3*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))

```
*(a^8 + 4*b^8 - 61*a^2*b^6 + 155*a^4*b^4 - 67*a^6*b^2))/(a^8 + 4*b^8 - 11*a
^2*b^6 + 15*a^4*b^4 + 31*a^6*b^2)^2 + (2*a*b*(7*a^6 - 10*b^6 + 59*a^2*b^4 -
 68*a^4*b^2)*((32*(a*b^7 - 3*a^7*b + 3*a^3*b^5 - 17*a^5*b^3))/(a^12 + b^12
+ 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + ((6*a^2
*b - 2*b^3)*((32*(3*a*b^10 - a^11 - 21*a^3*b^8 - 34*a^5*b^6 + 6*a^7*b^4 + 1
5*a^9*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^
4 + 6*a^10*b^2) - ((6*a^2*b - 2*b^3)*((32*(a*b^13 - 8*a^13*b + 9*a^3*b^11 +
 18*a^5*b^9 + 2*a^7*b^7 - 27*a^9*b^5 - 27*a^11*b^3))/(a^12 + b^12 + 6*a^2*b
^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + (16*(6*a^2*b - 2
*b^3)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 +
63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^
2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^1
0*b^2))))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(2*(a^6 + b^6 + 3*a^2*b
^4 + 3*a^4*b^2)) + (a*(a^2 - 3*b^2)*((a*(a^2 - 3*b^2)*((32*(a*b^13 - 8*a^13
*b + 9*a^3*b^11 + 18*a^5*b^9 + 2*a^7*b^7 - 27*a^9*b^5 - 27*a^11*b^3))/(a^12
 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) +
 (16*(6*a^2*b - 2*b^3)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10
 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*...
```

3.133 $\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=119

$$\frac{(2a^2-b^2)\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}d}-\frac{b\left((4a^2+b^2)\cos(c+dx)+3ab\sin(c+dx)\right)}{2\left(a^2+b^2\right)^2 d(a\cos(c+dx)+b\sin(c+dx))^2}$$

```
[Out] (2*a^2-b^2)*arctanh((-b+a*tan(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5
/2)/d-1/2*b*((4*a^2+b^2)*cos(d*x+c)+3*a*b*sin(d*x+c))/(a^2+b^2)^2/d/(a*cos(
d*x+c)+b*sin(d*x+c))^2
```

Rubi [A]
time = 0.43, antiderivative size = 225, normalized size of antiderivative = 1.89, number of steps used = 6, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1674, 12, 632, 212}

$$-\frac{(2a^2-b^2)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{5/2}}+\frac{2b^2\left((a^2+2b^2)\tan\left(\frac{1}{2}(c+dx)\right)+ab\right)}{a^3d\left(a^2+b^2\right)\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{b\left(4a^4+ab(5a^2+2b^2)\tan\left(\frac{1}{2}(c+dx)\right)+3a^2b^2+2b^4\right)}{a^3d\left(a^2+b^2\right)^2\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] -(((2*a^2 - b^2)*ArcTanh[(b - a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a^2 +
 b^2)^(5/2)*d)) + (2*b^2*(a*b + (a^2 + 2*b^2)*Tan[(c + d*x)/2]))/(a^3*(a^2
+ b^2)*d*(a + 2*b*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/2]^2)^2) - (b*(4*a^4 +
 3*a^2*b^2 + 2*b^4 + a*b*(5*a^2 + 2*b^2)*Tan[(c + d*x)/2]))/(a^3*(a^2 + b^2
)^2*d*(a + 2*b*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/2]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{2\text{Subst}\left(\int \frac{(1-x^2)^2}{(a+2bx-ax^2)^3}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

$$= \frac{2b^2\left(ab+(a^2+2b^2)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^3(a^2+b^2)\,d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)-a\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^2} - \text{Sub}$$

$$= \frac{2b^2\left(ab+(a^2+2b^2)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^3(a^2+b^2)\,d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)-a\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{b}{a^3(}$$

$$= \frac{2b^2\left(ab+(a^2+2b^2)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^3(a^2+b^2)\,d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)-a\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{b}{a^3(}$$

$$= \frac{2b^2\left(ab+(a^2+2b^2)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a^3(a^2+b^2)\,d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)-a\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{b}{a^3(}$$

$$= -\frac{(2a^2-b^2)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}\,d} + \frac{2b^2(ab+(a^2+}{a^3(a^2+b^2)\,d\left(a+2b\tan\left(\right.\right.}$$

Mathematica [A]
time = 0.73, size = 119, normalized size = 1.00

$$\frac{\frac{2(2a^2-b^2)\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{b\left((4a^2+b^2)\cos(c+dx)+3ab\sin(c+dx)\right)}{(a^2+b^2)^2(a\cos(c+dx)+b\sin(c+dx))^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] ((2*(2*a^2 - b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2
+ b^2)^(5/2) - (b*((4*a^2 + b^2)*Cos[c + d*x] + 3*a*b*Sin[c + d*x]))/((a^2
+ b^2)^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^2))/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(112) = 224$.
time = 0.58, size = 280, normalized size = 2.35

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{b^2(5a^2+2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a\left(a^4+2a^2b^2+b^4\right)}-\frac{b\left(4a^4-7a^2b^2-2b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^4+2a^2b^2+b^4\right)a^2}+\frac{b^2\left(11a^2+2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^4+2a^2b^2+b^4\right)a}+\frac{b\left(4a^2+b^2\right)}{2a^4+4a^2b^2+2b^4}\right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+\cdots}{d}$
default	$\frac{-\frac{2\left(-\frac{b^2(5a^2+2b^2)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a\left(a^4+2a^2b^2+b^4\right)}-\frac{b\left(4a^4-7a^2b^2-2b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^4+2a^2b^2+b^4\right)a^2}+\frac{b^2\left(11a^2+2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^4+2a^2b^2+b^4\right)a}+\frac{b\left(4a^2+b^2\right)}{2a^4+4a^2b^2+2b^4}\right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+\cdots}{d}$
risch	$\frac{b\left(-3iab\,e^{3i(dx+c)}+4a^2e^{3i(dx+c)}+b^2e^{3i(dx+c)}+3iab\,e^{i(dx+c)}+4a^2e^{i(dx+c)}+b^2e^{i(dx+c)}\right)}{(-ia+b)^2\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)^2d(ia+b)^2}+\frac{\ln\left(e^{i(dx+c)}+\frac{ia^5+2ia^3b^2+\cdots}{\cdots}\right)}{(a^2+b^2\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/2*b^2*(5*a^2+2*b^2)/a/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)^3-
1/2*b*(4*a^4-7*a^2*b^2-2*b^4)/(a^4+2*a^2*b^2+b^4)/a^2*tan(1/2*d*x+1/2*c)^2+
1/2*b^2*(11*a^2+2*b^2)/(a^4+2*a^2*b^2+b^4)/a*tan(1/2*d*x+1/2*c)+1/2*b*(4*a^
2+b^2)/(a^4+2*a^2*b^2+b^4))/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-
a)^2+(2*a^2-b^2)/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1
/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(112) = 224$.
time = 0.50, size = 412, normalized size = 3.46

$$-\frac{\frac{(2a^2-b^2)\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}}+\frac{2\left(4a^4b+a^2b^3+\frac{(11a^3b^2+2ab^4)\sin(dx+c)}{\cos(dx+c)+1}-\frac{(4a^4b-7a^2b^3-2b^5)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{(5a^3b^2+2ab^4)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^8+2a^6b^2+a^4b^4+\frac{4(a^7b+2a^5b^3+a^3b^5)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2(a^8-3a^4b^4-2a^2b^6)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4(a^7b+2a^5b^3+a^3b^5)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{(a^8+2a^6b^2+a^4b^4)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((2*a^2 - b^2)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 +
 b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2
*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2*(4*a^4*b + a^2*b^3 + (11*a^3*b^2 + 2*a
*b^4)*sin(d*x + c)/(cos(d*x + c) + 1) - (4*a^4*b - 7*a^2*b^3 - 2*b^5)*sin(d
```

```
*x + c)^2/(cos(d*x + c) + 1)^2 - (5*a^3*b^2 + 2*a*b^4)*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3)/(a^8 + 2*a^6*b^2 + a^4*b^4 + 4*(a^7*b + 2*a^5*b^3 + a^3*b^
5)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*(a^8 - 3*a^4*b^4 - 2*a^2*b^6)*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 - 4*(a^7*b + 2*a^5*b^3 + a^3*b^5)*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + (a^8 + 2*a^6*b^2 + a^4*b^4)*sin(d*x + c)^4/(cos(
d*x + c) + 1)^4))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(112) = 224$.
time = 1.96, size = 352, normalized size = 2.96

$$-\frac{\left(2\,a^2b^2-b^4+\left(2\,a^4-3\,a^2b^2+b^4\right)\cos\left(dx+c\right)^2+2\left(2\,a^3b-ab^3\right)\cos\left(dx+c\right)\sin\left(dx+c\right)\right)\sqrt{a^2+b^2}\,\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)+2\left(4\,a^4b+5\,a^2b^3+b^5\right)\cos\left(dx+c\right)+6\left(a^3b^2+ab^4\right)\sin\left(dx+c\right)}{4\left(\left(a^8+2\,a^6b^2-2\,a^2b^6-b^8\right)d\cos\left(dx+c\right)^2+2\left(a^7b+3\,a^5b^3+3\,a^3b^5+ab^7\right)d\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^6b^2+3\,a^4b^4+3\,a^2b^6+b^8\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*((2*a^2*b^2 - b^4 + (2*a^4 - 3*a^2*b^2 + b^4)*cos(d*x + c)^2 + 2*(2*a^
3*b - a*b^3)*cos(d*x + c)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x
+ c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 +
 b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) +
 (a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 2*(4*a^4*b + 5*a^2*b^3 + b^5)*cos(d*x
 + c) + 6*(a^3*b^2 + a*b^4)*sin(d*x + c))/((a^8 + 2*a^6*b^2 - 2*a^2*b^6 - b
^8)*d*cos(d*x + c)^2 + 2*(a^7*b + 3*a^5*b^3 + 3*a^3*b^5 + a*b^7)*d*cos(d*x
+ c)*sin(d*x + c) + (a^6*b^2 + 3*a^4*b^4 + 3*a^2*b^6 + b^8)*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(112) = 224$.
time = 0.55, size = 293, normalized size = 2.46

$$-\frac{\frac{\left(2\,a^2-b^2\right)\log\left(\left|\frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}}{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}}\right|\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}-\frac{2\left(5\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,a^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-7\,a^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-11\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,a^4b-a^2b^3\right)}{\left(a^6+2\,a^4b^2+a^2b^4\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*((2*a^2 - b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b
^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2
*b^2 + b^4)*sqrt(a^2 + b^2)) - 2*(5*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*a*b^
4*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^2 - 7*a^2*b^3*tan(1
/2*d*x + 1/2*c)^2 - 2*b^5*tan(1/2*d*x + 1/2*c)^2 - 11*a^3*b^2*tan(1/2*d*x +
 1/2*c) - 2*a*b^4*tan(1/2*d*x + 1/2*c) - 4*a^4*b - a^2*b^3)/((a^6 + 2*a^4*b
^2 + a^2*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2))
/d
```

Mupad [B]
time = 1.72, size = 443, normalized size = 3.72

$$\frac{\ln\left(\left(a^2+b^2\right)^{5/2}-a^4\,b-b^5-2\,a^2\,b^3+a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\,b^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+2\,a^3\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a^2-\frac{b^2}{2}\right)}{d\,\left(a^2+b^2\right)^{5/2}}-\frac{\ln\left(\left(a^2+b^2\right)^{5/2}+a^4\,b+b^5+2\,a^2\,b^3-a^5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,b^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,a^3\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a^2-b^2\right)}{2\,d\,\left(a^2+b^2\right)^{5/2}}-\frac{\frac{4\,a^2\,b+b^3}{a^4+2\,a^2\,b^2+b^4}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(a^2-2\,b^2\right)\,\left(4\,a^2\,b+b^3\right)}{a^2\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(11\,a^2\,b+2\,b^3\right)}{a\,\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(5\,a^2\,b+2\,b^3\right)}{a\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left(a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(2\,a^2-4\,b^2\right)+a^2-4\,a\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+4\,a\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

```
[Out] (log((a^2 + b^2)^(5/2) - a^4*b - b^5 - 2*a^2*b^3 + a^5*tan(c/2 + (d*x)/2) +
 a*b^4*tan(c/2 + (d*x)/2) + 2*a^3*b^2*tan(c/2 + (d*x)/2))*(a^2 - b^2/2))/(d
*(a^2 + b^2)^(5/2)) - (log((a^2 + b^2)^(5/2) + a^4*b + b^5 + 2*a^2*b^3 - a^
5*tan(c/2 + (d*x)/2) - a*b^4*tan(c/2 + (d*x)/2) - 2*a^3*b^2*tan(c/2 + (d*x)
/2))*(2*a^2 - b^2))/(2*d*(a^2 + b^2)^(5/2)) - ((4*a^2*b + b^3)/(a^4 + b^4 +
 2*a^2*b^2) - (tan(c/2 + (d*x)/2)^2*(a^2 - 2*b^2)*(4*a^2*b + b^3))/(a^2*(a^
4 + b^4 + 2*a^2*b^2)) + (b*tan(c/2 + (d*x)/2)*(11*a^2*b + 2*b^3))/(a*(a^4 +
 b^4 + 2*a^2*b^2)) - (b*tan(c/2 + (d*x)/2)^3*(5*a^2*b + 2*b^3))/(a*(a^4 + b
^4 + 2*a^2*b^2)))/(d*(a^2*tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^2*(2*a^
2 - 4*b^2) + a^2 - 4*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/2 + (d*x)/2)))
```

3.134 $\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=22

$$-\frac{1}{2bd(a+b\tan(c+dx))^2}$$

```
[Out] -1/2/b/d/(a+b*tan(d*x+c))^2
```

Rubi [A]
time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.36, number of steps used = 2, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{3167, 37\}$

$$-\frac{\cot^2(c+dx)}{2bd(a\cot(c+dx)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/2*Cot[c + d*x]^2/(b*d*(b + a*Cot[c + d*x])^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{x}{(b+ax)^3}\,dx, x, \cot(c+dx)\right)}{d}\\ &= -\frac{\cot^2(c+dx)}{2bd(b+a\cot(c+dx))^2}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 57 vs. $2(22) = 44$.
time = 0.14, size = 57, normalized size = 2.59

$$\frac{-b\cos(2(c+dx)) + a\sin(2(c+dx))}{2\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (-(b*Cos[2*(c + d*x)]) + a*Sin[2*(c + d*x)])/(2*(a^2 + b^2)*d*(a*Cos[c + d*
x] + b*Sin[c + d*x])^2)
```

Maple [A]
time = 0.38, size = 21, normalized size = 0.95

method	result	size
derivativedivides	$-\frac{1}{2bd(a+b\tan(dx+c))^2}$	21
default	$-\frac{1}{2bd(a+b\tan(dx+c))^2}$	21
risch	$\frac{2ia\,\mathrm{e}^{2i(dx+c)}+2b\,\mathrm{e}^{2i(dx+c)}+2ia}{\left(b\,\mathrm{e}^{2i(dx+c)}+ia\,\mathrm{e}^{2i(dx+c)}-b+ia\right)^2 d(ia+b)^2}$	77
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}+\frac{2b\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}$	125

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/b/d/(a+b*tan(d*x+c))^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(20) = 40$.
time = 0.28, size = 171, normalized size = 7.77

$$\frac{2\left(\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\frac{b\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{a\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\left(a^4+\frac{4a^3b\sin(dx+c)}{\cos(dx+c)+1}-\frac{4a^3b\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{2\left(a^4-2a^2b^2\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 2*(a*sin(d*x + c)/(cos(d*x + c) + 1) + b*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 - a*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/((a^4 + 4*a^3*b*sin(d*x + c)/(co
s(d*x + c) + 1) - 4*a^3*b*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + a^4*sin(d*x
```

```
 + c)^4/(cos(d*x + c) + 1)^4 - 2*(a^4 - 2*a^2*b^2)*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(20) = 40.
time = 2.61, size = 142, normalized size = 6.45

$$-\frac{4 a^2 b \cos(dx+c)^2 - a^2 b + b^3 - 2(a^3 - ab^2)\cos(dx+c)\sin(dx+c)}{2\left((a^6 + a^4b^2 - a^2b^4 - b^6)d\cos(dx+c)^2 + 2(a^5b + 2a^3b^3 + ab^5)d\cos(dx+c)\sin(dx+c) + (a^4b^2 + 2a^2b^4 + b^6)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^2*b*cos(d*x + c)^2 - a^2*b + b^3 - 2*(a^3 - a*b^2)*cos(d*x + c)*s
in(d*x + c))/((a^6 + a^4*b^2 - a^2*b^4 - b^6)*d*cos(d*x + c)^2 + 2*(a^5*b +
 2*a^3*b^3 + a*b^5)*d*cos(d*x + c)*sin(d*x + c) + (a^4*b^2 + 2*a^2*b^4 + b^
6)*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.52, size = 20, normalized size = 0.91

$$-\frac{1}{2\left(b\tan(dx+c)+a\right)^2 bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2/((b*tan(d*x + c) + a)^2*b*d)
```

Mupad [B]
time = 0.61, size = 85, normalized size = 3.86

$$-\frac{b\left(\frac{\cos(2c+2dx)}{2}-\frac{1}{2}\right)-a\sin(2c+2dx)}{a^2 d\left(a^2+b^2+a^2\cos(2c+2dx)-b^2\cos(2c+2dx)+2ab\sin(2c+2dx)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

```
[Out] -(b*(cos(2*c + 2*d*x)/2 - 1/2) - a*sin(2*c + 2*d*x))/(a^2*d*(a^2 + b^2 + a^
2*cos(2*c + 2*d*x) - b^2*cos(2*c + 2*d*x) + 2*a*b*sin(2*c + 2*d*x)))
```

3.135 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=103

$$-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}d}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^2}$$

```
[Out] -1/2*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)/d
+1/2*(-b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^2
```

Rubi [A]

time = 0.04, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3155, 3153, 212}

$$-\frac{b\cos(c+dx)-a\sin(c+dx)}{2d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2}-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-3),x]
```

```
[Out] -1/2*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]/((a^2 + b^2
)^(3/2)*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])/(2*(a^2 + b^2)*d*(a*Cos[c +
d*x] + b*Sin[c + d*x])^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3155

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
```

```
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{b\cos(c+dx)-a\sin(c+dx)}{2\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^2} + \frac{\int \frac{1}{a\cos(c+dx)+b\sin(c+d}}{2\left(a^2+b^2\right)}$$

$$= -\frac{b\cos(c+dx)-a\sin(c+dx)}{2\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^2} - \frac{\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,d\right.}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{3/2}d} - \frac{b\cos(c+dx)-a\sin(c+}{2\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.30, size = 132, normalized size = 1.28

$$\frac{\left(a^2+b^2\right)(-b\cos(c+dx)+a\sin(c+dx))+2\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)(a\cos(c+dx)+b\sin(c+dx))^2}{2(a-ib)^2(a+ib)^2d(a\cos(c+dx)+b\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-3),x]
```

```
[Out] ((a^2 + b^2)*(-(b*Cos[c + d*x]) + a*Sin[c + d*x]) + 2*Sqrt[a^2 + b^2]*ArcTa
nh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d
*x])^2)/(2*(a - I*b)^2*(a + I*b)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)
```

Maple [A]
time = 0.41, size = 191, normalized size = 1.85

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{\left(a^2+2b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^2+b^2\right)a}-\frac{b\left(a^2-2b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^2+b^2\right)a^2}-\frac{\left(a^2-2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^2+b^2\right)a}+\frac{b}{2a^2+2b^2}\right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+\frac{\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2}{2\sqrt{a^2+b^2}}\right.}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{d}$
default	$\frac{-\frac{2\left(-\frac{\left(a^2+2b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^2+b^2\right)a}-\frac{b\left(a^2-2b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^2+b^2\right)a^2}-\frac{\left(a^2-2b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^2+b^2\right)a}+\frac{b}{2a^2+2b^2}\right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+\frac{\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2}{2\sqrt{a^2+b^2}}\right.}{\left(a^2+b^2\right)^{\frac{3}{2}}}}{d}$
risch	$\frac{ia\,e^{3i(dx+c)}+b\,e^{3i(dx+c)}-ia\,e^{i(dx+c)}+b\,e^{i(dx+c)}}{\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)^2(-ia+b)d(ia+b)}+\frac{\ln\left(e^{i(dx+c)}+\frac{ia^3+iab^2-a^2b-b^3}{\left(a^2+b^2\right)^{\frac{3}{2}}}\right)}{2\left(a^2+b^2\right)^{\frac{3}{2}}d}-\frac{\ln\left(e^{i(dx+c)}-\frac{ia^3+iab^2-a^2}{\left(a^2+b^2\right)}\right.}{2\left(a^2+b^2\right)^{\frac{3}{2}}d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/2*(a^2+2*b^2)/(a^2+b^2)/a*tan(1/2*d*x+1/2*c)^3-1/2*b*(a^2-2*b^2
)/(a^2+b^2)/a^2*tan(1/2*d*x+1/2*c)^2-1/2*(a^2-2*b^2)/(a^2+b^2)/a*tan(1/2*d*
x+1/2*c)+1/2*b/(a^2+b^2))/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)
^2+1/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/
2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(95) = 190$.
time = 0.49, size = 326, normalized size = 3.17

$$-\frac{\frac{2\left(a^{2}b-\frac{\left(a^{3}-2\,ab^{2}\right)\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{\left(a^{2}b-2\,b^{3}\right)\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}-\frac{\left(a^{3}+2\,ab^{2}\right)\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}\right)}{a^{6}+a^{4}b^{2}+\frac{4\left(a^{5}b+a^{3}b^{3}\right)\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{2\left(a^{6}-a^{4}b^{2}-2\,a^{2}b^{4}\right)\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}-\frac{4\left(a^{5}b+a^{3}b^{3}\right)\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}+\frac{\left(a^{6}+a^{4}b^{2}\right)\sin\left(dx+c\right)^{4}}{\left(\cos\left(dx+c\right)+1\right)^{4}}}+\frac{\log\left(\frac{b-\frac{a\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\sqrt{a^{2}+b^{2}}}{b-\frac{a\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\sqrt{a^{2}+b^{2}}}\right)}{\left(a^{2}+b^{2}\right)^{\frac{3}{2}}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(2*(a^2*b - (a^3 - 2*a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) - (a^2*b -
 2*b^3)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - (a^3 + 2*a*b^2)*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3)/(a^6 + a^4*b^2 + 4*(a^5*b + a^3*b^3)*sin(d*x + c)/(
cos(d*x + c) + 1) - 2*(a^6 - a^4*b^2 - 2*a^2*b^4)*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 4*(a^5*b + a^3*b^3)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + (a^6
 + a^4*b^2)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + log((b - a*sin(d*x + c)/
(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1
) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(95) = 190$.
time = 1.89, size = 294, normalized size = 2.85

$$\frac{\left(2\,ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^{2}-b^{2}\right)\cos\left(dx+c\right)^{2}+b^{2}\right)\sqrt{a^{2}+b^{2}}\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^{2}-b^{2}\right)\cos(dx+c)^{2}-2\,a^{2}-b^{2}+2\,\sqrt{a^{2}+b^{2}}\left(b\cos(dx+c)-a\sin(dx+c)\right)}{2\,ab\cos(dx+c)\sin(dx+c)+\left(a^{2}-b^{2}\right)\cos(dx+c)^{2}+b^{2}}\right)-2\left(a^{2}b+b^{3}\right)\cos\left(dx+c\right)+2\left(a^{3}+ab^{2}\right)\sin\left(dx+c\right)}{4\left(\left(a^{6}+a^{4}b^{2}-a^{2}b^{4}-b^{6}\right)d\cos\left(dx+c\right)^{2}+2\left(a^{5}b+2\,a^{3}b^{3}+ab^{5}\right)d\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^{4}b^{2}+2\,a^{2}b^{4}+b^{6}\right)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)*s
qrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x
+ c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))
/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2)) - 2*
```

```
(a^2*b + b^3)*cos(d*x + c) + 2*(a^3 + a*b^2)*sin(d*x + c))/((a^6 + a^4*b^2
- a^2*b^4 - b^6)*d*cos(d*x + c)^2 + 2*(a^5*b + 2*a^3*b^3 + a*b^5)*d*cos(d*x
 + c)*sin(d*x + c) + (a^4*b^2 + 2*a^2*b^4 + b^6)*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(95) = 190.
time = 0.44, size = 221, normalized size = 2.15

$$-\frac{\frac{\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a^2b\right)}{(a^4+a^2b^2)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*t
an(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a^3*
tan(1/2*d*x + 1/2*c)^3 + 2*a*b^2*tan(1/2*d*x + 1/2*c)^3 + a^2*b*tan(1/2*d*x
 + 1/2*c)^2 - 2*b^3*tan(1/2*d*x + 1/2*c)^2 + a^3*tan(1/2*d*x + 1/2*c) - 2*a
*b^2*tan(1/2*d*x + 1/2*c) - a^2*b)/((a^4 + a^2*b^2)*(a*tan(1/2*d*x + 1/2*c)
^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2))/d
```

Mupad [B]
time = 2.73, size = 260, normalized size = 2.52

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^2-2\,b^2)}{a\,(a^2+b^2)}-\frac{b}{a^2+b^2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(a^2+2\,b^2)}{a\,(a^2+b^2)}+\frac{b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(a^2-2\,b^2)}{a^2\,(a^2+b^2)}}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(2\,a^2-4\,b^2)+a^2-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{\mathrm{atanh}\left(\frac{\left(2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{2\,a^2\,b+2\,b^3}{a^2+b^2}\right)\,\left(\frac{a^2}{2}+\frac{b^2}{2}\right)}{(a^2+b^2)^{3/2}}\right)}{d\,(a^2+b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)*(a^2 - 2*b^2))/(a*(a^2 + b^2)) - b/(a^2 + b^2) + (tan(
c/2 + (d*x)/2)^3*(a^2 + 2*b^2))/(a*(a^2 + b^2)) + (b*tan(c/2 + (d*x)/2)^2*(
a^2 - 2*b^2))/(a^2*(a^2 + b^2)))/(d*(a^2*tan(c/2 + (d*x)/2)^4 - tan(c/2 + (
d*x)/2)^2*(2*a^2 - 4*b^2) + a^2 - 4*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/
2 + (d*x)/2))) + atanh(((2*a*tan(c/2 + (d*x)/2) - (2*a^2*b + 2*b^3)/(a^2 +
b^2))*(a^2/2 + b^2/2))/(a^2 + b^2)^(3/2))/(d*(a^2 + b^2)^(3/2))
```

3.136 $\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=86

$$-\frac{\frac{1}{b}+\frac{b}{a^2}}{2d(b+a\cot(c+dx))^2}+\frac{\frac{1}{a^2}-\frac{1}{b^2}}{d(b+a\cot(c+dx))}+\frac{\log(b+a\cot(c+dx))}{b^3d}+\frac{\log(\tan(c+dx))}{b^3d}$$

```
[Out] 1/2*(-1/b-b/a^2)/d/(b+a*cot(d*x+c))^2+(1/a^2-1/b^2)/d/(b+a*cot(d*x+c))+ln(b
+a*cot(d*x+c))/b^3/d+ln(tan(d*x+c))/b^3/d
```

Rubi [A]
time = 0.07, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {3167, 908}

$$\frac{\frac{1}{a^2}-\frac{1}{b^2}}{d(a\cot(c+dx)+b)}-\frac{\frac{b}{a^2}+\frac{1}{b}}{2d(a\cot(c+dx)+b)^2}+\frac{\log(a\cot(c+dx)+b)}{b^3d}+\frac{\log(\tan(c+dx))}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/2*(b^(-1) + b/a^2)/(d*(b + a*Cot[c + d*x])^2) + (a^(-2) - b^(-2))/(d*(b
+ a*Cot[c + d*x])) + Log[b + a*Cot[c + d*x]]/(b^3*d) + Log[Tan[c + d*x]]/(b
^3*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1+x^2}{x(b+ax)^3}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{b^3x} + \frac{-a^2-b^2}{ab(b+ax)^3} + \frac{-a^2+b^2}{ab^2(b+ax)^2} - \frac{a}{b^3(b+ax)}\right)\,dx, x, \cot(c+dx\right.}{d}$$
$$= -\frac{\frac{1}{b}+\frac{b}{a^2}}{2d(b+a\cot(c+dx))^2} + \frac{\frac{1}{a^2}-\frac{1}{b^2}}{d(b+a\cot(c+dx))} + \frac{\log(b+a\cot(c+}{b^3d}$$

Mathematica [A]
time = 0.67, size = 115, normalized size = 1.34

$$-\frac{b^2\sec^2(c+dx)+2(a+b\tan(c+dx))(a(\log(\cos(c+dx))-\log(a\cos(c+dx)+b\sin(c+dx)))+b(1+\log(\cos(c+dx))-\log(a\cos(c+dx)+b\sin(c+dx)))\tan(c+dx))}{2b^3d(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/2*(b^2*Sec[c + d*x]^2 + 2*(a + b*Tan[c + d*x])*(a*(Log[Cos[c + d*x]] - L
og[a*Cos[c + d*x] + b*Sin[c + d*x]]) + b*(1 + Log[Cos[c + d*x]] - Log[a*Cos
[c + d*x] + b*Sin[c + d*x]])*Tan[c + d*x]))/(b^3*d*(a + b*Tan[c + d*x])^2)
```

Maple [A]
time = 0.54, size = 63, normalized size = 0.73

method	result
derivativedivides	$\frac{\frac{\ln(a+b\tan(dx+c))}{b^3}+\frac{2a}{b^3(a+b\tan(dx+c))}-\frac{a^2+b^2}{2b^3(a+b\tan(dx+c))^2}}{d}$
default	$\frac{\frac{\ln(a+b\tan(dx+c))}{b^3}+\frac{2a}{b^3(a+b\tan(dx+c))}-\frac{a^2+b^2}{2b^3(a+b\tan(dx+c))^2}}{d}$
risch	$\frac{-2a^2e^{2i(dx+c)}+2b^2e^{2i(dx+c)}+4iab\,e^{2i(dx+c)}-2a^2-2iab}{b^2(ia+b)\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)^2d}+\frac{\ln\left(e^{2i(dx+c)}-\frac{ib+a}{ib-a}\right)}{b^3d}-\frac{\ln(e^{2i(dx+c)}+1)}{b^3d}$
norman	$\frac{-\frac{2\left(a^2-b^2\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^2da}+\frac{2\left(a^2-b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2da}-\frac{2\left(3a^2-b^2\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{bd\,a^2}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^2}+\frac{\ln\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{b^3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^3*ln(a+b*tan(d*x+c))+2*a/b^3/(a+b*tan(d*x+c))-1/2*(a^2+b^2)/b^3/(a
+b*tan(d*x+c))^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(84) = 168$.
time = 0.32, size = 315, normalized size = 3.66

$$-\frac{\frac{2\left(\frac{(a^3-ab^2)\sin(dx+c)}{\cos(dx+c)+1}+\frac{(3a^2b-b^3)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{(a^3-ab^2)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4b^2+\frac{4a^3b^3\sin(dx+c)}{\cos(dx+c)+1}-\frac{4a^3b^3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{a^4b^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{2(a^4b^2-2a^2b^4)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}-\frac{\log\left(-a-\frac{2b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{b^3}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^3}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(2*((a^3 - a*b^2)*sin(d*x + c)/(cos(d*x + c) + 1) + (3*a^2*b - b^3)*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 - (a^3 - a*b^2)*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/(a^4*b^2 + 4*a^3*b^3*sin(d*x + c)/(cos(d*x + c) + 1) - 4*a^3*b^3*si
n(d*x + c)^3/(cos(d*x + c) + 1)^3 + a^4*b^2*sin(d*x + c)^4/(cos(d*x + c) +
1)^4 - 2*(a^4*b^2 - 2*a^2*b^4)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2) - log(-
a - 2*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x + c) +
1)^2)/b^3 + log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^3 + log(sin(d*x + c)
/(cos(d*x + c) + 1) - 1)/b^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(84) = 168$.
time = 2.20, size = 284, normalized size = 3.30

$$\frac{4a^2b^2\cos(dx+c)^2-3a^2b^2-b^4-2(a^3b-ab^3)\cos(dx+c)\sin(dx+c)+(a^2b^2+b^4+(a^4-b^4)\cos(dx+c)^2+2(a^3b+ab^3)\cos(dx+c)\sin(dx+c))\log(2ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2)-(a^2b^2+b^4+(a^4-b^4)\cos(dx+c)^2+2(a^3b+ab^3)\cos(dx+c)\sin(dx+c))\log(\cos(dx+c)^2)}{2((a^4b^3-b^7)d\cos(dx+c)^2+2(a^3b^4+ab^6)d\cos(dx+c)\sin(dx+c)+(a^2b^5+b^7)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*a^2*b^2*cos(d*x + c)^2 - 3*a^2*b^2 - b^4 - 2*(a^3*b - a*b^3)*cos(d*x
 + c)*sin(d*x + c) + (a^2*b^2 + b^4 + (a^4 - b^4)*cos(d*x + c)^2 + 2*(a^3*b
 + a*b^3)*cos(d*x + c)*sin(d*x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) +
(a^2 - b^2)*cos(d*x + c)^2 + b^2) - (a^2*b^2 + b^4 + (a^4 - b^4)*cos(d*x +
c)^2 + 2*(a^3*b + a*b^3)*cos(d*x + c)*sin(d*x + c))*log(cos(d*x + c)^2))/((
a^4*b^3 - b^7)*d*cos(d*x + c)^2 + 2*(a^3*b^4 + a*b^6)*d*cos(d*x + c)*sin(d*
x + c) + (a^2*b^5 + b^7)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)/(a*cos(c + d*x) + b*sin(c + d*x))**3, x)
```

Giac [A]
time = 0.49, size = 62, normalized size = 0.72

$$\frac{\frac{2\log(|b\tan(dx+c)+a|)}{b^3}-\frac{3\,b\tan(dx+c)^2+2\,a\tan(dx+c)+b}{(b\tan(dx+c)+a)^2b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*log(abs(b*tan(d*x + c) + a))/b^3 - (3*b*tan(d*x + c)^2 + 2*a*tan(d*x
 + c) + b)/((b*tan(d*x + c) + a)^2*b^2))/d
```

Mupad [B]
time = 2.59, size = 396, normalized size = 4.60

$$\frac{2\,\mathrm{atanh}\left(\frac{16\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{16\,a+\frac{32\,a^3}{b^2}-16\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\frac{32\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{b^2}+\frac{32\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b}}-\frac{16\,a}{16\,a+\frac{32\,a^3}{b^2}-16\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\frac{32\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{b^2}+\frac{32\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b}}+\frac{32\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,a\,b+\frac{32\,a^3}{b}+32\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{32\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{b}-16\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)}{b^3\,d}-\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(3\,a^2-b^2)}{a^2\,b}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(a^2-b^2)}{a\,b^2}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a^2-b^2)}{a\,b^2}}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(2\,a^2-4\,b^2)+a^2-4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+4\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a*cos(c + d*x) + b*sin(c + d*x))^3),x)
```

```
[Out] (2*atanh((16*a*tan(c/2 + (d*x)/2)^2)/(16*a + (32*a^3)/b^2 - 16*a*tan(c/2 +
(d*x)/2)^2 - (32*a^3*tan(c/2 + (d*x)/2)^2)/b^2 + (32*a^2*tan(c/2 + (d*x)/2)
)/b) - (16*a)/(16*a + (32*a^3)/b^2 - 16*a*tan(c/2 + (d*x)/2)^2 - (32*a^3*ta
n(c/2 + (d*x)/2)^2)/b^2 + (32*a^2*tan(c/2 + (d*x)/2))/b) + (32*a^2*tan(c/2
+ (d*x)/2))/(16*a*b + (32*a^3)/b + 32*a^2*tan(c/2 + (d*x)/2) - (32*a^3*tan(
c/2 + (d*x)/2)^2)/b - 16*a*b*tan(c/2 + (d*x)/2)^2)))/(b^3*d) - ((2*tan(c/2
+ (d*x)/2)^2*(3*a^2 - b^2))/(a^2*b) - (2*tan(c/2 + (d*x)/2)^3*(a^2 - b^2))/
(a*b^2) + (2*tan(c/2 + (d*x)/2)*(a^2 - b^2))/(a*b^2))/(d*(a^2*tan(c/2 + (d*
x)/2)^4 - tan(c/2 + (d*x)/2)^2*(2*a^2 - 4*b^2) + a^2 - 4*a*b*tan(c/2 + (d*x
)/2)^3 + 4*a*b*tan(c/2 + (d*x)/2)))
```

3.137 $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=260

$$-\frac{3a\tanh^{-1}(\sin(c+dx))}{b^4d}-\frac{2a^2\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4\sqrt{a^2+b^2}\,d}-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2b^2\sqrt{a^2+b^2}\,d}-\frac{\sqrt{a^2+b^2}\,\text{ta}}{}$$

```
[Out] -3*a*arctanh(sin(d*x+c))/b^4/d+sec(d*x+c)/b^3/d+1/2*(-b*cos(d*x+c)+a*sin(d*
x+c))/b^2/d/(a*cos(d*x+c)+b*sin(d*x+c))^2+2*a/b^3/d/(a*cos(d*x+c)+b*sin(d*x
+c))-2*a^2*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/b^4/d/(a^2+
b^2)^(1/2)-1/2*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/b^2/d/(
a^2+b^2)^(1/2)-arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))*(a^2+b^
2)^(1/2)/b^4/d
```

Rubi [A]

time = 0.20, antiderivative size = 260, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3185, 3155, 3153, 212, 3183, 3855, 3173}

$$-\frac{\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2b^2d\sqrt{a^2+b^2}}-\frac{2a^2\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4d\sqrt{a^2+b^2}}-\frac{\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4d}-\frac{3a\tanh^{-1}(\sin(c+dx))}{b^4d}+\frac{2a}{b^3d(a\cos(c+dx)+b\sin(c+dx))}-\frac{b\cos(c+dx)-a\sin(c+dx)}{2b^2d(a\cos(c+dx)+b\sin(c+dx))^2}+\frac{\sec(c+dx)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (-3*a*ArcTanh[Sin[c + d*x]])/(b^4*d) - (2*a^2*ArcTanh[(b*Cos[c + d*x] - a*S
in[c + d*x])/Sqrt[a^2 + b^2]])/(b^4*Sqrt[a^2 + b^2]*d) - ArcTanh[(b*Cos[c +
 d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]]/(2*b^2*Sqrt[a^2 + b^2]*d) - (Sqrt[
a^2 + b^2]*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^4
*d) + Sec[c + d*x]/(b^3*d) - (b*Cos[c + d*x] - a*Sin[c + d*x])/(2*b^2*d*(a*
Cos[c + d*x] + b*Sin[c + d*x])^2) + (2*a)/(b^3*d*(a*Cos[c + d*x] + b*Sin[c
+ d*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3155

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3173

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/co
s[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^
(n + 1)/(b*d*(n + 1)), x] + (Dist[1/b^2, Int[(a*Cos[c + d*x] + b*Sin[c + d*
x])^(n + 2)/Cos[c + d*x], x], x] - Dist[a/b^2, Int[(a*Cos[c + d*x] + b*Sin[
c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &
& LtQ[n, -1]
```

Rule 3183

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3185

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2 + b^2)/b^2, Int[Cos[c +
d*x]^(m + 2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^n, x], x] + (Dist[1/b^2, Int
[Cos[c + d*x]^m*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] - Dist[2*
(a/b^2), Int[Cos[c + d*x]^(m + 1)*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 1)
, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && L
tQ[m, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{b^2} - \frac{(2a)\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx}{b^2} + \frac{(a^2 +$$

$$= \frac{\sec(c+dx)}{b^3d} - \frac{b\cos(c+dx)-a\sin(c+dx)}{2b^2d(a\cos(c+dx)+b\sin(c+dx))^2} + \frac{}{b^3d(a\cos(c +}$$

$$= -\frac{3a\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{\sec(c+dx)}{b^3d} - \frac{b\cos(c+dx)-a\sin}{2b^2d(a\cos(c+dx)+b\,\mathrm{s}}$$

$$= -\frac{3a\tanh^{-1}(\sin(c+dx))}{b^4d} - \frac{2a^2\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4\sqrt{a^2+b^2}\,d} - \frac{\mathrm{t}}{}$$

Mathematica [A]

time = 2.56, size = 396, normalized size = 1.52

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(\frac{b^2(a^2+b^2)\sin(c+dx)}{a}+\frac{(2a-b)b(2a+b)(a\cos(c+dx)+b\sin(c+dx))}{a}+2b(a\cos(c+dx)+b\sin(c+dx))^2+\frac{6(2a^2+b^2)\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)(a\cos(c+dx)+b\sin(c+dx))^2}{\sqrt{a^2+b^2}}+6a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^2-6a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^2+\frac{2b\sin\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b\sin(c+dx))^2}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}-\frac{2b\sin\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b\sin(c+dx))^2}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}\right)}{2b^4d(a+b\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])*((b^2*(a^2 + b^2)*Sin[c +
 d*x])/a + ((2*a - b)*b*(2*a + b)*(a*Cos[c + d*x] + b*Sin[c + d*x]))/a + 2*
b*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 + (6*(2*a^2 + b^2)*ArcTanh[(-b + a*Ta
n[(c + d*x)/2])/Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/Sqrt[
a^2 + b^2] + 6*a*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^2 - 6*a*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a*Cos[c
+ d*x] + b*Sin[c + d*x])^2 + (2*b*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[
c + d*x])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (2*b*Sin[(c + d*x)/2]*
(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
)/(2*b^4*d*(a + b*Tan[c + d*x])^3)
```

Maple [A]

time = 0.80, size = 269, normalized size = 1.03

method	result
derivativedivides	$\frac{-\frac{1}{b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^4}+\frac{1}{b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^4}-\frac{2\left(\frac{b^2\left(3a^2-2b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{2a}\right.}{}}{d}$

default	$\frac{-\frac{1}{b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^4}+\frac{1}{b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{3a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^4}-\frac{2\left(\frac{b^2\left(3a^2-2b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}\right.}{}}{d}$
risch	$\frac{-9iab\,e^{5i(dx+c)}+6a^2e^{5i(dx+c)}-3b^2e^{5i(dx+c)}+12a^2e^{3i(dx+c)}+2b^2e^{3i(dx+c)}+9iab\,e^{i(dx+c)}+6a^2e^{i(dx+c)}-3b^2e^{i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)\left(-ib\,e^{2i(dx+c)}+a\,e^{2i(dx+c)}+ib+a\right)^2b^3d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^3/(tan(1/2*d*x+1/2*c)-1)+3*a/b^4*ln(tan(1/2*d*x+1/2*c)-1)+1/b^3/(
tan(1/2*d*x+1/2*c)+1)-3*a/b^4*ln(tan(1/2*d*x+1/2*c)+1)-2/b^4*((1/2*b^2*(3*a
^2-2*b^2)/a*tan(1/2*d*x+1/2*c)^3+1/2*b*(4*a^4-9*a^2*b^2+2*b^4)/a^2*tan(1/2*
d*x+1/2*c)^2-1/2*b^2*(13*a^2-2*b^2)/a*tan(1/2*d*x+1/2*c)-2*a^2*b+1/2*b^3)/(
a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)^2-3/2*(2*a^2+b^2)/(a^2+b^2
)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(244) = 488$.
time = 0.50, size = 518, normalized size = 1.99

$$\frac{\frac{2\left(6a^4-a^2b^2+\frac{\left(21a^3b-2ab^3\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\left(6a^4-9a^2b^2+b^4\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4\left(6a^3b-ab^3\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\left(6a^4-9a^2b^2+2b^4\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\left(3a^3b-2ab^3\right)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^4b^3+\frac{4a^3b^4\sin(dx+c)}{\cos(dx+c)+1}-\frac{8a^3b^4\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{4a^3b^4\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{a^4b^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{\left(3a^4b^3-4a^2b^5\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\left(3a^4b^3-4a^2b^5\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{6a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^4}+\frac{6a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^4}-\frac{3\left(2a^2+b^2\right)\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b^4}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(6*a^4 - a^2*b^2 + (21*a^3*b - 2*a*b^3)*sin(d*x + c)/(cos(d*x + c) +
 1) - 2*(6*a^4 - 9*a^2*b^2 + b^4)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4*(
6*a^3*b - a*b^3)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + (6*a^4 - 9*a^2*b^2 +
 2*b^4)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (3*a^3*b - 2*a*b^3)*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5)/(a^4*b^3 + 4*a^3*b^4*sin(d*x + c)/(cos(d*x + c)
 + 1) - 8*a^3*b^4*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4*a^3*b^4*sin(d*x +
 c)^5/(cos(d*x + c) + 1)^5 - a^4*b^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 -
(3*a^4*b^3 - 4*a^2*b^5)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + (3*a^4*b^3 -
4*a^2*b^5)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 6*a*log(sin(d*x + c)/(cos
(d*x + c) + 1) + 1)/b^4 + 6*a*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/b^4
- 3*(2*a^2 + b^2)*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b
^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 +
 b^2)*b^4))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(244) = 488$.

time = 3.50, size = 513, normalized size = 1.97

$$\frac{4\,a^{2}b^{3}+4\,b^{5}+6\,(2\,a^{4}b+a^{2}b^{3}-b^{5})\cos\left(dx+c\right)^{2}+18\,(a^{3}b^{2}+ab^{4})\cos\left(dx+c\right)\sin\left(dx+c\right)+3\,\left((2\,a^{4}-a^{2}b^{2}-b^{4})\cos\left(dx+c\right)^{3}+2\,(2\,a^{3}b+ab^{3})\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)+(2\,a^{2}b^{2}+b^{4})\cos\left(dx+c\right)\right)\sqrt{a^{2}+b^{2}}\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^{2}-b^{2})\cos(dx+c)^{2}-2\,a^{2}-b^{2}+2\,\sqrt{a^{2}+b^{2}}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^{2}-b^{2})\cos(dx+c)^{2}+b^{2}}\right)-6\,\left((a^{5}-ab^{4})\cos\left(dx+c\right)^{3}+2\,(a^{4}b+a^{2}b^{3})\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)+(a^{3}b^{2}+ab^{4})\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)+6\,\left((a^{5}-ab^{4})\cos\left(dx+c\right)^{3}+2\,(a^{4}b+a^{2}b^{3})\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)+(a^{3}b^{2}+ab^{4})\cos\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)}{4\,\left((a^{4}b^{4}-b^{8})d\cos\left(dx+c\right)^{3}+2\,(a^{3}b^{5}+ab^{7})d\cos\left(dx+c\right)^{2}\sin\left(dx+c\right)+(a^{2}b^{6}+b^{8})d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^2*b^3 + 4*b^5 + 6*(2*a^4*b + a^2*b^3 - b^5)*cos(d*x + c)^2 + 18*(a
^3*b^2 + a*b^4)*cos(d*x + c)*sin(d*x + c) + 3*((2*a^4 - a^2*b^2 - b^4)*cos(
d*x + c)^3 + 2*(2*a^3*b + a*b^3)*cos(d*x + c)^2*sin(d*x + c) + (2*a^2*b^2 +
 b^4)*cos(d*x + c))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) +
 (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x +
c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*
x + c)^2 + b^2)) - 6*((a^5 - a*b^4)*cos(d*x + c)^3 + 2*(a^4*b + a^2*b^3)*co
s(d*x + c)^2*sin(d*x + c) + (a^3*b^2 + a*b^4)*cos(d*x + c))*log(sin(d*x + c
) + 1) + 6*((a^5 - a*b^4)*cos(d*x + c)^3 + 2*(a^4*b + a^2*b^3)*cos(d*x + c)
^2*sin(d*x + c) + (a^3*b^2 + a*b^4)*cos(d*x + c))*log(-sin(d*x + c) + 1))/(
(a^4*b^4 - b^8)*d*cos(d*x + c)^3 + 2*(a^3*b^5 + a*b^7)*d*cos(d*x + c)^2*sin
(d*x + c) + (a^2*b^6 + b^8)*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{2}\left(c+dx\right)}{\left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*cos(c + d*x) + b*sin(c + d*x))**3, x)
```

Giac [A]
time = 0.54, size = 314, normalized size = 1.21

$$-\frac{\frac{6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^{4}}-\frac{6\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^{4}}+\frac{3\,(2\,a^{2}+b^{2})\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\sqrt{a^{2}+b^{2}}\,b^{4}}+\frac{4}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)b^{3}}+\frac{2\,\left(3\,a^{3}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2\,ab^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+4\,a^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-9\,a^{2}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+2\,b^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-13\,a^{3}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,ab^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,a^{4}+a^{2}b^{2}\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)^{2}a^{2}b^{3}}}{2\,d}$$

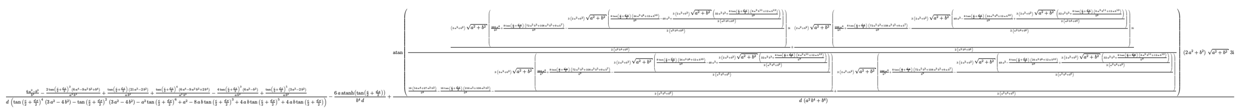
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(6*a*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 6*a*log(abs(tan(1/2*d*x
+ 1/2*c) - 1))/b^4 + 3*(2*a^2 + b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b
 - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2
)))/(sqrt(a^2 + b^2)*b^4) + 4/((tan(1/2*d*x + 1/2*c)^2 - 1)*b^3) + 2*(3*a^3
*b*tan(1/2*d*x + 1/2*c)^3 - 2*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 4*a^4*tan(1/2*
```

```
d*x + 1/2*c)^2 - 9*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 + 2*b^4*tan(1/2*d*x + 1/2
*c)^2 - 13*a^3*b*tan(1/2*d*x + 1/2*c) + 2*a*b^3*tan(1/2*d*x + 1/2*c) - 4*a^
4 + a^2*b^2)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2*a
^2*b^3))/d
```

Mupad [B]
time = 2.62, size = 1311, normalized size = 5.04



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a*cos(c + d*x) + b*sin(c + d*x))^3),x)
```

```
[Out] ((6*a^2 - b^2)/b^3 - (2*tan(c/2 + (d*x)/2)^2*(6*a^4 + b^4 - 9*a^2*b^2))/(a^
2*b^3) + (tan(c/2 + (d*x)/2)*(21*a^2 - 2*b^2))/(a*b^2) + (tan(c/2 + (d*x)/2
)^4*(6*a^4 + 2*b^4 - 9*a^2*b^2))/(a^2*b^3) - (4*tan(c/2 + (d*x)/2)^3*(6*a^2
 - b^2))/(a*b^2) + (tan(c/2 + (d*x)/2)^5*(3*a^2 - 2*b^2))/(a*b^2))/(d*(tan(
c/2 + (d*x)/2)^4*(3*a^2 - 4*b^2) - tan(c/2 + (d*x)/2)^2*(3*a^2 - 4*b^2) - a
^2*tan(c/2 + (d*x)/2)^6 + a^2 - 8*a*b*tan(c/2 + (d*x)/2)^3 + 4*a*b*tan(c/2
+ (d*x)/2)^5 + 4*a*b*tan(c/2 + (d*x)/2))) - (6*a*atanh(tan(c/2 + (d*x)/2)))
/(b^4*d) + (atan((((2*a^2 + b^2)*(a^2 + b^2)^(1/2)*((288*a^4)/b^5 + (8*tan(
c/2 + (d*x)/2)*(9*a*b^7 + 108*a^3*b^5 + 72*a^5*b^3))/b^9 - (3*(2*a^2 + b^2)
*(a^2 + b^2)^(1/2)*((8*tan(c/2 + (d*x)/2)*(12*a*b^10 + 24*a^3*b^8))/b^9 - 4
8*a^2 + (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)
/2)*(12*a*b^13 + 8*a^3*b^11))/b^9))/(2*(b^6 + a^2*b^4))))/(2*(b^6 + a^2*b^4
)))*3i)/(2*(b^6 + a^2*b^4)) + ((2*a^2 + b^2)*(a^2 + b^2)^(1/2)*((288*a^4)/b
^5 + (8*tan(c/2 + (d*x)/2)*(9*a*b^7 + 108*a^3*b^5 + 72*a^5*b^3))/b^9 - (3*(
2*a^2 + b^2)*(a^2 + b^2)^(1/2)*(48*a^2 - (8*tan(c/2 + (d*x)/2)*(12*a*b^10 +
 24*a^3*b^8))/b^9 + (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*(32*a^2*b^3 + (8*tan
(c/2 + (d*x)/2)*(12*a*b^13 + 8*a^3*b^11))/b^9))/(2*(b^6 + a^2*b^4))))/(2*(b
^6 + a^2*b^4)))*3i)/(2*(b^6 + a^2*b^4)))/((16*(54*a^4 + 27*a^2*b^2))/b^8 -
(16*tan(c/2 + (d*x)/2)*(216*a^5 + 108*a^3*b^2))/b^9 - (3*(2*a^2 + b^2)*(a^2
 + b^2)^(1/2)*((288*a^4)/b^5 + (8*tan(c/2 + (d*x)/2)*(9*a*b^7 + 108*a^3*b^5
 + 72*a^5*b^3))/b^9 - (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*((8*tan(c/2 + (d*x
)/2)*(12*a*b^10 + 24*a^3*b^8))/b^9 - 48*a^2 + (3*(2*a^2 + b^2)*(a^2 + b^2)^
(1/2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(12*a*b^13 + 8*a^3*b^11))/b^9))/(
2*(b^6 + a^2*b^4))))/(2*(b^6 + a^2*b^4))))/(2*(b^6 + a^2*b^4)) + (3*(2*a^2
+ b^2)*(a^2 + b^2)^(1/2)*((288*a^4)/b^5 + (8*tan(c/2 + (d*x)/2)*(9*a*b^7 +
108*a^3*b^5 + 72*a^5*b^3))/b^9 - (3*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*(48*a^2
 - (8*tan(c/2 + (d*x)/2)*(12*a*b^10 + 24*a^3*b^8))/b^9 + (3*(2*a^2 + b^2)*(
a^2 + b^2)^(1/2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(12*a*b^13 + 8*a^3*b^1
1))/b^9))/(2*(b^6 + a^2*b^4))))/(2*(b^6 + a^2*b^4))))/(2*(b^6 + a^2*b^4)))
*(2*a^2 + b^2)*(a^2 + b^2)^(1/2)*3i)/(d*(b^6 + a^2*b^4))
```

3.138 $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=161

$$-\frac{(a^2+b^2)^2}{2a^2b^3d(b+a\cot(c+dx))^2}-\frac{(3a^2-b^2)(a^2+b^2)}{a^2b^4d(b+a\cot(c+dx))}+\frac{2(3a^2+b^2)\log(b+a\cot(c+dx))}{b^5d}+\frac{2(3a^2+b^2)\log}{b^5}$$

```
[Out] -1/2*(a^2+b^2)^2/a^2/b^3/d/(b+a*cot(d*x+c))^2-(3*a^2-b^2)*(a^2+b^2)/a^2/b^4
/d/(b+a*cot(d*x+c))+2*(3*a^2+b^2)*ln(b+a*cot(d*x+c))/b^5/d+2*(3*a^2+b^2)*ln
(tan(d*x+c))/b^5/d-3*a*tan(d*x+c)/b^4/d+1/2*tan(d*x+c)^2/b^3/d
```

Rubi [A]
time = 0.13, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3167, 908}

$$\frac{2(3a^2+b^2)\log(\tan(c+dx))}{b^5d}+\frac{2(3a^2+b^2)\log(a\cot(c+dx)+b)}{b^5d}-\frac{(3a^2-b^2)(a^2+b^2)}{a^2b^4d(a\cot(c+dx)+b)}-\frac{(a^2+b^2)^2}{2a^2b^3d(a\cot(c+dx)+b)^2}-\frac{3a\tan(c+dx)}{b^4d}+\frac{\tan^2(c+dx)}{2b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/2*(a^2 + b^2)^2/(a^2*b^3*d*(b + a*Cot[c + d*x])^2) - ((3*a^2 - b^2)*(a^2
 + b^2))/(a^2*b^4*d*(b + a*Cot[c + d*x])) + (2*(3*a^2 + b^2)*Log[b + a*Cot[
c + d*x]])/(b^5*d) + (2*(3*a^2 + b^2)*Log[Tan[c + d*x]])/(b^5*d) - (3*a*Tan
[c + d*x])/(b^4*d) + Tan[c + d*x]^2/(2*b^3*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^3(b+ax)^3}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{b^3x^3} - \frac{3a}{b^4x^2} + \frac{2(3a^2+b^2)}{b^5x} - \frac{(a^2+b^2)^2}{ab^3(b+ax)^3} + \frac{-3a^4-2a^2b^2+b^4}{ab^4(b+ax)^2} - \frac{2a(}{b^5}\right.\right.}{d}$$

$$= -\frac{(a^2+b^2)^2}{2a^2b^3d(b+a\cot(c+dx))^2} - \frac{(3a^2-b^2)(a^2+b^2)}{a^2b^4d(b+a\cot(c+dx))} + \frac{2(3a^2+}{}$$

Mathematica [A]
time = 2.39, size = 191, normalized size = 1.19

$$-\frac{b^3\sec^4(c+dx)(b\cos(2(c+dx))-a\sin(2(c+dx)))+2(a+b\tan(c+dx))\left(2a(3a^2+b^2)\left(\log(\cos(c+dx))-\log(a\cos(c+dx)+b\sin(c+dx))\right)+b(3(2a^2+b^2)+2(3a^2+b^2)\log(\cos(c+dx))-2(3a^2+b^2)\log(a\cos(c+dx)+b\sin(c+dx)))\tan(c+dx)+3ab^2\tan^2(c+dx)\right)}{2b^5d(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/2*(b^3*Sec[c + d*x]^4*(b*Cos[2*(c + d*x)] - a*Sin[2*(c + d*x)]) + 2*(a +
 b*Tan[c + d*x])*(2*a*(3*a^2 + b^2)*(Log[Cos[c + d*x]] - Log[a*Cos[c + d*x]
 + b*Sin[c + d*x]]) + b*(3*(2*a^2 + b^2) + 2*(3*a^2 + b^2)*Log[Cos[c + d*x]
] - 2*(3*a^2 + b^2)*Log[a*Cos[c + d*x] + b*Sin[c + d*x]])*Tan[c + d*x] + 3*
a*b^2*Tan[c + d*x]^2))/(b^5*d*(a + b*Tan[c + d*x])^2)
```

Maple [A]
time = 0.73, size = 115, normalized size = 0.71

method	result
derivativedivides	$-\frac{\frac{-\frac{b\left(\tan^2(dx+c)\right)}{2}+3a\tan(dx+c)}{b^4}+\frac{\left(6a^2+2b^2\right)\ln(a+b\tan(dx+c))}{b^5}+\frac{4a\left(a^2+b^2\right)}{b^5(a+b\tan(dx+c))}-\frac{a^4+2a^2b^2+b^4}{2b^5(a+b\tan(dx+c))^2}}{d}$
default	$-\frac{\frac{-\frac{b\left(\tan^2(dx+c)\right)}{2}+3a\tan(dx+c)}{b^4}+\frac{\left(6a^2+2b^2\right)\ln(a+b\tan(dx+c))}{b^5}+\frac{4a\left(a^2+b^2\right)}{b^5(a+b\tan(dx+c))}-\frac{a^4+2a^2b^2+b^4}{2b^5(a+b\tan(dx+c))^2}}{d}$
risch	$\frac{12a^2b\,e^{6i(dx+c)}+12ia^3e^{6i(dx+c)}+36ia^3e^{4i(dx+c)}+12ia^3-12iab^2-24a^2b+4b^3e^{2i(dx+c)}-4iab^2e^{2i(dx+c)}+4iab^2e^{6i(dx+c)}}{\left(e^{2i(dx+c)}+1\right)^2\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)^2b^4d}$
norman	$\frac{-\frac{2\left(18a^4+6a^2b^2-b^4\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d\,b^3}-\frac{2\left(18a^4+6a^2b^2-b^4\right)\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d\,b^3}+\frac{2\left(18a^4-2a^2b^2-3b^4\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\,b^4}-\frac{2\left(18a^4-2a^2\right.}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(a\left(\tan\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^4*(-1/2*b*tan(d*x+c)^2+3*a*tan(d*x+c))+(6*a^2+2*b^2)/b^5*ln(a+b*t
an(d*x+c))+4*a/b^5*(a^2+b^2)/(a+b*tan(d*x+c))-1/2/b^5*(a^4+2*a^2*b^2+b^4)/(
a+b*tan(d*x+c))^2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 652 vs. $2(157) = 314$.
time = 0.33, size = 652, normalized size = 4.05

$$-\frac{2\left(\frac{\frac{(6a^5+2a^3b^2-ab^4)\sin(dx+c)}{\cos(dx+c)+1}+\frac{(18a^4b+6a^2b^3-b^5)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{(18a^5-2a^3b^2-3ab^4)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{2(18a^4b+8a^2b^3-b^5)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{(18a^5-2a^3b^2-3ab^4)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{(18a^4b+6a^2b^3-b^5)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{(6a^5+2a^3b^2-ab^4)\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4b^4+\frac{4a^3b^5\sin(dx+c)}{\cos(dx+c)+1}-\frac{12a^3b^5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{12a^3b^5\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{4a^3b^5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{a^4b^4\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{4(a^4b^4-a^2b^6)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2(3a^4b^4-4a^2b^6)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4(a^4b^4-a^2b^6)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{(3a^2+b^2)\log\left(-a-\frac{2b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{b^5}+\frac{(3a^2+b^2)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^5}+\frac{(3a^2+b^2)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^5}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -2*(((6*a^5 + 2*a^3*b^2 - a*b^4)*sin(d*x + c)/(cos(d*x + c) + 1) + (18*a^4*
b + 6*a^2*b^3 - b^5)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - (18*a^5 - 2*a^3*
b^2 - 3*a*b^4)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 2*(18*a^4*b + 8*a^2*b^
3 - b^5)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (18*a^5 - 2*a^3*b^2 - 3*a*b^
4)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + (18*a^4*b + 6*a^2*b^3 - b^5)*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 - (6*a^5 + 2*a^3*b^2 - a*b^4)*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/(a^4*b^4 + 4*a^3*b^5*sin(d*x + c)/(cos(d*x + c) + 1)
 - 12*a^3*b^5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 12*a^3*b^5*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5 - 4*a^3*b^5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + a
^4*b^4*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 4*(a^4*b^4 - a^2*b^6)*sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 + 2*(3*a^4*b^4 - 4*a^2*b^6)*sin(d*x + c)^4/(cos
(d*x + c) + 1)^4 - 4*(a^4*b^4 - a^2*b^6)*sin(d*x + c)^6/(cos(d*x + c) + 1)^
6) - (3*a^2 + b^2)*log(-a - 2*b*sin(d*x + c)/(cos(d*x + c) + 1) + a*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2)/b^5 + (3*a^2 + b^2)*log(sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)/b^5 + (3*a^2 + b^2)*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1
)/b^5)/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(157) = 314$.
time = 2.28, size = 354, normalized size = 2.20

$$\frac{24a^2b^2\cos(dx+c)^4+b^4-2(9a^2b^2+b^4)\cos(dx+c)^2+2\left((3a^4-2a^2b^2-b^4)\cos(dx+c)^4+2(3a^3b+ab^3)\cos(dx+c)^3\sin(dx+c)+(3a^2b^2+b^4)\cos(dx+c)^2\right)\log\left(2ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2\right)-2\left((3a^4-2a^2b^2-b^4)\cos(dx+c)^4+2(3a^3b+ab^3)\cos(dx+c)^3\sin(dx+c)+(3a^2b^2+b^4)\cos(dx+c)^2\right)\log\left(\cos(dx+c)^2\right)-4\left(ab^3\cos(dx+c)+3(a^3b-ab^3)\cos(dx+c)^3\right)\sin(dx+c)}{2\left(2ab^6d\cos(dx+c)^3\sin(dx+c)+b^7d\cos(dx+c)^2+(a^2b^5-b^7)d\cos(dx+c)^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(24*a^2*b^2*cos(d*x + c)^4 + b^4 - 2*(9*a^2*b^2 + b^4)*cos(d*x + c)^2 +
 2*((3*a^4 - 2*a^2*b^2 - b^4)*cos(d*x + c)^4 + 2*(3*a^3*b + a*b^3)*cos(d*x
+ c)^3*sin(d*x + c) + (3*a^2*b^2 + b^4)*cos(d*x + c)^2)*log(2*a*b*cos(d*x +
 c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 2*((3*a^4 - 2*a^2*b^
2 - b^4)*cos(d*x + c)^4 + 2*(3*a^3*b + a*b^3)*cos(d*x + c)^3*sin(d*x + c) +
 (3*a^2*b^2 + b^4)*cos(d*x + c)^2)*log(cos(d*x + c)^2) - 4*(a*b^3*cos(d*x +
```

c) + 3*(a^3*b - a*b^3)*cos(d*x + c)^3)*sin(d*x + c))/(2*a*b^6*d*cos(d*x + c)^3*sin(d*x + c) + b^7*d*cos(d*x + c)^2 + (a^2*b^5 - b^7)*d*cos(d*x + c)^4)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)

[Out] Integral(sec(c + d*x)**3/(a*cos(c + d*x) + b*sin(c + d*x))**3, x)

Giac [A]
time = 0.51, size = 140, normalized size = 0.87

$$\frac{\frac{4(3a^2+b^2)\log(|b\tan(dx+c)+a|)}{b^5}+\frac{b^3\tan(dx+c)^2-6ab^2\tan(dx+c)}{b^6}-\frac{18a^2b^2\tan(dx+c)^2+6b^4\tan(dx+c)^2+28a^3b\tan(dx+c)+4ab^3\tan(dx+c)+11a^4+b^4}{(b\tan(dx+c)+a)^2b^5}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")

[Out] 1/2*(4*(3*a^2 + b^2)*log(abs(b*tan(d*x + c) + a))/b^5 + (b^3*tan(d*x + c)^2 - 6*a*b^2*tan(d*x + c))/b^6 - (18*a^2*b^2*tan(d*x + c)^2 + 6*b^4*tan(d*x + c)^2 + 28*a^3*b*tan(d*x + c) + 4*a*b^3*tan(d*x + c) + 11*a^4 + b^4)/((b*tan(d*x + c) + a)^2*b^5))/d

Mupad [B]
time = 4.74, size = 1204, normalized size = 7.48

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(c + d*x)^3*(a*cos(c + d*x) + b*sin(c + d*x))^3),x)

[Out] - ((2*tan(c/2 + (d*x)/2)*(6*a^4 - b^4 + 2*a^2*b^2))/(a*b^4) - (2*tan(c/2 + (d*x)/2)^7*(6*a^4 - b^4 + 2*a^2*b^2))/(a*b^4) + (2*tan(c/2 + (d*x)/2)^3*(3*b^4 - 18*a^4 + 2*a^2*b^2))/(a*b^4) + (2*tan(c/2 + (d*x)/2)^2*(18*a^4 - b^4 + 6*a^2*b^2))/(a^2*b^3) - (2*tan(c/2 + (d*x)/2)^5*(3*b^4 - 18*a^4 + 2*a^2*b^2))/(a*b^4) - (4*tan(c/2 + (d*x)/2)^4*(18*a^4 - b^4 + 8*a^2*b^2))/(a^2*b^3) + (2*tan(c/2 + (d*x)/2)^6*(18*a^4 - b^4 + 6*a^2*b^2))/(a^2*b^3))/(d*(tan(c/2 + (d*x)/2)^4*(6*a^2 - 8*b^2) - tan(c/2 + (d*x)/2)^6*(4*a^2 - 4*b^2) - tan(c/2 + (d*x)/2)^2*(4*a^2 - 4*b^2) + a^2*tan(c/2 + (d*x)/2)^8 + a^2 - 12*a*b*tan(c/2 + (d*x)/2)^3 + 12*a*b*tan(c/2 + (d*x)/2)^5 - 4*a*b*tan(c/2 + (d*

```
x)/2)^7 + 4*a*b*tan(c/2 + (d*x)/2))) - (atan((((3*a^2 + b^2)*((2*(3*a^2 + b
^2)*((4*(a*b^10 + 4*a^3*b^8))/b^8 - (4*tan(c/2 + (d*x)/2)^2*(3*a*b^10 + 4*a
^3*b^8))/b^8 + 16*a^2*b*tan(c/2 + (d*x)/2)))/b^5 - (4*(4*a*b^7 + 12*a^3*b^5
))/b^8 + (4*tan(c/2 + (d*x)/2)^2*(4*a*b^7 + 12*a^3*b^5))/b^8 + (16*tan(c/2
+ (d*x)/2)*(6*a^4 + 2*a^2*b^2))/b^4)*2i)/b^5 - ((3*a^2 + b^2)*((4*(4*a*b^7
+ 12*a^3*b^5))/b^8 + (2*(3*a^2 + b^2)*((4*(a*b^10 + 4*a^3*b^8))/b^8 - (4*ta
n(c/2 + (d*x)/2)^2*(3*a*b^10 + 4*a^3*b^8))/b^8 + 16*a^2*b*tan(c/2 + (d*x)/2
)))/b^5 - (4*tan(c/2 + (d*x)/2)^2*(4*a*b^7 + 12*a^3*b^5))/b^8 - (16*tan(c/2
 + (d*x)/2)*(6*a^4 + 2*a^2*b^2))/b^4)*2i)/b^5)/((8*(4*a*b^4 + 36*a^5 + 24*a
^3*b^2))/b^8 + (2*(3*a^2 + b^2)*((2*(3*a^2 + b^2)*((4*(a*b^10 + 4*a^3*b^8))
/b^8 - (4*tan(c/2 + (d*x)/2)^2*(3*a*b^10 + 4*a^3*b^8))/b^8 + 16*a^2*b*tan(c
/2 + (d*x)/2)))/b^5 - (4*(4*a*b^7 + 12*a^3*b^5))/b^8 + (4*tan(c/2 + (d*x)/2
)^2*(4*a*b^7 + 12*a^3*b^5))/b^8 + (16*tan(c/2 + (d*x)/2)*(6*a^4 + 2*a^2*b^2
))/b^4))/b^5 + (2*(3*a^2 + b^2)*((4*(4*a*b^7 + 12*a^3*b^5))/b^8 + (2*(3*a^2
 + b^2)*((4*(a*b^10 + 4*a^3*b^8))/b^8 - (4*tan(c/2 + (d*x)/2)^2*(3*a*b^10 +
 4*a^3*b^8))/b^8 + 16*a^2*b*tan(c/2 + (d*x)/2)))/b^5 - (4*tan(c/2 + (d*x)/2
)^2*(4*a*b^7 + 12*a^3*b^5))/b^8 - (16*tan(c/2 + (d*x)/2)*(6*a^4 + 2*a^2*b^2
))/b^4))/b^5 + (8*tan(c/2 + (d*x)/2)^2*(4*a*b^4 + 36*a^5 + 24*a^3*b^2))/b^8
))*(3*a^2 + b^2)*4i)/(b^5*d)
```

3.139 $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=383

$$-\frac{4a^3\tanh^{-1}(\sin(c+dx))}{b^6d}-\frac{3a\tanh^{-1}(\sin(c+dx))}{2b^4d}-\frac{6a(a^2+b^2)\tanh^{-1}(\sin(c+dx))}{b^6d}-\frac{8a^2\sqrt{a^2+b^2}\,\tanh^{-1}}{b}$$

```
[Out] -4*a^3*arctanh(sin(d*x+c))/b^6/d-3/2*a*arctanh(sin(d*x+c))/b^4/d-6*a*(a^2+b
^2)*arctanh(sin(d*x+c))/b^6/d-2*(a^2+b^2)^(3/2)*arctanh((b*cos(d*x+c)-a*sin
(d*x+c))/(a^2+b^2)^(1/2))/b^6/d+4*a^2*sec(d*x+c)/b^5/d+2*(a^2+b^2)*sec(d*x+
c)/b^5/d+1/3*sec(d*x+c)^3/b^3/d-1/2*(a^2+b^2)*(b*cos(d*x+c)-a*sin(d*x+c))/b
^4/d/(a*cos(d*x+c)+b*sin(d*x+c))^2+4*a*(a^2+b^2)/b^5/d/(a*cos(d*x+c)+b*sin(
d*x+c))-8*a^2*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))*(a^2+b^2
)^(1/2)/b^6/d-1/2*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))*(a^2
+b^2)^(1/2)/b^4/d-3/2*a*sec(d*x+c)*tan(d*x+c)/b^4/d
```

Rubi [A]
time = 0.58, antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 31, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3185, 3155, 3153, 212, 3183, 3855, 3173, 3853}

$$-\frac{4a^3\tanh^{-1}(\sin(c+dx))}{b^6d}+\frac{4a^2\sec(c+dx)}{b^5d}-\frac{6a(a^2+b^2)\tanh^{-1}(\sin(c+dx))}{b^6d}-\frac{8a^2\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^6d}-\frac{2(a^2+b^2)^{3/2}\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^6d}+\frac{2(a^2+b^2)\sec(c+dx)}{b^5d}+\frac{4a(a^2+b^2)}{b^5d(a\cos(c+dx)+b\sin(c+dx))}-\frac{(a^2+b^2)(b\cos(c+dx)-a\sin(c+dx))}{2b^4d(a\cos(c+dx)+b\sin(c+dx))^2}-\frac{\sqrt{a^2+b^2}\,\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2b^4d}-\frac{3a\tanh^{-1}(\sin(c+dx))}{2b^4d}-\frac{3a\tan(c+dx)\sec(c+dx)}{2b^4d}+\frac{\sec^3(c+dx)}{3b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (-4*a^3*ArcTanh[Sin[c + d*x]])/(b^6*d) - (3*a*ArcTanh[Sin[c + d*x]])/(2*b^4
*d) - (6*a*(a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(b^6*d) - (8*a^2*Sqrt[a^2 + b
^2]*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^6*d) - (
Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])
/(2*b^4*d) - (2*(a^2 + b^2)^(3/2)*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])
/Sqrt[a^2 + b^2]])/(b^6*d) + (4*a^2*Sec[c + d*x])/(b^5*d) + (2*(a^2 + b^2)*
Sec[c + d*x])/(b^5*d) + Sec[c + d*x]^3/(3*b^3*d) - ((a^2 + b^2)*(b*Cos[c +
d*x] - a*Sin[c + d*x]))/(2*b^4*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) + (4*
a*(a^2 + b^2))/(b^5*d*(a*Cos[c + d*x] + b*Sin[c + d*x])) - (3*a*Sec[c + d*x
]*Tan[c + d*x])/(2*b^4*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3155

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3173

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/co
s[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^
(n + 1)/(b*d*(n + 1)), x] + (Dist[1/b^2, Int[(a*Cos[c + d*x] + b*Sin[c + d*
x])^(n + 2)/Cos[c + d*x], x], x] - Dist[a/b^2, Int[(a*Cos[c + d*x] + b*Sin[
c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &
& LtQ[n, -1]
```

Rule 3183

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3185

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2 + b^2)/b^2, Int[Cos[c +
d*x]^(m + 2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^n, x], x] + (Dist[1/b^2, Int
[Cos[c + d*x]^m*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] - Dist[2*
(a/b^2), Int[Cos[c + d*x]^(m + 1)*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 1)
, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && L
tQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = \frac{\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+b\sin(c+dx)}\,dx}{b^2} - \frac{(2a)\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx}{b^2} + \frac{(a^2+b \ldots}{\ldots}$$

$$= \frac{\sec^3(c+dx)}{3b^3d} - \frac{a\int \sec^3(c+dx)\,dx}{b^4} - \frac{(2a)\int \sec^3(c+dx)\,dx}{b^4} + \frac{(4a^2 \ldots}{\ldots}$$

$$= \frac{4a^2\sec(c+dx)}{b^5d} + \frac{\sec^3(c+dx)}{3b^3d} - \frac{(a^2+b^2)\,(b\cos(c+dx)-a\sin(c \ldots}{2b^4d(a\cos(c+dx)+b\sin(c+d \ldots}$$

$$= -\frac{4a^3\tanh^{-1}(\sin(c+dx))}{b^6d} - \frac{3a\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{4a^2\sec(c+ \ldots}{b^5d}$$

$$= -\frac{4a^3\tanh^{-1}(\sin(c+dx))}{b^6d} - \frac{3a\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{4a^2\sqrt{a^2+b^2 \ldots}}{\ldots}$$

Mathematica [**C**] Result contains complex when optimal does not.
time = 2.58, size = 688, normalized size = 1.80

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])*((6*b^2*(a^2 + b^2)^2*Sin
[c + d*x])/a + (6*(a - I*b)*(a + I*b)*b*(8*a^2 - b^2)*(a*Cos[c + d*x] + b*S
in[c + d*x]))/a + 2*b*(36*a^2 + 13*b^2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2
 + 60*Sqrt[a^2 + b^2]*(4*a^2 + b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[
a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 + 30*a*(4*a^2 + 3*b^2)*Log[
Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 -
30*a*(4*a^2 + 3*b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a*Cos[c + d*
x] + b*Sin[c + d*x])^2 + (b^2*(-9*a + b)*(a*Cos[c + d*x] + b*Sin[c + d*x])^
2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (2*b^3*Sin[(c + d*x)/2]*(a*Cos
[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3 + (2
*b*(36*a^2 + 13*b^2)*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (2*b^3*Sin[(c + d*x)/2]*(a*Cos[c +
```

```
d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3 + (b^2*(9
*a + b)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^2 - (2*b*(36*a^2 + 13*b^2)*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin
[c + d*x])^2)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(12*b^6*d*(a + b*Tan[
c + d*x])^3)
```

Maple [A]
time = 1.06, size = 444, normalized size = 1.16 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3/b^3/(tan(1/2*d*x+1/2*c)-1)^3-1/2*(3*a+b)/b^4/(tan(1/2*d*x+1/2*c)-
1)^2-1/2*(12*a^2+3*a*b+5*b^2)/b^5/(tan(1/2*d*x+1/2*c)-1)+5/2*a*(4*a^2+3*b^2
)/b^6*ln(tan(1/2*d*x+1/2*c)-1)+1/3/b^3/(tan(1/2*d*x+1/2*c)+1)^3-1/2*(-3*a+b
)/b^4/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(-12*a^2+3*a*b-5*b^2)/b^5/(tan(1/2*d*x+1
/2*c)+1)-5/2*a*(4*a^2+3*b^2)/b^6*ln(tan(1/2*d*x+1/2*c)+1)-2/b^6*((1/2*b^2*(
7*a^4+5*a^2*b^2-2*b^4)/a*tan(1/2*d*x+1/2*c)^3+1/2*b*(8*a^6-9*a^4*b^2-15*a^2
*b^4+2*b^6)/a^2*tan(1/2*d*x+1/2*c)^2-1/2*b^2*(25*a^4+23*a^2*b^2-2*b^4)/a*ta
n(1/2*d*x+1/2*c)-4*b*a^4-7/2*b^3*a^2+1/2*b^5)/(a*tan(1/2*d*x+1/2*c)^2-2*b*t
an(1/2*d*x+1/2*c)-a)^2-5/2*(4*a^4+5*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/
2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 902 vs. 2(361) = 722.
time = 0.51, size = 902, normalized size = 2.36

$$\frac{\frac{2\left(60a^6+35a^4b^2-3a^2b^4+\frac{(210a^5b+125a^3b^3-6ab^5)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2(120a^6-10a^4b^2-55a^2b^4+3b^6)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2(330a^5b+205a^3b^3-12ab^5)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2(180a^6-95a^4b^2-120a^2b^4+9b^6)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{12(60a^5b+35a^3b^3-3ab^5)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{6(40a^6-30a^4b^2-35a^2b^4+3b^6)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{6(50a^5b+25a^3b^3-4ab^5)\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{3(20a^6-15a^4b^2-15a^2b^4+2b^6)\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{3(10a^5b+5a^3b^3-2ab^5)\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right)}{a^4b^5+\frac{4a^3b^6\sin(dx+c)}{\cos(dx+c)+1}-\frac{16a^3b^6\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{24a^3b^6\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{16a^3b^6\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{4a^3b^6\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{a^4b^5\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{(5a^4b^5-4a^2b^7)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{2(5a^4b^5-6a^2b^7)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{2(5a^4b^5-6a^2b^7)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{(5a^4b^5-4a^2b^7)\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{15(4a^3+3ab^2)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^6}+\frac{15(4a^3+3ab^2)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^6}-\frac{15(4a^4+5a^2b^2+b^4)\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*(60*a^6 + 35*a^4*b^2 - 3*a^2*b^4 + (210*a^5*b + 125*a^3*b^3 - 6*a*b^
5)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*(120*a^6 - 10*a^4*b^2 - 55*a^2*b^4 +
 3*b^6)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(330*a^5*b + 205*a^3*b^3 -
12*a*b^5)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*(180*a^6 - 95*a^4*b^2 - 1
20*a^2*b^4 + 9*b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 12*(60*a^5*b + 35
*a^3*b^3 - 3*a*b^5)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 6*(40*a^6 - 30*a^
4*b^2 - 35*a^2*b^4 + 3*b^6)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 6*(50*a^5
*b + 25*a^3*b^3 - 4*a*b^5)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3*(20*a^6
- 15*a^4*b^2 - 15*a^2*b^4 + 2*b^6)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 3*
(10*a^5*b + 5*a^3*b^3 - 2*a*b^5)*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/(a^4*
b^5 + 4*a^3*b^6*sin(d*x + c)/(cos(d*x + c) + 1) - 16*a^3*b^6*sin(d*x + c)^3
/(cos(d*x + c) + 1)^3 + 24*a^3*b^6*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 16
*a^3*b^6*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 4*a^3*b^6*sin(d*x + c)^9/(co
s(d*x + c) + 1)^9 - a^4*b^5*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 - (5*a^4*
```

```
b^5 - 4*a^2*b^7)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 2*(5*a^4*b^5 - 6*a^2
*b^7)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 2*(5*a^4*b^5 - 6*a^2*b^7)*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 + (5*a^4*b^5 - 4*a^2*b^7)*sin(d*x + c)^8/(co
s(d*x + c) + 1)^8) - 15*(4*a^3 + 3*a*b^2)*log(sin(d*x + c)/(cos(d*x + c) +
1) + 1)/b^6 + 15*(4*a^3 + 3*a*b^2)*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)
/b^6 - 15*(4*a^4 + 5*a^2*b^2 + b^4)*log((b - a*sin(d*x + c)/(cos(d*x + c) +
 1) + sqrt(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 +
b^2)))/(sqrt(a^2 + b^2)*b^6))/d
```

Fricas [A]
time = 2.49, size = 564, normalized size = 1.47

$$\frac{4\,b^5+30\,(4\,a^4b+a^2b^3-b^5)\cos(dx+c)^4+20\,(2\,a^2b^3+b^5)\cos(dx+c)^2+15\,\left((4\,a^4-3\,a^2b^2-b^4)\cos(dx+c)^5+2\,(4\,a^3b+ab^3)\cos(dx+c)^4\sin(dx+c)+(4\,a^2b^2+b^4)\cos(dx+c)^3\right)\sqrt{a^2+b^2}\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)-15\,\left((4\,a^5-a^3b^2-3\,ab^4)\cos(dx+c)^5+2\,(4\,a^4b+3\,a^2b^3)\cos(dx+c)^4\sin(dx+c)+(4\,a^3b^2+3\,ab^4)\cos(dx+c)^3\right)\log(\sin(dx+c)+1)+15\,\left((4\,a^5-a^3b^2-3\,ab^4)\cos(dx+c)^5+2\,(4\,a^4b+3\,a^2b^3)\cos(dx+c)^4\sin(dx+c)+(4\,a^3b^2+3\,ab^4)\cos(dx+c)^3\right)\log(-\sin(dx+c)+1)-10\,\left(ab^4\cos(dx+c)-6\,(3\,a^3b^2+2\,ab^4)\cos(dx+c)^3\right)\sin(dx+c)}{12\,\left(2\,ab^7d\cos(dx+c)^4\sin(dx+c)+b^8d\cos(dx+c)^3+(a^2b^6-b^8)d\cos(dx+c)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*b^5 + 30*(4*a^4*b + a^2*b^3 - b^5)*cos(d*x + c)^4 + 20*(2*a^2*b^3 +
 b^5)*cos(d*x + c)^2 + 15*((4*a^4 - 3*a^2*b^2 - b^4)*cos(d*x + c)^5 + 2*(4*
a^3*b + a*b^3)*cos(d*x + c)^4*sin(d*x + c) + (4*a^2*b^2 + b^4)*cos(d*x + c)
^3)*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos
(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*sin(d*x +
 c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2))
 - 15*((4*a^5 - a^3*b^2 - 3*a*b^4)*cos(d*x + c)^5 + 2*(4*a^4*b + 3*a^2*b^3)
*cos(d*x + c)^4*sin(d*x + c) + (4*a^3*b^2 + 3*a*b^4)*cos(d*x + c)^3)*log(si
n(d*x + c) + 1) + 15*((4*a^5 - a^3*b^2 - 3*a*b^4)*cos(d*x + c)^5 + 2*(4*a^4
*b + 3*a^2*b^3)*cos(d*x + c)^4*sin(d*x + c) + (4*a^3*b^2 + 3*a*b^4)*cos(d*x
 + c)^3)*log(-sin(d*x + c) + 1) - 10*(a*b^4*cos(d*x + c) - 6*(3*a^3*b^2 + 2
*a*b^4)*cos(d*x + c)^3)*sin(d*x + c))/(2*a*b^7*d*cos(d*x + c)^4*sin(d*x + c
) + b^8*d*cos(d*x + c)^3 + (a^2*b^6 - b^8)*d*cos(d*x + c)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a*cos(c + d*x) + b*sin(c + d*x))**3, x)
```

Giac [A]
time = 0.59, size = 510, normalized size = 1.33

$$-\frac{\frac{15\,(4\,a^3+3\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^6}-\frac{15\,(4\,a^3+3\,ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^6}+\frac{15\,(4\,a^4+5\,a^2b^2+b^4)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^6}+\frac{2\left(9\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+36\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+18\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-72\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-24\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+36\,a^2+14\,b^2\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3b^5}+\frac{6\left(7\,a^5b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,a^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,ab^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+8\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-9\,a^4b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,a^2b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+2\,b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-25\,a^5b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-23\,a^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,ab^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-8\,a^6-7\,a^4b^2+a^2b^4\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)^2a^2b^5}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(15*(4*a^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^6 - 15*(4*a
^3 + 3*a*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^6 + 15*(4*a^4 + 5*a^2*b^
2 + b^4)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*
a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^6) +
2*(9*a*b*tan(1/2*d*x + 1/2*c)^5 + 36*a^2*tan(1/2*d*x + 1/2*c)^4 + 18*b^2*ta
n(1/2*d*x + 1/2*c)^4 - 72*a^2*tan(1/2*d*x + 1/2*c)^2 - 24*b^2*tan(1/2*d*x +
 1/2*c)^2 - 9*a*b*tan(1/2*d*x + 1/2*c) + 36*a^2 + 14*b^2)/((tan(1/2*d*x + 1
/2*c)^2 - 1)^3*b^5) + 6*(7*a^5*b*tan(1/2*d*x + 1/2*c)^3 + 5*a^3*b^3*tan(1/2
*d*x + 1/2*c)^3 - 2*a*b^5*tan(1/2*d*x + 1/2*c)^3 + 8*a^6*tan(1/2*d*x + 1/2*
c)^2 - 9*a^4*b^2*tan(1/2*d*x + 1/2*c)^2 - 15*a^2*b^4*tan(1/2*d*x + 1/2*c)^2
 + 2*b^6*tan(1/2*d*x + 1/2*c)^2 - 25*a^5*b*tan(1/2*d*x + 1/2*c) - 23*a^3*b^
3*tan(1/2*d*x + 1/2*c) + 2*a*b^5*tan(1/2*d*x + 1/2*c) - 8*a^6 - 7*a^4*b^2 +
 a^2*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^2*a^2*
b^5))/d
```

Mupad [B]
time = 3.82, size = 1203, normalized size = 3.14

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a*cos(c + d*x) + b*sin(c + d*x))^3),x)
```

```
[Out] ((60*a^4 - 3*b^4 + 35*a^2*b^2)/(3*b^5) + (tan(c/2 + (d*x)/2)*(210*a^4 - 6*b
^4 + 125*a^2*b^2))/(3*a*b^4) + (tan(c/2 + (d*x)/2)^8*(20*a^6 + 2*b^6 - 15*a
^2*b^4 - 15*a^4*b^2))/(a^2*b^5) - (2*tan(c/2 + (d*x)/2)^6*(40*a^6 + 3*b^6 -
 35*a^2*b^4 - 30*a^4*b^2))/(a^2*b^5) - (2*tan(c/2 + (d*x)/2)^2*(120*a^6 + 3
*b^6 - 55*a^2*b^4 - 10*a^4*b^2))/(3*a^2*b^5) + (2*tan(c/2 + (d*x)/2)^4*(180
*a^6 + 9*b^6 - 120*a^2*b^4 - 95*a^4*b^2))/(3*a^2*b^5) + (tan(c/2 + (d*x)/2)
^9*(10*a^4 - 2*b^4 + 5*a^2*b^2))/(a*b^4) - (2*tan(c/2 + (d*x)/2)^7*(50*a^4
- 4*b^4 + 25*a^2*b^2))/(a*b^4) + (4*tan(c/2 + (d*x)/2)^5*(60*a^4 - 3*b^4 +
35*a^2*b^2))/(a*b^4) - (2*tan(c/2 + (d*x)/2)^3*(330*a^4 - 12*b^4 + 205*a^2*
b^2))/(3*a*b^4))/(d*(tan(c/2 + (d*x)/2)^8*(5*a^2 - 4*b^2) - tan(c/2 + (d*x)
/2)^2*(5*a^2 - 4*b^2) + tan(c/2 + (d*x)/2)^4*(10*a^2 - 12*b^2) - tan(c/2 +
(d*x)/2)^6*(10*a^2 - 12*b^2) - a^2*tan(c/2 + (d*x)/2)^10 + a^2 - 16*a*b*tan
(c/2 + (d*x)/2)^3 + 24*a*b*tan(c/2 + (d*x)/2)^5 - 16*a*b*tan(c/2 + (d*x)/2)
^7 + 4*a*b*tan(c/2 + (d*x)/2)^9 + 4*a*b*tan(c/2 + (d*x)/2))) - (atanh((3000
*a^2*tan(c/2 + (d*x)/2))/(3000*a^2 + (7000*a^4)/b^2 + (4000*a^6)/b^4) + (70
00*a^4*tan(c/2 + (d*x)/2))/(7000*a^4 + 3000*a^2*b^2 + (4000*a^6)/b^2) + (40
00*a^6*tan(c/2 + (d*x)/2))/(4000*a^6 + 3000*a^2*b^4 + 7000*a^4*b^2))*(15*a*
b^2 + 20*a^3))/(b^6*d) + (5*atanh((1000*a^2*(a^2 + b^2)^(1/2))/(1000*a^2*b
+ (5000*a^4)/b + (4000*a^6)/b^3 + 10000*a^3*tan(c/2 + (d*x)/2) + 2000*a*b^2
```

```
*tan(c/2 + (d*x)/2) + (8000*a^5*tan(c/2 + (d*x)/2))/b^2) + (4000*a^4*(a^2 +
 b^2)^(1/2))/(5000*a^4*b + 1000*a^2*b^3 + (4000*a^6)/b + 8000*a^5*tan(c/2 +
 (d*x)/2) + 2000*a*b^4*tan(c/2 + (d*x)/2) + 10000*a^3*b^2*tan(c/2 + (d*x)/2
)) + (9000*a^3*tan(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2))/(5000*a^4 + 1000*a^2*b
^2 + (4000*a^6)/b^2 + 2000*a*b^3*tan(c/2 + (d*x)/2) + 10000*a^3*b*tan(c/2 +
 (d*x)/2) + (8000*a^5*tan(c/2 + (d*x)/2))/b) + (4000*a^5*tan(c/2 + (d*x)/2)
*(a^2 + b^2)^(1/2))/(4000*a^6 + 1000*a^2*b^4 + 5000*a^4*b^2 + 2000*a*b^5*ta
n(c/2 + (d*x)/2) + 8000*a^5*b*tan(c/2 + (d*x)/2) + 10000*a^3*b^3*tan(c/2 +
(d*x)/2)) + (2000*a*tan(c/2 + (d*x)/2)*(a^2 + b^2)^(1/2))/(1000*a^2 + (5000
*a^4)/b^2 + (4000*a^6)/b^4 + (10000*a^3*tan(c/2 + (d*x)/2))/b + (8000*a^5*t
an(c/2 + (d*x)/2))/b^3 + 2000*a*b*tan(c/2 + (d*x)/2)))*(4*a^2 + b^2)*(a^2 +
 b^2)^(1/2))/(b^6*d)
```

3.140 $\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$

Optimal. Leaf size=232

$$-\frac{(a^2+b^2)^3}{2a^2b^5d(b+a\cot(c+dx))^2}-\frac{(5a^2-b^2)(a^2+b^2)^2}{a^2b^6d(b+a\cot(c+dx))}+\frac{3(a^2+b^2)(5a^2+b^2)\log(b+a\cot(c+dx))}{b^7d}+\frac{3(a^2$$

```
[Out] -1/2*(a^2+b^2)^3/a^2/b^5/d/(b+a*cot(d*x+c))^2-(5*a^2-b^2)*(a^2+b^2)^2/a^2/b
^6/d/(b+a*cot(d*x+c))+3*(a^2+b^2)*(5*a^2+b^2)*ln(b+a*cot(d*x+c))/b^7/d+3*(a
^2+b^2)*(5*a^2+b^2)*ln(tan(d*x+c))/b^7/d-a*(10*a^2+9*b^2)*tan(d*x+c)/b^6/d+
3/2*(2*a^2+b^2)*tan(d*x+c)^2/b^5/d-a*tan(d*x+c)^3/b^4/d+1/4*tan(d*x+c)^4/b^
3/d
```

Rubi [A]
time = 0.17, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3167, 908}

$$\frac{3(a^2+b^2)(5a^2+b^2)\log(\tan(c+dx))}{b^7d}+\frac{3(a^2+b^2)(5a^2+b^2)\log(a\cot(c+dx)+b)}{b^7d}-\frac{a(10a^2+9b^2)\tan(c+dx)}{b^6d}-\frac{(5a^2-b^2)(a^2+b^2)^2}{a^2b^6d(a\cot(c+dx)+b)}+\frac{3(2a^2+b^2)\tan^2(c+dx)}{2b^5d}-\frac{(a^2+b^2)^3}{2a^2b^5d(a\cot(c+dx)+b)^2}-\frac{a\tan^3(c+dx)}{b^4d}+\frac{\tan^4(c+dx)}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] -1/2*(a^2 + b^2)^3/(a^2*b^5*d*(b + a*Cot[c + d*x])^2) - ((5*a^2 - b^2)*(a^2
 + b^2)^2)/(a^2*b^6*d*(b + a*Cot[c + d*x])) + (3*(a^2 + b^2)*(5*a^2 + b^2)*
Log[b + a*Cot[c + d*x]])/(b^7*d) + (3*(a^2 + b^2)*(5*a^2 + b^2)*Log[Tan[c +
 d*x]])/(b^7*d) - (a*(10*a^2 + 9*b^2)*Tan[c + d*x])/(b^6*d) + (3*(2*a^2 + b
^2)*Tan[c + d*x]^2)/(2*b^5*d) - (a*Tan[c + d*x]^3)/(b^4*d) + Tan[c + d*x]^4
/(4*b^3*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^5(b+ax)^3}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{b^3x^5} - \frac{3a}{b^4x^4} + \frac{3(2a^2+b^2)}{b^5x^3} + \frac{-10a^3-9ab^2}{b^6x^2} + \frac{3(5a^4+6a^2b^2+b^4)}{b^7x} - \cdots\right.\right.}{d}$$

$$= -\frac{(a^2+b^2)^3}{2a^2b^5d(b+a\cot(c+dx))^2} - \frac{(5a^2-b^2)(a^2+b^2)^2}{a^2b^6d(b+a\cot(c+dx))} + \frac{3(a^2+b\cdots}{\cdots}$$

Mathematica [**A**]
time = 5.29, size = 302, normalized size = 1.30

$$\frac{b^4\sec^4(c+dx)(a+b\tan(c+dx))^2-4(a+b\tan(c+dx))\left(3a(5a^4+6a^2b^2+b^4)\left(\log(\cos(c+dx))-\log(a\cos(c+dx)+b\sin(c+dx))\right)+b\left(15a^4+18a^2b^2+5b^4+3(5a^4+6a^2b^2+b^4)\log(\cos(c+dx))-3(5a^4+6a^2b^2+b^4)\log(a\cos(c+dx)+b\sin(c+dx))\right)\tan(c+dx)+2ab^2(5a^2+4b^2)\tan^2(c+dx)\right)+2b^2\sec^2(c+dx)\left(5a^4-b^4+2ab(5a^2+2b^2)\tan(c+dx)+2b^2(a^2+b^2)\tan^2(c+dx)-2ab^3\tan^3(c+dx)\right)}{4b^7d(a+b\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a*Cos[c + d*x] + b*Sin[c + d*x])^3,x]
```

```
[Out] (b^4*Sec[c + d*x]^4*(a + b*Tan[c + d*x])^2 - 4*(a + b*Tan[c + d*x])*(3*a*(5
*a^4 + 6*a^2*b^2 + b^4)*(Log[Cos[c + d*x]] - Log[a*Cos[c + d*x] + b*Sin[c +
 d*x]]) + b*(15*a^4 + 18*a^2*b^2 + 5*b^4 + 3*(5*a^4 + 6*a^2*b^2 + b^4)*Log[
Cos[c + d*x]] - 3*(5*a^4 + 6*a^2*b^2 + b^4)*Log[a*Cos[c + d*x] + b*Sin[c +
d*x]])*Tan[c + d*x] + 2*a*b^2*(5*a^2 + 4*b^2)*Tan[c + d*x]^2) + 2*b^2*Sec[c
 + d*x]^2*(5*a^4 - b^4 + 2*a*b*(5*a^2 + 2*b^2)*Tan[c + d*x] + 2*b^2*(a^2 +
b^2)*Tan[c + d*x]^2 - 2*a*b^3*Tan[c + d*x]^3))/(4*b^7*d*(a + b*Tan[c + d*x]
)^2)
```

Maple [**A**]
time = 0.95, size = 195, normalized size = 0.84 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^6*(-1/4*tan(d*x+c)^4*b^3+a*tan(d*x+c)^3*b^2-3*a^2*b*tan(d*x+c)^2-
3/2*b^3*tan(d*x+c)^2+10*a^3*tan(d*x+c)+9*a*b^2*tan(d*x+c))+(15*a^4+18*a^2*b
^2+3*b^4)/b^7*ln(a+b*tan(d*x+c))-1/2/b^7*(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(a+b
*tan(d*x+c))^2+6*a/b^7*(a^4+2*a^2*b^2+b^4)/(a+b*tan(d*x+c)))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1053 vs. 2(226) = 452.
time = 0.31, size = 1053, normalized size = 4.54

$$-\frac{\frac{2\left(\frac{(15a^7+18a^5b^2+3a^3b^4-ab^6)\sin(dx+c)}{\cos(dx+c)+1}+\frac{(45a^6b+54a^4b^3+9a^2b^5-b^7)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{(75a^7+70a^5b^2-9a^3b^4-5ab^6)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{2(90a^6b+113a^4b^3+24a^2b^5-2b^7)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{2(75a^7+60a^5b^2-17a^3b^4-5ab^6)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{2(135a^6b+172a^4b^3+35a^2b^5-3b^7)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{2(75a^7+60a^5b^2-17a^3b^4-5ab^6)\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{2(90a^6b+113a^4b^3+24a^2b^5-2b^7)\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{(75a^7+70a^5b^2-9a^3b^4-5ab^6)\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{(45a^6b+54a^4b^3+9a^2b^5-b^7)\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{(15a^7+18a^5b^2+3a^3b^4-ab^6)\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)}{a^4b^6+\frac{4a^3b^7\sin(dx+c)}{\cos(dx+c)+1}-\frac{20a^3b^7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{40a^3b^7\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{40a^3b^7\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{20a^3b^7\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{4a^3b^7\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{a^4b^6\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}-\frac{2(3a^4b^6-2a^2b^8)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{(15a^4b^6-16a^2b^8)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4(5a^4b^6-6a^2b^8)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{(15a^4b^6-16a^2b^8)\sin(dx+c)^8}{(\cos(dx+c)+1)^8}-\frac{2(3a^4b^6-2a^2b^8)\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{3(5a^4+6a^2b^2+b^4)\log\left(-a-\frac{2b\sin(dx+c)}{\cos(dx+c)+1}+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{b^7}+\frac{3(5a^4+6a^2b^2+b^4)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^7}+\frac{3(5a^4+6a^2b^2+b^4)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^7}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -(2*((15*a^7 + 18*a^5*b^2 + 3*a^3*b^4 - a*b^6)*sin(d*x + c)/(cos(d*x + c) +
 1) + (45*a^6*b + 54*a^4*b^3 + 9*a^2*b^5 - b^7)*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - (75*a^7 + 70*a^5*b^2 - 9*a^3*b^4 - 5*a*b^6)*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 - 2*(90*a^6*b + 113*a^4*b^3 + 24*a^2*b^5 - 2*b^7)*sin(d*x +
 c)^4/(cos(d*x + c) + 1)^4 + 2*(75*a^7 + 60*a^5*b^2 - 17*a^3*b^4 - 5*a*b^6)
*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2*(135*a^6*b + 172*a^4*b^3 + 35*a^2*
b^5 - 3*b^7)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 2*(75*a^7 + 60*a^5*b^2 -
 17*a^3*b^4 - 5*a*b^6)*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 2*(90*a^6*b +
113*a^4*b^3 + 24*a^2*b^5 - 2*b^7)*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + (75
*a^7 + 70*a^5*b^2 - 9*a^3*b^4 - 5*a*b^6)*sin(d*x + c)^9/(cos(d*x + c) + 1)^
9 + (45*a^6*b + 54*a^4*b^3 + 9*a^2*b^5 - b^7)*sin(d*x + c)^10/(cos(d*x + c)
 + 1)^10 - (15*a^7 + 18*a^5*b^2 + 3*a^3*b^4 - a*b^6)*sin(d*x + c)^11/(cos(d
*x + c) + 1)^11)/(a^4*b^6 + 4*a^3*b^7*sin(d*x + c)/(cos(d*x + c) + 1) - 20*
a^3*b^7*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 40*a^3*b^7*sin(d*x + c)^5/(co
s(d*x + c) + 1)^5 - 40*a^3*b^7*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 20*a^3
*b^7*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 4*a^3*b^7*sin(d*x + c)^11/(cos(d
*x + c) + 1)^11 + a^4*b^6*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 - 2*(3*a^4*
b^6 - 2*a^2*b^8)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + (15*a^4*b^6 - 16*a^2
*b^8)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*(5*a^4*b^6 - 6*a^2*b^8)*sin(d
*x + c)^6/(cos(d*x + c) + 1)^6 + (15*a^4*b^6 - 16*a^2*b^8)*sin(d*x + c)^8/(
cos(d*x + c) + 1)^8 - 2*(3*a^4*b^6 - 2*a^2*b^8)*sin(d*x + c)^10/(cos(d*x +
c) + 1)^10) - 3*(5*a^4 + 6*a^2*b^2 + b^4)*log(-a - 2*b*sin(d*x + c)/(cos(d*
x + c) + 1) + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/b^7 + 3*(5*a^4 + 6*a^2
*b^2 + b^4)*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^7 + 3*(5*a^4 + 6*a^2
*b^2 + b^4)*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/b^7)/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(226) = 452$.
time = 2.10, size = 476, normalized size = 2.05

$$\frac{8\left(15a^4b^2+13a^2b^4\right)\cos\left(dx+c\right)^6+b^6-2\left(45a^4b^2+44a^2b^4+3b^6\right)\cos\left(dx+c\right)^4+\left(5a^2b^4+3b^6\right)\cos\left(dx+c\right)^2+6\left(\left(5a^6+a^4b^2-5a^2b^4-b^6\right)\cos\left(dx+c\right)^6+2\left(5a^5b+6a^3b^3+ab^5\right)\cos\left(dx+c\right)^5\sin\left(dx+c\right)+\left(5a^4b^2+6a^2b^4+b^6\right)\cos\left(dx+c\right)^4\right)\log\left(2ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)-6\left(\left(5a^6+a^4b^2-5a^2b^4-b^6\right)\cos\left(dx+c\right)^6+2\left(5a^5b+6a^3b^3+ab^5\right)\cos\left(dx+c\right)^5\sin\left(dx+c\right)+\left(5a^4b^2+6a^2b^4+b^6\right)\cos\left(dx+c\right)^4\right)\log\left(\cos\left(dx+c\right)^2\right)-2\left(ab^5\cos\left(dx+c\right)+2\left(15a^5b-2a^3b^3-13ab^5\right)\cos\left(dx+c\right)^5+10\left(a^3b^3+ab^5\right)\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)}{4\left(2ab^8d\cos\left(dx+c\right)^5\sin\left(dx+c\right)+b^9d\cos\left(dx+c\right)^4+\left(a^2b^7-b^9\right)d\cos\left(dx+c\right)^6\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(8*(15*a^4*b^2 + 13*a^2*b^4)*cos(d*x + c)^6 + b^6 - 2*(45*a^4*b^2 + 44*
a^2*b^4 + 3*b^6)*cos(d*x + c)^4 + (5*a^2*b^4 + 3*b^6)*cos(d*x + c)^2 + 6*((
5*a^6 + a^4*b^2 - 5*a^2*b^4 - b^6)*cos(d*x + c)^6 + 2*(5*a^5*b + 6*a^3*b^3
+ a*b^5)*cos(d*x + c)^5*sin(d*x + c) + (5*a^4*b^2 + 6*a^2*b^4 + b^6)*cos(d*
x + c)^4)*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2
+ b^2) - 6*((5*a^6 + a^4*b^2 - 5*a^2*b^4 - b^6)*cos(d*x + c)^6 + 2*(5*a^5*b
 + 6*a^3*b^3 + a*b^5)*cos(d*x + c)^5*sin(d*x + c) + (5*a^4*b^2 + 6*a^2*b^4
```

```
+ b^6)*cos(d*x + c)^4)*log(cos(d*x + c)^2) - 2*(a*b^5*cos(d*x + c) + 2*(15*
a^5*b - 2*a^3*b^3 - 13*a*b^5)*cos(d*x + c)^5 + 10*(a^3*b^3 + a*b^5)*cos(d*x
 + c)^3)*sin(d*x + c))/(2*a*b^8*d*cos(d*x + c)^5*sin(d*x + c) + b^9*d*cos(d
*x + c)^4 + (a^2*b^7 - b^9)*d*cos(d*x + c)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a*cos(d*x+c)+b*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**5/(a*cos(c + d*x) + b*sin(c + d*x))**3, x)
```

Giac [A]
time = 0.53, size = 243, normalized size = 1.05

$$\frac{\frac{12\,(5\,a^4+6\,a^2b^2+b^4)\log(|b\tan(dx+c)+a|)}{b^7}-\frac{2\left(45\,a^4b^2\tan(dx+c)^2+54\,a^2b^4\tan(dx+c)^2+9\,b^6\tan(dx+c)^2+78\,a^5b\tan(dx+c)+84\,a^3b^3\tan(dx+c)+6\,ab^5\tan(dx+c)+34\,a^6+33\,a^4b^2+b^6\right)}{(b\tan(dx+c)+a)^2b^7}+\frac{b^9\tan(dx+c)^4-4\,ab^8\tan(dx+c)^3+12\,a^2b^7\tan(dx+c)^2+6\,b^9\tan(dx+c)^2-40\,a^3b^6\tan(dx+c)-36\,ab^8\tan(dx+c)}{b^{12}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+b*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/4*(12*(5*a^4 + 6*a^2*b^2 + b^4)*log(abs(b*tan(d*x + c) + a))/b^7 - 2*(45*
a^4*b^2*tan(d*x + c)^2 + 54*a^2*b^4*tan(d*x + c)^2 + 9*b^6*tan(d*x + c)^2 +
 78*a^5*b*tan(d*x + c) + 84*a^3*b^3*tan(d*x + c) + 6*a*b^5*tan(d*x + c) + 3
4*a^6 + 33*a^4*b^2 + b^6)/((b*tan(d*x + c) + a)^2*b^7) + (b^9*tan(d*x + c)^
4 - 4*a*b^8*tan(d*x + c)^3 + 12*a^2*b^7*tan(d*x + c)^2 + 6*b^9*tan(d*x + c)
^2 - 40*a^3*b^6*tan(d*x + c) - 36*a*b^8*tan(d*x + c))/b^12)/d
```

Mupad [B]
time = 7.64, size = 1712, normalized size = 7.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a*cos(c + d*x) + b*sin(c + d*x))^3),x)
```

```
[Out] - ((2*tan(c/2 + (d*x)/2)*(15*a^6 - b^6 + 3*a^2*b^4 + 18*a^4*b^2))/(a*b^6) -
 (2*tan(c/2 + (d*x)/2)^11*(15*a^6 - b^6 + 3*a^2*b^4 + 18*a^4*b^2))/(a*b^6)
+ (2*tan(c/2 + (d*x)/2)^2*(45*a^6 - b^6 + 9*a^2*b^4 + 54*a^4*b^2))/(a^2*b^5
) + (2*tan(c/2 + (d*x)/2)^10*(45*a^6 - b^6 + 9*a^2*b^4 + 54*a^4*b^2))/(a^2*
b^5) - (2*tan(c/2 + (d*x)/2)^3*(75*a^6 - 5*b^6 - 9*a^2*b^4 + 70*a^4*b^2))/(
a*b^6) + (4*tan(c/2 + (d*x)/2)^5*(75*a^6 - 5*b^6 - 17*a^2*b^4 + 60*a^4*b^2)
)/(a*b^6) - (4*tan(c/2 + (d*x)/2)^7*(75*a^6 - 5*b^6 - 17*a^2*b^4 + 60*a^4*b
```

```
^2))/(a*b^6) + (2*tan(c/2 + (d*x)/2)^9*(75*a^6 - 5*b^6 - 9*a^2*b^4 + 70*a^4
*b^2))/(a*b^6) - (4*tan(c/2 + (d*x)/2)^4*(90*a^6 - 2*b^6 + 24*a^2*b^4 + 113
*a^4*b^2))/(a^2*b^5) - (4*tan(c/2 + (d*x)/2)^8*(90*a^6 - 2*b^6 + 24*a^2*b^4
 + 113*a^4*b^2))/(a^2*b^5) + (4*tan(c/2 + (d*x)/2)^6*(135*a^6 - 3*b^6 + 35*
a^2*b^4 + 172*a^4*b^2))/(a^2*b^5))/(d*(tan(c/2 + (d*x)/2)^4*(15*a^2 - 16*b^
2) - tan(c/2 + (d*x)/2)^10*(6*a^2 - 4*b^2) - tan(c/2 + (d*x)/2)^2*(6*a^2 -
4*b^2) + tan(c/2 + (d*x)/2)^8*(15*a^2 - 16*b^2) - tan(c/2 + (d*x)/2)^6*(20*
a^2 - 24*b^2) + a^2*tan(c/2 + (d*x)/2)^12 + a^2 - 20*a*b*tan(c/2 + (d*x)/2)
^3 + 40*a*b*tan(c/2 + (d*x)/2)^5 - 40*a*b*tan(c/2 + (d*x)/2)^7 + 20*a*b*tan
(c/2 + (d*x)/2)^9 - 4*a*b*tan(c/2 + (d*x)/2)^11 + 4*a*b*tan(c/2 + (d*x)/2))
) - (atan((((5*a^2 + b^2)*(a^2 + b^2)*((16*tan(c/2 + (d*x)/2)*(15*a^6 + 3*a
^2*b^4 + 18*a^4*b^2))/b^6 - (4*(6*a*b^11 + 36*a^3*b^9 + 30*a^5*b^7))/b^12 +
 (4*tan(c/2 + (d*x)/2)^2*(6*a*b^11 + 36*a^3*b^9 + 30*a^5*b^7))/b^12 + (3*(5
*a^2 + b^2)*(a^2 + b^2)*((4*(a*b^14 + 4*a^3*b^12))/b^12 - (4*tan(c/2 + (d*x
)/2)^2*(3*a*b^14 + 4*a^3*b^12))/b^12 + 16*a^2*b*tan(c/2 + (d*x)/2)))/b^7)*3
i)/b^7 - ((5*a^2 + b^2)*(a^2 + b^2)*((4*(6*a*b^11 + 36*a^3*b^9 + 30*a^5*b^7
))/b^12 - (16*tan(c/2 + (d*x)/2)*(15*a^6 + 3*a^2*b^4 + 18*a^4*b^2))/b^6 - (
4*tan(c/2 + (d*x)/2)^2*(6*a*b^11 + 36*a^3*b^9 + 30*a^5*b^7))/b^12 + (3*(5*a
^2 + b^2)*(a^2 + b^2)*((4*(a*b^14 + 4*a^3*b^12))/b^12 - (4*tan(c/2 + (d*x)/
2)^2*(3*a*b^14 + 4*a^3*b^12))/b^12 + 16*a^2*b*tan(c/2 + (d*x)/2)))/b^7)*3i)
/b^7)/((8*(9*a*b^8 + 225*a^9 + 108*a^3*b^6 + 414*a^5*b^4 + 540*a^7*b^2))/b^
12 + (8*tan(c/2 + (d*x)/2)^2*(9*a*b^8 + 225*a^9 + 108*a^3*b^6 + 414*a^5*b^4
 + 540*a^7*b^2))/b^12 + (3*(5*a^2 + b^2)*(a^2 + b^2)*((16*tan(c/2 + (d*x)/2
)*(15*a^6 + 3*a^2*b^4 + 18*a^4*b^2))/b^6 - (4*(6*a*b^11 + 36*a^3*b^9 + 30*a
^5*b^7))/b^12 + (4*tan(c/2 + (d*x)/2)^2*(6*a*b^11 + 36*a^3*b^9 + 30*a^5*b^7
))/b^12 + (3*(5*a^2 + b^2)*(a^2 + b^2)*((4*(a*b^14 + 4*a^3*b^12))/b^12 - (4
*tan(c/2 + (d*x)/2)^2*(3*a*b^14 + 4*a^3*b^12))/b^12 + 16*a^2*b*tan(c/2 + (d
*x)/2)))/b^7))/b^7 + (3*(5*a^2 + b^2)*(a^2 + b^2)*((4*(6*a*b^11 + 36*a^3*b^
9 + 30*a^5*b^7))/b^12 - (16*tan(c/2 + (d*x)/2)*(15*a^6 + 3*a^2*b^4 + 18*a^4
*b^2))/b^6 - (4*tan(c/2 + (d*x)/2)^2*(6*a*b^11 + 36*a^3*b^9 + 30*a^5*b^7))/
b^12 + (3*(5*a^2 + b^2)*(a^2 + b^2)*((4*(a*b^14 + 4*a^3*b^12))/b^12 - (4*ta
n(c/2 + (d*x)/2)^2*(3*a*b^14 + 4*a^3*b^12))/b^12 + 16*a^2*b*tan(c/2 + (d*x)
/2)))/b^7))/b^7))*(5*a^2 + b^2)*(a^2 + b^2)*6i)/(b^7*d)
```

3.141 $\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=165

$$\frac{(a^4-6a^2b^2+b^4)\,x}{(a^2+b^2)^4}+\frac{4ab(a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{(a^2+b^2)^4\,d}-\frac{b}{3\,(a^2+b^2)\,d(a+b\tan(c+dx))^3}-\frac{}{(a^2}$$

```
[Out] (a^4-6*a^2*b^2+b^4)*x/(a^2+b^2)^4+4*a*b*(a^2-b^2)*ln(a*cos(d*x+c)+b*sin(d*x
+c))/(a^2+b^2)^4/d-1/3*b/(a^2+b^2)/d/(a+b*tan(d*x+c))^3-a*b/(a^2+b^2)^2/d/(
a+b*tan(d*x+c))^2-b*(3*a^2-b^2)/(a^2+b^2)^3/d/(a+b*tan(d*x+c))
```

Rubi [A]
time = 0.21, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$,
Rules used = {3165, 3564, 3610, 3612, 3611}

$$-\frac{b(3a^2-b^2)}{d\,(a^2+b^2)^3\,(a+b\tan(c+dx))}-\frac{ab}{d\,(a^2+b^2)^2\,(a+b\tan(c+dx))^2}-\frac{b}{3d\,(a^2+b^2)\,(a+b\tan(c+dx))^3}+\frac{4ab(a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx))}{d\,(a^2+b^2)^4}+\frac{x(a^4-6a^2b^2+b^4)}{(a^2+b^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] ((a^4 - 6*a^2*b^2 + b^4)*x)/(a^2 + b^2)^4 + (4*a*b*(a^2 - b^2)*Log[a*Cos[c
+ d*x] + b*Sin[c + d*x]])/((a^2 + b^2)^4*d) - b/(3*(a^2 + b^2)*d*(a + b*Tan
[c + d*x])^3) - (a*b)/((a^2 + b^2)^2*d*(a + b*Tan[c + d*x])^2) - (b*(3*a^2
- b^2))/((a^2 + b^2)^3*d*(a + b*Tan[c + d*x]))
```

Rule 3165

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3610

```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*tan[(e_.) +
 (f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*((a + b*Tan[e + f*x])^(m + 1)/
(f*(m + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2), Int[(a + b*Tan[e + f*x])
^(m + 1)*Simp[a*c + b*d - (b*c - a*d)*Tan[e + f*x], x], x], x] /; FreeQ[{a,
```

```
b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx &= \int \frac{1}{(a+b\tan(c+dx))^4}\,dx \\
&= -\frac{b}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^3}+\frac{\int \frac{a-b\tan(c+dx)}{(a+b\tan(c+dx))^3}\,dx}{a^2+b^2} \\
&= -\frac{b}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^3}-\frac{ab}{\left(a^2+b^2\right)^2 d(a+b\tan(c+dx \\
&= -\frac{b}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^3}-\frac{ab}{\left(a^2+b^2\right)^2 d(a+b\tan(c+dx \\
&= \frac{\left(a^4-6a^2b^2+b^4\right)x}{\left(a^2+b^2\right)^4}-\frac{b}{3\left(a^2+b^2\right)d(a+b\tan(c+dx))^3}-\frac{}{(a^2+b^2 \\
&= \frac{\left(a^4-6a^2b^2+b^4\right)x}{\left(a^2+b^2\right)^4}+\frac{4ab(a^2-b^2)\log(a\cos(c+dx)+b\sin(c+dx}{\left(a^2+b^2\right)^4 d}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.24, size = 419, normalized size = 2.54

$$
\frac{\left(a^2-2ab-b^2\right)\left(a^2+2ab-b^2\right)(c+dx)}{(a-ib)^4(a+ib)^4d}+\frac{4\left(ia^{10}b+a^9b^2+2ia^8b^3+2a^7b^4-2ia^4b^7-2a^3b^8-ia^2b^9-ab^{10}\right)(c+dx)}{(a-ib)^8(a+ib)^7d}-\frac{4i\left(a^3b-ab^3\right)\mathrm{ArcTan}(\tan(c+dx))}{\left(a^2+b^2\right)^4d}+\frac{2\left(a^3b-ab^3\right)\log\left((a\cos(c+dx)+b\sin(c+dx))^2\right)}{\left(a^2+b^2\right)^4d}+\frac{b^4\sin(c+dx)}{3a(a-ib)^2(a+ib)^2d(a\cos(c+dx)+b\sin(c+dx))^3}-\frac{b^3\left(6a^2+b^2\right)}{3a(a-ib)^3(a+ib)^3d(a\cos(c+dx)+b\sin(c+dx))^2}+\frac{2\left(9a^2b^2\sin(c+dx)-2b^4\sin(c+dx)\right)}{3a(a-ib)^3(a+ib)^3d(a\cos(c+dx)+b\sin(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] ((a^2 - 2*a*b - b^2)*(a^2 + 2*a*b - b^2)*(c + d*x))/((a - I*b)^4*(a + I*b)^
4*d) + (4*(I*a^10*b + a^9*b^2 + (2*I)*a^8*b^3 + 2*a^7*b^4 - (2*I)*a^4*b^7 -
 2*a^3*b^8 - I*a^2*b^9 - a*b^10)*(c + d*x))/((a - I*b)^8*(a + I*b)^7*d) - (
(4*I)*(a^3*b - a*b^3)*ArcTan[Tan[c + d*x]])/((a^2 + b^2)^4*d) + (2*(a^3*b -
 a*b^3)*Log[(a*Cos[c + d*x] + b*Sin[c + d*x])^2])/((a^2 + b^2)^4*d) + (b^4*
Sin[c + d*x])/(3*a*(a - I*b)^2*(a + I*b)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*
x])^3) - (b^3*(6*a^2 + b^2))/(3*a*(a - I*b)^3*(a + I*b)^3*d*(a*Cos[c + d*x]
 + b*Sin[c + d*x])^2) + (2*(9*a^2*b^2*Sin[c + d*x] - 2*b^4*Sin[c + d*x]))/(
3*a*(a - I*b)^3*(a + I*b)^3*d*(a*Cos[c + d*x] + b*Sin[c + d*x]))
```

Maple [A]
time = 0.94, size = 183, normalized size = 1.11

method	result
derivativedivides	$\frac{-\frac{b}{3\left(a^2+b^2\right)(a+b\tan(dx+c))^3}-\frac{b\left(3a^2-b^2\right)}{\left(a^2+b^2\right)^3(a+b\tan(dx+c))}-\frac{ba}{\left(a^2+b^2\right)^2(a+b\tan(dx+c))^2}+\frac{4ab\left(a^2-b^2\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^4}+\frac{\left(-4a^3\right.}{}}{d}$
default	$\frac{-\frac{b}{3\left(a^2+b^2\right)(a+b\tan(dx+c))^3}-\frac{b\left(3a^2-b^2\right)}{\left(a^2+b^2\right)^3(a+b\tan(dx+c))}-\frac{ba}{\left(a^2+b^2\right)^2(a+b\tan(dx+c))^2}+\frac{4ab\left(a^2-b^2\right)\ln(a+b\tan(dx+c))}{\left(a^2+b^2\right)^4}+\frac{\left(-4a^3\right.}{}}{d}$
risch	$-\frac{x}{4ia^3b-4iab^3-a^4+6a^2b^2-b^4}-\frac{8ia^3bx}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}+\frac{8iab^3x}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}-\frac{8ia^3}{d(a^8+4a^6b^2+6a^4}$
norman	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*b/(a^2+b^2)/(a+b*tan(d*x+c))^3-b*(3*a^2-b^2)/(a^2+b^2)^3/(a+b*tan
(d*x+c))-b*a/(a^2+b^2)^2/(a+b*tan(d*x+c))^2+4*a*b*(a^2-b^2)/(a^2+b^2)^4*ln(
a+b*tan(d*x+c))+1/(a^2+b^2)^4*(1/2*(-4*a^3*b+4*a*b^3)*ln(tan(d*x+c)^2+1)+(a
^4-6*a^2*b^2+b^4)*arctan(tan(d*x+c))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(163) = 326.
time = 0.50, size = 385, normalized size = 2.33

$$\frac{\frac{3\left(a^4-6a^2b^2+b^4\right)(dx+c)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}+\frac{12\left(a^3b-ab^3\right)\log(b\tan(dx+c)+a)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}-\frac{6\left(a^3b-ab^3\right)\log\left(\tan(dx+c)^2+1\right)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}-\frac{13a^4b+2a^2b^3+b^5+3\left(3a^2b^3-b^5\right)\tan(dx+c)^2+3\left(7a^3b^2-ab^4\right)\tan(dx+c)}{a^9+3a^7b^2+3a^5b^4+a^3b^6+\left(a^6b^3+3a^4b^5+3a^2b^7+b^9\right)\tan(dx+c)^3+3\left(a^7b^2+3a^5b^4+3a^3b^6+ab^8\right)\tan(dx+c)^2+3\left(a^8b+3a^6b^3+3a^4b^5+a^2b^7\right)\tan(dx+c)}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a
^2*b^6 + b^8) + 12*(a^3*b - a*b^3)*log(b*tan(d*x + c) + a)/(a^8 + 4*a^6*b^2
 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 6*(a^3*b - a*b^3)*log(tan(d*x + c)^2 + 1)
/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - (13*a^4*b + 2*a^2*b^3 +
```

```
b^5 + 3*(3*a^2*b^3 - b^5)*tan(d*x + c)^2 + 3*(7*a^3*b^2 - a*b^4)*tan(d*x +
c))/(a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*b^6 + (a^6*b^3 + 3*a^4*b^5 + 3*a^2*b
^7 + b^9)*tan(d*x + c)^3 + 3*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*tan(
d*x + c)^2 + 3*(a^8*b + 3*a^6*b^3 + 3*a^4*b^5 + a^2*b^7)*tan(d*x + c)))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 575 vs. $2(163) = 326$.
time = 3.09, size = 575, normalized size = 3.48

$$-\frac{(54\,a^4b^3-30\,a^2b^5+4\,b^7-3\,(a^7-9\,a^5b^2+19\,a^3b^4-3\,ab^6)dx)\cos(dx+c)^3-3\,(10\,a^4b^3-11\,a^2b^5+b^7+3\,(a^5b^2-6\,a^3b^4+ab^6)dx)\cos(dx+c)-6\left((a^6b-4\,a^4b^3+3\,a^2b^5)\cos(dx+c)^3+3\,(a^4b^3-a^2b^5)\cos(dx+c)+(a^3b^4-ab^6+(3\,a^5b^2-4\,a^3b^4+ab^6)\cos(dx+c)^2)\sin(dx+c)\right)\log\left(2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2\right)-\left(13\,a^3b^4-9\,ab^6+3\,(a^4b^3-6\,a^2b^5+b^7)dx+(18\,a^5b^2-58\,a^3b^4+12\,ab^6+3\,(3\,a^6b-19\,a^4b^3+9\,a^2b^5-b^7)dx)\cos(dx+c)^2\right)\sin(dx+c)}{3\left((a^{11}+a^9b^2-6\,a^7b^4-14\,a^5b^6-11\,a^3b^8-3\,ab^{10})d\cos(dx+c)^3+3\,(a^9b^2+4\,a^7b^4+6\,a^5b^6+4\,a^3b^8+ab^{10})d\cos(dx+c)+\left((3\,a^{10}b+11\,a^8b^3+14\,a^6b^5+6\,a^4b^7-a^2b^9-b^{11})d\cos(dx+c)^2+(a^8b^3+4\,a^6b^5+6\,a^4b^7+4\,a^2b^9+b^{11})d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*((54*a^4*b^3 - 30*a^2*b^5 + 4*b^7 - 3*(a^7 - 9*a^5*b^2 + 19*a^3*b^4 -
3*a*b^6)*d*x)*cos(d*x + c)^3 - 3*(10*a^4*b^3 - 11*a^2*b^5 + b^7 + 3*(a^5*b^
2 - 6*a^3*b^4 + a*b^6)*d*x)*cos(d*x + c) - 6*((a^6*b - 4*a^4*b^3 + 3*a^2*b^
5)*cos(d*x + c)^3 + 3*(a^4*b^3 - a^2*b^5)*cos(d*x + c) + (a^3*b^4 - a*b^6 +
 (3*a^5*b^2 - 4*a^3*b^4 + a*b^6)*cos(d*x + c)^2)*sin(d*x + c))*log(2*a*b*co
s(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - (13*a^3*b^4 -
 9*a*b^6 + 3*(a^4*b^3 - 6*a^2*b^5 + b^7)*d*x + (18*a^5*b^2 - 58*a^3*b^4 + 1
2*a*b^6 + 3*(3*a^6*b - 19*a^4*b^3 + 9*a^2*b^5 - b^7)*d*x)*cos(d*x + c)^2)*s
in(d*x + c))/((a^11 + a^9*b^2 - 6*a^7*b^4 - 14*a^5*b^6 - 11*a^3*b^8 - 3*a*b
^10)*d*cos(d*x + c)^3 + 3*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 + 4*a^3*b^8 + a*
b^10)*d*cos(d*x + c) + ((3*a^10*b + 11*a^8*b^3 + 14*a^6*b^5 + 6*a^4*b^7 - a
^2*b^9 - b^11)*d*cos(d*x + c)^2 + (a^8*b^3 + 4*a^6*b^5 + 6*a^4*b^7 + 4*a^2*
b^9 + b^11)*d)*sin(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Exception raised: AttributeError >> 'NoneType' object has no attribute 'pri
mitive'
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(163) = 326$.
time = 0.51, size = 370, normalized size = 2.24

$$\frac{\frac{3\,(a^4-6\,a^2b^2+b^4)(dx+c)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}-\frac{6\,(a^3b-ab^3)\log\left(\tan(dx+c)^2+1\right)}{a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8}+\frac{12\,(a^3b^2-ab^4)\log(|b\tan(dx+c)+a|)}{a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9}-\frac{22\,a^3b^4\tan(dx+c)^3-22\,ab^6\tan(dx+c)^3+75\,a^4b^3\tan(dx+c)^2-60\,a^2b^5\tan(dx+c)^2-3\,b^7\tan(dx+c)^2+87\,a^5b^2\tan(dx+c)-48\,a^3b^4\tan(dx+c)-3\,ab^6\tan(dx+c)+35\,a^6b-7\,a^4b^3+3\,a^2b^5+b^7}{(a^8+4\,a^6b^2+6\,a^4b^4+4\,a^2b^6+b^8)(b\tan(dx+c)+a)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(a^4 - 6*a^2*b^2 + b^4)*(d*x + c)/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a
^2*b^6 + b^8) - 6*(a^3*b - a*b^3)*log(tan(d*x + c)^2 + 1)/(a^8 + 4*a^6*b^2
+ 6*a^4*b^4 + 4*a^2*b^6 + b^8) + 12*(a^3*b^2 - a*b^4)*log(abs(b*tan(d*x + c
) + a))/(a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9) - (22*a^3*b^4*tan
(d*x + c)^3 - 22*a*b^6*tan(d*x + c)^3 + 75*a^4*b^3*tan(d*x + c)^2 - 60*a^2*
b^5*tan(d*x + c)^2 - 3*b^7*tan(d*x + c)^2 + 87*a^5*b^2*tan(d*x + c) - 48*a^
3*b^4*tan(d*x + c) - 3*a*b^6*tan(d*x + c) + 35*a^6*b - 7*a^4*b^3 + 3*a^2*b^
5 + b^7)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(b*tan(d*x + c) +
 a)^3))/d
```

Mupad [B]
time = 12.55, size = 2500, normalized size = 15.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

```
[Out] ((4*tan(c/2 + (d*x)/2)^2*(b^7 + 3*a^2*b^5 + 10*a^4*b^3))/(a^2*(a^6 + b^6 +
3*a^2*b^4 + 3*a^4*b^2)) - (4*tan(c/2 + (d*x)/2)^4*(b^7 + 3*a^2*b^5 + 10*a^4
*b^3))/(a^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (2*b*tan(c/2 + (d*x)/2)^
5*(6*a^4*b + b^5 + 3*a^2*b^3))/(a*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (4
*b*tan(c/2 + (d*x)/2)^3*(2*b^7 - 18*a^6*b + a^2*b^5 + 17*a^4*b^3))/(3*a^3*(
a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (2*b*tan(c/2 + (d*x)/2)*(6*a^4*b + b^
5 + 3*a^2*b^3))/(a*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))/(d*(tan(c/2 + (d*x
)/2)^2*(12*a*b^2 - 3*a^3) - a^3*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^4
*(12*a*b^2 - 3*a^3) - tan(c/2 + (d*x)/2)^3*(12*a^2*b - 8*b^3) + a^3 + 6*a^2
*b*tan(c/2 + (d*x)/2) + 6*a^2*b*tan(c/2 + (d*x)/2)^5)) - (log(a + 2*b*tan(c
/2 + (d*x)/2) - a*tan(c/2 + (d*x)/2)^2)*(4*a*b^3 - 4*a^3*b))/(d*(a^8 + b^8
+ 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (log((((-(a^4 + b^4 - 6*a^2*b^2)^2/
(a^2 + b^2)^8)^(1/2) - (4*a*b*(a^2 - b^2))/(a^2 + b^2)^4)*(((-(a^4 + b^4 -
6*a^2*b^2)^2/(a^2 + b^2)^8)^(1/2) - (4*a*b*(a^2 - b^2))/(a^2 + b^2)^4)*((32
*a*(a^6 - b^6 + 11*a^2*b^4 - 11*a^4*b^2))/(a^2 + b^2)^3 + 96*a*b*((-(a^4 +
b^4 - 6*a^2*b^2)^2/(a^2 + b^2)^8)^(1/2) - (4*a*b*(a^2 - b^2))/(a^2 + b^2)^4
)*(a + b*tan(c/2 + (d*x)/2))*(a^2 + b^2) - (64*a^2*b*tan(c/2 + (d*x)/2)*(b^
4 - 5*a^4 + 8*a^2*b^2))/(a^2 + b^2)^3) - (32*a^2*b*(7*a^4 + 7*b^4 - 18*a^2*
b^2))/(a^2 + b^2)^5 + (32*a*tan(c/2 + (d*x)/2)*(a^8 + 2*b^8 - 57*a^2*b^6 +
105*a^4*b^4 - 27*a^6*b^2))/(a^2 + b^2)^6) + (128*a^3*b^2*(3*a^6 - 3*b^6 + 1
3*a^2*b^4 - 13*a^4*b^2))/(a^2 + b^2)^9 - (128*a^2*b*tan(c/2 + (d*x)/2)*(a^8
 - 2*b^8 + 5*a^2*b^6 - 15*a^4*b^4 + 11*a^6*b^2))/(a^2 + b^2)^9)*(((-(a^4 +
b^4 - 6*a^2*b^2)^2/(a^2 + b^2)^8)^(1/2) + (4*a*b*(a^2 - b^2))/(a^2 + b^2)^4
)*(((-(a^4 + b^4 - 6*a^2*b^2)^2/(a^2 + b^2)^8)^(1/2) + (4*a*b*(a^2 - b^2))/
(a^2 + b^2)^4)*(96*a*b*((-(a^4 + b^4 - 6*a^2*b^2)^2/(a^2 + b^2)^8)^(1/2) +
(4*a*b*(a^2 - b^2))/(a^2 + b^2)^4)*(a + b*tan(c/2 + (d*x)/2))*(a^2 + b^2) -
```

(32*a*(a^6 - b^6 + 11*a^2*b^4 - 11*a^4*b^2))/(a^2 + b^2)^3 + (64*a^2*b*tan
(c/2 + (d*x)/2)*(b^4 - 5*a^4 + 8*a^2*b^2))/(a^2 + b^2)^3) - (32*a^2*b*(7*a^
4 + 7*b^4 - 18*a^2*b^2))/(a^2 + b^2)^5 + (32*a*tan(c/2 + (d*x)/2)*(a^8 + 2*
b^8 - 57*a^2*b^6 + 105*a^4*b^4 - 27*a^6*b^2))/(a^2 + b^2)^6) - (128*a^3*b^2
*(3*a^6 - 3*b^6 + 13*a^2*b^4 - 13*a^4*b^2))/(a^2 + b^2)^9 + (128*a^2*b*tan(
c/2 + (d*x)/2)*(a^8 - 2*b^8 + 5*a^2*b^6 - 15*a^4*b^4 + 11*a^6*b^2))/(a^2 +
b^2)^9))*(8*a*b^3 - 8*a^3*b))/(2*d*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a
^6*b^2)) - (2*atan((tan(c/2 + (d*x)/2)*((((32*(4*a^10*b - 8*a^2*b^9 + 20*a^
4*b^7 - 60*a^6*b^5 + 44*a^8*b^3))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 +
84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*
a^16*b^2) - (((32*(2*a*b^14 + a^15 - 51*a^3*b^12 - 60*a^5*b^10 + 119*a^7*b^
8 + 178*a^9*b^6 + 27*a^11*b^4 - 24*a^13*b^2))/(a^18 + b^18 + 9*a^2*b^16 + 3
6*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a
^14*b^4 + 9*a^16*b^2) - ((8*a*b^3 - 8*a^3*b)*((32*(2*a^2*b^17 - 10*a^18*b +
28*a^4*b^15 + 116*a^6*b^13 + 220*a^8*b^11 + 200*a^10*b^9 + 52*a^12*b^7 - 5
2*a^14*b^5 - 44*a^16*b^3))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6
*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^
2) - (16*(8*a*b^3 - 8*a^3*b)*(3*a*b^22 + 30*a^3*b^20 + 135*a^5*b^18 + 360*a
^7*b^16 + 630*a^9*b^14 + 756*a^11*b^12 + 630*a^13*b^10 + 360*a^15*b^8 + 135
*a^17*b^6 + 30*a^19*b^4 + 3*a^21*b^2))/((a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4
+ 4*a^6*b^2)*(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^
8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2))))/(2*(a^8
+ b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)))*(8*a*b^3 - 8*a^3*b))/(2*(a^8 +
b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) - (((((32*(2*a^2*b^17 - 10*a^18*
b + 28*a^4*b^15 + 116*a^6*b^13 + 220*a^8*b^11 + 200*a^10*b^9 + 52*a^12*b^7
- 52*a^14*b^5 - 44*a^16*b^3))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*
a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16
b^2) - (16(8*a*b^3 - 8*a^3*b)*(3*a*b^22 + 30*a^3*b^20 + 135*a^5*b^18 + 36
0*a^7*b^16 + 630*a^9*b^14 + 756*a^11*b^12 + 630*a^13*b^10 + 360*a^15*b^8 +
135*a^17*b^6 + 30*a^19*b^4 + 3*a^21*b^2))/((a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b
^4 + 4*a^6*b^2)*(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126
*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2)))*(2*a*b
- a^2 + b^2)*(2*a*b + a^2 - b^2))/(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a
^6*b^2) - (16*(8*a*b^3 - 8*a^3*b)*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2)*(
3*a*b^22 + 30*a^3*b^20 + 135*a^5*b^18 + 360*a^7*b^16 + 630*a^9*b^14 + 756*a
^11*b^12 + 630*a^13*b^10 + 360*a^15*b^8 + 135*a^17*b^6 + 30*a^19*b^4 + 3*a^
21*b^2))/((a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)^2*(a^18 + b^18 +
9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a
^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2)))*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^
2))/(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2) + (16*(8*a*b^3 - 8*a^3*
b)*(2*a*b - a^2 + b^2)^2*(2*a*b + a^2 - b^2)^2*...

3.142 $\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=157

$$\frac{a(2a^2-3b^2)\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{7/2}d}+\frac{-3(3a^4b-a^2b^3+b^5)\cos(2(c+dx))+\frac{1}{2}b(-9a^2+b^2)\left(2(a^2+b^2)+\right.}{6\left(a^2+b^2\right)^3 d(a\cos(c+dx)+b\sin(c+dx))^3}$$

```
[Out] a*(2*a^2-3*b^2)*arctanh((-b+a*tan(1/2*d*x+1/2*c))/(a^2+b^2)^(1/2))/(a^2+b^2
)^(7/2)/d+1/6*(-3*(3*a^4*b-a^2*b^3+b^5)*cos(2*d*x+2*c)+1/2*b*(-9*a^2+b^2)*(
2*a^2+2*b^2+3*a*b*sin(2*d*x+2*c)))/(a^2+b^2)^3/d/(a*cos(d*x+c)+b*sin(d*x+c)
)^3
```

Rubi [B] Leaf count is larger than twice the leaf count of optimal. 362 vs. $2(157) = 314$.
time = 0.84, antiderivative size = 362, normalized size of antiderivative = 2.31, number of steps used = 7, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1674, 12, 632, 212}

$$-\frac{a(2a^2-3b^2)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{d\left(a^2+b^2\right)^{7/2}}-\frac{8b^3\left(b(3a^2+4b^2)\tan\left(\frac{1}{2}(c+dx)\right)+a(a^2+2b^2)\right)}{3a^5d\left(a^2+b^2\right)\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)^3}-\frac{b\left(6a^6+9a^4b^2+12a^2b^4+ab(9a^4+6a^2b^2+2b^4)\tan\left(\frac{1}{2}(c+dx)\right)+4b^6\right)}{a^4d\left(a^2+b^2\right)^3\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{2b^2\left(a(9a^4+30a^2b^2+16b^4)\tan\left(\frac{1}{2}(c+dx)\right)+b(15a^4+18a^2b^2+8b^4)\right)}{3a^5d\left(a^2+b^2\right)^2\left(-a\tan^2\left(\frac{1}{2}(c+dx)\right)+a+2b\tan\left(\frac{1}{2}(c+dx)\right)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] -((a*(2*a^2 - 3*b^2)*ArcTanh[(b - a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/((a
^2 + b^2)^(7/2)*d)) - (8*b^3*(a*(a^2 + 2*b^2) + b*(3*a^2 + 4*b^2)*Tan[(c +
d*x)/2]))/(3*a^5*(a^2 + b^2)*d*(a + 2*b*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/
2]^2)^3) + (2*b^2*(b*(15*a^4 + 18*a^2*b^2 + 8*b^4) + a*(9*a^4 + 30*a^2*b^2
+ 16*b^4)*Tan[(c + d*x)/2]))/(3*a^5*(a^2 + b^2)^2*d*(a + 2*b*Tan[(c + d*x)/
2] - a*Tan[(c + d*x)/2]^2)^2) - (b*(6*a^6 + 9*a^4*b^2 + 12*a^2*b^4 + 4*b^6
+ a*b*(9*a^4 + 6*a^2*b^2 + 2*b^4)*Tan[(c + d*x)/2]))/(a^4*(a^2 + b^2)^3*d*(
a + 2*b*Tan[(c + d*x)/2] - a*Tan[(c + d*x)/2]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1674

```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q =
PolynomialQuotient[Pq, a + b*x + c*x^2, x], f = Coeff[PolynomialRemainder[P
q, a + b*x + c*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x +
c*x^2, x], x, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^
(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[(p + 1)*(b^2 - 4*a*c)*Q - (2*p + 3)*(
2*c*f - b*g), x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x] && NeQ[b^2
 - 4*a*c, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \frac{2\text{Subst}\left(\int \frac{(1-x^2)^3}{(a+2bx-ax^2)^4}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

$$= -\frac{8b^3\left(a(a^2+2b^2)+b(3a^2+4b^2)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^5\left(a^2+b^2\right)d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)-a\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^3} -$$

$$= -\frac{8b^3\left(a(a^2+2b^2)+b(3a^2+4b^2)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^5\left(a^2+b^2\right)d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)-a\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^3} +$$

$$= -\frac{8b^3\left(a(a^2+2b^2)+b(3a^2+4b^2)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^5\left(a^2+b^2\right)d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)-a\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^3} +$$

$$= -\frac{8b^3\left(a(a^2+2b^2)+b(3a^2+4b^2)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^5\left(a^2+b^2\right)d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)-a\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^3} +$$

$$= -\frac{8b^3\left(a(a^2+2b^2)+b(3a^2+4b^2)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^5\left(a^2+b^2\right)d\left(a+2b\tan\left(\frac{1}{2}(c+dx)\right)-a\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^3} +$$

$$= -\frac{a(2a^2-3b^2)\tanh^{-1}\left(\frac{b-a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}d} - \frac{8b^3\left(a(a^2+2b^2)\right.}{3a^5\left(a^2+b^2\right)d\left(a+2b\right.}$$

Mathematica [C] Result contains complex when optimal does not.

time = 1.20, size = 165, normalized size = 1.05

$$\frac{\frac{6a(2a^2-3b^2)\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}}+\frac{-3(3a^4b-a^2b^3+b^5)\cos(2(c+dx))+\frac{1}{2}b(-9a^2+b^2)\left(2(a^2+b^2)+3ab\sin(2(c+dx))\right)}{(a-ib)^3(a+ib)^3(a\cos(c+dx)+b\sin(c+dx))^3}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] ((6*a*(2*a^2 - 3*b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(
a^2 + b^2)^(7/2) + (-3*(3*a^4*b - a^2*b^3 + b^5)*Cos[2*(c + d*x)] + (b*(-9*
a^2 + b^2)*(2*(a^2 + b^2) + 3*a*b*Sin[2*(c + d*x)]))/2)/((a - I*b)^3*(a + I
*b)^3*(a*Cos[c + d*x] + b*Sin[c + d*x])^3))/(6*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 493 vs. 2(151) = 302.
time = 0.91, size = 494, normalized size = 3.15

method	result
derivativedivides	$-\frac{2\left(-\frac{b^2(9a^4+6a^2b^2+2b^4)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a(a^6+3a^4b^2+3a^2b^4+b^6)}-\frac{b(6a^6-27a^4b^2-12a^2b^4-4b^6)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2(a^6+3a^4b^2+3a^2b^4+b^6)}+\frac{b^2(54a^6-21a^4b^2-4a^2b^4-4b^6)\left(\tan^3 \cdots\right.}{3a^3(a^6+3a^4b^2+3a^2b^4+b^6 \cdots}\right.}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b \cdots\right.}$
default	$-\frac{2\left(-\frac{b^2(9a^4+6a^2b^2+2b^4)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a(a^6+3a^4b^2+3a^2b^4+b^6)}-\frac{b(6a^6-27a^4b^2-12a^2b^4-4b^6)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2(a^6+3a^4b^2+3a^2b^4+b^6)}+\frac{b^2(54a^6-21a^4b^2-4a^2b^4-4b^6)\left(\tan^3 \cdots\right.}{3a^3(a^6+3a^4b^2+3a^2b^4+b^6 \cdots}\right.}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b \cdots\right.}$
risch	$\frac{ib\left(-27ia^3b\,e^{5i(dx+c)}+3ia\,b^3e^{5i(dx+c)}+18a^4e^{5i(dx+c)}-6a^2b^2e^{5i(dx+c)}+6b^4e^{5i(dx+c)}+36a^4e^{3i(dx+c)}+32a^2b^2e^{3i(dx+c)}- \cdots\right.}{3(-ia+b)^3\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia \cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/2*b^2*(9*a^4+6*a^2*b^2+2*b^4)/a/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*t
an(1/2*d*x+1/2*c)^5-1/2*b*(6*a^6-27*a^4*b^2-12*a^2*b^4-4*b^6)/a^2/(a^6+3*a^
4*b^2+3*a^2*b^4+b^6)*tan(1/2*d*x+1/2*c)^4+1/3/a^3*b^2*(54*a^6-21*a^4*b^2-4*
a^2*b^4-4*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*tan(1/2*d*x+1/2*c)^3+1/a^2*b*(
6*a^6-20*a^4*b^2-3*a^2*b^4-2*b^6)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*tan(1/2*d*x
+1/2*c)^2-1/2/a*b^2*(27*a^4+4*a^2*b^2+2*b^4)/(a^6+3*a^4*b^2+3*a^2*b^4+b^6)*
tan(1/2*d*x+1/2*c)-1/6*b*(18*a^4+5*a^2*b^2+2*b^4)/(a^6+3*a^4*b^2+3*a^2*b^4+
b^6))/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)^3+a*(2*a^2-3*b^2)/(
a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1
/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 724 vs. $2(152) = 304$.
time = 0.51, size = 724, normalized size = 4.61

$$-\frac{\frac{3\,(2\,a^2-3\,b^2)a\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}}+\frac{2\left(18\,a^7b+5\,a^5b^3+2\,a^3b^5+\frac{3\,(27\,a^6b^2+4\,a^4b^4+2\,a^2b^6)\sin(dx+c)}{\cos(dx+c)+1}-\frac{6\,(6\,a^7b-20\,a^5b^3-3\,a^3b^5-2\,ab^7)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2\,(54\,a^6b^2-21\,a^4b^4-4\,a^2b^6-4\,b^8)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,(6\,a^7b-27\,a^5b^3-12\,a^3b^5-4\,ab^7)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{3\,(9\,a^6b^2+6\,a^4b^4+2\,a^2b^6)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^{12}+3\,a^{10}b^2+3\,a^8b^4+a^6b^6+\frac{6\,(a^{11}b+3\,a^9b^3+3\,a^7b^5+a^5b^7)\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\,(a^{12}-a^{10}b^2-9\,a^8b^4-11\,a^6b^6-4\,a^4b^8)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4\,(3\,a^{11}b+7\,a^9b^3+3\,a^7b^5-3\,a^5b^7-2\,a^3b^9)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,(a^{12}-a^{10}b^2-9\,a^8b^4-11\,a^6b^6-4\,a^4b^8)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{6\,(a^{11}b+3\,a^9b^3+3\,a^7b^5+a^5b^7)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{(a^{12}+3\,a^{10}b^2+3\,a^8b^4+a^6b^6)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(2*a^2 - 3*b^2)*a*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt
(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a
^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) + 2*(18*a^7*b + 5*a^5*b^
3 + 2*a^3*b^5 + 3*(27*a^6*b^2 + 4*a^4*b^4 + 2*a^2*b^6)*sin(d*x + c)/(cos(d*
x + c) + 1) - 6*(6*a^7*b - 20*a^5*b^3 - 3*a^3*b^5 - 2*a*b^7)*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 - 2*(54*a^6*b^2 - 21*a^4*b^4 - 4*a^2*b^6 - 4*b^8)*sin
(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*(6*a^7*b - 27*a^5*b^3 - 12*a^3*b^5 - 4
*a*b^7)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 3*(9*a^6*b^2 + 6*a^4*b^4 + 2*
a^2*b^6)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^12 + 3*a^10*b^2 + 3*a^8*b^
4 + a^6*b^6 + 6*(a^11*b + 3*a^9*b^3 + 3*a^7*b^5 + a^5*b^7)*sin(d*x + c)/(co
s(d*x + c) + 1) - 3*(a^12 - a^10*b^2 - 9*a^8*b^4 - 11*a^6*b^6 - 4*a^4*b^8)*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4*(3*a^11*b + 7*a^9*b^3 + 3*a^7*b^5 -
 3*a^5*b^7 - 2*a^3*b^9)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*(a^12 - a^1
0*b^2 - 9*a^8*b^4 - 11*a^6*b^6 - 4*a^4*b^8)*sin(d*x + c)^4/(cos(d*x + c) +
1)^4 + 6*(a^11*b + 3*a^9*b^3 + 3*a^7*b^5 + a^5*b^7)*sin(d*x + c)^5/(cos(d*x
 + c) + 1)^5 - (a^12 + 3*a^10*b^2 + 3*a^8*b^4 + a^6*b^6)*sin(d*x + c)^6/(co
s(d*x + c) + 1)^6))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 524 vs. $2(152) = 304$.
time = 1.94, size = 524, normalized size = 3.34

$$-\frac{22\,a^4b^3+14\,a^2b^5-8\,b^7+12\,(3\,a^6b+2\,a^4b^3+b^7)\cos(dx+c)^2+6\,(9\,a^5b^2+8\,a^3b^4-ab^6)\cos(dx+c)\sin(dx+c)+3\,\left((2\,a^6-9\,a^4b^2+9\,a^2b^4)\cos(dx+c)^3+3\,(2\,a^4b^2-3\,a^2b^4)\cos(dx+c)+(2\,a^3b^3-3\,ab^5+(6\,a^5b-11\,a^3b^3+3\,ab^5)\cos(dx+c)^2)\sin(dx+c)\right)\sqrt{a^2+b^2}\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)}{12\,\left((a^{11}+a^9b^2-6\,a^7b^4-14\,a^5b^6-11\,a^3b^8-3\,ab^{10})d\cos(dx+c)^3+3\,(a^9b^2+4\,a^7b^4+6\,a^5b^6+4\,a^3b^8+ab^{10})d\cos(dx+c)+((3\,a^{10}b+11\,a^8b^3+14\,a^6b^5+6\,a^4b^7-a^2b^9-b^{11})d\cos(dx+c)^2+(a^8b^3+4\,a^6b^5+6\,a^4b^7+4\,a^2b^9+b^{11})d)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/12*(22*a^4*b^3 + 14*a^2*b^5 - 8*b^7 + 12*(3*a^6*b + 2*a^4*b^3 + b^7)*cos
(d*x + c)^2 + 6*(9*a^5*b^2 + 8*a^3*b^4 - a*b^6)*cos(d*x + c)*sin(d*x + c) +
 3*((2*a^6 - 9*a^4*b^2 + 9*a^2*b^4)*cos(d*x + c)^3 + 3*(2*a^4*b^2 - 3*a^2*b
^4)*cos(d*x + c) + (2*a^3*b^3 - 3*a*b^5 + (6*a^5*b - 11*a^3*b^3 + 3*a*b^5)*
cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x + c)*sin(d
*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*c
os(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^
2)*cos(d*x + c)^2 + b^2)))/((a^11 + a^9*b^2 - 6*a^7*b^4 - 14*a^5*b^6 - 11*a
```

```
^3*b^8 - 3*a*b^10)*d*cos(d*x + c)^3 + 3*(a^9*b^2 + 4*a^7*b^4 + 6*a^5*b^6 +
4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + ((3*a^10*b + 11*a^8*b^3 + 14*a^6*b^5 +
 6*a^4*b^7 - a^2*b^9 - b^11)*d*cos(d*x + c)^2 + (a^8*b^3 + 4*a^6*b^5 + 6*a^
4*b^7 + 4*a^2*b^9 + b^11)*d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 524 vs. 2(152) = 304.
time = 0.56, size = 524, normalized size = 3.34

$$-\frac{\frac{3\,(2a^3-3ab^2)\log\left(\frac{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{(a^6+3a^4b^2+3a^2b^4+b^6)\sqrt{a^2+b^2}}-\frac{2\left(27a^6b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+18a^4b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+6a^2b^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+18a^7b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-81a^5b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-36a^3b^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-12ab^7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-108a^6b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+42a^4b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+8a^2b^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+8b^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-36a^7b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+120a^5b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+18a^3b^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+12ab^7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+81a^6b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+12a^4b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+6a^2b^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+18a^7b+5a^5b^3+2a^3b^5\right)}{(a^9+3a^7b^2+3a^5b^4+a^3b^6)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*(2*a^3 - 3*a*b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a
^2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 +
 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2 + b^2)) - 2*(27*a^6*b^2*tan(1/2*d*x
+ 1/2*c)^5 + 18*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*a^2*b^6*tan(1/2*d*x + 1/
2*c)^5 + 18*a^7*b*tan(1/2*d*x + 1/2*c)^4 - 81*a^5*b^3*tan(1/2*d*x + 1/2*c)^
4 - 36*a^3*b^5*tan(1/2*d*x + 1/2*c)^4 - 12*a*b^7*tan(1/2*d*x + 1/2*c)^4 - 1
08*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 42*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 + 8*a
^2*b^6*tan(1/2*d*x + 1/2*c)^3 + 8*b^8*tan(1/2*d*x + 1/2*c)^3 - 36*a^7*b*tan
(1/2*d*x + 1/2*c)^2 + 120*a^5*b^3*tan(1/2*d*x + 1/2*c)^2 + 18*a^3*b^5*tan(1
/2*d*x + 1/2*c)^2 + 12*a*b^7*tan(1/2*d*x + 1/2*c)^2 + 81*a^6*b^2*tan(1/2*d*
x + 1/2*c) + 12*a^4*b^4*tan(1/2*d*x + 1/2*c) + 6*a^2*b^6*tan(1/2*d*x + 1/2*
c) + 18*a^7*b + 5*a^5*b^3 + 2*a^3*b^5)/((a^9 + 3*a^7*b^2 + 3*a^5*b^4 + a^3*
b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^3))/d
```

Mupad [**B**]
time = 2.69, size = 764, normalized size = 4.87

$$\frac{\ln\left({\left(a^2+b^2\right)}^{7/2}+a^6\,b+b^7+3\,a^2\,b^5+3\,a^4\,b^3-a^7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,b^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-3\,a^3\,b^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-3\,a^5\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{3\,a\,b^2}{2}-a^3\right)}{d\,{\left(a^2+b^2\right)}^{7/2}}-\frac{\frac{18\,a^4\,b+5\,a^2\,b^3+2\,b^5}{3\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(-6\,a^6\,b+20\,a^4\,b^3+3\,a^2\,b^5+2\,b^7\right)}{a^2\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(-6\,a^6\,b+27\,a^4\,b^3+12\,a^2\,b^5+4\,b^7\right)}{a^2\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(27\,a^4\,b+4\,a^2\,b^3+2\,b^5\right)}{a\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(9\,a^4\,b+6\,a^2\,b^3+2\,b^5\right)}{a\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}-\frac{2\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(3\,a^2-2\,b^2\right)\,\left(18\,a^4\,b+5\,a^2\,b^3+2\,b^5\right)}{3\,a^3\,\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(12\,a\,b^2-3\,a^3\right)-a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(12\,a\,b^2-3\,a^3\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(12\,a^2\,b-8\,b^3\right)+a^3+6\,a^2\,b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6\,a^2\,b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\right)}+\frac{a\,\ln\left({\left(a^2+b^2\right)}^{7/2}-a^6\,b-b^7-3\,a^2\,b^5-3\,a^4\,b^3+a^7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+a\,b^6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+3\,a^3\,b^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+3\,a^5\,b^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a^2-3\,b^2\right)}{2\,d\,{\left(a^2+b^2\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

```
[Out] (log((a^2 + b^2)^(7/2) + a^6*b + b^7 + 3*a^2*b^5 + 3*a^4*b^3 - a^7*tan(c/2
+ (d*x)/2) - a*b^6*tan(c/2 + (d*x)/2) - 3*a^3*b^4*tan(c/2 + (d*x)/2) - 3*a^
5*b^2*tan(c/2 + (d*x)/2))*((3*a*b^2)/2 - a^3))/(d*(a^2 + b^2)^(7/2)) - ((18
*a^4*b + 2*b^5 + 5*a^2*b^3)/(3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (2*ta
n(c/2 + (d*x)/2)^2*(2*b^7 - 6*a^6*b + 3*a^2*b^5 + 20*a^4*b^3))/(a^2*(a^6 +
b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (tan(c/2 + (d*x)/2)^4*(4*b^7 - 6*a^6*b + 12
*a^2*b^5 + 27*a^4*b^3))/(a^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (b*tan(
c/2 + (d*x)/2)*(27*a^4*b + 2*b^5 + 4*a^2*b^3))/(a*(a^6 + b^6 + 3*a^2*b^4 +
3*a^4*b^2)) + (b*tan(c/2 + (d*x)/2)^5*(9*a^4*b + 2*b^5 + 6*a^2*b^3))/(a*(a^
6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (2*b*tan(c/2 + (d*x)/2)^3*(3*a^2 - 2*b^
2)*(18*a^4*b + 2*b^5 + 5*a^2*b^3))/(3*a^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^
2)))/(d*(tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 3*a^3) - a^3*tan(c/2 + (d*x)/2)^6
 - tan(c/2 + (d*x)/2)^4*(12*a*b^2 - 3*a^3) - tan(c/2 + (d*x)/2)^3*(12*a^2*b
 - 8*b^3) + a^3 + 6*a^2*b*tan(c/2 + (d*x)/2) + 6*a^2*b*tan(c/2 + (d*x)/2)^5
)) + (a*log((a^2 + b^2)^(7/2) - a^6*b - b^7 - 3*a^2*b^5 - 3*a^4*b^3 + a^7*t
an(c/2 + (d*x)/2) + a*b^6*tan(c/2 + (d*x)/2) + 3*a^3*b^4*tan(c/2 + (d*x)/2)
 + 3*a^5*b^2*tan(c/2 + (d*x)/2))*(2*a^2 - 3*b^2))/(2*d*(a^2 + b^2)^(7/2))
```

3.143 $\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=30

$$-\frac{\cot^3(c+dx)}{3bd(b+a\cot(c+dx))^3}$$

```
[Out] -1/3*cot(d*x+c)^3/b/d/(b+a*cot(d*x+c))^3
```

Rubi [A]
time = 0.04, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3167, 37}

$$-\frac{\cot^3(c+dx)}{3bd(a\cot(c+dx)+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] -1/3*Cot[c + d*x]^3/(b*d*(b + a*Cot[c + d*x])^3)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx &= -\frac{\text{Subst}\left(\int \frac{x^2}{(b+ax)^4}\,dx, x, \cot(c+dx)\right)}{d}\\ &= -\frac{\cot^3(c+dx)}{3bd(b+a\cot(c+dx))^3}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 124 vs. 2(30) = 60.
time = 0.71, size = 124, normalized size = 4.13

$$\frac{-6ab(a^2+b^2)\cos(c+dx)+(-6a^3b+2ab^3)\cos(3(c+dx))+2(a^2-b^2)\left(3a^2+b^2+(3a^2-b^2)\cos(2(c+dx))\right)\sin(c+dx)}{12a\left(a^2+b^2\right)^2d(a\cos(c+dx)+b\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (-6*a*b*(a^2 + b^2)*Cos[c + d*x] + (-6*a^3*b + 2*a*b^3)*Cos[3*(c + d*x)] +
2*(a^2 - b^2)*(3*a^2 + b^2 + (3*a^2 - b^2)*Cos[2*(c + d*x)])*Sin[c + d*x])/
(12*a*(a^2 + b^2)^2*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)
```

Maple [A]
time = 0.64, size = 21, normalized size = 0.70

method	result	size
derivativedivides	$-\frac{1}{3db\left(a+b\tan(dx+c)\right)^3}$	21
default	$-\frac{1}{3db\left(a+b\tan(dx+c)\right)^3}$	21
risch	$\frac{2i\left(6iab\,e^{4i(dx+c)}-3a^2e^{4i(dx+c)}+3b^2e^{4i(dx+c)}+6iab\,e^{2i(dx+c)}-6a^2e^{2i(dx+c)}-3a^2+b^2\right)}{3\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)^3d(ia+b)^3}$	128
norman	$\frac{\frac{1}{3bd}-\frac{\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)}{3bd}-\frac{\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)}{3bd}+\frac{\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)}{3db}-\frac{2\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3db}+\frac{2\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3db}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^3}$	152

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/d/b/(a+b*tan(d*x+c))^3
```

Maxima [A]
time = 0.31, size = 53, normalized size = 1.77

$$-\frac{1}{3\left(b^4\tan\left(dx+c\right)^3+3\,ab^3\tan\left(dx+c\right)^2+3\,a^2b^2\tan\left(dx+c\right)+a^3b\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3/((b^4*tan(d*x + c)^3 + 3*a*b^3*tan(d*x + c)^2 + 3*a^2*b^2*tan(d*x + c)
 + a^3*b)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(28) = 56.
time = 2.48, size = 255, normalized size = 8.50

$$-\frac{(9\,a^4b-6\,a^2b^3+b^5)\cos\left(dx+c\right)^3-3\,(a^4b-3\,a^2b^3)\cos\left(dx+c\right)-\left(a^3b^2-3\,ab^4+(3\,a^5-10\,a^3b^2+3\,ab^4)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{3\left((a^9-6\,a^5b^4-8\,a^3b^6-3\,ab^8)d\cos\left(dx+c\right)^3+3\,(a^7b^2+3\,a^5b^4+3\,a^3b^6+ab^8)d\cos\left(dx+c\right)+\left((3\,a^8b+8\,a^6b^3+6\,a^4b^5-b^9)d\cos\left(dx+c\right)^2+(a^6b^3+3\,a^4b^5+3\,a^2b^7+b^9)d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*((9*a^4*b - 6*a^2*b^3 + b^5)*cos(d*x + c)^3 - 3*(a^4*b - 3*a^2*b^3)*co
s(d*x + c) - (a^3*b^2 - 3*a*b^4 + (3*a^5 - 10*a^3*b^2 + 3*a*b^4)*cos(d*x +
c)^2)*sin(d*x + c))/((a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d*cos(d*x + c)
^3 + 3*(a^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*d*cos(d*x + c) + ((3*a^8*b
 + 8*a^6*b^3 + 6*a^4*b^5 - b^9)*d*cos(d*x + c)^2 + (a^6*b^3 + 3*a^4*b^5 + 3
*a^2*b^7 + b^9)*d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.48, size = 20, normalized size = 0.67

$$-\frac{1}{3\left(b\tan\left(dx+c\right)+a\right)^{3}bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3/((b*tan(d*x + c) + a)^3*b*d)
```

Mupad [**B**]
time = 1.29, size = 224, normalized size = 7.47

$$\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{a}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a}+\frac{4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^2}-\frac{4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{a^2}-\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(3\,a^2-2\,b^2\right)}{3\,a^3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(12\,a\,b^2-3\,a^3\right)-a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(12\,a\,b^2-3\,a^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(12\,a^2\,b-8\,b^3\right)+a^3+6\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^5)/a + (2*tan(c/2 + (d*x)/2))/a + (4*b*tan(c/2 + (d*
x)/2)^2)/a^2 - (4*b*tan(c/2 + (d*x)/2)^4)/a^2 - (4*tan(c/2 + (d*x)/2)^3*(3*
a^2 - 2*b^2))/(3*a^3))/(d*(tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 3*a^3) - a^3*ta
n(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^4*(12*a*b^2 - 3*a^3) - tan(c/2 + (d
*x)/2)^3*(12*a^2*b - 8*b^3) + a^3 + 6*a^2*b*tan(c/2 + (d*x)/2) + 6*a^2*b*ta
n(c/2 + (d*x)/2)^5))
```

3.144 $\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=141

$$-\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{5/2}d}-\frac{b}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^3}-\frac{a(b\cos(c+dx)-a$$

```
[Out] -1/2*a*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)
/d-1/3*b/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^3-1/2*a*(b*cos(d*x+c)-a*si
n(d*x+c))/(a^2+b^2)^2/d/(a*cos(d*x+c)+b*sin(d*x+c))^2
```

Rubi [A]
time = 0.08, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3237, 12, 3155, 3153, 212}

$$-\frac{b}{3d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^3}-\frac{a(b\cos(c+dx)-a\sin(c+dx))}{2d\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^2}-\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2d\left(a^2+b^2\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] -1/2*(a*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/((a^2 +
 b^2)^(5/2)*d) - b/(3*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3) -
(a*(b*Cos[c + d*x] - a*Sin[c + d*x]))/(2*(a^2 + b^2)^2*d*(a*Cos[c + d*x] +
b*Sin[c + d*x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3155

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3237

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))*((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^(n_), x_Symbol] :> Simp[(-(c*B + c
*A*Cos[d + e*x] + (a*B - b*A)*Sin[d + e*x]))*((a + b*Cos[d + e*x] + c*Sin[d
 + e*x])^(n + 1)/(e*(n + 1)*(a^2 - b^2 - c^2))), x] + Dist[1/((n + 1)*(a^2
- b^2 - c^2)), Int[(a + b*Cos[d + e*x] + c*Sin[d + e*x])^(n + 1)*Simp[(n +
1)*(a*A - b*B) + (n + 2)*(a*B - b*A)*Cos[d + e*x] - (n + 2)*c*A*Sin[d + e*x
], x], x], x] /; FreeQ[{a, b, c, d, e, A, B}, x] && LtQ[n, -1] && NeQ[a^2 -
 b^2 - c^2, 0] && NeQ[n, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx &= -\frac{b}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{\int \frac{3a}{(a\cos(c+dx)+b\sin(c+\ldots}}{3\left(a^2+b^2\right)\ldots}\\
&= -\frac{b}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{a\int \frac{1}{(a\cos(c+dx)+b\sin(\ldots}}{a^2+b^2\ldots}\\
&= -\frac{b}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^3}-\frac{a(b\cos(c+\ldots}{2\left(a^2+b^2\right)^2d(a\cos\ldots}\\
&= -\frac{b}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^3}-\frac{a(b\cos(c+\ldots}{2\left(a^2+b^2\right)^2d(a\cos\ldots}\\
&= -\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2\left(a^2+b^2\right)^{5/2}d}-\frac{b}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\,\mathrm{s}\ldots}
\end{aligned}
$$

Mathematica [A]
time = 0.80, size = 128, normalized size = 0.91

$$
\frac{\frac{6a\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^{5/2}}+\frac{-4b\left(a^2+b^2\right)-6a^2b\cos(2(c+dx))+3\left(a^3-ab^2\right)\sin(2(c+dx))}{2\left(a^2+b^2\right)^2(a\cos(c+dx)+b\sin(c+dx))^3}}{6d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] ((6*a*ArcTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2)
 + (-4*b*(a^2 + b^2) - 6*a^2*b*Cos[2*(c + d*x)] + 3*(a^3 - a*b^2)*Sin[2*(c
+ d*x)])/(2*(a^2 + b^2)^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3))/(6*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(131) = 262$.
time = 0.66, size = 383, normalized size = 2.72

method	result
risch	$-\frac{i\left(-8ib^3 e^{3i(dx+c)}-8ia^2b\, e^{3i(dx+c)}+3a\, b^2 e^{i(dx+c)}-6ia^2b\, e^{i(dx+c)}+3a^3 e^{5i(dx+c)}-6ia^2b\, e^{5i(dx+c)}-3a\, b^2 e^{5i(dx+c)}-3a \cdots\right.}{3\left(-ib\, e^{2i(dx+c)}+a\, e^{2i(dx+c)}+ib+a\right)^3 (ib+a)^2 d(-ib+a)^2}$
derivativedivides	$-\frac{\frac{2\left(-\frac{\left(a^4+4a^2b^2+2b^4\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a\left(a^4+2a^2b^2+b^4\right)}-\frac{b\left(a^4-8a^2b^2-4b^4\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2\left(a^4+2a^2b^2+b^4\right)}+\frac{b^2\left(15a^4-4a^2b^2-4b^4\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a^3\left(a^4+2a^2b^2+b^4\right)}+\frac{b\left(2a^4-5 \cdots\right.}{\cdots}\right.}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^3}\cdots}{d}$
default	$-\frac{\frac{2\left(-\frac{\left(a^4+4a^2b^2+2b^4\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a\left(a^4+2a^2b^2+b^4\right)}-\frac{b\left(a^4-8a^2b^2-4b^4\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a^2\left(a^4+2a^2b^2+b^4\right)}+\frac{b^2\left(15a^4-4a^2b^2-4b^4\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3a^3\left(a^4+2a^2b^2+b^4\right)}+\frac{b\left(2a^4-5 \cdots\right.}{\cdots}\right.}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^3}\cdots}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/2*(a^4+4*a^2*b^2+2*b^4)/a/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c
)^5-1/2*b*(a^4-8*a^2*b^2-4*b^4)/a^2/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)^
4+1/3/a^3*b^2*(15*a^4-4*a^2*b^2-4*b^4)/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*
c)^3+1/a^2*b*(2*a^4-5*a^2*b^2-2*b^4)/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)
^2+1/2/a*(a^4-6*a^2*b^2-2*b^4)/(a^4+2*a^2*b^2+b^4)*tan(1/2*d*x+1/2*c)-1/6*b
*(5*a^2+2*b^2)/(a^4+2*a^2*b^2+b^4))/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x
+1/2*c)-a)^3+a/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2
*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 606 vs. $2(131) = 262$.
time = 0.51, size = 606, normalized size = 4.30

$$-\frac{\frac{3\,a\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}}+\frac{2\left(5\,a^5b+2\,a^3b^3-\frac{3\left(a^6-6\,a^4b^2-2\,a^2b^4\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{6\left(2\,a^5b-5\,a^3b^3-2\,ab^5\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2\left(15\,a^4b^2-4\,a^2b^4-4\,b^6\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\left(a^5b-8\,a^3b^3-4\,ab^5\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{3\left(a^6+4\,a^4b^2+2\,a^2b^4\right)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^{10}+2\,a^8b^2+a^6b^4+\frac{6\left(a^9b+2\,a^7b^3+a^5b^5\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\left(a^{10}-2\,a^8b^2-7\,a^6b^4-4\,a^4b^6\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4\left(3\,a^9b+4\,a^7b^3-a^5b^5-2\,a^3b^7\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\left(a^{10}-2\,a^8b^2-7\,a^6b^4-4\,a^4b^6\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{6\left(a^9b+2\,a^7b^3+a^5b^5\right)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{\left(a^{10}+2\,a^8b^2+a^6b^4\right)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/6*(3*a*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(a^2 + b^2))/(b
- a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 +
```

```
b^4)*sqrt(a^2 + b^2)) + 2*(5*a^5*b + 2*a^3*b^3 - 3*(a^6 - 6*a^4*b^2 - 2*a^
2*b^4)*sin(d*x + c)/(cos(d*x + c) + 1) - 6*(2*a^5*b - 5*a^3*b^3 - 2*a*b^5)*
sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(15*a^4*b^2 - 4*a^2*b^4 - 4*b^6)*si
n(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*(a^5*b - 8*a^3*b^3 - 4*a*b^5)*sin(d*x
 + c)^4/(cos(d*x + c) + 1)^4 + 3*(a^6 + 4*a^4*b^2 + 2*a^2*b^4)*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5)/(a^10 + 2*a^8*b^2 + a^6*b^4 + 6*(a^9*b + 2*a^7*b^3
 + a^5*b^5)*sin(d*x + c)/(cos(d*x + c) + 1) - 3*(a^10 - 2*a^8*b^2 - 7*a^6*b
^4 - 4*a^4*b^6)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 4*(3*a^9*b + 4*a^7*b^
3 - a^5*b^5 - 2*a^3*b^7)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*(a^10 - 2*
a^8*b^2 - 7*a^6*b^4 - 4*a^4*b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 6*(a
^9*b + 2*a^7*b^3 + a^5*b^5)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - (a^10 + 2
*a^8*b^2 + a^6*b^4)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(131) = 262$.
time $= 2.45$, size $= 420$, normalized size $= 2.98$

$$\frac{2\,a^4b-2\,a^2b^3-4\,b^5-12\,(a^4b+a^2b^3)\cos(dx+c)^2+6\,(a^5-ab^4)\cos(dx+c)\sin(dx+c)+3\,\left(3\,a^2b^2\cos(dx+c)+(a^4-3\,a^2b^2)\cos(dx+c)^3+\left(ab^3+(3\,a^3b-ab^3)\cos(dx+c)^2\right)\sin(dx+c)\right)\sqrt{a^2+b^2}\,\log\left(-\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\,(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2}\right)}{12\,\left((a^9-6\,a^5b^4-8\,a^3b^6-3\,ab^8)d\cos(dx+c)^3+3\,(a^7b^2+3\,a^5b^4+3\,a^3b^6+ab^8)d\cos(dx+c)+\left((3\,a^8b+8\,a^6b^3+6\,a^4b^5-b^9)d\cos(dx+c)^2+(a^6b^3+3\,a^4b^5+3\,a^2b^7+b^9)d\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/12*(2*a^4*b - 2*a^2*b^3 - 4*b^5 - 12*(a^4*b + a^2*b^3)*cos(d*x + c)^2 + 6
*(a^5 - a*b^4)*cos(d*x + c)*sin(d*x + c) + 3*(3*a^2*b^2*cos(d*x + c) + (a^4
 - 3*a^2*b^2)*cos(d*x + c)^3 + (a*b^3 + (3*a^3*b - a*b^3)*cos(d*x + c)^2)*s
in(d*x + c))*sqrt(a^2 + b^2)*log(-(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 -
 b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(d*x + c) - a*
sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^
2 + b^2)))/((a^9 - 6*a^5*b^4 - 8*a^3*b^6 - 3*a*b^8)*d*cos(d*x + c)^3 + 3*(a
^7*b^2 + 3*a^5*b^4 + 3*a^3*b^6 + a*b^8)*d*cos(d*x + c) + ((3*a^8*b + 8*a^6*
b^3 + 6*a^4*b^5 - b^9)*d*cos(d*x + c)^2 + (a^6*b^3 + 3*a^4*b^5 + 3*a^2*b^7
+ b^9)*d)*sin(d*x + c))
```

Sympy [**F(-1)**] Timed out
time $= 0.00$, size $= 0$, normalized size $= 0.00$

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(131) = 262$.
time $= 0.51$, size $= 426$, normalized size $= 3.02$

$$-\frac{\frac{3\,a\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}}-\frac{2\left(3\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,a^4b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,a^2b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,a^5b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-24\,a^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-12\,ab^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-30\,a^4b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+8\,a^2b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+8\,b^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-12\,a^5b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+30\,a^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,ab^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+18\,a^4b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,a^2b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,a^5b+2\,a^3b^3\right)}{(a^7+2\,a^5b^2+a^3b^4)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/6*(3*a*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2
*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4
)*sqrt(a^2 + b^2)) - 2*(3*a^6*tan(1/2*d*x + 1/2*c)^5 + 12*a^4*b^2*tan(1/2*d
*x + 1/2*c)^5 + 6*a^2*b^4*tan(1/2*d*x + 1/2*c)^5 + 3*a^5*b*tan(1/2*d*x + 1/
2*c)^4 - 24*a^3*b^3*tan(1/2*d*x + 1/2*c)^4 - 12*a*b^5*tan(1/2*d*x + 1/2*c)^
4 - 30*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 + 8*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 +
8*b^6*tan(1/2*d*x + 1/2*c)^3 - 12*a^5*b*tan(1/2*d*x + 1/2*c)^2 + 30*a^3*b^3
*tan(1/2*d*x + 1/2*c)^2 + 12*a*b^5*tan(1/2*d*x + 1/2*c)^2 - 3*a^6*tan(1/2*d
*x + 1/2*c) + 18*a^4*b^2*tan(1/2*d*x + 1/2*c) + 6*a^2*b^4*tan(1/2*d*x + 1/2
*c) + 5*a^5*b + 2*a^3*b^3)/((a^7 + 2*a^5*b^2 + a^3*b^4)*(a*tan(1/2*d*x + 1/
2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a)^3))/d
```

Mupad [B]
time = 3.82, size = 505, normalized size = 3.58

$$-\frac{\frac{\frac{5\,a^2\,b}{3}+\frac{2\,b^3}{3}}{a^4+2\,a^2\,b^2+b^4}-\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(-a^4\,b+8\,a^2\,b^3+4\,b^5\right)}{a^2\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(-4\,a^4\,b+10\,a^2\,b^3+4\,b^5\right)}{a^2\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(-a^4+6\,a^2\,b^2+2\,b^4\right)}{a\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(a^4+4\,a^2\,b^2+2\,b^4\right)}{a\,\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{2\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(5\,a^2\,b+2\,b^3\right)\,\left(3\,a^2-2\,b^2\right)}{3\,a^3\,\left(a^4+2\,a^2\,b^2+b^4\right)}}{d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(12\,a\,b^2-3\,a^3\right)-a^3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(12\,a\,b^2-3\,a^3\right)-{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(12\,a^2\,b-8\,b^3\right)+a^3+6\,a^2\,b\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6\,a^2\,b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\right)}-\frac{a\,\mathrm{atanh}\left(\frac{a^4\,b+2\,a^2\,b^3+b^5}{{\left(a^2+b^2\right)}^{5/2}}-\frac{a\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}{{\left(a^2+b^2\right)}^{5/2}}\right)}{d\,{\left(a^2+b^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

```
[Out] - (((5*a^2*b)/3 + (2*b^3)/3)/(a^4 + b^4 + 2*a^2*b^2) - (tan(c/2 + (d*x)/2)^
4*(4*b^5 - a^4*b + 8*a^2*b^3))/(a^2*(a^4 + b^4 + 2*a^2*b^2)) + (tan(c/2 + (
d*x)/2)^2*(4*b^5 - 4*a^4*b + 10*a^2*b^3))/(a^2*(a^4 + b^4 + 2*a^2*b^2)) + (
tan(c/2 + (d*x)/2)*(2*b^4 - a^4 + 6*a^2*b^2))/(a*(a^4 + b^4 + 2*a^2*b^2)) +
 (tan(c/2 + (d*x)/2)^5*(a^4 + 2*b^4 + 4*a^2*b^2))/(a*(a^4 + b^4 + 2*a^2*b^2
)) - (2*b*tan(c/2 + (d*x)/2)^3*(5*a^2*b + 2*b^3)*(3*a^2 - 2*b^2))/(3*a^3*(a
^4 + b^4 + 2*a^2*b^2)))/(d*(tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 3*a^3) - a^3*t
an(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^4*(12*a*b^2 - 3*a^3) - tan(c/2 + (
d*x)/2)^3*(12*a^2*b - 8*b^3) + a^3 + 6*a^2*b*tan(c/2 + (d*x)/2) + 6*a^2*b*t
an(c/2 + (d*x)/2)^5)) - (a*atanh((a^4*b + b^5 + 2*a^2*b^3)/(a^2 + b^2)^(5/2
) - (a*tan(c/2 + (d*x)/2)*(a^4 + b^4 + 2*a^2*b^2))/(a^2 + b^2)^(5/2)))/(d*(
a^2 + b^2)^(5/2))
```

3.145 $\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=98

$$-\frac{b\cos(c+dx)-a\sin(c+dx)}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{2\sin(c+dx)}{3a\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))}$$

```
[Out] 1/3*(-b*cos(d*x+c)+a*sin(d*x+c))/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^3+
2/3*sin(d*x+c)/a/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))
```

Rubi [A]
time = 0.03, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}=0.105$, Rules used = {3155, 3154}

$$\frac{2\sin(c+dx)}{3ad\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))}-\frac{b\cos(c+dx)-a\sin(c+dx)}{3d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-4),x]
```

```
[Out] -1/3*(b*Cos[c + d*x] - a*Sin[c + d*x])/((a^2 + b^2)*d*(a*Cos[c + d*x] + b*S
in[c + d*x])^3) + (2*Sin[c + d*x])/(3*a*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*S
in[c + d*x]))
```

Rule 3154

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3155

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx &= -\frac{b\cos(c+dx)-a\sin(c+dx)}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{2\int \frac{1}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx}{3\left(a^2+b^2\right)}\\ &= -\frac{b\cos(c+dx)-a\sin(c+dx)}{3\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{2\sin(c+dx)}{3a\left(a^2+b^2\right)d(a\cos(c+dx)+b\sin(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.32, size = 85, normalized size = 0.87

$$\frac{-ab\cos(3(c+dx))+(2a^2+b^2+(a^2-b^2)\cos(2(c+dx)))\sin(c+dx)}{3a(a^2+b^2)\,d(a\cos(c+dx)+b\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + b*Sin[c + d*x])^(-4),x]
[Out] (-(a*b*Cos[3*(c + d*x)]) + (2*a^2 + b^2 + (a^2 - b^2)*Cos[2*(c + d*x)])*Sin
[c + d*x])/(3*a*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)
```

Maple [A]
time = 0.45, size = 64, normalized size = 0.65

method	result	size
derivativedivides	$\frac{-\frac{1}{b^3(a+b\tan(dx+c))}+\frac{a}{b^3(a+b\tan(dx+c))^2}-\frac{a^2+b^2}{3b^3(a+b\tan(dx+c))^3}}{d}$	64
default	$\frac{-\frac{1}{b^3(a+b\tan(dx+c))}+\frac{a}{b^3(a+b\tan(dx+c))^2}-\frac{a^2+b^2}{3b^3(a+b\tan(dx+c))^3}}{d}$	64
risch	$\frac{4i\left(3ia\,e^{2i(dx+c)}+3b\,e^{2i(dx+c)}+ia-b\right)}{3\left(b\,e^{2i(dx+c)}+ia\,e^{2i(dx+c)}-b+ia\right)^3 d(ia+b)^2}$	82
norman	$\frac{\frac{1}{3bd}-\frac{\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)}{3db}-\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{bd}+\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{db}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-a\right)^3}$	117

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/b^3/(a+b*tan(d*x+c))+a/b^3/(a+b*tan(d*x+c))^2-1/3*(a^2+b^2)/b^3/(a+
b*tan(d*x+c))^3)
```

Maxima [A]
time = 0.29, size = 85, normalized size = 0.87

$$-\frac{3\,b^2\tan\left(dx+c\right)^2+3\,ab\tan\left(dx+c\right)+a^2+b^2}{3\left(b^6\tan\left(dx+c\right)^3+3\,ab^5\tan\left(dx+c\right)^2+3\,a^2b^4\tan\left(dx+c\right)+a^3b^3\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
[Out] -1/3*(3*b^2*tan(d*x + c)^2 + 3*a*b*tan(d*x + c) + a^2 + b^2)/((b^6*tan(d*x
+ c)^3 + 3*a*b^5*tan(d*x + c)^2 + 3*a^2*b^4*tan(d*x + c) + a^3*b^3)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(94) = 188$.
time = 3.31, size = 217, normalized size = 2.21

$$-\frac{2\,(3\,a^2b-b^3)\cos\left(dx+c\right)^3-3\,(a^2b-b^3)\cos\left(dx+c\right)-\left(a^3+3\,ab^2+2\,(a^3-3\,ab^2)\cos\left(dx+c\right)^2\right)\sin\left(dx+c\right)}{3\left((a^7-a^5b^2-5\,a^3b^4-3\,ab^6)d\cos\left(dx+c\right)^3+3\,(a^5b^2+2\,a^3b^4+ab^6)d\cos\left(dx+c\right)+\left((3\,a^6b+5\,a^4b^3+a^2b^5-b^7)d\cos\left(dx+c\right)^2+(a^4b^3+2\,a^2b^5+b^7)d\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(2*(3*a^2*b - b^3)*cos(d*x + c)^3 - 3*(a^2*b - b^3)*cos(d*x + c) - (a^
3 + 3*a*b^2 + 2*(a^3 - 3*a*b^2)*cos(d*x + c)^2)*sin(d*x + c))/((a^7 - a^5*b
^2 - 5*a^3*b^4 - 3*a*b^6)*d*cos(d*x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^4 + a*b^6
)*d*cos(d*x + c) + ((3*a^6*b + 5*a^4*b^3 + a^2*b^5 - b^7)*d*cos(d*x + c)^2
+ (a^4*b^3 + 2*a^2*b^5 + b^7)*d)*sin(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.43, size = 50, normalized size = 0.51

$$-\frac{3\,b^2\tan\left(dx+c\right)^2+3\,ab\tan\left(dx+c\right)+a^2+b^2}{3\left(b\tan\left(dx+c\right)+a\right)^3b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(3*b^2*tan(d*x + c)^2 + 3*a*b*tan(d*x + c) + a^2 + b^2)/((b*tan(d*x +
c) + a)^3*b^3*d)
```

Mupad [B]
time = 1.25, size = 222, normalized size = 2.27

$$\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{a}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a}-\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(a^2-2\,b^2)}{3\,a^3}+\frac{4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{a^2}-\frac{4\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{a^2}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(12\,a\,b^2-3\,a^3)-a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,(12\,a\,b^2-3\,a^3)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(12\,a^2\,b-8\,b^3)+a^3+6\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+6\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + b*sin(c + d*x))^4,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^5)/a + (2*tan(c/2 + (d*x)/2))/a - (4*tan(c/2 + (d*x)
/2)^3*(a^2 - 2*b^2))/(3*a^3) + (4*b*tan(c/2 + (d*x)/2)^2)/a^2 - (4*b*tan(c/
2 + (d*x)/2)^4)/a^2)/(d*(tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 3*a^3) - a^3*tan(
c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^4*(12*a*b^2 - 3*a^3) - tan(c/2 + (d*x
)/2)^3*(12*a^2*b - 8*b^3) + a^3 + 6*a^2*b*tan(c/2 + (d*x)/2) + 6*a^2*b*tan(
c/2 + (d*x)/2)^5))
```

3.146 $\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=231

$$\frac{\tanh^{-1}(\sin(c+dx))}{b^4 d}+\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2b^2\left(a^2+b^2\right)^{3/2}d}+\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4\sqrt{a^2+b^2}\,d}-\frac{1}{3bd(a\cos(c+dx)-}$$

```
[Out] arctanh(sin(d*x+c))/b^4/d+1/2*a*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+b^
2)^(1/2))/b^2/(a^2+b^2)^(3/2)/d-1/3/b/d/(a*cos(d*x+c)+b*sin(d*x+c))^3+1/2*a
*(b*cos(d*x+c)-a*sin(d*x+c))/b^2/(a^2+b^2)/d/(a*cos(d*x+c)+b*sin(d*x+c))^2-
1/b^3/d/(a*cos(d*x+c)+b*sin(d*x+c))+a*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(
a^2+b^2)^(1/2))/b^4/d/(a^2+b^2)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3173, 3855, 3153, 212, 3155}

$$\frac{a(b\cos(c+dx)-a\sin(c+dx))}{2b^2d\left(a^2+b^2\right)(a\cos(c+dx)+b\sin(c+dx))^2}+\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2b^2d\left(a^2+b^2\right)^{3/2}}+\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^4d\sqrt{a^2+b^2}}-\frac{1}{b^3d(a\cos(c+dx)+b\sin(c+dx))}-\frac{1}{3bd(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{\tanh^{-1}(\sin(c+dx))}{b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(b^4*d) + (a*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x]
)/Sqrt[a^2 + b^2]])/(2*b^2*(a^2 + b^2)^(3/2)*d) + (a*ArcTanh[(b*Cos[c + d*x
] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^4*Sqrt[a^2 + b^2]*d) - 1/(3*b*d*(a
*Cos[c + d*x] + b*Sin[c + d*x])^3) + (a*(b*Cos[c + d*x] - a*Sin[c + d*x]))/
(2*b^2*(a^2 + b^2)*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - 1/(b^3*d*(a*Cos
[c + d*x] + b*Sin[c + d*x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3155

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3173

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/co
s[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^
(n + 1)/(b*d*(n + 1)), x] + (Dist[1/b^2, Int[(a*Cos[c + d*x] + b*Sin[c + d*
x])^(n + 2)/Cos[c + d*x], x], x] - Dist[a/b^2, Int[(a*Cos[c + d*x] + b*Sin[
c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &
& LtQ[n, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx &= -\frac{1}{3bd(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx}{b^2} \\ &= -\frac{1}{3bd(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{a(b\cos(c+dx)-a}{2b^2\left(a^2+b^2\right)d(a\cos(c+d} \\ &= \frac{\tanh^{-1}(\sin(c+dx))}{b^4d}-\frac{1}{3bd(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{}{2b^2(} \\ &= \frac{\tanh^{-1}(\sin(c+dx))}{b^4d}+\frac{a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2b^2\left(a^2+b^2\right)^{3/2}d}+\frac{a\tanh^{-}}{}\end{aligned}$$

Mathematica [A]
time = 3.48, size = 290, normalized size = 1.26

$$-\frac{\sec^3(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(2b^3\sec(c+dx)+3b^2(a\cos(c+dx)+b\sin(c+dx))\tan(c+dx)+\frac{3b(2a^2+b^2)\cos(c+dx)(a+b\tan(c+dx))^2}{a^2+b^2}+\frac{6a(2a^2+3b^2)\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)\cos^2(c+dx)(a+b\tan(c+dx))^3}{(a^2+b^2)^{3/2}}+6\cos^2(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3-6\cos^2(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\tan(c+dx))^3\right)}{6b^4d(a+b\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] -1/6*(Sec[c + d*x]^3*(a*Cos[c + d*x] + b*Sin[c + d*x])*(2*b^3*Sec[c + d*x]
+ 3*b^2*(a*Cos[c + d*x] + b*Sin[c + d*x])*Tan[c + d*x] + (3*b*(2*a^2 + b^2)
*Cos[c + d*x]*(a + b*Tan[c + d*x])^2)/(a^2 + b^2) + (6*a*(2*a^2 + 3*b^2)*Ar
cTanh[(-b + a*Tan[(c + d*x)/2])/Sqrt[a^2 + b^2]]*Cos[c + d*x]^2*(a + b*Tan[
c + d*x])^3)/(a^2 + b^2)^(3/2) + 6*Cos[c + d*x]^2*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3 - 6*Cos[c + d*x]^2*Log[Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]]*(a + b*Tan[c + d*x])^3))/(b^4*d*(a + b*Tan[c + d*x]
)^4)
```

Maple [**A**]
time = 1.07, size = 411, normalized size = 1.78

method	result
derivativedivides	$-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^4}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^4}+\frac{2\left(\frac{b^2\left(a^4+2a^2b^2+2b^4\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^2+b^2\right)a}+\frac{b\left(2a^6-3a^4b^2-4a^2b^4-4b^6\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^2+b^2\right)a^2}\right.}{}$ (formula cut off at the page edge)
default	$-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^4}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^4}+\frac{2\left(\frac{b^2\left(a^4+2a^2b^2+2b^4\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^2+b^2\right)a}+\frac{b\left(2a^6-3a^4b^2-4a^2b^4-4b^6\right)\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2\left(a^2+b^2\right)a^2}\right.}{}$ (formula cut off at the page edge)
risch	$-\frac{12a^4e^{3i(dx+c)}+20b^4e^{3i(dx+c)}-6b^4e^{5i(dx+c)}+6a^4e^{i(dx+c)}-15ia^3b\,e^{5i(dx+c)}+6a^4e^{5i(dx+c)}+32a^2b^2e^{3i(dx+c)}-6a^2b^2e^{i(}}{3(ib+a)b^3\left(-ib\,e^{2i(dx+c)}+a\,e^{2i(dx+c)}+ib+a\right)}$ (formula cut off at the page edge)

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/b^4*ln(tan(1/2*d*x+1/2*c)-1)+1/b^4*ln(tan(1/2*d*x+1/2*c)+1)+2/b^4*(
(1/2*b^2*(a^4+2*a^2*b^2+2*b^4)/(a^2+b^2)/a*tan(1/2*d*x+1/2*c)^5+1/2*b*(2*a^
6-3*a^4*b^2-4*a^2*b^4-4*b^6)/(a^2+b^2)/a^2*tan(1/2*d*x+1/2*c)^4-1/3/a^3*b^2
*(18*a^6+3*a^4*b^2-4*a^2*b^4-4*b^6)/(a^2+b^2)*tan(1/2*d*x+1/2*c)^3-1/a^2*b*
(2*a^6-8*a^4*b^2-7*a^2*b^4-2*b^6)/(a^2+b^2)*tan(1/2*d*x+1/2*c)^2+1/2/a*b^2*
(11*a^4+8*a^2*b^2+2*b^4)/(a^2+b^2)*tan(1/2*d*x+1/2*c)+1/6*b*(6*a^4+5*a^2*b^
2+2*b^4)/(a^2+b^2))/(a*tan(1/2*d*x+1/2*c)^2-2*b*tan(1/2*d*x+1/2*c)-a)^3-1/2
*a*(2*a^2+3*b^2)/(a^2+b^2)^(3/2)*arctanh(1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(
a^2+b^2)^(1/2))))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 661 vs. 2(217) = 434.
time = 0.49, size = 661, normalized size = 2.86

$$\frac{\frac{3\,(2\,a^2+3\,b^2)a\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{(a^2b^4+b^6)\sqrt{a^2+b^2}}-\frac{2\left(6\,a^7+5\,a^5b^2+2\,a^3b^4+\frac{3\left(11\,a^6b+8\,a^4b^3+2\,a^2b^5\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{6\left(2\,a^7-8\,a^5b^2-7\,a^3b^4-2\,ab^6\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2\left(18\,a^6b+3\,a^4b^3-4\,a^2b^5-4\,b^7\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\left(2\,a^7-3\,a^5b^2-4\,a^3b^4-4\,ab^6\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{3\left(a^6b+2\,a^4b^3+2\,a^2b^5\right)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^8b^3+a^6b^5+\frac{6\left(a^7b^4+a^5b^6\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\left(a^8b^3-3\,a^6b^5-4\,a^4b^7\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{4\left(3\,a^7b^4+a^5b^6-2\,a^3b^8\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\left(a^8b^3-3\,a^6b^5-4\,a^4b^7\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{6\left(a^7b^4+a^5b^6\right)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{\left(a^8b^3+a^6b^5\right)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{6\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^4}-\frac{6\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(2*a^2 + 3*b^2)*a*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt(
a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/((a^
2*b^4 + b^6)*sqrt(a^2 + b^2)) - 2*(6*a^7 + 5*a^5*b^2 + 2*a^3*b^4 + 3*(11*a^
6*b + 8*a^4*b^3 + 2*a^2*b^5)*sin(d*x + c)/(cos(d*x + c) + 1) - 6*(2*a^7 - 8
*a^5*b^2 - 7*a^3*b^4 - 2*a*b^6)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(18
*a^6*b + 3*a^4*b^3 - 4*a^2*b^5 - 4*b^7)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
 + 3*(2*a^7 - 3*a^5*b^2 - 4*a^3*b^4 - 4*a*b^6)*sin(d*x + c)^4/(cos(d*x + c)
 + 1)^4 + 3*(a^6*b + 2*a^4*b^3 + 2*a^2*b^5)*sin(d*x + c)^5/(cos(d*x + c) +
1)^5)/(a^8*b^3 + a^6*b^5 + 6*(a^7*b^4 + a^5*b^6)*sin(d*x + c)/(cos(d*x + c)
 + 1) - 3*(a^8*b^3 - 3*a^6*b^5 - 4*a^4*b^7)*sin(d*x + c)^2/(cos(d*x + c) +
1)^2 - 4*(3*a^7*b^4 + a^5*b^6 - 2*a^3*b^8)*sin(d*x + c)^3/(cos(d*x + c) + 1
)^3 + 3*(a^8*b^3 - 3*a^6*b^5 - 4*a^4*b^7)*sin(d*x + c)^4/(cos(d*x + c) + 1)
^4 + 6*(a^7*b^4 + a^5*b^6)*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - (a^8*b^3 +
 a^6*b^5)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + 6*log(sin(d*x + c)/(cos(d*
x + c) + 1) + 1)/b^4 - 6*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/b^4)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 745 vs. $2(217) = 434$.
time = 3.12, size = 745, normalized size = 3.23

$$-\frac{22\,a^{4}b^{3}+38\,a^{2}b^{5}+16\,b^{7}+12\,(a^{6}b-2\,a^{2}b^{5}-b^{7})\cos(dx+c)^{2}+6\,(5\,a^{5}b^{2}+8\,a^{3}b^{4}+3\,ab^{6})\cos(dx+c)\sin(dx+c)-3\,\left((2\,a^{6}-3\,a^{4}b^{2}-9\,a^{2}b^{4})\cos(dx+c)^{3}+3\,(2\,a^{4}b^{2}+3\,a^{2}b^{4})\cos(dx+c)+(2\,a^{3}b^{3}+3\,ab^{5}+(6\,a^{5}b+7\,a^{3}b^{3}-3\,ab^{5})\cos(dx+c)^{2})\sin(dx+c)\right)\sqrt{a^{2}+b^{2}}\log\left(\frac{2\,ab\cos(dx+c)\sin(dx+c)+(a^{2}-b^{2})\cos(dx+c)^{2}-2\,a^{2}-b^{2}-2\,\sqrt{a^{2}+b^{2}}(b\cos(dx+c)-a\sin(dx+c))}{2\,ab\cos(dx+c)\sin(dx+c)+(a^{2}-b^{2})\cos(dx+c)^{2}+b^{2}}\right)-6\,\left((a^{7}-a^{5}b^{2}-5\,a^{3}b^{4}-3\,ab^{6})\cos(dx+c)^{3}+3\,(a^{5}b^{2}+2\,a^{3}b^{4}+ab^{6})\cos(dx+c)+(a^{4}b^{3}+2\,a^{2}b^{5}+b^{7}+(3\,a^{6}b+5\,a^{4}b^{3}+a^{2}b^{5}-b^{7})\cos(dx+c)^{2})\sin(dx+c)\right)\log(\sin(dx+c)+1)+6\,\left((a^{7}-a^{5}b^{2}-5\,a^{3}b^{4}-3\,ab^{6})\cos(dx+c)^{3}+3\,(a^{5}b^{2}+2\,a^{3}b^{4}+ab^{6})\cos(dx+c)+(a^{4}b^{3}+2\,a^{2}b^{5}+b^{7}+(3\,a^{6}b+5\,a^{4}b^{3}+a^{2}b^{5}-b^{7})\cos(dx+c)^{2})\sin(dx+c)\right)\log(-\sin(dx+c)+1)}{12\,\left((a^{7}b^{4}-a^{5}b^{6}-5\,a^{3}b^{8}-3\,ab^{10})d\cos(dx+c)^{3}+3\,(a^{5}b^{6}+2\,a^{3}b^{8}+ab^{10})d\cos(dx+c)+((3\,a^{6}b^{5}+5\,a^{4}b^{7}+a^{2}b^{9}-b^{11})d\cos(dx+c)^{2}+(a^{4}b^{7}+2\,a^{2}b^{9}+b^{11})d)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/12*(22*a^4*b^3 + 38*a^2*b^5 + 16*b^7 + 12*(a^6*b - 2*a^2*b^5 - b^7)*cos(
d*x + c)^2 + 6*(5*a^5*b^2 + 8*a^3*b^4 + 3*a*b^6)*cos(d*x + c)*sin(d*x + c)
- 3*((2*a^6 - 3*a^4*b^2 - 9*a^2*b^4)*cos(d*x + c)^3 + 3*(2*a^4*b^2 + 3*a^2*
b^4)*cos(d*x + c) + (2*a^3*b^3 + 3*a*b^5 + (6*a^5*b + 7*a^3*b^3 - 3*a*b^5)*
cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x + c)*sin(d
*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*c
os(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^
2)*cos(d*x + c)^2 + b^2)) - 6*((a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6)*cos(d*
x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cos(d*x + c) + (a^4*b^3 + 2*a^2*
b^5 + b^7 + (3*a^6*b + 5*a^4*b^3 + a^2*b^5 - b^7)*cos(d*x + c)^2)*sin(d*x +
 c))*log(sin(d*x + c) + 1) + 6*((a^7 - a^5*b^2 - 5*a^3*b^4 - 3*a*b^6)*cos(d
*x + c)^3 + 3*(a^5*b^2 + 2*a^3*b^4 + a*b^6)*cos(d*x + c) + (a^4*b^3 + 2*a^2
*b^5 + b^7 + (3*a^6*b + 5*a^4*b^3 + a^2*b^5 - b^7)*cos(d*x + c)^2)*sin(d*x
+ c))*log(-sin(d*x + c) + 1))/((a^7*b^4 - a^5*b^6 - 5*a^3*b^8 - 3*a*b^10)*d
*cos(d*x + c)^3 + 3*(a^5*b^6 + 2*a^3*b^8 + a*b^10)*d*cos(d*x + c) + ((3*a^6
*b^5 + 5*a^4*b^7 + a^2*b^9 - b^11)*d*cos(d*x + c)^2 + (a^4*b^7 + 2*a^2*b^9
+ b^11)*d)*sin(d*x + c))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)/(a*cos(c + d*x) + b*sin(c + d*x))**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 527 vs. 2(217) = 434.
time = 0.52, size = 527, normalized size = 2.28

$$\frac{\frac{3\left(2a^3+3ab^2\right)\log\left(\frac{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{\left(a^2b^4+b^6\right)\sqrt{a^2+b^2}}+\frac{2\left(3a^6b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+6a^4b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+6a^2b^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+6a^7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-9a^5b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-12a^3b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-12ab^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-36a^6b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-6a^4b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+8a^2b^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+8b^7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-12a^7\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+48a^5b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+42a^3b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+12ab^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+33a^6b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+24a^4b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+6a^2b^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+6a^7+5a^5b^2+2a^3b^4\right)}{\left(a^5b^3+a^3b^5\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-a\right)^3}+\frac{6\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^4}-\frac{6\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^4}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(3*(2*a^3 + 3*a*b^2)*log(abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^
2 + b^2))/abs(2*a*tan(1/2*d*x + 1/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/((a^2*b^
4 + b^6)*sqrt(a^2 + b^2)) + 2*(3*a^6*b*tan(1/2*d*x + 1/2*c)^5 + 6*a^4*b^3*t
an(1/2*d*x + 1/2*c)^5 + 6*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 6*a^7*tan(1/2*d*
x + 1/2*c)^4 - 9*a^5*b^2*tan(1/2*d*x + 1/2*c)^4 - 12*a^3*b^4*tan(1/2*d*x +
1/2*c)^4 - 12*a*b^6*tan(1/2*d*x + 1/2*c)^4 - 36*a^6*b*tan(1/2*d*x + 1/2*c)^
3 - 6*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 + 8*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 8
*b^7*tan(1/2*d*x + 1/2*c)^3 - 12*a^7*tan(1/2*d*x + 1/2*c)^2 + 48*a^5*b^2*ta
n(1/2*d*x + 1/2*c)^2 + 42*a^3*b^4*tan(1/2*d*x + 1/2*c)^2 + 12*a*b^6*tan(1/2
*d*x + 1/2*c)^2 + 33*a^6*b*tan(1/2*d*x + 1/2*c) + 24*a^4*b^3*tan(1/2*d*x +
1/2*c) + 6*a^2*b^5*tan(1/2*d*x + 1/2*c) + 6*a^7 + 5*a^5*b^2 + 2*a^3*b^4)/((
a^5*b^3 + a^3*b^5)*(a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) - a
)^3) + 6*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 - 6*log(abs(tan(1/2*d*x + 1
/2*c) - 1))/b^4)/d
```

Mupad [B]
time = 4.83, size = 2848, normalized size = 12.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a*cos(c + d*x) + b*sin(c + d*x))^4),x)
```

```
[Out] (2*atanh((64*a*b^5*tan(c/2 + (d*x)/2))/((176*a^3*b^15)/(b^12 + 2*a^2*b^10 +
 a^4*b^8) + (160*a^5*b^13)/(b^12 + 2*a^2*b^10 + a^4*b^8) + (48*a^7*b^11)/(b
^12 + 2*a^2*b^10 + a^4*b^8) + (64*a*b^17)/(b^12 + 2*a^2*b^10 + a^4*b^8)) +
```

(48*a^3*b^3*tan(c/2 + (d*x)/2))/((176*a^3*b^15)/(b^12 + 2*a^2*b^10 + a^4*b^8) + (160*a^5*b^13)/(b^12 + 2*a^2*b^10 + a^4*b^8) + (48*a^7*b^11)/(b^12 + 2*a^2*b^10 + a^4*b^8) + (64*a*b^17)/(b^12 + 2*a^2*b^10 + a^4*b^8))))/(b^4*d) - ((6*a^4 + 2*b^4 + 5*a^2*b^2)/(3*b^3*(a^2 + b^2)) + (tan(c/2 + (d*x)/2)*(11*a^4 + 2*b^4 + 8*a^2*b^2))/(a*b^2*(a^2 + b^2)) + (tan(c/2 + (d*x)/2)^5*(a^4 + 2*b^4 + 2*a^2*b^2))/(a*b^2*(a^2 + b^2)) - (tan(c/2 + (d*x)/2)^4*(4*b^6 - 2*a^6 + 4*a^2*b^4 + 3*a^4*b^2))/(a^2*b^3*(a^2 + b^2)) + (2*tan(c/2 + (d*x)/2)^2*(2*b^6 - 2*a^6 + 7*a^2*b^4 + 8*a^4*b^2))/(a^2*b^3*(a^2 + b^2)) - (2*tan(c/2 + (d*x)/2)^3*(3*a^2 - 2*b^2)*(6*a^4 + 2*b^4 + 5*a^2*b^2))/(3*a^3*b^2*(a^2 + b^2)))/(d*(tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 3*a^3) - a^3*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^4*(12*a*b^2 - 3*a^3) - tan(c/2 + (d*x)/2)^3*(12*a^2*b - 8*b^3) + a^3 + 6*a^2*b*tan(c/2 + (d*x)/2) + 6*a^2*b*tan(c/2 + (d*x)/2)^5)) - (a*atan(((a*((a^2 + b^2)^3)^(1/2)*(2*a^2 + 3*b^2)*((8*(4*a^2*b^7 + 8*a^4*b^5 + 4*a^6*b^3))/(b^12 + 2*a^2*b^10 + a^4*b^8) + (8*tan(c/2 + (d*x)/2)*(8*a*b^9 + 29*a^3*b^7 + 28*a^5*b^5 + 8*a^7*b^3))/(b^13 + 2*a^2*b^11 + a^4*b^9) - (a*((a^2 + b^2)^3)^(1/2)*(2*a^2 + 3*b^2)*((8*tan(c/2 + (d*x)/2)*(12*a^2*b^12 + 20*a^4*b^10 + 8*a^6*b^8))/(b^13 + 2*a^2*b^11 + a^4*b^9) - (8*(4*a*b^12 + 6*a^3*b^10 + 2*a^5*b^8))/(b^12 + 2*a^2*b^10 + a^4*b^8) + (a*((a^2 + b^2)^3)^(1/2)*(2*a^2 + 3*b^2)*((8*(4*a^2*b^15 + 8*a^4*b^13 + 4*a^6*b^11))/(b^12 + 2*a^2*b^10 + a^4*b^8) + (8*tan(c/2 + (d*x)/2)*(12*a*b^17 + 32*a^3*b^15 + 28*a^5*b^13 + 8*a^7*b^11))/(b^13 + 2*a^2*b^11 + a^4*b^9)))/(2*(b^10 + 3*a^2*b^8 + 3*a^4*b^6 + a^6*b^4))))/(2*(b^10 + 3*a^2*b^8 + 3*a^4*b^6 + a^6*b^4)))*1i)/(2*(b^10 + 3*a^2*b^8 + 3*a^4*b^6 + a^6*b^4)) + (a*((a^2 + b^2)^3)^(1/2)*(2*a^2 + 3*b^2)*((8*(4*a^2*b^7 + 8*a^4*b^5 + 4*a^6*b^3))/(b^12 + 2*a^2*b^10 + a^4*b^8) + (8*tan(c/2 + (d*x)/2)*(8*a*b^9 + 29*a^3*b^7 + 28*a^5*b^5 + 8*a^7*b^3))/(b^13 + 2*a^2*b^11 + a^4*b^9) - (a*((a^2 + b^2)^3)^(1/2)*(2*a^2 + 3*b^2)*((8*(4*a*b^12 + 6*a^3*b^10 + 2*a^5*b^8))/(b^12 + 2*a^2*b^10 + a^4*b^8) - (8*tan(c/2 + (d*x)/2)*(12*a^2*b^12 + 20*a^4*b^10 + 8*a^6*b^8))/(b^13 + 2*a^2*b^11 + a^4*b^9) + (a*((a^2 + b^2)^3)^(1/2)*(2*a^2 + 3*b^2)*((8*(4*a^2*b^15 + 8*a^4*b^13 + 4*a^6*b^11))/(b^12 + 2*a^2*b^10 + a^4*b^8) + (8*tan(c/2 + (d*x)/2)*(12*a*b^17 + 32*a^3*b^15 + 28*a^5*b^13 + 8*a^7*b^11))/(b^13 + 2*a^2*b^11 + a^4*b^9)))/(2*(b^10 + 3*a^2*b^8 + 3*a^4*b^6 + a^6*b^4))))/(2*(b^10 + 3*a^2*b^8 + 3*a^4*b^6 + a^6*b^4)))*1i)/(2*(b^10 + 3*a^2*b^8 + 3*a^4*b^6 + a^6*b^4)))/((16*(2*a^5 + 3*a^3*b^2))/(b^12 + 2*a^2*b^10 + a^4*b^8) - (16*tan(c/2 + (d*x)/2)*(8*a^6 + 12*a^2*b^4 + 20*a^4*b^2))/(b^13 + 2*a^2*b^11 + a^4*b^9) - (a*((a^2 + b^2)^3)^(1/2)*(2*a^2 + 3*b^2)*((8*(4*a^2*b^7 + 8*a^4*b^5 + 4*a^6*b^3))/(b^12 + 2*a^2*b^10 + a^4*b^8) + (8*tan(c/2 + (d*x)/2)*(8*a*b^9 + 29*a^3*b^7 + 28*a^5*b^5 + 8*a^7*b^3))/(b^13 + 2*a^2*b^11 + a^4*b^9) - (a*((a^2 + b^2)^3)^(1/2)*(2*a^2 + 3*b^2)*((8*tan(c/2 + (d*x)/2)*(12*a^2*b^12 + 20*a^4*b^10 + 8*a^6*b^8))/(b^13 + 2*a^2*b^11 + a^4*b^9) - (8*(4*a*b^12 + 6*a^3*b^10 + 2*a^5*b^8))/(b^12 + 2*a^2*b^10 + a^4*b^8) + (a*((a^2 + b^2)^3)^(1/2)*(2*a^2 + 3*b^2)*((8*(4*a^2*b^15 + 8*a^4*b^13 + 4*a^6*b^11))/(b^12 + 2*a^2*b^10 + a^4*b^8) + (8*tan(c/2 + (d*x)/2)*(12*a*b^17 + 32*a^3*b^15 + 28*a^5*b^13 + 8*a^7*b^11))/(b^13 + 2*a^2*b^11 + a^4*b^9)))/(2*(b^10 + 3*a^2*b^8 + 3*a^4*b^6 + a^6*b^4))))/(2*(b^10 + 3*a

^2*b^8 + 3*a^4*b^6 + a^6*b^4))))/(2*(b^10 + 3*a^2*b^8 + 3*a^4*b^6 + a^6*b^4
)) + (a*((a^2 + b^2)^3)^(1/2)*(2*a^2 + 3*b^2)*((8*(4*a^2*b^7 + 8*a^4*b^5 +
4*a^6*b^3))/(b^12 + 2*a^2*b^10 + a^4*b^8) + (8*tan(c/2 + (d*x)/2)*(8*a*b^9
+ 29*a^3*b^7 + 28*a^5*b^5 + 8*a^7*b^3))/(b^13 + 2*a^2*b^11 + a^4*b^9) - (a*
((a^2 + b^2)^3)^(1/2)*(2*a^2 + 3*b^2)*((8*(4*a*b^12 + 6*a^3*b^10 + 2*a^5*b^
8))/(b^12 + 2*a^2*b^10 + a^4*b^8) - (8*tan(c/2 + (d*x)/2)*(12*a^2*b^12 + 20
*a^4*b^10 + 8*a^6*b^8))/(b^13 + 2*a^2*b^11 + a^4*b^9) + (a*((a^2 + b^2)^3)^
(1/2)*(2*a^2 + 3*b^2)*((8*(4*a^2*b^15 + 8*a^4*b^13 + 4*a^6*b^11))/(b^12 + 2
*a^2*b^10 + a^4*b^8) + (8*tan(c/2 + (d*x)/2)*(12*a*b^17 + 32*a^3*b^15 + 28*
a^5*b^13 + 8*a^7*b^11))/(b^13 + 2*a^2*b^11 + a^4*b^9)))/(2*(b^10 + 3*a^2*b^
8 + 3*a^4*b^6 + a^6*b^4))))/(2*(b^10 + 3*a^2*b^8 + 3*a^4*b^6 + a^6*b^4))))/
(2*(b^10 + 3*a^2*b^8 + 3*a^4*b^6 + a^6*b^4))))*((a^2 + b^2)^3)^(1/2)*(2*a^2
 + 3*b^2)*1i)/(d*(b^10 + 3*a^2*b^8 + 3*a^4*b^6 + a^6*b^4))

3.147 $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=138

$$\frac{(a^2+b^2)^2}{3a^3b^2d(b+a\cot(c+dx))^3}+\frac{\frac{a}{b^3}-\frac{b}{a^3}}{d(b+a\cot(c+dx))^2}+\frac{\frac{1}{a^3}+\frac{3a}{b^4}}{d(b+a\cot(c+dx))}-\frac{4a\log(b+a\cot(c+dx))}{b^5d}-\frac{4a\log}{}$$

```
[Out] 1/3*(a^2+b^2)^2/a^3/b^2/d/(b+a*cot(d*x+c))^3+(a/b^3-b/a^3)/d/(b+a*cot(d*x+c
))^2+(1/a^3+3*a/b^4)/d/(b+a*cot(d*x+c))-4*a*ln(b+a*cot(d*x+c))/b^5/d-4*a*ln
(tan(d*x+c))/b^5/d+tan(d*x+c)/b^4/d
```

Rubi [A]
time = 0.11, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3167, 908}

$$\frac{\frac{1}{a^3}+\frac{3a}{b^4}}{d(a\cot(c+dx)+b)}+\frac{\frac{a}{b^3}-\frac{b}{a^3}}{d(a\cot(c+dx)+b)^2}+\frac{(a^2+b^2)^2}{3a^3b^2d(a\cot(c+dx)+b)^3}-\frac{4a\log(\tan(c+dx))}{b^5d}-\frac{4a\log(a\cot(c+dx)+b)}{b^5d}+\frac{\tan(c+dx)}{b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (a^2 + b^2)^2/(3*a^3*b^2*d*(b + a*Cot[c + d*x])^3) + (a/b^3 - b/a^3)/(d*(b
+ a*Cot[c + d*x])^2) + (a^(-3) + (3*a)/b^4)/(d*(b + a*Cot[c + d*x])) - (4*a
*Log[b + a*Cot[c + d*x]])/(b^5*d) - (4*a*Log[Tan[c + d*x]])/(b^5*d) + Tan[c
 + d*x]/(b^4*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = -\frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^2(b+ax)^4}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{b^4x^2} - \frac{4a}{b^5x} + \frac{(a^2+b^2)^2}{a^2b^2(b+ax)^4} + \frac{2(a^4-b^4)}{a^2b^3(b+ax)^3} + \frac{3a^4+b^4}{a^2b^4(b+ax)^2} + \frac{4a}{b^5(b+ \cdots\right.\right.}{d}$$

$$= \frac{(a^2+b^2)^2}{3a^3b^2d(b+a\cot(c+dx))^3} + \frac{\frac{a}{b^3}-\frac{b}{a^3}}{d(b+a\cot(c+dx))^2} + \frac{\frac{1}{a^3}+\frac{3a}{b^4} \cdots}{d(b+a\cot(c \cdots}$$

Mathematica [A]

time = 1.31, size = 175, normalized size = 1.27

$$\frac{ab^2\sec^2(c+dx)\left(3a^2-b^2+4ab\tan(c+dx)\right)+(a+b\tan(c+dx))^2\left(12a^3(\log(\cos(c+dx))-\log(a\cos(c+dx)+b\sin(c+dx)))+b(12a^2+5b^2+12a^2\log(\cos(c+dx))-12a^2\log(a\cos(c+dx)+b\sin(c+dx)))\tan(c+dx)+3ab^2\tan^2(c+dx)\right)}{3ab^5d(a+b\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (a*b^2*Sec[c + d*x]^2*(3*a^2 - b^2 + 4*a*b*Tan[c + d*x]) + (a + b*Tan[c + d
*x])^2*(12*a^3*(Log[Cos[c + d*x]] - Log[a*Cos[c + d*x] + b*Sin[c + d*x]]) +
 b*(12*a^2 + 5*b^2 + 12*a^2*Log[Cos[c + d*x]] - 12*a^2*Log[a*Cos[c + d*x] +
 b*Sin[c + d*x]])*Tan[c + d*x] + 3*a*b^2*Tan[c + d*x]^2))/(3*a*b^5*d*(a + b
*Tan[c + d*x])^3)
```

Maple [A]

time = 0.85, size = 118, normalized size = 0.86

method	result
derivativedivides	$\frac{\frac{\tan(dx+c)}{b^4}-\frac{4a\ln(a+b\tan(dx+c))}{b^5}+\frac{2a\left(a^2+b^2\right)}{b^5(a+b\tan(dx+c))^2}-\frac{6a^2+2b^2}{b^5(a+b\tan(dx+c))}-\frac{a^4+2a^2b^2+b^4}{3b^5(a+b\tan(dx+c))^3}}{d}$
default	$\frac{\frac{\tan(dx+c)}{b^4}-\frac{4a\ln(a+b\tan(dx+c))}{b^5}+\frac{2a\left(a^2+b^2\right)}{b^5(a+b\tan(dx+c))^2}-\frac{6a^2+2b^2}{b^5(a+b\tan(dx+c))}-\frac{a^4+2a^2b^2+b^4}{3b^5(a+b\tan(dx+c))^3}}{d}$
risch	$\frac{8i\left(-6iab^3e^{4i(dx+c)}+3iab^3e^{6i(dx+c)}+3a^4e^{6i(dx+c)}-9a^2b^2e^{6i(dx+c)}-9ia^3b\,e^{6i(dx+c)}+6ia^3b+9a^4e^{4i(dx+c)}+3a^2b^2e^{4i(dx \cdots\right.}{3\left(e^{2i(dx+c)}+1\right)\left(b\,e^{2i(dx+c)} \cdots\right.}$
norman	$\frac{-\frac{8a^4+2b^4}{6b^5d}-\frac{\left(8a^4+2b^4\right)\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6b^5d}+\frac{\left(144a^3+16a\,b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d\,a^2b^2}-\frac{\left(144a^3+16a\,b^2\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d\,a^2b^2}-\frac{\left(8a^5+48a^3b^2-14a \cdots\right.}{d \cdots}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2b \cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^4*tan(d*x+c)-4*a/b^5*ln(a+b*tan(d*x+c))+2*a/b^5*(a^2+b^2)/(a+b*tan
(d*x+c))^2-(6*a^2+2*b^2)/b^5/(a+b*tan(d*x+c))-1/3/b^5*(a^4+2*a^2*b^2+b^4)/(
a+b*tan(d*x+c))^3)
```

Maxima [A]
time = 0.27, size = 144, normalized size = 1.04

$$-\frac{\frac{13\,a^4+2\,a^2b^2+b^4+6\,(3\,a^2b^2+b^4)\,\tan(dx+c)^2+6\,(5\,a^3b+ab^3)\,\tan(dx+c)}{b^8\,\tan(dx+c)^3+3\,ab^7\,\tan(dx+c)^2+3\,a^2b^6\,\tan(dx+c)+a^3b^5}+\frac{12\,a\log(b\tan(dx+c)+a)}{b^5}-\frac{3\,\tan(dx+c)}{b^4}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*((13*a^4 + 2*a^2*b^2 + b^4 + 6*(3*a^2*b^2 + b^4)*tan(d*x + c)^2 + 6*(5
*a^3*b + a*b^3)*tan(d*x + c))/(b^8*tan(d*x + c)^3 + 3*a*b^7*tan(d*x + c)^2
+ 3*a^2*b^6*tan(d*x + c) + a^3*b^5) + 12*a*log(b*tan(d*x + c) + a)/b^5 - 3*
tan(d*x + c)/b^4)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 537 vs. 2(136) = 272.
time = 3.42, size = 537, normalized size = 3.89

$$\frac{3\,a^2b^4+3\,b^6-4\,(9\,a^4b^2+3\,a^2b^4-2\,b^6)\cos(dx+c)^4+6\,(5\,a^4b^2+a^2b^4-2\,b^6)\cos(dx+c)^2-6\,\left((a^6-2\,a^4b^2-3\,a^2b^4)\cos(dx+c)^4+3\,(a^4b^2+a^2b^4)\cos(dx+c)^2+\left((3\,a^5b+2\,a^3b^3-ab^5)\cos(dx+c)^3+(a^3b^3+ab^5)\cos(dx+c)\right)\sin(dx+c)\right)\log\left(2\,ab\cos(dx+c)\sin(dx+c)+(a^2-b^2)\cos(dx+c)^2+b^2\right)+6\,\left((a^6-2\,a^4b^2-3\,a^2b^4)\cos(dx+c)^4+3\,(a^4b^2+a^2b^4)\cos(dx+c)^2+\left((3\,a^5b+2\,a^3b^3-ab^5)\cos(dx+c)^3+(a^3b^3+ab^5)\cos(dx+c)\right)\sin(dx+c)\right)\log\left(\cos(dx+c)^2\right)+2\,\left(2\,(3\,a^5b-7\,a^3b^3-6\,ab^5)\cos(dx+c)^3+(11\,a^3b^3+9\,ab^5)\cos(dx+c)\right)\sin(dx+c)}{3\,\left((a^5b^5-2\,a^3b^7-3\,ab^9)d\cos(dx+c)^4+3\,(a^3b^7+ab^9)d\cos(dx+c)^2+\left((3\,a^4b^6+2\,a^2b^8-b^{10})d\cos(dx+c)^3+(a^2b^8+b^{10})d\cos(dx+c)\right)\sin(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/3*(3*a^2*b^4 + 3*b^6 - 4*(9*a^4*b^2 + 3*a^2*b^4 - 2*b^6)*cos(d*x + c)^4 +
 6*(5*a^4*b^2 + a^2*b^4 - 2*b^6)*cos(d*x + c)^2 - 6*((a^6 - 2*a^4*b^2 - 3*a
^2*b^4)*cos(d*x + c)^4 + 3*(a^4*b^2 + a^2*b^4)*cos(d*x + c)^2 + ((3*a^5*b +
 2*a^3*b^3 - a*b^5)*cos(d*x + c)^3 + (a^3*b^3 + a*b^5)*cos(d*x + c))*sin(d*
x + c))*log(2*a*b*cos(d*x + c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 +
b^2) + 6*((a^6 - 2*a^4*b^2 - 3*a^2*b^4)*cos(d*x + c)^4 + 3*(a^4*b^2 + a^2*b
^4)*cos(d*x + c)^2 + ((3*a^5*b + 2*a^3*b^3 - a*b^5)*cos(d*x + c)^3 + (a^3*b
^3 + a*b^5)*cos(d*x + c))*sin(d*x + c))*log(cos(d*x + c)^2) + 2*(2*(3*a^5*b
 - 7*a^3*b^3 - 6*a*b^5)*cos(d*x + c)^3 + (11*a^3*b^3 + 9*a*b^5)*cos(d*x + c
))*sin(d*x + c))/((a^5*b^5 - 2*a^3*b^7 - 3*a*b^9)*d*cos(d*x + c)^4 + 3*(a^3
*b^7 + a*b^9)*d*cos(d*x + c)^2 + ((3*a^4*b^6 + 2*a^2*b^8 - b^10)*d*cos(d*x
+ c)^3 + (a^2*b^8 + b^10)*d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*cos(c + d*x) + b*sin(c + d*x))**4, x)
```

Giac [A]
time = 0.48, size = 138, normalized size = 1.00

$$-\frac{\frac{12\,a\log\left(|b\tan\left(dx+c\right)+a|\right)}{b^5}-\frac{3\,\tan\left(dx+c\right)}{b^4}-\frac{22\,ab^3\tan\left(dx+c\right)^3+48\,a^2b^2\tan\left(dx+c\right)^2-6\,b^4\tan\left(dx+c\right)^2+36\,a^3b\tan\left(dx+c\right)-6\,ab^3\tan\left(dx+c\right)+9\,a^4-2\,a^2b^2-b^4}{\left(b\tan\left(dx+c\right)+a\right)^3b^5}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(12*a*log(abs(b*tan(d*x + c) + a))/b^5 - 3*tan(d*x + c)/b^4 - (22*a*b^
3*tan(d*x + c)^3 + 48*a^2*b^2*tan(d*x + c)^2 - 6*b^4*tan(d*x + c)^2 + 36*a^
3*b*tan(d*x + c) - 6*a*b^3*tan(d*x + c) + 9*a^4 - 2*a^2*b^2 - b^4)/((b*tan(
d*x + c) + a)^3*b^5))/d
```

Mupad [B]
time = 4.44, size = 666, normalized size = 4.83

$$\frac{\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(10\,a^4+b^4\right)}{a^2\,b^3}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(4\,a^4+b^4\right)}{a\,b^4}-\frac{8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(10\,a^4-2\,a^2\,b^2+b^4\right)}{a^2\,b^3}+\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(10\,a^4+b^4\right)}{a^2\,b^3}-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(36\,a^6-88\,a^4\,b^2+a^2\,b^4-4\,b^6\right)}{3\,a^3\,b^4}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(36\,a^6-88\,a^4\,b^2+a^2\,b^4-4\,b^6\right)}{3\,a^3\,b^4}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(4\,a^4+b^4\right)}{a\,b^4}}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(12\,a\,b^2-4\,a^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(12\,a\,b^2-4\,a^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(24\,a\,b^2-6\,a^3\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(18\,a^2\,b-8\,b^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(18\,a^2\,b-8\,b^3\right)+a^3+6\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-6\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\right)}-\frac{8\,a\,\mathrm{atanh}\left(\frac{256\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{256\,a^3-256\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{512\,a^5}{b^2}-\frac{512\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{b^2}+\frac{512\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b}}-\frac{256\,a^3}{256\,a^3-256\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{512\,a^5}{b^2}-\frac{512\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{b^2}+\frac{512\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b}}+\frac{512\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{256\,a^3\,b+\frac{512\,a^5}{b}+512\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\frac{512\,a^5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{b}-256\,a^3\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}\right)}{b^5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a*cos(c + d*x) + b*sin(c + d*x))^4),x)
```

```
[Out] ((4*tan(c/2 + (d*x)/2)^2*(10*a^4 + b^4))/(a^2*b^3) - (2*tan(c/2 + (d*x)/2)^
7*(4*a^4 + b^4))/(a*b^4) - (8*tan(c/2 + (d*x)/2)^4*(10*a^4 + b^4 - 2*a^2*b^
2))/(a^2*b^3) + (4*tan(c/2 + (d*x)/2)^6*(10*a^4 + b^4))/(a^2*b^3) - (2*tan(
c/2 + (d*x)/2)^3*(36*a^6 - 4*b^6 + a^2*b^4 - 88*a^4*b^2))/(3*a^3*b^4) + (2*
tan(c/2 + (d*x)/2)^5*(36*a^6 - 4*b^6 + a^2*b^4 - 88*a^4*b^2))/(3*a^3*b^4) +
 (2*tan(c/2 + (d*x)/2)*(4*a^4 + b^4))/(a*b^4))/(d*(a^3*tan(c/2 + (d*x)/2)^8
 + tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 4*a^3) + tan(c/2 + (d*x)/2)^6*(12*a*b^2
 - 4*a^3) - tan(c/2 + (d*x)/2)^4*(24*a*b^2 - 6*a^3) - tan(c/2 + (d*x)/2)^3*
(18*a^2*b - 8*b^3) + tan(c/2 + (d*x)/2)^5*(18*a^2*b - 8*b^3) + a^3 + 6*a^2*
b*tan(c/2 + (d*x)/2) - 6*a^2*b*tan(c/2 + (d*x)/2)^7)) - (8*a*atanh((256*a^3
*tan(c/2 + (d*x)/2)^2)/(256*a^3 - 256*a^3*tan(c/2 + (d*x)/2)^2 + (512*a^5)/
b^2 - (512*a^5*tan(c/2 + (d*x)/2)^2)/b^2 + (512*a^4*tan(c/2 + (d*x)/2))/b)
- (256*a^3)/(256*a^3 - 256*a^3*tan(c/2 + (d*x)/2)^2 + (512*a^5)/b^2 - (512*
a^5*tan(c/2 + (d*x)/2)^2)/b^2 + (512*a^4*tan(c/2 + (d*x)/2))/b) + (512*a^4*
tan(c/2 + (d*x)/2))/(256*a^3*b + (512*a^5)/b + 512*a^4*tan(c/2 + (d*x)/2) -
 (512*a^5*tan(c/2 + (d*x)/2)^2)/b - 256*a^3*b*tan(c/2 + (d*x)/2)^2)))/(b^5*
d)
```

3.148 $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=400

$$\frac{8a^2\tanh^{-1}(\sin(c+dx))}{b^6d}+\frac{\tanh^{-1}(\sin(c+dx))}{2b^4d}+\frac{2(a^2+b^2)\tanh^{-1}(\sin(c+dx))}{b^6d}+\frac{4a^3\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin}{\sqrt{a^2+b}}\right.}{b^6\sqrt{a^2+b^2}\,d}$$

```
[Out] 8*a^2*arctanh(sin(d*x+c))/b^6/d+1/2*arctanh(sin(d*x+c))/b^4/d+2*(a^2+b^2)*a
rctanh(sin(d*x+c))/b^6/d-4*a*sec(d*x+c)/b^5/d+1/3*(-a^2-b^2)/b^3/d/(a*cos(d
*x+c)+b*sin(d*x+c))^3+3/2*a*(b*cos(d*x+c)-a*sin(d*x+c))/b^4/d/(a*cos(d*x+c)
+b*sin(d*x+c))^2-4*a^2/b^5/d/(a*cos(d*x+c)+b*sin(d*x+c))-2*(a^2+b^2)/b^5/d/
(a*cos(d*x+c)+b*sin(d*x+c))+4*a^3*arctanh((b*cos(d*x+c)-a*sin(d*x+c))/(a^2+
b^2)^(1/2))/b^6/d/(a^2+b^2)^(1/2)+3/2*a*arctanh((b*cos(d*x+c)-a*sin(d*x+c))
/(a^2+b^2)^(1/2))/b^4/d/(a^2+b^2)^(1/2)+6*a*arctanh((b*cos(d*x+c)-a*sin(d*x
+c))/(a^2+b^2)^(1/2))*(a^2+b^2)^(1/2)/b^6/d+1/2*sec(d*x+c)*tan(d*x+c)/b^4/d
```

Rubi [A]

time = 0.58, antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3185, 3173, 3855, 3153, 212, 3155, 3853, 3183}

$$\frac{8a^2\tanh^{-1}(\sin(c+dx))}{b^6d}-\frac{4a^2}{b^5d(a\cos(c+dx)+b\sin(c+dx))}+\frac{2(a^2+b^2)\tanh^{-1}(\sin(c+dx))}{b^6d}+\frac{6a\sqrt{a^2+b^2}\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^6d}-\frac{2(a^2+b^2)}{b^5d(a\cos(c+dx)+b\sin(c+dx))}+\frac{3a\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{2b^4d\sqrt{a^2+b^2}}-\frac{a^2+b^2}{3b^3d(a\cos(c+dx)+b\sin(c+dx))^3}+\frac{4a^3\tanh^{-1}\left(\frac{b\cos(c+dx)-a\sin(c+dx)}{\sqrt{a^2+b^2}}\right)}{b^6d\sqrt{a^2+b^2}}-\frac{4a\sec(c+dx)}{b^5d}+\frac{3a(b\cos(c+dx)-a\sin(c+dx))}{2b^4d(a\cos(c+dx)+b\sin(c+dx))^2}+\frac{\tanh^{-1}(\sin(c+dx))}{2b^4d}+\frac{\tan(c+dx)\sec(c+dx)}{2b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (8*a^2*ArcTanh[Sin[c + d*x]])/(b^6*d) + ArcTanh[Sin[c + d*x]]/(2*b^4*d) + (
2*(a^2 + b^2)*ArcTanh[Sin[c + d*x]])/(b^6*d) + (4*a^3*ArcTanh[(b*Cos[c + d*
x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(b^6*Sqrt[a^2 + b^2]*d) + (3*a*ArcTa
nh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt[a^2 + b^2]])/(2*b^4*Sqrt[a^2 + b^
2]*d) + (6*a*Sqrt[a^2 + b^2]*ArcTanh[(b*Cos[c + d*x] - a*Sin[c + d*x])/Sqrt
[a^2 + b^2]])/(b^6*d) - (4*a*Sec[c + d*x])/(b^5*d) - (a^2 + b^2)/(3*b^3*d*(
a*Cos[c + d*x] + b*Sin[c + d*x])^3) + (3*a*(b*Cos[c + d*x] - a*Sin[c + d*x]
))/(2*b^4*d*(a*Cos[c + d*x] + b*Sin[c + d*x])^2) - (4*a^2)/(b^5*d*(a*Cos[c
+ d*x] + b*Sin[c + d*x])) - (2*(a^2 + b^2))/(b^5*d*(a*Cos[c + d*x] + b*Sin[
c + d*x])) + (Sec[c + d*x]*Tan[c + d*x])/(2*b^4*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3155

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[(b*Cos[c + d*x] - a*Sin[c + d*x])*((a*Cos[c + d*x] + b*Sin
[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[(n + 2)/((n + 1)*(a^
2 + b^2)), Int[(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{
a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && NeQ[n, -2]
```

Rule 3173

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_)/co
s[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(a*Cos[c + d*x] + b*Sin[c + d*x])^
(n + 1)/(b*d*(n + 1)), x] + (Dist[1/b^2, Int[(a*Cos[c + d*x] + b*Sin[c + d*
x])^(n + 2)/Cos[c + d*x], x], x] - Dist[a/b^2, Int[(a*Cos[c + d*x] + b*Sin[
c + d*x])^(n + 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &
& LtQ[n, -1]
```

Rule 3183

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[-Cos[c + d*x]^(m + 1)/(b*d*(m + 1)
), x] + (-Dist[a/b^2, Int[Cos[c + d*x]^(m + 1), x], x] + Dist[(a^2 + b^2)/b
^2, Int[Cos[c + d*x]^(m + 2)/(a*Cos[c + d*x] + b*Sin[c + d*x]), x], x]) /;
FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[m, -1]
```

Rule 3185

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^2 + b^2)/b^2, Int[Cos[c +
d*x]^(m + 2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^n, x], x] + (Dist[1/b^2, Int
[Cos[c + d*x]^m*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 2), x], x] - Dist[2*
(a/b^2), Int[Cos[c + d*x]^(m + 1)*(a*Cos[c + d*x] + b*Sin[c + d*x])^(n + 1)
, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1] && L
tQ[m, -1]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx = \frac{\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^2}\,dx}{b^2} - \frac{(2a)\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^3}\,dx}{b^2} + \frac{(a^2}{}$$

$$= -\frac{a^2+b^2}{3b^3d(a\cos(c+dx)+b\sin(c+dx))^3} + \frac{\int \sec^3(c+dx)\,dx}{b^4} - 2\frac{(2a}{}$$

$$= -\frac{a^2+b^2}{3b^3d(a\cos(c+dx)+b\sin(c+dx))^3} + \frac{3a(b\cos(c+dx)-a\sin}{2b^4d(a\cos(c+dx)+b\sin}$$

$$= \frac{4a^2\tanh^{-1}(\sin(c+dx))}{b^6d} + \frac{\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{a}{3b^3d(a\cos(c+}$$

$$= \frac{4a^2\tanh^{-1}(\sin(c+dx))}{b^6d} + \frac{\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{4a^3\tanh^{-1}\left(\frac{b\cos}{}\right.}{b^6\sqrt{a}}$$

Mathematica [A]
time = 3.68, size = 538, normalized size = 1.34

$$-\frac{\sec^4(c+dx)(a\cos(c+dx)+b\sin(c+dx))\left(4b^3(a^2+b^2)+18b^2(a^2+b^2)\sin(c+dx)(a\cos(c+dx)+b\sin(c+dx))+6b(12a^2+b^2)(a\cos(c+dx)+b\sin(c+dx))^2+48ab(a\cos(c+dx)+b\sin(c+dx))^3+\frac{60a(4a^2+3b^2)\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2+b^2}}\right)(a\cos(c+dx)+b\sin(c+dx))^3}{\sqrt{a^2+b^2}}+30(4a^2+b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^3-30(4a^2+b^2)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b\sin(c+dx))^3-\frac{3b^2(a\cos(c+dx)+b\sin(c+dx))^3}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{48ab\sin\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b\sin(c+dx))^3}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+\frac{3b^2(a\cos(c+dx)+b\sin(c+dx))^3}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{48ab\sin\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b\sin(c+dx))^3}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}\right)}{12b^6d(a+b\tan(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] -1/12*(Sec[c + d*x]^4*(a*Cos[c + d*x] + b*Sin[c + d*x])*(4*b^3*(a^2 + b^2)
+ 18*b^2*(a^2 + b^2)*Sin[c + d*x]*(a*Cos[c + d*x] + b*Sin[c + d*x]) + 6*b*(
12*a^2 + b^2)*(a*Cos[c + d*x] + b*Sin[c + d*x])^2 + 48*a*b*(a*Cos[c + d*x]
+ b*Sin[c + d*x])^3 + (60*a*(4*a^2 + 3*b^2)*ArcTanh[(-b + a*Tan[(c + d*x)/2
])/Sqrt[a^2 + b^2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)/Sqrt[a^2 + b^2] +
30*(4*a^2 + b^2)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*(a*Cos[c + d*x] +
 b*Sin[c + d*x])^3 - 30*(4*a^2 + b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3 - (3*b^2*(a*Cos[c + d*x] + b*Sin[c
+ d*x])^3)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (48*a*b*Sin[(c + d*x)/
2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]) + (3*b^2*(a*Cos[c + d*x] + b*Sin[c + d*x])^3)/(Cos[(c + d*x)/2] + Sin[(c
```

```
 + d*x)/2])^2 - (48*a*b*Sin[(c + d*x)/2]*(a*Cos[c + d*x] + b*Sin[c + d*x])^
3)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(b^6*d*(a + b*Tan[c + d*x])^4)
```

Maple [A]
time = 1.35, size = 452, normalized size = 1.13 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/b^4/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(-8*a-b)/b^5/(tan(1/2*d*x+1/2*c)-
1)+1/2/b^6*(-20*a^2-5*b^2)*ln(tan(1/2*d*x+1/2*c)-1)-1/2/b^4/(tan(1/2*d*x+1/
2*c)+1)^2-1/2*(8*a-b)/b^5/(tan(1/2*d*x+1/2*c)+1)+1/2/b^6*(20*a^2+5*b^2)*ln(
tan(1/2*d*x+1/2*c)+1)+2/b^6*((1/2*b^2*(9*a^4+2*b^4)/a*tan(1/2*d*x+1/2*c)^5+
1/2*b*(12*a^6-39*a^4*b^2-4*b^6)/a^2*tan(1/2*d*x+1/2*c)^4-1/3/a^3*b^2*(108*a
^6-57*a^4*b^2-4*a^2*b^4-4*b^6)*tan(1/2*d*x+1/2*c)^3-1/a^2*b*(12*a^6-50*a^4*
b^2-9*a^2*b^4-2*b^6)*tan(1/2*d*x+1/2*c)^2+1/2/a*b^2*(63*a^4+10*a^2*b^2+2*b^
4)*tan(1/2*d*x+1/2*c)+1/6*b*(36*a^4+5*a^2*b^2+2*b^4))/(a*tan(1/2*d*x+1/2*c)
^2-2*b*tan(1/2*d*x+1/2*c)-a)^3-5/2*a*(4*a^2+3*b^2)/(a^2+b^2)^(1/2)*arctanh(
1/2*(2*a*tan(1/2*d*x+1/2*c)-2*b)/(a^2+b^2)^(1/2))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 936 vs. 2(378) = 756.
time = 0.53, size = 936, normalized size = 2.34

$$-\frac{\frac{2\left(60a^7+5a^5b^2+2a^3b^4+\frac{6\left(55a^6b+5a^4b^3+a^2b^5\right)\sin(dx+c)}{\cos(dx+c)+1}-\frac{2\left(120a^7-280a^5b^2-25a^3b^4-6ab^6\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2\left(510a^6b-105a^4b^3+2a^2b^5-4b^7\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2\left(180a^7-635a^5b^2-65a^3b^4-18ab^6\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{2\left(540a^6b-195a^4b^3-2a^2b^5-8b^7\right)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{6\left(40a^7-140a^5b^2-5a^3b^4-6ab^6\right)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{2\left(210a^6b-75a^4b^3+2a^2b^5-4b^7\right)\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{3\left(20a^7-45a^5b^2-4ab^6\right)\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6\left(5a^6b+a^2b^5\right)\sin(dx+c)^9}{(\cos(dx+c)+1)^9}\right)}{a^6b^5+\frac{6a^5b^6\sin(dx+c)}{\cos(dx+c)+1}+\frac{6a^5b^6\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{a^6b^5\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{\left(5a^6b^5-12a^4b^7\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{8\left(3a^5b^6-a^3b^8\right)\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2\left(5a^6b^5-18a^4b^7\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\left(9a^5b^6-4a^3b^8\right)\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{2\left(5a^6b^5-18a^4b^7\right)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{8\left(3a^5b^6-a^3b^8\right)\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{\left(5a^6b^5-12a^4b^7\right)\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{15\left(4a^2+3b^2\right)a\log\left(\frac{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(dx+c)}{\cos(dx+c)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b^6}-\frac{15\left(4a^2+b^2\right)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^6}+\frac{15\left(4a^2+b^2\right)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/6*(2*(60*a^7 + 5*a^5*b^2 + 2*a^3*b^4 + 6*(55*a^6*b + 5*a^4*b^3 + a^2*b^5
)*sin(d*x + c)/(cos(d*x + c) + 1) - 2*(120*a^7 - 280*a^5*b^2 - 25*a^3*b^4 -
 6*a*b^6)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(510*a^6*b - 105*a^4*b^3
+ 2*a^2*b^5 - 4*b^7)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*(180*a^7 - 635
*a^5*b^2 - 65*a^3*b^4 - 18*a*b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 2*(
540*a^6*b - 195*a^4*b^3 - 2*a^2*b^5 - 8*b^7)*sin(d*x + c)^5/(cos(d*x + c) +
 1)^5 - 6*(40*a^7 - 140*a^5*b^2 - 5*a^3*b^4 - 6*a*b^6)*sin(d*x + c)^6/(cos(
d*x + c) + 1)^6 - 2*(210*a^6*b - 75*a^4*b^3 + 2*a^2*b^5 - 4*b^7)*sin(d*x +
c)^7/(cos(d*x + c) + 1)^7 + 3*(20*a^7 - 45*a^5*b^2 - 4*a*b^6)*sin(d*x + c)^
8/(cos(d*x + c) + 1)^8 + 6*(5*a^6*b + a^2*b^5)*sin(d*x + c)^9/(cos(d*x + c)
 + 1)^9)/(a^6*b^5 + 6*a^5*b^6*sin(d*x + c)/(cos(d*x + c) + 1) + 6*a^5*b^6*s
in(d*x + c)^9/(cos(d*x + c) + 1)^9 - a^6*b^5*sin(d*x + c)^10/(cos(d*x + c)
+ 1)^10 - (5*a^6*b^5 - 12*a^4*b^7)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 8*
(3*a^5*b^6 - a^3*b^8)*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 2*(5*a^6*b^5 -
18*a^4*b^7)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*(9*a^5*b^6 - 4*a^3*b^8)
*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 2*(5*a^6*b^5 - 18*a^4*b^7)*sin(d*x +
 c)^6/(cos(d*x + c) + 1)^6 - 8*(3*a^5*b^6 - a^3*b^8)*sin(d*x + c)^7/(cos(d*
```

```
x + c) + 1)^7 + (5*a^6*b^5 - 12*a^4*b^7)*sin(d*x + c)^8/(cos(d*x + c) + 1)^
8) - 15*(4*a^2 + 3*b^2)*a*log((b - a*sin(d*x + c)/(cos(d*x + c) + 1) + sqrt
(a^2 + b^2))/(b - a*sin(d*x + c)/(cos(d*x + c) + 1) - sqrt(a^2 + b^2)))/(sq
rt(a^2 + b^2)*b^6) - 15*(4*a^2 + b^2)*log(sin(d*x + c)/(cos(d*x + c) + 1) +
 1)/b^6 + 15*(4*a^2 + b^2)*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/b^6)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 820 vs. $2(378) = 756$.
time = 2.64, size = 820, normalized size = 2.05

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/12*(6*a^2*b^5 + 6*b^7 - 30*(4*a^6*b - 3*a^4*b^3 - 8*a^2*b^5 - b^7)*cos(d*
x + c)^4 - 20*(11*a^4*b^3 + 13*a^2*b^5 + 2*b^7)*cos(d*x + c)^2 + 15*((4*a^6
 - 9*a^4*b^2 - 9*a^2*b^4)*cos(d*x + c)^5 + 3*(4*a^4*b^2 + 3*a^2*b^4)*cos(d*
x + c)^3 + ((12*a^5*b + 5*a^3*b^3 - 3*a*b^5)*cos(d*x + c)^4 + (4*a^3*b^3 +
3*a*b^5)*cos(d*x + c)^2)*sin(d*x + c))*sqrt(a^2 + b^2)*log((2*a*b*cos(d*x +
 c)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 +
b^2)*(b*cos(d*x + c) - a*sin(d*x + c)))/(2*a*b*cos(d*x + c)*sin(d*x + c) +
(a^2 - b^2)*cos(d*x + c)^2 + b^2)) + 15*((4*a^7 - 7*a^5*b^2 - 14*a^3*b^4 -
3*a*b^6)*cos(d*x + c)^5 + 3*(4*a^5*b^2 + 5*a^3*b^4 + a*b^6)*cos(d*x + c)^3
+ ((12*a^6*b + 11*a^4*b^3 - 2*a^2*b^5 - b^7)*cos(d*x + c)^4 + (4*a^4*b^3 +
5*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin(d*x + c))*log(sin(d*x + c) + 1) - 15*(
(4*a^7 - 7*a^5*b^2 - 14*a^3*b^4 - 3*a*b^6)*cos(d*x + c)^5 + 3*(4*a^5*b^2 +
5*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + ((12*a^6*b + 11*a^4*b^3 - 2*a^2*b^5 - b
^7)*cos(d*x + c)^4 + (4*a^4*b^3 + 5*a^2*b^5 + b^7)*cos(d*x + c)^2)*sin(d*x
+ c))*log(-sin(d*x + c) + 1) - 30*(10*(a^5*b^2 + a^3*b^4)*cos(d*x + c)^3 +
(a^3*b^4 + a*b^6)*cos(d*x + c))*sin(d*x + c))/((a^5*b^6 - 2*a^3*b^8 - 3*a*b
^10)*d*cos(d*x + c)^5 + 3*(a^3*b^8 + a*b^10)*d*cos(d*x + c)^3 + ((3*a^4*b^7
 + 2*a^2*b^9 - b^11)*d*cos(d*x + c)^4 + (a^2*b^9 + b^11)*d*cos(d*x + c)^2)*
sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c + dx)}{(a\cos(c + dx) + b\sin(c + dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*cos(c + d*x) + b*sin(c + d*x))**4, x)
```

Giac [A]
time = 0.53, size = 548, normalized size = 1.37

$$\frac{\frac{15\,(4\,a^2+b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^6}-\frac{15\,(4\,a^2+b^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^6}+\frac{15\,(4\,a^3+3\,ab^2)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b^6}+\frac{6\left(b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+8\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-8\,a\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2b^5}+\frac{2\left(27\,a^6b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,a^2b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+36\,a^7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-117\,a^5b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-12\,ab^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-216\,a^6b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+114\,a^4b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+8\,a^2b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+8\,b^7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-72\,a^7\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+300\,a^5b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+54\,a^3b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+12\,ab^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+189\,a^6b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+30\,a^4b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,a^2b^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+36\,a^7+5\,a^5b^2+2\,a^3b^4\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-a\right)^3a^3b^5}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/6*(15*(4*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^6 - 15*(4*a^2 +
b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^6 + 15*(4*a^3 + 3*a*b^2)*log(abs(
2*a*tan(1/2*d*x + 1/2*c) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*d*x + 1
/2*c) - 2*b + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b^6) + 6*(b*tan(1/2*d*x
+ 1/2*c)^3 + 8*a*tan(1/2*d*x + 1/2*c)^2 + b*tan(1/2*d*x + 1/2*c) - 8*a)/((t
an(1/2*d*x + 1/2*c)^2 - 1)^2*b^5) + 2*(27*a^6*b*tan(1/2*d*x + 1/2*c)^5 + 6*
a^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 36*a^7*tan(1/2*d*x + 1/2*c)^4 - 117*a^5*b^
2*tan(1/2*d*x + 1/2*c)^4 - 12*a*b^6*tan(1/2*d*x + 1/2*c)^4 - 216*a^6*b*tan(
1/2*d*x + 1/2*c)^3 + 114*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 + 8*a^2*b^5*tan(1/2
*d*x + 1/2*c)^3 + 8*b^7*tan(1/2*d*x + 1/2*c)^3 - 72*a^7*tan(1/2*d*x + 1/2*c
)^2 + 300*a^5*b^2*tan(1/2*d*x + 1/2*c)^2 + 54*a^3*b^4*tan(1/2*d*x + 1/2*c)^
2 + 12*a*b^6*tan(1/2*d*x + 1/2*c)^2 + 189*a^6*b*tan(1/2*d*x + 1/2*c) + 30*a
^4*b^3*tan(1/2*d*x + 1/2*c) + 6*a^2*b^5*tan(1/2*d*x + 1/2*c) + 36*a^7 + 5*a
^5*b^2 + 2*a^3*b^4)/((a*tan(1/2*d*x + 1/2*c)^2 - 2*b*tan(1/2*d*x + 1/2*c) -
 a)^3*a^3*b^5))/d
```

Mupad [B]
time = 4.59, size = 1961, normalized size = 4.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a*cos(c + d*x) + b*sin(c + d*x))^4),x)
```

```
[Out] (atanh((4000*a^3*tan(c/2 + (d*x)/2))/(1000*a*b^2 + 4000*a^3) + (1000*a*tan(
c/2 + (d*x)/2))/(1000*a + (4000*a^3)/b^2))*(20*a^2 + 5*b^2))/(b^6*d) - ((60
*a^4 + 2*b^4 + 5*a^2*b^2)/(3*b^5) + (2*tan(c/2 + (d*x)/2)^9*(5*a^4 + b^4))/
(a*b^4) + (2*tan(c/2 + (d*x)/2)^6*(6*b^6 - 40*a^6 + 5*a^2*b^4 + 140*a^4*b^2
))/(a^2*b^5) - (2*tan(c/2 + (d*x)/2)^7*(210*a^6 - 4*b^6 + 2*a^2*b^4 - 75*a^
4*b^2))/(3*a^3*b^4) + (2*tan(c/2 + (d*x)/2)^2*(6*b^6 - 120*a^6 + 25*a^2*b^4
 + 280*a^4*b^2))/(3*a^2*b^5) - (2*tan(c/2 + (d*x)/2)^3*(510*a^6 - 4*b^6 + 2
*a^2*b^4 - 105*a^4*b^2))/(3*a^3*b^4) - (2*tan(c/2 + (d*x)/2)^4*(18*b^6 - 18
0*a^6 + 65*a^2*b^4 + 635*a^4*b^2))/(3*a^2*b^5) - (tan(c/2 + (d*x)/2)^8*(4*b
^6 - 20*a^6 + 45*a^4*b^2))/(a^2*b^5) + (2*tan(c/2 + (d*x)/2)*(55*a^4 + b^4
+ 5*a^2*b^2))/(a*b^4) + (2*tan(c/2 + (d*x)/2)^5*(9*a^2 - 4*b^2)*(60*a^4 + 2
*b^4 + 5*a^2*b^2))/(3*a^3*b^4))/(d*(tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 5*a^3)
 - a^3*tan(c/2 + (d*x)/2)^10 - tan(c/2 + (d*x)/2)^8*(12*a*b^2 - 5*a^3) - ta
```

```
n(c/2 + (d*x)/2)^4*(36*a*b^2 - 10*a^3) + tan(c/2 + (d*x)/2)^6*(36*a*b^2 - 1
0*a^3) - tan(c/2 + (d*x)/2)^3*(24*a^2*b - 8*b^3) - tan(c/2 + (d*x)/2)^7*(24
*a^2*b - 8*b^3) + tan(c/2 + (d*x)/2)^5*(36*a^2*b - 16*b^3) + a^3 + 6*a^2*b*
tan(c/2 + (d*x)/2) + 6*a^2*b*tan(c/2 + (d*x)/2)^9)) - (a*atan(((a*(4*a^2 +
3*b^2)*(a^2 + b^2)^(1/2)*((8*(25*a^2*b^9 + 200*a^4*b^7 + 400*a^6*b^5))/b^14
 + (8*tan(c/2 + (d*x)/2)*(50*a*b^11 + 650*a^3*b^9 + 1600*a^5*b^7 + 800*a^7*
b^5))/b^15 - (5*a*(4*a^2 + 3*b^2)*(a^2 + b^2)^(1/2)*((8*tan(c/2 + (d*x)/2)*
(60*a^2*b^14 + 80*a^4*b^12))/b^15 - (8*(10*a*b^14 + 20*a^3*b^12))/b^14 + (5
*a*(4*a^2 + 3*b^2)*(a^2 + b^2)^(1/2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(1
2*a*b^19 + 8*a^3*b^17))/b^15))/(2*(b^8 + a^2*b^6))))/(2*(b^8 + a^2*b^6)))*5
i)/(2*(b^8 + a^2*b^6)) + (a*(4*a^2 + 3*b^2)*(a^2 + b^2)^(1/2)*((8*(25*a^2*b
^9 + 200*a^4*b^7 + 400*a^6*b^5))/b^14 + (8*tan(c/2 + (d*x)/2)*(50*a*b^11 +
650*a^3*b^9 + 1600*a^5*b^7 + 800*a^7*b^5))/b^15 - (5*a*(4*a^2 + 3*b^2)*(a^2
 + b^2)^(1/2)*((8*(10*a*b^14 + 20*a^3*b^12))/b^14 - (8*tan(c/2 + (d*x)/2)*(
60*a^2*b^14 + 80*a^4*b^12))/b^15 + (5*a*(4*a^2 + 3*b^2)*(a^2 + b^2)^(1/2)*(
32*a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(12*a*b^19 + 8*a^3*b^17))/b^15))/(2*(b^8
 + a^2*b^6))))/(2*(b^8 + a^2*b^6)))*5i)/(2*(b^8 + a^2*b^6)))/((16*(2000*a^7
 + 375*a^3*b^4 + 2000*a^5*b^2))/b^14 - (16*tan(c/2 + (d*x)/2)*(8000*a^8 + 3
75*a^2*b^6 + 3500*a^4*b^4 + 10000*a^6*b^2))/b^15 - (5*a*(4*a^2 + 3*b^2)*(a^
2 + b^2)^(1/2)*((8*(25*a^2*b^9 + 200*a^4*b^7 + 400*a^6*b^5))/b^14 + (8*tan(
c/2 + (d*x)/2)*(50*a*b^11 + 650*a^3*b^9 + 1600*a^5*b^7 + 800*a^7*b^5))/b^15
 - (5*a*(4*a^2 + 3*b^2)*(a^2 + b^2)^(1/2)*((8*tan(c/2 + (d*x)/2)*(60*a^2*b^
14 + 80*a^4*b^12))/b^15 - (8*(10*a*b^14 + 20*a^3*b^12))/b^14 + (5*a*(4*a^2
+ 3*b^2)*(a^2 + b^2)^(1/2)*(32*a^2*b^3 + (8*tan(c/2 + (d*x)/2)*(12*a*b^19 +
 8*a^3*b^17))/b^15))/(2*(b^8 + a^2*b^6))))/(2*(b^8 + a^2*b^6))))/(2*(b^8 +
a^2*b^6)) + (5*a*(4*a^2 + 3*b^2)*(a^2 + b^2)^(1/2)*((8*(25*a^2*b^9 + 200*a^
4*b^7 + 400*a^6*b^5))/b^14 + (8*tan(c/2 + (d*x)/2)*(50*a*b^11 + 650*a^3*b^9
 + 1600*a^5*b^7 + 800*a^7*b^5))/b^15 - (5*a*(4*a^2 + 3*b^2)*(a^2 + b^2)^(1/
2)*((8*(10*a*b^14 + 20*a^3*b^12))/b^14 - (8*tan(c/2 + (d*x)/2)*(60*a^2*b^14
 + 80*a^4*b^12))/b^15 + (5*a*(4*a^2 + 3*b^2)*(a^2 + b^2)^(1/2)*(32*a^2*b^3
+ (8*tan(c/2 + (d*x)/2)*(12*a*b^19 + 8*a^3*b^17))/b^15))/(2*(b^8 + a^2*b^6)
)))/(2*(b^8 + a^2*b^6))))/(2*(b^8 + a^2*b^6))))*(4*a^2 + 3*b^2)*(a^2 + b^2)
^(1/2)*5i)/(d*(b^8 + a^2*b^6))
```

3.149 $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx$

Optimal. Leaf size=232

$$\frac{(a^2+b^2)^3}{3a^3b^4d(b+a\cot(c+dx))^3}+\frac{2a^6+3a^4b^2-b^6}{a^3b^5d(b+a\cot(c+dx))^2}+\frac{10a^6+9a^4b^2+b^6}{a^3b^6d(b+a\cot(c+dx))}-\frac{4a(5a^2+3b^2)\log(b+a\cot(c+dx))}{b^7d}$$

```
[Out] 1/3*(a^2+b^2)^3/a^3/b^4/d/(b+a*cot(d*x+c))^3+(2*a^6+3*a^4*b^2-b^6)/a^3/b^5/
d/(b+a*cot(d*x+c))^2+(10*a^6+9*a^4*b^2+b^6)/a^3/b^6/d/(b+a*cot(d*x+c))-4*a*
(5*a^2+3*b^2)*ln(b+a*cot(d*x+c))/b^7/d-4*a*(5*a^2+3*b^2)*ln(tan(d*x+c))/b^7
/d+(10*a^2+3*b^2)*tan(d*x+c)/b^6/d-2*a*tan(d*x+c)^2/b^5/d+1/3*tan(d*x+c)^3/
b^4/d
```

Rubi [A]
time = 0.17, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3167, 908}

$$-\frac{4a(5a^2+3b^2)\log(\tan(c+dx))}{b^7d}-\frac{4a(5a^2+3b^2)\log(a\cot(c+dx)+b)}{b^7d}+\frac{(10a^2+3b^2)\tan(c+dx)}{b^6d}+\frac{(a^2+b^2)^3}{3a^3b^4d(a\cot(c+dx)+b)^3}+\frac{10a^6+9a^4b^2+b^6}{a^3b^6d(a\cot(c+dx)+b)}+\frac{2a^6+3a^4b^2-b^6}{a^3b^5d(a\cot(c+dx)+b)^2}-\frac{2a\tan^2(c+dx)}{b^5d}+\frac{\tan^3(c+dx)}{3b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (a^2 + b^2)^3/(3*a^3*b^4*d*(b + a*Cot[c + d*x])^3) + (2*a^6 + 3*a^4*b^2 - b
^6)/(a^3*b^5*d*(b + a*Cot[c + d*x])^2) + (10*a^6 + 9*a^4*b^2 + b^6)/(a^3*b^
6*d*(b + a*Cot[c + d*x])) - (4*a*(5*a^2 + 3*b^2)*Log[b + a*Cot[c + d*x]])/(
b^7*d) - (4*a*(5*a^2 + 3*b^2)*Log[Tan[c + d*x]])/(b^7*d) + ((10*a^2 + 3*b^2
)*Tan[c + d*x])/(b^6*d) - (2*a*Tan[c + d*x]^2)/(b^5*d) + Tan[c + d*x]^3/(3*
b^4*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+b\sin(c+dx))^4}\,dx &= -\frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^4(b+ax)^4}\,dx, x, \cot(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(\frac{1}{b^4x^4}-\frac{4a}{b^5x^3}+\frac{10a^2+3b^2}{b^6x^2}-\frac{4(5a^3+3ab^2)}{b^7x}+\frac{(a^2+b^2)^3}{a^2b^4(b+ax)^4}+\frac{2(2a}{a^2}\right.\right.}{d}\\ &= \frac{(a^2+b^2)^3}{3a^3b^4d(b+a\cot(c+dx))^3}+\frac{2a^6+3a^4b^2-b^6}{a^3b^5d(b+a\cot(c+dx))^2}+\frac{10a^6}{a^3b^6d(b}\end{aligned}$$

Mathematica [A]
time = 1.89, size = 366, normalized size = 1.58

$$\frac{b\sec^6(c+dx)\left(150a^5b+130a^3b^3+24ab^5+3(25a^5b+25a^3b^3-4ab^5)\cos(2(c+dx))-6(25a^5b+15a^3b^3+4ab^5)\cos(4(c+dx))-75a^5b\cos(6(c+dx))-35a^3b^3\cos(6(c+dx))-4ab^5\cos(6(c+dx))+120a^6\sin(4(c+dx))+72a^4b^2\sin(4(c+dx))+36a^2b^4\sin(4(c+dx))+30a^6\sin(6(c+dx))-37a^4b^2\sin(6(c+dx))-27a^2b^4\sin(6(c+dx))-4b^6\sin(6(c+dx))\right)+6b(50a^6+85a^4b^2+43a^2b^4+4b^6)\sec^4(c+dx)\tan(c+dx)+192a^2(5a^2+3b^2)\left(\log(\cos(c+dx))-\log(a\cos(c+dx)+b\sin(c+dx))\right)(a+b\tan(c+dx))^3}{48ab^7d(a+b\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a*Cos[c + d*x] + b*Sin[c + d*x])^4,x]
```

```
[Out] (b*Sec[c + d*x]^6*(150*a^5*b + 130*a^3*b^3 + 24*a*b^5 + 3*(25*a^5*b + 25*a^
3*b^3 - 4*a*b^5)*Cos[2*(c + d*x)] - 6*(25*a^5*b + 15*a^3*b^3 + 4*a*b^5)*Cos
[4*(c + d*x)] - 75*a^5*b*Cos[6*(c + d*x)] - 35*a^3*b^3*Cos[6*(c + d*x)] - 4
*a*b^5*Cos[6*(c + d*x)] + 120*a^6*Sin[4*(c + d*x)] + 72*a^4*b^2*Sin[4*(c +
d*x)] + 36*a^2*b^4*Sin[4*(c + d*x)] + 30*a^6*Sin[6*(c + d*x)] - 37*a^4*b^2*
Sin[6*(c + d*x)] - 27*a^2*b^4*Sin[6*(c + d*x)] - 4*b^6*Sin[6*(c + d*x)]) +
6*b*(50*a^6 + 85*a^4*b^2 + 43*a^2*b^4 + 4*b^6)*Sec[c + d*x]^4*Tan[c + d*x]
+ 192*a^2*(5*a^2 + 3*b^2)*(Log[Cos[c + d*x]] - Log[a*Cos[c + d*x] + b*Sin[c
 + d*x]])*(a + b*Tan[c + d*x])^3)/(48*a*b^7*d*(a + b*Tan[c + d*x])^3)
```

Maple [A]
time = 1.17, size = 195, normalized size = 0.84 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/b^6*(1/3*b^2*tan(d*x+c)^3-2*a*b*tan(d*x+c)^2+10*a^2*tan(d*x+c)+3*b^2
*tan(d*x+c))-4*a/b^7*(5*a^2+3*b^2)*ln(a+b*tan(d*x+c))-(15*a^4+18*a^2*b^2+3*
b^4)/b^7/(a+b*tan(d*x+c))-1/3/b^7*(a^6+3*a^4*b^2+3*a^2*b^4+b^6)/(a+b*tan(d*
x+c))^3+3*a/b^7*(a^4+2*a^2*b^2+b^4)/(a+b*tan(d*x+c))^2)
```

Maxima [A]
time = 0.28, size = 217, normalized size = 0.94

$$-\frac{\frac{37\,a^6+39\,a^4b^2+3\,a^2b^4+b^6+9\,(5\,a^4b^2+6\,a^2b^4+b^6)\tan(dx+c)^2+9\,(9\,a^5b+10\,a^3b^3+ab^5)\tan(dx+c)}{b^{10}\tan(dx+c)^3+3\,ab^9\tan(dx+c)^2+3\,a^2b^8\tan(dx+c)+a^3b^7}-\frac{b^2\tan(dx+c)^3-6\,ab\tan(dx+c)^2+3\,(10\,a^2+3\,b^2)\tan(dx+c)}{b^6}+\frac{12\,(5\,a^3+3\,ab^2)\log(b\tan(dx+c)+a)}{b^7}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/3*((37*a^6 + 39*a^4*b^2 + 3*a^2*b^4 + b^6 + 9*(5*a^4*b^2 + 6*a^2*b^4 + b
^6)*tan(d*x + c)^2 + 9*(9*a^5*b + 10*a^3*b^3 + a*b^5)*tan(d*x + c))/(b^10*t
an(d*x + c)^3 + 3*a*b^9*tan(d*x + c)^2 + 3*a^2*b^8*tan(d*x + c) + a^3*b^7)
- (b^2*tan(d*x + c)^3 - 6*a*b*tan(d*x + c)^2 + 3*(10*a^2 + 3*b^2)*tan(d*x +
 c))/b^6 + 12*(5*a^3 + 3*a*b^2)*log(b*tan(d*x + c) + a)/b^7)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 553 vs. $2(228) = 456$.
time = 3.16, size = 553, normalized size = 2.38

$$-\frac{4\left(45a^4b^2-3a^2b^4-4b^6\right)\cos\left(dx+c\right)^6-b^6-6\left(25a^4b^2-5a^2b^4-4b^6\right)\cos\left(dx+c\right)^4-3\left(5a^2b^4+2b^6\right)\cos\left(dx+c\right)^2+6\left(\left(5a^6-12a^4b^2-9a^2b^4\right)\cos\left(dx+c\right)^6+3\left(5a^4b^2+3a^2b^4\right)\cos\left(dx+c\right)^4+\left(\left(15a^5b+4a^3b^3-3ab^5\right)\cos\left(dx+c\right)^5+\left(5a^3b^3+3ab^5\right)\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)\right)\log\left(2ab\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)-6\left(\left(5a^6-12a^4b^2-9a^2b^4\right)\cos\left(dx+c\right)^6+3\left(5a^4b^2+3a^2b^4\right)\cos\left(dx+c\right)^4+\left(\left(15a^5b+4a^3b^3-3ab^5\right)\cos\left(dx+c\right)^5+\left(5a^3b^3+3ab^5\right)\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)\right)\log\left(\cos\left(dx+c\right)^2\right)+\left(3ab^5\cos\left(dx+c\right)-4\left(15a^5b-41a^3b^3-12ab^5\right)\cos\left(dx+c\right)^5-2\left(55a^3b^3+21ab^5\right)\cos\left(dx+c\right)^3\right)\sin\left(dx+c\right)}{3\left(3ab^9d\cos\left(dx+c\right)^4+\left(a^3b^7-3ab^9\right)d\cos\left(dx+c\right)^6+\left(b^{10}d\cos\left(dx+c\right)^3+\left(3a^2b^8-b^{10}\right)d\cos\left(dx+c\right)^5\right)\sin\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] -1/3*(4*(45*a^4*b^2 - 3*a^2*b^4 - 4*b^6)*cos(d*x + c)^6 - b^6 - 6*(25*a^4*b
^2 - 5*a^2*b^4 - 4*b^6)*cos(d*x + c)^4 - 3*(5*a^2*b^4 + 2*b^6)*cos(d*x + c)
^2 + 6*((5*a^6 - 12*a^4*b^2 - 9*a^2*b^4)*cos(d*x + c)^6 + 3*(5*a^4*b^2 + 3*
a^2*b^4)*cos(d*x + c)^4 + ((15*a^5*b + 4*a^3*b^3 - 3*a*b^5)*cos(d*x + c)^5
+ (5*a^3*b^3 + 3*a*b^5)*cos(d*x + c)^3)*sin(d*x + c))*log(2*a*b*cos(d*x + c
)*sin(d*x + c) + (a^2 - b^2)*cos(d*x + c)^2 + b^2) - 6*((5*a^6 - 12*a^4*b^2
 - 9*a^2*b^4)*cos(d*x + c)^6 + 3*(5*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^4 + (
(15*a^5*b + 4*a^3*b^3 - 3*a*b^5)*cos(d*x + c)^5 + (5*a^3*b^3 + 3*a*b^5)*cos
(d*x + c)^3)*sin(d*x + c))*log(cos(d*x + c)^2) + (3*a*b^5*cos(d*x + c) - 4*
(15*a^5*b - 41*a^3*b^3 - 12*a*b^5)*cos(d*x + c)^5 - 2*(55*a^3*b^3 + 21*a*b^
5)*cos(d*x + c)^3)*sin(d*x + c))/(3*a*b^9*d*cos(d*x + c)^4 + (a^3*b^7 - 3*a
*b^9)*d*cos(d*x + c)^6 + (b^10*d*cos(d*x + c)^3 + (3*a^2*b^8 - b^10)*d*cos(
d*x + c)^5)*sin(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4\left(c+dx\right)}{\left(a\cos\left(c+dx\right)+b\sin\left(c+dx\right)\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a*cos(d*x+c)+b*sin(d*x+c))**4,x)
```

```
[Out] Integral(sec(c + d*x)**4/(a*cos(c + d*x) + b*sin(c + d*x))**4, x)
```

Giac [A]
time = 0.49, size = 249, normalized size = 1.07

$$-\frac{\frac{12\left(5a^3+3ab^2\right)\log(|b\tan(dx+c)+a|)}{b^7}-\frac{110a^3b^3\tan(dx+c)^3+66ab^5\tan(dx+c)^3+285a^4b^2\tan(dx+c)^2+144a^2b^4\tan(dx+c)^2-9b^6\tan(dx+c)^2+249a^5b\tan(dx+c)+108a^3b^3\tan(dx+c)-9ab^5\tan(dx+c)+73a^6+27a^4b^2-3a^2b^4-b^6}{(b\tan(dx+c)+a)^3b^7}-\frac{b^8\tan(dx+c)^3-6ab^7\tan(dx+c)^2+30a^2b^6\tan(dx+c)+9b^8\tan(dx+c)}{b^{12}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+b*sin(d*x+c))^4,x, algorithm="giac")
```

```
[Out] -1/3*(12*(5*a^3 + 3*a*b^2)*log(abs(b*tan(d*x + c) + a))/b^7 - (110*a^3*b^3*
tan(d*x + c)^3 + 66*a*b^5*tan(d*x + c)^3 + 285*a^4*b^2*tan(d*x + c)^2 + 144
*a^2*b^4*tan(d*x + c)^2 - 9*b^6*tan(d*x + c)^2 + 249*a^5*b*tan(d*x + c) + 1
08*a^3*b^3*tan(d*x + c) - 9*a*b^5*tan(d*x + c) + 73*a^6 + 27*a^4*b^2 - 3*a^
2*b^4 - b^6)/((b*tan(d*x + c) + a)^3*b^7) - (b^8*tan(d*x + c)^3 - 6*a*b^7*t
an(d*x + c)^2 + 30*a^2*b^6*tan(d*x + c) + 9*b^8*tan(d*x + c))/b^12)/d
```

Mupad [B]
time = 7.97, size = 1599, normalized size = 6.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a*cos(c + d*x) + b*sin(c + d*x))^4),x)
```

```
[Out] ((4*tan(c/2 + (d*x)/2)^2*(50*a^6 + b^6 + 30*a^4*b^2))/(a^2*b^5) - (16*tan(c
/2 + (d*x)/2)^8*(50*a^6 + b^6 - 3*a^2*b^4 + 25*a^4*b^2))/(a^2*b^5) - (2*tan
(c/2 + (d*x)/2)^11*(20*a^6 + b^6 + 12*a^4*b^2))/(a*b^6) - (16*tan(c/2 + (d*
x)/2)^4*(50*a^6 + b^6 - 3*a^2*b^4 + 25*a^4*b^2))/(a^2*b^5) + (4*tan(c/2 + (
d*x)/2)^10*(50*a^6 + b^6 + 30*a^4*b^2))/(a^2*b^5) - (4*tan(c/2 + (d*x)/2)^5
*(2*b^8 - 100*a^8 + a^2*b^6 + 140*a^4*b^4 + 160*a^6*b^2))/(a^3*b^6) + (4*ta
n(c/2 + (d*x)/2)^7*(2*b^8 - 100*a^8 + a^2*b^6 + 140*a^4*b^4 + 160*a^6*b^2))
/(a^3*b^6) + (2*tan(c/2 + (d*x)/2)^3*(4*b^8 - 300*a^8 - 3*a^2*b^6 + 264*a^4
*b^4 + 260*a^6*b^2))/(3*a^3*b^6) - (2*tan(c/2 + (d*x)/2)^9*(4*b^8 - 300*a^8
 - 3*a^2*b^6 + 264*a^4*b^4 + 260*a^6*b^2))/(3*a^3*b^6) + (8*tan(c/2 + (d*x)
/2)^6*(450*a^6 + 9*b^6 - 28*a^2*b^4 + 210*a^4*b^2))/(3*a^2*b^5) + (2*tan(c/
2 + (d*x)/2)*(20*a^6 + b^6 + 12*a^4*b^2))/(a*b^6))/(d*(a^3*tan(c/2 + (d*x)/
2)^12 + tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 6*a^3) + tan(c/2 + (d*x)/2)^10*(12
*a*b^2 - 6*a^3) - tan(c/2 + (d*x)/2)^4*(48*a*b^2 - 15*a^3) - tan(c/2 + (d*x
)/2)^8*(48*a*b^2 - 15*a^3) + tan(c/2 + (d*x)/2)^6*(72*a*b^2 - 20*a^3) - tan
(c/2 + (d*x)/2)^3*(30*a^2*b - 8*b^3) + tan(c/2 + (d*x)/2)^9*(30*a^2*b - 8*b
^3) + tan(c/2 + (d*x)/2)^5*(60*a^2*b - 24*b^3) - tan(c/2 + (d*x)/2)^7*(60*a
^2*b - 24*b^3) + a^3 + 6*a^2*b*tan(c/2 + (d*x)/2) - 6*a^2*b*tan(c/2 + (d*x)
/2)^11)) + (a*atan(((a*(5*a^2 + 3*b^2)*((16*tan(c/2 + (d*x)/2)*(20*a^5 + 12
*a^3*b^2))/b^6 - (4*(24*a^2*b^9 + 40*a^4*b^7))/b^12 + (4*tan(c/2 + (d*x)/2)
^2*(24*a^2*b^9 + 40*a^4*b^7))/b^12 + (4*a*(5*a^2 + 3*b^2)*((4*(a*b^14 + 4*a
^3*b^12))/b^12 - (4*tan(c/2 + (d*x)/2)^2*(3*a*b^14 + 4*a^3*b^12))/b^12 + 16
*a^2*b*tan(c/2 + (d*x)/2)))/b^7)*4i)/b^7 - (a*(5*a^2 + 3*b^2)*((4*(24*a^2*b
^9 + 40*a^4*b^7))/b^12 - (16*tan(c/2 + (d*x)/2)*(20*a^5 + 12*a^3*b^2))/b^6
 - (4*tan(c/2 + (d*x)/2)^2*(24*a^2*b^9 + 40*a^4*b^7))/b^12 + (4*a*(5*a^2 + 3
*b^2)*((4*(a*b^14 + 4*a^3*b^12))/b^12 - (4*tan(c/2 + (d*x)/2)^2*(3*a*b^14 +
 4*a^3*b^12))/b^12 + 16*a^2*b*tan(c/2 + (d*x)/2)))/b^7)*4i)/b^7)/((8*(400*a
```

```
^7 + 144*a^3*b^4 + 480*a^5*b^2))/b^12 + (8*tan(c/2 + (d*x)/2)^2*(400*a^7 +
144*a^3*b^4 + 480*a^5*b^2))/b^12 + (4*a*(5*a^2 + 3*b^2)*((16*tan(c/2 + (d*x
)/2)*(20*a^5 + 12*a^3*b^2))/b^6 - (4*(24*a^2*b^9 + 40*a^4*b^7))/b^12 + (4*t
an(c/2 + (d*x)/2)^2*(24*a^2*b^9 + 40*a^4*b^7))/b^12 + (4*a*(5*a^2 + 3*b^2)*
((4*(a*b^14 + 4*a^3*b^12))/b^12 - (4*tan(c/2 + (d*x)/2)^2*(3*a*b^14 + 4*a^3
*b^12))/b^12 + 16*a^2*b*tan(c/2 + (d*x)/2)))/b^7))/b^7 + (4*a*(5*a^2 + 3*b^
2)*((4*(24*a^2*b^9 + 40*a^4*b^7))/b^12 - (16*tan(c/2 + (d*x)/2)*(20*a^5 + 1
2*a^3*b^2))/b^6 - (4*tan(c/2 + (d*x)/2)^2*(24*a^2*b^9 + 40*a^4*b^7))/b^12 +
 (4*a*(5*a^2 + 3*b^2)*((4*(a*b^14 + 4*a^3*b^12))/b^12 - (4*tan(c/2 + (d*x)/
2)^2*(3*a*b^14 + 4*a^3*b^12))/b^12 + 16*a^2*b*tan(c/2 + (d*x)/2)))/b^7))/b^
7))*(5*a^2 + 3*b^2)*8i)/(b^7*d)
```

3.150 $\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=99

$$\frac{5x}{16a}+\frac{i\cos^6(c+dx)}{6ad}+\frac{5\cos(c+dx)\sin(c+dx)}{16ad}+\frac{5\cos^3(c+dx)\sin(c+dx)}{24ad}+\frac{\cos^5(c+dx)\sin(c+dx)}{6ad}$$

```
[Out] 5/16*x/a+1/6*I*cos(d*x+c)^6/a/d+5/16*cos(d*x+c)*sin(d*x+c)/a/d+5/24*cos(d*x
+c)^3*sin(d*x+c)/a/d+1/6*cos(d*x+c)^5*sin(d*x+c)/a/d
```

Rubi [A]
time = 0.11, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3171, 3169, 2715, 8, 2645, 30}

$$\frac{i\cos^6(c+dx)}{6ad}+\frac{\sin(c+dx)\cos^5(c+dx)}{6ad}+\frac{5\sin(c+dx)\cos^3(c+dx)}{24ad}+\frac{5\sin(c+dx)\cos(c+dx)}{16ad}+\frac{5x}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] (5*x)/(16*a) + ((I/6)*Cos[c + d*x]^6)/(a*d) + (5*Cos[c + d*x]*Sin[c + d*x])
/(16*a*d) + (5*Cos[c + d*x]^3*Sin[c + d*x])/(24*a*d) + (Cos[c + d*x]^5*Sin[
c + d*x])/(6*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
```

```
*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3171

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \cos^5(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2}\\
&= -\frac{i\int \left(ia\cos^6(c+dx)+a\cos^5(c+dx)\sin(c+dx)\right)\,dx}{a^2}\\
&= -\frac{i\int \cos^5(c+dx)\sin(c+dx)\,dx}{a}+\frac{\int \cos^6(c+dx)\,dx}{a}\\
&= \frac{\cos^5(c+dx)\sin(c+dx)}{6ad}+\frac{5\int \cos^4(c+dx)\,dx}{6a}+\frac{i\text{Subst}\left(\int x^5\,dx,x,c\right.}{ad}\\
&= \frac{i\cos^6(c+dx)}{6ad}+\frac{5\cos^3(c+dx)\sin(c+dx)}{24ad}+\frac{\cos^5(c+dx)\sin(c+dx)}{6ad}\\
&= \frac{i\cos^6(c+dx)}{6ad}+\frac{5\cos(c+dx)\sin(c+dx)}{16ad}+\frac{5\cos^3(c+dx)\sin(c+dx)}{24ad}\\
&= \frac{5x}{16a}+\frac{i\cos^6(c+dx)}{6ad}+\frac{5\cos(c+dx)\sin(c+dx)}{16ad}+\frac{5\cos^3(c+dx)\sin}{24ad}
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 82, normalized size = 0.83

$$\frac{60c+60dx+15i\cos(2(c+dx))+6i\cos(4(c+dx))+i\cos(6(c+dx))+45\sin(2(c+dx))+9\sin(4(c+dx))+\sin(6(c+dx))}{192ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

[Out] (60*c + 60*d*x + (15*I)*Cos[2*(c + d*x)] + (6*I)*Cos[4*(c + d*x)] + I*Cos[6 *(c + d*x)] + 45*Sin[2*(c + d*x)] + 9*Sin[4*(c + d*x)] + Sin[6*(c + d*x)])/ (192*a*d)

Maple [A]
time = 0.70, size = 102, normalized size = 1.03

method	result
risch	$\frac{5x}{16a}+\frac{i\mathrm{e}^{-6i(dx+c)}}{192ad}+\frac{i\cos(4dx+4c)}{32ad}+\frac{3\sin(4dx+4c)}{64ad}+\frac{5i\cos(2dx+2c)}{64ad}+\frac{15\sin(2dx+2c)}{64ad}$
derivativedivides	$\frac{-\frac{5i\ln(\tan(dx+c)-i)}{32}-\frac{3i}{32(\tan(dx+c)-i)^2}-\frac{1}{24(\tan(dx+c)-i)^3}+\frac{3}{16(\tan(dx+c)-i)}+\frac{i}{32(\tan(dx+c)+i)^2}+\frac{5i\ln(\tan(dx+c)+i)}{32}+\frac{\cdots}{8\tan\cdots}}{da}$
default	$\frac{-\frac{5i\ln(\tan(dx+c)-i)}{32}-\frac{3i}{32(\tan(dx+c)-i)^2}-\frac{1}{24(\tan(dx+c)-i)^3}+\frac{3}{16(\tan(dx+c)-i)}+\frac{i}{32(\tan(dx+c)+i)^2}+\frac{5i\ln(\tan(dx+c)+i)}{32}+\frac{\cdots}{8\tan\cdots}}{da}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d/a*(-5/32*I*ln(tan(d*x+c)-I)-3/32*I/(tan(d*x+c)-I)^2-1/24/(tan(d*x+c)-I) ^3+3/16/(tan(d*x+c)-I)+1/32*I/(tan(d*x+c)+I)^2+5/32*I*ln(tan(d*x+c)+I)+1/8/ (tan(d*x+c)+I))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: expt: undefined: 0 to a negative exponent.

Fricas [A]
time = 2.08, size = 76, normalized size = 0.77

$$\frac{\left(120\,dxe^{(6i\,dx+6i\,c)}-3i\,e^{(10i\,dx+10i\,c)}-30i\,e^{(8i\,dx+8i\,c)}+60i\,e^{(4i\,dx+4i\,c)}+15i\,e^{(2i\,dx+2i\,c)}+2i\right)e^{(-6i\,dx-6i\,c)}}{384\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")

[Out] 1/384*(120*d*x*e^(6*I*d*x + 6*I*c) - 3*I*e^(10*I*d*x + 10*I*c) - 30*I*e^(8* I*d*x + 8*I*c) + 60*I*e^(4*I*d*x + 4*I*c) + 15*I*e^(2*I*d*x + 2*I*c) + 2*I) *e^(-6*I*d*x - 6*I*c)/(a*d)

Sympy [A]
time = 0.22, size = 219, normalized size = 2.21

$$\begin{cases} \frac{\left(-50331648 i a^{4} d^{4} e^{16 i c} e^{4 i d x} - 503316480 i a^{4} d^{4} e^{14 i c} e^{2 i d x} + 1006632960 i a^{4} d^{4} e^{10 i c} e^{- 2 i d x} + 251658240 i a^{4} d^{4} e^{8 i c} e^{- 4 i d x} + 33554432 i a^{4} d^{4} e^{6 i c} e^{- 6 i d x}\right) e^{- 12 i c}}{6442450944 a^{5} d^{5}} & \text{for}\: a^{5} d^{5} e^{12 i c} \neq 0 \\ x \left(\frac{\left(e^{10 i c} + 5 e^{8 i c} + 10 e^{6 i c} + 10 e^{4 i c} + 5 e^{2 i c} + 1\right) e^{- 6 i c}}{32 a} - \frac{5}{16 a}\right) & \text{otherwise} \end{cases} + \frac{5 x}{16 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Piecewise(((-50331648*I*a**4*d**4*exp(16*I*c)*exp(4*I*d*x) - 503316480*I*a*
*4*d**4*exp(14*I*c)*exp(2*I*d*x) + 1006632960*I*a**4*d**4*exp(10*I*c)*exp(-
2*I*d*x) + 251658240*I*a**4*d**4*exp(8*I*c)*exp(-4*I*d*x) + 33554432*I*a**4
*d**4*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*I*c)/(6442450944*a**5*d**5), Ne(a**
5*d**5*exp(12*I*c), 0)), (x*((exp(10*I*c) + 5*exp(8*I*c) + 10*exp(6*I*c) +
10*exp(4*I*c) + 5*exp(2*I*c) + 1)*exp(-6*I*c)/(32*a) - 5/(16*a)), True)) +
5*x/(16*a)
```

Giac [A]
time = 0.41, size = 116, normalized size = 1.17

$$-\frac{-\frac{30i \log(\tan(dx+c)+i)}{a} + \frac{30i \log(\tan(dx+c)-i)}{a} + \frac{3\left(-15i \tan(dx+c)^2+38 \tan(dx+c)+25i\right)}{a(-i \tan(dx+c)+1)^2} - \frac{55i \tan(dx+c)^3+201 \tan(dx+c)^2-255i \tan(dx+c)-117}{a(\tan(dx+c)-i)^3}}{192\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/192*(-30*I*log(tan(d*x + c) + I)/a + 30*I*log(tan(d*x + c) - I)/a + 3*(-
15*I*tan(d*x + c)^2 + 38*tan(d*x + c) + 25*I)/(a*(-I*tan(d*x + c) + 1)^2) -
 (55*I*tan(d*x + c)^3 + 201*tan(d*x + c)^2 - 255*I*tan(d*x + c) - 117)/(a*(
tan(d*x + c) - I)^3))/d
```

Mupad [B]
time = 5.14, size = 164, normalized size = 1.66

$$\frac{5\,x}{16\,a}+\frac{\frac{11\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{8}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,3{}\mathrm{i}}{4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,1{}\mathrm{i}}{12}+\frac{13\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,1{}\mathrm{i}}{12}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,3{}\mathrm{i}}{4}+\frac{11\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{a\,d\,{\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)}^4\,{\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}\right)}^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a*cos(c + d*x) + a*sin(c + d*x)*1i),x)
```

```
[Out] (5*x)/(16*a) + ((11*tan(c/2 + (d*x)/2))/8 + (tan(c/2 + (d*x)/2)^2*3i)/4 - t
an(c/2 + (d*x)/2)^3/3 + (tan(c/2 + (d*x)/2)^4*1i)/12 + (13*tan(c/2 + (d*x)/
2)^5)/4 - (tan(c/2 + (d*x)/2)^6*1i)/12 - tan(c/2 + (d*x)/2)^7/3 - (tan(c/2
+ (d*x)/2)^8*3i)/4 + (11*tan(c/2 + (d*x)/2)^9)/8)/(a*d*(tan(c/2 + (d*x)/2)
+ 1i)^4*(tan(c/2 + (d*x)/2)*1i + 1)^6)
```

3.151 $\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=70

$$\frac{i\cos^5(c+dx)}{5ad}+\frac{\sin(c+dx)}{ad}-\frac{2\sin^3(c+dx)}{3ad}+\frac{\sin^5(c+dx)}{5ad}$$

```
[Out] 1/5*I*cos(d*x+c)^5/a/d+sin(d*x+c)/a/d-2/3*sin(d*x+c)^3/a/d+1/5*sin(d*x+c)^5
/a/d
```

Rubi [A]
time = 0.09, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}=0.161$, Rules used = {3171, 3169, 2713, 2645, 30}

$$\frac{\sin^5(c+dx)}{5ad}-\frac{2\sin^3(c+dx)}{3ad}+\frac{\sin(c+dx)}{ad}+\frac{i\cos^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((I/5)*Cos[c + d*x]^5)/(a*d) + Sin[c + d*x]/(a*d) - (2*Sin[c + d*x]^3)/(3*a
*d) + Sin[c + d*x]^5/(5*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
```

```
gerQ[m] && IGtQ[n, 0]
```

Rule 3171

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \cos^4(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2}\\ &= -\frac{i\int \left(ia\cos^5(c+dx)+a\cos^4(c+dx)\sin(c+dx)\right)\,dx}{a^2}\\ &= -\frac{i\int \cos^4(c+dx)\sin(c+dx)\,dx}{a}+\frac{\int \cos^5(c+dx)\,dx}{a}\\ &= \frac{i\text{Subst}\left(\int x^4\,dx,x,\cos(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx,x,-\sin(c+dx)\right)}{ad}\\ &= \frac{i\cos^5(c+dx)}{5ad}+\frac{\sin(c+dx)}{ad}-\frac{2\sin^3(c+dx)}{3ad}+\frac{\sin^5(c+dx)}{5ad}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 111, normalized size = 1.59

$$\frac{i\cos(c+dx)}{8ad}+\frac{i\cos(3(c+dx))}{16ad}+\frac{i\cos(5(c+dx))}{80ad}+\frac{5\sin(c+dx)}{8ad}+\frac{5\sin(3(c+dx))}{48ad}+\frac{\sin(5(c+dx))}{80ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((I/8)*Cos[c + d*x])/(a*d) + ((I/16)*Cos[3*(c + d*x)])/(a*d) + ((I/80)*Cos[
5*(c + d*x)])/(a*d) + (5*Sin[c + d*x])/(8*a*d) + (5*Sin[3*(c + d*x)])/(48*a
*d) + Sin[5*(c + d*x)]/(80*a*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(63) = 126.
time = 0.49, size = 141, normalized size = 2.01

method	result
risch	$\frac{i e^{-5i(dx+c)}}{80ad}+\frac{i\cos(dx+c)}{8ad}+\frac{5\sin(dx+c)}{8ad}+\frac{i\cos(3dx+3c)}{16ad}+\frac{5\sin(3dx+3c)}{48ad}$

derivativedivides	$\frac{-\frac{i}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{2}}-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{3}}+\frac{5}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\frac{i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{3i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{2}{5\left(-i+\tan\left(\frac{dx}{2}\right.\right.}}{ad}$
default	$\frac{-\frac{i}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{2}}-\frac{1}{6\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^{3}}+\frac{5}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}-\frac{i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{3i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{2}{5\left(-i+\tan\left(\frac{dx}{2}\right.\right.}}{ad}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a*(-1/8*I/(tan(1/2*d*x+1/2*c)+I)^2-1/12/(tan(1/2*d*x+1/2*c)+I)^3+5/16/(
tan(1/2*d*x+1/2*c)+I)-1/2*I/(-I+tan(1/2*d*x+1/2*c))^4+3/4*I/(-I+tan(1/2*d*x
+1/2*c))^2+1/5/(-I+tan(1/2*d*x+1/2*c))^5-5/6/(-I+tan(1/2*d*x+1/2*c))^3+11/1
6/(-I+tan(1/2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 1.79, size = 63, normalized size = 0.90

$$\frac{\left(-5i\,e^{(8i\,dx+8i\,c)}-60i\,e^{(6i\,dx+6i\,c)}+90i\,e^{(4i\,dx+4i\,c)}+20i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-5i\,dx-5i\,c)}}{240\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/240*(-5*I*e^(8*I*d*x + 8*I*c) - 60*I*e^(6*I*d*x + 6*I*c) + 90*I*e^(4*I*d*
x + 4*I*c) + 20*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-5*I*d*x - 5*I*c)/(a*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(53) = 106$.
time = 0.23, size = 196, normalized size = 2.80

$$\begin{cases} \frac{\left(-30720ia^4d^4e^{12ic}e^{3idx}-368640ia^4d^4e^{10ic}e^{idx}+552960ia^4d^4e^{8ic}e^{-idx}+122880ia^4d^4e^{6ic}e^{-3idx}+18432ia^4d^4e^{4ic}e^{-5idx}\right)e^{-9ic}}{1474560a^5d^5} & \text{for } a^5d^5e^{9ic}\neq 0 \\ \frac{x\left(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1\right)e^{-5ic}}{16a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Piecewise(((-30720*I*a**4*d**4*exp(12*I*c)*exp(3*I*d*x) - 368640*I*a**4*d**
4*exp(10*I*c)*exp(I*d*x) + 552960*I*a**4*d**4*exp(8*I*c)*exp(-I*d*x) + 1228
80*I*a**4*d**4*exp(6*I*c)*exp(-3*I*d*x) + 18432*I*a**4*d**4*exp(4*I*c)*exp(
-5*I*d*x))*exp(-9*I*c)/(1474560*a**5*d**5), Ne(a**5*d**5*exp(9*I*c), 0)), (
x*(exp(8*I*c) + 4*exp(6*I*c) + 6*exp(4*I*c) + 4*exp(2*I*c) + 1)*exp(-5*I*c)
/(16*a), True))
```

Giac [A]

time = 0.44, size = 119, normalized size = 1.70

$$\frac{\frac{5\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+24i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-13\right)}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^3}+\frac{165\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-480i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-650\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+400i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+113}{a\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^5}}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/120*(5*(15*tan(1/2*d*x + 1/2*c)^2 + 24*I*tan(1/2*d*x + 1/2*c) - 13)/(a*(t
an(1/2*d*x + 1/2*c) + I)^3) + (165*tan(1/2*d*x + 1/2*c)^4 - 480*I*tan(1/2*d
*x + 1/2*c)^3 - 650*tan(1/2*d*x + 1/2*c)^2 + 400*I*tan(1/2*d*x + 1/2*c) + 1
13)/(a*(tan(1/2*d*x + 1/2*c) - I)^5))/d
```

Mupad [B]

time = 2.09, size = 134, normalized size = 1.91

$$-\frac{\left(-15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\,15i-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\,25i-13\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\,21i+9\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+3i\right)\,2i}{15\,a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1i\right)^3\left(1+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,1i\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a*cos(c + d*x) + a*sin(c + d*x)*1i),x)
```

```
[Out] -((9*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*21i - 13*tan(c/2 + (d*x)/2)^
3 + tan(c/2 + (d*x)/2)^4*25i - 5*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2)^
6*15i - 15*tan(c/2 + (d*x)/2)^7 + 3i)*2i)/(15*a*d*(tan(c/2 + (d*x)/2) + 1i)
^3*(tan(c/2 + (d*x)/2)*1i + 1)^5)
```

3.152 $\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{3x}{8a}+\frac{i\cos^4(c+dx)}{4ad}+\frac{3\cos(c+dx)\sin(c+dx)}{8ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}$$

```
[Out] 3/8*x/a+1/4*I*cos(d*x+c)^4/a/d+3/8*cos(d*x+c)*sin(d*x+c)/a/d+1/4*cos(d*x+c)
^3*sin(d*x+c)/a/d
```

Rubi [A]
time = 0.09, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}=0.194$, Rules used = {3171, 3169, 2715, 8, 2645, 30}

$$\frac{i\cos^4(c+dx)}{4ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}+\frac{3\sin(c+dx)\cos(c+dx)}{8ad}+\frac{3x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] (3*x)/(8*a) + ((I/4)*Cos[c + d*x]^4)/(a*d) + (3*Cos[c + d*x]*Sin[c + d*x])/
(8*a*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(4*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3171

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \cos^3(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2}\\ &= -\frac{i\int \left(ia\cos^4(c+dx)+a\cos^3(c+dx)\sin(c+dx)\right)\,dx}{a^2}\\ &= -\frac{i\int \cos^3(c+dx)\sin(c+dx)\,dx}{a}+\frac{\int \cos^4(c+dx)\,dx}{a}\\ &= \frac{\cos^3(c+dx)\sin(c+dx)}{4ad}+\frac{3\int \cos^2(c+dx)\,dx}{4a}+\frac{i\text{Subst}\left(\int x^3\,dx,x,c\right.}{ad}\\ &= \frac{i\cos^4(c+dx)}{4ad}+\frac{3\cos(c+dx)\sin(c+dx)}{8ad}+\frac{\cos^3(c+dx)\sin(c+dx)}{4ad}\\ &= \frac{3x}{8a}+\frac{i\cos^4(c+dx)}{4ad}+\frac{3\cos(c+dx)\sin(c+dx)}{8ad}+\frac{\cos^3(c+dx)\sin(c}{4ad}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 60, normalized size = 0.80

$$\frac{12c+12dx+4i\cos(2(c+dx))+i\cos(4(c+dx))+8\sin(2(c+dx))+\sin(4(c+dx))}{32ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] (12*c + 12*d*x + (4*I)*Cos[2*(c + d*x)] + I*Cos[4*(c + d*x)] + 8*Sin[2*(c +
 d*x)] + Sin[4*(c + d*x)])/(32*a*d)
```

Maple [A]
time = 0.38, size = 75, normalized size = 1.00

method	result	size
risch	$\frac{3x}{8a}+\frac{i\mathrm{e}^{-4i(dx+c)}}{32ad}+\frac{i\cos(2dx+2c)}{8ad}+\frac{\sin(2dx+2c)}{4ad}$	61
derivativedivides	$\frac{-\frac{3i\ln(\tan(dx+c)-i)}{16}-\frac{i}{8(\tan(dx+c)-i)^2}+\frac{1}{4\tan(dx+c)-4i}+\frac{3i\ln(\tan(dx+c)+i)}{16}+\frac{1}{8\tan(dx+c)+8i}}{da}$	75
default	$\frac{-\frac{3i\ln(\tan(dx+c)-i)}{16}-\frac{i}{8(\tan(dx+c)-i)^2}+\frac{1}{4\tan(dx+c)-4i}+\frac{3i\ln(\tan(dx+c)+i)}{16}+\frac{1}{8\tan(dx+c)+8i}}{da}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-3/16*I*ln(tan(d*x+c)-I)-1/8*I/(tan(d*x+c)-I)^2+1/4/(tan(d*x+c)-I)+3
/16*I*ln(tan(d*x+c)+I)+1/8/(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 2.73, size = 54, normalized size = 0.72

$$\frac{\left(12\,dxe^{(4i\,dx+4i\,c)}-2i\,e^{(6i\,dx+6i\,c)}+6i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-4i\,dx-4i\,c)}}{32\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/32*(12*d*x*e^(4*I*d*x + 4*I*c) - 2*I*e^(6*I*d*x + 6*I*c) + 6*I*e^(2*I*d*x
 + 2*I*c) + I)*e^(-4*I*d*x - 4*I*c)/(a*d)
```

Sympy [A]
time = 0.15, size = 151, normalized size = 2.01

$$\begin{cases}\frac{\left(-512ia^2d^2e^{8ic}e^{2idx}+1536ia^2d^2e^{4ic}e^{-2idx}+256ia^2d^2e^{2ic}e^{-4idx}\right)e^{-6ic}}{8192a^3d^3} & \text{for}\ a^3d^3e^{6ic}\neq 0\\ x\left(\frac{\left(e^{6ic}+3e^{4ic}+3e^{2ic}+1\right)e^{-4ic}}{8a}-\frac{3}{8a}\right) & \text{otherwise}\end{cases}+\frac{3x}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Piecewise(((-512*I*a**2*d**2*exp(8*I*c)*exp(2*I*d*x) + 1536*I*a**2*d**2*exp
(4*I*c)*exp(-2*I*d*x) + 256*I*a**2*d**2*exp(2*I*c)*exp(-4*I*d*x))*exp(-6*I*
c)/(8192*a**3*d**3), Ne(a**3*d**3*exp(6*I*c), 0)), (x*((exp(6*I*c) + 3*exp(
4*I*c) + 3*exp(2*I*c) + 1)*exp(-4*I*c)/(8*a) - 3/(8*a)), True)) + 3*x/(8*a)
```

Giac [A]
time = 0.43, size = 99, normalized size = 1.32

$$-\frac{\frac{6i\log(i\tan(dx+c)+1)}{a}-\frac{6i\log(i\tan(dx+c)-1)}{a}+\frac{2(3\tan(dx+c)+5i)}{a(-i\tan(dx+c)+1)}+\frac{-9i\tan(dx+c)^2-26\tan(dx+c)+21i}{a(\tan(dx+c)-i)^2}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/32*(6*I*log(I*tan(d*x + c) + 1)/a - 6*I*log(I*tan(d*x + c) - 1)/a + 2*(3
*tan(d*x + c) + 5*I)/(a*(-I*tan(d*x + c) + 1)) + (-9*I*tan(d*x + c)^2 - 26*
tan(d*x + c) + 21*I)/(a*(tan(d*x + c) - I)^2))/d
```

Mupad [B]
time = 3.43, size = 111, normalized size = 1.48

$$\frac{3\,x}{8\,a}-\frac{\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,1{}\mathrm{i}}{2}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,1{}\mathrm{i}}{2}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)^2\,\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a*cos(c + d*x) + a*sin(c + d*x)*1i),x)
```

```
[Out] (3*x)/(8*a) - ((5*tan(c/2 + (d*x)/2))/4 + (tan(c/2 + (d*x)/2)^2*1i)/2 - tan
(c/2 + (d*x)/2)^3/2 - (tan(c/2 + (d*x)/2)^4*1i)/2 + (5*tan(c/2 + (d*x)/2)^5
)/4)/(a*d*(tan(c/2 + (d*x)/2) + 1i)^2*(tan(c/2 + (d*x)/2)*1i + 1)^4)
```

3.153 $\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=52

$$\frac{i\cos^3(c+dx)}{3ad}+\frac{\sin(c+dx)}{ad}-\frac{\sin^3(c+dx)}{3ad}$$

```
[Out] 1/3*I*cos(d*x+c)^3/a/d+sin(d*x+c)/a/d-1/3*sin(d*x+c)^3/a/d
```

Rubi [A]
time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3171, 3169, 2713, 2645, 30}

$$-\frac{\sin^3(c+dx)}{3ad}+\frac{\sin(c+dx)}{ad}+\frac{i\cos^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((I/3)*Cos[c + d*x]^3)/(a*d) + Sin[c + d*x]/(a*d) - Sin[c + d*x]^3/(3*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3171

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \cos^2(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2}\\ &= -\frac{i\int \left(ia\cos^3(c+dx)+a\cos^2(c+dx)\sin(c+dx)\right)\,dx}{a^2}\\ &= -\frac{i\int \cos^2(c+dx)\sin(c+dx)\,dx}{a}+\frac{\int \cos^3(c+dx)\,dx}{a}\\ &= \frac{i\text{Subst}\left(\int x^2\,dx,x,\cos(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(c+d\right.}{ad}\\ &= \frac{i\cos^3(c+dx)}{3ad}+\frac{\sin(c+dx)}{ad}-\frac{\sin^3(c+dx)}{3ad}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 73, normalized size = 1.40

$$\frac{i\cos(c+dx)}{4ad}+\frac{i\cos(3(c+dx))}{12ad}+\frac{3\sin(c+dx)}{4ad}+\frac{\sin(3(c+dx))}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
[Out] ((I/4)*Cos[c + d*x])/(a*d) + ((I/12)*Cos[3*(c + d*x)])/(a*d) + (3*Sin[c + d
*x])/(4*a*d) + Sin[3*(c + d*x)]/(12*a*d)
```

Maple [A]
time = 0.27, size = 75, normalized size = 1.44

method	result	size
risch	$\frac{i e^{-3i(dx+c)}}{12ad}+\frac{i\cos(dx+c)}{4ad}+\frac{3\sin(dx+c)}{4ad}$	49
derivativedivides	$\frac{\frac{2}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4i}-\frac{2}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{3}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{ad}$	75
default	$\frac{\frac{2}{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4i}-\frac{2}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{3}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{ad}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a*(1/4/(tan(1/2*d*x+1/2*c)+I)-1/3/(-I+tan(1/2*d*x+1/2*c))^3+1/2*I/(-I+t
an(1/2*d*x+1/2*c))^2+3/4/(-I+tan(1/2*d*x+1/2*c)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 2.82, size = 41, normalized size = 0.79

$$\frac{\left(-3i\,e^{(4i\,dx+4i\,c)}+6i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-3i\,dx-3i\,c)}}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(-3*I*e^(4*I*d*x + 4*I*c) + 6*I*e^(2*I*d*x + 2*I*c) + I)*e^(-3*I*d*x -
 3*I*c)/(a*d)
```

Sympy [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(37) = 74$.
time = 0.16, size = 126, normalized size = 2.42

$$\begin{cases} \frac{\left(-24ia^2d^2e^{5ic}e^{idx}+48ia^2d^2e^{3ic}e^{-idx}+8ia^2d^2e^{ic}e^{-3idx}\right)e^{-4ic}}{96a^3d^3} & \text{for } a^3d^3e^{4ic}\neq 0\\ \frac{x\left(e^{4ic}+2e^{2ic}+1\right)e^{-3ic}}{4a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Piecewise(((-24*I*a**2*d**2*exp(5*I*c)*exp(I*d*x) + 48*I*a**2*d**2*exp(3*I*
c)*exp(-I*d*x) + 8*I*a**2*d**2*exp(I*c)*exp(-3*I*d*x))*exp(-4*I*c)/(96*a**3
*d**3), Ne(a**3*d**3*exp(4*I*c), 0)), (x*(exp(4*I*c) + 2*exp(2*I*c) + 1)*ex
p(-3*I*c)/(4*a), True))
```

Giac [A]
time = 0.42, size = 67, normalized size = 1.29

$$\frac{\frac{3}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)}+\frac{9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-12i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-7}{a\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3/(a*(tan(1/2*d*x + 1/2*c) + I)) + (9*tan(1/2*d*x + 1/2*c)^2 - 12*I*ta
n(1/2*d*x + 1/2*c) - 7)/(a*(tan(1/2*d*x + 1/2*c) - I)^3))/d
```

Mupad [B]
time = 0.77, size = 78, normalized size = 1.50

$$\frac{\left(-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,3{}\mathrm{i}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)\,2{}\mathrm{i}}{3\,a\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)\,{\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a*cos(c + d*x) + a*sin(c + d*x)*1i),x)
```

```
[Out] ((tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*3i - 3*tan(c/2 + (d*x)/2)^3 + 1
i)*2i)/(3*a*d*(tan(c/2 + (d*x)/2) + 1i)*(tan(c/2 + (d*x)/2)*1i + 1)^3)
```

3.154 $\int \frac{\cos(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=46

$$\frac{x}{2a}+\frac{i\cos(c+dx)}{2d(a\cos(c+dx)+ia\sin(c+dx))}$$

```
[Out] 1/2*x/a+1/2*I*cos(d*x+c)/d/(a*cos(d*x+c)+I*a*sin(d*x+c))
```

Rubi [A]
time = 0.02, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {3161, 8}

$$\frac{x}{2a}+\frac{i\cos(c+dx)}{2d(a\cos(c+dx)+ia\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] x/(2*a) + ((I/2)*Cos[c + d*x])/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3161

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*((a*Cos[c + d*x] + b*Si
n[c + d*x])^n/(2*a*d*n*Cos[c + d*x]^n)), x] + Dist[1/(2*a), Int[(a*Cos[c +
d*x] + b*Sin[c + d*x])^(n + 1)/Cos[c + d*x]^(n + 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && EqQ[m + n, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= \frac{i\cos(c+dx)}{2d(a\cos(c+dx)+ia\sin(c+dx))}+\frac{\int 1\,dx}{2a}\\ &= \frac{x}{2a}+\frac{i\cos(c+dx)}{2d(a\cos(c+dx)+ia\sin(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 38, normalized size = 0.83

$$\frac{2(c+dx)+i\cos(2(c+dx))+\sin(2(c+dx))}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] (2*(c + d*x) + I*Cos[2*(c + d*x)] + Sin[2*(c + d*x)])/(4*a*d)
```

Maple [A]
time = 0.20, size = 48, normalized size = 1.04

method	result	size
risch	$\frac{x}{2a}+\frac{i\mathrm{e}^{-2i(dx+c)}}{4ad}$	26
derivativedivides	$\frac{-\frac{i\ln(\tan(dx+c)-i)}{4}+\frac{1}{2\tan(dx+c)-2i}+\frac{i\ln(\tan(dx+c)+i)}{4}}{da}$	48
default	$\frac{-\frac{i\ln(\tan(dx+c)-i)}{4}+\frac{1}{2\tan(dx+c)-2i}+\frac{i\ln(\tan(dx+c)+i)}{4}}{da}$	48

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-1/4*I*ln(tan(d*x+c)-I)+1/2/(tan(d*x+c)-I)+1/4*I*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 4.12, size = 32, normalized size = 0.70

$$\frac{\left(2\,dxe^{(2i\,dx+2i\,c)}+i\right)e^{(-2i\,dx-2i\,c)}}{4\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*d*x*e^(2*I*d*x + 2*I*c) + I)*e^(-2*I*d*x - 2*I*c)/(a*d)
```

Sympy [A]
time = 0.08, size = 60, normalized size = 1.30

$$\begin{cases}\frac{ie^{-2ic}e^{-2idx}}{4ad} & \text{for } ade^{2ic}\neq 0\\ x\left(\frac{(e^{2ic}+1)e^{-2ic}}{2a}-\frac{1}{2a}\right) & \text{otherwise}\end{cases}+\frac{x}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Piecewise((I*exp(-2*I*c)*exp(-2*I*d*x)/(4*a*d), Ne(a*d*exp(2*I*c), 0)), (x*
((exp(2*I*c) + 1)*exp(-2*I*c)/(2*a) - 1/(2*a)), True)) + x/(2*a)
```

Giac [A]
time = 0.41, size = 60, normalized size = 1.30

$$-\frac{\frac{i\log(\tan(dx+c)-i)}{a}-\frac{i\log(-i\tan(dx+c)+1)}{a}+\frac{-i\tan(dx+c)-3}{a(\tan(dx+c)-i)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(I*log(tan(d*x + c) - I)/a - I*log(-I*tan(d*x + c) + 1)/a + (-I*tan(d*
x + c) - 3)/(a*(tan(d*x + c) - I)))/d
```

Mupad [B]
time = 0.71, size = 39, normalized size = 0.85

$$\frac{x}{2a}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a\,d\,{\left(1+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,1{}\mathrm{i}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a*cos(c + d*x) + a*sin(c + d*x)*1i),x)
```

```
[Out] x/(2*a) + tan(c/2 + (d*x)/2)/(a*d*(tan(c/2 + (d*x)/2)*1i + 1)^2)
```

3.155 $\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=29

$$\frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

```
[Out] I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))
```

Rubi [A]
time = 0.01, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{3150\}$

$$\frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-1),x]
```

```
[Out] I/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x]))
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = \frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

Mathematica [A]
time = 0.04, size = 29, normalized size = 1.00

$$\frac{i}{d(a\cos(c+dx)+ia\sin(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-1),x]
```

```
[Out] I/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x]))
```

Maple [A]
time = 0.16, size = 23, normalized size = 0.79

method	result	size
risch	$\frac{i e^{-i(dx+c)}}{ad}$	19
derivativedivides	$\frac{2}{da\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	23
default	$\frac{2}{da\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$	23
norman	$\frac{-\frac{2i\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a/(-I+tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.27, size = 29, normalized size = 1.00

$$\frac{2}{\left(-i\,a+\frac{a\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/((-I*a + a*sin(d*x + c)/(cos(d*x + c) + 1))*d)
```

Fricas [A]
time = 3.98, size = 17, normalized size = 0.59

$$\frac{i\,e^{\left(-i\,dx-i\,c\right)}}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] I*e^(-I*d*x - I*c)/(a*d)
```

Sympy [A]
time = 0.06, size = 31, normalized size = 1.07

$$\begin{cases} \frac{ie^{-ic}e^{-idx}}{ad} & \text{for}\: ade^{ic} \neq 0 \\ \frac{xe^{-ic}}{a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Piecewise((I*exp(-I*c)*exp(-I*d*x)/(a*d), Ne(a*d*exp(I*c), 0)), (x*exp(-I*c
)/a, True))
```

Giac [A]
time = 0.44, size = 21, normalized size = 0.72

$$\frac{2}{ad\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 2/(a*d*(tan(1/2*d*x + 1/2*c) - I))
```

Mupad [B]
time = 0.61, size = 25, normalized size = 0.86

$$\frac{2\mathrm{i}}{a\,d\,\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i),x)
```

```
[Out] 2i/(a*d*(tan(c/2 + (d*x)/2)*1i + 1))
```

3.156 $\int \frac{\sec(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=23

$$\frac{x}{a}+\frac{i\log(\cos(c+dx))}{ad}$$

[Out] x/a+I*ln(cos(d*x+c))/a/d

Rubi [A]
time = 0.05, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.103, Rules used = {3171, 3169, 3556}

$$\frac{x}{a}+\frac{i\log(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]

[Out] x/a + (I*Log[Cos[c + d*x]])/(a*d)

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3171

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx = -\frac{i\int \sec(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2}$$
$$= -\frac{i\int (ia+a\tan(c+dx))\,dx}{a^2}$$
$$= \frac{x}{a}-\frac{i\int \tan(c+dx)\,dx}{a}$$
$$= \frac{x}{a}+\frac{i\log(\cos(c+dx))}{ad}$$

Mathematica [A]
time = 0.06, size = 23, normalized size = 1.00

$$\frac{c+dx+i\log(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] (c + d*x + I*Log[Cos[c + d*x]])/(a*d)
```

Maple [A]
time = 0.24, size = 22, normalized size = 0.96

method	result	size
derivativedivides	$-\frac{i\ln(i\tan(dx+c)+1)}{da}$	22
default	$-\frac{i\ln(i\tan(dx+c)+1)}{da}$	22
risch	$\frac{2x}{a}+\frac{2c}{ad}+\frac{i\ln\left(e^{2i(dx+c)}+1\right)}{ad}$	38
norman	$\frac{x}{a}+\frac{i\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}+\frac{i\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}-\frac{i\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -I/d/a*ln(I*tan(d*x+c)+1)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(21) = 42.
time = 0.28, size = 101, normalized size = 4.39

$$-\frac{-\frac{i\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{i\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{i\log\left(-\frac{2i\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(-I*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - I*log(sin(d*x + c)/(cos(d
*x + c) + 1) - 1)/a + I*log(-2*I*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x
+ c)^2/(cos(d*x + c) + 1)^2 - 1)/a)/d
```

Fricas [A]
time = 4.20, size = 26, normalized size = 1.13

$$\frac{2\,dx + i\,\log\left(e^{(2i\,dx+2i\,c)} + 1\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] (2*d*x + I*log(e^(2*I*d*x + 2*I*c) + 1))/(a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec\left(c+dx\right)}{i\sin\left(c+dx\right)+\cos\left(c+dx\right)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(I*sin(c + d*x) + cos(c + d*x)), x)/a
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 57 vs. 2(21) = 42.
time = 0.43, size = 57, normalized size = 2.48

$$-\frac{-\frac{i\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a} + \frac{2i\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}{a} - \frac{i\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(-I*log(tan(1/2*d*x + 1/2*c) + 1)/a + 2*I*log(tan(1/2*d*x + 1/2*c) - I)/a
- I*log(tan(1/2*d*x + 1/2*c) - 1)/a)/d
```

Mupad [B]
time = 0.74, size = 41, normalized size = 1.78

$$-\frac{\left(2\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)-\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\right)\,1\mathrm{i}}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a*cos(c + d*x) + a*sin(c + d*x)*1i)),x)
```

```
[Out] -((2*log(tan(c/2 + (d*x)/2) - 1i) - log(tan(c/2 + (d*x)/2)^2 - 1))*1i)/(a*d
)
```

3.157 $\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{i\sec(c+dx)}{ad}$$

```
[Out] arctanh(sin(d*x+c))/a/d-I*sec(d*x+c)/a/d
```

Rubi [A]
time = 0.07, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.161, Rules used = {3171, 3169, 3855, 2686, 8}

$$\frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{i\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(a*d) - (I*Sec[c + d*x])/(a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3171

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \sec^2(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2}\\ &= -\frac{i\int (ia\sec(c+dx)+a\sec(c+dx)\tan(c+dx))\,dx}{a^2}\\ &= -\frac{i\int \sec(c+dx)\tan(c+dx)\,dx}{a}+\frac{\int \sec(c+dx)\,dx}{a}\\ &= \frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{i\text{Subst}(\int 1\,dx,x,\sec(c+dx))}{ad}\\ &= \frac{\tanh^{-1}(\sin(c+dx))}{ad}-\frac{i\sec(c+dx)}{ad}\end{aligned}$$

Mathematica [A]
time = 0.24, size = 35, normalized size = 1.13

$$-\frac{i\left(2i\tanh^{-1}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)+\sec(c+dx)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
[Out] ((-I)*((2*I)*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] + Sec[c + d*x]))/(a*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(30) = 60.
time = 0.28, size = 70, normalized size = 2.26

method	result	size
norman	$\frac{2i}{ad\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}$	65
derivativedivides	$\frac{\frac{2i}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{i}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}$	70
default	$\frac{\frac{2i}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2}-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{i}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}$	70

risch	$-\frac{2i e^{i(dx+c)}}{da\left(e^{2i(dx+c)}+1\right)}-\frac{\ln\left(e^{i(dx+c)}-i\right)}{ad}+\frac{\ln\left(e^{i(dx+c)}+i\right)}{ad}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a*(1/2*I/(tan(1/2*d*x+1/2*c)-1)-1/2*ln(tan(1/2*d*x+1/2*c)-1)-1/2*I/(tan
(1/2*d*x+1/2*c)+1)+1/2*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(29) = 58$.
time = 0.28, size = 83, normalized size = 2.68

$$\frac{\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{2}{-i\,a+\frac{i\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] (log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c
) + 1) - 1)/a - 2/(-I*a + I*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2))/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(29) = 58$.
time = 2.47, size = 80, normalized size = 2.58

$$\frac{\left(e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-\left(e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-2i\,e^{(i\,dx+i\,c)}}{ade^{(2i\,dx+2i\,c)}+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] ((e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - (e^(2*I*d*x + 2*I*c)
+ 1)*log(e^(I*d*x + I*c) - I) - 2*I*e^(I*d*x + I*c))/(a*d*e^(2*I*d*x + 2*I*
c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^2(c+dx)}{i\sin(c+dx)+\cos(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(I*sin(c + d*x) + cos(c + d*x)), x)/a
```

Giac [A]
time = 0.43, size = 58, normalized size = 1.87

$$\frac{\frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a}-\frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a}+\frac{2i}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
[Out] (log(tan(1/2*d*x + 1/2*c) + 1)/a - log(tan(1/2*d*x + 1/2*c) - 1)/a + 2*I/((
tan(1/2*d*x + 1/2*c)^2 - 1)*a))/d
```

Mupad [B]
time = 0.67, size = 43, normalized size = 1.39

$$\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}+\frac{2\mathrm{i}}{a\,d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a*cos(c + d*x) + a*sin(c + d*x)*1i)),x)
[Out] (2*atanh(tan(c/2 + (d*x)/2)))/(a*d) + 2i/(a*d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.158 $\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=34

$$-\frac{i\sec^2(c+dx)}{2ad}+\frac{\tan(c+dx)}{ad}$$

```
[Out] -1/2*I*sec(d*x+c)^2/a/d+tan(d*x+c)/a/d
```

Rubi [A]
time = 0.08, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}=0.194$, Rules used = {3171, 3169, 3852, 8, 2686, 30}

$$\frac{\tan(c+dx)}{ad}-\frac{i\sec^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((-1/2*I)*Sec[c + d*x]^2)/(a*d) + Tan[c + d*x]/(a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3171

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
```

```
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \sec^3(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2}\\ &= -\frac{i\int \left(ia\sec^2(c+dx)+a\sec^2(c+dx)\tan(c+dx)\right)\,dx}{a^2}\\ &= -\frac{i\int \sec^2(c+dx)\tan(c+dx)\,dx}{a}+\frac{\int \sec^2(c+dx)\,dx}{a}\\ &= -\frac{i\text{Subst}(\int x\,dx,x,\sec(c+dx))}{ad}-\frac{\text{Subst}(\int 1\,dx,x,-\tan(c+dx))}{ad}\\ &= -\frac{i\sec^2(c+dx)}{2ad}+\frac{\tan(c+dx)}{ad}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 35, normalized size = 1.03

$$-\frac{i\sec(c+dx)(\sec(c+dx)+2i\sec(c)\sin(dx))}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
[Out] ((-1/2*I)*Sec[c + d*x]*(Sec[c + d*x] + (2*I)*Sec[c]*Sin[d*x]))/(a*d)
```

Maple [A]
time = 0.30, size = 30, normalized size = 0.88

method	result	size
risch	$\frac{2i}{ad\left(e^{2i(dx+c)}+1\right)^2}$	23
derivativedivides	$-\frac{i\left(\frac{\left(\tan^2(dx+c)\right)}{2}+i\tan(dx+c)\right)}{da}$	30

default	$-\frac{i\left(\frac{\left(\tan^2(dx+c)\right)}{2}+i\tan(dx+c)\right)}{da}$	30
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{2i\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}$	74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -I/d/a*(1/2*tan(d*x+c)^2+I*tan(d*x+c))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(30) = 60$.
time = 0.28, size = 108, normalized size = 3.18

$$\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{\left(a-\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2*(sin(d*x + c)/(cos(d*x + c) + 1) - I*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
- sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/((a - 2*a*sin(d*x + c)^2/(cos(d*x +
c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)*d)
```

Fricas [A]
time = 3.07, size = 33, normalized size = 0.97

$$\frac{2i}{ade^{(4i\,dx+4i\,c)}+2\,ade^{(2i\,dx+2i\,c)}+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 2*I/(a*d*e^(4*I*d*x + 4*I*c) + 2*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec^3(c+dx)}{i\sin(c+dx)+\cos(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(I*sin(c + d*x) + cos(c + d*x)), x)/a
```

Giac [A]
time = 0.45, size = 27, normalized size = 0.79

$$-\frac{i\tan\left(dx+c\right)^{2}-2\tan\left(dx+c\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
[Out] -1/2*(I*tan(d*x + c)^2 - 2*tan(d*x + c))/(a*d)
```

Mupad [B]
time = 0.68, size = 25, normalized size = 0.74

$$-\frac{\tan(c+d\,x)\,\left(-2+\tan(c+d\,x)\,1\mathrm{i}\right)}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a*cos(c + d*x) + a*sin(c + d*x)*1i)),x)
[Out] -(tan(c + d*x)*(tan(c + d*x)*1i - 2))/(2*a*d)
```

3.159 $\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=60

$$\frac{\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{i\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)\tan(c+dx)}{2ad}$$

```
[Out] 1/2*arctanh(sin(d*x+c))/a/d-1/3*I*sec(d*x+c)^3/a/d+1/2*sec(d*x+c)*tan(d*x+c
)/a/d
```

Rubi [A]
time = 0.09, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}=0.194$, Rules used = {3171, 3169, 3853, 3855, 2686, 30}

$$-\frac{i\sec^3(c+dx)}{3ad}+\frac{\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{\tan(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(2*a*d) - ((I/3)*Sec[c + d*x]^3)/(a*d) + (Sec[c + d*x
]*Tan[c + d*x])/(2*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3171

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
```

```
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \sec^4(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2}\\ &= -\frac{i\int (ia\sec^3(c+dx)+a\sec^3(c+dx)\tan(c+dx))\,dx}{a^2}\\ &= -\frac{i\int \sec^3(c+dx)\tan(c+dx)\,dx}{a}+\frac{\int \sec^3(c+dx)\,dx}{a}\\ &= \frac{\sec(c+dx)\tan(c+dx)}{2ad}+\frac{\int \sec(c+dx)\,dx}{2a}-\frac{i\text{Subst}\left(\int x^2\,dx,x,\sec(c\right.}{ad}\\ &= \frac{\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{i\sec^3(c+dx)}{3ad}+\frac{\sec(c+dx)\tan(c+dx)}{2ad}\end{aligned}$$

Mathematica [A]
time = 0.28, size = 54, normalized size = 0.90

$$-\frac{i\left(12i\tanh^{-1}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)+\sec^3(c+dx)(4+3i\sin(2(c+dx)))\right)}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((-1/12*I)*((12*I)*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] + Sec[c + d*x]^3*(
4 + (3*I)*Sin[2*(c + d*x)])))/(a*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(53) = 106.
time = 0.35, size = 138, normalized size = 2.30

method	result
risch	$-\frac{i\left(3\,\mathrm{e}^{5i(dx+c)}+8\,\mathrm{e}^{3i(dx+c)}-3\,\mathrm{e}^{i(dx+c)}\right)}{3da\left(\mathrm{e}^{2i(dx+c)}+1\right)^{3}}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2ad}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2ad}$
norman	$\frac{\frac{\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{2i}{3ad}+\frac{2i\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2ad}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2ad}$
derivativedivides	$\frac{\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(\frac{1}{4}+\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(\frac{1}{4}+\frac{i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{2\left(\frac{1}{4}-\frac{i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\cdots}{ad}$
default	$\frac{\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(\frac{1}{4}+\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(\frac{1}{4}+\frac{i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}-\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{2\left(\frac{1}{4}-\frac{i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\cdots}{ad}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a*(1/6*I/(tan(1/2*d*x+1/2*c)-1)^3+(1/4+1/4*I)/(tan(1/2*d*x+1/2*c)-1)^2+
(1/4+1/4*I)/(tan(1/2*d*x+1/2*c)-1)-1/4*ln(tan(1/2*d*x+1/2*c)-1)-1/6*I/(tan(
1/2*d*x+1/2*c)+1)^3+(1/4-1/4*I)/(tan(1/2*d*x+1/2*c)+1)+(-1/4+1/4*I)/(tan(1/
2*d*x+1/2*c)+1)^2+1/4*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(52) = 104$.
time = 0.29, size = 186, normalized size = 3.10

$$\frac{\frac{4\left(\frac{3i\sin(dx+c)}{\cos(dx+c)+1}+\frac{6\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{3i\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+2\right)}{6i\,a-\frac{18i\,a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{18i\,a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{6i\,a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(4*(3*I*sin(d*x + c)/(cos(d*x + c) + 1) + 6*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 - 3*I*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 2)/(6*I*a - 18*I*a*sin
(d*x + c)^2/(cos(d*x + c) + 1)^2 + 18*I*a*sin(d*x + c)^4/(cos(d*x + c) + 1)
^4 - 6*I*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + log(sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(52) = 104$.
time = 3.24, size = 174, normalized size = 2.90

$$\frac{3\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-6i\,e^{(5i\,dx+5i\,c)}-16i\,e^{(3i\,dx+3i\,c)}+6i\,e^{(i\,dx+i\,c)}}{6\left(ade^{(6i\,dx+6i\,c)}+3\,ade^{(4i\,dx+4i\,c)}+3\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c)
 + 1)*log(e^(I*d*x + I*c) + I) - 3*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*
I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) - I) - 6*I*e^(5*I*d*x
 + 5*I*c) - 16*I*e^(3*I*d*x + 3*I*c) + 6*I*e^(I*d*x + I*c))/(a*d*e^(6*I*d*x
 + 6*I*c) + 3*a*d*e^(4*I*d*x + 4*I*c) + 3*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^4(c+dx)}{i\sin(c+dx)+\cos(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**4/(I*sin(c + d*x) + cos(c + d*x)), x)/a
```

Giac [A]
time = 0.44, size = 99, normalized size = 1.65

$$\frac{\frac{3\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a}-\frac{3\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a}+\frac{2\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+6i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2i\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3a}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(3*log(tan(1/2*d*x + 1/2*c) + 1)/a - 3*log(tan(1/2*d*x + 1/2*c) - 1)/a
+ 2*(3*tan(1/2*d*x + 1/2*c)^5 + 6*I*tan(1/2*d*x + 1/2*c)^4 - 3*tan(1/2*d*x
+ 1/2*c) + 2*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a))/d
```

Mupad [B]
time = 2.55, size = 116, normalized size = 1.93

$$\frac{\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{ad}+\frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5}{a}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{a}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 2\mathrm{i}}{a}+\frac{2\mathrm{i}}{3a}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a*cos(c + d*x) + a*sin(c + d*x)*1i)),x)
```

```
[Out] atanh(tan(c/2 + (d*x)/2))/(a*d) + ((tan(c/2 + (d*x)/2)^4*2i)/a + tan(c/2 +
(d*x)/2)^5/a + 2i/(3*a) - tan(c/2 + (d*x)/2)/a)/(d*(3*tan(c/2 + (d*x)/2)^2
- 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.160 $\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=52

$$-\frac{i\sec^4(c+dx)}{4ad}+\frac{\tan(c+dx)}{ad}+\frac{\tan^3(c+dx)}{3ad}$$

```
[Out] -1/4*I*sec(d*x+c)^4/a/d+tan(d*x+c)/a/d+1/3*tan(d*x+c)^3/a/d
```

Rubi [A]
time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3171, 3169, 3852, 2686, 30}

$$\frac{\tan^3(c+dx)}{3ad}+\frac{\tan(c+dx)}{ad}-\frac{i\sec^4(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((-1/4*I)*Sec[c + d*x]^4)/(a*d) + Tan[c + d*x]/(a*d) + Tan[c + d*x]^3/(3*a*
d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3171

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
```

```
EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^5(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \sec^5(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2}\\ &= -\frac{i\int \left(ia\sec^4(c+dx)+a\sec^4(c+dx)\tan(c+dx)\right)\,dx}{a^2}\\ &= -\frac{i\int \sec^4(c+dx)\tan(c+dx)\,dx}{a}+\frac{\int \sec^4(c+dx)\,dx}{a}\\ &= -\frac{i\text{Subst}\left(\int x^3\,dx,x,\sec(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\tan(c+dx)\right)}{ad}\\ &= -\frac{i\sec^4(c+dx)}{4ad}+\frac{\tan(c+dx)}{ad}+\frac{\tan^3(c+dx)}{3ad}\end{aligned}$$

Mathematica [A]
time = 0.31, size = 53, normalized size = 1.02

$$-\frac{i\sec^4(c+dx)(3+i\sec(c)(4\sin(c+2dx)+\sin(3c+4dx))-3i\tan(c))}{12ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
[Out] ((-1/12*I)*Sec[c + d*x]^4*(3 + I*Sec[c]*(4*Sin[c + 2*d*x] + Sin[3*c + 4*d*x
]) - (3*I)*Tan[c]))/(a*d)
```

Maple [A]
time = 0.37, size = 51, normalized size = 0.98

method	result	size
risch	$\frac{4i\left(4\,e^{2i(dx+c)}+1\right)}{3ad\left(e^{2i(dx+c)}+1\right)^4}$	36
derivativedivides	$\frac{i\left(-i\tan(dx+c)-\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{i\left(\tan^3(dx+c)\right)}{3}-\frac{\left(\tan^2(dx+c)\right)}{2}\right)}{da}$	51

default	$\frac{i\left(-i\tan(dx+c)-\frac{\left(\tan^4(dx+c)\right)}{4}-\frac{i\left(\tan^3(dx+c)\right)}{3}-\frac{\left(\tan^2(dx+c)\right)}{2}\right)}{da}$	51
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{10\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{10\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2i\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2i\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}$	132

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] I/d/a*(-I*tan(d*x+c)-1/4*tan(d*x+c)^4-1/3*I*tan(d*x+c)^3-1/2*tan(d*x+c)^2)
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(46) = 92$.

time = 0.28, size = 211, normalized size = 4.06

$$\frac{2\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{3i\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{5\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3i\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{3\left(a-\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/3*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 3*I*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 - 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 5*sin(d*x + c)^5/(cos(d*x +
 c) + 1)^5 - 3*I*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 3*sin(d*x + c)^7/(co
s(d*x + c) + 1)^7)/((a - 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(
d*x + c)^4/(cos(d*x + c) + 1)^4 - 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 +
 a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8)*d)
```

Fricas [A]

time = 2.42, size = 72, normalized size = 1.38

$$-\frac{4\left(-4i\,e^{(2i\,dx+2i\,c)}-i\right)}{3\left(ade^{(8i\,dx+8i\,c)}+4\,ade^{(6i\,dx+6i\,c)}+6\,ade^{(4i\,dx+4i\,c)}+4\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -4/3*(-4*I*e^(2*I*d*x + 2*I*c) - I)/(a*d*e^(8*I*d*x + 8*I*c) + 4*a*d*e^(6*I
*d*x + 6*I*c) + 6*a*d*e^(4*I*d*x + 4*I*c) + 4*a*d*e^(2*I*d*x + 2*I*c) + a*d
)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^5(c+dx)}{i\sin(c+dx)+\cos(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**5/(I*sin(c + d*x) + cos(c + d*x)), x)/a
```

Giac [A]
time = 0.46, size = 47, normalized size = 0.90

$$-\frac{3i\tan\left(dx+c\right)^4-4\tan\left(dx+c\right)^3+6i\tan\left(dx+c\right)^2-12\tan\left(dx+c\right)}{12\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(3*I*tan(d*x + c)^4 - 4*tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - 12*tan(
d*x + c))/(a*d)
```

Mupad [B]
time = 1.29, size = 99, normalized size = 1.90

$$-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,3{}\mathrm{i}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,3{}\mathrm{i}-3\right)}{3\,a\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a*cos(c + d*x) + a*sin(c + d*x)*1i)),x)
```

```
[Out] -(2*tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)*3i + 5*tan(c/2 + (d*x)/2)^2 - 5*
tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^5*3i + 3*tan(c/2 + (d*x)/2)^6 - 3
))/(3*a*d*(tan(c/2 + (d*x)/2)^2 - 1)^4)
```

3.161 $\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=84

$$\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}-\frac{i\sec^5(c+dx)}{5ad}+\frac{3\sec(c+dx)\tan(c+dx)}{8ad}+\frac{\sec^3(c+dx)\tan(c+dx)}{4ad}$$

```
[Out] 3/8*arctanh(sin(d*x+c))/a/d-1/5*I*sec(d*x+c)^5/a/d+3/8*sec(d*x+c)*tan(d*x+c
)/a/d+1/4*sec(d*x+c)^3*tan(d*x+c)/a/d
```

Rubi [A]
time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3171, 3169, 3853, 3855, 2686, 30}

$$-\frac{i\sec^5(c+dx)}{5ad}+\frac{3\tanh^{-1}(\sin(c+dx))}{8ad}+\frac{\tan(c+dx)\sec^3(c+dx)}{4ad}+\frac{3\tan(c+dx)\sec(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(8*a*d) - ((I/5)*Sec[c + d*x]^5)/(a*d) + (3*Sec[c
 + d*x]*Tan[c + d*x])/(8*a*d) + (Sec[c + d*x]^3*Tan[c + d*x])/(4*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3171

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
```

```
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^6(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \sec^6(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2}\\ &= -\frac{i\int \left(ia\sec^5(c+dx)+a\sec^5(c+dx)\tan(c+dx)\right)\,dx}{a^2}\\ &= -\frac{i\int \sec^5(c+dx)\tan(c+dx)\,dx}{a}+\frac{\int \sec^5(c+dx)\,dx}{a}\\ &= \frac{\sec^3(c+dx)\tan(c+dx)}{4ad}+\frac{3\int \sec^3(c+dx)\,dx}{4a}-\frac{i\text{Subst}\left(\int x^4\,dx,x,\text{s}\right.}{ad}\\ &= -\frac{i\sec^5(c+dx)}{5ad}+\frac{3\sec(c+dx)\tan(c+dx)}{8ad}+\frac{\sec^3(c+dx)\tan(c+d}{4ad}\\ &= \frac{3\tanh^{-1}(\sin(c+dx))}{8ad}-\frac{i\sec^5(c+dx)}{5ad}+\frac{3\sec(c+dx)\tan(c+dx)}{8ad}+\end{aligned}$$

Mathematica [A]
time = 0.52, size = 66, normalized size = 0.79

$$-\frac{i\left(240i\tanh^{-1}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)+\sec^5(c+dx)(64+70i\sin(2(c+dx))+15i\sin(4(c+dx)))\right)}{320ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((-1/320*I)*((240*I)*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] + Sec[c + d*x]^5
*(64 + (70*I)*Sin[2*(c + d*x)] + (15*I)*Sin[4*(c + d*x)])))/(a*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(75) = 150.
time = 0.43, size = 206, normalized size = 2.45

method	result
risch	$-\frac{i\left(15\,\mathrm{e}^{9i(dx+c)}+70\,\mathrm{e}^{7i(dx+c)}+128\,\mathrm{e}^{5i(dx+c)}-70\,\mathrm{e}^{3i(dx+c)}-15\,\mathrm{e}^{i(dx+c)}\right)}{20da\left(\mathrm{e}^{2i(dx+c)}+1\right)^{5}}-\frac{3\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{8ad}+\frac{3\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{8ad}$
norman	$\frac{-\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2da}-\frac{\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{5\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}+\frac{2i}{5ad}+\frac{4i\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2i\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}-\frac{3\ln\left(\mathrm{ta}\right.}{}$
derivativedivides	$\frac{\frac{i}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}+\frac{2\left(\frac{7}{16}+\frac{5i}{16}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(\frac{5}{16}+\frac{3i}{16}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(\frac{1}{4}+\frac{3i}{8}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(\frac{1}{8}+\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}-}{ad}$
default	$\frac{\frac{i}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}+\frac{2\left(\frac{7}{16}+\frac{5i}{16}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(\frac{5}{16}+\frac{3i}{16}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(\frac{1}{4}+\frac{3i}{8}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(\frac{1}{8}+\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}-}{ad}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a*(1/10*I/(tan(1/2*d*x+1/2*c)-1)^5+(7/16+5/16*I)/(tan(1/2*d*x+1/2*c)-1)
^2+(5/16+3/16*I)/(tan(1/2*d*x+1/2*c)-1)+(1/4+3/8*I)/(tan(1/2*d*x+1/2*c)-1)^
3+(1/8+1/4*I)/(tan(1/2*d*x+1/2*c)-1)^4-3/16*ln(tan(1/2*d*x+1/2*c)-1)-1/10*I
/(tan(1/2*d*x+1/2*c)+1)^5+(5/16-3/16*I)/(tan(1/2*d*x+1/2*c)+1)+(1/4-3/8*I)/
(tan(1/2*d*x+1/2*c)+1)^3+(-1/8+1/4*I)/(tan(1/2*d*x+1/2*c)+1)^4+(-7/16+5/16*
I)/(tan(1/2*d*x+1/2*c)+1)^2+3/16*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(74) = 148.
time = 0.29, size = 288, normalized size = 3.43

$$-\frac{3\left(\frac{16\left(\frac{25i\sin(dx+c)}{\cos(dx+c)+1}-\frac{10i\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{80\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{10i\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{40\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{25i\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}+8\right)}{-120ia+\frac{600ia\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{1200ia\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{1200ia\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{600ia\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\frac{120ia\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -3/8*(16*(25*I*sin(d*x + c)/(cos(d*x + c) + 1) - 10*I*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 80*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 10*I*sin(d*x + c)
^7/(cos(d*x + c) + 1)^7 + 40*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 - 25*I*sin
(d*x + c)^9/(cos(d*x + c) + 1)^9 + 8)/(-120*I*a + 600*I*a*sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 - 1200*I*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1200*I*
a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 600*I*a*sin(d*x + c)^8/(cos(d*x + c
) + 1)^8 + 120*I*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10) - log(sin(d*x + c
)/(cos(d*x + c) + 1) + 1)/a + log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(74) = 148$.
time = 1.63, size = 266, normalized size = 3.17

$$\frac{15\left(e^{(10i\,dx+10i\,c)}+5\,e^{(8i\,dx+8i\,c)}+10\,e^{(6i\,dx+6i\,c)}+10\,e^{(4i\,dx+4i\,c)}+5\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(e^{(10i\,dx+10i\,c)}+5\,e^{(8i\,dx+8i\,c)}+10\,e^{(6i\,dx+6i\,c)}+10\,e^{(4i\,dx+4i\,c)}+5\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-30i\,e^{(9i\,dx+9i\,c)}-140i\,e^{(7i\,dx+7i\,c)}-256i\,e^{(5i\,dx+5i\,c)}+140i\,e^{(3i\,dx+3i\,c)}+30i\,e^{(i\,dx+i\,c)}}{40\left(ade^{(10i\,dx+10i\,c)}+5\,ade^{(8i\,dx+8i\,c)}+10\,ade^{(6i\,dx+6i\,c)}+10\,ade^{(4i\,dx+4i\,c)}+5\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/40*(15*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x + 6
*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x +
I*c) + I) - 15*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d
*x + 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*
d*x + I*c) - I) - 30*I*e^(9*I*d*x + 9*I*c) - 140*I*e^(7*I*d*x + 7*I*c) - 25
6*I*e^(5*I*d*x + 5*I*c) + 140*I*e^(3*I*d*x + 3*I*c) + 30*I*e^(I*d*x + I*c))
/(a*d*e^(10*I*d*x + 10*I*c) + 5*a*d*e^(8*I*d*x + 8*I*c) + 10*a*d*e^(6*I*d*x
 + 6*I*c) + 10*a*d*e^(4*I*d*x + 4*I*c) + 5*a*d*e^(2*I*d*x + 2*I*c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{6}(c+dx)}{i\sin(c+dx)+\cos(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**6/(I*sin(c + d*x) + cos(c + d*x)), x)/a
```

Giac [A]
time = 0.43, size = 138, normalized size = 1.64

$$\frac{\frac{15\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a}-\frac{15\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a}+\frac{2\left(25\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+40i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{8}-10\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+80i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+10\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-25\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8i\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{5}a}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/40*(15*log(tan(1/2*d*x + 1/2*c) + 1)/a - 15*log(tan(1/2*d*x + 1/2*c) - 1)
/a + 2*(25*tan(1/2*d*x + 1/2*c)^9 + 40*I*tan(1/2*d*x + 1/2*c)^8 - 10*tan(1/
2*d*x + 1/2*c)^7 + 80*I*tan(1/2*d*x + 1/2*c)^4 + 10*tan(1/2*d*x + 1/2*c)^3
- 25*tan(1/2*d*x + 1/2*c) + 8*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*a))/d
```

Mupad [B]
time = 4.25, size = 193, normalized size = 2.30

$$\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a\,d}+\frac{\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{2\,a}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}{2\,a}+\frac{5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^9}{4\,a}-\frac{5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4\,a}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,4{}\mathrm{i}}{a}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8\,2{}\mathrm{i}}{a}+\frac{2{}\mathrm{i}}{5\,a}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a*cos(c + d*x) + a*sin(c + d*x)*1i)),x)
```

```
[Out] (3*atanh(tan(c/2 + (d*x)/2)))/(4*a*d) + (tan(c/2 + (d*x)/2)^3/(2*a) + (tan(
c/2 + (d*x)/2)^4*4i)/a - tan(c/2 + (d*x)/2)^7/(2*a) + (tan(c/2 + (d*x)/2)^8
*2i)/a + (5*tan(c/2 + (d*x)/2)^9)/(4*a) + 2i/(5*a) - (5*tan(c/2 + (d*x)/2))
/(4*a))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 +
 (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.162 $\int \frac{\sec^7(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx$

Optimal. Leaf size=70

$$-\frac{i\sec^6(c+dx)}{6ad}+\frac{\tan(c+dx)}{ad}+\frac{2\tan^3(c+dx)}{3ad}+\frac{\tan^5(c+dx)}{5ad}$$

```
[Out] -1/6*I*sec(d*x+c)^6/a/d+tan(d*x+c)/a/d+2/3*tan(d*x+c)^3/a/d+1/5*tan(d*x+c)^
5/a/d
```

Rubi [A]
time = 0.09, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {3171, 3169, 3852, 2686, 30}

$$\frac{\tan^5(c+dx)}{5ad}+\frac{2\tan^3(c+dx)}{3ad}+\frac{\tan(c+dx)}{ad}-\frac{i\sec^6(c+dx)}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^7/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
```

```
[Out] ((-1/6*I)*Sec[c + d*x]^6)/(a*d) + Tan[c + d*x]/(a*d) + (2*Tan[c + d*x]^3)/(
3*a*d) + Tan[c + d*x]^5/(5*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3171

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
```

```
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^7(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx &= -\frac{i\int \sec^7(c+dx)(ia\cos(c+dx)+a\sin(c+dx))\,dx}{a^2}\\ &= -\frac{i\int \left(ia\sec^6(c+dx)+a\sec^6(c+dx)\tan(c+dx)\right)\,dx}{a^2}\\ &= -\frac{i\int \sec^6(c+dx)\tan(c+dx)\,dx}{a}+\frac{\int \sec^6(c+dx)\,dx}{a}\\ &= -\frac{i\mathrm{Subst}\left(\int x^5\,dx,x,\sec(c+dx)\right)}{ad}-\frac{\mathrm{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,-\right.}{ad}\\ &= -\frac{i\sec^6(c+dx)}{6ad}+\frac{\tan(c+dx)}{ad}+\frac{2\tan^3(c+dx)}{3ad}+\frac{\tan^5(c+dx)}{5ad}\end{aligned}$$

Mathematica [A]
time = 0.39, size = 67, normalized size = 0.96

$$-\frac{i\sec(c)\sec^6(c+dx)(10\cos(c)-i(10\sin(c)-15\sin(c+2dx)-6\sin(3c+4dx)-\sin(5c+6dx)))}{60ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^7/(a*Cos[c + d*x] + I*a*Sin[c + d*x]),x]
[Out] ((-1/60*I)*Sec[c]*Sec[c + d*x]^6*(10*Cos[c] - I*(10*Sin[c] - 15*Sin[c + 2*d
*x] - 6*Sin[3*c + 4*d*x] - Sin[5*c + 6*d*x])))/(a*d)
```

Maple [A]
time = 0.45, size = 72, normalized size = 1.03

method	result
risch	$\frac{16i\left(15\,\mathrm{e}^{4i(dx+c)}+6\,\mathrm{e}^{2i(dx+c)}+1\right)}{15da\left(\mathrm{e}^{2i(dx+c)}+1\right)^6}$
derivativedivides	$-\frac{i\left(\frac{\left(\tan^6(dx+c)\right)}{6}+\frac{\left(\tan^4(dx+c)\right)}{2}+\frac{i\left(\tan^5(dx+c)\right)}{5}+\frac{\left(\tan^2(dx+c)\right)}{2}+\frac{2i\left(\tan^3(dx+c)\right)}{3}+i\tan(dx+c)\right)}{da}$

default	$-\frac{i\left(\frac{\left(\tan^{6}(dx+c)\right)}{6}+\frac{\left(\tan^{4}(dx+c)\right)}{2}+\frac{i\left(\tan^{5}(dx+c)\right)}{5}+\frac{\left(\tan^{2}(dx+c)\right)}{2}+\frac{2i\left(\tan^{3}(dx+c)\right)}{3}+i\tan(dx+c)\right)}{da}$
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{14\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{52\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}-\frac{52\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}+\frac{14\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{2\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2i\left(\tan^{2}\left(\cdots\right.\right.}{a\cdots}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{6}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^7/(a*cos(d*x+c)+I*a*sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -I/d/a*(1/6*tan(d*x+c)^6+1/2*tan(d*x+c)^4+1/5*I*tan(d*x+c)^5+1/2*tan(d*x+c)
^2+2/3*I*tan(d*x+c)^3+I*tan(d*x+c))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(62) = 124$.
time = 0.29, size = 313, normalized size = 4.47

$$\frac{2\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{15i\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{35\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{78\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{50i\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}-\frac{78\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{35\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{15i\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}-\frac{15\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}\right)}{15\left(a-\frac{6a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{15a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{20a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{15a\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}-\frac{6a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 2/15*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 15*I*sin(d*x + c)^2/(cos(d*x + c
) + 1)^2 - 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 78*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5 - 50*I*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 78*sin(d*x + c
)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 15*I*si
n(d*x + c)^10/(cos(d*x + c) + 1)^10 - 15*sin(d*x + c)^11/(cos(d*x + c) + 1)
^11)/((a - 6*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 15*a*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4 - 20*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 15*a*sin(d*
x + c)^8/(cos(d*x + c) + 1)^8 - 6*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 +
 a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12)*d)
```

Fricas [**A**]
time = 1.89, size = 109, normalized size = 1.56

$$-\frac{16\left(-15i\,e^{(4i\,dx+4i\,c)}-6i\,e^{(2i\,dx+2i\,c)}-i\right)}{15\left(ade^{(12i\,dx+12i\,c)}+6\,ade^{(10i\,dx+10i\,c)}+15\,ade^{(8i\,dx+8i\,c)}+20\,ade^{(6i\,dx+6i\,c)}+15\,ade^{(4i\,dx+4i\,c)}+6\,ade^{(2i\,dx+2i\,c)}+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -16/15*(-15*I*e^(4*I*d*x + 4*I*c) - 6*I*e^(2*I*d*x + 2*I*c) - I)/(a*d*e^(12
*I*d*x + 12*I*c) + 6*a*d*e^(10*I*d*x + 10*I*c) + 15*a*d*e^(8*I*d*x + 8*I*c)
 + 20*a*d*e^(6*I*d*x + 6*I*c) + 15*a*d*e^(4*I*d*x + 4*I*c) + 6*a*d*e^(2*I*d
*x + 2*I*c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^7(c+dx)}{i\sin(c+dx)+\cos(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**7/(a*cos(d*x+c)+I*a*sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**7/(I*sin(c + d*x) + cos(c + d*x)), x)/a
```

Giac [A]
time = 0.44, size = 67, normalized size = 0.96

$$-\frac{5i\tan(dx+c)^6-6\tan(dx+c)^5+15i\tan(dx+c)^4-20\tan(dx+c)^3+15i\tan(dx+c)^2-30\tan(dx+c)}{30\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^7/(a*cos(d*x+c)+I*a*sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/30*(5*I*tan(d*x + c)^6 - 6*tan(d*x + c)^5 + 15*I*tan(d*x + c)^4 - 20*tan
(d*x + c)^3 + 15*I*tan(d*x + c)^2 - 30*tan(d*x + c))/(a*d)
```

Mupad [B]
time = 1.89, size = 139, normalized size = 1.99

$$-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9 15\mathrm{i}-35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+78\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 50\mathrm{i}-78\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)15\mathrm{i}-15\right)}{15\,a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^7*(a*cos(c + d*x) + a*sin(c + d*x)*1i)),x)
```

```
[Out] -(2*tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2)*15i + 35*tan(c/2 + (d*x)/2)^2 -
78*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^5*50i + 78*tan(c/2 + (d*x)/2)^
6 - 35*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^9*15i + 15*tan(c/2 + (d*x)
/2)^10 - 15))/(15*a*d*(tan(c/2 + (d*x)/2)^2 - 1)^6)
```

3.163 $\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=85

$$\frac{2i\cos^7(c+dx)}{7a^2d}+\frac{\sin(c+dx)}{a^2d}-\frac{4\sin^3(c+dx)}{3a^2d}+\frac{\sin^5(c+dx)}{a^2d}-\frac{2\sin^7(c+dx)}{7a^2d}$$

```
[Out] 2/7*I*cos(d*x+c)^7/a^2/d+sin(d*x+c)/a^2/d-4/3*sin(d*x+c)^3/a^2/d+sin(d*x+c)
^5/a^2/d-2/7*sin(d*x+c)^7/a^2/d
```

Rubi [A]
time = 0.13, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3171, 3169, 2713, 2645, 30, 2644, 276}

$$-\frac{2\sin^7(c+dx)}{7a^2d}+\frac{\sin^5(c+dx)}{a^2d}-\frac{4\sin^3(c+dx)}{3a^2d}+\frac{\sin(c+dx)}{a^2d}+\frac{2i\cos^7(c+dx)}{7a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (((2*I)/7)*Cos[c + d*x]^7)/(a^2*d) + Sin[c + d*x]/(a^2*d) - (4*Sin[c + d*x]
^3)/(3*a^2*d) + Sin[c + d*x]^5/(a^2*d) - (2*Sin[c + d*x]^7)/(7*a^2*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3171

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx &= -\frac{\int \cos^5(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^2\,dx}{a^4} \\
&= -\frac{\int \left(-a^2\cos^7(c+dx)+2ia^2\cos^6(c+dx)\sin(c+dx)+a^2\cos^5(c\right.}{a^4} \\
&= -\frac{(2i)\int \cos^6(c+dx)\sin(c+dx)\,dx}{a^2}+\frac{\int \cos^7(c+dx)\,dx}{a^2}-\frac{\int \cos^5}{} \\
&= \frac{(2i)\mathrm{Subst}\left(\int x^6\,dx,x,\cos(c+dx)\right)}{a^2d}-\frac{\mathrm{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx,x,\right.}{a^2d} \\
&= \frac{2i\cos^7(c+dx)}{7a^2d}+\frac{\sin(c+dx)}{a^2d}-\frac{\sin^3(c+dx)}{a^2d}+\frac{3\sin^5(c+dx)}{5a^2d}- \\
&= \frac{2i\cos^7(c+dx)}{7a^2d}+\frac{\sin(c+dx)}{a^2d}-\frac{4\sin^3(c+dx)}{3a^2d}+\frac{\sin^5(c+dx)}{a^2d}-
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 149, normalized size = 1.75

$$\frac{5i\cos(c+dx)}{32a^2d}+\frac{3i\cos(3(c+dx))}{32a^2d}+\frac{i\cos(5(c+dx))}{32a^2d}+\frac{i\cos(7(c+dx))}{224a^2d}+\frac{15\sin(c+dx)}{32a^2d}+\frac{11\sin(3(c+dx))}{96a^2d}+\frac{\sin(5(c+dx))}{32a^2d}+\frac{\sin(7(c+dx))}{224a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (((5*I)/32)*Cos[c + d*x])/(a^2*d) + (((3*I)/32)*Cos[3*(c + d*x)])/(a^2*d) +
 ((I/32)*Cos[5*(c + d*x)])/(a^2*d) + ((I/224)*Cos[7*(c + d*x)])/(a^2*d) + (
15*Sin[c + d*x])/(32*a^2*d) + (11*Sin[3*(c + d*x)])/(96*a^2*d) + Sin[5*(c +
 d*x)]/(32*a^2*d) + Sin[7*(c + d*x)]/(224*a^2*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(78) = 156$.
time = 0.71, size = 174, normalized size = 2.05

method	result
risch	$\frac{i\mathrm{e}^{-5i(dx+c)}}{32d\,a^2}+\frac{i\mathrm{e}^{-7i(dx+c)}}{224d\,a^2}+\frac{5i\cos(dx+c)}{32d\,a^2}+\frac{15\sin(dx+c)}{32a^2d}+\frac{3i\cos(3dx+3c)}{32d\,a^2}+\frac{11\sin(3dx+3c)}{96d\,a^2}$
derivativedivides	$\frac{-\frac{i}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}+\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{5i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{23i}{8\left(-i+\tan\left(\frac{dx}{2}+\cdots\right.\right.}}{d\,a^2}$
default	$\frac{-\frac{i}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^2}-\frac{1}{12\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)^3}+\frac{3}{8\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+i\right)}+\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{5i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{23i}{8\left(-i+\tan\left(\frac{dx}{2}+\cdots\right.\right.}}{d\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^2*(-1/16*I/(tan(1/2*d*x+1/2*c)+I)^2-1/24/(tan(1/2*d*x+1/2*c)+I)^3+3/1
6/(tan(1/2*d*x+1/2*c)+I)+I/(-I+tan(1/2*d*x+1/2*c))^6-5/2*I/(-I+tan(1/2*d*x+
1/2*c))^4+23/16*I/(-I+tan(1/2*d*x+1/2*c))^2-2/7/(-I+tan(1/2*d*x+1/2*c))^7+2
/(-I+tan(1/2*d*x+1/2*c))^5-55/24/(-I+tan(1/2*d*x+1/2*c))^3+13/16/(-I+tan(1/
2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 2.05, size = 74, normalized size = 0.87

$$\frac{\left(-7i\,e^{(10i\,dx+10i\,c)}-105i\,e^{(8i\,dx+8i\,c)}+210i\,e^{(6i\,dx+6i\,c)}+70i\,e^{(4i\,dx+4i\,c)}+21i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-7i\,dx-7i\,c)}}{672\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/672*(-7*I*e^(10*I*d*x + 10*I*c) - 105*I*e^(8*I*d*x + 8*I*c) + 210*I*e^(6*
I*d*x + 6*I*c) + 70*I*e^(4*I*d*x + 4*I*c) + 21*I*e^(2*I*d*x + 2*I*c) + 3*I)
*e^(-7*I*d*x - 7*I*c)/(a^2*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(76) = 152$.
time = 0.32, size = 231, normalized size = 2.72

$$\begin{cases} \frac{\left(-176160768ia^{10}d^{5}e^{19ic}e^{3idx} - 2642411520ia^{10}d^{5}e^{17ic}e^{idx} + 5284823040ia^{10}d^{5}e^{15ic}e^{-idx} + 1761607680ia^{10}d^{5}e^{13ic}e^{-3idx} + 528482304ia^{10}d^{5}e^{11ic}e^{-5idx} + 75497472ia^{10}d^{5}e^{9ic}e^{-7idx}\right)e^{-16ic}}{16911433728a^{12}d^{6}} & \text{for } a^{12}d^{6}e^{16ic} \neq 0 \\ \frac{x\left(e^{10ic} + 5e^{8ic} + 10e^{6ic} + 10e^{4ic} + 5e^{2ic} + 1\right)e^{-7ic}}{32a^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise(((-176160768*I*a**10*d**5*exp(19*I*c)*exp(3*I*d*x) - 2642411520*I
*a**10*d**5*exp(17*I*c)*exp(I*d*x) + 5284823040*I*a**10*d**5*exp(15*I*c)*ex
p(-I*d*x) + 1761607680*I*a**10*d**5*exp(13*I*c)*exp(-3*I*d*x) + 528482304*I
*a**10*d**5*exp(11*I*c)*exp(-5*I*d*x) + 75497472*I*a**10*d**5*exp(9*I*c)*ex
p(-7*I*d*x))*exp(-16*I*c)/(16911433728*a**12*d**6), Ne(a**12*d**6*exp(16*I*
c), 0)), (x*(exp(10*I*c) + 5*exp(8*I*c) + 10*exp(6*I*c) + 10*exp(4*I*c) + 5
*exp(2*I*c) + 1)*exp(-7*I*c)/(32*a**2), True))
```

Giac [A]
time = 0.42, size = 145, normalized size = 1.71

$$\frac{\frac{7\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+15i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-8\right)}{a^{2}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+i\right)^{3}}+\frac{273\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}-1155i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-2450\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+2870i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+2037\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-791i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-152}{a^{2}\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^{7}}}{168\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/168*(7*(9*tan(1/2*d*x + 1/2*c)^2 + 15*I*tan(1/2*d*x + 1/2*c) - 8)/(a^2*(t
an(1/2*d*x + 1/2*c) + I)^3) + (273*tan(1/2*d*x + 1/2*c)^6 - 1155*I*tan(1/2*
d*x + 1/2*c)^5 - 2450*tan(1/2*d*x + 1/2*c)^4 + 2870*I*tan(1/2*d*x + 1/2*c)^
3 + 2037*tan(1/2*d*x + 1/2*c)^2 - 791*I*tan(1/2*d*x + 1/2*c) - 152)/(a^2*(t
an(1/2*d*x + 1/2*c) - I)^7))/d
```

Mupad [B]
time = 4.11, size = 161, normalized size = 1.89

$$\frac{\left(-21\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{8}42\mathrm{i}+28\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{6}56\mathrm{i}+42\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}28\mathrm{i}+76\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}24\mathrm{i}+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-6\mathrm{i}\right)2\mathrm{i}}{21\,a^{2}\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)^{3}\left(1+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)1\mathrm{i}\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2,x)
```

```
[Out] ((3*tan(c/2 + (d*x)/2) - tan(c/2 + (d*x)/2)^2*24i + 76*tan(c/2 + (d*x)/2)^3
 + tan(c/2 + (d*x)/2)^4*28i + 42*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2)^
6*56i + 28*tan(c/2 + (d*x)/2)^7 + tan(c/2 + (d*x)/2)^8*42i - 21*tan(c/2 + (
d*x)/2)^9 - 6i)*2i)/(21*a^2*d*(tan(c/2 + (d*x)/2) + 1i)^3*(tan(c/2 + (d*x)/
2)*1i + 1)^7)
```

3.164 $\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=101

$$\frac{x}{4a^2}-\frac{1}{16a^2d(i-\cot(c+dx))}-\frac{1}{12a^2d(i+\cot(c+dx))^3}-\frac{3i}{8a^2d(i+\cot(c+dx))^2}+\frac{11}{16a^2d(i+\cot(c+dx))}$$

```
[Out] 1/4*x/a^2-1/16/a^2/d/(I-cot(d*x+c))-1/12/a^2/d/(I+cot(d*x+c))^3-3/8*I/a^2/d
/(I+cot(d*x+c))^2+11/16/a^2/d/(I+cot(d*x+c))
```

Rubi [A]
time = 0.07, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {3167, 862, 90, 209}

$$-\frac{1}{16a^2d(-\cot(c+dx)+i)}+\frac{11}{16a^2d(\cot(c+dx)+i)}-\frac{3i}{8a^2d(\cot(c+dx)+i)^2}-\frac{1}{12a^2d(\cot(c+dx)+i)^3}+\frac{x}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] x/(4*a^2) - 1/(16*a^2*d*(I - Cot[c + d*x])) - 1/(12*a^2*d*(I + Cot[c + d*x]
)^3) - ((3*I)/8)/(a^2*d*(I + Cot[c + d*x])^2) + 11/(16*a^2*d*(I + Cot[c + d
*x]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx &= -\frac{\mathrm{Subst}\left(\int \frac{x^4}{(ia+ax)^2(1+x^2)^2}\,dx, x, \cot(c+dx)\right)}{d} \\
&= -\frac{\mathrm{Subst}\left(\int \frac{x^4}{\left(-\frac{i}{a}+\frac{x}{a}\right)^2(ia+ax)^4}\,dx, x, \cot(c+dx)\right)}{d} \\
&= -\frac{\mathrm{Subst}\left(\int \left(\frac{1}{16a^2(-i+x)^2}-\frac{1}{4a^2(i+x)^4}-\frac{3i}{4a^2(i+x)^3}+\frac{11}{16a^2(i+x)^2}+\frac{1}{4a^2(1+x^2)}\right.\right.}{d} \\
&= -\frac{1}{16a^2d(i-\cot(c+dx))}-\frac{1}{12a^2d(i+\cot(c+dx))^3}-\frac{3}{8a^2d(i+\cot(c+dx))^2} \\
&= \frac{x}{4a^2}-\frac{1}{16a^2d(i-\cot(c+dx))}-\frac{1}{12a^2d(i+\cot(c+dx))^3}-\frac{}{8a^2d(i}
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 82, normalized size = 0.81

$$\frac{24c+24dx+15i\cos(2(c+dx))+6i\cos(4(c+dx))+i\cos(6(c+dx))+21\sin(2(c+dx))+6\sin(4(c+dx))+\sin(6(c+dx))}{96a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (24*c + 24*d*x + (15*I)*Cos[2*(c + d*x)] + (6*I)*Cos[4*(c + d*x)] + I*Cos[6
*(c + d*x)] + 21*Sin[2*(c + d*x)] + 6*Sin[4*(c + d*x)] + Sin[6*(c + d*x)])/
(96*a^2*d)
```

Maple [A]
time = 0.49, size = 88, normalized size = 0.87

method	result	size
risch	$\frac{x}{4a^2}+\frac{i e^{-4i(dx+c)}}{16d\,a^2}+\frac{i e^{-6i(dx+c)}}{96d\,a^2}+\frac{5i\cos(2dx+2c)}{32d\,a^2}+\frac{7\sin(2dx+2c)}{32d\,a^2}$	79
derivativedivides	$\frac{-\frac{i\ln(\tan(dx+c)-i)}{8}-\frac{i}{8(\tan(dx+c)-i)^2}-\frac{1}{12(\tan(dx+c)-i)^3}+\frac{3}{16(\tan(dx+c)-i)}+\frac{i\ln(\tan(dx+c)+i)}{8}+\frac{1}{16\tan(dx+c)+16i}}{d\,a^2}$	88

default	$\frac{-\frac{i\ln(\tan(dx+c)-i)}{8}-\frac{i}{8(\tan(dx+c)-i)^2}-\frac{1}{12(\tan(dx+c)-i)^3}+\frac{3}{16(\tan(dx+c)-i)}+\frac{i\ln(\tan(dx+c)+i)}{8}+\frac{1}{16\tan(dx+c)+16i}}{d\,a^2}$	88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-1/8*I*ln(tan(d*x+c)-I)-1/8*I/(tan(d*x+c)-I)^2-1/12/(tan(d*x+c)-I)
^3+3/16/(tan(d*x+c)-I)+1/8*I*ln(tan(d*x+c)+I)+1/16/(tan(d*x+c)+I))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [**A**]
time = 2.14, size = 65, normalized size = 0.64

$$\frac{\left(24\,dxe^{(6i\,dx+6i\,c)}-3i\,e^{(8i\,dx+8i\,c)}+18i\,e^{(4i\,dx+4i\,c)}+6i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-6i\,dx-6i\,c)}}{96\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/96*(24*d*x*e^(6*I*d*x + 6*I*c) - 3*I*e^(8*I*d*x + 8*I*c) + 18*I*e^(4*I*d*
x + 4*I*c) + 6*I*e^(2*I*d*x + 2*I*c) + I)*e^(-6*I*d*x - 6*I*c)/(a^2*d)
```

Sympy [**A**]
time = 0.20, size = 189, normalized size = 1.87

$$\begin{cases}\frac{\left(-24576ia^6d^3e^{14ic}e^{2idx}+147456ia^6d^3e^{10ic}e^{-2idx}+49152ia^6d^3e^{8ic}e^{-4idx}+8192ia^6d^3e^{6ic}e^{-6idx}\right)e^{-12ic}}{786432a^8d^4} & \text{for } a^8d^4e^{12ic}\neq 0\\ x\left(\frac{\left(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1\right)e^{-6ic}}{16a^2}-\frac{1}{4a^2}\right) & \text{otherwise}\end{cases}+\frac{x}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise(((-24576*I*a**6*d**3*exp(14*I*c)*exp(2*I*d*x) + 147456*I*a**6*d**
3*exp(10*I*c)*exp(-2*I*d*x) + 49152*I*a**6*d**3*exp(8*I*c)*exp(-4*I*d*x) +
```

```
8192*I*a**6*d**3*exp(6*I*c)*exp(-6*I*d*x))*exp(-12*I*c)/(786432*a**8*d**4),
 Ne(a**8*d**4*exp(12*I*c), 0)), (x*((exp(8*I*c) + 4*exp(6*I*c) + 6*exp(4*I*
c) + 4*exp(2*I*c) + 1)*exp(-6*I*c)/(16*a**2) - 1/(4*a**2)), True)) + x/(4*a
**2)
```

Giac [A]
time = 0.43, size = 103, normalized size = 1.02

$$-\frac{-\frac{6i\log(\tan(dx+c)+i)}{a^2}+\frac{6i\log(\tan(dx+c)-i)}{a^2}+\frac{3(2i\tan(dx+c)-3)}{a^2(\tan(dx+c)+i)}+\frac{-11i\tan(dx+c)^3-42\tan(dx+c)^2+57i\tan(dx+c)+30}{a^2(\tan(dx+c)-i)^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/48*(-6*I*log(tan(d*x + c) + I)/a^2 + 6*I*log(tan(d*x + c) - I)/a^2 + 3*(
2*I*tan(d*x + c) - 3)/(a^2*(tan(d*x + c) + I)) + (-11*I*tan(d*x + c)^3 - 42
*tan(d*x + c)^2 + 57*I*tan(d*x + c) + 30)/(a^2*(tan(d*x + c) - I)^3))/d
```

Mupad [B]
time = 4.74, size = 138, normalized size = 1.37

$$\frac{x}{4\,a^2}-\frac{-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,2{}\mathrm{i}+\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{6}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,4{}\mathrm{i}}{3}-\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,2{}\mathrm{i}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1{}\mathrm{i}\right)^2\,\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2,x)
```

```
[Out] x/(4*a^2) - ((3*tan(c/2 + (d*x)/2))/2 + tan(c/2 + (d*x)/2)^2*2i - (7*tan(c/
2 + (d*x)/2)^3)/6 + (tan(c/2 + (d*x)/2)^4*4i)/3 + (7*tan(c/2 + (d*x)/2)^5)/
6 + tan(c/2 + (d*x)/2)^6*2i - (3*tan(c/2 + (d*x)/2)^7)/2)/(a^2*d*(tan(c/2 +
 (d*x)/2) + 1i)^2*(tan(c/2 + (d*x)/2)*1i + 1)^6)
```

3.165 $\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=68

$$\frac{2i\cos^5(c+dx)}{5a^2d}+\frac{\sin(c+dx)}{a^2d}-\frac{\sin^3(c+dx)}{a^2d}+\frac{2\sin^5(c+dx)}{5a^2d}$$

```
[Out] 2/5*I*cos(d*x+c)^5/a^2/d+sin(d*x+c)/a^2/d-sin(d*x+c)^3/a^2/d+2/5*sin(d*x+c)
^5/a^2/d
```

Rubi [A]
time = 0.13, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3171, 3169, 2713, 2645, 30, 2644, 14}

$$\frac{2\sin^5(c+dx)}{5a^2d}-\frac{\sin^3(c+dx)}{a^2d}+\frac{\sin(c+dx)}{a^2d}+\frac{2i\cos^5(c+dx)}{5a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (((2*I)/5)*Cos[c + d*x]^5)/(a^2*d) + Sin[c + d*x]/(a^2*d) - Sin[c + d*x]^3/
(a^2*d) + (2*Sin[c + d*x]^5)/(5*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3171

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx &= -\frac{\int \cos^3(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^2\,dx}{a^4}\\
&= -\frac{\int \left(-a^2\cos^5(c+dx)+2ia^2\cos^4(c+dx)\sin(c+dx)+a^2\cos^3(c+\right.}{a^4}\\
&= -\frac{(2i)\int \cos^4(c+dx)\sin(c+dx)\,dx}{a^2}+\frac{\int \cos^5(c+dx)\,dx}{a^2}-\frac{\int \cos^3(}{}\\
&= \frac{(2i)\mathrm{Subst}\left(\int x^4\,dx,x,\cos(c+dx)\right)}{a^2d}-\frac{\mathrm{Subst}\left(\int x^2(1-x^2)\,dx,x,\sin\right.}{a^2d}\\
&= \frac{2i\cos^5(c+dx)}{5a^2d}+\frac{\sin(c+dx)}{a^2d}-\frac{2\sin^3(c+dx)}{3a^2d}+\frac{\sin^5(c+dx)}{5a^2d}-\frac{\mathrm{S}}{}\\
&= \frac{2i\cos^5(c+dx)}{5a^2d}+\frac{\sin(c+dx)}{a^2d}-\frac{\sin^3(c+dx)}{a^2d}+\frac{2\sin^5(c+dx)}{5a^2d}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 111, normalized size = 1.63

$$\frac{i\cos(c+dx)}{4a^2d}+\frac{i\cos(3(c+dx))}{8a^2d}+\frac{i\cos(5(c+dx))}{40a^2d}+\frac{\sin(c+dx)}{2a^2d}+\frac{\sin(3(c+dx))}{8a^2d}+\frac{\sin(5(c+dx))}{40a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] ((I/4)*Cos[c + d*x])/(a^2*d) + ((I/8)*Cos[3*(c + d*x)])/(a^2*d) + ((I/40)*C
os[5*(c + d*x)])/(a^2*d) + Sin[c + d*x]/(2*a^2*d) + Sin[3*(c + d*x)]/(8*a^2
*d) + Sin[5*(c + d*x)]/(40*a^2*d)
```

Maple [A]
time = 0.40, size = 108, normalized size = 1.59

method	result
risch	$\frac{i e^{-3i(dx+c)}}{8d\,a^2}+\frac{i e^{-5i(dx+c)}}{40d\,a^2}+\frac{i\cos(dx+c)}{4d\,a^2}+\frac{\sin(dx+c)}{2a^2d}$
derivativedivides	$\frac{\frac{2}{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+8i}-\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{5i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{4}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{3}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{7}{4\left(-i+\tan\left(\frac{dx}{2}\right.\right.}}{d\,a^2}$
default	$\frac{\frac{2}{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+8i}-\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{5i}{2\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{4}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}-\frac{3}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}+\frac{7}{4\left(-i+\tan\left(\frac{dx}{2}\right.\right.}}{d\,a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^2*(1/8/(tan(1/2*d*x+1/2*c)+I)-I/(-I+tan(1/2*d*x+1/2*c))^4+5/4*I/(-I+t
an(1/2*d*x+1/2*c))^2+2/5/(-I+tan(1/2*d*x+1/2*c))^5-3/2/(-I+tan(1/2*d*x+1/2*
c))^3+7/8/(-I+tan(1/2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 2.40, size = 52, normalized size = 0.76

$$\frac{\left(-5i\,e^{(6i\,dx+6i\,c)}+15i\,e^{(4i\,dx+4i\,c)}+5i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-5i\,dx-5i\,c)}}{40\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/40*(-5*I*e^(6*I*d*x + 6*I*c) + 15*I*e^(4*I*d*x + 4*I*c) + 5*I*e^(2*I*d*x
+ 2*I*c) + I)*e^(-5*I*d*x - 5*I*c)/(a^2*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(60) = 120$.
time = 0.27, size = 163, normalized size = 2.40

$$\begin{cases} \frac{(-2560ia^6d^3e^{10ic}e^{idx}+7680ia^6d^3e^{8ic}e^{-idx}+2560ia^6d^3e^{6ic}e^{-3idx}+512ia^6d^3e^{4ic}e^{-5idx})e^{-9ic}}{20480a^8d^4} & \text{for } a^8d^4e^{9ic} \neq 0 \\ \frac{x(e^{6ic}+3e^{4ic}+3e^{2ic}+1)e^{-5ic}}{8a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise(((-2560*I*a**6*d**3*exp(10*I*c)*exp(I*d*x) + 7680*I*a**6*d**3*exp
(8*I*c)*exp(-I*d*x) + 2560*I*a**6*d**3*exp(6*I*c)*exp(-3*I*d*x) + 512*I*a**
6*d**3*exp(4*I*c)*exp(-5*I*d*x))*exp(-9*I*c)/(20480*a**8*d**4), Ne(a**8*d**
4*exp(9*I*c), 0)), (x*(exp(6*I*c) + 3*exp(4*I*c) + 3*exp(2*I*c) + 1)*exp(-5
*I*c)/(8*a**2), True))
```

Giac [A]
time = 0.42, size = 93, normalized size = 1.37

$$\frac{\frac{5}{a^2(\tan(\frac{1}{2}dx+\frac{1}{2}c)+i)} + \frac{35\tan(\frac{1}{2}dx+\frac{1}{2}c)^4-90i\tan(\frac{1}{2}dx+\frac{1}{2}c)^3-120\tan(\frac{1}{2}dx+\frac{1}{2}c)^2+70i\tan(\frac{1}{2}dx+\frac{1}{2}c)+21}{a^2(\tan(\frac{1}{2}dx+\frac{1}{2}c)-i)^5}}{20d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/20*(5/(a^2*(tan(1/2*d*x + 1/2*c) + I)) + (35*tan(1/2*d*x + 1/2*c)^4 - 90*
I*tan(1/2*d*x + 1/2*c)^3 - 120*tan(1/2*d*x + 1/2*c)^2 + 70*I*tan(1/2*d*x +
1/2*c) + 21)/(a^2*(tan(1/2*d*x + 1/2*c) - I)^5))/d
```

Mupad [B]
time = 1.02, size = 90, normalized size = 1.32

$$-\frac{2\left(-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 10\mathrm{i}+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-2\mathrm{i}\right)}{5a^2d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-\mathrm{i}\right)^5\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2,x)
```

```
[Out] -(2*(3*tan(c/2 + (d*x)/2) + 10*tan(c/2 + (d*x)/2)^3 + tan(c/2 + (d*x)/2)^4*
10i - 5*tan(c/2 + (d*x)/2)^5 - 2i))/(5*a^2*d*(tan(c/2 + (d*x)/2) - 1i)^5*(t
an(c/2 + (d*x)/2) + 1i))
```

3.166 $\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=89

$$\frac{x}{4a^2}+\frac{i\cos^2(c+dx)}{4d(a\cos(c+dx)+ia\sin(c+dx))^2}+\frac{i\cos(c+dx)}{4d\left(a^2\cos(c+dx)+ia^2\sin(c+dx)\right)}$$

```
[Out] 1/4*x/a^2+1/4*I*cos(d*x+c)^2/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^2+1/4*I*cos(d*
x+c)/d/(a^2*cos(d*x+c)+I*a^2*sin(d*x+c))
```

Rubi [A]

time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.065, Rules used = {3161, 8}

$$\frac{i\cos(c+dx)}{4d\left(a^2\cos(c+dx)+ia^2\sin(c+dx)\right)}+\frac{x}{4a^2}+\frac{i\cos^2(c+dx)}{4d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] x/(4*a^2) + ((I/4)*Cos[c + d*x]^2)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2
) + ((I/4)*Cos[c + d*x])/(d*(a^2*Cos[c + d*x] + I*a^2*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3161

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*((a*Cos[c + d*x] + b*Si
n[c + d*x])^n/(2*a*d*n*Cos[c + d*x]^n)), x] + Dist[1/(2*a), Int[(a*Cos[c +
d*x] + b*Sin[c + d*x])^(n + 1)/Cos[c + d*x]^(n + 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && EqQ[m + n, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx &= \frac{i\cos^2(c+dx)}{4d(a\cos(c+dx)+ia\sin(c+dx))^2}+\frac{\int \frac{\cos(c+dx)}{a\cos(c+dx)+ia\sin(c+dx)}\,dx}{2a}\\ &= \frac{i\cos^2(c+dx)}{4d(a\cos(c+dx)+ia\sin(c+dx))^2}+\frac{i\cos(c+dx)}{4d\left(a^2\cos(c+dx)+ia^2\sin\right.}\\ &= \frac{x}{4a^2}+\frac{i\cos^2(c+dx)}{4d(a\cos(c+dx)+ia\sin(c+dx))^2}+\frac{i\cos(c+}{4d\left(a^2\cos(c+dx)+\right.}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 60, normalized size = 0.67

$$\frac{4c+4dx+4i\cos(2(c+dx))+i\cos(4(c+dx))+4\sin(2(c+dx))+\sin(4(c+dx))}{16a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (4*c + 4*d*x + (4*I)*Cos[2*(c + d*x)] + I*Cos[4*(c + d*x)] + 4*Sin[2*(c + d
*x)] + Sin[4*(c + d*x)])/(16*a^2*d)
```

Maple [A]
time = 0.30, size = 62, normalized size = 0.70

method	result	size
risch	$\frac{x}{4a^2}+\frac{i\mathrm{e}^{-2i(dx+c)}}{4d\,a^2}+\frac{i\mathrm{e}^{-4i(dx+c)}}{16d\,a^2}$	44
derivativedivides	$\frac{-\frac{i\ln(\tan(dx+c)-i)}{8}-\frac{i}{4(\tan(dx+c)-i)^2}+\frac{1}{4\tan(dx+c)-4i}+\frac{i\ln(\tan(dx+c)+i)}{8}}{d\,a^2}$	62
default	$\frac{-\frac{i\ln(\tan(dx+c)-i)}{8}-\frac{i}{4(\tan(dx+c)-i)^2}+\frac{1}{4\tan(dx+c)-4i}+\frac{i\ln(\tan(dx+c)+i)}{8}}{d\,a^2}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-1/8*I*ln(tan(d*x+c)-I)-1/4*I/(tan(d*x+c)-I)^2+1/4/(tan(d*x+c)-I)+
1/8*I*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 3.45, size = 43, normalized size = 0.48

$$\frac{\left(4\,dxe^{(4i\,dx+4i\,c)}+4i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-4i\,dx-4i\,c)}}{16\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/16*(4*d*x*e^(4*I*d*x + 4*I*c) + 4*I*e^(2*I*d*x + 2*I*c) + I)*e^(-4*I*d*x
- 4*I*c)/(a^2*d)
```

Sympy [A]
time = 0.13, size = 117, normalized size = 1.31

$$\begin{cases} \frac{(16ia^2de^{4ic}e^{-2idx}+4ia^2de^{2ic}e^{-4idx})e^{-6ic}}{64a^4d^2} & \text{for } a^4d^2e^{6ic} \neq 0 \\ x\left(\frac{(e^{4ic}+2e^{2ic}+1)e^{-4ic}}{4a^2}-\frac{1}{4a^2}\right) & \text{otherwise} \end{cases} + \frac{x}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise(((16*I*a**2*d*exp(4*I*c)*exp(-2*I*d*x) + 4*I*a**2*d*exp(2*I*c)*ex
p(-4*I*d*x))*exp(-6*I*c)/(64*a**4*d**2), Ne(a**4*d**2*exp(6*I*c), 0)), (x*(
(exp(4*I*c) + 2*exp(2*I*c) + 1)*exp(-4*I*c)/(4*a**2) - 1/(4*a**2)), True))
+ x/(4*a**2)
```

Giac [A]
time = 0.43, size = 72, normalized size = 0.81

$$-\frac{\frac{2i\log(i\tan(dx+c)+1)}{a^2}-\frac{2i\log(i\tan(dx+c)-1)}{a^2}+\frac{-3i\tan(dx+c)^2-10\tan(dx+c)+11i}{a^2(\tan(dx+c)-i)^2}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/16*(2*I*log(I*tan(d*x + c) + 1)/a^2 - 2*I*log(I*tan(d*x + c) - 1)/a^2 +
(-3*I*tan(d*x + c)^2 - 10*tan(d*x + c) + 11*I)/(a^2*(tan(d*x + c) - I)^2))/
d
```

Mupad [B]
time = 1.76, size = 69, normalized size = 0.78

$$\frac{x}{4\,a^2}+\frac{-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,2\mathrm{i}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}}{a^2\,d\,\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2,x)
```

```
[Out] x/(4*a^2) + ((3*tan(c/2 + (d*x)/2))/2 + tan(c/2 + (d*x)/2)^2*2i - (3*tan(c/
2 + (d*x)/2)^3)/2)/(a^2*d*(tan(c/2 + (d*x)/2)*1i + 1)^4)
```

3.167 $\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=52

$$\frac{2i\cos^3(c+dx)}{3a^2d}+\frac{\sin(c+dx)}{a^2d}-\frac{2\sin^3(c+dx)}{3a^2d}$$

```
[Out] 2/3*I*cos(d*x+c)^3/a^2/d+sin(d*x+c)/a^2/d-2/3*sin(d*x+c)^3/a^2/d
```

Rubi [A]
time = 0.08, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {3171, 3169, 2713, 2645, 30, 2644}

$$-\frac{2\sin^3(c+dx)}{3a^2d}+\frac{\sin(c+dx)}{a^2d}+\frac{2i\cos^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (((2*I)/3)*Cos[c + d*x]^3)/(a^2*d) + Sin[c + d*x]/(a^2*d) - (2*Sin[c + d*x]
^3)/(3*a^2*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3171

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx &= -\frac{\int \cos(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^2\,dx}{a^4} \\
&= -\frac{\int \left(-a^2\cos^3(c+dx)+2ia^2\cos^2(c+dx)\sin(c+dx)+a^2\cos(c-\right.}{a^4} \\
&= -\frac{(2i)\int \cos^2(c+dx)\sin(c+dx)\,dx}{a^2}+\frac{\int \cos^3(c+dx)\,dx}{a^2}-\frac{\int \cos(}{} \\
&= \frac{(2i)\mathrm{Subst}\left(\int x^2\,dx, x, \cos(c+dx)\right)}{a^2d}-\frac{\mathrm{Subst}\left(\int x^2\,dx, x, \sin(c+dx\right.}{a^2d} \\
&= \frac{2i\cos^3(c+dx)}{3a^2d}+\frac{\sin(c+dx)}{a^2d}-\frac{2\sin^3(c+dx)}{3a^2d}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 73, normalized size = 1.40

$$\frac{i\cos(c+dx)}{2a^2d}+\frac{i\cos(3(c+dx))}{6a^2d}+\frac{\sin(c+dx)}{2a^2d}+\frac{\sin(3(c+dx))}{6a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] ((I/2)*Cos[c + d*x])/(a^2*d) + ((I/6)*Cos[3*(c + d*x)])/(a^2*d) + Sin[c + d
*x]/(2*a^2*d) + Sin[3*(c + d*x)]/(6*a^2*d)
```

Maple [A]
time = 0.26, size = 57, normalized size = 1.10

method	result	size
risch	$\frac{i e^{-i(dx+c)}}{2d a^2}+\frac{i e^{-3i(dx+c)}}{6d a^2}$	38
derivativedivides	$\frac{\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{4}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}}{d a^2}$	57
default	$\frac{\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{4}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}}{d a^2}$	57
norman	$\frac{\frac{4i\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{4i}{3ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{a\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^2*(1/(-I+tan(1/2*d*x+1/2*c))+I/(-I+tan(1/2*d*x+1/2*c))^2-2/3/(-I+tan(
1/2*d*x+1/2*c))^3)
```

Maxima [A]
time = 0.27, size = 45, normalized size = 0.87

$$\frac{i\cos\left(3\,dx+3\,c\right)+3i\cos\left(dx+c\right)+\sin\left(3\,dx+3\,c\right)+3\sin\left(dx+c\right)}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(I*cos(3*d*x + 3*c) + 3*I*cos(d*x + c) + sin(3*d*x + 3*c) + 3*sin(d*x +
 c))/(a^2*d)
```

Fricas [A]
time = 2.54, size = 30, normalized size = 0.58

$$\frac{\left(3i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-3i\,dx-3i\,c)}}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(3*I*e^(2*I*d*x + 2*I*c) + I)*e^(-3*I*d*x - 3*I*c)/(a^2*d)
```

Sympy [A]
time = 0.12, size = 92, normalized size = 1.77

$$\begin{cases}\frac{\left(6ia^2de^{3ic}e^{-idx}+2ia^2de^{ic}e^{-3idx}\right)e^{-4ic}}{12a^4d^2} & \text{for } a^4d^2e^{4ic}\neq 0\\ \frac{x\left(e^{2ic}+1\right)e^{-3ic}}{2a^2} & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise(((6*I*a**2*d*exp(3*I*c)*exp(-I*d*x) + 2*I*a**2*d*exp(I*c)*exp(-3*
I*d*x))*exp(-4*I*c)/(12*a**4*d**2), Ne(a**4*d**2*exp(4*I*c), 0)), (x*(exp(2
*I*c) + 1)*exp(-3*I*c)/(2*a**2), True))
```

Giac [A]
time = 0.43, size = 47, normalized size = 0.90

$$\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\right)}{3\,a^{2}d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2/3*(3*tan(1/2*d*x + 1/2*c)^2 - 3*I*tan(1/2*d*x + 1/2*c) - 2)/(a^2*d*(tan(1
/2*d*x + 1/2*c) - I)^3)
```

Mupad [B]
time = 0.65, size = 79, normalized size = 1.52

$$-\frac{2\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\,3\mathrm{i}+3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\mathrm{i}\right)}{3\,a^{2}\,d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}\,1\mathrm{i}-3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,3\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2,x)
```

```
[Out] -(2*(3*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*3i - 2i))/(3*a^2*d*(tan(c/
2 + (d*x)/2)*3i - 3*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*1i + 1))
```

3.168 $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=31

$$\frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

```
[Out] 1/2*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^2
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{3150\}$

$$\frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-2),x]
```

```
[Out] (I/2)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = \frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

Mathematica [A]
time = 0.04, size = 31, normalized size = 1.00

$$\frac{i}{2d(a\cos(c+dx)+ia\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-2),x]
```

```
[Out] (I/2)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2)
```

Maple [A]
time = 0.20, size = 23, normalized size = 0.74

method	result	size
risch	$\frac{i\mathrm{e}^{-2i(dx+c)}}{2d\,a^2}$	19
derivativedivides	$\frac{i}{d\,a^2(i\tan(dx+c)+1)}$	23
default	$\frac{i}{d\,a^2(i\tan(dx+c)+1)}$	23
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{4i\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{a\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] I/d/a^2/(I*tan(d*x+c)+1)
```

Maxima [A]
time = 0.26, size = 22, normalized size = 0.71

$$\frac{1}{(a^2\tan(dx+c)-i\,a^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/((a^2*tan(d*x + c) - I*a^2)*d)
```

Fricas [A]
time = 3.25, size = 17, normalized size = 0.55

$$\frac{i\,e^{(-2i\,dx-2i\,c)}}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*I*e^(-2*I*d*x - 2*I*c)/(a^2*d)
```

Sympy [A]
time = 0.09, size = 44, normalized size = 1.42

$$\begin{cases} \frac{ie^{-2ic}e^{-2idx}}{2a^2d} & \text{for } a^2de^{2ic}\neq 0 \\ \frac{xe^{-2ic}}{a^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Piecewise((I*exp(-2*I*c)*exp(-2*I*d*x)/(2*a**2*d), Ne(a**2*d*exp(2*I*c), 0)
), (x*exp(-2*I*c)/a**2, True))
```

Giac [A]
time = 0.40, size = 30, normalized size = 0.97

$$-\frac{2 \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{a^2 d\left(\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - i\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*tan(1/2*d*x + 1/2*c)/(a^2*d*(tan(1/2*d*x + 1/2*c) - I)^2)
```

Mupad [B]
time = 0.61, size = 31, normalized size = 1.00

$$-\frac{2 \tan\left(\frac{c}{2} + \frac{d x}{2}\right)}{a^2 d \left(\tan\left(\frac{c}{2} + \frac{d x}{2}\right) - \mathrm{i}\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2,x)
```

```
[Out] -(2*tan(c/2 + (d*x)/2))/(a^2*d*(tan(c/2 + (d*x)/2) - 1i)^2)
```

3.169 $\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=46

$$-\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{2i\cos(c+dx)}{a^2d}+\frac{2\sin(c+dx)}{a^2d}$$

```
[Out] -arctanh(sin(d*x+c))/a^2/d+2*I*cos(d*x+c)/a^2/d+2*sin(d*x+c)/a^2/d
```

Rubi [A]
time = 0.08, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {3171, 3169, 2717, 2718, 2672, 327, 212}

$$\frac{2\sin(c+dx)}{a^2d}+\frac{2i\cos(c+dx)}{a^2d}-\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] -(ArcTanh[Sin[c + d*x]]/(a^2*d)) + ((2*I)*Cos[c + d*x])/(a^2*d) + (2*Sin[c
+ d*x])/(a^2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3171

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx &= -\frac{\int \sec(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^2\,dx}{a^4}\\
&= -\frac{\int \left(-a^2\cos(c+dx)+2ia^2\sin(c+dx)+a^2\sin(c+dx)\tan(c+dx\right.}{a^4}\\
&= -\frac{(2i)\int \sin(c+dx)\,dx}{a^2}+\frac{\int \cos(c+dx)\,dx}{a^2}-\frac{\int \sin(c+dx)\tan(c+}{a^2}\\
&= \frac{2i\cos(c+dx)}{a^2d}+\frac{\sin(c+dx)}{a^2d}-\frac{\operatorname{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\sin(c+dx)\right)}{a^2d}\\
&= \frac{2i\cos(c+dx)}{a^2d}+\frac{2\sin(c+dx)}{a^2d}-\frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sin(c+dx)\right)}{a^2d}\\
&= -\frac{\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{2i\cos(c+dx)}{a^2d}+\frac{2\sin(c+dx)}{a^2d}
\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 184 vs. 2(46) = 92.
time = 0.27, size = 184, normalized size = 4.00

$$-\frac{\sec^2(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)\left(2i+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\left(2+i\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-i\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{3}{2}(c+dx)\right)+i\sin\left(\frac{3}{2}(c+dx)\right)\right)}{a^2d(-i+\tan(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] -((Sec[c + d*x]^2*(Cos[(c + d*x)/2]*(2*I + Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (2 + I*Log[Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]] - I*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*S
in[(c + d*x)/2])*(Cos[(3*(c + d*x))/2] + I*Sin[(3*(c + d*x))/2]))/(a^2*d*(-
I + Tan[c + d*x])^2))
```

Maple [A]
time = 0.30, size = 54, normalized size = 1.17

method	result	size
derivativedivides	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{4}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{a^2 d}$	54
default	$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{4}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{a^2 d}$	54
risch	$\frac{2i\,\mathrm{e}^{-i(dx+c)}}{d\,a^2}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{a^2 d}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^2 d}$	61
norman	$\frac{\frac{4i}{ad}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}}{a\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2 d}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2 d}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^2*(1/2*ln(tan(1/2*d*x+1/2*c)-1)-1/2*ln(tan(1/2*d*x+1/2*c)+1)+2/(-I+ta
n(1/2*d*x+1/2*c)))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(44) = 88.
time = 0.47, size = 117, normalized size = 2.54

$$-\frac{-2i\arctan\left(\cos\left(dx+c\right),\sin\left(dx+c\right)+1\right)-2i\arctan\left(\cos\left(dx+c\right),-\sin\left(dx+c\right)+1\right)-4i\cos\left(dx+c\right)+\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\sin\left(dx+c\right)+1\right)-\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2-2\sin\left(dx+c\right)+1\right)-4\sin\left(dx+c\right)}{2a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(-2*I*arctan2(cos(d*x + c), sin(d*x + c) + 1) - 2*I*arctan2(cos(d*x +
c), -sin(d*x + c) + 1) - 4*I*cos(d*x + c) + log(cos(d*x + c)^2 + sin(d*x +
c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*
x + c) + 1) - 4*sin(d*x + c))/(a^2*d)
```

Fricas [A]
time = 2.07, size = 64, normalized size = 1.39

$$-\frac{\left(e^{(i\,dx+i\,c)}\log\left(e^{(i\,dx+i\,c)}+i\right)-e^{(i\,dx+i\,c)}\log\left(e^{(i\,dx+i\,c)}-i\right)-2i\right)e^{(-i\,dx-i\,c)}}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(e^(I*d*x + I*c)*log(e^(I*d*x + I*c) + I) - e^(I*d*x + I*c)*log(e^(I*d*x +
 I*c) - I) - 2*I)*e^(-I*d*x - I*c)/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(c+dx)}{-\sin^2(c+dx)+2i\sin(c+dx)\cos(c+dx)+\cos^2(c+dx)}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)/(-sin(c + d*x)**2 + 2*I*sin(c + d*x)*cos(c + d*x) + c
os(c + d*x)**2), x)/a**2
```

Giac [A]
time = 0.45, size = 57, normalized size = 1.24

$$-\frac{\frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^2}-\frac{\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^2}-\frac{4}{a^2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -(log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - log(tan(1/2*d*x + 1/2*c) - 1)/a^2 - 4
/(a^2*(tan(1/2*d*x + 1/2*c) - I)))/d
```

Mupad [B]
time = 0.67, size = 44, normalized size = 0.96

$$-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}+\frac{4\mathrm{i}}{a^2\,d\,\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2),x)
```

```
[Out] 4i/(a^2*d*(tan(c/2 + (d*x)/2)*1i + 1)) - (2*atanh(tan(c/2 + (d*x)/2)))/(a^2
*d)
```

3.170 $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$\frac{2x}{a^2}+\frac{2i\log(\sin(c+dx))}{a^2d}-\frac{2i\log(\tan(c+dx))}{a^2d}-\frac{\tan(c+dx)}{a^2d}$$

```
[Out] 2*x/a^2+2*I*ln(sin(d*x+c))/a^2/d-2*I*ln(tan(d*x+c))/a^2/d-tan(d*x+c)/a^2/d
```

Rubi [A]
time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3167, 862, 78}

$$-\frac{\tan(c+dx)}{a^2d}+\frac{2i\log(\sin(c+dx))}{a^2d}-\frac{2i\log(\tan(c+dx))}{a^2d}+\frac{2x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (2*x)/a^2 + ((2*I)*Log[Sin[c + d*x]])/(a^2*d) - ((2*I)*Log[Tan[c + d*x]])/(
a^2*d) - Tan[c + d*x]/(a^2*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx &= -\frac{\text{Subst}\left(\int \frac{1+x^2}{x^2(ia+ax)^2}\,dx, x, \cot(c+dx)\right)}{d} \\
&= -\frac{\text{Subst}\left(\int \frac{-\frac{i}{a}+\frac{x}{a}}{x^2(ia+ax)}\,dx, x, \cot(c+dx)\right)}{d} \\
&= -\frac{\text{Subst}\left(\int \left(-\frac{1}{a^2x^2}-\frac{2i}{a^2x}+\frac{2i}{a^2(i+x)}\right)\,dx, x, \cot(c+dx)\right)}{d} \\
&= \frac{2x}{a^2}+\frac{2i\log(\sin(c+dx))}{a^2d}-\frac{2i\log(\tan(c+dx))}{a^2d}-\frac{\tan(c+dx)}{a^2d}
\end{aligned}
$$

Mathematica [A]
time = 0.48, size = 71, normalized size = 1.29

$$
\frac{4\text{ArcTan}(\tan(dx))+i\sec(c)\sec(c+dx)\left(\cos(dx)\log\left(\cos^2(c+dx)\right)+\cos(2c+dx)\log\left(\cos^2(c+dx)\right)+2i\sin(dx)\right)}{2a^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (4*ArcTan[Tan[d*x]] + I*Sec[c]*Sec[c + d*x]*(Cos[d*x]*Log[Cos[c + d*x]^2] +
 Cos[2*c + d*x]*Log[Cos[c + d*x]^2] + (2*I)*Sin[d*x]))/(2*a^2*d)
```

Maple [A]
time = 0.32, size = 30, normalized size = 0.55

method	result
derivativedivides	$\frac{-\tan(dx+c)-2i\ln(\tan(dx+c)-i)}{d\,a^2}$
default	$\frac{-\tan(dx+c)-2i\ln(\tan(dx+c)-i)}{d\,a^2}$
risch	$\frac{4x}{a^2}+\frac{4c}{a^2d}-\frac{2i}{a^2d\left(e^{2i(dx+c)}+1\right)}+\frac{2i\ln\left(e^{2i(dx+c)}+1\right)}{a^2d}$
norman	$\frac{-\frac{2x}{a}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2i\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^2d}+\frac{2i\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^2d}-\frac{2i\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^2*(-tan(d*x+c)-2*I*ln(tan(d*x+c)-I))
```

Maxima [A]
time = 0.28, size = 30, normalized size = 0.55

$$\frac{-\frac{2i\log(\tan(dx+c)-i)}{a^2}-\frac{\tan(dx+c)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] (-2*I*log(tan(d*x + c) - I)/a^2 - tan(d*x + c)/a^2)/d
```

Fricas [A]
time = 2.77, size = 70, normalized size = 1.27

$$\frac{2\left(2\,dxe^{(2i\,dx+2i\,c)}+2\,dx-\left(-i\,e^{(2i\,dx+2i\,c)}-i\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-i\right)}{a^2de^{(2i\,dx+2i\,c)}+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 2*(2*d*x*e^(2*I*d*x + 2*I*c) + 2*d*x - (-I*e^(2*I*d*x + 2*I*c) - I)*log(e^(
2*I*d*x + 2*I*c) + 1) - I)/(a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^2(c+dx)}{-\sin^2(c+dx)+2i\sin(c+dx)\cos(c+dx)+\cos^2(c+dx)}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(-sin(c + d*x)**2 + 2*I*sin(c + d*x)*cos(c + d*x)
+ cos(c + d*x)**2), x)/a**2
```

Giac [A]
time = 0.42, size = 100, normalized size = 1.82

$$\frac{2\left(\frac{i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^2}-\frac{2i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}{a^2}+\frac{i\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^2}+\frac{-i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*(I*log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - 2*I*log(tan(1/2*d*x + 1/2*c) - I)/
a^2 + I*log(tan(1/2*d*x + 1/2*c) - 1)/a^2 + (-I*tan(1/2*d*x + 1/2*c)^2 + ta
n(1/2*d*x + 1/2*c) + I)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a^2))/d
```

Mupad [B]
time = 0.78, size = 83, normalized size = 1.51

$$\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^2\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)\,4\mathrm{i}}{a^2\,d}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\,2\mathrm{i}}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2),x)
```

```
[Out] (2*tan(c/2 + (d*x)/2))/(d*(a^2*tan(c/2 + (d*x)/2)^2 - a^2)) - (log(tan(c/2
+ (d*x)/2) - 1i)*4i)/(a^2*d) + (log(tan(c/2 + (d*x)/2)^2 - 1)*2i)/(a^2*d)
```

3.171 $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=56

$$\frac{3\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{2i\sec(c+dx)}{a^2d}-\frac{\sec(c+dx)\tan(c+dx)}{2a^2d}$$

```
[Out] 3/2*arctanh(sin(d*x+c))/a^2/d-2*I*sec(d*x+c)/a^2/d-1/2*sec(d*x+c)*tan(d*x+c
)/a^2/d
```

Rubi [A]
time = 0.10, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3171, 3169, 3855, 2686, 8, 2691}

$$-\frac{2i\sec(c+dx)}{a^2d}+\frac{3\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{\tan(c+dx)\sec(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (3*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - ((2*I)*Sec[c + d*x])/(a^2*d) - (Sec[c
 + d*x]*Tan[c + d*x])/(2*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
```

```
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3171

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx &= -\frac{\int \sec^3(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^2\,dx}{a^4}\\
&= -\frac{\int \left(-a^2\sec(c+dx)+2ia^2\sec(c+dx)\tan(c+dx)+a^2\sec(c+d\right.}{a^4}\\
&= -\frac{(2i)\int \sec(c+dx)\tan(c+dx)\,dx}{a^2}+\frac{\int \sec(c+dx)\,dx}{a^2}-\frac{\int \sec(c+}{}\\
&= \frac{\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{\sec(c+dx)\tan(c+dx)}{2a^2d}+\frac{\int \sec(c+dx)\,dx}{2a^2}\\
&= \frac{3\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{2i\sec(c+dx)}{a^2d}-\frac{\sec(c+dx)\tan(c+dx)}{2a^2d}
\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 146 vs. $2(56) = 112$.
time = 0.46, size = 146, normalized size = 2.61

$$-\frac{\sec^2(c+dx)\left(8i\cos(c+dx)+3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+3\cos(2(c+dx))\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sin(c+dx)\right)}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] -1/4*(Sec[c + d*x]^2*((8*I)*Cos[c + d*x] + 3*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] + 3*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
 Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 3*Log[Cos[(c + d*x)/2] + Sin[(
c + d*x)/2]] + 2*Sin[c + d*x]))/(a^2*d)
```

Maple [A]
time = 0.37, size = 102, normalized size = 1.82

method	result
risch	$-\frac{i\left(3\,\mathrm{e}^{3i(dx+c)}+5\,\mathrm{e}^{i(dx+c)}\right)}{d\,a^{2}\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}+\frac{3\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2a^{2}d}-\frac{3\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2a^{2}d}$
derivativedivides	$\frac{\frac{2\left(-\frac{1}{4}+i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{2\left(-\frac{1}{4}-i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}}{a^{2}d}$
default	$\frac{\frac{2\left(-\frac{1}{4}+i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{2\left(-\frac{1}{4}-i\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+\frac{1}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2}}{a^{2}d}$
norman	$\frac{-\frac{4i}{ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{da}+\frac{4i\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a^{2}d}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2a^{2}d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^2*((-1/4+I)/(tan(1/2*d*x+1/2*c)-1)-1/4/(tan(1/2*d*x+1/2*c)-1)^2-3/4*l
n(tan(1/2*d*x+1/2*c)-1)-(1/4+I)/(tan(1/2*d*x+1/2*c)+1)+1/4/(tan(1/2*d*x+1/2
*c)+1)^2+3/4*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(50) = 100.
time = 0.28, size = 167, normalized size = 2.98

$$-\frac{\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{4i\,\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+4i\right)}{a^{2}-\frac{2\,a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}-\frac{3\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^{2}}+\frac{3\,\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/2*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 4*I*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2 + sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 4*I)/(a^2 - 2*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*l
og(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 3*log(sin(d*x + c)/(cos(d*x +
 c) + 1) - 1)/a^2)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(50) = 100.
time = 4.04, size = 134, normalized size = 2.39

$$\frac{3\left(e^{(4i\,dx+4i\,c)}+2\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(e^{(4i\,dx+4i\,c)}+2\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-6i\,e^{(3i\,dx+3i\,c)}-10i\,e^{(i\,dx+i\,c)}}{2\left(a^{2}de^{(4i\,dx+4i\,c)}+2\,a^{2}de^{(2i\,dx+2i\,c)}+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/2*(3*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c
) + I) - 3*(e^(4*I*d*x + 4*I*c) + 2*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x +
 I*c) - I) - 6*I*e^(3*I*d*x + 3*I*c) - 10*I*e^(I*d*x + I*c))/(a^2*d*e^(4*I*
d*x + 4*I*c) + 2*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^3(c+dx)}{-\sin^2(c+dx)+2i\sin(c+dx)\cos(c+dx)+\cos^2(c+dx)}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(-sin(c + d*x)**2 + 2*I*sin(c + d*x)*cos(c + d*x)
+ cos(c + d*x)**2), x)/a**2
```

Giac [A]
time = 0.45, size = 95, normalized size = 1.70

$$\frac{\frac{3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^2}-\frac{3\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^2}-\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4i\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4i\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - 3*log(tan(1/2*d*x + 1/2*c) - 1)/
a^2 - 2*(tan(1/2*d*x + 1/2*c)^3 - 4*I*tan(1/2*d*x + 1/2*c)^2 + tan(1/2*d*x
+ 1/2*c) + 4*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2))/d
```

Mupad [B]
time = 1.18, size = 104, normalized size = 1.86

$$\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}-\frac{\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{a^2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^2}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,4{}\mathrm{i}}{a^2}+\frac{4{}\mathrm{i}}{a^2}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2),x)
```

```
[Out] (3*atanh(tan(c/2 + (d*x)/2)))/(a^2*d) - (tan(c/2 + (d*x)/2)^3/a^2 - (tan(c/
2 + (d*x)/2)^2*4i)/a^2 + 4i/a^2 + tan(c/2 + (d*x)/2)/a^2)/(d*(tan(c/2 + (d*
x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1))
```

3.172 $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=34

$$-\frac{i(i-\cot(c+dx))^3\tan^3(c+dx)}{3a^2d}$$

```
[Out] -1/3*I*(I-cot(d*x+c))^3*tan(d*x+c)^3/a^2/d
```

Rubi [A]
time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3167, 862, 37}

$$-\frac{i\tan^3(c+dx)(-\cot(c+dx)+i)^3}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] ((-1/3*I)*(I - Cot[c + d*x])^3*Tan[c + d*x]^3)/(a^2*d)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^4(ia+ax)^2}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{\left(-\frac{i}{a}+\frac{x}{a}\right)^2}{x^4}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{i(i-\cot(c+dx))^3\tan^3(c+dx)}{3a^2d}$$

Mathematica [A]
time = 0.26, size = 68, normalized size = 2.00

$$-\frac{\sec(c)\sec^3(c+dx)(3i\cos(dx)+3i\cos(2c+dx)-3\sin(dx)+3\sin(2c+dx)-2\sin(2c+3dx))}{6a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] -1/6*(Sec[c]*Sec[c + d*x]^3*((3*I)*Cos[d*x] + (3*I)*Cos[2*c + d*x] - 3*Sin[
d*x] + 3*Sin[2*c + d*x] - 2*Sin[2*c + 3*d*x]))/(a^2*d)
```

Maple [A]
time = 0.38, size = 20, normalized size = 0.59

method	result	size
derivativedivides	$-\frac{(\tan(dx+c)+i)^3}{3d\,a^2}$	20
default	$-\frac{(\tan(dx+c)+i)^3}{3d\,a^2}$	20
risch	$\frac{8i}{3a^2d\left(e^{2i(dx+c)}+1\right)^3}$	23
norman	$\frac{\frac{4i\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{20\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}-\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{4i\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}$	127

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/d/a^2*(tan(d*x+c)+I)^3
```

Maxima [A]
time = 0.30, size = 35, normalized size = 1.03

$$-\frac{\tan\left(dx+c\right)^3+3i\,\tan\left(dx+c\right)^2-3\,\tan\left(dx+c\right)}{3\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/3*(tan(d*x + c)^3 + 3*I*tan(d*x + c)^2 - 3*tan(d*x + c))/(a^2*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(26) = 52$.
time = 3.25, size = 54, normalized size = 1.59

$$\frac{8i}{3\left(a^2de^{(6i\,dx+6i\,c)} + 3\,a^2de^{(4i\,dx+4i\,c)} + 3\,a^2de^{(2i\,dx+2i\,c)} + a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 8/3*I/(a^2*d*e^(6*I*d*x + 6*I*c) + 3*a^2*d*e^(4*I*d*x + 4*I*c) + 3*a^2*d*e^
(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^4(c+dx)}{-\sin^2(c+dx)+2i\sin(c+dx)\cos(c+dx)+\cos^2(c+dx)}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**4/(-sin(c + d*x)**2 + 2*I*sin(c + d*x)*cos(c + d*x)
+ cos(c + d*x)**2), x)/a**2
```

Giac [A]
time = 0.45, size = 35, normalized size = 1.03

$$-\frac{\tan(dx+c)^3 + 3i\,\tan(dx+c)^2 - 3\,\tan(dx+c)}{3\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(tan(d*x + c)^3 + 3*I*tan(d*x + c)^2 - 3*tan(d*x + c))/(a^2*d)
```

Mupad [B]
time = 0.73, size = 49, normalized size = 1.44

$$-\frac{\sin(c+d\,x)\,\left(-4\cos(c+d\,x)^2+3\mathrm{i}\,\sin(c+d\,x)\,\cos(c+d\,x)+1\right)}{3\,a^2\,d\cos(c+d\,x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2),x)
```

```
[Out] -(sin(c + d*x)*(cos(c + d*x)*sin(c + d*x)*3i - 4*cos(c + d*x)^2 + 1))/(3*a^
2*d*cos(c + d*x)^3)
```

3.173 $\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=84

$$\frac{5\tanh^{-1}(\sin(c+dx))}{8a^2d}-\frac{2i\sec^3(c+dx)}{3a^2d}+\frac{5\sec(c+dx)\tan(c+dx)}{8a^2d}-\frac{\sec^3(c+dx)\tan(c+dx)}{4a^2d}$$

```
[Out] 5/8*arctanh(sin(d*x+c))/a^2/d-2/3*I*sec(d*x+c)^3/a^2/d+5/8*sec(d*x+c)*tan(d
*x+c)/a^2/d-1/4*sec(d*x+c)^3*tan(d*x+c)/a^2/d
```

Rubi [A]
time = 0.14, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3171, 3169, 3853, 3855, 2686, 30, 2691}

$$-\frac{2i\sec^3(c+dx)}{3a^2d}+\frac{5\tanh^{-1}(\sin(c+dx))}{8a^2d}-\frac{\tan(c+dx)\sec^3(c+dx)}{4a^2d}+\frac{5\tan(c+dx)\sec(c+dx)}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(8*a^2*d) - (((2*I)/3)*Sec[c + d*x]^3)/(a^2*d) +
(5*Sec[c + d*x]*Tan[c + d*x])/(8*a^2*d) - (Sec[c + d*x]^3*Tan[c + d*x])/(4*
a^2*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3171

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx &= -\frac{\int \sec^5(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^2\,dx}{a^4} \\
&= -\frac{\int \left(-a^2\sec^3(c+dx)+2ia^2\sec^3(c+dx)\tan(c+dx)+a^2\sec^3(c+\right.}{a^4} \\
&= -\frac{(2i)\int \sec^3(c+dx)\tan(c+dx)\,dx}{a^2}+\frac{\int \sec^3(c+dx)\,dx}{a^2}-\frac{\int \sec^3(}{} \\
&= \frac{\sec(c+dx)\tan(c+dx)}{2a^2d}-\frac{\sec^3(c+dx)\tan(c+dx)}{4a^2d}+\frac{\int \sec^3(c+d}{4a^2} \\
&= \frac{\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{2i\sec^3(c+dx)}{3a^2d}+\frac{5\sec(c+dx)\tan(c+dx)}{8a^2d} \\
&= \frac{5\tanh^{-1}(\sin(c+dx))}{8a^2d}-\frac{2i\sec^3(c+dx)}{3a^2d}+\frac{5\sec(c+dx)\tan(c+dx}{8a^2d}
\end{aligned}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 215 vs. $2(84) = 168$.
time = 1.05, size = 215, normalized size = 2.56

$$-\frac{\sec^4(c+dx)\left(128i\cos(c+dx)+45\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+60\cos(2(c+dx))\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+15\cos(4(c+dx))\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-45\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+18\sin(c+dx)-30\sin(3(c+dx))\right)}{192a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] -1/192*(Sec[c + d*x]^4*((128*I)*Cos[c + d*x] + 45*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]] + 60*Cos[2*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 15*Cos[4*(c + d*x)]*(Log
[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]]) - 45*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 18*Sin[c + d*x] - 30
*Sin[3*(c + d*x)]))/(a^2*d)
```

Maple [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(75) = 150$.
time = 0.44, size = 170, normalized size = 2.02

method	result
risch	$-\frac{i\left(15\,\mathrm{e}^{7i(dx+c)}+55\,\mathrm{e}^{5i(dx+c)}+73\,\mathrm{e}^{3i(dx+c)}-15\,\mathrm{e}^{i(dx+c)}\right)}{12d\,a^{2}\left(\mathrm{e}^{2i(dx+c)}+1\right)^{4}}+\frac{5\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{8a^{2}d}-\frac{5\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{8a^{2}d}$
derivativedivides	$\frac{\frac{2\left(\frac{3}{16}+\frac{i}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(\frac{1}{16}+\frac{i}{2}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(-\frac{1}{4}+\frac{i}{3}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}+\frac{2\left(\frac{3}{16}-\frac{i}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+}{a^{2}d}$
default	$\frac{\frac{2\left(\frac{3}{16}+\frac{i}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(\frac{1}{16}+\frac{i}{2}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(-\frac{1}{4}+\frac{i}{3}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{1}{4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}+\frac{2\left(\frac{3}{16}-\frac{i}{2}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+}{a^{2}d}$
norman	$\frac{\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{11\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4da}-\frac{11\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}+\frac{3\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4ad}-\frac{4i}{3ad}+\frac{4i\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{4i\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+}{a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^2*((3/16+1/2*I)/(tan(1/2*d*x+1/2*c)-1)+(1/16+1/2*I)/(tan(1/2*d*x+1/2*
c)-1)^2+(-1/4+1/3*I)/(tan(1/2*d*x+1/2*c)-1)^3-1/8/(tan(1/2*d*x+1/2*c)-1)^4-
5/16*ln(tan(1/2*d*x+1/2*c)-1)+(3/16-1/2*I)/(tan(1/2*d*x+1/2*c)+1)+(-1/16+1/
2*I)/(tan(1/2*d*x+1/2*c)+1)^2-(1/4+1/3*I)/(tan(1/2*d*x+1/2*c)+1)^3+1/8/(tan
(1/2*d*x+1/2*c)+1)^4+5/16*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(74) = 148$.
time = 0.29, size = 295, normalized size = 3.51

$$\frac{\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{16i\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{33\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{48i\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{33\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{48i\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{9\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-16i\right)}{a^{2}-\frac{4a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{6a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{4a^{2}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{a^{2}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}}+\frac{15\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^{2}}-\frac{15\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^{2}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] 1/24*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*I*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 - 33*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 48*I*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 33*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 48*I*sin(d*x
 + c)^6/(cos(d*x + c) + 1)^6 + 9*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 16*I
)/(a^2 - 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^2*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4 - 4*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^2*sin(d
*x + c)^8/(cos(d*x + c) + 1)^8) + 15*log(sin(d*x + c)/(cos(d*x + c) + 1) +
1)/a^2 - 15*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(74) = 148$.
time = 2.75, size = 230, normalized size = 2.74

$$\frac{15\left(e^{(8i\,dx+8i\,c)}+4\,e^{(6i\,dx+6i\,c)}+6\,e^{(4i\,dx+4i\,c)}+4\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(e^{(8i\,dx+8i\,c)}+4\,e^{(6i\,dx+6i\,c)}+6\,e^{(4i\,dx+4i\,c)}+4\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-30i\,e^{(7i\,dx+7i\,c)}-110i\,e^{(5i\,dx+5i\,c)}-146i\,e^{(3i\,dx+3i\,c)}+30i\,e^{(i\,dx+i\,c)}}{24\left(a^2de^{(8i\,dx+8i\,c)}+4\,a^2de^{(6i\,dx+6i\,c)}+6\,a^2de^{(4i\,dx+4i\,c)}+4\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] 1/24*(15*(e^(8*I*d*x + 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*
c) + 4*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) + I) - 15*(e^(8*I*d*x +
 8*I*c) + 4*e^(6*I*d*x + 6*I*c) + 6*e^(4*I*d*x + 4*I*c) + 4*e^(2*I*d*x + 2*
I*c) + 1)*log(e^(I*d*x + I*c) - I) - 30*I*e^(7*I*d*x + 7*I*c) - 110*I*e^(5*
I*d*x + 5*I*c) - 146*I*e^(3*I*d*x + 3*I*c) + 30*I*e^(I*d*x + I*c))/(a^2*d*e
^(8*I*d*x + 8*I*c) + 4*a^2*d*e^(6*I*d*x + 6*I*c) + 6*a^2*d*e^(4*I*d*x + 4*I
*c) + 4*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^5(c+dx)}{-\sin^2(c+dx)+2i\sin(c+dx)\cos(c+dx)+\cos^2(c+dx)}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**5/(-sin(c + d*x)**2 + 2*I*sin(c + d*x)*cos(c + d*x)
+ cos(c + d*x)**2), x)/a**2
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(74) = 148$.
time = 0.45, size = 151, normalized size = 1.80

$$\frac{\frac{15\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^2}-\frac{15\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^2}+\frac{2\left(9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+48i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-33\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-48i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-33\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+16i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+9\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-16i\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4a^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(15*log(tan(1/2*d*x + 1/2*c) + 1)/a^2 - 15*log(tan(1/2*d*x + 1/2*c) -
1)/a^2 + 2*(9*tan(1/2*d*x + 1/2*c)^7 + 48*I*tan(1/2*d*x + 1/2*c)^6 - 33*tan
(1/2*d*x + 1/2*c)^5 - 48*I*tan(1/2*d*x + 1/2*c)^4 - 33*tan(1/2*d*x + 1/2*c)
^3 + 16*I*tan(1/2*d*x + 1/2*c)^2 + 9*tan(1/2*d*x + 1/2*c) - 16*I)/((tan(1/2
*d*x + 1/2*c)^2 - 1)^4*a^2))/d
```

Mupad [B]
time = 3.21, size = 136, normalized size = 1.62

$$\frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a^2\,d}+\frac{\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,4\mathrm{i}-\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,4\mathrm{i}-\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,4\mathrm{i}}{3}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}-\frac{4}{3}\mathrm{i}}{a^2\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2),x)
```

```
[Out] (5*atanh(tan(c/2 + (d*x)/2)))/(4*a^2*d) + ((3*tan(c/2 + (d*x)/2))/4 + (tan(
c/2 + (d*x)/2)^2*4i)/3 - (11*tan(c/2 + (d*x)/2)^3)/4 - tan(c/2 + (d*x)/2)^4
*4i - (11*tan(c/2 + (d*x)/2)^5)/4 + tan(c/2 + (d*x)/2)^6*4i + (3*tan(c/2 +
(d*x)/2)^7)/4 - 4i/3)/(a^2*d*(tan(c/2 + (d*x)/2)^2 - 1)^4)
```

3.174 $\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx$

Optimal. Leaf size=70

$$\frac{\tan(c+dx)}{a^2d}-\frac{i\tan^2(c+dx)}{a^2d}-\frac{i\tan^4(c+dx)}{2a^2d}-\frac{\tan^5(c+dx)}{5a^2d}$$

```
[Out] tan(d*x+c)/a^2/d-I*tan(d*x+c)^2/a^2/d-1/2*I*tan(d*x+c)^4/a^2/d-1/5*tan(d*x+
c)^5/a^2/d
```

Rubi [A]
time = 0.05, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3167, 862, 76}

$$-\frac{\tan^5(c+dx)}{5a^2d}-\frac{i\tan^4(c+dx)}{2a^2d}-\frac{i\tan^2(c+dx)}{a^2d}+\frac{\tan(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
```

```
[Out] Tan[c + d*x]/(a^2*d) - (I*Tan[c + d*x]^2)/(a^2*d) - ((I/2)*Tan[c + d*x]^4)/
(a^2*d) - Tan[c + d*x]^5/(5*a^2*d)
```

Rule 76

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{(1+x^2)^3}{x^6(ia+ax)^2}\,dx, x, \cot(c+dx)\right)}{d} \\
&= -\frac{\operatorname{Subst}\left(\int \frac{\left(-\frac{i}{a}+\frac{x}{a}\right)^3(ia+ax)}{x^6}\,dx, x, \cot(c+dx)\right)}{d} \\
&= -\frac{\operatorname{Subst}\left(\int \left(-\frac{1}{a^2x^6}-\frac{2i}{a^2x^5}-\frac{2i}{a^2x^3}+\frac{1}{a^2x^2}\right)\,dx, x, \cot(c+dx)\right)}{d} \\
&= \frac{\tan(c+dx)}{a^2d}-\frac{i\tan^2(c+dx)}{a^2d}-\frac{i\tan^4(c+dx)}{2a^2d}-\frac{\tan^5(c+dx)}{5a^2d}
\end{aligned}
$$

Mathematica [A]

time = 0.43, size = 77, normalized size = 1.10

$$
\frac{\sec(c)\sec^5(c+dx)(-5i\cos(dx)-5i\cos(2c+dx)+5\sin(dx)-5\sin(2c+dx)+5\sin(2c+3dx)+\sin(4c+5dx))}{20a^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2,x]
[Out] (Sec[c]*Sec[c + d*x]^5*((-5*I)*Cos[d*x] - (5*I)*Cos[2*c + d*x] + 5*Sin[d*x]
 - 5*Sin[2*c + d*x] + 5*Sin[2*c + 3*d*x] + Sin[4*c + 5*d*x]))/(20*a^2*d)
```

Maple [A]

time = 0.46, size = 47, normalized size = 0.67

method	result
risch	$\frac{8i\left(5\,\mathrm{e}^{2i(dx+c)}+1\right)}{5d\,a^2\left(\mathrm{e}^{2i(dx+c)}+1\right)^5}$
derivativedivides	$\frac{\tan(dx+c)-\frac{\left(\tan^5(dx+c)\right)}{5}-\frac{i\left(\tan^4(dx+c)\right)}{2}-i\left(\tan^2(dx+c)\right)}{d\,a^2}$
default	$\frac{\tan(dx+c)-\frac{\left(\tan^5(dx+c)\right)}{5}-\frac{i\left(\tan^4(dx+c)\right)}{2}-i\left(\tan^2(dx+c)\right)}{d\,a^2}$
norman	$\frac{\frac{4i\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{28\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5ad}+\frac{8\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{4i\left(\tan^4\left(\frac{dx}{2}\right.\right.}{ad}}{a\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x,method=_RETURNVERBOSE)
[Out] 1/d/a^2*(tan(d*x+c)-1/5*tan(d*x+c)^5-1/2*I*tan(d*x+c)^4-I*tan(d*x+c)^2)
```

Maxima [A]
time = 0.30, size = 47, normalized size = 0.67

$$-\frac{2\tan(dx+c)^5+5i\tan(dx+c)^4+10i\tan(dx+c)^2-10\tan(dx+c)}{10\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="maxima
")
```

```
[Out] -1/10*(2*tan(d*x + c)^5 + 5*I*tan(d*x + c)^4 + 10*I*tan(d*x + c)^2 - 10*tan
(d*x + c))/(a^2*d)
```

Fricas [A]
time = 2.14, size = 97, normalized size = 1.39

$$-\frac{8\left(-5i\,e^{(2i\,dx+2i\,c)}-i\right)}{5\left(a^2de^{(10i\,dx+10i\,c)}+5\,a^2de^{(8i\,dx+8i\,c)}+10\,a^2de^{(6i\,dx+6i\,c)}+10\,a^2de^{(4i\,dx+4i\,c)}+5\,a^2de^{(2i\,dx+2i\,c)}+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="fricas
")
```

```
[Out] -8/5*(-5*I*e^(2*I*d*x + 2*I*c) - I)/(a^2*d*e^(10*I*d*x + 10*I*c) + 5*a^2*d*
e^(8*I*d*x + 8*I*c) + 10*a^2*d*e^(6*I*d*x + 6*I*c) + 10*a^2*d*e^(4*I*d*x +
4*I*c) + 5*a^2*d*e^(2*I*d*x + 2*I*c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec^6(c+dx)}{-\sin^2(c+dx)+2i\sin(c+dx)\cos(c+dx)+\cos^2(c+dx)}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a*cos(d*x+c)+I*a*sin(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**6/(-sin(c + d*x)**2 + 2*I*sin(c + d*x)*cos(c + d*x)
+ cos(c + d*x)**2), x)/a**2
```

Giac [A]
time = 0.47, size = 47, normalized size = 0.67

$$-\frac{2\tan(dx+c)^5+5i\tan(dx+c)^4+10i\tan(dx+c)^2-10\tan(dx+c)}{10\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/10*(2*tan(d*x + c)^5 + 5*I*tan(d*x + c)^4 + 10*I*tan(d*x + c)^2 - 10*tan
(d*x + c))/(a^2*d)
```

Mupad [B]
time = 0.92, size = 76, normalized size = 1.09

$$-\frac{\sin(c+dx)\left(-4\cos(c+dx)^4+\frac{5\mathrm{i}\,\sin(c+dx)\cos(c+dx)^3}{2}-2\cos(c+dx)^2+\frac{5\mathrm{i}\,\sin(c+dx)\,\cos(c+dx)}{2}+1\right)}{5\,a^2\,d\cos(c+dx)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^2),x)
```

```
[Out] -(sin(c + d*x)*((cos(c + d*x)*sin(c + d*x)*5i)/2 + (cos(c + d*x)^3*sin(c +
d*x)*5i)/2 - 2*cos(c + d*x)^2 - 4*cos(c + d*x)^4 + 1))/(5*a^2*d*cos(c + d*x
)^5)
```

3.175 $\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=125

$$\frac{5x}{32a^3}-\frac{1}{32a^3d(i-\cot(c+dx))}+\frac{i}{16a^3d(i+\cot(c+dx))^4}-\frac{1}{3a^3d(i+\cot(c+dx))^3}-\frac{23i}{32a^3d(i+\cot(c+dx))^2}+$$

```
[Out] 5/32*x/a^3-1/32/a^3/d/(I-cot(d*x+c))+1/16*I/a^3/d/(I+cot(d*x+c))^4-1/3/a^3/
d/(I+cot(d*x+c))^3-23/32*I/a^3/d/(I+cot(d*x+c))^2+13/16/a^3/d/(I+cot(d*x+c)
)
```

Rubi [A]
time = 0.08, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {3167, 862, 90, 209}

$$-\frac{1}{32a^3d(-\cot(c+dx)+i)}+\frac{13}{16a^3d(\cot(c+dx)+i)}-\frac{23i}{32a^3d(\cot(c+dx)+i)^2}-\frac{1}{3a^3d(\cot(c+dx)+i)^3}+\frac{i}{16a^3d(\cot(c+dx)+i)^4}+\frac{5x}{32a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] (5*x)/(32*a^3) - 1/(32*a^3*d*(I - Cot[c + d*x])) + (I/16)/(a^3*d*(I + Cot[c
 + d*x])^4) - 1/(3*a^3*d*(I + Cot[c + d*x])^3) - ((23*I)/32)/(a^3*d*(I + Co
t[c + d*x])^2) + 13/(16*a^3*d*(I + Cot[c + d*x]))
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{x^5}{(ia+ax)^3(1+x^2)^2}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^5}{\left(-\frac{i}{a}+\frac{x}{a}\right)^2(ia+ax)^5}\,dx, x, \cot(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{32a^3(-i+x)^2}+\frac{i}{4a^3(i+x)^5}-\frac{1}{a^3(i+x)^4}-\frac{23i}{16a^3(i+x)^3}+\frac{13}{16a^3(i+}\right.\right.}{d}$$

$$= -\frac{1}{32a^3d(i-\cot(c+dx))}+\frac{i}{16a^3d(i+\cot(c+dx))^4}-\frac{}{3a^3d(i+c}$$

$$= \frac{5x}{32a^3}-\frac{1}{32a^3d(i-\cot(c+dx))}+\frac{i}{16a^3d(i+\cot(c+dx))^4}-\frac{}{3a^3d}$$

Mathematica [A]
time = 0.18, size = 106, normalized size = 0.85

$$\frac{120c+120dx+108i\cos(2(c+dx))+60i\cos(4(c+dx))+20i\cos(6(c+dx))+3i\cos(8(c+dx))+132\sin(2(c+dx))+60\sin(4(c+dx))+20\sin(6(c+dx))+3\sin(8(c+dx))}{768a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] (120*c + 120*d*x + (108*I)*Cos[2*(c + d*x)] + (60*I)*Cos[4*(c + d*x)] + (20
*I)*Cos[6*(c + d*x)] + (3*I)*Cos[8*(c + d*x)] + 132*Sin[2*(c + d*x)] + 60*S
in[4*(c + d*x)] + 20*Sin[6*(c + d*x)] + 3*Sin[8*(c + d*x)])/(768*a^3*d)
```

Maple [A]
time = 0.70, size = 102, normalized size = 0.82

method	result
risch	$\frac{5x}{32a^3}+\frac{5i\,e^{-4i(dx+c)}}{64a^3d}+\frac{5i\,e^{-6i(dx+c)}}{192a^3d}+\frac{i\,e^{-8i(dx+c)}}{256a^3d}+\frac{9i\cos(2dx+2c)}{64a^3d}+\frac{11\sin(2dx+2c)}{64a^3d}$
derivativedivides	$\frac{-\frac{5i\ln(\tan(dx+c)-i)}{64}+\frac{i}{16(\tan(dx+c)-i)^4}-\frac{3i}{32(\tan(dx+c)-i)^2}-\frac{1}{12(\tan(dx+c)-i)^3}+\frac{1}{8\tan(dx+c)-8i}+\frac{5i\ln(\tan(dx+c)+i)}{64}+\frac{}{32\tan}}{d\,a^3}$

default	$\frac{-\frac{5i\ln(\tan(dx+c)-i)}{64}+\frac{i}{16(\tan(dx+c)-i)^4}-\frac{3i}{32(\tan(dx+c)-i)^2}-\frac{1}{12(\tan(dx+c)-i)^3}+\frac{1}{8\tan(dx+c)-8i}+\frac{5i\ln(\tan(dx+c)+i)}{64}+\frac{}{32\tan(}}{d\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-5/64*I*ln(tan(d*x+c)-I)+1/16*I/(tan(d*x+c)-I)^4-3/32*I/(tan(d*x+c
)-I)^2-1/12/(tan(d*x+c)-I)^3+1/8/(tan(d*x+c)-I)+5/64*I*ln(tan(d*x+c)+I)+1/3
2/(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 2.54, size = 76, normalized size = 0.61

$$\frac{\left(120\,dxe^{(8i\,dx+8i\,c)}-12i\,e^{(10i\,dx+10i\,c)}+120i\,e^{(6i\,dx+6i\,c)}+60i\,e^{(4i\,dx+4i\,c)}+20i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-8i\,dx-8i\,c)}}{768\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/768*(120*d*x*e^(8*I*d*x + 8*I*c) - 12*I*e^(10*I*d*x + 10*I*c) + 120*I*e^(
6*I*d*x + 6*I*c) + 60*I*e^(4*I*d*x + 4*I*c) + 20*I*e^(2*I*d*x + 2*I*c) + 3*
I)*e^(-8*I*d*x - 8*I*c)/(a^3*d)
```

Sympy [A]
time = 0.23, size = 224, normalized size = 1.79

$$\begin{cases}\frac{\left(-100663296ia^{12}d^4e^{22ic}e^{2idx}+1006632960ia^{12}d^4e^{18ic}e^{-2idx}+503316480ia^{12}d^4e^{16ic}e^{-4idx}+167772160ia^{12}d^4e^{14ic}e^{-6idx}+25165824ia^{12}d^4e^{12ic}e^{-8idx}\right)e^{-20ic}}{6442450944a^{15}d^5} & \text{for}\ a^{15}d^5e^{20ic}\neq 0\\ x\left(\frac{\left(e^{10ic}+5e^{8ic}+10e^{6ic}+10e^{4ic}+5e^{2ic}+1\right)e^{-8ic}}{32a^3}-\frac{5}{32a^3}\right) & \text{otherwise}\end{cases}+\frac{5x}{32a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise(((-100663296*I*a**12*d**4*exp(22*I*c)*exp(2*I*d*x) + 1006632960*I
*a**12*d**4*exp(18*I*c)*exp(-2*I*d*x) + 503316480*I*a**12*d**4*exp(16*I*c)*
exp(-4*I*d*x) + 167772160*I*a**12*d**4*exp(14*I*c)*exp(-6*I*d*x) + 25165824
*I*a**12*d**4*exp(12*I*c)*exp(-8*I*d*x))*exp(-20*I*c)/(6442450944*a**15*d**
5), Ne(a**15*d**5*exp(20*I*c), 0)), (x*((exp(10*I*c) + 5*exp(8*I*c) + 10*ex
p(6*I*c) + 10*exp(4*I*c) + 5*exp(2*I*c) + 1)*exp(-8*I*c)/(32*a**3) - 5/(32*
a**3)), True)) + 5*x/(32*a**3)
```

Giac [A]
time = 0.47, size = 119, normalized size = 0.95

$$-\frac{-\frac{60i\log(-i\tan(dx+c)+1)}{a^3}+\frac{60i\log(-i\tan(dx+c)-1)}{a^3}-\frac{12(5\tan(dx+c)+7i)}{a^3(i\tan(dx+c)-1)}+\frac{-125i\tan(dx+c)^4-596\tan(dx+c)^3+1110i\tan(dx+c)^2+996\tan(dx+c)-405i}{a^3(\tan(dx+c)-i)^4}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/768*(-60*I*log(-I*tan(d*x + c) + 1)/a^3 + 60*I*log(-I*tan(d*x + c) - 1)/
a^3 - 12*(5*tan(d*x + c) + 7*I)/(a^3*(I*tan(d*x + c) - 1)) + (-125*I*tan(d*
x + c)^4 - 596*tan(d*x + c)^3 + 1110*I*tan(d*x + c)^2 + 996*tan(d*x + c) -
405*I)/(a^3*(tan(d*x + c) - I)^4))/d
```

Mupad [B]
time = 4.79, size = 164, normalized size = 1.31

$$\frac{5\,x}{32\,a^3}+\frac{-\frac{27\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{16}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,33\mathrm{i}}{8}+\frac{31\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{6}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,9\mathrm{i}}{8}+\frac{89\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{24}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,9\mathrm{i}}{8}+\frac{31\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,33\mathrm{i}}{8}-\frac{27\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16}}{a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)^2\,\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3,x)
```

```
[Out] (5*x)/(32*a^3) + ((31*tan(c/2 + (d*x)/2)^3)/6 - (tan(c/2 + (d*x)/2)^2*33i)/
8 - (27*tan(c/2 + (d*x)/2))/16 - (tan(c/2 + (d*x)/2)^4*9i)/8 + (89*tan(c/2
+ (d*x)/2)^5)/24 + (tan(c/2 + (d*x)/2)^6*9i)/8 + (31*tan(c/2 + (d*x)/2)^7)/
6 + (tan(c/2 + (d*x)/2)^8*33i)/8 - (27*tan(c/2 + (d*x)/2)^9)/16)/(a^3*d*(ta
n(c/2 + (d*x)/2) + 1i)^2*(tan(c/2 + (d*x)/2)*1i + 1)^8)
```

3.176 $\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=106

$$-\frac{i\cos^5(c+dx)}{5a^3d}+\frac{4i\cos^7(c+dx)}{7a^3d}+\frac{\sin(c+dx)}{a^3d}-\frac{2\sin^3(c+dx)}{a^3d}+\frac{9\sin^5(c+dx)}{5a^3d}-\frac{4\sin^7(c+dx)}{7a^3d}$$

```
[Out] -1/5*I*cos(d*x+c)^5/a^3/d+4/7*I*cos(d*x+c)^7/a^3/d+sin(d*x+c)/a^3/d-2*sin(d
*x+c)^3/a^3/d+9/5*sin(d*x+c)^5/a^3/d-4/7*sin(d*x+c)^7/a^3/d
```

Rubi [A]
time = 0.17, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {3171, 3169, 2713, 2645, 30, 2644, 276, 14}

$$-\frac{4\sin^7(c+dx)}{7a^3d}+\frac{9\sin^5(c+dx)}{5a^3d}-\frac{2\sin^3(c+dx)}{a^3d}+\frac{\sin(c+dx)}{a^3d}+\frac{4i\cos^7(c+dx)}{7a^3d}-\frac{i\cos^5(c+dx)}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((-1/5*I)*Cos[c + d*x]^5)/(a^3*d) + (((4*I)/7)*Cos[c + d*x]^7)/(a^3*d) + Si
n[c + d*x]/(a^3*d) - (2*Sin[c + d*x]^3)/(a^3*d) + (9*Sin[c + d*x]^5)/(5*a^3
*d) - (4*Sin[c + d*x]^7)/(7*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3171

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx &= \frac{i\int \cos^4(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^3\,dx}{a^6} \\
&= \frac{i\int \left(-ia^3\cos^7(c+dx)-3a^3\cos^6(c+dx)\sin(c+dx)+3ia^3\cos^5(\right.}{a^6} \\
&= \frac{i\int \cos^4(c+dx)\sin^3(c+dx)\,dx}{a^3} - \frac{(3i)\int \cos^6(c+dx)\sin(c+dx)}{a^3} \\
&= -\frac{i\mathrm{Subst}\left(\int x^4(1-x^2)\,dx,x,\cos(c+dx)\right)}{a^3d} + \frac{(3i)\mathrm{Subst}\left(\int x^6\,dx,x\right.}{a^3d} \\
&= \frac{3i\cos^7(c+dx)}{7a^3d} + \frac{\sin(c+dx)}{a^3d} - \frac{\sin^3(c+dx)}{a^3d} + \frac{3\sin^5(c+dx)}{5a^3d} - \\
&= -\frac{i\cos^5(c+dx)}{5a^3d} + \frac{4i\cos^7(c+dx)}{7a^3d} + \frac{\sin(c+dx)}{a^3d} - \frac{2\sin^3(c+dx)}{a^3d}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 149, normalized size = 1.41

$$\frac{3i\cos(c+dx)}{16a^3d}+\frac{i\cos(3(c+dx))}{8a^3d}+\frac{i\cos(5(c+dx))}{20a^3d}+\frac{i\cos(7(c+dx))}{112a^3d}+\frac{5\sin(c+dx)}{16a^3d}+\frac{\sin(3(c+dx))}{8a^3d}+\frac{\sin(5(c+dx))}{20a^3d}+\frac{\sin(7(c+dx))}{112a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] (((3*I)/16)*Cos[c + d*x])/(a^3*d) + ((I/8)*Cos[3*(c + d*x)])/(a^3*d) + ((I/
20)*Cos[5*(c + d*x)])/(a^3*d) + ((I/112)*Cos[7*(c + d*x)])/(a^3*d) + (5*Sin
[c + d*x])/(16*a^3*d) + Sin[3*(c + d*x)]/(8*a^3*d) + Sin[5*(c + d*x)]/(20*a
^3*d) + Sin[7*(c + d*x)]/(112*a^3*d)
```

Maple [A]
time = 0.55, size = 141, normalized size = 1.33

method	result
risch	$\frac{i e^{-3i(dx+c)}}{8a^3d}+\frac{i e^{-5i(dx+c)}}{20a^3d}+\frac{i e^{-7i(dx+c)}}{112a^3d}+\frac{3i\cos(dx+c)}{16a^3d}+\frac{5\sin(dx+c)}{16a^3d}$
derivativedivides	$\frac{\frac{2}{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16i}+\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{9i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{17i}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{8}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+\frac{38}{5\left(-i+\tan\left(\frac{dx}{2}\right.\right.}\cdots}{a^3d}$
default	$\frac{\frac{2}{16\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+16i}+\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{9i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{17i}{4\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{8}{7\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7}+\frac{38}{5\left(-i+\tan\left(\frac{dx}{2}\right.\right.}\cdots}{a^3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^3*(1/16/(tan(1/2*d*x+1/2*c)+I)+2*I/(-I+tan(1/2*d*x+1/2*c))^6-9/2*I/(-
I+tan(1/2*d*x+1/2*c))^4+17/8*I/(-I+tan(1/2*d*x+1/2*c))^2-4/7/(-I+tan(1/2*d*
x+1/2*c))^7+19/5/(-I+tan(1/2*d*x+1/2*c))^5-15/4/(-I+tan(1/2*d*x+1/2*c))^3+1
5/16/(-I+tan(1/2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 2.31, size = 63, normalized size = 0.59

$$\frac{\left(-35i\,e^{(8i\,dx+8i\,c)}+140i\,e^{(6i\,dx+6i\,c)}+70i\,e^{(4i\,dx+4i\,c)}+28i\,e^{(2i\,dx+2i\,c)}+5i\right)e^{(-7i\,dx-7i\,c)}}{560\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/560*(-35*I*e^(8*I*d*x + 8*I*c) + 140*I*e^(6*I*d*x + 6*I*c) + 70*I*e^(4*I*
d*x + 4*I*c) + 28*I*e^(2*I*d*x + 2*I*c) + 5*I)*e^(-7*I*d*x - 7*I*c)/(a^3*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(95) = 190$.
time = 0.26, size = 197, normalized size = 1.86

$$\begin{cases} \frac{\left(-71680ia^{12}d^4e^{17ic}e^{idx}+286720ia^{12}d^4e^{15ic}e^{-idx}+143360ia^{12}d^4e^{13ic}e^{-3idx}+57344ia^{12}d^4e^{11ic}e^{-5idx}+10240ia^{12}d^4e^{9ic}e^{-7idx}\right)e^{-16ic}}{1146880a^{15}d^5} & \text{for } a^{15}d^5e^{16ic}\neq 0 \\ \frac{x\left(e^{8ic}+4e^{6ic}+6e^{4ic}+4e^{2ic}+1\right)e^{-7ic}}{16a^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise(((-71680*I*a**12*d**4*exp(17*I*c)*exp(I*d*x) + 286720*I*a**12*d**
4*exp(15*I*c)*exp(-I*d*x) + 143360*I*a**12*d**4*exp(13*I*c)*exp(-3*I*d*x) +
 57344*I*a**12*d**4*exp(11*I*c)*exp(-5*I*d*x) + 10240*I*a**12*d**4*exp(9*I*
c)*exp(-7*I*d*x))*exp(-16*I*c)/(1146880*a**15*d**5), Ne(a**15*d**5*exp(16*I
*c), 0)), (x*(exp(8*I*c) + 4*exp(6*I*c) + 6*exp(4*I*c) + 4*exp(2*I*c) + 1)*
exp(-7*I*c)/(16*a**3), True))
```

Giac [A]
time = 0.46, size = 119, normalized size = 1.12

$$\frac{\frac{35}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+i\right)}+\frac{525\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6-1960i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-4025\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+4480i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3143\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1176i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-243}{a^3\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^7}}{280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/280*(35/(a^3*(tan(1/2*d*x + 1/2*c) + I)) + (525*tan(1/2*d*x + 1/2*c)^6 -
1960*I*tan(1/2*d*x + 1/2*c)^5 - 4025*tan(1/2*d*x + 1/2*c)^4 + 4480*I*tan(1/
2*d*x + 1/2*c)^3 + 3143*tan(1/2*d*x + 1/2*c)^2 - 1176*I*tan(1/2*d*x + 1/2*c
) - 243)/(a^3*(tan(1/2*d*x + 1/2*c) - I)^7))/d
```

Mupad [B]
time = 3.17, size = 134, normalized size = 1.26

$$-\frac{\left(35\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\,105{}\mathrm{i}-175\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\,105{}\mathrm{i}-7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\,77{}\mathrm{i}+43\tan\left(\frac{c}{2}+\frac{dx}{2}\right)-13{}\mathrm{i}\right)\,2{}\mathrm{i}}{35\,a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+1{}\mathrm{i}\right)\,{\left(1+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,1{}\mathrm{i}\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3,x)
```

```
[Out] -((43*tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*77i - 7*tan(c/2 + (d*x)/2)^
3 + tan(c/2 + (d*x)/2)^4*105i - 175*tan(c/2 + (d*x)/2)^5 - tan(c/2 + (d*x)/
2)^6*105i + 35*tan(c/2 + (d*x)/2)^7 - 13i)*2i)/(35*a^3*d*(tan(c/2 + (d*x)/2
) + 1i)*(tan(c/2 + (d*x)/2)*1i + 1)^7)
```

3.177 $\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=131

$$\frac{x}{8a^3}+\frac{i\cos^3(c+dx)}{6d(a\cos(c+dx)+ia\sin(c+dx))^3}+\frac{i\cos^2(c+dx)}{8ad(a\cos(c+dx)+ia\sin(c+dx))^2}+\frac{i\cos(c+dx)}{8d\,(a^3\cos(c+dx)+ia^3\,\mathrm{si}}$$

```
[Out] 1/8*x/a^3+1/6*I*cos(d*x+c)^3/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^3+1/8*I*cos(d*
x+c)^2/a/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^2+1/8*I*cos(d*x+c)/d/(a^3*cos(d*x+
c)+I*a^3*sin(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {3161, 8}

$$\frac{i\cos(c+dx)}{8d\,(a^3\cos(c+dx)+ia^3\sin(c+dx))}+\frac{x}{8a^3}+\frac{i\cos^3(c+dx)}{6d(a\cos(c+dx)+ia\sin(c+dx))^3}+\frac{i\cos^2(c+dx)}{8ad(a\cos(c+dx)+ia\sin(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] x/(8*a^3) + ((I/6)*Cos[c + d*x]^3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3
) + ((I/8)*Cos[c + d*x]^2)/(a*d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^2) + ((
I/8)*Cos[c + d*x])/(d*(a^3*Cos[c + d*x] + I*a^3*Sin[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3161

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*((a*Cos[c + d*x] + b*Si
n[c + d*x])^n/(2*a*d*n*Cos[c + d*x]^n)), x] + Dist[1/(2*a), Int[(a*Cos[c +
d*x] + b*Sin[c + d*x])^(n + 1)/Cos[c + d*x]^(n + 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && EqQ[m + n, 0] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i\cos^3(c+dx)}{6d(a\cos(c+dx)+ia\sin(c+dx))^3} + \frac{\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^2}\,dx}{2a}$$
$$= \frac{i\cos^3(c+dx)}{6d(a\cos(c+dx)+ia\sin(c+dx))^3} + \frac{i\cos^2(c+dx)}{8ad(a\cos(c+dx)+ia\sin(c$$
$$= \frac{i\cos^3(c+dx)}{6d(a\cos(c+dx)+ia\sin(c+dx))^3} + \frac{i\cos^2(c+dx)}{8ad(a\cos(c+dx)+ia\sin(c$$
$$= \frac{x}{8a^3} + \frac{i\cos^3(c+dx)}{6d(a\cos(c+dx)+ia\sin(c+dx))^3} + \frac{i\cos^2(c+}{8ad(a\cos(c+dx)+i}$$

Mathematica [A]
time = 0.11, size = 84, normalized size = 0.64

$$\frac{12c+12dx+18i\cos(2(c+dx))+9i\cos(4(c+dx))+2i\cos(6(c+dx))+18\sin(2(c+dx))+9\sin(4(c+dx))+2\sin(6(c+dx))}{96a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] (12*c + 12*d*x + (18*I)*Cos[2*(c + d*x)] + (9*I)*Cos[4*(c + d*x)] + (2*I)*C
os[6*(c + d*x)] + 18*Sin[2*(c + d*x)] + 9*Sin[4*(c + d*x)] + 2*Sin[6*(c + d
*x)])/(96*a^3*d)
```

Maple [A]
time = 0.40, size = 75, normalized size = 0.57

method	result	size
risch	$\frac{x}{8a^3}+\frac{3i e^{-2i(dx+c)}}{16a^3d}+\frac{3i e^{-4i(dx+c)}}{32a^3d}+\frac{i e^{-6i(dx+c)}}{48a^3d}$	62
derivativedivides	$\frac{-\frac{i\ln(\tan(dx+c)-i)}{16}-\frac{i}{8(\tan(dx+c)-i)^2}-\frac{1}{6(\tan(dx+c)-i)^3}+\frac{1}{8\tan(dx+c)-8i}+\frac{i\ln(\tan(dx+c)+i)}{16}}{d\,a^3}$	75
default	$\frac{-\frac{i\ln(\tan(dx+c)-i)}{16}-\frac{i}{8(\tan(dx+c)-i)^2}-\frac{1}{6(\tan(dx+c)-i)^3}+\frac{1}{8\tan(dx+c)-8i}+\frac{i\ln(\tan(dx+c)+i)}{16}}{d\,a^3}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-1/16*I*ln(tan(d*x+c)-I)-1/8*I/(tan(d*x+c)-I)^2-1/6/(tan(d*x+c)-I)
^3+1/8/(tan(d*x+c)-I)+1/16*I*ln(tan(d*x+c)+I))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 expt: undefined: 0 to a negative exponent.
```

Fricas [A]
time = 2.57, size = 54, normalized size = 0.41

$$\frac{\left(12\,dxe^{(6i\,dx+6i\,c)}+18i\,e^{(4i\,dx+4i\,c)}+9i\,e^{(2i\,dx+2i\,c)}+2i\right)e^{(-6i\,dx-6i\,c)}}{96\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/96*(12*d*x*e^(6*I*d*x + 6*I*c) + 18*I*e^(4*I*d*x + 4*I*c) + 9*I*e^(2*I*d*
x + 2*I*c) + 2*I)*e^(-6*I*d*x - 6*I*c)/(a^3*d)
```

Sympy [A]
time = 0.17, size = 155, normalized size = 1.18

$$\begin{cases}\frac{\left(4608ia^6d^2e^{10ic}e^{-2idx}+2304ia^6d^2e^{8ic}e^{-4idx}+512ia^6d^2e^{6ic}e^{-6idx}\right)e^{-12ic}}{24576a^9d^3} & \text{for } a^9d^3e^{12ic}\neq 0\\ x\left(\frac{\left(e^{6ic}+3e^{4ic}+3e^{2ic}+1\right)e^{-6ic}}{8a^3}-\frac{1}{8a^3}\right) & \text{otherwise}\end{cases}+\frac{x}{8a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise(((4608*I*a**6*d**2*exp(10*I*c)*exp(-2*I*d*x) + 2304*I*a**6*d**2*e
xp(8*I*c)*exp(-4*I*d*x) + 512*I*a**6*d**2*exp(6*I*c)*exp(-6*I*d*x))*exp(-12
*I*c)/(24576*a**9*d**3), Ne(a**9*d**3*exp(12*I*c), 0)), (x*((exp(6*I*c) + 3
*exp(4*I*c) + 3*exp(2*I*c) + 1)*exp(-6*I*c)/(8*a**3) - 1/(8*a**3)), True))
+ x/(8*a**3)
```

Giac [A]
time = 0.47, size = 80, normalized size = 0.61

$$-\frac{\frac{6i\,\log(\tan(dx+c)-i)}{a^3}-\frac{6i\,\log(i\,\tan(dx+c)-1)}{a^3}+\frac{-11i\,\tan(dx+c)^3-45\,\tan(dx+c)^2+69i\,\tan(dx+c)+51}{a^3(\tan(dx+c)-i)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/96*(6*I*log(tan(d*x + c) - I)/a^3 - 6*I*log(I*tan(d*x + c) - 1)/a^3 + (-
11*I*tan(d*x + c)^3 - 45*tan(d*x + c)^2 + 69*I*tan(d*x + c) + 51)/(a^3*(tan
(d*x + c) - I)^3))/d
```

Mupad [B]
time = 3.49, size = 96, normalized size = 0.73

$$\frac{x}{8\,a^3}+\frac{\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,9\mathrm{i}}{2}-\frac{41\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,9\mathrm{i}}{2}+\frac{7\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{a^3\,d\left(1+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3,x)
```

```
[Out] x/(8*a^3) + ((7*tan(c/2 + (d*x)/2))/4 + (tan(c/2 + (d*x)/2)^2*9i)/2 - (41*t
an(c/2 + (d*x)/2)^3)/6 - (tan(c/2 + (d*x)/2)^4*9i)/2 + (7*tan(c/2 + (d*x)/2
)^5)/4)/(a^3*d*(tan(c/2 + (d*x)/2)*1i + 1)^6)
```

3.178 $\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=90

$$-\frac{i\cos^3(c+dx)}{3a^3d}+\frac{4i\cos^5(c+dx)}{5a^3d}+\frac{\sin(c+dx)}{a^3d}-\frac{5\sin^3(c+dx)}{3a^3d}+\frac{4\sin^5(c+dx)}{5a^3d}$$

```
[Out] -1/3*I*cos(d*x+c)^3/a^3/d+4/5*I*cos(d*x+c)^5/a^3/d+sin(d*x+c)/a^3/d-5/3*sin
(d*x+c)^3/a^3/d+4/5*sin(d*x+c)^5/a^3/d
```

Rubi [A]
time = 0.15, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3171, 3169, 2713, 2645, 30, 2644, 14}

$$\frac{4\sin^5(c+dx)}{5a^3d}-\frac{5\sin^3(c+dx)}{3a^3d}+\frac{\sin(c+dx)}{a^3d}+\frac{4i\cos^5(c+dx)}{5a^3d}-\frac{i\cos^3(c+dx)}{3a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((-1/3*I)*Cos[c + d*x]^3)/(a^3*d) + (((4*I)/5)*Cos[c + d*x]^5)/(a^3*d) + Si
n[c + d*x]/(a^3*d) - (5*Sin[c + d*x]^3)/(3*a^3*d) + (4*Sin[c + d*x]^5)/(5*a
^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
```

```
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3171

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx &= \frac{i\int \cos^2(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^3\,dx}{a^6}\\ &= \frac{i\int \left(-ia^3\cos^5(c+dx)-3a^3\cos^4(c+dx)\sin(c+dx)+3ia^3\cos^3(c\right.}{a^6}\\ &= \frac{i\int \cos^2(c+dx)\sin^3(c+dx)\,dx}{a^3}-\frac{(3i)\int \cos^4(c+dx)\sin(c+dx)\,d}{a^3}\\ &= -\frac{i\text{Subst}\left(\int x^2(1-x^2)\,dx,x,\cos(c+dx)\right)}{a^3d}+\frac{(3i)\text{Subst}\left(\int x^4\,dx,x,\right.}{a^3d}\\ &= \frac{3i\cos^5(c+dx)}{5a^3d}+\frac{\sin(c+dx)}{a^3d}-\frac{2\sin^3(c+dx)}{3a^3d}+\frac{\sin^5(c+dx)}{5a^3d}-\\ &= -\frac{i\cos^3(c+dx)}{3a^3d}+\frac{4i\cos^5(c+dx)}{5a^3d}+\frac{\sin(c+dx)}{a^3d}-\frac{5\sin^3(c+dx)}{3a^3d}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 111, normalized size = 1.23

$$\frac{i\cos(c+dx)}{4a^3d}+\frac{i\cos(3(c+dx))}{6a^3d}+\frac{i\cos(5(c+dx))}{20a^3d}+\frac{\sin(c+dx)}{4a^3d}+\frac{\sin(3(c+dx))}{6a^3d}+\frac{\sin(5(c+dx))}{20a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((I/4)*Cos[c + d*x])/(a^3*d) + ((I/6)*Cos[3*(c + d*x)])/(a^3*d) + ((I/20)*C
os[5*(c + d*x)])/(a^3*d) + Sin[c + d*x]/(4*a^3*d) + Sin[3*(c + d*x)]/(6*a^3
*d) + Sin[5*(c + d*x)]/(20*a^3*d)
```

Maple [A]
time = 0.40, size = 90, normalized size = 1.00

method	result
risch	$\frac{i e^{-i(dx+c)}}{4a^3d}+\frac{i e^{-3i(dx+c)}}{6a^3d}+\frac{i e^{-5i(dx+c)}}{20a^3d}$
derivativedivides	$\frac{\frac{8}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}+\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{16}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}}{a^3d}$
default	$\frac{\frac{8}{5\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}+\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{2}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{4i}{\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{16}{3\left(-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}}{a^3d}$
norman	$\frac{\frac{6i\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{16\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3da}+\frac{164\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15ad}-\frac{16\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{2\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{14i}{15ad}-}{a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^3*(4/5/(-I+tan(1/2*d*x+1/2*c))^5+2*I/(-I+tan(1/2*d*x+1/2*c))^2+1/(-I+
tan(1/2*d*x+1/2*c))-2*I/(-I+tan(1/2*d*x+1/2*c))^4-8/3/(-I+tan(1/2*d*x+1/2*c
))^3)
```

Maxima [A]
time = 0.29, size = 69, normalized size = 0.77

$$\frac{3i\cos(5dx+5c)+10i\cos(3dx+3c)+15i\cos(dx+c)+3\sin(5dx+5c)+10\sin(3dx+3c)+15\sin(dx+c)}{60a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/60*(3*I*cos(5*d*x + 5*c) + 10*I*cos(3*d*x + 3*c) + 15*I*cos(d*x + c) + 3*
sin(5*d*x + 5*c) + 10*sin(3*d*x + 3*c) + 15*sin(d*x + c))/(a^3*d)
```

Fricas [A]
time = 2.96, size = 41, normalized size = 0.46

$$\frac{\left(15i\,e^{(4i\,dx+4i\,c)}+10i\,e^{(2i\,dx+2i\,c)}+3i\right)e^{(-5i\,dx-5i\,c)}}{60\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/60*(15*I*e^(4*I*d*x + 4*I*c) + 10*I*e^(2*I*d*x + 2*I*c) + 3*I)*e^(-5*I*d*
x - 5*I*c)/(a^3*d)
```

Sympy [A]
time = 0.18, size = 131, normalized size = 1.46

$$\begin{cases} \frac{(120ia^6d^2e^{8ic}e^{-idx}+80ia^6d^2e^{6ic}e^{-3idx}+24ia^6d^2e^{4ic}e^{-5idx})e^{-9ic}}{480a^9d^3} & \text{for } a^9d^3e^{9ic} \neq 0 \\ \frac{x(e^{4ic}+2e^{2ic}+1)e^{-5ic}}{4a^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise(((120*I*a**6*d**2*exp(8*I*c)*exp(-I*d*x) + 80*I*a**6*d**2*exp(6*I
*c)*exp(-3*I*d*x) + 24*I*a**6*d**2*exp(4*I*c)*exp(-5*I*d*x))*exp(-9*I*c)/(4
80*a**9*d**3), Ne(a**9*d**3*exp(9*I*c), 0)), (x*(exp(4*I*c) + 2*exp(2*I*c)
+ 1)*exp(-5*I*c)/(4*a**3), True))
```

Giac [A]
time = 0.45, size = 73, normalized size = 0.81

$$\frac{2\left(15\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-30i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-40\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+20i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+7\right)}{15a^3d\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2/15*(15*tan(1/2*d*x + 1/2*c)^4 - 30*I*tan(1/2*d*x + 1/2*c)^3 - 40*tan(1/2*
d*x + 1/2*c)^2 + 20*I*tan(1/2*d*x + 1/2*c) + 7)/(a^3*d*(tan(1/2*d*x + 1/2*c
) - I)^5)
```

Mupad [B]
time = 0.87, size = 133, normalized size = 1.48

$$\frac{2\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4 15\mathrm{i}+30\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2 40\mathrm{i}-20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)+7\mathrm{i}\right)}{15a^3d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5 1\mathrm{i}+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 10\mathrm{i}-10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)5\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3,x)
```

```
[Out] (2*(30*tan(c/2 + (d*x)/2)^3 - tan(c/2 + (d*x)/2)^2*40i - 20*tan(c/2 + (d*x)
/2) + tan(c/2 + (d*x)/2)^4*15i + 7i))/(15*a^3*d*(tan(c/2 + (d*x)/2)*5i - 10
*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*10i + 5*tan(c/2 + (d*x)/2)^4 +
 tan(c/2 + (d*x)/2)^5*1i + 1))
```

3.179 $\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=32

$$\frac{i\cot^2(c+dx)}{2a^3d(i+\cot(c+dx))^2}$$

```
[Out] 1/2*I*cot(d*x+c)^2/a^3/d/(I+cot(d*x+c))^2
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {3167, 37}

$$\frac{i\cot^2(c+dx)}{2a^3d(\cot(c+dx)+i)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((I/2)*Cot[c + d*x]^2)/(a^3*d*(I + Cot[c + d*x])^2)
```

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx &= -\frac{\text{Subst}\left(\int \frac{x}{(ia+ax)^3}\,dx, x, \cot(c+dx)\right)}{d}\\ &= \frac{i\cot^2(c+dx)}{2a^3d(i+\cot(c+dx))^2}\end{aligned}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 77 vs. $2(32) = 64$.
time = 0.06, size = 77, normalized size = 2.41

$$\frac{i\cos(2(c+dx))}{4a^3d}+\frac{i\cos(4(c+dx))}{8a^3d}+\frac{\sin(2(c+dx))}{4a^3d}+\frac{\sin(4(c+dx))}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((I/4)*Cos[2*(c + d*x)])/(a^3*d) + ((I/8)*Cos[4*(c + d*x)])/(a^3*d) + Sin[2
*(c + d*x)]/(4*a^3*d) + Sin[4*(c + d*x)]/(8*a^3*d)
```

Maple [A]
time = 0.32, size = 23, normalized size = 0.72

method	result
derivativedivides	$\frac{i}{2d\,a^3(i\tan(dx+c)+1)^2}$
default	$\frac{i}{2d\,a^3(i\tan(dx+c)+1)^2}$
risch	$\frac{i\mathrm{e}^{-2i(dx+c)}}{4a^3d}+\frac{i\mathrm{e}^{-4i(dx+c)}}{8a^3d}$
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{6\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{6\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{6i\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{6i\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{4i\left(\tan^4\left(\frac{dx}{2}+\cdots\right.\right.}{ad}}{a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I/d/a^3/(I*tan(d*x+c)+1)^2
```

Maxima [A]
time = 0.28, size = 51, normalized size = 1.59

$$\frac{i\,\cos\left(4\,dx+4\,c\right)+2i\,\cos\left(2\,dx+2\,c\right)+\sin\left(4\,dx+4\,c\right)+2\,\sin\left(2\,dx+2\,c\right)}{8\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(I*cos(4*d*x + 4*c) + 2*I*cos(2*d*x + 2*c) + sin(4*d*x + 4*c) + 2*sin(2
*d*x + 2*c))/(a^3*d)
```

Fricas [A]
time = 2.82, size = 30, normalized size = 0.94

$$\frac{\left(2i\,e^{(2i\,dx+2i\,c)}+i\right)e^{(-4i\,dx-4i\,c)}}{8\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*I*e^(2*I*d*x + 2*I*c) + I)*e^(-4*I*d*x - 4*I*c)/(a^3*d)
```

Sympy [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(26) = 52$.
time = 0.13, size = 95, normalized size = 2.97

$$\begin{cases} \frac{(8ia^3de^{4ic}e^{-2idx}+4ia^3de^{2ic}e^{-4idx})e^{-6ic}}{32a^6d^2} & \text{for } a^6d^2e^{6ic} \neq 0 \\ \frac{x(e^{2ic}+1)e^{-4ic}}{2a^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise(((8*I*a**3*d*exp(4*I*c)*exp(-2*I*d*x) + 4*I*a**3*d*exp(2*I*c)*exp
(-4*I*d*x))*exp(-6*I*c)/(32*a**6*d**2), Ne(a**6*d**2*exp(6*I*c), 0)), (x*(e
xp(2*I*c) + 1)*exp(-4*I*c)/(2*a**3), True))
```

Giac [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(26) = 52$.
time = 0.47, size = 57, normalized size = 1.78

$$-\frac{2\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^3d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -2*(tan(1/2*d*x + 1/2*c)^3 - I*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c
))/(a^3*d*(tan(1/2*d*x + 1/2*c) - I)^4)
```

Mupad [B]
time = 0.75, size = 100, normalized size = 3.12

$$-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,1\mathrm{i}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)}{a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,1\mathrm{i}+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,6\mathrm{i}-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3,x)
```

```
[Out] -(2*tan(c/2 + (d*x)/2)*(tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^2*1i - 1i))
/(a^3*d*(4*tan(c/2 + (d*x)/2)^3 - tan(c/2 + (d*x)/2)^2*6i - 4*tan(c/2 + (d*
x)/2) + tan(c/2 + (d*x)/2)^4*1i + 1i))
```

3.180 $\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=31

$$\frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

```
[Out] 1/3*I/d/(a*cos(d*x+c)+I*a*sin(d*x+c))^3
```

Rubi [A]
time = 0.01, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = $\{3150\}$

$$\frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-3),x]
```

```
[Out] (I/3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)
```

Rule 3150

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x
_Symbol] :> Simp[a*((a*Cos[c + d*x] + b*Sin[c + d*x])^n/(b*d*n)), x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 + b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

Mathematica [A]
time = 0.05, size = 31, normalized size = 1.00

$$\frac{i}{3d(a\cos(c+dx)+ia\sin(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^(-3),x]
```

```
[Out] (I/3)/(d*(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 56 vs. 2(27) = 54.
time = 0.28, size = 57, normalized size = 1.84

method	result	size
risch	$\frac{i e^{-3 i (d x+c)}}{3 a^{3} d}$	19
derivativedivides	$\frac{\frac{2}{-i+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)}-\frac{8}{3\left(-i+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{4 i}{\left(-i+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{2}}}{a^{3} d}$	57
default	$\frac{\frac{2}{-i+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)}-\frac{8}{3\left(-i+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{3}}+\frac{4 i}{\left(-i+\tan \left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{2}}}{a^{3} d}$	57
norman	$\frac{-\frac{4 i \left(\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{a d}+\frac{2 \tan \left(\frac{d x}{2}+\frac{c}{2}\right)}{a d}-\frac{20 \left(\tan^{3}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{3 d a}+\frac{2 \left(\tan^{5}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{a d}+\frac{2 i}{3 a d}+\frac{6 i \left(\tan^{4}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{a d}}{\left(1+\tan^{2}\left(\frac{d x}{2}+\frac{c}{2}\right)\right)^{3} a^{2}}$	125

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
[Out] 2/d/a^3*(1/(-I+tan(1/2*d*x+1/2*c))-4/3/(-I+tan(1/2*d*x+1/2*c))^3+2*I/(-I+ta
n(1/2*d*x+1/2*c))^2)
```

Maxima [A]
time = 0.30, size = 29, normalized size = 0.94

$$\frac{i \cos \left(3 \, d x + 3 \, c\right) + \sin \left(3 \, d x + 3 \, c\right)}{3 \, a^{3} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima")
[Out] 1/3*(I*cos(3*d*x + 3*c) + sin(3*d*x + 3*c))/(a^3*d)
```

Fricas [A]
time = 2.39, size = 17, normalized size = 0.55

$$\frac{i \, e^{\left(-3 i \, d x - 3 i \, c\right)}}{3 \, a^{3} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas")
[Out] 1/3*I*e^(-3*I*d*x - 3*I*c)/(a^3*d)
```

Sympy [A]
time = 0.08, size = 44, normalized size = 1.42

$$\begin{cases} \frac{i e^{-3 i c} e^{-3 i d x}}{3 a^{3} d} & \text{for}\: a^{3} d e^{3 i c} \neq 0 \\ \frac{x e^{-3 i c}}{a^{3}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Piecewise((I*exp(-3*I*c)*exp(-3*I*d*x)/(3*a**3*d), Ne(a**3*d*exp(3*I*c), 0)
), (x*exp(-3*I*c)/a**3, True))
```

Giac [A]
time = 0.42, size = 36, normalized size = 1.16

$$\frac{2\left(3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)}{3\,a^{3}d\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2/3*(3*tan(1/2*d*x + 1/2*c)^2 - 1)/(a^3*d*(tan(1/2*d*x + 1/2*c) - I)^3)
```

Mupad [B]
time = 0.63, size = 68, normalized size = 2.19

$$-\frac{2\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}\,3\mathrm{i}-\mathrm{i}\right)}{3\,a^{3}\,d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}\,1\mathrm{i}-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,3\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3,x)
```

```
[Out] -(2*(tan(c/2 + (d*x)/2)^2*3i - 1i))/(3*a^3*d*(tan(c/2 + (d*x)/2)*3i - 3*tan
(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*1i + 1))
```

3.181 $\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=61

$$-\frac{x}{a^3}+\frac{2}{a^3d(i+\cot(c+dx))}-\frac{i\log(\sin(c+dx))}{a^3d}+\frac{i\log(\tan(c+dx))}{a^3d}$$

```
[Out] -x/a^3+2/a^3/d/(I+cot(d*x+c))-I*ln(sin(d*x+c))/a^3/d+I*ln(tan(d*x+c))/a^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {3167, 862, 78}

$$\frac{2}{a^3d(\cot(c+dx)+i)}-\frac{i\log(\sin(c+dx))}{a^3d}+\frac{i\log(\tan(c+dx))}{a^3d}-\frac{x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] -(x/a^3) + 2/(a^3*d*(I + Cot[c + d*x])) - (I*Log[Sin[c + d*x]])/(a^3*d) + (
I*Log[Tan[c + d*x]])/(a^3*d)
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1+x^2}{x(ia+ax)^3}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{-\frac{i}{a}+\frac{x}{a}}{x(ia+ax)^2}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{i}{a^3x}+\frac{2}{a^3(i+x)^2}-\frac{i}{a^3(i+x)}\right)dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{x}{a^3}+\frac{2}{a^3d(i+\cot(c+dx))}-\frac{i\log(\sin(c+dx))}{a^3d}+\frac{i\log(\tan(c+d}{a^3d}$$

Mathematica [A]
time = 0.33, size = 91, normalized size = 1.49

$$\frac{i\sec^2(c+dx)(-i\cos(2(c+dx))+\sin(2(c+dx)))(-1-idx+\log(\cos(c+dx))+(i+dx+i\log(\cos(c+dx)))\tan(c+dx))}{a^3d(-i+\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] (I*Sec[c + d*x]^2*((-I)*Cos[2*(c + d*x)] + Sin[2*(c + d*x)])*(-1 - I*d*x +
Log[Cos[c + d*x]] + (I + d*x + I*Log[Cos[c + d*x]])*Tan[c + d*x]))/(a^3*d*(
-I + Tan[c + d*x])^3)
```

Maple [A]
time = 0.42, size = 35, normalized size = 0.57

method	result
derivativedivides	$\frac{i\ln(\tan(dx+c)-i)+\frac{2}{\tan(dx+c)-i}}{d\,a^3}$
default	$\frac{i\ln(\tan(dx+c)-i)+\frac{2}{\tan(dx+c)-i}}{d\,a^3}$
risch	$\frac{ie^{-2i(dx+c)}}{a^3d}-\frac{2x}{a^3}-\frac{2c}{a^3d}-\frac{i\ln\left(e^{2i(dx+c)}+1\right)}{a^3d}$
norman	$\frac{-\frac{x}{a}+\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{4\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{2x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{8i\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2a^2}+\frac{i\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{a^3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(I*ln(tan(d*x+c)-I)+2/(tan(d*x+c)-I))
```

Maxima [A]
time = 0.49, size = 99, normalized size = 1.62

$$-\frac{4\,dx+4\,c-2\,\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)+1\right)-2i\,\cos\left(2\,dx+2\,c\right)+i\,\log\left(\cos\left(2\,dx+2\,c\right)^{2}+\sin\left(2\,dx+2\,c\right)^{2}+2\,\cos\left(2\,dx+2\,c\right)+1\right)-2\,\sin\left(2\,dx+2\,c\right)}{2\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*(4*d*x + 4*c - 2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1) - 2*I
*cos(2*d*x + 2*c) + I*log(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1) - 2*sin(2*d*x + 2*c))/(a^3*d)
```

Fricas [A]
time = 3.50, size = 55, normalized size = 0.90

$$-\frac{\left(2\,dxe^{(2i\,dx+2i\,c)}+i\,e^{(2i\,dx+2i\,c)}\log\left(e^{(2i\,dx+2i\,c)}+1\right)-i\right)e^{(-2i\,dx-2i\,c)}}{a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(2*d*x*e^(2*I*d*x + 2*I*c) + I*e^(2*I*d*x + 2*I*c)*log(e^(2*I*d*x + 2*I*c)
 + 1) - I)*e^(-2*I*d*x - 2*I*c)/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sec\left(c+dx\right)}{-i\sin^{3}\left(c+dx\right)-3\sin^{2}\left(c+dx\right)\cos\left(c+dx\right)+3i\sin\left(c+dx\right)\cos^{2}\left(c+dx\right)+\cos^{3}\left(c+dx\right)}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)/(-I*sin(c + d*x)**3 - 3*sin(c + d*x)**2*cos(c + d*x)
+ 3*I*sin(c + d*x)*cos(c + d*x)**2 + cos(c + d*x)**3), x)/a**3
```

Giac [A]
time = 0.45, size = 100, normalized size = 1.64

$$-\frac{\frac{i\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^{3}}-\frac{2i\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)}{a^{3}}+\frac{i\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^{3}}+\frac{3i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+10\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3i}{a^{3}\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-i\right)^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(I*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 2*I*log(tan(1/2*d*x + 1/2*c) - I)/a
^3 + I*log(tan(1/2*d*x + 1/2*c) - 1)/a^3 + (3*I*tan(1/2*d*x + 1/2*c)^2 + 10
*tan(1/2*d*x + 1/2*c) - 3*I)/(a^3*(tan(1/2*d*x + 1/2*c) - I)^2))/d
```

Mupad [B]
time = 0.77, size = 101, normalized size = 1.66

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,4{}\mathrm{i}}{d\,\left(a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,1{}\mathrm{i}+2\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a^3\,1{}\mathrm{i}\right)}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)\,2{}\mathrm{i}}{a^3\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\,1{}\mathrm{i}}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3),x)
```

```
[Out] (log(tan(c/2 + (d*x)/2) - 1i)*2i)/(a^3*d) - (tan(c/2 + (d*x)/2)*4i)/(d*(a^3
*tan(c/2 + (d*x)/2)^2*1i - a^3*1i + 2*a^3*tan(c/2 + (d*x)/2))) - (log(tan(c
/2 + (d*x)/2)^2 - 1)*1i)/(a^3*d)
```

3.182 $\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=62

$$-\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{4i\cos(c+dx)}{a^3d}+\frac{i\sec(c+dx)}{a^3d}+\frac{4\sin(c+dx)}{a^3d}$$

```
[Out] -3*arctanh(sin(d*x+c))/a^3/d+4*I*cos(d*x+c)/a^3/d+I*sec(d*x+c)/a^3/d+4*sin(
d*x+c)/a^3/d
```

Rubi [A]
time = 0.11, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {3171, 3169, 2717, 2718, 2672, 327, 212, 2670, 14}

$$\frac{4\sin(c+dx)}{a^3d}+\frac{4i\cos(c+dx)}{a^3d}+\frac{i\sec(c+dx)}{a^3d}-\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] (-3*ArcTanh[Sin[c + d*x]])/(a^3*d) + ((4*I)*Cos[c + d*x])/(a^3*d) + (I*Sec[
c + d*x])/(a^3*d) + (4*Sin[c + d*x])/(a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2670

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3171

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = \frac{i\int \sec^2(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^3\,dx}{a^6}$$
$$= \frac{i\int \left(-ia^3\cos(c+dx)-3a^3\sin(c+dx)+3ia^3\sin(c+dx)\tan(c+\right.}{a^6}$$
$$= \frac{i\int \sin(c+dx)\tan^2(c+dx)\,dx}{a^3} - \frac{(3i)\int \sin(c+dx)\,dx}{a^3} + \frac{\int \cos(c}{}$$
$$= \frac{3i\cos(c+dx)}{a^3d} + \frac{\sin(c+dx)}{a^3d} - \frac{i\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cos(c+dx)\right)}{a^3d}$$
$$= \frac{3i\cos(c+dx)}{a^3d} + \frac{4\sin(c+dx)}{a^3d} - \frac{i\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)\,dx, x, \cos(c\right.}{a^3d}$$
$$= -\frac{3\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{4i\cos(c+dx)}{a^3d} + \frac{i\sec(c+dx)}{a^3d} + \frac{4\sin(}{}$$

Mathematica [A]
time = 0.37, size = 109, normalized size = 1.76

$$-\frac{i\sec^3(c+dx)(\cos(dx)+i\sin(dx))^3\left(6\tanh^{-1}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)(\cos(3c)+i\sin(3c))+(\cos(2c-dx)+i\sin(2c-dx))(-5i+\tan(c+dx))\right)}{a^3d(-i+\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((-I)*Sec[c + d*x]^3*(Cos[d*x] + I*Sin[d*x])^3*(6*ArcTanh[Sin[c] + Cos[c]*T
an[(d*x)/2]]*(Cos[3*c] + I*Sin[3*c]) + (Cos[2*c - d*x] + I*Sin[2*c - d*x])*
(-5*I + Tan[c + d*x])))/(a^3*d*(-I + Tan[c + d*x])^3)
```

Maple [A]
time = 0.41, size = 86, normalized size = 1.39

method	result	size
derivativedivides	$\frac{-\frac{i}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{2i}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2}-3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{8}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{a^3d}$	86
default	$\frac{-\frac{i}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{2i}{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2}-3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{8}{-i+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{a^3d}$	86
risch	$\frac{4i\mathrm{e}^{-i(dx+c)}}{a^3d}+\frac{2i\mathrm{e}^{i(dx+c)}}{d\,a^3\left(\mathrm{e}^{2i(dx+c)}+1\right)}+\frac{3\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{a^3d}-\frac{3\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{a^3d}$	93
norman	$\frac{-\frac{8\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{10i}{ad}+\frac{6i\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{a^3d}-\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{a^3d}$	142

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^3*(-1/2*I/(tan(1/2*d*x+1/2*c)-1)+3/2*ln(tan(1/2*d*x+1/2*c)-1)+1/2*I/(
tan(1/2*d*x+1/2*c)+1)-3/2*ln(tan(1/2*d*x+1/2*c)+1)+4/(-I+tan(1/2*d*x+1/2*c)
))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(58) = 116$.
time = 0.50, size = 319, normalized size = 5.15

$$\frac{6\left(\cos\left(3\,dx+3\,c\right)+\cos\left(dx+c\right)+i\,\sin\left(3\,dx+3\,c\right)+i\,\sin\left(dx+c\right)\right)\arctan\left(\cos\left(dx+c\right),\sin\left(dx+c\right)+1\right)+6\left(\cos\left(3\,dx+3\,c\right)+\cos\left(dx+c\right)+i\,\sin\left(3\,dx+3\,c\right)+i\,\sin\left(dx+c\right)\right)\arctan\left(\cos\left(dx+c\right),-\sin\left(dx+c\right)+1\right)+3\left(i\,\cos\left(3\,dx+3\,c\right)+i\,\cos\left(dx+c\right)-\sin\left(3\,dx+3\,c\right)-\sin\left(dx+c\right)\right)\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2+2\,\sin\left(dx+c\right)+1\right)+3\left(-i\,\cos\left(3\,dx+3\,c\right)-i\,\cos\left(dx+c\right)+\sin\left(3\,dx+3\,c\right)+\sin\left(dx+c\right)\right)\log\left(\cos\left(dx+c\right)^2+\sin\left(dx+c\right)^2-2\,\sin\left(dx+c\right)+1\right)+12\,\cos\left(2\,dx+2\,c\right)+12i\,\sin\left(2\,dx+2\,c\right)+8}{-2\left(i\,a^3\cos\left(3\,dx+3\,c\right)+i\,a^3\cos\left(dx+c\right)-a^3\sin\left(3\,dx+3\,c\right)-a^3\sin\left(dx+c\right)\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] (6*(cos(3*d*x + 3*c) + cos(d*x + c) + I*sin(3*d*x + 3*c) + I*sin(d*x + c))*
arctan2(cos(d*x + c), sin(d*x + c) + 1) + 6*(cos(3*d*x + 3*c) + cos(d*x + c
) + I*sin(3*d*x + 3*c) + I*sin(d*x + c))*arctan2(cos(d*x + c), -sin(d*x + c
) + 1) + 3*(I*cos(3*d*x + 3*c) + I*cos(d*x + c) - sin(3*d*x + 3*c) - sin(d*
x + c))*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) + 3*(-I*c
os(3*d*x + 3*c) - I*cos(d*x + c) + sin(3*d*x + 3*c) + sin(d*x + c))*log(cos
(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) + 12*cos(2*d*x + 2*c) +
12*I*sin(2*d*x + 2*c) + 8)/((-2*I*a^3*cos(3*d*x + 3*c) - 2*I*a^3*cos(d*x +
c) + 2*a^3*sin(3*d*x + 3*c) + 2*a^3*sin(d*x + c))*d)
```

Fricas [A]
time = 3.72, size = 112, normalized size = 1.81

$$-\frac{3\left(e^{(3i\,dx+3i\,c)}+e^{(i\,dx+i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-3\left(e^{(3i\,dx+3i\,c)}+e^{(i\,dx+i\,c)}\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-6i\,e^{(2i\,dx+2i\,c)}-4i}{a^3de^{(3i\,dx+3i\,c)}+a^3de^{(i\,dx+i\,c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] -(3*(e^(3*I*d*x + 3*I*c) + e^(I*d*x + I*c))*log(e^(I*d*x + I*c) + I) - 3*(e
^(3*I*d*x + 3*I*c) + e^(I*d*x + I*c))*log(e^(I*d*x + I*c) - I) - 6*I*e^(2*I
*d*x + 2*I*c) - 4*I)/(a^3*d*e^(3*I*d*x + 3*I*c) + a^3*d*e^(I*d*x + I*c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^2\left(c+dx\right)}{-i\sin^3\left(c+dx\right)-3\sin^2\left(c+dx\right)\cos\left(c+dx\right)+3i\sin\left(c+dx\right)\cos^2\left(c+dx\right)+\cos^3\left(c+dx\right)}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**2/(-I*sin(c + d*x)**3 - 3*sin(c + d*x)**2*cos(c + d*
x) + 3*I*sin(c + d*x)*cos(c + d*x)**2 + cos(c + d*x)**3), x)/a**3
```

Giac [A]
time = 0.47, size = 110, normalized size = 1.77

$$-\frac{\frac{3 \log\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+1\right)}{a^3}-\frac{3 \log\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-1\right)}{a^3}-\frac{2\left(4 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-i \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)-5\right)}{\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3-i \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^2-\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)+i\right)a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -(3*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 3*log(tan(1/2*d*x + 1/2*c) - 1)/a^3
 - 2*(4*tan(1/2*d*x + 1/2*c)^2 - I*tan(1/2*d*x + 1/2*c) - 5)/((tan(1/2*d*x
+ 1/2*c)^3 - I*tan(1/2*d*x + 1/2*c)^2 - tan(1/2*d*x + 1/2*c) + I)*a^3))/d
```

Mupad [B]
time = 0.99, size = 105, normalized size = 1.69

$$-\frac{6\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}-\frac{\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^3}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,8\mathrm{i}}{a^3}-\frac{10\mathrm{i}}{a^3}}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,1\mathrm{i}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1\mathrm{i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3),x)
```

```
[Out] - (6*atanh(tan(c/2 + (d*x)/2)))/(a^3*d) - ((tan(c/2 + (d*x)/2)^2*8i)/a^3 -
10i/a^3 + (2*tan(c/2 + (d*x)/2))/a^3)/(d*(tan(c/2 + (d*x)/2)*1i - tan(c/2 +
 (d*x)/2)^2 - tan(c/2 + (d*x)/2)^3*1i + 1))
```

3.183 $\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=75

$$\frac{4x}{a^3}+\frac{4i\log(\sin(c+dx))}{a^3d}-\frac{4i\log(\tan(c+dx))}{a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{i\tan^2(c+dx)}{2a^3d}$$

```
[Out] 4*x/a^3+4*I*ln(sin(d*x+c))/a^3/d-4*I*ln(tan(d*x+c))/a^3/d-3*tan(d*x+c)/a^3/
d+1/2*I*tan(d*x+c)^2/a^3/d
```

Rubi [A]
time = 0.06, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3167, 862, 90}

$$\frac{i\tan^2(c+dx)}{2a^3d}-\frac{3\tan(c+dx)}{a^3d}+\frac{4i\log(\sin(c+dx))}{a^3d}-\frac{4i\log(\tan(c+dx))}{a^3d}+\frac{4x}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] (4*x)/a^3 + ((4*I)*Log[Sin[c + d*x]])/(a^3*d) - ((4*I)*Log[Tan[c + d*x]])/(
a^3*d) - (3*Tan[c + d*x])/(a^3*d) + ((I/2)*Tan[c + d*x]^2)/(a^3*d)
```

Rule 90

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^3(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{(1+x^2)^2}{x^3(ia+ax)^3}\,dx, x, \cot(c+dx)\right)}{d}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{\left(-\frac{i}{a}+\frac{x}{a}\right)^2}{x^3(ia+ax)}\,dx, x, \cot(c+dx)\right)}{d}\\
&= -\frac{\operatorname{Subst}\left(\int \left(\frac{i}{a^3x^3}-\frac{3}{a^3x^2}-\frac{4i}{a^3x}+\frac{4i}{a^3(i+x)}\right)\,dx, x, \cot(c+dx)\right)}{d}\\
&= \frac{4x}{a^3}+\frac{4i\log(\sin(c+dx))}{a^3d}-\frac{4i\log(\tan(c+dx))}{a^3d}-\frac{3\tan(c+dx)}{a^3d}
\end{aligned}
$$

Mathematica [A]
time = 0.69, size = 110, normalized size = 1.47

$$
\frac{i\sec(c)\sec^2(c+dx)(\cos(c)(1-4idx+4\log(\cos(c+dx)))-i(2\cos(c+2dx)(dx+i\log(\cos(c+dx)))+2\cos(3c+2dx)(dx+i\log(\cos(c+dx)))-6\cos(c+dx)\sin(dx)))}{2a^3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((I/2)*Sec[c]*Sec[c + d*x]^2*(Cos[c]*(1 - (4*I)*d*x + 4*Log[Cos[c + d*x]])
- I*(2*Cos[c + 2*d*x]*(d*x + I*Log[Cos[c + d*x]]) + 2*Cos[3*c + 2*d*x]*(d*x
 + I*Log[Cos[c + d*x]]) - 6*Cos[c + d*x]*Sin[d*x])))/(a^3*d)
```

Maple [A]
time = 0.44, size = 41, normalized size = 0.55

method	result
derivativedivides	$\frac{-3\tan(dx+c)+\frac{i\left(\tan^2(dx+c)\right)}{2}-4i\ln(\tan(dx+c)-i)}{d\,a^3}$
default	$\frac{-3\tan(dx+c)+\frac{i\left(\tan^2(dx+c)\right)}{2}-4i\ln(\tan(dx+c)-i)}{d\,a^3}$
risch	$\frac{8x}{a^3}+\frac{8c}{a^3d}-\frac{2i\left(2\,e^{2i(dx+c)}+3\right)}{a^3d\left(e^{2i(dx+c)}+1\right)^2}+\frac{4i\ln\left(e^{2i(dx+c)}+1\right)}{a^3d}$
norman	$\frac{\frac{4x}{a}-\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{6\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}-\frac{8x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{4x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{2i\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{4i\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}{a^3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a^3*(-3*tan(d*x+c)+1/2*I*tan(d*x+c)^2-4*I*ln(tan(d*x+c)-I))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(67) = 134$.
time = 0.51, size = 301, normalized size = 4.01

$$-\frac{2\left(4i\,dx+2\left(-i\,\cos\left(4\,dx+4\,c\right)-2i\,\cos\left(2\,dx+2\,c\right)+\sin\left(4\,dx+4\,c\right)+2\,\sin\left(2\,dx+2\,c\right)-i\right)\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)+1\right)+4\left(i\,dx+i\,c\right)\cos\left(4\,dx+4\,c\right)+2\left(4i\,dx+4i\,c+1\right)\cos\left(2\,dx+2\,c\right)-\left(\cos\left(4\,dx+4\,c\right)+2\,\cos\left(2\,dx+2\,c\right)+i\,\sin\left(4\,dx+4\,c\right)+2i\,\sin\left(2\,dx+2\,c\right)+1\right)\log\left(\cos\left(2\,dx+2\,c\right)^2+\sin\left(2\,dx+2\,c\right)^2+2\,\cos\left(2\,dx+2\,c\right)+1\right)-4\left(dx+c\right)\sin\left(4\,dx+4\,c\right)-2\left(4\,dx+4\,c-i\right)\sin\left(2\,dx+2\,c\right)+4i\,c+3\right)}{\left(-i\,a^3\cos\left(4\,dx+4\,c\right)-2i\,a^3\cos\left(2\,dx+2\,c\right)+a^3\sin\left(4\,dx+4\,c\right)+2\,a^3\sin\left(2\,dx+2\,c\right)-i\,a^3\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] -2*(4*I*d*x + 2*(-I*cos(4*d*x + 4*c) - 2*I*cos(2*d*x + 2*c) + sin(4*d*x + 4
*c) + 2*sin(2*d*x + 2*c) - I)*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1) + 4*(I*d*x + I*c)*cos(4*d*x + 4*c) + 2*(4*I*d*x + 4*I*c + 1)*cos(2*d*x +
 2*c) - (cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + I*sin(4*d*x + 4*c) + 2*I*s
in(2*d*x + 2*c) + 1)*log(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1) - 4*(d*x + c)*sin(4*d*x + 4*c) - 2*(4*d*x + 4*c - I)*sin(2*
d*x + 2*c) + 4*I*c + 3)/((-I*a^3*cos(4*d*x + 4*c) - 2*I*a^3*cos(2*d*x + 2*c
) + a^3*sin(4*d*x + 4*c) + 2*a^3*sin(2*d*x + 2*c) - I*a^3)*d)
```

Fricas [A]
time = 2.71, size = 113, normalized size = 1.51

$$\frac{2\left(4\,dxe^{(4i\,dx+4i\,c)}+4\,dx+2\,(4\,dx-i)e^{(2i\,dx+2i\,c)}-2\left(-i\,e^{(4i\,dx+4i\,c)}-2i\,e^{(2i\,dx+2i\,c)}-i\right)\log\left(e^{(2i\,dx+2i\,c)}+1\right)-3i\right)}{a^3de^{(4i\,dx+4i\,c)}+2\,a^3de^{(2i\,dx+2i\,c)}+a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 2*(4*d*x*e^(4*I*d*x + 4*I*c) + 4*d*x + 2*(4*d*x - I)*e^(2*I*d*x + 2*I*c) -
2*(-I*e^(4*I*d*x + 4*I*c) - 2*I*e^(2*I*d*x + 2*I*c) - I)*log(e^(2*I*d*x + 2
*I*c) + 1) - 3*I)/(a^3*d*e^(4*I*d*x + 4*I*c) + 2*a^3*d*e^(2*I*d*x + 2*I*c)
+ a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^3(c+dx)}{-i\sin^3(c+dx)-3\sin^2(c+dx)\cos(c+dx)+3i\sin(c+dx)\cos^2(c+dx)+\cos^3(c+dx)}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**3/(-I*sin(c + d*x)**3 - 3*sin(c + d*x)**2*cos(c + d*
x) + 3*I*sin(c + d*x)*cos(c + d*x)**2 + cos(c + d*x)**3), x)/a**3
```

Giac [A]
time = 0.46, size = 128, normalized size = 1.71

$$\frac{2\left(\frac{2i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3}-\frac{4i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-i\right)}{a^3}+\frac{2i\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^3}+\frac{-3i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+7i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3i}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2*(2*I*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 4*I*log(tan(1/2*d*x + 1/2*c) - I
)/a^3 + 2*I*log(tan(1/2*d*x + 1/2*c) - 1)/a^3 + (-3*I*tan(1/2*d*x + 1/2*c)^
4 + 3*tan(1/2*d*x + 1/2*c)^3 + 7*I*tan(1/2*d*x + 1/2*c)^2 - 3*tan(1/2*d*x +
 1/2*c) - 3*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3))/d
```

Mupad [B]
time = 0.92, size = 104, normalized size = 1.39

$$-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\mathrm{i}\right)\,8\mathrm{i}-\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)\,4\mathrm{i}}{a^3\,d}+\frac{6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,2\mathrm{i}-6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*2i - 6*tan(c/2 + (d*x)/2) + 6*tan(c/2 + (d*x)/2)^3)/(
a^3*d*(tan(c/2 + (d*x)/2)^2 - 1)^2) - (log(tan(c/2 + (d*x)/2) - 1i)*8i - lo
g(tan(c/2 + (d*x)/2)^2 - 1)*4i)/(a^3*d)
```

3.184 $\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=76

$$\frac{5\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{4i\sec(c+dx)}{a^3d}+\frac{i\sec^3(c+dx)}{3a^3d}-\frac{3\sec(c+dx)\tan(c+dx)}{2a^3d}$$

```
[Out] 5/2*arctanh(sin(d*x+c))/a^3/d-4*I*sec(d*x+c)/a^3/d+1/3*I*sec(d*x+c)^3/a^3/d
-3/2*sec(d*x+c)*tan(d*x+c)/a^3/d
```

Rubi [A]
time = 0.12, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3171, 3169, 3855, 2686, 8, 2691}

$$\frac{i\sec^3(c+dx)}{3a^3d}-\frac{4i\sec(c+dx)}{a^3d}+\frac{5\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{3\tan(c+dx)\sec(c+dx)}{2a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] (5*ArcTanh[Sin[c + d*x]])/(2*a^3*d) - ((4*I)*Sec[c + d*x])/(a^3*d) + ((I/3)
*Sec[c + d*x]^3)/(a^3*d) - (3*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
```

```
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3171

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx &= \frac{i\int \sec^4(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^3\,dx}{a^6}\\ &= \frac{i\int \left(-ia^3\sec(c+dx)-3a^3\sec(c+dx)\tan(c+dx)+3ia^3\sec(c+\right.}{a^6}\\ &= \frac{i\int \sec(c+dx)\tan^3(c+dx)\,dx}{a^3}-\frac{(3i)\int \sec(c+dx)\tan(c+dx)\,d}{a^3}\\ &= \frac{\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{3\sec(c+dx)\tan(c+dx)}{2a^3d}+\frac{3\int \sec(c+dx}{2a^3}\\ &= \frac{5\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{4i\sec(c+dx)}{a^3d}+\frac{i\sec^3(c+dx)}{3a^3d}-\frac{3\sec(c}{}\end{aligned}$$

Mathematica [A]
time = 0.52, size = 64, normalized size = 0.84

$$\frac{i\left(-60i\tanh^{-1}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)+\sec^3(c+dx)(-20-24\cos(2(c+dx))+9i\sin(2(c+dx)))\right)}{12a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
[Out] ((I/12)*((-60*I)*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]] + Sec[c + d*x]^3*(-2
0 - 24*Cos[2*(c + d*x)] + (9*I)*Sin[2*(c + d*x)])))/(a^3*d)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 137 vs. 2(68) = 136.
time = 0.52, size = 138, normalized size = 1.82

method	result
risch	$-\frac{i\left(15\,\mathrm{e}^{5i(dx+c)}+40\,\mathrm{e}^{3i(dx+c)}+33\,\mathrm{e}^{i(dx+c)}\right)}{3d\,a^{3}\left(\mathrm{e}^{2i(dx+c)}+1\right)^{3}}-\frac{5\ln\left(\mathrm{e}^{i(dx+c)}-i\right)}{2a^{3}d}+\frac{5\ln\left(\mathrm{e}^{i(dx+c)}+i\right)}{2a^{3}d}$
derivativedivides	$\frac{-\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(-\frac{3}{4}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(-\frac{3}{4}+\frac{7i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{2\left(\frac{3}{4}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+}{a^{3}d}$
default	$\frac{-\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{2\left(-\frac{3}{4}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{2\left(-\frac{3}{4}+\frac{7i}{4}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}-\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2}+\frac{i}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}+\frac{2\left(\frac{3}{4}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{2}}+}{a^{3}d}$
norman	$\frac{-\frac{16i\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{3\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{22i}{3ad}+\frac{6i\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{a^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{3}}-\frac{5\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2a^{3}d}+\frac{5\ln\left(\tan\left(\frac{d}{}\right.\right.}{2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^3*(-1/6*I/(tan(1/2*d*x+1/2*c)-1)^3-(3/4+1/4*I)/(tan(1/2*d*x+1/2*c)-1)
^2+(-3/4+7/4*I)/(tan(1/2*d*x+1/2*c)-1)-5/4*ln(tan(1/2*d*x+1/2*c)-1)+1/6*I/(
tan(1/2*d*x+1/2*c)+1)^3+(3/4-1/4*I)/(tan(1/2*d*x+1/2*c)+1)^2-(3/4+7/4*I)/(t
an(1/2*d*x+1/2*c)+1)+5/4*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(66) = 132$.
time = 0.29, size = 215, normalized size = 2.83

$$\frac{\frac{4\left(-\frac{9i\sin(dx+c)}{\cos(dx+c)+1}-\frac{48\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{18\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{9i\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+22\right)}{6i\,a^{3}-\frac{18i\,a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{18i\,a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{6i\,a^{3}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}}+\frac{5\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^{3}}-\frac{5\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^{3}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/2*(4*(-9*I*sin(d*x + c)/(cos(d*x + c) + 1) - 48*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 18*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 9*I*sin(d*x + c)^5/(c
os(d*x + c) + 1)^5 + 22)/(6*I*a^3 - 18*I*a^3*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 18*I*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 6*I*a^3*sin(d*x + c)
^6/(cos(d*x + c) + 1)^6) + 5*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 -
 5*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(66) = 132$.
time = 2.19, size = 182, normalized size = 2.39

$$\frac{15\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-15\left(e^{(6i\,dx+6i\,c)}+3\,e^{(4i\,dx+4i\,c)}+3\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-30i\,e^{(5i\,dx+5i\,c)}-80i\,e^{(3i\,dx+3i\,c)}-66i\,e^{(i\,dx+i\,c)}}{6\left(a^{3}de^{(6i\,dx+6i\,c)}+3\,a^{3}de^{(4i\,dx+4i\,c)}+3\,a^{3}de^{(2i\,dx+2i\,c)}+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/6*(15*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x + 4*I*c) + 3*e^(2*I*d*x + 2*I*c
) + 1)*log(e^(I*d*x + I*c) + I) - 15*(e^(6*I*d*x + 6*I*c) + 3*e^(4*I*d*x +
4*I*c) + 3*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x + I*c) - I) - 30*I*e^(5*I*
d*x + 5*I*c) - 80*I*e^(3*I*d*x + 3*I*c) - 66*I*e^(I*d*x + I*c))/(a^3*d*e^(6
*I*d*x + 6*I*c) + 3*a^3*d*e^(4*I*d*x + 4*I*c) + 3*a^3*d*e^(2*I*d*x + 2*I*c)
 + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^4(c+dx)}{-i\sin^3(c+dx)-3\sin^2(c+dx)\cos(c+dx)+3i\sin(c+dx)\cos^2(c+dx)+\cos^3(c+dx)}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**4/(-I*sin(c + d*x)**3 - 3*sin(c + d*x)**2*cos(c + d*
x) + 3*I*sin(c + d*x)*cos(c + d*x)**2 + cos(c + d*x)**3), x)/a**3
```

Giac [A]
time = 0.47, size = 112, normalized size = 1.47

$$\frac{\frac{15\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)}{a^3}-\frac{15\log\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right)}{a^3}-\frac{2\left(9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-18i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+48i\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-9\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-22i\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3a^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(15*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 15*log(tan(1/2*d*x + 1/2*c) - 1
)/a^3 - 2*(9*tan(1/2*d*x + 1/2*c)^5 - 18*I*tan(1/2*d*x + 1/2*c)^4 + 48*I*ta
n(1/2*d*x + 1/2*c)^2 - 9*tan(1/2*d*x + 1/2*c) - 22*I)/((tan(1/2*d*x + 1/2*c
)^2 - 1)^3*a^3))/d
```

Mupad [B]
time = 2.72, size = 135, normalized size = 1.78

$$\frac{5\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}+\frac{\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^3}-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{a^3}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,16\mathrm{i}}{a^3}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,6\mathrm{i}}{a^3}+\frac{22\mathrm{i}}{3\,a^3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3),x)
```

```
[Out] (5*atanh(tan(c/2 + (d*x)/2)))/(a^3*d) + ((tan(c/2 + (d*x)/2)^4*6i)/a^3 - (t
an(c/2 + (d*x)/2)^2*16i)/a^3 - (3*tan(c/2 + (d*x)/2)^5)/a^3 + 22i/(3*a^3) +
 (3*tan(c/2 + (d*x)/2))/a^3)/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)
/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.185 $\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=34

$$\frac{i(i-\cot(c+dx))^4\tan^4(c+dx)}{4a^3d}$$

[Out] 1/4*I*(I-cot(d*x+c))^4*tan(d*x+c)^4/a^3/d

Rubi [A]
time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3167, 862, 37}

$$\frac{i\tan^4(c+dx)(-\cot(c+dx)+i)^4}{4a^3d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]

[Out] ((I/4)*(I - Cot[c + d*x])^4*Tan[c + d*x]^4)/(a^3*d)

Rule 37

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[m + n + 2, 0] && NeQ[m, -
1]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 3167

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[x^m*((b +
 a*x)^n/(1 + x^2)^((m + n + 2)/2)), x], x, Cot[c + d*x]], x] /; FreeQ[{a, b
, c, d}, x] && IntegerQ[n] && IntegerQ[(m + n)/2] && NeQ[n, -1] &&  !(GtQ[n
, 0] && GtQ[m, 1])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx = -\frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^5(ia+ax)^3}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{\left(-\frac{i}{a}+\frac{x}{a}\right)^3}{x^5}\,dx, x, \cot(c+dx)\right)}{d}$$
$$= \frac{i(i-\cot(c+dx))^4\tan^4(c+dx)}{4a^3d}$$

Mathematica [**B**] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 90 vs. $2(34) = 68$.
time = 0.50, size = 90, normalized size = 2.65

$$-\frac{i\sec(c)\sec^4(c+dx)(3\cos(c)+2\cos(c+2dx)+2\cos(3c+2dx)-3i\sin(c)+2i\sin(c+2dx)-2i\sin(3c+2dx)+i\sin(3c+4dx))}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((-1/4*I)*Sec[c]*Sec[c + d*x]^4*(3*Cos[c] + 2*Cos[c + 2*d*x] + 2*Cos[3*c +
2*d*x] - (3*I)*Sin[c] + (2*I)*Sin[c + 2*d*x] - (2*I)*Sin[3*c + 2*d*x] + I*S
in[3*c + 4*d*x]))/(a^3*d)
```

Maple [**A**]
time = 0.47, size = 21, normalized size = 0.62

method	result
derivativedivides	$\frac{i(\tan(dx+c)+i)^4}{4d\,a^3}$
default	$\frac{i(\tan(dx+c)+i)^4}{4d\,a^3}$
risch	$\frac{4i}{d\,a^3\left(e^{2i(dx+c)}+1\right)^4}$
norman	$\frac{\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{14\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}+\frac{14\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{16i\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{6i\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{6i\left(\tan^6\left(\ldots\right.\right.}{ad}}{a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*I/d/a^3*(tan(d*x+c)+I)^4
```

Maxima [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(26) = 52$.

time = 0.29, size = 240, normalized size = 7.06

$$\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1} - \frac{3i\sin(dx+c)^2}{(\cos(dx+c)+1)^2} - \frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3} + \frac{8i\sin(dx+c)^4}{(\cos(dx+c)+1)^4} + \frac{7\sin(dx+c)^5}{(\cos(dx+c)+1)^5} - \frac{3i\sin(dx+c)^6}{(\cos(dx+c)+1)^6} - \frac{\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{\left(a^3 - \frac{4a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2} + \frac{6a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4} - \frac{4a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6} + \frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*I*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 - 7*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 8*I*sin(d*x + c)^4/(cos(d*x + c
) + 1)^4 + 7*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*I*sin(d*x + c)^6/(cos(
d*x + c) + 1)^6 - sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/((a^3 - 4*a^3*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 -
 4*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a^3*sin(d*x + c)^8/(cos(d*x +
c) + 1)^8)*d)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(26) = 52.
time = 3.53, size = 69, normalized size = 2.03

$$\frac{4i}{a^3de^{(8i\,dx+8i\,c)} + 4\,a^3de^{(6i\,dx+6i\,c)} + 6\,a^3de^{(4i\,dx+4i\,c)} + 4\,a^3de^{(2i\,dx+2i\,c)} + a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 4*I/(a^3*d*e^(8*I*d*x + 8*I*c) + 4*a^3*d*e^(6*I*d*x + 6*I*c) + 6*a^3*d*e^(4
*I*d*x + 4*I*c) + 4*a^3*d*e^(2*I*d*x + 2*I*c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^5(c+dx)}{-i\sin^3(c+dx) - 3\sin^2(c+dx)\cos(c+dx) + 3i\sin(c+dx)\cos^2(c+dx) + \cos^3(c+dx)}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**5/(-I*sin(c + d*x)**3 - 3*sin(c + d*x)**2*cos(c + d*
x) + 3*I*sin(c + d*x)*cos(c + d*x)**2 + cos(c + d*x)**3), x)/a**3
```

Giac [A]
time = 0.47, size = 47, normalized size = 1.38

$$-\frac{-i\,\tan(dx+c)^4 + 4\,\tan(dx+c)^3 + 6i\,\tan(dx+c)^2 - 4\,\tan(dx+c)}{4\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/4*(-I*tan(d*x + c)^4 + 4*tan(d*x + c)^3 + 6*I*tan(d*x + c)^2 - 4*tan(d*x
 + c))/(a^3*d)
```

Mupad [B]
time = 0.88, size = 55, normalized size = 1.62

$$\frac{\sin\left(c+d\,x\right)^2\,1\mathrm{i}-\frac{\sin\left(2\,c+2\,d\,x\right)^2\,7\mathrm{i}}{4}+\sin\left(4\,c+4\,d\,x\right)}{4\,a^3\,d\,{\left(\sin\left(c+d\,x\right)^2-1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3),x)
```

```
[Out] (sin(4*c + 4*d*x) - (sin(2*c + 2*d*x)^2*7i)/4 + sin(c + d*x)^2*1i)/(4*a^3*d
*(sin(c + d*x)^2 - 1)^2)
```

3.186 $\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx$

Optimal. Leaf size=104

$$\frac{7\tanh^{-1}(\sin(c+dx))}{8a^3d}-\frac{4i\sec^3(c+dx)}{3a^3d}+\frac{i\sec^5(c+dx)}{5a^3d}+\frac{7\sec(c+dx)\tan(c+dx)}{8a^3d}-\frac{3\sec^3(c+dx)\tan(c+}{4a^3d}$$

```
[Out] 7/8*arctanh(sin(d*x+c))/a^3/d-4/3*I*sec(d*x+c)^3/a^3/d+1/5*I*sec(d*x+c)^5/a
^3/d+7/8*sec(d*x+c)*tan(d*x+c)/a^3/d-3/4*sec(d*x+c)^3*tan(d*x+c)/a^3/d
```

Rubi [A]

time = 0.17, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.258, Rules used = {3171, 3169, 3853, 3855, 2686, 30, 2691, 14}

$$\frac{i\sec^5(c+dx)}{5a^3d}-\frac{4i\sec^3(c+dx)}{3a^3d}+\frac{7\tanh^{-1}(\sin(c+dx))}{8a^3d}-\frac{3\tan(c+dx)\sec^3(c+dx)}{4a^3d}+\frac{7\tan(c+dx)\sec(c+dx)}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] (7*ArcTanh[Sin[c + d*x]])/(8*a^3*d) - (((4*I)/3)*Sec[c + d*x]^3)/(a^3*d) +
((I/5)*Sec[c + d*x]^5)/(a^3*d) + (7*Sec[c + d*x]*Tan[c + d*x])/(8*a^3*d) -
(3*Sec[c + d*x]^3*Tan[c + d*x])/(4*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2691

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
```

```
 + n - 1))), x] - Dist[b^2*((n - 1)/(m + n - 1)), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3169

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[ExpandTrig[cos[c + d*x]^m*(a
*cos[c + d*x] + b*sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d}, x] && Inte
gerQ[m] && IGtQ[n, 0]
```

Rule 3171

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[a^n*b^n, Int[Cos[c + d*x]^m/
(b*Cos[c + d*x] + a*Sin[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, m}, x] &&
 EqQ[a^2 + b^2, 0] && ILtQ[n, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^6(c+dx)}{(a\cos(c+dx)+ia\sin(c+dx))^3}\,dx &= \frac{i\int \sec^6(c+dx)(ia\cos(c+dx)+a\sin(c+dx))^3\,dx}{a^6}\\
&= \frac{i\int \left(-ia^3\sec^3(c+dx)-3a^3\sec^3(c+dx)\tan(c+dx)+3ia^3\sec^3(c\right.}{a^6}\\
&= \frac{i\int \sec^3(c+dx)\tan^3(c+dx)\,dx}{a^3}-\frac{(3i)\int \sec^3(c+dx)\tan(c+dx)\,c}{a^3}\\
&= \frac{\sec(c+dx)\tan(c+dx)}{2a^3d}-\frac{3\sec^3(c+dx)\tan(c+dx)}{4a^3d}+\frac{\int \sec(c+c}{2a^3}\\
&= \frac{\tanh^{-1}(\sin(c+dx))}{2a^3d}-\frac{i\sec^3(c+dx)}{a^3d}+\frac{7\sec(c+dx)\tan(c+dx)}{8a^3d}\\
&= \frac{7\tanh^{-1}(\sin(c+dx))}{8a^3d}-\frac{4i\sec^3(c+dx)}{3a^3d}+\frac{i\sec^5(c+dx)}{5a^3d}+\frac{7\sec(c}{}
\end{aligned}$$

Mathematica [A]
time = 0.47, size = 115, normalized size = 1.11

$$\frac{i\sec^8(c+dx)(-i\cos(3(c+dx))+\sin(3(c+dx)))\left(448+1680i\tanh^{-1}\left(\sin(c)+\cos(c)\tan\left(\frac{dx}{2}\right)\right)\cos^5(c+dx)+640\cos(2(c+dx))-150i\sin(2(c+dx))+105i\sin(4(c+dx))\right)}{960a^3d(-i+\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a*Cos[c + d*x] + I*a*Sin[c + d*x])^3,x]
```

```
[Out] ((I/960)*Sec[c + d*x]^8*((-I)*Cos[3*(c + d*x)] + Sin[3*(c + d*x)])*(448 + (
1680*I)*ArcTanh[Sin[c] + Cos[c]*Tan[(d*x)/2]]*Cos[c + d*x]^5 + 640*Cos[2*(c
 + d*x)] - (150*I)*Sin[2*(c + d*x)] + (105*I)*Sin[4*(c + d*x)]))/(a^3*d*(-I
 + Tan[c + d*x])^3)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 205 vs. 2(92) = 184.
time = 0.52, size = 206, normalized size = 1.98

method	result
risch	$-\frac{i\left(105\,e^{9i(dx+c)}+490\,e^{7i(dx+c)}+896\,e^{5i(dx+c)}+790\,e^{3i(dx+c)}-105\,e^{i(dx+c)}\right)}{60d\,a^3\left(e^{2i(dx+c)}+1\right)^5}-\frac{7\ln\left(e^{i(dx+c)}-i\right)}{8a^3d}+\frac{7\ln\left(e^{i(dx+c)}+i\right.}{8a^3d}$
derivativedivides	$\frac{-\frac{i}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}+\frac{2\left(\frac{1}{16}+\frac{13i}{16}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(-\frac{3}{8}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{2\left(-\frac{5}{16}+\frac{11i}{16}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{2\left(-\frac{3}{4}+\frac{7i}{24}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}}{a^3}$
default	$\frac{-\frac{i}{5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}+\frac{2\left(\frac{1}{16}+\frac{13i}{16}\right)}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{2\left(-\frac{3}{8}-\frac{i}{4}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{2\left(-\frac{5}{16}+\frac{11i}{16}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{2\left(-\frac{3}{4}+\frac{7i}{24}\right)}{\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{7\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{8}}{a^3}$
norman	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{13\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2da}-\frac{13\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad}+\frac{\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{34i}{15ad}-\frac{16i\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{6i\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-16i}{a^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 2/d/a^3*(-1/10*I/(tan(1/2*d*x+1/2*c)-1)^5+(1/16+13/16*I)/(tan(1/2*d*x+1/2*c
)-1)-(3/8+1/4*I)/(tan(1/2*d*x+1/2*c)-1)^4+(-5/16+11/16*I)/(tan(1/2*d*x+1/2*
c)-1)^2+(-3/4+7/24*I)/(tan(1/2*d*x+1/2*c)-1)^3-7/16*ln(tan(1/2*d*x+1/2*c)-1
)+1/10*I/(tan(1/2*d*x+1/2*c)+1)^5+(5/16+11/16*I)/(tan(1/2*d*x+1/2*c)+1)^2+(
3/8-1/4*I)/(tan(1/2*d*x+1/2*c)+1)^4+(1/16-13/16*I)/(tan(1/2*d*x+1/2*c)+1)-(
3/4+7/24*I)/(tan(1/2*d*x+1/2*c)+1)^3+7/16*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(90) = 180.
time = 0.30, size = 341, normalized size = 3.28

$$\frac{\frac{16\left(-\frac{15i\sin(dx+c)}{\cos(dx+c)+1}+\frac{320\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{390i\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{400\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{960\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{390i\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{360\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{15i\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-136\right)}{-120i\,a^3+\frac{600i\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{1200i\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1200i\,a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}-\frac{600i\,a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{120i\,a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}}+\frac{7\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^3}-\frac{7\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="maxima
")
```

```
[Out] 1/8*(16*(-15*I*sin(d*x + c)/(cos(d*x + c) + 1) + 320*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + 390*I*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 - 400*sin(d*x + c
)^4/(cos(d*x + c) + 1)^4 + 960*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 390*I*
sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 360*sin(d*x + c)^8/(cos(d*x + c) + 1)
^8 + 15*I*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 136)/(-120*I*a^3 + 600*I*a^
3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1200*I*a^3*sin(d*x + c)^4/(cos(d*x
+ c) + 1)^4 + 1200*I*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 - 600*I*a^3*si
n(d*x + c)^8/(cos(d*x + c) + 1)^8 + 120*I*a^3*sin(d*x + c)^10/(cos(d*x + c)
 + 1)^10) + 7*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 - 7*log(sin(d*x
+ c)/(cos(d*x + c) + 1) - 1)/a^3)/d
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(90) = 180$.
time = 2.68, size = 278, normalized size = 2.67

$$\frac{105\left(e^{(10i\,dx+10i\,c)}+5\,e^{(8i\,dx+8i\,c)}+10\,e^{(6i\,dx+6i\,c)}+10\,e^{(4i\,dx+4i\,c)}+5\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}+i\right)-105\left(e^{(10i\,dx+10i\,c)}+5\,e^{(8i\,dx+8i\,c)}+10\,e^{(6i\,dx+6i\,c)}+10\,e^{(4i\,dx+4i\,c)}+5\,e^{(2i\,dx+2i\,c)}+1\right)\log\left(e^{(i\,dx+i\,c)}-i\right)-210i\,e^{(9i\,dx+9i\,c)}-980i\,e^{(7i\,dx+7i\,c)}-1792i\,e^{(5i\,dx+5i\,c)}-1580i\,e^{(3i\,dx+3i\,c)}+210i\,e^{(i\,dx+i\,c)}}{120\left(a^3de^{(10i\,dx+10i\,c)}+5\,a^3de^{(8i\,dx+8i\,c)}+10\,a^3de^{(6i\,dx+6i\,c)}+10\,a^3de^{(4i\,dx+4i\,c)}+5\,a^3de^{(2i\,dx+2i\,c)}+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="fricas
")
```

```
[Out] 1/120*(105*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*I*d*x +
 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^(I*d*x
+ I*c) + I) - 105*(e^(10*I*d*x + 10*I*c) + 5*e^(8*I*d*x + 8*I*c) + 10*e^(6*
I*d*x + 6*I*c) + 10*e^(4*I*d*x + 4*I*c) + 5*e^(2*I*d*x + 2*I*c) + 1)*log(e^
(I*d*x + I*c) - I) - 210*I*e^(9*I*d*x + 9*I*c) - 980*I*e^(7*I*d*x + 7*I*c)
- 1792*I*e^(5*I*d*x + 5*I*c) - 1580*I*e^(3*I*d*x + 3*I*c) + 210*I*e^(I*d*x
+ I*c))/(a^3*d*e^(10*I*d*x + 10*I*c) + 5*a^3*d*e^(8*I*d*x + 8*I*c) + 10*a^3
*d*e^(6*I*d*x + 6*I*c) + 10*a^3*d*e^(4*I*d*x + 4*I*c) + 5*a^3*d*e^(2*I*d*x
+ 2*I*c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^6(c+dx)}{-i\sin^3(c+dx)-3\sin^2(c+dx)\cos(c+dx)+3i\sin(c+dx)\cos^2(c+dx)+\cos^3(c+dx)}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a*cos(d*x+c)+I*a*sin(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**6/(-I*sin(c + d*x)**3 - 3*sin(c + d*x)**2*cos(c + d*
x) + 3*I*sin(c + d*x)*cos(c + d*x)**2 + cos(c + d*x)**3), x)/a**3
```

Giac [A]
time = 0.46, size = 164, normalized size = 1.58

$$\frac{\frac{105\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)}{a^3}-\frac{105\,\log\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right)}{a^3}+\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+360i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^8-390\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-960i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+400i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+390\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-320i\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+136i\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^5a^3}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a*cos(d*x+c)+I*a*sin(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/120*(105*log(tan(1/2*d*x + 1/2*c) + 1)/a^3 - 105*log(tan(1/2*d*x + 1/2*c)
 - 1)/a^3 + 2*(15*tan(1/2*d*x + 1/2*c)^9 + 360*I*tan(1/2*d*x + 1/2*c)^8 - 3
90*tan(1/2*d*x + 1/2*c)^7 - 960*I*tan(1/2*d*x + 1/2*c)^6 + 400*I*tan(1/2*d*
x + 1/2*c)^4 + 390*tan(1/2*d*x + 1/2*c)^3 - 320*I*tan(1/2*d*x + 1/2*c)^2 -
15*tan(1/2*d*x + 1/2*c) + 136*I)/((tan(1/2*d*x + 1/2*c)^2 - 1)^5*a^3))/d
```

Mupad [B]
time = 3.29, size = 150, normalized size = 1.44

$$\frac{7\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{4\,a^3\,d}+\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,6{}\mathrm{i}-\frac{13\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,16{}\mathrm{i}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,20{}\mathrm{i}}{3}+\frac{13\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,16{}\mathrm{i}}{3}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}+\frac{34}{15}{}\mathrm{i}}{a^3\,d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a*cos(c + d*x) + a*sin(c + d*x)*1i)^3),x)
```

```
[Out] (7*atanh(tan(c/2 + (d*x)/2)))/(4*a^3*d) + ((13*tan(c/2 + (d*x)/2)^3)/2 - (t
an(c/2 + (d*x)/2)^2*16i)/3 - tan(c/2 + (d*x)/2)/4 + (tan(c/2 + (d*x)/2)^4*2
0i)/3 - tan(c/2 + (d*x)/2)^6*16i - (13*tan(c/2 + (d*x)/2)^7)/2 + tan(c/2 +
(d*x)/2)^8*6i + tan(c/2 + (d*x)/2)^9/4 + 34i/15)/(a^3*d*(tan(c/2 + (d*x)/2)
^2 - 1)^5)
```

3.187 $\int \cos^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx$

Optimal. Leaf size=66

$$-\frac{i\cos^{-n}(c+dx)\,{}_2F_1\left(1,n;1+n;\frac{1}{2}(1+i\tan(c+dx))\right)(a\cos(c+dx)+ia\sin(c+dx))^n}{2dn}$$

```
[Out] -1/2*I*hypergeom([1, n],[1+n],1/2+1/2*I*tan(d*x+c))*(a*cos(d*x+c)+I*a*sin(d
*x+c))^n/d/n/(cos(d*x+c)^n)
```

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.030$, Rules used = {3163}

$$-\frac{i\cos^{-n}(c+dx)\,{}_2F_1\left(1,n;n+1;\frac{1}{2}(i\tan(c+dx)+1)\right)(a\cos(c+dx)+ia\sin(c+dx))^n}{2dn}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n/Cos[c + d*x]^n,x]
```

```
[Out] ((-1/2*I)*Hypergeometric2F1[1, n, 1 + n, (1 + I*Tan[c + d*x])/2]*(a*Cos[c +
 d*x] + I*a*Sin[c + d*x])^n)/(d*n*Cos[c + d*x]^n)
```

Rule 3163

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*si
n[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*((a*Cos[c + d*x] + b*Si
n[c + d*x])^n/(2*a*d*n*Cos[c + d*x]^n))*Hypergeometric2F1[1, n, n + 1, (a +
 b*Tan[c + d*x])/(2*a)], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[m + n, 0] &
& EqQ[a^2 + b^2, 0] &&  !IntegerQ[n]
```

Rubi steps

$$\int \cos^{-n}(c+dx)(a\cos(c+dx)+ia\sin(c+dx))^n\,dx = -\frac{i\cos^{-n}(c+dx)\,{}_2F_1\left(1,n;1+n;\frac{1}{2}(1+i\tan(c+dx)\right)}{2dn}$$

Mathematica [A]
time = 2.34, size = 90, normalized size = 1.36

$$\frac{\cos^{-n}(c+dx)(a(\cos(c+dx)+i\sin(c+dx)))^n\left(-2i(1+n)+n\,{}_2F_1\left(1,1+n;2+n;\frac{1}{2}(1+i\tan(c+dx))\right)(-i+\tan(c+dx))\right)}{4dn(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Cos[c + d*x] + I*a*Sin[c + d*x])^n/Cos[c + d*x]^n,x]
```

```
[Out] ((a*(Cos[c + d*x] + I*Sin[c + d*x]))^n*((-2*I)*(1 + n) + n*Hypergeometric2F
1[1, 1 + n, 2 + n, (1 + I*Tan[c + d*x])/2]*(-I + Tan[c + d*x])))/(4*d*n*(1
+ n)*Cos[c + d*x]^n)
```

Maple [F]
time = 0.17, size = 0, normalized size = 0.00

$$\int \left(a \cos \left(d x +c \right)+i a \sin \left(d x +c \right)\right)^{n} \left(\cos^{-n}\left(d x +c \right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(cos(d*x+c)^n),x)
```

```
[Out] int((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(cos(d*x+c)^n),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(cos(d*x+c)^n),x, algorithm="maxi
ma")
```

```
[Out] integrate((a*cos(d*x + c) + I*a*sin(d*x + c))^n*cos(d*x + c)^(-n), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(cos(d*x+c)^n),x, algorithm="fric
as")
```

```
[Out] integral(e^(I*d*n*x + I*c*n + n*log(a))/(1/2*(e^(2*I*d*x + 2*I*c) + 1)*e^(-
I*d*x - I*c))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(i \sin{\left(c + d x \right)} + \cos{\left(c + d x \right)}\right)\right)^{n} \cos^{- n}{\left(c + d x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))**n/(cos(d*x+c)**n),x)
```

```
[Out] Integral((a*(I*sin(c + d*x) + cos(c + d*x)))**n/cos(c + d*x)**n, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c)+I*a*sin(d*x+c))^n/(cos(d*x+c)^n),x, algorithm="giac
")
```

```
[Out] integrate((a*cos(d*x + c) + I*a*sin(d*x + c))^n/cos(d*x + c)^n, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a\,\cos\left(c+d\,x\right)+a\,\sin\left(c+d\,x\right)\,1{}\mathrm{i}\right)^n}{\cos\left(c+d\,x\right)^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^n/cos(c + d*x)^n,x)
```

```
[Out] int((a*cos(c + d*x) + a*sin(c + d*x)*1i)^n/cos(c + d*x)^n, x)
```

3.188 $\int \frac{1}{\sec(x)+\tan(x)}\,dx$

Optimal. Leaf size=5

$$\log(1+\sin(x))$$

[Out] ln(1+sin(x))

Rubi [A]
time = 0.02, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3238, 2746, 31}

$$\log(\sin(x)+1)$$

Antiderivative was successfully verified.

[In] Int[(Sec[x] + Tan[x])^(-1),x]

[Out] Log[1 + Sin[x]]

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 3238

```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sec(x)+\tan(x)}\,dx &= \int \frac{\cos(x)}{1+\sin(x)}\,dx\\ &= \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sin(x)\right)\\ &= \log(1+\sin(x))\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 16 vs. $2(5) = 10$.
time = 0.02, size = 16, normalized size = 3.20

$$2\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] + Tan[x])^(-1),x]
```

```
[Out] 2*Log[Cos[x/2] + Sin[x/2]]
```

Maple [A]
time = 0.10, size = 6, normalized size = 1.20

method	result	size
default	$\ln(1+\sin(x))$	6
risch	$-ix+2\ln(e^{ix}+i)$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)+tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(1+sin(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(5) = 10$.
time = 0.27, size = 31, normalized size = 6.20

$$2\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)-\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x)),x, algorithm="maxima")
```

```
[Out] 2*log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A]
time = 2.66, size = 5, normalized size = 1.00

$$\log(\sin(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x)),x, algorithm="fricas")
```

```
[Out] log(sin(x) + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(5) = 10$.
time = 0.06, size = 17, normalized size = 3.40

$$\log(\tan(x) + \sec(x)) - \frac{\log(\tan^2(x) + 1)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x)),x)
```

```
[Out] log(tan(x) + sec(x)) - log(tan(x)**2 + 1)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(5) = 10$.
time = 0.40, size = 22, normalized size = 4.40

$$-\log\left(\tan\left(\frac{1}{2}x\right)^2 + 1\right) + 2\log\left(\left|\tan\left(\frac{1}{2}x\right) + 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)+tan(x)),x, algorithm="giac")
```

```
[Out] -log(tan(1/2*x)^2 + 1) + 2*log(abs(tan(1/2*x) + 1))
```

Mupad [B]
time = 1.11, size = 21, normalized size = 4.20

$$2\ln\left(\tan\left(\frac{x}{2}\right) + 1\right) - \ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(tan(x) + 1/cos(x)),x)
```

```
[Out] 2*log(tan(x/2) + 1) - log(tan(x/2)^2 + 1)
```

3.189 $\int \frac{\sin(x)}{\sec(x)+\tan(x)}\,dx$

Optimal. Leaf size=10

$$-\log(1+\sin(x))+\sin(x)$$

```
[Out] -ln(1+sin(x))+sin(x)
```

Rubi [A]
time = 0.05, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4476, 2912, 45}

$$\sin(x)-\log(\sin(x)+1)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(Sec[x] + Tan[x]),x]
```

```
[Out] -Log[1 + Sin[x]] + Sin[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin(x)}{\sec(x)+\tan(x)}\,dx &= \int \frac{\cos(x)\sin(x)}{1+\sin(x)}\,dx \\
&= \text{Subst}\left(\int \frac{x}{1+x}\,dx, x, \sin(x)\right) \\
&= \text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)\,dx, x, \sin(x)\right) \\
&= -\log(1+\sin(x))+\sin(x)
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 19, normalized size = 1.90

$$-2\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(Sec[x] + Tan[x]),x]
```

```
[Out] -2*Log[Cos[x/2] + Sin[x/2]] + Sin[x]
```

Maple [A]
time = 0.14, size = 11, normalized size = 1.10

method	result	size
derivativedivides	$-\ln\left(1+\sin\left(x\right)\right)+\sin\left(x\right)$	11
default	$-\ln\left(1+\sin\left(x\right)\right)+\sin\left(x\right)$	11
risch	$ix-\frac{i\mathrm{e}^{ix}}{2}+\frac{i\mathrm{e}^{-ix}}{2}-2\ln\left(\mathrm{e}^{ix}+i\right)$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(sec(x)+tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -ln(1+sin(x))+sin(x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(10) = 20$.
time = 0.47, size = 54, normalized size = 5.40

$$\frac{2\,\sin\left(x\right)}{\left(\frac{\sin\left(x\right)^{2}}{\left(\cos\left(x\right)+1\right)^{2}}+1\right)\left(\cos\left(x\right)+1\right)}-2\,\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}+1\right)+\log\left(\frac{\sin\left(x\right)^{2}}{\left(\cos\left(x\right)+1\right)^{2}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(sec(x)+tan(x)),x, algorithm="maxima")
```

```
[Out] 2*sin(x)/((sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)) - 2*log(sin(x)/(cos(x
) + 1) + 1) + log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A]
time = 2.49, size = 10, normalized size = 1.00

$$-\log\left(\sin\left(x\right)+1\right)+\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(sec(x)+tan(x)),x, algorithm="fricas")
```

```
[Out] -log(sin(x) + 1) + sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(x\right)}{\tan\left(x\right)+\sec\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(sec(x)+tan(x)),x)
```

```
[Out] Integral(sin(x)/(tan(x) + sec(x)), x)
```

Giac [A]
time = 0.41, size = 10, normalized size = 1.00

$$-\log\left(\sin\left(x\right)+1\right)+\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(sec(x)+tan(x)),x, algorithm="giac")
```

```
[Out] -log(sin(x) + 1) + sin(x)
```

Mupad [B]
time = 0.60, size = 21, normalized size = 2.10

$$\ln\left({\mathrm{tan}\left(\frac{x}{2}\right)}^2+1\right)-2\,\ln\left(\mathrm{tan}\left(\frac{x}{2}\right)+1\right)+\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(tan(x) + 1/cos(x)),x)
```

```
[Out] log(tan(x/2)^2 + 1) - 2*log(tan(x/2) + 1) + sin(x)
```

3.190 $\int \frac{\cos(x)}{\sec(x)+\tan(x)} dx$

Optimal. Leaf size=4

$$x + \cos(x)$$

[Out] x+cos(x)

Rubi [A]
time = 0.04, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4476, 2761, 8}

$$x + \cos(x)$$

Antiderivative was successfully verified.

[In] Int[Cos[x]/(Sec[x] + Tan[x]),x]

[Out] x + Cos[x]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x)}{\sec(x)+\tan(x)} dx &= \int \frac{\cos^2(x)}{1+\sin(x)} dx \\
&= \cos(x) + \int 1 \, dx \\
&= x + \cos(x)
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 4, normalized size = 1.00

$$x + \cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(Sec[x] + Tan[x]),x]
```

```
[Out] x + Cos[x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(4) = 8$.
time = 0.14, size = 21, normalized size = 5.25

method	result	size
risch	$x + \cos(x)$	5
default	$\frac{2}{1+\tan^2\left(\frac{x}{2}\right)} + 2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(sec(x)+tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/(1+tan(1/2*x)^2)+2*arctan(tan(1/2*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(4) = 8$.
time = 0.47, size = 30, normalized size = 7.50

$$\frac{2}{\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1} + 2\arctan\left(\frac{\sin(x)}{\cos(x) + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sec(x)+tan(x)),x, algorithm="maxima")
```

```
[Out] 2/(sin(x)^2/(cos(x) + 1)^2 + 1) + 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [A]
time = 2.97, size = 4, normalized size = 1.00

$$x + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sec(x)+tan(x)),x, algorithm="fricas")
```

```
[Out] x + cos(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(x)}{\tan(x) + \sec(x)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sec(x)+tan(x)),x)
[Out] Integral(cos(x)/(tan(x) + sec(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(4) = 8.
time = 0.40, size = 14, normalized size = 3.50

$$x + \frac{2}{\tan\left(\frac{1}{2}x\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sec(x)+tan(x)),x, algorithm="giac")
[Out] x + 2/(tan(1/2*x)^2 + 1)
```

Mupad [B]
time = 0.55, size = 4, normalized size = 1.00

$$x + \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(tan(x) + 1/cos(x)),x)
[Out] x + cos(x)
```

3.191 $\int \frac{\tan(x)}{\sec(x)+\tan(x)}\,dx$

Optimal. Leaf size=11

$$x+\frac{\cos(x)}{1+\sin(x)}$$

```
[Out] x+cos(x)/(1+sin(x))
```

Rubi [A]
time = 0.04, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4476, 2814, 2727}

$$x+\frac{\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(Sec[x] + Tan[x]),x]
```

```
[Out] x + Cos[x]/(1 + Sin[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\tan(x)}{\sec(x)+\tan(x)}\,dx = \int \frac{\sin(x)}{1+\sin(x)}\,dx$$
$$= x - \int \frac{1}{1+\sin(x)}\,dx$$
$$= x + \frac{\cos(x)}{1+\sin(x)}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.
time = 0.04, size = 25, normalized size = 2.27

$$x - \frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(Sec[x] + Tan[x]),x]
[Out] x - (2*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

Maple [A]
time = 0.09, size = 19, normalized size = 1.73

method	result	size
risch	$x+\frac{2}{e^{ix}+i}$	15
default	$\frac{2}{\tan\left(\frac{x}{2}\right)+1}+2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(sec(x)+tan(x)),x,method=_RETURNVERBOSE)
[Out] 2/(tan(1/2*x)+1)+2*arctan(tan(1/2*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(11) = 22.
time = 0.46, size = 28, normalized size = 2.55

$$\frac{2}{\frac{\sin(x)}{\cos(x)+1}+1} + 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(sec(x)+tan(x)),x, algorithm="maxima")
```

```
[Out] 2/(sin(x)/(cos(x) + 1) + 1) + 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(11) = 22$.
time = 2.21, size = 24, normalized size = 2.18

$$\frac{(x+1)\cos(x) + (x-1)\sin(x) + x + 1}{\cos(x) + \sin(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(sec(x)+tan(x)),x, algorithm="fricas")
```

```
[Out] ((x + 1)*cos(x) + (x - 1)*sin(x) + x + 1)/(cos(x) + sin(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(x)}{\tan(x) + \sec(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(sec(x)+tan(x)),x)
```

```
[Out] Integral(tan(x)/(tan(x) + sec(x)), x)
```

Giac [A]
time = 0.41, size = 12, normalized size = 1.09

$$x + \frac{2}{\tan\left(\frac{1}{2}\,x\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(sec(x)+tan(x)),x, algorithm="giac")
```

```
[Out] x + 2/(tan(1/2*x) + 1)
```

Mupad [B]
time = 0.58, size = 12, normalized size = 1.09

$$x + \frac{2}{\tan\left(\frac{x}{2}\right) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(tan(x) + 1/cos(x)),x)
```

```
[Out] x + 2/(tan(x/2) + 1)
```

3.192 $\int \frac{\cot(x)}{\sec(x)+\tan(x)}\,dx$

Optimal. Leaf size=9

$$-x - \tanh^{-1}(\cos(x))$$

```
[Out] -x-arctanh(cos(x))
```

Rubi [A]
time = 0.06, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4476, 2918, 3855, 8}

$$-x - \tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(Sec[x] + Tan[x]),x]
```

```
[Out] -x - ArcTanh[Cos[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\cot(x)}{\sec(x)+\tan(x)}\,dx = \int \frac{\cos(x)\cot(x)}{1+\sin(x)}\,dx$$
$$= -\int 1\,dx + \int \csc(x)\,dx$$
$$= -x - \tanh^{-1}(\cos(x))$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 20 vs. 2(9) = 18.
time = 0.03, size = 20, normalized size = 2.22

$$-x-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(Sec[x] + Tan[x]),x]
```

```
[Out] -x - Log[Cos[x/2]] + Log[Sin[x/2]]
```

Maple [A]
time = 0.16, size = 14, normalized size = 1.56

method	result	size
default	$-2\arctan\left(\tan\left(\frac{x}{2}\right)\right)+\ln\left(\tan\left(\frac{x}{2}\right)\right)$	14
risch	$-x+\ln\left(e^{ix}-1\right)-\ln\left(e^{ix}+1\right)$	23

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(sec(x)+tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*arctan(tan(1/2*x))+ln(tan(1/2*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. 2(9) = 18.
time = 0.46, size = 23, normalized size = 2.56

$$-2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)+\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(sec(x)+tan(x)),x, algorithm="maxima")
```

```
[Out] -2*arctan(sin(x)/(cos(x) + 1)) + log(sin(x)/(cos(x) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(9) = 18.
time = 2.90, size = 22, normalized size = 2.44

$$-x - \frac{1}{2}\log\left(\frac{1}{2}\cos\left(x\right) + \frac{1}{2}\right) + \frac{1}{2}\log\left(-\frac{1}{2}\cos\left(x\right) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(sec(x)+tan(x)),x, algorithm="fricas")
```

```
[Out] -x - 1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(x\right)}{\tan\left(x\right) + \sec\left(x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(sec(x)+tan(x)),x)
```

```
[Out] Integral(cot(x)/(tan(x) + sec(x)), x)
```

Giac [A]
time = 0.40, size = 10, normalized size = 1.11

$$-x + \log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(sec(x)+tan(x)),x, algorithm="giac")
```

```
[Out] -x + log(abs(tan(1/2*x)))
```

Mupad [B]
time = 0.59, size = 23, normalized size = 2.56

$$2\,\mathrm{atan}\left(\frac{8}{4\tan\left(\frac{x}{2}\right)+4}-1\right)+\ln\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(tan(x) + 1/cos(x)),x)
```

```
[Out] 2*atan(8/(4*tan(x/2) + 4) - 1) + log(tan(x/2))
```

3.193 $\int \frac{\sec(x)}{\sec(x)+\tan(x)}\,dx$

Optimal. Leaf size=10

$$-\frac{\cos(x)}{1+\sin(x)}$$

```
[Out] -cos(x)/(1+sin(x))
```

Rubi [A]
time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3244, 2727}

$$-\frac{\cos(x)}{\sin(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(Sec[x] + Tan[x]),x]
```

```
[Out] -(Cos[x]/(1 + Sin[x]))
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 3244

```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] +
(c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Int[1/(b + a*Cos[d + e*x]
 + c*Sin[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(x)}{\sec(x)+\tan(x)}\,dx &= \int \frac{1}{1+\sin(x)}\,dx\\ &= -\frac{\cos(x)}{1+\sin(x)}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 23 vs. 2(10) = 20.

time = 0.02, size = 23, normalized size = 2.30

$$\frac{2 \sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(Sec[x] + Tan[x]),x]
[Out] (2*Sin[x/2])/(Cos[x/2] + Sin[x/2])
```

Maple [A]
time = 0.10, size = 11, normalized size = 1.10

method	result	size
default	$-\frac{2}{\tan\left(\frac{x}{2}\right)+1}$	11
risch	$-\frac{2}{e^{ix}+i}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(sec(x)+tan(x)),x,method=_RETURNVERBOSE)
[Out] -2/(tan(1/2*x)+1)
```

Maxima [A]
time = 0.29, size = 15, normalized size = 1.50

$$-\frac{2}{\frac{\sin(x)}{\cos(x)+1} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(sec(x)+tan(x)),x, algorithm="maxima")
[Out] -2/(sin(x)/(cos(x) + 1) + 1)
```

Fricas [A]
time = 2.88, size = 18, normalized size = 1.80

$$-\frac{\cos(x) - \sin(x) + 1}{\cos(x) + \sin(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(sec(x)+tan(x)),x, algorithm="fricas")
[Out] -(cos(x) - sin(x) + 1)/(cos(x) + sin(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(x)}{\tan(x)+\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(sec(x)+tan(x)),x)
```

```
[Out] Integral(sec(x)/(tan(x) + sec(x)), x)
```

Giac [A]
time = 0.39, size = 10, normalized size = 1.00

$$-\frac{2}{\tan\left(\frac{1}{2}\,x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(sec(x)+tan(x)),x, algorithm="giac")
```

```
[Out] -2/(tan(1/2*x) + 1)
```

Mupad [B]
time = 0.55, size = 10, normalized size = 1.00

$$-\frac{2}{\tan\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)*(tan(x) + 1/cos(x))),x)
```

```
[Out] -2/(tan(x/2) + 1)
```

3.194 $\int \frac{\csc(x)}{\sec(x)+\tan(x)} \, dx$

Optimal. Leaf size=11

$$\log(\sin(x)) - \log(1 + \sin(x))$$

```
[Out] ln(sin(x))-ln(1+sin(x))
```

Rubi [A]
time = 0.04, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4476, 2786, 36, 29, 31}

$$\log(\sin(x)) - \log(\sin(x) + 1)$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(Sec[x] + Tan[x]),x]
```

```
[Out] Log[Sin[x]] - Log[1 + Sin[x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x, x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(x)}{\sec(x)+\tan(x)}\,dx &= \int \frac{\cot(x)}{1+\sin(x)}\,dx\\ &= \text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, \sin(x)\right)\\ &= \text{Subst}\left(\int \frac{1}{x}\,dx, x, \sin(x)\right) - \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \sin(x)\right)\\ &= \log(\sin(x)) - \log(1+\sin(x))\end{aligned}$$

Mathematica [A]
time = 0.03, size = 20, normalized size = 1.82

$$-2\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)+\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(Sec[x] + Tan[x]),x]
[Out] -2*Log[Cos[x/2] + Sin[x/2]] + Log[Sin[x]]
```

Maple [A]
time = 0.15, size = 8, normalized size = 0.73

method	result	size
derivativedivides	$-\ln\left(\csc\left(x\right)+1\right)$	8
default	$-\ln\left(\csc\left(x\right)+1\right)$	8
risch	$-2\ln\left(\mathrm{e}^{ix}+i\right)+\ln\left(\mathrm{e}^{2ix}-1\right)$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(sec(x)+tan(x)),x,method=_RETURNVERBOSE)
[Out] -ln(csc(x)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.
time = 0.27, size = 25, normalized size = 2.27

$$-2\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}+1\right)+\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(sec(x)+tan(x)),x, algorithm="maxima")
```

```
[Out] -2*log(sin(x)/(cos(x) + 1) + 1) + log(sin(x)/(cos(x) + 1))
```

Fricas [A]
time = 2.86, size = 13, normalized size = 1.18

$$\log\left(\frac{1}{2}\sin\left(x\right)\right) - \log\left(\sin\left(x\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(sec(x)+tan(x)),x, algorithm="fricas")
```

```
[Out] log(1/2*sin(x)) - log(sin(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc\left(x\right)}{\tan\left(x\right) + \sec\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(sec(x)+tan(x)),x)
```

```
[Out] Integral(csc(x)/(tan(x) + sec(x)), x)
```

Giac [A]
time = 0.40, size = 12, normalized size = 1.09

$$-\log\left(\sin\left(x\right) + 1\right) + \log\left(\left|\sin\left(x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(sec(x)+tan(x)),x, algorithm="giac")
```

```
[Out] -log(sin(x) + 1) + log(abs(sin(x)))
```

Mupad [B]
time = 0.57, size = 15, normalized size = 1.36

$$\ln\left(\tan\left(\frac{x}{2}\right)\right) - 2\,\ln\left(\tan\left(\frac{x}{2}\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(tan(x) + 1/cos(x))),x)
```

```
[Out] log(tan(x/2)) - 2*log(tan(x/2) + 1)
```

3.195 $\int \frac{1}{\sec(x)-\tan(x)}\,dx$

Optimal. Leaf size=9

$$-\log(1-\sin(x))$$

```
[Out] -ln(1-sin(x))
```

Rubi [A]

time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3238, 2746, 31}

$$-\log(1-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[x] - Tan[x])^(-1),x]
```

```
[Out] -Log[1 - Sin[x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2746

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)
^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && In
tegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/2
])
```

Rule 3238

```
Int[((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])
^(-1), x_Symbol] :> Int[Cos[d + e*x]/(b + a*Cos[d + e*x] + c*Sin[d + e*x]),
 x] /; FreeQ[{a, b, c, d, e}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sec(x)-\tan(x)}\,dx &= \int \frac{\cos(x)}{1-\sin(x)}\,dx\\ &= -\text{Subst}\left(\int \frac{1}{1+x}\,dx, x, -\sin(x)\right)\\ &= -\log(1-\sin(x))\end{aligned}$$

Mathematica [A]
time = 0.02, size = 18, normalized size = 2.00

$$-2\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[x] - Tan[x])^(-1),x]
[Out] -2*Log[Cos[x/2] - Sin[x/2]]
```

Maple [A]
time = 0.10, size = 8, normalized size = 0.89

method	result	size
default	$-\ln\left(\sin\left(x\right)-1\right)$	8
risch	$ix-2\ln\left(\mathrm{e}^{ix}-i\right)$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sec(x)-tan(x)),x,method=_RETURNVERBOSE)
[Out] -ln(sin(x)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(9) = 18$.
time = 0.27, size = 29, normalized size = 3.22

$$-2\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}-1\right)+\log\left(\frac{\sin\left(x\right)^{2}}{\left(\cos\left(x\right)+1\right)^{2}}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)-tan(x)),x, algorithm="maxima")
[Out] -2*log(sin(x)/(cos(x) + 1) - 1) + log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A]
time = 3.05, size = 9, normalized size = 1.00

$$-\log\left(-\sin\left(x\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)-tan(x)),x, algorithm="fricas")
[Out] -log(-sin(x) + 1)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 17 vs. $2(7) = 14$.
time = 0.06, size = 17, normalized size = 1.89

$$-\log\left(-\tan\left(x\right)+\sec\left(x\right)\right)+\frac{\log\left(\tan^{2}\left(x\right)+1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)-tan(x)),x)
```

```
[Out] -log(-tan(x) + sec(x)) + log(tan(x)**2 + 1)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(9) = 18$.
time = 0.42, size = 20, normalized size = 2.22

$$\log\left(\tan\left(\frac{1}{2}\,x\right)^{2}+1\right)-2\,\log\left(\left|\tan\left(\frac{1}{2}\,x\right)-1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sec(x)-tan(x)),x, algorithm="giac")
```

```
[Out] log(tan(1/2*x)^2 + 1) - 2*log(abs(tan(1/2*x) - 1))
```

Mupad [B]
time = 0.96, size = 19, normalized size = 2.11

$$\ln\left(\tan\left(\frac{x}{2}\right)^{2}+1\right)-2\,\ln\left(\tan\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(tan(x) - 1/cos(x)),x)
```

```
[Out] log(tan(x/2)^2 + 1) - 2*log(tan(x/2) - 1)
```

3.196 $\int \frac{\sin(x)}{\sec(x)-\tan(x)}\,dx$

Optimal. Leaf size=14

$$-\log(1-\sin(x))-\sin(x)$$

```
[Out] -ln(1-sin(x))-sin(x)
```

Rubi [A]

time = 0.05, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4476, 2912, 45}

$$-\sin(x)-\log(1-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(Sec[x] - Tan[x]),x]
```

```
[Out] -Log[1 - Sin[x]] - Sin[x]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\sin(x)}{\sec(x)-\tan(x)}\,dx = \int \frac{\cos(x)\sin(x)}{1-\sin(x)}\,dx$$
$$= \text{Subst}\left(\int \frac{x}{1+x}\,dx, x, -\sin(x)\right)$$
$$= \text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)\,dx, x, -\sin(x)\right)$$
$$= -\log(1-\sin(x))-\sin(x)$$

Mathematica [A]
time = 0.03, size = 23, normalized size = 1.64

$$-2\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\sin(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(Sec[x] - Tan[x]),x]
[Out] -2*Log[Cos[x/2] - Sin[x/2]] - Sin[x]
```

Maple [A]
time = 0.14, size = 13, normalized size = 0.93

method	result	size
derivativedivides	$-\sin(x)-\ln(\sin(x)-1)$	13
default	$-\sin(x)-\ln(\sin(x)-1)$	13
risch	$ix+\frac{i e^{ix}}{2}-\frac{i e^{-ix}}{2}-2\ln(e^{ix}-i)$	33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(sec(x)-tan(x)),x,method=_RETURNVERBOSE)
[Out] -sin(x)-ln(sin(x)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(14) = 28$.
time = 0.46, size = 54, normalized size = 3.86

$$-\frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)}-2\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)+\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(sec(x)-tan(x)),x, algorithm="maxima")
```

```
[Out] -2*sin(x)/((sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)) - 2*log(sin(x)/(cos(
x) + 1) - 1) + log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [**A**]
time = 3.04, size = 14, normalized size = 1.00

$$-\log\left(-\sin\left(x\right)+1\right)-\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(sec(x)-tan(x)),x, algorithm="fricas")
```

```
[Out] -log(-sin(x) + 1) - sin(x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(x\right)}{-\tan\left(x\right)+\sec\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(sec(x)-tan(x)),x)
```

```
[Out] Integral(sin(x)/(-tan(x) + sec(x)), x)
```

Giac [**A**]
time = 0.41, size = 14, normalized size = 1.00

$$-\log\left(-\sin\left(x\right)+1\right)-\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(sec(x)-tan(x)),x, algorithm="giac")
```

```
[Out] -log(-sin(x) + 1) - sin(x)
```

Mupad [**B**]
time = 0.61, size = 23, normalized size = 1.64

$$\ln\left({\mathrm{tan}\left(\frac{x}{2}\right)}^2+1\right)-2\,\ln\left(\mathrm{tan}\left(\frac{x}{2}\right)-1\right)-\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sin(x)/(tan(x) - 1/cos(x)),x)
```

```
[Out] log(tan(x/2)^2 + 1) - 2*log(tan(x/2) - 1) - sin(x)
```

3.197 $\int \frac{\cos(x)}{\sec(x)-\tan(x)}\,dx$

Optimal. Leaf size=6

$$x-\cos(x)$$

```
[Out] x-cos(x)
```

Rubi [A]
time = 0.04, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4476, 2761, 8}

$$x-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(Sec[x] - Tan[x]),x]
```

```
[Out] x - Cos[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(x)}{\sec(x)-\tan(x)}\,dx &= \int \frac{\cos^2(x)}{1-\sin(x)}\,dx\\
&= -\cos(x)+\int 1\,dx\\
&= x-\cos(x)
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 6, normalized size = 1.00

$$x - \cos(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(Sec[x] - Tan[x]),x]
```

```
[Out] x - Cos[x]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(6) = 12$.
time = 0.14, size = 21, normalized size = 3.50

method	result	size
risch	$x - \cos(x)$	7
default	$-\frac{2}{1+\tan^2\left(\frac{x}{2}\right)} + 2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(sec(x)-tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(1+tan(1/2*x)^2)+2*arctan(tan(1/2*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. $2(6) = 12$.
time = 0.48, size = 30, normalized size = 5.00

$$-\frac{2}{\frac{\sin(x)^2}{(\cos(x)+1)^2} + 1} + 2\arctan\left(\frac{\sin(x)}{\cos(x) + 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sec(x)-tan(x)),x, algorithm="maxima")
```

```
[Out] -2/(sin(x)^2/(cos(x) + 1)^2 + 1) + 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [A]
time = 2.69, size = 6, normalized size = 1.00

$$x - \cos(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sec(x)-tan(x)),x, algorithm="fricas")
```

```
[Out] x - cos(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos{\left(x \right)}}{- \tan{\left(x \right)} + \sec{\left(x \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sec(x)-tan(x)),x)
```

```
[Out] Integral(cos(x)/(-tan(x) + sec(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(6) = 12.
time = 0.41, size = 14, normalized size = 2.33

$$x - \frac{2}{\tan\left(\frac{1}{2} \, x\right)^{2} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(sec(x)-tan(x)),x, algorithm="giac")
```

```
[Out] x - 2/(tan(1/2*x)^2 + 1)
```

Mupad [B]
time = 0.58, size = 6, normalized size = 1.00

$$x-\cos\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(x)/(tan(x) - 1/cos(x)),x)
```

```
[Out] x - cos(x)
```

3.198 $\int \frac{\tan(x)}{\sec(x)-\tan(x)} \, dx$

Optimal. Leaf size=15

$$-x+\frac{\cos(x)}{1-\sin(x)}$$

```
[Out] -x+cos(x)/(1-sin(x))
```

Rubi [A]
time = 0.04, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4476, 2814, 2727}

$$\frac{\cos(x)}{1-\sin(x)}-x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(Sec[x] - Tan[x]),x]
```

```
[Out] -x + Cos[x]/(1 - Sin[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\tan(x)}{\sec(x)-\tan(x)}\,dx = \int \frac{\sin(x)}{1-\sin(x)}\,dx$$
$$= -x + \int \frac{1}{1-\sin(x)}\,dx$$
$$= -x + \frac{\cos(x)}{1-\sin(x)}$$

Mathematica [A]
time = 0.04, size = 29, normalized size = 1.93

$$-x+\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(Sec[x] - Tan[x]),x]
[Out] -x + (2*Sin[x/2])/(Cos[x/2] - Sin[x/2])
```

Maple [A]
time = 0.09, size = 19, normalized size = 1.27

method	result	size
risch	$-x+\frac{2}{e^{ix}-i}$	17
default	$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}-2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(sec(x)-tan(x)),x,method=_RETURNVERBOSE)
[Out] -2/(tan(1/2*x)-1)-2*arctan(tan(1/2*x))
```

Maxima [A]
time = 0.47, size = 28, normalized size = 1.87

$$-\frac{2}{\frac{\sin(x)}{\cos(x)+1}-1}-2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(sec(x)-tan(x)),x, algorithm="maxima")
[Out] -2/(sin(x)/(cos(x) + 1) - 1) - 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [A]
time = 2.95, size = 28, normalized size = 1.87

$$-\frac{(x-1)\cos(x)-(x+1)\sin(x)+x-1}{\cos(x)-\sin(x)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(sec(x)-tan(x)),x, algorithm="fricas")
[Out] -((x - 1)*cos(x) - (x + 1)*sin(x) + x - 1)/(cos(x) - sin(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan(x)}{-\tan(x)+\sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(sec(x)-tan(x)),x)
[Out] Integral(tan(x)/(-tan(x) + sec(x)), x)
```

Giac [A]
time = 0.40, size = 14, normalized size = 0.93

$$-x-\frac{2}{\tan\left(\frac{1}{2}x\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(sec(x)-tan(x)),x, algorithm="giac")
[Out] -x - 2/(tan(1/2*x) - 1)
```

Mupad [B]
time = 0.58, size = 14, normalized size = 0.93

$$-x-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-tan(x)/(tan(x) - 1/cos(x)),x)
[Out] - x - 2/(tan(x/2) - 1)
```

3.199 $\int \frac{\cot(x)}{\sec(x)-\tan(x)}\,dx$

Optimal. Leaf size=7

$$x - \tanh^{-1}(\cos(x))$$

```
[Out] x-arctanh(cos(x))
```

Rubi [A]
time = 0.06, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4476, 2918, 3855, 8}

$$x - \tanh^{-1}(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(Sec[x] - Tan[x]),x]
```

```
[Out] x - ArcTanh[Cos[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\cot(x)}{\sec(x)-\tan(x)}\,dx = \int \frac{\cos(x)\cot(x)}{1-\sin(x)}\,dx$$
$$= \int 1\,dx + \int \csc(x)\,dx$$
$$= x - \tanh^{-1}(\cos(x))$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 18 vs. $2(7) = 14$.
time = 0.03, size = 18, normalized size = 2.57

$$x - \log\left(\cos\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(Sec[x] - Tan[x]),x]
```

```
[Out] x - Log[Cos[x/2]] + Log[Sin[x/2]]
```

Maple [A]
time = 0.16, size = 14, normalized size = 2.00

method	result	size
default	$2\arctan\left(\tan\left(\frac{x}{2}\right)\right)+\ln\left(\tan\left(\frac{x}{2}\right)\right)$	14
risch	$x+\ln\left(e^{ix}-1\right)-\ln\left(e^{ix}+1\right)$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(sec(x)-tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*arctan(tan(1/2*x))+ln(tan(1/2*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(7) = 14$.
time = 0.47, size = 23, normalized size = 3.29

$$2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right) + \log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(sec(x)-tan(x)),x, algorithm="maxima")
```

```
[Out] 2*arctan(sin(x)/(cos(x) + 1)) + log(sin(x)/(cos(x) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(7) = 14$.
time = 3.44, size = 20, normalized size = 2.86

$$x - \frac{1}{2}\log\left(\frac{1}{2}\cos(x) + \frac{1}{2}\right) + \frac{1}{2}\log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(sec(x)-tan(x)),x, algorithm="fricas")
[Out] x - 1/2*log(1/2*cos(x) + 1/2) + 1/2*log(-1/2*cos(x) + 1/2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{-\tan(x) + \sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(sec(x)-tan(x)),x)
[Out] Integral(cot(x)/(-tan(x) + sec(x)), x)
```

Giac [A]
time = 0.42, size = 8, normalized size = 1.14

$$x + \log\left(\left|\tan\left(\frac{1}{2}x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(sec(x)-tan(x)),x, algorithm="giac")
[Out] x + log(abs(tan(1/2*x)))
```

Mupad [B]
time = 0.61, size = 23, normalized size = 3.29

$$\ln\left(\tan\left(\frac{x}{2}\right)\right) - 2\,\mathrm{atan}\left(\frac{8}{4\tan\left(\frac{x}{2}\right) - 4} + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cot(x)/(tan(x) - 1/cos(x)),x)
[Out] log(tan(x/2)) - 2*atan(8/(4*tan(x/2) - 4) + 1)
```

3.200 $\int \frac{\sec(x)}{\sec(x)-\tan(x)}\,dx$

Optimal. Leaf size=11

$$\frac{\cos(x)}{1-\sin(x)}$$

```
[Out] cos(x)/(1-sin(x))
```

Rubi [A]
time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3244, 2727}

$$\frac{\cos(x)}{1-\sin(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(Sec[x] - Tan[x]),x]
```

```
[Out] Cos[x]/(1 - Sin[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c + d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3244

```
Int[sec[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + (b_.)*sec[(d_.) + (e_.)*(x_)] + (c_.)*tan[(d_.) + (e_.)*(x_)])^(m_), x_Symbol] :> Int[1/(b + a*Cos[d + e*x] + c*Sin[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(x)}{\sec(x)-\tan(x)}\,dx &= \int \frac{1}{1-\sin(x)}\,dx\\ &= \frac{\cos(x)}{1-\sin(x)}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

time = 0.02, size = 25, normalized size = 2.27

$$\frac{2\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(Sec[x] - Tan[x]),x]
```

```
[Out] (2*Sin[x/2])/(Cos[x/2] - Sin[x/2])
```

Maple [A]
time = 0.10, size = 11, normalized size = 1.00

method	result	size
default	$-\frac{2}{\tan\left(\frac{x}{2}\right)-1}$	11
risch	$\frac{2}{e^{ix}-i}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(sec(x)-tan(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/(tan(1/2*x)-1)
```

Maxima [A]
time = 0.26, size = 15, normalized size = 1.36

$$-\frac{2}{\frac{\sin(x)}{\cos(x)+1}-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(sec(x)-tan(x)),x, algorithm="maxima")
```

```
[Out] -2/(sin(x)/(cos(x) + 1) - 1)
```

Fricas [A]
time = 2.05, size = 17, normalized size = 1.55

$$\frac{\cos\left(x\right)+\sin\left(x\right)+1}{\cos\left(x\right)-\sin\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(sec(x)-tan(x)),x, algorithm="fricas")
```

```
[Out] (cos(x) + sin(x) + 1)/(cos(x) - sin(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(x)}{-\tan(x) + \sec(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(sec(x)-tan(x)),x)
[Out] Integral(sec(x)/(-tan(x) + sec(x)), x)
```

Giac [A]
time = 0.40, size = 10, normalized size = 0.91

$$-\frac{2}{\tan\left(\frac{1}{2}\,x\right) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(sec(x)-tan(x)),x, algorithm="giac")
[Out] -2/(tan(1/2*x) - 1)
```

Mupad [B]
time = 0.56, size = 10, normalized size = 0.91

$$-\frac{2}{\tan\left(\frac{x}{2}\right) - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x)*(tan(x) - 1/cos(x))),x)
[Out] -2/(tan(x/2) - 1)
```

3.201 $\int \frac{\csc(x)}{\sec(x)-\tan(x)}\,dx$

Optimal. Leaf size=13

$$-\log(1-\sin(x))+\log(\sin(x))$$

```
[Out] -ln(1-sin(x))+ln(sin(x))
```

Rubi [A]
time = 0.04, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {4476, 2786, 36, 29, 31}

$$\log(\sin(x))-\log(1-\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(Sec[x] - Tan[x]),x]
```

```
[Out] -Log[1 - Sin[x]] + Log[Sin[x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 4476

```
Int[(u_.)*((b_.)*sec[(c_.) + (d_.)*(x_)]^(n_.) + (a_.)*tan[(c_.) + (d_.)*(x
_)]^(n_.))^(p_), x_Symbol] :> Int[ActivateTrig[u]*Sec[c + d*x]^(n*p)*(b + a
*Sin[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\csc(x)}{\sec(x)-\tan(x)}\,dx = \int \frac{\cot(x)}{1-\sin(x)}\,dx$$
$$= \text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, -\sin(x)\right)$$
$$= \text{Subst}\left(\int \frac{1}{x}\,dx, x, -\sin(x)\right) - \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, -\sin(x)\right)$$
$$= -\log(1-\sin(x)) + \log(\sin(x))$$

Mathematica [**A**]
time = 0.03, size = 22, normalized size = 1.69

$$-2\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(Sec[x] - Tan[x]),x]
[Out] -2*Log[Cos[x/2] - Sin[x/2]] + Log[Sin[x]]
```

Maple [**A**]
time = 0.15, size = 8, normalized size = 0.62

method	result	size
derivativedivides	$-\ln\left(-1+\csc\left(x\right)\right)$	8
default	$-\ln\left(-1+\csc\left(x\right)\right)$	8
risch	$-2\ln\left(e^{ix}-i\right)+\ln\left(e^{2ix}-1\right)$	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(sec(x)-tan(x)),x,method=_RETURNVERBOSE)
[Out] -ln(-1+csc(x))
```

Maxima [**A**]
time = 0.28, size = 25, normalized size = 1.92

$$-2\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}-1\right)+\log\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(sec(x)-tan(x)),x, algorithm="maxima")
```

```
[Out] -2*log(sin(x)/(cos(x) + 1) - 1) + log(sin(x)/(cos(x) + 1))
```

Fricas [A]
time = 3.09, size = 15, normalized size = 1.15

$$\log\left(\frac{1}{2}\sin\left(x\right)\right) - \log\left(-\sin\left(x\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(sec(x)-tan(x)),x, algorithm="fricas")
[Out] log(1/2*sin(x)) - log(-sin(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(x\right)}{-\tan\left(x\right) + \sec\left(x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(sec(x)-tan(x)),x)
[Out] Integral(csc(x)/(-tan(x) + sec(x)), x)
```

Giac [A]
time = 0.40, size = 14, normalized size = 1.08

$$-\log\left(-\sin\left(x\right) + 1\right) + \log\left(\left|\sin\left(x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(sec(x)-tan(x)),x, algorithm="giac")
[Out] -log(-sin(x) + 1) + log(abs(sin(x)))
```

Mupad [B]
time = 0.56, size = 15, normalized size = 1.15

$$\ln\left(\tan\left(\frac{x}{2}\right)\right) - 2\,\ln\left(\tan\left(\frac{x}{2}\right) - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(x)*(tan(x) - 1/cos(x))),x)
[Out] log(tan(x/2)) - 2*log(tan(x/2) - 1)
```

3.202 $\int \csc(c+dx)(\cot(c+dx)+\csc(c+dx))\,dx$

Optimal. Leaf size=23

$$-\frac{\cot(c+dx)}{d}-\frac{\csc(c+dx)}{d}$$

```
[Out] -cot(d*x+c)/d-csc(d*x+c)/d
```

Rubi [A]

time = 0.07, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4482, 2748, 3852, 8}

$$-\frac{\cot(c+dx)}{d}-\frac{\csc(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(Cot[c + d*x] + Csc[c + d*x]),x]
```

```
[Out] -(Cot[c + d*x]/d) - Csc[c + d*x]/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)(\cot(c+dx)+\csc(c+dx))\,dx &= \int (1+\cos(c+dx))\csc^2(c+dx)\,dx \\ &= -\frac{\csc(c+dx)}{d}+\int \csc^2(c+dx)\,dx \\ &= -\frac{\csc(c+dx)}{d}-\frac{\text{Subst}(\int 1\,dx,x,\cot(c+dx))}{d} \\ &= -\frac{\cot(c+dx)}{d}-\frac{\csc(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 15, normalized size = 0.65

$$-\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(Cot[c + d*x] + Csc[c + d*x]),x]
[Out] -(Cot[(c + d*x)/2]/d)
```

Maple [A]
time = 0.05, size = 24, normalized size = 1.04

method	result	size
risch	$-\frac{2i}{d\left(e^{i(dx+c)}-1\right)}$	20
derivativedivides	$\frac{-\frac{1}{\sin(dx+c)}-\cot(dx+c)}{d}$	24
default	$\frac{-\frac{1}{\sin(dx+c)}-\cot(dx+c)}{d}$	24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(cot(d*x+c)+csc(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/sin(d*x+c)-cot(d*x+c))
```

Maxima [A]
time = 0.27, size = 22, normalized size = 0.96

$$-\frac{\frac{1}{\sin(dx+c)}+\frac{1}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(cot(d*x+c)+csc(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(1/sin(d*x + c) + 1/tan(d*x + c))/d
```

Fricas [A]
time = 2.51, size = 21, normalized size = 0.91

$$-\frac{\cos(dx+c)+1}{d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(cot(d*x+c)+csc(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(cos(d*x + c) + 1)/(d*sin(d*x + c))
```

Sympy [A]
time = 1.18, size = 27, normalized size = 1.17

$$\begin{cases} \frac{-\cot(c+dx)-\csc(c+dx)}{d} & \text{for}\ d\neq 0 \\ x(\cot(c)+\csc(c))\csc(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(cot(d*x+c)+csc(d*x+c)),x)
```

```
[Out] Piecewise(((-cot(c + d*x) - csc(c + d*x))/d, Ne(d, 0)), (x*(cot(c) + csc(c)
)*csc(c), True))
```

Giac [A]
time = 0.41, size = 16, normalized size = 0.70

$$-\frac{1}{d\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(cot(d*x+c)+csc(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/(d*tan(1/2*d*x + 1/2*c))
```

Mupad [B]
time = 0.59, size = 14, normalized size = 0.61

$$-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cot(c + d*x) + 1/sin(c + d*x))/sin(c + d*x),x)
```

```
[Out] -cot(c/2 + (d*x)/2)/d
```

3.203 $\int \frac{\sin(x)}{\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=6

$$x-\sin(x)$$

```
[Out] x-sin(x)
```

Rubi [A]

time = 0.05, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4477, 2761, 8}

$$x-\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/(Cot[x] + Csc[x]),x]
```

```
[Out] x - Sin[x]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(x)}{\cot(x)+\csc(x)}\,dx &= \int \frac{\sin^2(x)}{1+\cos(x)}\,dx\\
&= -\sin(x)+\int 1\,dx\\
&= x-\sin(x)
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 14 vs. $2(6) = 12$.
time = 0.01, size = 14, normalized size = 2.33

$$2\left(\frac{x}{2}-\frac{\sin(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(Cot[x] + Csc[x]),x]
```

```
[Out] 2*(x/2 - Sin[x]/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(6) = 12$.
time = 0.12, size = 25, normalized size = 4.17

method	result	size
risch	$x-\sin\left(x\right)$	7
default	$-\frac{2\tan\left(\frac{x}{2}\right)}{1+\tan^{2}\left(\frac{x}{2}\right)}+2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cot(x)+csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -2*tan(1/2*x)/(1+tan(1/2*x)^2)+2*arctan(tan(1/2*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(6) = 12$.
time = 0.47, size = 38, normalized size = 6.33

$$-\frac{2\sin\left(x\right)}{\left(\frac{\sin\left(x\right)^{2}}{\left(\cos\left(x\right)+1\right)^{2}}+1\right)\left(\cos\left(x\right)+1\right)}+2\arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] -2*sin(x)/((sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)) + 2*arctan(sin(x)/(c
os(x) + 1))
```

Fricas [A]
time = 2.17, size = 6, normalized size = 1.00

$$x-\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cot(x)+csc(x)),x, algorithm="fricas")
```

```
[Out] x - sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)}{\cot(x)+\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cot(x)+csc(x)),x)
```

```
[Out] Integral(sin(x)/(cot(x) + csc(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(6) = 12.
time = 0.40, size = 18, normalized size = 3.00

$$x - \frac{2\,\tan\left(\frac{1}{2}\,x\right)}{\tan\left(\frac{1}{2}\,x\right)^{2} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(cot(x)+csc(x)),x, algorithm="giac")
```

```
[Out] x - 2*tan(1/2*x)/(tan(1/2*x)^2 + 1)
```

Mupad [B]
time = 1.05, size = 6, normalized size = 1.00

$$x-\sin\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(cot(x) + 1/sin(x)),x)
```

```
[Out] x - sin(x)
```

3.204 $\int \frac{\cos(x)}{\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=10

$$-\cos(x)+\log(1+\cos(x))$$

```
[Out] -cos(x)+ln(cos(x)+1)
```

Rubi [A]
time = 0.04, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4477, 2912, 45}

$$\log(\cos(x)+1)-\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(Cot[x] + Csc[x]),x]
```

```
[Out] -Cos[x] + Log[1 + Cos[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\cos(x)}{\cot(x)+\csc(x)}\,dx = \int \frac{\cos(x)\sin(x)}{1+\cos(x)}\,dx$$
$$= -\text{Subst}\left(\int \frac{x}{1+x}\,dx, x, \cos(x)\right)$$
$$= -\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)\,dx, x, \cos(x)\right)$$
$$= -\cos(x) + \log(1+\cos(x))$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 2.00

$$-2\cos^2\left(\frac{x}{2}\right) + 2\log\left(\cos\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(Cot[x] + Csc[x]),x]
```

```
[Out] -2*Cos[x/2]^2 + 2*Log[Cos[x/2]]
```

Maple [A]
time = 0.12, size = 11, normalized size = 1.10

method	result	size
derivativedivides	$-\cos(x)+\ln(1+\cos(x))$	11
default	$-\cos(x)+\ln(1+\cos(x))$	11
risch	$-ix-\frac{e^{ix}}{2}-\frac{e^{-ix}}{2}+2\ln(e^{ix}+1)$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(cot(x)+csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -cos(x)+ln(1+cos(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(10) = 20$.
time = 0.46, size = 34, normalized size = 3.40

$$-\frac{2}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1} - \log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(cot(x)+csc(x)),x, algorithm="maxima")
[Out] -2/(sin(x)^2/(cos(x) + 1)^2 + 1) - log(sin(x)^2/(cos(x) + 1)^2 + 1)
```

Fricas [A]
time = 2.67, size = 12, normalized size = 1.20

$$-\cos(x)+\log\left(\frac{1}{2}\cos(x)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(cot(x)+csc(x)),x, algorithm="fricas")
[Out] -cos(x) + log(1/2*cos(x) + 1/2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos(x)}{\cot(x)+\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(cot(x)+csc(x)),x)
[Out] Integral(cos(x)/(cot(x) + csc(x)), x)
```

Giac [A]
time = 0.40, size = 10, normalized size = 1.00

$$-\cos(x)+\log(\cos(x)+1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(cot(x)+csc(x)),x, algorithm="giac")
[Out] -cos(x) + log(cos(x) + 1)
```

Mupad [B]
time = 0.59, size = 24, normalized size = 2.40

$$-\ln\left(\tan\left(\frac{x}{2}\right)^2+1\right)-\frac{2}{\tan\left(\frac{x}{2}\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(cot(x) + 1/sin(x)),x)
[Out] - log(tan(x/2)^2 + 1) - 2/(tan(x/2)^2 + 1)
```

3.205 $\int \frac{\tan(x)}{\cot(x)+\csc(x)} dx$

Optimal. Leaf size=7

$$-x+\tanh^{-1}(\sin(x))$$

```
[Out] -x+arctanh(sin(x))
```

Rubi [A]
time = 0.06, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4477, 2918, 3855, 8}

$$\tanh^{-1}(\sin(x))-x$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(Cot[x] + Csc[x]),x]
```

```
[Out] -x + ArcTanh[Sin[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(x)}{\cot(x)+\csc(x)}\,dx &= \int \frac{\sin(x)\tan(x)}{1+\cos(x)}\,dx\\ &= -\int 1\,dx + \int \sec(x)\,dx\\ &= -x+\tanh^{-1}(\sin(x))\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 36 vs. $2(7) = 14$.
time = 0.03, size = 36, normalized size = 5.14

$$-x-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(Cot[x] + Csc[x]),x]
```

```
[Out] -x - Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]]
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(7) = 14$.
time = 0.15, size = 25, normalized size = 3.57

method	result	size
default	$\ln\left(\tan\left(\frac{x}{2}\right)+1\right)-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)-2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	25
risch	$-x-\ln\left(e^{ix}-i\right)+\ln\left(e^{ix}+i\right)$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(cot(x)+csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(tan(1/2*x)+1)-ln(tan(1/2*x)-1)-2*arctan(tan(1/2*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. $2(7) = 14$.
time = 0.48, size = 39, normalized size = 5.57

$$-2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)+\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)-\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] -2*arctan(sin(x)/(cos(x) + 1)) + log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)/
(cos(x) + 1) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(7) = 14$.
time = 3.19, size = 20, normalized size = 2.86

$$-x + \frac{1}{2} \log\left(\sin\left(x\right) + 1\right) - \frac{1}{2} \log\left(-\sin\left(x\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(cot(x)+csc(x)),x, algorithm="fricas")
[Out] -x + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(x\right)}{\cot\left(x\right) + \csc\left(x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(cot(x)+csc(x)),x)
[Out] Integral(tan(x)/(cot(x) + csc(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(7) = 14$.
time = 0.40, size = 22, normalized size = 3.14

$$-x + \log\left(\left|\tan\left(\frac{1}{2}\,x\right) + 1\right|\right) - \log\left(\left|\tan\left(\frac{1}{2}\,x\right) - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(cot(x)+csc(x)),x, algorithm="giac")
[Out] -x + log(abs(tan(1/2*x) + 1)) - log(abs(tan(1/2*x) - 1))
```

Mupad [B]
time = 0.57, size = 11, normalized size = 1.57

$$2\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right) - x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(cot(x) + 1/sin(x)),x)
[Out] 2*atanh(tan(x/2)) - x
```

3.206 $\int \frac{\cot(x)}{\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=12

$$x-\frac{\sin(x)}{1+\cos(x)}$$

[Out] x-sin(x)/(cos(x)+1)

Rubi [A]
time = 0.04, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4477, 2814, 2727}

$$x-\frac{\sin(x)}{\cos(x)+1}$$

Antiderivative was successfully verified.

[In] Int[Cot[x]/(Cot[x] + Csc[x]),x]

[Out] x - Sin[x]/(1 + Cos[x])

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\cot(x)}{\cot(x)+\csc(x)}\,dx = \int \frac{\cos(x)}{1+\cos(x)}\,dx$$
$$= x - \int \frac{1}{1+\cos(x)}\,dx$$
$$= x - \frac{\sin(x)}{1+\cos(x)}$$

Mathematica [A]
time = 0.02, size = 10, normalized size = 0.83

$$x - \tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(Cot[x] + Csc[x]),x]
```

```
[Out] x - Tan[x/2]
```

Maple [A]
time = 0.08, size = 15, normalized size = 1.25

method	result	size
default	$-\tan\left(\frac{x}{2}\right)+2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	15
risch	$x-\frac{2i}{e^{ix}+1}$	15

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(cot(x)+csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -tan(1/2*x)+2*arctan(tan(1/2*x))
```

Maxima [A]
time = 0.48, size = 23, normalized size = 1.92

$$-\frac{\sin(x)}{\cos(x)+1} + 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] -sin(x)/(cos(x) + 1) + 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [A]
time = 2.58, size = 17, normalized size = 1.42

$$\frac{x\cos(x) + x - \sin(x)}{\cos(x) + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(cot(x)+csc(x)),x, algorithm="fricas")
[Out] (x*cos(x) + x - sin(x))/(cos(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot(x)}{\cot(x) + \csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(cot(x)+csc(x)),x)
[Out] Integral(cot(x)/(cot(x) + csc(x)), x)
```

Giac [A]
time = 0.41, size = 8, normalized size = 0.67

$$x - \tan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(cot(x)+csc(x)),x, algorithm="giac")
[Out] x - tan(1/2*x)
```

Mupad [B]
time = 0.57, size = 8, normalized size = 0.67

$$x - \tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(cot(x) + 1/sin(x)),x)
[Out] x - tan(x/2)
```

3.207 $\int \frac{\sec(x)}{\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=11

$$-\log(\cos(x)) + \log(1 + \cos(x))$$

```
[Out] -ln(cos(x))+ln(cos(x)+1)
```

Rubi [A]
time = 0.04, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4477, 2786, 36, 29, 31}

$$\log(\cos(x) + 1) - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(Cot[x] + Csc[x]),x]
```

```
[Out] -Log[Cos[x]] + Log[1 + Cos[x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec(x)}{\cot(x)+\csc(x)}\,dx &= \int \frac{\tan(x)}{1+\cos(x)}\,dx \\
&= -\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, \cos(x)\right) \\
&= -\text{Subst}\left(\int \frac{1}{x}\,dx, x, \cos(x)\right) + \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, \cos(x)\right) \\
&= -\log(\cos(x)) + \log(1+\cos(x))
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.
time = 0.01, size = 25, normalized size = 2.27

$$2\log\left(\cos\left(\frac{x}{2}\right)\right) - \log\left(1 - 2\cos^2\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(Cot[x] + Csc[x]),x]
[Out] 2*Log[Cos[x/2]] - Log[1 - 2*Cos[x/2]^2]
```

Maple [A]
time = 0.13, size = 6, normalized size = 0.55

method	result	size
derivativedivides	$\ln(\sec(x)+1)$	6
default	$\ln(\sec(x)+1)$	6
risch	$2\ln(e^{ix}+1) - \ln(e^{2ix}+1)$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(cot(x)+csc(x)),x,method=_RETURNVERBOSE)
[Out] ln(sec(x)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(11) = 22$.
time = 0.27, size = 29, normalized size = 2.64

$$-\log\left(\frac{\sin(x)}{\cos(x)+1} + 1\right) - \log\left(\frac{\sin(x)}{\cos(x)+1} - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] -log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)/(cos(x) + 1) - 1)
```

Fricas [A]
time = 2.15, size = 15, normalized size = 1.36

$$-\log\left(-\cos\left(x\right)\right) + \log\left(\frac{1}{2}\cos\left(x\right) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(cot(x)+csc(x)),x, algorithm="fricas")
```

```
[Out] -log(-cos(x)) + log(1/2*cos(x) + 1/2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(x\right)}{\cot\left(x\right) + \csc\left(x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(cot(x)+csc(x)),x)
```

```
[Out] Integral(sec(x)/(cot(x) + csc(x)), x)
```

Giac [A]
time = 0.40, size = 12, normalized size = 1.09

$$\log\left(\cos\left(x\right) + 1\right) - \log\left(\left|\cos\left(x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(cot(x)+csc(x)),x, algorithm="giac")
```

```
[Out] log(cos(x) + 1) - log(abs(cos(x)))
```

Mupad [B]
time = 0.66, size = 11, normalized size = 1.00

$$-\ln\left(\tan\left(\frac{x}{2}\right)^2 - 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)*(cot(x) + 1/sin(x))),x)
```

```
[Out] -log(tan(x/2)^2 - 1)
```

3.208 $\int \frac{\csc(x)}{\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=9

$$\frac{\sin(x)}{1+\cos(x)}$$

```
[Out] sin(x)/(cos(x)+1)
```

Rubi [A]
time = 0.02, antiderivative size = 9, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3245, 2727}

$$\frac{\sin(x)}{\cos(x)+1}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[x]/(Cot[x] + Csc[x]),x]
```

```
[Out] Sin[x]/(1 + Cos[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 3245

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(x)}{\cot(x)+\csc(x)}\,dx &= \int \frac{1}{1+\cos(x)}\,dx\\ &= \frac{\sin(x)}{1+\cos(x)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 6, normalized size = 0.67

$$\tan\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(Cot[x] + Csc[x]),x]
```

```
[Out] Tan[x/2]
```

Maple [A]
time = 0.08, size = 5, normalized size = 0.56

method	result	size
default	$\tan\left(\frac{x}{2}\right)$	5
risch	$\frac{2i}{e^{ix}+1}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(cot(x)+csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] tan(1/2*x)
```

Maxima [A]
time = 0.27, size = 9, normalized size = 1.00

$$\frac{\sin\left(x\right)}{\cos\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] sin(x)/(cos(x) + 1)
```

Fricas [A]
time = 2.71, size = 9, normalized size = 1.00

$$\frac{\sin\left(x\right)}{\cos\left(x\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(cot(x)+csc(x)),x, algorithm="fricas")
```

```
[Out] sin(x)/(cos(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc\left(x\right)}{\cot\left(x\right)+\csc\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(cot(x)+csc(x)),x)
[Out] Integral(csc(x)/(cot(x) + csc(x)), x)
```

Giac [A]
time = 0.42, size = 4, normalized size = 0.44

$$\tan\left(\frac{1}{2}\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(cot(x)+csc(x)),x, algorithm="giac")
[Out] tan(1/2*x)
```

Mupad [B]
time = 0.54, size = 4, normalized size = 0.44

$$\tan\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)*(cot(x) + 1/sin(x))),x)
[Out] tan(x/2)
```

3.209 $\int \frac{\sin(x)}{-\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=4

$$x+\sin(x)$$

[Out] x+sin(x)

Rubi [A]
time = 0.05, antiderivative size = 4, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4477, 2761, 8}

$$x+\sin(x)$$

Antiderivative was successfully verified.

[In] Int[Sin[x]/(-Cot[x] + Csc[x]),x]

[Out] x + Sin[x]

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2761

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Di
st[g^2/a, Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x]
 && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{-\cot(x)+\csc(x)}\,dx &= \int \frac{\sin^2(x)}{1-\cos(x)}\,dx\\ &= \sin(x)+\int 1\,dx\\ &= x+\sin(x)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 14 vs. $2(4) = 8$.
time = 0.01, size = 14, normalized size = 3.50

$$2\left(\frac{x}{2}+\frac{\sin(x)}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/(-Cot[x] + Csc[x]),x]
```

```
[Out] 2*(x/2 + Sin[x]/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. $2(4) = 8$.
time = 0.13, size = 25, normalized size = 6.25

method	result	size
risch	$x+\sin(x)$	5
default	$\frac{2\tan(\frac{x}{2})}{1+\tan^2(\frac{x}{2})}+2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(-cot(x)+csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*tan(1/2*x)/(1+tan(1/2*x)^2)+2*arctan(tan(1/2*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(4) = 8$.
time = 0.46, size = 38, normalized size = 9.50

$$\frac{2\sin(x)}{\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)(\cos(x)+1)}+2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(-cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] 2*sin(x)/((sin(x)^2/(cos(x) + 1)^2 + 1)*(cos(x) + 1)) + 2*arctan(sin(x)/(co
s(x) + 1))
```

Fricas [A]
time = 3.08, size = 4, normalized size = 1.00

$$x+\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(-cot(x)+csc(x)),x, algorithm="fricas")
```

```
[Out] x + sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sin(x)}{\cot(x) - \csc(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(-cot(x)+csc(x)),x)
[Out] -Integral(sin(x)/(cot(x) - csc(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(4) = 8.
time = 0.41, size = 18, normalized size = 4.50

$$x + \frac{2\tan\left(\frac{1}{2}x\right)}{\tan\left(\frac{1}{2}x\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(-cot(x)+csc(x)),x, algorithm="giac")
[Out] x + 2*tan(1/2*x)/(tan(1/2*x)^2 + 1)
```

Mupad [B]
time = 1.03, size = 4, normalized size = 1.00

$$x + \sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sin(x)/(cot(x) - 1/sin(x)),x)
[Out] x + sin(x)
```

3.210 $\int \frac{\cos(x)}{-\cot(x)+\csc(x)} \, dx$

Optimal. Leaf size=10

$$\cos(x) + \log(1 - \cos(x))$$

```
[Out] cos(x)+ln(1-cos(x))
```

Rubi [A]

time = 0.05, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4477, 2912, 45}

$$\cos(x) + \log(1 - \cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(-Cot[x] + Csc[x]),x]
```

```
[Out] Cos[x] + Log[1 - Cos[x]]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2912

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d/b)*x)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{-\cot(x)+\csc(x)}\,dx &= \int \frac{\cos(x)\sin(x)}{1-\cos(x)}\,dx\\ &= -\text{Subst}\left(\int \frac{x}{1+x}\,dx, x, -\cos(x)\right)\\ &= -\text{Subst}\left(\int \left(1+\frac{1}{-1-x}\right)\,dx, x, -\cos(x)\right)\\ &= \cos(x)+\log(1-\cos(x))\end{aligned}$$

Mathematica [A]
time = 0.01, size = 20, normalized size = 2.00

$$2\log\left(\sin\left(\frac{x}{2}\right)\right)-2\sin^2\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(-Cot[x] + Csc[x]),x]
[Out] 2*Log[Sin[x/2]] - 2*Sin[x/2]^2
```

Maple [A]
time = 0.13, size = 9, normalized size = 0.90

method	result	size
derivativedivides	$\cos(x)+\ln(\cos(x)-1)$	9
default	$\cos(x)+\ln(\cos(x)-1)$	9
risch	$-ix+\frac{e^{ix}}{2}+\frac{e^{-ix}}{2}+2\ln(e^{ix}-1)$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(-cot(x)+csc(x)),x,method=_RETURNVERBOSE)
[Out] cos(x)+ln(cos(x)-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(10) = 20.
time = 0.48, size = 46, normalized size = 4.60

$$\frac{2}{\frac{\sin(x)^2}{(\cos(x)+1)^2}+1}+2\log\left(\frac{\sin(x)}{\cos(x)+1}\right)-\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(-cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] 2/(sin(x)^2/(cos(x) + 1)^2 + 1) + 2*log(sin(x)/(cos(x) + 1)) - log(sin(x)^2
/(cos(x) + 1)^2 + 1)
```

Fricas [A]
time = 3.18, size = 10, normalized size = 1.00

$$\cos(x) + \log\left(-\frac{1}{2}\cos(x) + \frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(-cot(x)+csc(x)),x, algorithm="fricas")
```

```
[Out] cos(x) + log(-1/2*cos(x) + 1/2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\cos(x)}{\cot(x) - \csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(-cot(x)+csc(x)),x)
```

```
[Out] -Integral(cos(x)/(cot(x) - csc(x)), x)
```

Giac [A]
time = 0.40, size = 10, normalized size = 1.00

$$\cos(x) + \log(-\cos(x) + 1)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(-cot(x)+csc(x)),x, algorithm="giac")
```

```
[Out] cos(x) + log(-cos(x) + 1)
```

Mupad [B]
time = 0.62, size = 31, normalized size = 3.10

$$2\,\ln\left(\tan\left(\frac{x}{2}\right)\right) - \ln\left(\tan\left(\frac{x}{2}\right)^2 + 1\right) + \frac{2}{\tan\left(\frac{x}{2}\right)^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(x)/(cot(x) - 1/sin(x)),x)
```

```
[Out] 2*log(tan(x/2)) - log(tan(x/2)^2 + 1) + 2/(tan(x/2)^2 + 1)
```

3.211 $\int \frac{\tan(x)}{-\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=5

$$x+\tanh^{-1}(\sin(x))$$

```
[Out] x+arctanh(sin(x))
```

Rubi [A]
time = 0.07, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {4477, 2918, 3855, 8}

$$x+\tanh^{-1}(\sin(x))$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(-Cot[x] + Csc[x]),x]
```

```
[Out] x + ArcTanh[Sin[x]]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2918

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\tan(x)}{-\cot(x)+\csc(x)}\,dx = \int \frac{\sin(x)\tan(x)}{1-\cos(x)}\,dx$$
$$= \int 1\,dx + \int \sec(x)\,dx$$
$$= x + \tanh^{-1}(\sin(x))$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 46 vs. 2(5) = 10.
time = 0.02, size = 46, normalized size = 9.20

$$2\left(\frac{x}{2}-\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\frac{1}{2}\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(-Cot[x] + Csc[x]),x]
```

```
[Out] 2*(x/2 - Log[Cos[x/2] - Sin[x/2]]/2 + Log[Cos[x/2] + Sin[x/2]]/2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(5) = 10.
time = 0.15, size = 25, normalized size = 5.00

method	result	size
risch	$x+\ln\left(e^{ix}+i\right)-\ln\left(e^{ix}-i\right)$	23
default	$\ln\left(\tan\left(\frac{x}{2}\right)+1\right)-\ln\left(\tan\left(\frac{x}{2}\right)-1\right)+2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	25

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(-cot(x)+csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(tan(1/2*x)+1)-ln(tan(1/2*x)-1)+2*arctan(tan(1/2*x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(5) = 10.
time = 0.47, size = 39, normalized size = 7.80

$$2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)+\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)-\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(-cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] 2*arctan(sin(x)/(cos(x) + 1)) + log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)/(
cos(x) + 1) - 1)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 18 vs. $2(5) = 10$.
time = 1.54, size = 18, normalized size = 3.60

$$x + \frac{1}{2}\log\left(\sin\left(x\right) + 1\right) - \frac{1}{2}\log\left(-\sin\left(x\right) + 1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(-cot(x)+csc(x)),x, algorithm="fricas")
```

```
[Out] x + 1/2*log(sin(x) + 1) - 1/2*log(-sin(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\tan\left(x\right)}{\cot\left(x\right) - \csc\left(x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(-cot(x)+csc(x)),x)
```

```
[Out] -Integral(tan(x)/(cot(x) - csc(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 20 vs. $2(5) = 10$.
time = 0.41, size = 20, normalized size = 4.00

$$x + \log\left(\left|\tan\left(\frac{1}{2}\,x\right) + 1\right|\right) - \log\left(\left|\tan\left(\frac{1}{2}\,x\right) - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(-cot(x)+csc(x)),x, algorithm="giac")
```

```
[Out] x + log(abs(tan(1/2*x) + 1)) - log(abs(tan(1/2*x) - 1))
```

Mupad [B]
time = 0.57, size = 9, normalized size = 1.80

$$x + 2\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-tan(x)/(cot(x) - 1/sin(x)),x)
```

```
[Out] x + 2*atanh(tan(x/2))
```

3.212 $\int \frac{\cot(x)}{-\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=16

$$-x-\frac{\sin(x)}{1-\cos(x)}$$

```
[Out] -x-sin(x)/(1-cos(x))
```

Rubi [A]
time = 0.04, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4477, 2814, 2727}

$$-x-\frac{\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(-Cot[x] + Csc[x]),x]
```

```
[Out] -x - Sin[x]/(1 - Cos[x])
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\int \frac{\cot(x)}{-\cot(x)+\csc(x)}\,dx = \int \frac{\cos(x)}{1-\cos(x)}\,dx$$
$$= -x + \int \frac{1}{1-\cos(x)}\,dx$$
$$= -x - \frac{\sin(x)}{1-\cos(x)}$$

Mathematica [A]
time = 0.02, size = 16, normalized size = 1.00

$$\frac{1}{2}\left(-2x - 2\cot\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(-Cot[x] + Csc[x]),x]
```

```
[Out] (-2*x - 2*Cot[x/2])/2
```

Maple [A]
time = 0.09, size = 17, normalized size = 1.06

method	result	size
default	$-\frac{1}{\tan\left(\frac{x}{2}\right)} - 2\arctan\left(\tan\left(\frac{x}{2}\right)\right)$	17
risch	$-x - \frac{2i}{e^{ix}-1}$	17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(-cot(x)+csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/tan(1/2*x)-2*arctan(tan(1/2*x))
```

Maxima [A]
time = 0.47, size = 23, normalized size = 1.44

$$-\frac{\cos(x)+1}{\sin(x)} - 2\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(-cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] -(cos(x) + 1)/sin(x) - 2*arctan(sin(x)/(cos(x) + 1))
```

Fricas [A]
time = 1.66, size = 14, normalized size = 0.88

$$-\frac{x\sin\left(x\right)+\cos\left(x\right)+1}{\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(-cot(x)+csc(x)),x, algorithm="fricas")
[Out] -(x*sin(x) + cos(x) + 1)/sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\cot\left(x\right)}{\cot\left(x\right)-\csc\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(-cot(x)+csc(x)),x)
[Out] -Integral(cot(x)/(cot(x) - csc(x)), x)
```

Giac [A]
time = 0.41, size = 12, normalized size = 0.75

$$-x-\frac{1}{\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(-cot(x)+csc(x)),x, algorithm="giac")
[Out] -x - 1/tan(1/2*x)
```

Mupad [B]
time = 0.58, size = 10, normalized size = 0.62

$$-x-\cot\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cot(x)/(cot(x) - 1/sin(x)),x)
[Out] - x - cot(x/2)
```

3.213 $\int \frac{\sec(x)}{-\cot(x)+\csc(x)}\,dx$

Optimal. Leaf size=13

$$\log(1-\cos(x)) - \log(\cos(x))$$

```
[Out] ln(1-cos(x))-ln(cos(x))
```

Rubi [A]
time = 0.04, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {4477, 2786, 36, 29, 31}

$$\log(1-\cos(x)) - \log(\cos(x))$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(-Cot[x] + Csc[x]),x]
```

```
[Out] Log[1 - Cos[x]] - Log[Cos[x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2786

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)^
((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 4477

```
Int[(cot[(c_.) + (d_.)*(x_)]^(n_.)*(a_.) + csc[(c_.) + (d_.)*(x_)]^(n_.)*(b
_.))^(p_)*(u_.), x_Symbol] :> Int[ActivateTrig[u]*Csc[c + d*x]^(n*p)*(b + a
*Cos[c + d*x]^n)^p, x] /; FreeQ[{a, b, c, d}, x] && IntegersQ[n, p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(x)}{-\cot(x)+\csc(x)}\,dx &= \int \frac{\tan(x)}{1-\cos(x)}\,dx\\
&= -\text{Subst}\left(\int \frac{1}{x(1+x)}\,dx, x, -\cos(x)\right)\\
&= -\text{Subst}\left(\int \frac{1}{x}\,dx, x, -\cos(x)\right) + \text{Subst}\left(\int \frac{1}{1+x}\,dx, x, -\cos(x)\right)\\
&= \log(1-\cos(x)) - \log(\cos(x))
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 1.92

$$2\log\left(\sin\left(\frac{x}{2}\right)\right) - \log\left(1-2\sin^2\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(-Cot[x] + Csc[x]),x]
```

```
[Out] 2*Log[Sin[x/2]] - Log[1 - 2*Sin[x/2]^2]
```

Maple [A]
time = 0.13, size = 6, normalized size = 0.46

method	result	size
derivativedivides	$\ln(-1+\sec(x))$	6
default	$\ln(-1+\sec(x))$	6
risch	$2\ln(e^{ix}-1)-\ln(e^{2ix}+1)$	22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(-cot(x)+csc(x)),x,method=_RETURNVERBOSE)
```

```
[Out] ln(-1+sec(x))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(13) = 26$.
time = 0.27, size = 41, normalized size = 3.15

$$-\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right) - \log\left(\frac{\sin(x)}{\cos(x)+1}-1\right) + 2\log\left(\frac{\sin(x)}{\cos(x)+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-cot(x)+csc(x)),x, algorithm="maxima")
```

```
[Out] -log(sin(x)/(cos(x) + 1) + 1) - log(sin(x)/(cos(x) + 1) - 1) + 2*log(sin(x)
/(cos(x) + 1))
```

Fricas [A]
time = 2.23, size = 15, normalized size = 1.15

$$-\log\left(-\cos\left(x\right)\right)+\log\left(-\frac{1}{2}\cos\left(x\right)+\frac{1}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-cot(x)+csc(x)),x, algorithm="fricas")
```

```
[Out] -log(-cos(x)) + log(-1/2*cos(x) + 1/2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sec\left(x\right)}{\cot\left(x\right)-\csc\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-cot(x)+csc(x)),x)
```

```
[Out] -Integral(sec(x)/(cot(x) - csc(x)), x)
```

Giac [A]
time = 0.41, size = 14, normalized size = 1.08

$$\log\left(-\cos\left(x\right)+1\right)-\log\left(\left|\cos\left(x\right)\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(-cot(x)+csc(x)),x, algorithm="giac")
```

```
[Out] log(-cos(x) + 1) - log(abs(cos(x)))
```

Mupad [B]
time = 0.61, size = 19, normalized size = 1.46

$$2\,\ln\left(\tan\left(\frac{x}{2}\right)\right)-\ln\left({\tan\left(\frac{x}{2}\right)}^2-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(x)*(cot(x) - 1/sin(x))),x)
```

```
[Out] 2*log(tan(x/2)) - log(tan(x/2)^2 - 1)
```

3.214 $\int \frac{\csc(x)}{-\cot(x)+\csc(x)} \, dx$

Optimal. Leaf size=12

$$-\frac{\sin(x)}{1-\cos(x)}$$

[Out] -sin(x)/(1-cos(x))

Rubi [A]
time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3245, 2727}

$$-\frac{\sin(x)}{1-\cos(x)}$$

Antiderivative was successfully verified.

[In] Int[Csc[x]/(-Cot[x] + Csc[x]),x]

[Out] -(Sin[x]/(1 - Cos[x]))

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 3245

```
Int[csc[(d_.) + (e_.)*(x_)]^(n_.)*((a_.) + csc[(d_.) + (e_.)*(x_)]*(b_.) +
cot[(d_.) + (e_.)*(x_)]*(c_.))^(m_), x_Symbol] :> Int[1/(b + a*Sin[d + e*x]
 + c*Cos[d + e*x])^n, x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + n, 0] && I
ntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(x)}{-\cot(x)+\csc(x)} \, dx &= \int \frac{1}{1-\cos(x)} \, dx\\ &= -\frac{\sin(x)}{1-\cos(x)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 8, normalized size = 0.67

$$-\cot\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[x]/(-Cot[x] + Csc[x]),x]
[Out] -Cot[x/2]
```

Maple [A]
time = 0.08, size = 9, normalized size = 0.75

method	result	size
default	$-\frac{1}{\tan\left(\frac{x}{2}\right)}$	9
risch	$-\frac{2i}{e^{ix}-1}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(x)/(-cot(x)+csc(x)),x,method=_RETURNVERBOSE)
[Out] -1/tan(1/2*x)
```

Maxima [A]
time = 0.27, size = 10, normalized size = 0.83

$$-\frac{\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(-cot(x)+csc(x)),x, algorithm="maxima")
[Out] -(cos(x) + 1)/sin(x)
```

Fricas [A]
time = 2.61, size = 10, normalized size = 0.83

$$-\frac{\cos(x)+1}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(-cot(x)+csc(x)),x, algorithm="fricas")
[Out] -(cos(x) + 1)/sin(x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\csc(x)}{\cot(x)-\csc(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(-cot(x)+csc(x)),x)
[Out] -Integral(csc(x)/(cot(x) - csc(x)), x)
```

Giac [A]
time = 0.42, size = 8, normalized size = 0.67

$$-\frac{1}{\tan\left(\frac{1}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(x)/(-cot(x)+csc(x)),x, algorithm="giac")
[Out] -1/tan(1/2*x)
```

Mupad [B]
time = 0.53, size = 6, normalized size = 0.50

$$-\cot\left(\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(x)*(cot(x) - 1/sin(x))),x)
[Out] -cot(x/2)
```

3.215 $\int \frac{1}{\csc(c+dx)+\sin(c+dx)}\,dx$

Optimal. Leaf size=23

$$-\frac{\tanh^{-1}\left(\frac{\cos(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}\,d}$$

```
[Out] -1/2*arctanh(1/2*cos(d*x+c)*2^(1/2))/d*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4482, 3265, 212}

$$-\frac{\tanh^{-1}\left(\frac{\cos(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(Csc[c + d*x] + Sin[c + d*x])^(-1),x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]/Sqrt[2]]/(Sqrt[2]*d))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3265

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, Dist[-ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{1}{\csc(c+dx)+\sin(c+dx)}\,dx = \int \frac{\sin(c+dx)}{1+\sin^2(c+dx)}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{\tanh^{-1}\left(\frac{\cos(c+dx)}{\sqrt{2}}\right)}{\sqrt{2}\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.22, size = 61, normalized size = 2.65

$$-\frac{\tanh^{-1}\left(\frac{\cos(c)-(-i+\sin(c))\tan\left(\frac{dx}{2}\right)}{\sqrt{2}}\right)+\tanh^{-1}\left(\frac{\cos(c)-(i+\sin(c))\tan\left(\frac{dx}{2}\right)}{\sqrt{2}}\right)}{\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x] + Sin[c + d*x])^(-1),x]
```

```
[Out] -((ArcTanh[(Cos[c] - (-I + Sin[c])*Tan[(d*x)/2])/Sqrt[2]] + ArcTanh[(Cos[c]
 - (I + Sin[c])*Tan[(d*x)/2])/Sqrt[2]])/(Sqrt[2]*d))
```

Maple [A]
time = 0.18, size = 21, normalized size = 0.91

method	result	size
derivativedivides	$-\frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{2}}{2}\right)\sqrt{2}}{2d}$	21
default	$-\frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)\sqrt{2}}{2}\right)\sqrt{2}}{2d}$	21
risch	$\frac{\sqrt{2}\,\ln\left(e^{2i(dx+c)}-2\sqrt{2}\,e^{i(dx+c)}+1\right)}{4d}-\frac{\sqrt{2}\,\ln\left(e^{2i(dx+c)}+2\sqrt{2}\,e^{i(dx+c)}+1\right)}{4d}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(d*x+c)+sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*arctanh(1/2*cos(d*x+c)*2^(1/2))/d*2^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(20) = 40.

time = 0.52, size = 176, normalized size = 7.65

$$\frac{\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}+1\right)\cos\left(dx+c\right)-\cos\left(dx+c\right)^{2}-\sin\left(dx+c\right)^{2}-2\sqrt{2}-3}{2\left(\sqrt{2}-1\right)\cos\left(dx+c\right)+\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}-2\sqrt{2}+3}\right)+\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}-1\right)\cos\left(dx+c\right)-\cos\left(dx+c\right)^{2}-\sin\left(dx+c\right)^{2}+2\sqrt{2}-3}{2\left(\sqrt{2}+1\right)\cos\left(dx+c\right)+\cos\left(dx+c\right)^{2}+\sin\left(dx+c\right)^{2}+2\sqrt{2}+3}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(d*x+c)+sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/8*(sqrt(2)*log(-(2*(sqrt(2) + 1)*cos(d*x + c) - cos(d*x + c)^2 - sin(d*x
+ c)^2 - 2*sqrt(2) - 3)/(2*(sqrt(2) - 1)*cos(d*x + c) + cos(d*x + c)^2 + si
n(d*x + c)^2 - 2*sqrt(2) + 3)) + sqrt(2)*log(-(2*(sqrt(2) - 1)*cos(d*x + c)
 - cos(d*x + c)^2 - sin(d*x + c)^2 + 2*sqrt(2) - 3)/(2*(sqrt(2) + 1)*cos(d*
x + c) + cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sqrt(2) + 3)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(20) = 40.
time = 2.65, size = 44, normalized size = 1.91

$$\frac{\sqrt{2}\log\left(-\frac{\cos\left(dx+c\right)^{2}-2\sqrt{2}\cos\left(dx+c\right)+2}{\cos\left(dx+c\right)^{2}-2}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(d*x+c)+sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*cos(d*x + c) + 2)/(cos(d*x + c
)^2 - 2))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sin\left(c+dx\right)+\csc\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] Integral(1/(sin(c + d*x) + csc(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(20) = 40.
time = 0.43, size = 68, normalized size = 2.96

$$\frac{\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}-\frac{2\left(\cos\left(dx+c\right)-1\right)}{\cos\left(dx+c\right)+1}+6\right|}{\left|4\sqrt{2}-\frac{2\left(\cos\left(dx+c\right)-1\right)}{\cos\left(dx+c\right)+1}+6\right|}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(d*x+c)+sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*log(abs(-4*sqrt(2) - 2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
6)/abs(4*sqrt(2) - 2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 6))/d
```

Mupad [B]
time = 0.66, size = 42, normalized size = 1.83

$$\frac{\sqrt{2}\ \mathrm{atanh}\left(\frac{2\sqrt{2}\ \sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x) + 1/sin(c + d*x)),x)
```

```
[Out] (2^(1/2)*atanh((2*2^(1/2)*sin(c/2 + (d*x)/2)^2)/(2*sin(c/2 + (d*x)/2)^2 + 1
)))/(2*d)
```

3.216 $\int \frac{\sin(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx$

Optimal. Leaf size=51

$$x-\frac{x}{\sqrt{2}}-\frac{\text{ArcTan}\left(\frac{\cos(c+dx)\sin(c+dx)}{1+\sqrt{2}+\sin^2(c+dx)}\right)}{\sqrt{2}\,d}$$

```
[Out] x-1/2*x*2^(1/2)-1/2*arctan(cos(d*x+c)*sin(d*x+c)/(1+sin(d*x+c)^2+2^(1/2)))/
d*2^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1144, 209}

$$-\frac{\text{ArcTan}\left(\frac{\sin(c+dx)\cos(c+dx)}{\sin^2(c+dx)+\sqrt{2}+1}\right)}{\sqrt{2}\,d}-\frac{x}{\sqrt{2}}+x$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] x - x/Sqrt[2] - ArcTan[(Cos[c + d*x]*Sin[c + d*x])/(1 + Sqrt[2] + Sin[c + d
*x]^2)]/(Sqrt[2]*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{1+3x^2+2x^4}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \tan(c+dx)\right)}{d} + \frac{2\text{Subst}\left(\int \frac{1}{2+2x^2}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= x - \frac{x}{\sqrt{2}} - \frac{\tan^{-1}\left(\frac{\cos(c+dx)\sin(c+dx)}{1+\sqrt{2}+\sin^2(c+dx)}\right)}{\sqrt{2}\,d}$$

Mathematica [A]
time = 0.08, size = 30, normalized size = 0.59

$$\frac{c}{d} + x - \frac{\text{ArcTan}\left(\sqrt{2}\tan(c+dx)\right)}{\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
[Out] c/d + x - ArcTan[Sqrt[2]*Tan[c + d*x]]/(Sqrt[2]*d)
```

Maple [A]
time = 0.19, size = 29, normalized size = 0.57

method	result	size
derivativedivides	$\frac{-\frac{\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{2}\right)}{2}+\arctan(\tan(dx+c))}{d}$	29
default	$\frac{-\frac{\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{2}\right)}{2}+\arctan(\tan(dx+c))}{d}$	29
risch	$x - \frac{i\sqrt{2}\,\ln\left(e^{2i(dx+c)}-2\sqrt{2}-3\right)}{4d} + \frac{i\sqrt{2}\,\ln\left(e^{2i(dx+c)}+2\sqrt{2}-3\right)}{4d}$	55

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/2*2^(1/2)*arctan(tan(d*x+c)*2^(1/2))+arctan(tan(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. 2(45) = 90.
time = 0.50, size = 252, normalized size = 4.94

$$\frac{4\,dx - \sqrt{2}\arctan\left(\frac{2\sqrt{2}\sin(dx+c)}{2\left(\sqrt{2}+1\right)\cos(dx+c)+\cos(dx+c)^2+\sin(dx+c)^2+2\sqrt{2}+3}, \frac{\cos(dx+c)^2+\sin(dx+c)^2+2\cos(dx+c)-1}{2\left(\sqrt{2}+1\right)\cos(dx+c)+\cos(dx+c)^2+\sin(dx+c)^2+2\sqrt{2}+3}\right) + \sqrt{2}\arctan\left(\frac{2\sqrt{2}\sin(dx+c)}{2\left(\sqrt{2}-1\right)\cos(dx+c)+\cos(dx+c)^2+\sin(dx+c)^2-2\sqrt{2}+3}, \frac{\cos(dx+c)^2+\sin(dx+c)^2-2\cos(dx+c)-1}{2\left(\sqrt{2}-1\right)\cos(dx+c)+\cos(dx+c)^2+\sin(dx+c)^2-2\sqrt{2}+3}\right) + 4\,c}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*d*x - sqrt(2)*arctan2(2*sqrt(2)*sin(d*x + c)/(2*(sqrt(2) + 1)*cos(d*
x + c) + cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sqrt(2) + 3), (cos(d*x + c)^2
+ sin(d*x + c)^2 + 2*cos(d*x + c) - 1)/(2*(sqrt(2) + 1)*cos(d*x + c) + cos(
d*x + c)^2 + sin(d*x + c)^2 + 2*sqrt(2) + 3)) + sqrt(2)*arctan2(2*sqrt(2)*s
in(d*x + c)/(2*(sqrt(2) - 1)*cos(d*x + c) + cos(d*x + c)^2 + sin(d*x + c)^2
 - 2*sqrt(2) + 3), (cos(d*x + c)^2 + sin(d*x + c)^2 - 2*cos(d*x + c) - 1)/(
2*(sqrt(2) - 1)*cos(d*x + c) + cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sqrt(2)
+ 3)) + 4*c)/d
```

Fricas [A]
time = 2.51, size = 52, normalized size = 1.02

$$\frac{4\,dx+\sqrt{2}\,\arctan\left(\frac{3\,\sqrt{2}\,\cos(dx+c)^2-2\,\sqrt{2}}{4\,\cos(dx+c)\sin(dx+c)}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*d*x + sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(d*x + c)^2 - 2*sqrt(2))/(cos
(d*x + c)*sin(d*x + c))))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(c+dx\right)}{\sin\left(c+dx\right)+\csc\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)/(sin(c + d*x) + csc(c + d*x)), x)
```

Giac [A]
time = 0.43, size = 82, normalized size = 1.61

$$\frac{2\,dx-\sqrt{2}\left(dx+c+\arctan\left(-\frac{\sqrt{2}\,\sin(2\,dx+2\,c)-2\,\sin(2\,dx+2\,c)}{\sqrt{2}\,\cos(2\,dx+2\,c)+\sqrt{2}\,-2\,\cos(2\,dx+2\,c)+2}\right)\right)+2\,c}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*d*x - sqrt(2)*(d*x + c + arctan(-(sqrt(2)*sin(2*d*x + 2*c) - 2*sin(2
*d*x + 2*c))/(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2) - 2*cos(2*d*x + 2*c) + 2))
) + 2*c)/d
```

Mupad [B]
time = 0.62, size = 62, normalized size = 1.22

$$x-\frac{\sqrt{2}\,\left(2\,\mathrm{atan}\left(\frac{\sqrt{2}\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{4}+\frac{7\,\sqrt{2}\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}\right)+2\,\mathrm{atan}\left(\frac{\sqrt{2}\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(sin(c + d*x) + 1/sin(c + d*x)),x)
```

```
[Out] x - (2^(1/2)*(2*atan((7*2^(1/2)*tan(c/2 + (d*x)/2))/4 + (2^(1/2)*tan(c/2 +
(d*x)/2)^3)/4) + 2*atan((2^(1/2)*tan(c/2 + (d*x)/2))/4)))/(4*d)
```

3.217 $\int \frac{\cos(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx$

Optimal. Leaf size=18

$$\frac{\log\left(1+\sin^2(c+dx)\right)}{2d}$$

```
[Out] 1/2*ln(1+sin(d*x+c)^2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4419, 266}

$$\frac{\log\left(\sin^2(c+dx)+1\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] Log[1 + Sin[c + d*x]^2]/(2*d)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4419

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{1+x^2}\,dx, x, \sin(c+dx)\right)}{d}\\ &= \frac{\log\left(1+\sin^2(c+dx)\right)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.12, size = 20, normalized size = 1.11

$$\frac{\log(3-\cos(2(c+dx)))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
[Out] Log[3 - Cos[2*(c + d*x)]]/(2*d)
```

Maple [A]
time = 0.11, size = 17, normalized size = 0.94

method	result	size
derivativedivides	$\frac{\ln\left(\cos^2(dx+c)-2\right)}{2d}$	17
default	$\frac{\ln\left(\cos^2(dx+c)-2\right)}{2d}$	17
risch	$-ix-\frac{2ic}{d}+\frac{\ln\left(e^{4i(dx+c)}-6\,e^{2i(dx+c)}+1\right)}{2d}$	41
norman	$-\frac{\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{\ln\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)+6\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)}{2d}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/2/d*ln(cos(d*x+c)^2-2)
```

Maxima [A]
time = 0.48, size = 16, normalized size = 0.89

$$\frac{\log\left(\sin\left(dx+c\right)^2+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="maxima")
[Out] 1/2*log(sin(d*x + c)^2 + 1)/d
```

Fricas [A]
time = 2.03, size = 18, normalized size = 1.00

$$\frac{\log\left(-\cos\left(dx+c\right)^2+2\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="fricas")
[Out] 1/2*log(-cos(d*x + c)^2 + 2)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(c+dx\right)}{\sin\left(c+dx\right)+\csc\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)/(sin(c + d*x) + csc(c + d*x)), x)
```

Giac [A]
time = 0.41, size = 16, normalized size = 0.89

$$\frac{\log\left(\sin\left(dx+c\right)^{2}+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*log(sin(d*x + c)^2 + 1)/d
```

Mupad [B]
time = 0.06, size = 16, normalized size = 0.89

$$\frac{\ln\left(\sin\left(c+d\,x\right)^{2}+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(sin(c + d*x) + 1/sin(c + d*x)),x)
```

```
[Out] log(sin(c + d*x)^2 + 1)/(2*d)
```

3.218 $\int \frac{\tan(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx$

Optimal. Leaf size=29

$$-\frac{\text{ArcTan}(\sin(c+dx))}{2d}+\frac{\tanh^{-1}(\sin(c+dx))}{2d}$$

```
[Out] -1/2*arctan(sin(d*x+c))/d+1/2*arctanh(sin(d*x+c))/d
```

Rubi [A]
time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {304, 209, 212}

$$\frac{\tanh^{-1}(\sin(c+dx))}{2d}-\frac{\text{ArcTan}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] -1/2*ArcTan[Sin[c + d*x]]/d + ArcTanh[Sin[c + d*x]]/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 304

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x], x
] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx,x,\sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sin(c+dx)\right)}{2d} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sin(c+dx)\right)}{2d}$$
$$= -\frac{\tan^{-1}(\sin(c+dx))}{2d} + \frac{\tanh^{-1}(\sin(c+dx))}{2d}$$

Mathematica [A]
time = 0.04, size = 24, normalized size = 0.83

$$\frac{-\text{ArcTan}(\sin(c+dx))+\tanh^{-1}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
[Out] (-ArcTan[Sin[c + d*x]] + ArcTanh[Sin[c + d*x]])/(2*d)
```

Maple [A]
time = 0.22, size = 37, normalized size = 1.28

method	result	size
derivativedivides	$\frac{-\frac{\ln(\sin(dx+c)-1)}{4}-\frac{\arctan(\sin(dx+c))}{2}+\frac{\ln(\sin(dx+c)+1)}{4}}{d}$	37
default	$\frac{-\frac{\ln(\sin(dx+c)-1)}{4}-\frac{\arctan(\sin(dx+c))}{2}+\frac{\ln(\sin(dx+c)+1)}{4}}{d}$	37
risch	$\frac{\ln\left(e^{i(dx+c)}+i\right)}{2d}-\frac{\ln\left(e^{i(dx+c)}-i\right)}{2d}+\frac{i\ln\left(e^{2i(dx+c)}+2\,e^{i(dx+c)}-1\right)}{4d}-\frac{i\ln\left(e^{2i(dx+c)}-2\,e^{i(dx+c)}-1\right)}{4d}$	96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(-1/4*ln(sin(d*x+c)-1)-1/2*arctan(sin(d*x+c))+1/4*ln(sin(d*x+c)+1))
```

Maxima [A]
time = 0.50, size = 35, normalized size = 1.21

$$-\frac{2\arctan\left(\sin\left(dx+c\right)\right)-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(2*arctan(sin(d*x + c)) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1
))/d
```

Fricas [A]
time = 2.44, size = 37, normalized size = 1.28

$$-\frac{2 \arctan \left(\sin \left(dx+c\right)\right)-\log \left(\sin \left(dx+c\right)+1\right)+\log \left(-\sin \left(dx+c\right)+1\right)}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*arctan(sin(d*x + c)) - log(sin(d*x + c) + 1) + log(-sin(d*x + c) +
1))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan \left(c+dx\right)}{\sin \left(c+dx\right)+\csc \left(c+dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)/(sin(c + d*x) + csc(c + d*x)), x)
```

Giac [A]
time = 0.46, size = 37, normalized size = 1.28

$$-\frac{2 \arctan \left(\sin \left(dx+c\right)\right)-\log \left(\left|\sin \left(dx+c\right)+1\right|\right)+\log \left(\left|\sin \left(dx+c\right)-1\right|\right)}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(2*arctan(sin(d*x + c)) - log(abs(sin(d*x + c) + 1)) + log(abs(sin(d*x
 + c) - 1)))/d
```

Mupad [B]
time = 0.69, size = 61, normalized size = 2.10

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{\mathrm{atan}\left(\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3}{2}+\frac{5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)-\mathrm{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(sin(c + d*x) + 1/sin(c + d*x)),x)
```

```
[Out] atanh(tan(c/2 + (d*x)/2))/d - (atan((5*tan(c/2 + (d*x)/2))/2 + tan(c/2 + (d
*x)/2)^3/2) - atan(tan(c/2 + (d*x)/2)/2))/(2*d)
```

3.219 $\int \frac{\cot(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx$

Optimal. Leaf size=11

$$\frac{\text{ArcTan}(\sin(c+dx))}{d}$$

```
[Out] arctan(sin(d*x+c))/d
```

Rubi [A]
time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4423, 209}

$$\frac{\text{ArcTan}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] ArcTan[Sin[c + d*x]]/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 4423

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sin(c+dx)\right)}{d}\\
&= \frac{\tan^{-1}(\sin(c+dx))}{d}
\end{aligned}$$

Mathematica [A]
time = 0.04, size = 11, normalized size = 1.00

$$\frac{\text{ArcTan}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] ArcTan[Sin[c + d*x]]/d
```

Maple [A]
time = 0.11, size = 13, normalized size = 1.18

method	result	size
derivativedivides	$-\frac{\arctan(\csc(dx+c))}{d}$	13
default	$-\frac{\arctan(\csc(dx+c))}{d}$	13
risch	$-\frac{i\ln\left(\mathrm{e}^{2i(dx+c)}+2\,\mathrm{e}^{i(dx+c)}-1\right)}{2d}+\frac{i\ln\left(\mathrm{e}^{2i(dx+c)}-2\,\mathrm{e}^{i(dx+c)}-1\right)}{2d}$	60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*arctan(csc(d*x+c))
```

Maxima [A]
time = 0.48, size = 11, normalized size = 1.00

$$\frac{\arctan\left(\sin\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] arctan(sin(d*x + c))/d
```

Fricas [A]
time = 2.99, size = 11, normalized size = 1.00

$$\frac{\arctan\left(\sin\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] arctan(sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot\left(c+dx\right)}{\sin\left(c+dx\right)+\csc\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)/(sin(c + d*x) + csc(c + d*x)), x)
```

Giac [A]
time = 0.42, size = 11, normalized size = 1.00

$$\frac{\arctan\left(\sin\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="giac")
```

```
[Out] arctan(sin(d*x + c))/d
```

Mupad [B]
time = 0.67, size = 45, normalized size = 4.09

$$\frac{\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{2}+\frac{5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)-\operatorname{atan}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(sin(c + d*x) + 1/sin(c + d*x)),x)
```

```
[Out] (atan((5*tan(c/2 + (d*x)/2))/2 + tan(c/2 + (d*x)/2)^3/2) - atan(tan(c/2 + (
d*x)/2)/2))/d
```

3.220 $\int \frac{\sec(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx$

Optimal. Leaf size=16

$$\frac{\tanh^{-1}\left(\sin^2(c+dx)\right)}{2d}$$

```
[Out] 1/2*arctanh(sin(d*x+c)^2)/d
```

Rubi [A]
time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {281, 212}

$$\frac{\tanh^{-1}\left(\sin^2(c+dx)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]^2]/(2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 281

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{1-x^4}\,dx, x, \sin(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin^2(c+dx)\right)}{2d}\\
&= \frac{\tanh^{-1}\left(\sin^2(c+dx)\right)}{2d}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 30, normalized size = 1.88

$$\frac{-2\log(\cos(c+dx))+\log\left(2-\cos^2(c+dx)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
[Out] (-2*Log[Cos[c + d*x]] + Log[2 - Cos[c + d*x]^2])/(4*d)
```

Maple [A]
time = 0.13, size = 19, normalized size = 1.19

method	result	size
derivativedivides	$\frac{\ln\left(2\left(\sec^2(dx+c)\right)-1\right)}{4d}$	19
default	$\frac{\ln\left(2\left(\sec^2(dx+c)\right)-1\right)}{4d}$	19
risch	$-\frac{\ln\left(e^{2i(dx+c)}+1\right)}{2d}+\frac{\ln\left(e^{4i(dx+c)}-6\,e^{2i(dx+c)}+1\right)}{4d}$	47
norman	$-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}+\frac{\ln\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)+6\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)}{4d}$	68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/4/d*ln(2*sec(d*x+c)^2-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 39 vs. 2(14) = 28.
time = 0.47, size = 39, normalized size = 2.44

$$\frac{\log\left(\sin\left(dx+c\right)^2+1\right)-\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="maxima")
[Out] 1/4*(log(sin(d*x + c)^2 + 1) - log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(14) = 28.
time = 3.61, size = 30, normalized size = 1.88

$$\frac{\log\left(-\cos\left(dx+c\right)^2+2\right)-2\,\log\left(-\cos\left(dx+c\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(log(-cos(d*x + c)^2 + 2) - 2*log(-cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{\sin(c+dx)+\csc(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(sin(c + d*x) + csc(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(14) = 28.
time = 0.44, size = 79, normalized size = 4.94

$$-\frac{2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\log\left(\left|-\frac{6(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+1\right|\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - log(abs(-6*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) + (cos(d*x + c) - 1)^2/(cos(d*x + c) +
 1)^2 + 1)))/d
```

Mupad [B]
time = 0.61, size = 14, normalized size = 0.88

$$\frac{\operatorname{atanh}\left(\sin(c+d\,x)^2\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(sin(c + d*x) + 1/sin(c + d*x))),x)
```

```
[Out] atanh(sin(c + d*x)^2)/(2*d)
```

3.221 $\int \frac{\csc(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx$

Optimal. Leaf size=48

$$\frac{x}{\sqrt{2}}+\frac{\text{ArcTan}\left(\frac{\cos(c+dx)\sin(c+dx)}{1+\sqrt{2}+\sin^2(c+dx)}\right)}{\sqrt{2}\,d}$$

```
[Out] 1/2*x*2^(1/2)+1/2*arctan(cos(d*x+c)*sin(d*x+c)/(1+sin(d*x+c)^2+2^(1/2)))/d*
2^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.045, Rules used = {209}

$$\frac{\text{ArcTan}\left(\frac{\sin(c+dx)\cos(c+dx)}{\sin^2(c+dx)+\sqrt{2}+1}\right)}{\sqrt{2}\,d}+\frac{x}{\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] x/Sqrt[2] + ArcTan[(Cos[c + d*x]*Sin[c + d*x])/(1 + Sqrt[2] + Sin[c + d*x]^
2)]/(Sqrt[2]*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(c+dx)}{\csc(c+dx)+\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \tan(c+dx)\right)}{d}\\ &= \frac{x}{\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\cos(c+dx)\sin(c+dx)}{1+\sqrt{2}+\sin^2(c+dx)}\right)}{\sqrt{2}\,d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 22, normalized size = 0.46

$$\frac{\text{ArcTan}\left(\sqrt{2}\,\tan(c+dx)\right)}{\sqrt{2}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(Csc[c + d*x] + Sin[c + d*x]),x]
```

```
[Out] ArcTan[Sqrt[2]*Tan[c + d*x]]/(Sqrt[2]*d)
```

Maple [A]
time = 0.17, size = 20, normalized size = 0.42

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{2}\right)}{2d}$	20
default	$\frac{\sqrt{2}\,\arctan\left(\tan(dx+c)\sqrt{2}\right)}{2d}$	20
risch	$\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2i(dx+c)}-2\sqrt{2}-3\right)}{4d}-\frac{i\sqrt{2}\,\ln\left(\mathrm{e}^{2i(dx+c)}+2\sqrt{2}-3\right)}{4d}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d*2^(1/2)*arctan(tan(d*x+c)*2^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 245 vs. 2(44) = 88.
time = 0.50, size = 245, normalized size = 5.10

$$\frac{\sqrt{2}\,\arctan\left(\frac{2\sqrt{2}\,\sin(dx+c)}{2\left(\sqrt{2}+1\right)\cos(dx+c)+\cos(dx+c)^2+\sin(dx+c)^2+2\sqrt{2}+3},\frac{\cos(dx+c)^2+\sin(dx+c)^2+2\cos(dx+c)-1}{2\left(\sqrt{2}+1\right)\cos(dx+c)+\cos(dx+c)^2+\sin(dx+c)^2+2\sqrt{2}+3}\right)-\sqrt{2}\,\arctan\left(\frac{2\sqrt{2}\,\sin(dx+c)}{2\left(\sqrt{2}-1\right)\cos(dx+c)+\cos(dx+c)^2+\sin(dx+c)^2-2\sqrt{2}+3},\frac{\cos(dx+c)^2+\sin(dx+c)^2-2\cos(dx+c)-1}{2\left(\sqrt{2}-1\right)\cos(dx+c)+\cos(dx+c)^2+\sin(dx+c)^2-2\sqrt{2}+3}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(sqrt(2)*arctan2(2*sqrt(2)*sin(d*x + c)/(2*(sqrt(2) + 1)*cos(d*x + c) +
 cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sqrt(2) + 3), (cos(d*x + c)^2 + sin(d*
x + c)^2 + 2*cos(d*x + c) - 1)/(2*(sqrt(2) + 1)*cos(d*x + c) + cos(d*x + c)
^2 + sin(d*x + c)^2 + 2*sqrt(2) + 3)) - sqrt(2)*arctan2(2*sqrt(2)*sin(d*x +
 c)/(2*(sqrt(2) - 1)*cos(d*x + c) + cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sqr
t(2) + 3), (cos(d*x + c)^2 + sin(d*x + c)^2 - 2*cos(d*x + c) - 1)/(2*(sqrt(
2) - 1)*cos(d*x + c) + cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sqrt(2) + 3)))/d
```

Fricas [A]
time = 2.77, size = 46, normalized size = 0.96

$$-\frac{\sqrt{2}\,\arctan\left(\frac{3\sqrt{2}\,\cos(dx+c)^2-2\sqrt{2}}{4\,\cos(dx+c)\sin(dx+c)}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(d*x + c)^2 - 2*sqrt(2))/(cos(d*x + c
)*sin(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)}{\sin(c+dx)+\csc(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)/(sin(c + d*x) + csc(c + d*x)), x)
```

Giac [A]
time = 0.41, size = 72, normalized size = 1.50

$$\frac{\sqrt{2}\left(dx+c+\arctan\left(-\frac{\sqrt{2}\sin(2\,dx+2\,c)-2\sin(2\,dx+2\,c)}{\sqrt{2}\cos(2\,dx+2\,c)+\sqrt{2}-2\cos(2\,dx+2\,c)+2}\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(csc(d*x+c)+sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(d*x + c + arctan(-(sqrt(2)*sin(2*d*x + 2*c) - 2*sin(2*d*x + 2*
c))/(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2) - 2*cos(2*d*x + 2*c) + 2)))/d
```

Mupad [B]
time = 0.71, size = 56, normalized size = 1.17

$$\frac{\sqrt{2}\left(\mathrm{atan}\left(\frac{\sqrt{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{7\sqrt{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}\right)+\mathrm{atan}\left(\frac{\sqrt{2}\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(sin(c + d*x) + 1/sin(c + d*x))),x)
```

```
[Out] (2^(1/2)*(atan((7*2^(1/2)*tan(c/2 + (d*x)/2))/4 + (2^(1/2)*tan(c/2 + (d*x)/
2)^3)/4) + atan((2^(1/2)*tan(c/2 + (d*x)/2))/4)))/(2*d)
```

3.222 $\int \frac{1}{\csc(c+dx)-\sin(c+dx)}\,dx$

Optimal. Leaf size=10

$$\frac{\sec(c+dx)}{d}$$

[Out] sec(d*x+c)/d

Rubi [A]

time = 0.02, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {4482, 2686, 8}

$$\frac{\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[(Csc[c + d*x] - Sin[c + d*x])^(-1),x]

[Out] Sec[c + d*x]/d

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\csc(c+dx)-\sin(c+dx)}\,dx &= \int \sec(c+dx)\tan(c+dx)\,dx\\ &= \frac{\text{Subst}\left(\int 1\,dx, x, \sec(c+dx)\right)}{d}\\ &= \frac{\sec(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 1.00

$$\frac{\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Csc[c + d*x] - Sin[c + d*x])^(-1),x]
```

```
[Out] Sec[c + d*x]/d
```

Maple [A]
time = 0.14, size = 13, normalized size = 1.30

method	result	size
derivativedivides	$\frac{1}{d\cos(dx+c)}$	13
default	$\frac{1}{d\cos(dx+c)}$	13
norman	$-\frac{2}{d\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	21
risch	$\frac{2\,e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)}$	28

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(csc(d*x+c)-sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/cos(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(10) = 20$.
time = 0.26, size = 28, normalized size = 2.80

$$-\frac{2}{d\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(d*x+c)-sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2/(d*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1))
```

Fricas [A]
time = 2.26, size = 12, normalized size = 1.20

$$\frac{1}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(d*x+c)-sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{-\sin(c+dx)+\csc(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(d*x+c)-sin(d*x+c)),x)
```

```
[Out] Integral(1/(-sin(c + d*x) + csc(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(10) = 20$.
time = 0.41, size = 28, normalized size = 2.80

$$\frac{2}{d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(csc(d*x+c)-sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 2/(d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))
```

Mupad [B]
time = 0.66, size = 20, normalized size = 2.00

$$-\frac{2}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(c + d*x) - 1/sin(c + d*x)),x)
```

```
[Out] -2/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.223 $\int \frac{\sin(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx$

Optimal. Leaf size=14

$$-x+\frac{\tan(c+dx)}{d}$$

```
[Out] -x+tan(d*x+c)/d
```

Rubi [A]
time = 0.10, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {327, 209}

$$\frac{\tan(c+dx)}{d}-x$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

```
[Out] -x + Tan[c + d*x]/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}\\
&= \frac{\tan(c+dx)}{d}-\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}\\
&= -x+\frac{\tan(c+dx)}{d}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.64

$$-\frac{\text{ArcTan}(\tan(c+dx))}{d}+\frac{\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
[Out] -(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d
```

Maple [A]
time = 0.16, size = 21, normalized size = 1.50

method	result	size
derivativedivides	$\frac{\tan(dx+c)-\arctan(\tan(dx+c))}{d}$	21
default	$\frac{\tan(dx+c)-\arctan(\tan(dx+c))}{d}$	21
risch	$-x+\frac{2i}{d\left(e^{2i(dx+c)}+1\right)}$	24
norman	$\frac{x-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(tan(d*x+c)-arctan(tan(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 64 vs. $2(14) = 28$.
time = 0.47, size = 64, normalized size = 4.57

$$-\frac{2\left(\frac{\sin(dx+c)}{\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)}+\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="maxima")
[Out] -2*(sin(d*x + c)/((sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) +
 1)) + arctan(sin(d*x + c)/(cos(d*x + c) + 1)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.
time = 2.55, size = 31, normalized size = 2.21

$$-\frac{dx\cos(dx+c)-\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(d*x*cos(d*x + c) - sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)}{-\sin(c+dx)+\csc(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)/(-sin(c + d*x) + csc(c + d*x)), x)
```

Giac [A]
time = 0.44, size = 18, normalized size = 1.29

$$-\frac{dx+c-\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -(d*x + c - tan(d*x + c))/d
```

Mupad [B]
time = 0.62, size = 33, normalized size = 2.36

$$-x-\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-sin(c + d*x)/(sin(c + d*x) - 1/sin(c + d*x)),x)
```

```
[Out] - x - (2*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.224 $\int \frac{\cos(c+dx)}{\csc(c+dx)-\sin(c+dx)} \, dx$

Optimal. Leaf size=12

$$-\frac{\log(\cos(c+dx))}{d}$$

```
[Out] -ln(cos(d*x+c))/d
```

Rubi [A]
time = 0.02, antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4419, 266}

$$-\frac{\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

```
[Out] -(Log[Cos[c + d*x]]/d)
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 4419

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[d/(b*c), Subst[Int[SubstFor[1, Sin[c*(a + b*
x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*x)
]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cos] || EqQ[F, cos])
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)}{\csc(c+dx)-\sin(c+dx)} \, dx &= \frac{\text{Subst}\left(\int \frac{x}{1-x^2} \, dx, x, \sin(c+dx)\right)}{d}\\ &= -\frac{\log(\cos(c+dx))}{d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 12, normalized size = 1.00

$$-\frac{\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
[Out] -(Log[Cos[c + d*x]]/d)
```

Maple [A]
time = 0.11, size = 13, normalized size = 1.08

method	result	size
derivativedivides	$-\frac{\ln(\cos(dx+c))}{d}$	13
default	$-\frac{\ln(\cos(dx+c))}{d}$	13
risch	$ix+\frac{2ic}{d}-\frac{\ln\left(e^{2i(dx+c)}+1\right)}{d}$	30
norman	$\frac{\ln\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -ln(cos(d*x+c))/d
```

Maxima [A]
time = 0.27, size = 24, normalized size = 2.00

$$-\frac{\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="maxima")
[Out] -1/2*(log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1))/d
```

Fricas [A]
time = 2.08, size = 14, normalized size = 1.17

$$-\frac{\log\left(-\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="fricas")
[Out] -log(-cos(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(c+dx\right)}{-\sin\left(c+dx\right)+\csc\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
[Out] Integral(cos(c + d*x)/(-sin(c + d*x) + csc(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(12) = 24$.
time = 0.44, size = 26, normalized size = 2.17

$$-\frac{\log\left(\left|\sin\left(dx+c\right)+1\right|\right)+\log\left(\left|\sin\left(dx+c\right)-1\right|\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="giac")
[Out] -1/2*(log(abs(sin(d*x + c) + 1)) + log(abs(sin(d*x + c) - 1)))/d
```

Mupad [B]
time = 0.06, size = 14, normalized size = 1.17

$$-\frac{\ln\left(\cos\left(c+d\,x\right)^2\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cos(c + d*x)/(sin(c + d*x) - 1/sin(c + d*x)),x)
[Out] -log(cos(c + d*x)^2)/(2*d)
```

3.225 $\int \frac{\tan(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx$

Optimal. Leaf size=34

$$-\frac{\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] -1/2*arctanh(sin(d*x+c))/d+1/2*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}=0.083$, Rules used = {294, 212}

$$\frac{\tan(c+dx)\sec(c+dx)}{2d}-\frac{\tanh^{-1}(\sin(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

```
[Out] -1/2*ArcTanh[Sin[c + d*x]]/d + (Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x^2)^2}\,dx,x,\sin(c+dx)\right)}{d}\\ &= \frac{\sec(c+dx)\tan(c+dx)}{2d}-\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sin(c+dx)\right)}{2d}\\ &= -\frac{\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\sec(c+dx)\tan(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 34, normalized size = 1.00

$$-\frac{\tanh^{-1}(\sin(c+dx))}{2d}+\frac{\sec(c+dx)\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

```
[Out] -1/2*ArcTanh[Sin[c + d*x]]/d + (Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A]
time = 0.22, size = 52, normalized size = 1.53

method	result	size
derivativedivides	$\frac{-\frac{1}{4(\sin(dx+c)-1)}+\frac{\ln(\sin(dx+c)-1)}{4}-\frac{1}{4(\sin(dx+c)+1)}-\frac{\ln(\sin(dx+c)+1)}{4}}{d}$	52
default	$\frac{-\frac{1}{4(\sin(dx+c)-1)}+\frac{\ln(\sin(dx+c)-1)}{4}-\frac{1}{4(\sin(dx+c)+1)}-\frac{\ln(\sin(dx+c)+1)}{4}}{d}$	52
risch	$-\frac{i\left(e^{3i(dx+c)}-e^{i(dx+c)}\right)}{d\left(e^{2i(dx+c)}+1\right)^{2}}-\frac{\ln\left(e^{i(dx+c)}+i\right)}{2d}+\frac{\ln\left(e^{i(dx+c)}-i\right)}{2d}$	78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/4/(sin(d*x+c)-1)+1/4*ln(sin(d*x+c)-1)-1/4/(sin(d*x+c)+1)-1/4*ln(sin
(d*x+c)+1))
```

Maxima [A]
time = 0.26, size = 46, normalized size = 1.35

$$-\frac{\frac{2\sin(dx+c)}{\sin(dx+c)^{2}-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(sin
(d*x + c) - 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(30) = 60.
time = 2.66, size = 61, normalized size = 1.79

$$-\frac{\cos\left(dx+c\right)^{2}\log\left(\sin\left(dx+c\right)+1\right)-\cos\left(dx+c\right)^{2}\log\left(-\sin\left(dx+c\right)+1\right)-2\,\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(cos(d*x + c)^2*log(sin(d*x + c) + 1) - cos(d*x + c)^2*log(-sin(d*x +
c) + 1) - 2*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(c+dx)}{-\sin(c+dx)+\csc(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

```
[Out] Integral(tan(c + d*x)/(-sin(c + d*x) + csc(c + d*x)), x)
```

Giac [A]
time = 0.46, size = 48, normalized size = 1.41

$$-\frac{\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+\log(|\sin(dx+c)+1|)-\log(|\sin(dx+c)-1|)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(abs(sin(d*x + c) + 1)) - lo
g(abs(sin(d*x + c) - 1)))/d
```

Mupad [B]
time = 1.00, size = 69, normalized size = 2.03

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}-\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-tan(c + d*x)/(sin(c + d*x) - 1/sin(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2) + tan(c/2 + (d*x)/2)^3)/(d*(tan(c/2 + (d*x)/2)^4 - 2*ta
n(c/2 + (d*x)/2)^2 + 1)) - atanh(tan(c/2 + (d*x)/2))/d
```

3.226 $\int \frac{\cot(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx$

Optimal. Leaf size=11

$$\frac{\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] arctanh(sin(d*x+c))/d
```

Rubi [A]
time = 0.02, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {4423, 212}

$$\frac{\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/d
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 4423

```
Int[(u_)*(F_)[(c_.)*((a_.) + (b_.)*(x_))], x_Symbol] :> With[{d = FreeFacto
rs[Sin[c*(a + b*x)], x]}, Dist[1/(b*c), Subst[Int[SubstFor[1/x, Sin[c*(a +
b*x)]/d, u, x], x], x, Sin[c*(a + b*x)]/d], x] /; FunctionOfQ[Sin[c*(a + b*
x)]/d, u, x, True]] /; FreeQ[{a, b, c}, x] && (EqQ[F, Cot] || EqQ[F, cot])
```

Rubi steps

$$\begin{aligned}\int \frac{\cot(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(c+dx)\right)}{d}\\ &= \frac{\tanh^{-1}(\sin(c+dx))}{d}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$\frac{\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/d
```

Maple [A]
time = 0.15, size = 12, normalized size = 1.09

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}(\sin(dx+c))}{d}$	12
default	$\frac{\operatorname{arctanh}(\sin(dx+c))}{d}$	12
risch	$-\frac{\ln(\mathrm{e}^{i(dx+c)}-i)}{d}+\frac{\ln(\mathrm{e}^{i(dx+c)}+i)}{d}$	37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] arctanh(sin(d*x+c))/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(11) = 22$.
time = 0.28, size = 26, normalized size = 2.36

$$\frac{\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(11) = 22$.
time = 3.55, size = 28, normalized size = 2.55

$$\frac{\log\left(\sin\left(dx+c\right)+1\right)-\log\left(-\sin\left(dx+c\right)+1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(log(sin(d*x + c) + 1) - log(-sin(d*x + c) + 1))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(c+dx\right)}{-\sin\left(c+dx\right)+\csc\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

```
[Out] Integral(cot(c + d*x)/(-sin(c + d*x) + csc(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(11) = 22$.
time = 0.42, size = 28, normalized size = 2.55

$$\frac{\log\left(|\sin\left(dx+c\right)+1|\right)-\log\left(|\sin\left(dx+c\right)-1|\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(log(abs(sin(d*x + c) + 1)) - log(abs(sin(d*x + c) - 1)))/d
```

Mupad [B]
time = 0.64, size = 15, normalized size = 1.36

$$\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-cot(c + d*x)/(sin(c + d*x) - 1/sin(c + d*x)),x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2)))/d
```

3.227 $\int \frac{\sec(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx$

Optimal. Leaf size=15

$$\frac{\sec^2(c+dx)}{2d}$$

```
[Out] 1/2*sec(d*x+c)^2/d
```

Rubi [A]
time = 0.02, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{267\}$

$$\frac{\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

```
[Out] Sec[c + d*x]^2/(2*d)
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{(1-x^2)^2}\,dx, x, \sin(c+dx)\right)}{d}\\ &= \frac{\sec^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 15, normalized size = 1.00

$$\frac{\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]
```

```
[Out] Sec[c + d*x]^2/(2*d)
```

Maple [A]
time = 0.12, size = 14, normalized size = 0.93

method	result	size
derivativedivides	$\frac{\sec^2(dx+c)}{2d}$	14
default	$\frac{\sec^2(dx+c)}{2d}$	14
risch	$\frac{2\,\mathrm{e}^{2i(dx+c)}}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}$	28
norman	$\frac{2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}$	32

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*sec(d*x+c)^2/d
```

Maxima [A]
time = 0.27, size = 17, normalized size = 1.13

$$-\frac{1}{2\left(\sin\left(dx+c\right)^2-1\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2/((sin(d*x + c)^2 - 1)*d)
```

Fricas [A]
time = 2.21, size = 13, normalized size = 0.87

$$\frac{1}{2\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(c+dx\right)}{-\sin\left(c+dx\right)+\csc\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(-sin(c + d*x) + csc(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(13) = 26$.
time = 0.41, size = 46, normalized size = 3.07

$$-\frac{2\left(\cos\left(dx+c\right)-1\right)}{d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^{2}\left(\cos\left(dx+c\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="giac")
```

```
[Out] -2*(cos(d*x + c) - 1)/(d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2*(cos
(d*x + c) + 1))
```

Mupad [B]
time = 0.04, size = 13, normalized size = 0.87

$$\frac{1}{2\,d\cos\left(c+d\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(cos(c + d*x)*(sin(c + d*x) - 1/sin(c + d*x))),x)
```

```
[Out] 1/(2*d*cos(c + d*x)^2)
```

3.228 $\int \frac{\csc(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx$

Optimal. Leaf size=10

$$\frac{\tan(c+dx)}{d}$$

[Out] tan(d*x+c)/d

Rubi [A]
time = 0.07, antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{8\}$

$$\frac{\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]

[Out] Tan[c + d*x]/d

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(c+dx)}{\csc(c+dx)-\sin(c+dx)}\,dx &= \frac{\text{Subst}\left(\int 1\,dx, x, \tan(c+dx)\right)}{d}\\ &= \frac{\tan(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 10, normalized size = 1.00

$$\frac{\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Csc[c + d*x]/(Csc[c + d*x] - Sin[c + d*x]),x]

[Out] Tan[c + d*x]/d

Maple [A]
time = 0.14, size = 11, normalized size = 1.10

method	result	size
derivativedivides	$\frac{\tan(dx+c)}{d}$	11
default	$\frac{\tan(dx+c)}{d}$	11
risch	$\frac{2i}{d\left(e^{2i(dx+c)}+1\right)}$	20
norman	$-\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] tan(d*x+c)/d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(10) = 20$.
time = 0.26, size = 44, normalized size = 4.40

$$-\frac{2\sin\left(dx+c\right)}{d\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-1\right)\left(\cos\left(dx+c\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="maxima")
```

```
[Out] -2*sin(d*x + c)/(d*(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c)
+ 1))
```

Fricas [A]
time = 2.54, size = 18, normalized size = 1.80

$$\frac{\sin\left(dx+c\right)}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="fricas")
```

```
[Out] sin(d*x + c)/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc\left(c+dx\right)}{-\sin\left(c+dx\right)+\csc\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x)

[Out] Integral(csc(c + d*x)/(-sin(c + d*x) + csc(c + d*x)), x)

Giac [A]
time = 0.40, size = 10, normalized size = 1.00

$$\frac{\tan(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)/(csc(d*x+c)-sin(d*x+c)),x, algorithm="giac")

[Out] tan(d*x + c)/d

Mupad [B]
time = 0.58, size = 29, normalized size = 2.90

$$-\frac{2\tan\left(\frac{c}{2} + \frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2} + \frac{dx}{2}\right)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(-1/(sin(c + d*x)*(sin(c + d*x) - 1/sin(c + d*x))),x)

[Out] -(2*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (d*x)/2)^2 - 1))

3.229 $\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

Optimal. Leaf size=33

$$-\frac{b\cos^3(c+dx)}{3d}-\frac{a\cos^4(c+dx)}{4d}$$

```
[Out] -1/3*b*cos(d*x+c)^3/d-1/4*a*cos(d*x+c)^4/d
```

Rubi [A]
time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {4462, 12, 2645, 30}

$$-\frac{a\cos^4(c+dx)}{4d}-\frac{b\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] -1/3*(b*Cos[c + d*x]^3)/d - (a*Cos[c + d*x]^4)/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4462

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx = a\int \cos^3(c+dx)\sin(c+dx)\,dx + \int b\cos^2(c+dx)\sin($$
$$= b\int \cos^2(c+dx)\sin(c+dx)\,dx - \frac{a\text{Subst}\left(\int x^3\,dx, x, \text{co}\right.}{d}$$
$$= -\frac{a\cos^4(c+dx)}{4d} - \frac{b\text{Subst}\left(\int x^2\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{b\cos^3(c+dx)}{3d} - \frac{a\cos^4(c+dx)}{4d}$$

Mathematica [A]
time = 0.02, size = 33, normalized size = 1.00

$$-\frac{b\cos^3(c+dx)}{3d} - \frac{a\cos^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
[Out] -1/3*(b*Cos[c + d*x]^3)/d - (a*Cos[c + d*x]^4)/(4*d)
```

Maple [A]
time = 0.12, size = 29, normalized size = 0.88

method	result	size
derivativedivides	$-\frac{\frac{a\left(\cos^4(dx+c)\right)}{4}+\frac{b\left(\cos^3(dx+c)\right)}{3}}{d}$	29
default	$-\frac{\frac{a\left(\cos^4(dx+c)\right)}{4}+\frac{b\left(\cos^3(dx+c)\right)}{3}}{d}$	29
risch	$-\frac{b\cos(dx+c)}{4d}-\frac{a\cos(4dx+4c)}{32d}-\frac{b\cos(3dx+3c)}{12d}-\frac{a\cos(2dx+2c)}{8d}$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -1/d*(1/4*a*cos(d*x+c)^4+1/3*b*cos(d*x+c)^3)
```

Maxima [A]
time = 0.27, size = 28, normalized size = 0.85

$$-\frac{3\,a\cos\left(dx+c\right)^4+4\,b\cos\left(dx+c\right)^3}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*a*cos(d*x + c)^4 + 4*b*cos(d*x + c)^3)/d
```

Fricas [**A**]
time = 2.40, size = 28, normalized size = 0.85

$$-\frac{3\,a\cos\left(dx+c\right)^4+4\,b\cos\left(dx+c\right)^3}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/12*(3*a*cos(d*x + c)^4 + 4*b*cos(d*x + c)^3)/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)\cos^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))*cos(c + d*x)**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 11588 vs. $2(29) = 58$.
time = 2.23, size = 11588, normalized size = 351.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/32*a*cos(4*d*x + 4*c)/d - 1/8*a*cos(2*d*x + 2*c)/d + 1/96*(3*pi*b*sgn(ta
n(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 -
tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/
2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*t
an(1/2*d*x)^6*tan(1/2*c)^6 + 3*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan
(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x
)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 - 3*pi
*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*
d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 3
*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1
/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^6
```

- 6*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 6*pi*b*tan(1/2*d*x)^6*tan(1/2*c)^6 - 6*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 6*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 6*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 + 6*b*arctan((tan(1/2*d*x)*tan(1/2*c) - tan(1/2*d*x) - tan(1/2*c) - 1)/(tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) + tan(1/2*c) - 1))*tan(1/2*d*x)^6*tan(1/2*c)^6 - 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 - 18*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^6*tan(1/2*c)^4 - 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 - 18*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - tan(1/2*d*x)^2 - 4*tan(1/2*d*x)*tan(1/2*c) - tan(1/2*c)^2 + 1)*tan(1/2*d*x)^4*tan(1/2*c)^6 - 32*b*tan(1/2*d*x)^6*tan(1/2*c)^6 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 9*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 27*pi*b*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2

```
*d*x)^2*tan(1/2*c) + tan(1/2*d*x)^2 - tan(1/2*c)^2 + 2*tan(1/2*c) - 1)*sgn(
tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2
+ tan(1/2*c)^2 + 2*tan(1/2*d*x) - 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 27*pi*b*
sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^2*tan(1/2*c) + tan(1/2*d*x
)^2 - tan(1/2*c)^2 - 2*tan(1/2*c) - 1)*sgn(tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*
tan(1/2*d*x)*tan(1/2*c)^2 - tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x)
- 1)*tan(1/2*d*x)^4*tan(1/2*c)^4 + 18*pi*b*tan(1/2*d*x)^6*tan(1/2*c)^4 - 18
*b*arctan((tan(1/2*d*x)*tan(1/2*c) + tan(1/2*d*x) - tan(1/2*c) + 1)/(tan(1/
2*d*x)*tan(1/2*c) - tan(1/2*d*x) + tan(1/2*c) + 1))*tan(1/2*d*x)^6*tan(1/2*
c)^4 - 18*b*arctan((tan(1/2*d*x)*tan(1/2*c) - t...
```

Mupad [B]
time = 0.68, size = 29, normalized size = 0.88

$$-\frac{a\cos\left(c+d\,x\right)^4}{4\,d}-\frac{b\cos\left(c+d\,x\right)^3}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a*sin(c + d*x) + b*tan(c + d*x)),x)
[Out] - (a*cos(c + d*x)^4)/(4*d) - (b*cos(c + d*x)^3)/(3*d)
```

3.230 $\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

Optimal. Leaf size=33

$$-\frac{a\cos^3(c+dx)}{3d}+\frac{b\sin^2(c+dx)}{2d}$$

```
[Out] -1/3*a*cos(d*x+c)^3/d+1/2*b*sin(d*x+c)^2/d
```

Rubi [A]

time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {4462, 12, 2644, 30, 2645}

$$\frac{b\sin^2(c+dx)}{2d}-\frac{a\cos^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] -1/3*(a*Cos[c + d*x]^3)/d + (b*Sin[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4462

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] && !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx = a\int \cos^2(c+dx)\sin(c+dx)\,dx + \int b\cos(c+dx)\sin(c-$$
$$= b\int \cos(c+dx)\sin(c+dx)\,dx - \frac{a\text{Subst}\left(\int x^2\,dx, x, \cos(\right.}{d}$$
$$= -\frac{a\cos^3(c+dx)}{3d} + \frac{b\text{Subst}(\int x\,dx, x, \sin(c+dx))}{d}$$
$$= -\frac{a\cos^3(c+dx)}{3d} + \frac{b\sin^2(c+dx)}{2d}$$

Mathematica [A]
time = 0.14, size = 38, normalized size = 1.15

$$-\frac{3a\cos(c+dx)+3b\cos(2(c+dx))+a\cos(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
[Out] -1/12*(3*a*Cos[c + d*x] + 3*b*Cos[2*(c + d*x)] + a*Cos[3*(c + d*x)])/d
```

Maple [A]
time = 0.10, size = 29, normalized size = 0.88

method	result	size
derivativedivides	$-\frac{\frac{a\left(\cos^3(dx+c)\right)}{3}+\frac{\left(\cos^2(dx+c)\right)b}{2}}{d}$	29
default	$-\frac{\frac{a\left(\cos^3(dx+c)\right)}{3}+\frac{\left(\cos^2(dx+c)\right)b}{2}}{d}$	29
risch	$-\frac{a\cos(dx+c)}{4d}-\frac{a\cos(3dx+3c)}{12d}-\frac{b\cos(2dx+2c)}{4d}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -1/d*(1/3*a*cos(d*x+c)^3+1/2*cos(d*x+c)^2*b)
```

Maxima [A]
time = 0.25, size = 28, normalized size = 0.85

$$-\frac{2\,a\cos\left(dx+c\right)^{3}-3\,b\sin\left(dx+c\right)^{2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
[Out] -1/6*(2*a*cos(d*x + c)^3 - 3*b*sin(d*x + c)^2)/d
```

Fricas [A]
time = 3.59, size = 28, normalized size = 0.85

$$-\frac{2\,a\cos\left(dx+c\right)^{3}+3\,b\cos\left(dx+c\right)^{2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
[Out] -1/6*(2*a*cos(d*x + c)^3 + 3*b*cos(d*x + c)^2)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)\cos^{2}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*sin(d*x+c)+b*tan(d*x+c)),x)
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))*cos(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(29) = 58.
time = 0.48, size = 99, normalized size = 3.00

$$-\frac{a\cos\left(3\,dx+3\,c\right)}{12\,d}-\frac{a\cos\left(dx+c\right)}{4\,d}-\frac{b\tan\left(dx\right)^{2}\tan\left(c\right)^{2}-b\tan\left(dx\right)^{2}-4\,b\tan\left(dx\right)\tan\left(c\right)-b\tan\left(c\right)^{2}+b}{4\,\left(d\tan\left(dx\right)^{2}\tan\left(c\right)^{2}+d\tan\left(dx\right)^{2}+d\tan\left(c\right)^{2}+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
[Out] -1/12*a*cos(3*d*x + 3*c)/d - 1/4*a*cos(d*x + c)/d - 1/4*(b*tan(d*x)^2*tan(c
)^2 - b*tan(d*x)^2 - 4*b*tan(d*x)*tan(c) - b*tan(c)^2 + b)/(d*tan(d*x)^2*ta
n(c)^2 + d*tan(d*x)^2 + d*tan(c)^2 + d)
```

Mupad [B]
time = 0.64, size = 49, normalized size = 1.48

$$-\frac{(\cos(c+dx)+1)\,\left(2a-3b-2a\,\cos(c+dx)+3b\,\cos(c+dx)+2a\cos(c+dx)^2\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a*sin(c + d*x) + b*tan(c + d*x)),x)
[Out] -((cos(c + d*x) + 1)*(2*a - 3*b - 2*a*cos(c + d*x) + 3*b*cos(c + d*x) + 2*a
*cos(c + d*x)^2))/(6*d)
```

3.231 $\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

Optimal. Leaf size=22

$$-\frac{(b+a\cos(c+dx))^2}{2ad}$$

```
[Out] -1/2*(b+a*cos(d*x+c))^2/a/d
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.32, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4462, 12, 2718, 2644, 30}

$$\frac{a\sin^2(c+dx)}{2d} - \frac{b\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] -((b*Cos[c + d*x])/d) + (a*Sin[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 4462

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
```

```
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx &= a\int \cos(c+dx)\sin(c+dx)\,dx+\int b\sin(c+dx)\,dx\\ &= b\int \sin(c+dx)\,dx+\frac{a\text{Subst}(\int x\,dx,x,\sin(c+dx))}{d}\\ &= -\frac{b\cos(c+dx)}{d}+\frac{a\sin^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 40, normalized size = 1.82

$$-\frac{b\cos(c)\cos(dx)}{d}-\frac{a\cos^2(c+dx)}{2d}+\frac{b\sin(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
[Out] -((b*Cos[c]*Cos[d*x])/d) - (a*Cos[c + d*x]^2)/(2*d) + (b*Sin[c]*Sin[d*x])/d
```

Maple [A]
time = 0.07, size = 26, normalized size = 1.18

method	result	size
derivativedivides	$-\frac{\frac{(\cos^2(dx+c))a}{2}+b\cos(dx+c)}{d}$	26
default	$-\frac{\frac{(\cos^2(dx+c))a}{2}+b\cos(dx+c)}{d}$	26
risch	$-\frac{b\cos(dx+c)}{d}-\frac{a\cos(2dx+2c)}{4d}$	29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -1/d*(1/2*cos(d*x+c)^2*a+b*cos(d*x+c))
```

Maxima [A]
time = 0.26, size = 25, normalized size = 1.14

$$-\frac{a\cos(dx+c)^2+2\,b\cos(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
[Out] -1/2*(a*cos(d*x + c)^2 + 2*b*cos(d*x + c))/d
```

Fricas [A]
time = 3.38, size = 25, normalized size = 1.14

$$-\frac{a\cos\left(dx+c\right)^{2}+2\,b\cos\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
[Out] -1/2*(a*cos(d*x + c)^2 + 2*b*cos(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)\cos\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x)
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))*cos(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. 2(20) = 40.
time = 0.44, size = 102, normalized size = 4.64

$$-\frac{a\cos\left(2\,dx+2\,c\right)}{4\,d}-\frac{b\tan\left(\frac{1}{2}\,dx\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}-b\tan\left(\frac{1}{2}\,dx\right)^{2}-4\,b\tan\left(\frac{1}{2}\,dx\right)\tan\left(\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,c\right)^{2}+b}{d\tan\left(\frac{1}{2}\,dx\right)^{2}\tan\left(\frac{1}{2}\,c\right)^{2}+d\tan\left(\frac{1}{2}\,dx\right)^{2}+d\tan\left(\frac{1}{2}\,c\right)^{2}+d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
[Out] -1/4*a*cos(2*d*x + 2*c)/d - (b*tan(1/2*d*x)^2*tan(1/2*c)^2 - b*tan(1/2*d*x)
^2 - 4*b*tan(1/2*d*x)*tan(1/2*c) - b*tan(1/2*c)^2 + b)/(d*tan(1/2*d*x)^2*ta
n(1/2*c)^2 + d*tan(1/2*d*x)^2 + d*tan(1/2*c)^2 + d)
```

Mupad [B]
time = 0.63, size = 28, normalized size = 1.27

$$-\frac{\left(\cos\left(c+d\,x\right)+1\right)\,\left(2\,b-a+a\,\cos\left(c+d\,x\right)\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a*sin(c + d*x) + b*tan(c + d*x)),x)
[Out] -((cos(c + d*x) + 1)*(2*b - a + a*cos(c + d*x)))/(2*d)
```

3.232 $\int (a\sin(c+dx)+b\tan(c+dx))\,dx$

Optimal. Leaf size=26

$$-\frac{a\cos(c+dx)}{d}-\frac{b\log(\cos(c+dx))}{d}$$

[Out] -a*cos(d*x+c)/d-b*ln(cos(d*x+c))/d

Rubi [A]
time = 0.01, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.118$, Rules used = {2718, 3556}

$$-\frac{a\cos(c+dx)}{d}-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

[In] Int[a*Sin[c + d*x] + b*Tan[c + d*x],x]

[Out] -((a*Cos[c + d*x])/d) - (b*Log[Cos[c + d*x]])/d

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int (a\sin(c+dx)+b\tan(c+dx))\,dx&=a\int \sin(c+dx)\,dx+b\int \tan(c+dx)\,dx\\&=-\frac{a\cos(c+dx)}{d}-\frac{b\log(\cos(c+dx))}{d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 37, normalized size = 1.42

$$-\frac{a\cos(c)\cos(dx)}{d}-\frac{b\log(\cos(c+dx))}{d}+\frac{a\sin(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a*Sin[c + d*x] + b*Tan[c + d*x],x]
[Out] -((a*Cos[c]*Cos[d*x])/d) - (b*Log[Cos[c + d*x]])/d + (a*Sin[c]*Sin[d*x])/d
```

Maple [A]
time = 0.06, size = 31, normalized size = 1.19

method	result	size
derivativedivides	$\frac{-a\cos(dx+c)-b\ln(\cos(dx+c))}{d}$	25
default	$-\frac{a\cos(dx+c)}{d}+\frac{b\ln(\tan^2(dx+c)+1)}{2d}$	31
risch	$ibx+\frac{2ibc}{d}-\frac{b\ln(e^{2i(dx+c)}+1)}{d}-\frac{a\cos(dx+c)}{d}$	45
norman	$\frac{2a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{b\ln(\tan^2(dx+c)+1)}{2d}$	51

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*sin(d*x+c)+b*tan(d*x+c),x,method=_RETURNVERBOSE)
[Out] -a*cos(d*x+c)/d+1/2*b/d*ln(tan(d*x+c)^2+1)
```

Maxima [A]
time = 0.26, size = 25, normalized size = 0.96

$$-\frac{a\cos(dx+c)}{d}+\frac{b\log(\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*sin(d*x+c)+b*tan(d*x+c),x, algorithm="maxima")
[Out] -a*cos(d*x + c)/d + b*log(sec(d*x + c))/d
```

Fricas [A]
time = 3.84, size = 25, normalized size = 0.96

$$-\frac{a\cos(dx+c)+b\log(-\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*sin(d*x+c)+b*tan(d*x+c),x, algorithm="fricas")
[Out] -(a*cos(d*x + c) + b*log(-cos(d*x + c)))/d
```

Sympy [A]
time = 0.07, size = 37, normalized size = 1.42

$$a\left(\begin{cases}-\frac{\cos(c+dx)}{d} & \text{for } d\neq 0\\ x\sin(c) & \text{otherwise}\end{cases}\right)+b\left(\begin{cases}\frac{\log(\tan^2(c+dx)+1)}{2d} & \text{for } d\neq 0\\ x\tan(c) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*sin(d*x+c)+b*tan(d*x+c),x)
```

```
[Out] a*Piecewise((-cos(c + d*x)/d, Ne(d, 0)), (x*sin(c), True)) + b*Piecewise((l
og(tan(c + d*x)**2 + 1)/(2*d), Ne(d, 0)), (x*tan(c), True))
```

Giac [A]
time = 0.41, size = 27, normalized size = 1.04

$$-\frac{a\cos\left(dx+c\right)}{d}-\frac{b\log\left(\left|\cos\left(dx+c\right)\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a*sin(d*x+c)+b*tan(d*x+c),x, algorithm="giac")
```

```
[Out] -a*cos(d*x + c)/d - b*log(abs(cos(d*x + c)))/d
```

Mupad [B]
time = 0.65, size = 40, normalized size = 1.54

$$\frac{2\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{2\,a}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a*sin(c + d*x) + b*tan(c + d*x),x)
```

```
[Out] (2*b*atanh(tan(c/2 + (d*x)/2)^2))/d - (2*a)/(d*(tan(c/2 + (d*x)/2)^2 + 1))
```

3.233 $\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

Optimal. Leaf size=25

$$-\frac{a\log(\cos(c+dx))}{d}+\frac{b\sec(c+dx)}{d}$$

```
[Out] -a*ln(cos(d*x+c))/d+b*sec(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {4462, 12, 2686, 8, 3556}

$$\frac{b\sec(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] -((a*Log[Cos[c + d*x]])/d) + (b*Sec[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4462

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
```

```
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx &= a\int \tan(c+dx)\,dx+\int b\sec(c+dx)\tan(c+dx)\,dx\\ &= -\frac{a\log(\cos(c+dx))}{d}+b\int \sec(c+dx)\tan(c+dx)\,dx\\ &= -\frac{a\log(\cos(c+dx))}{d}+\frac{b\text{Subst}(\int 1\,dx,x,\sec(c+dx))}{d}\\ &= -\frac{a\log(\cos(c+dx))}{d}+\frac{b\sec(c+dx)}{d}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 25, normalized size = 1.00

$$-\frac{a\log(\cos(c+dx))}{d}+\frac{b\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
[Out] -((a*Log[Cos[c + d*x]])/d) + (b*Sec[c + d*x])/d
```

Maple [A]
time = 0.09, size = 23, normalized size = 0.92

method	result	size
derivativedivides	$\frac{b\sec(dx+c)+a\ln(\sec(dx+c))}{d}$	23
default	$\frac{b\sec(dx+c)+a\ln(\sec(dx+c))}{d}$	23
risch	$iax+\frac{2iac}{d}+\frac{2b\,e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)}-\frac{a\ln\left(e^{2i(dx+c)}+1\right)}{d}$	61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(b*sec(d*x+c)+a*ln(sec(d*x+c)))
```

Maxima [A]
time = 0.27, size = 32, normalized size = 1.28

$$-\frac{a\log\left(-\sin\left(dx+c\right)^2+1\right)-\frac{2\,b}{\cos(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(a*log(-sin(d*x + c)^2 + 1) - 2*b/cos(d*x + c))/d
```

Fricas [A]
time = 2.49, size = 34, normalized size = 1.36

$$-\frac{a\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)-b}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(a*cos(d*x + c)*log(-cos(d*x + c)) - b)/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))*sec(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(25) = 50.
time = 0.47, size = 107, normalized size = 4.28

$$\frac{a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{a+2b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - a*log(abs(-(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a + 2*b + a*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

Mupad [B]
time = 0.66, size = 40, normalized size = 1.60

$$\frac{2\,a\,\mathrm{atanh}\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{2\,b}{d\,\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b*tan(c + d*x))/cos(c + d*x),x)
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)^2))/d - (2*b)/(d*(tan(c/2 + (d*x)/2)^2 - 1))
```

3.234 $\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

Optimal. Leaf size=28

$$\frac{a\sec(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}$$

```
[Out] a*sec(d*x+c)/d+1/2*b*sec(d*x+c)^2/d
```

Rubi [A]
time = 0.04, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {4462, 12, 2686, 30, 8}

$$\frac{a\sec(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x])/d + (b*Sec[c + d*x]^2)/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4462

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
```

```
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx &= a\int \sec(c+dx)\tan(c+dx)\,dx+\int b\sec^2(c+dx)\tan(c\\ &= b\int \sec^2(c+dx)\tan(c+dx)\,dx+\frac{a\text{Subst}(\int 1\,dx,x,\sec(c}{d}\\ &= \frac{a\sec(c+dx)}{d}+\frac{b\text{Subst}(\int x\,dx,x,\sec(c+dx))}{d}\\ &= \frac{a\sec(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 28, normalized size = 1.00

$$\frac{a\sec(c+dx)}{d}+\frac{b\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
[Out] (a*Sec[c + d*x])/d + (b*Sec[c + d*x]^2)/(2*d)
```

Maple [A]
time = 0.11, size = 25, normalized size = 0.89

method	result	size
derivativedivides	$\frac{\frac{b\left(\sec^2(dx+c)\right)}{2}+a\sec(dx+c)}{d}$	25
default	$\frac{\frac{b\left(\sec^2(dx+c)\right)}{2}+a\sec(dx+c)}{d}$	25
risch	$\frac{2a\,e^{3i(dx+c)}+2b\,e^{2i(dx+c)}+2a\,e^{i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)^2}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(1/2*b*sec(d*x+c)^2+a*sec(d*x+c))
```

Maxima [A]
time = 0.26, size = 27, normalized size = 0.96

$$\frac{b\tan\left(dx+c\right)^2+\frac{2\,a}{\cos(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
[Out] 1/2*(b*tan(d*x + c)^2 + 2*a/cos(d*x + c))/d
```

Fricas [A]
time = 1.26, size = 24, normalized size = 0.86

$$\frac{2\,a\cos\left(dx+c\right)+b}{2\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
[Out] 1/2*(2*a*cos(d*x + c) + b)/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a*sin(d*x+c)+b*tan(d*x+c)),x)
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))*sec(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(26) = 52.
time = 0.46, size = 71, normalized size = 2.54

$$\frac{2\left(a+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
[Out] 2*(a + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1))/(d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)
```

Mupad [B]
time = 0.63, size = 24, normalized size = 0.86

$$\frac{\frac{b}{2}+a\,\cos\left(c+d\,x\right)}{d\cos\left(c+d\,x\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b*tan(c + d*x))/cos(c + d*x)^2,x)
[Out] (b/2 + a*cos(c + d*x))/(d*cos(c + d*x)^2)
```

3.235 $\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

Optimal. Leaf size=33

$$\frac{a\sec^2(c+dx)}{2d}+\frac{b\sec^3(c+dx)}{3d}$$

```
[Out] 1/2*a*sec(d*x+c)^2/d+1/3*b*sec(d*x+c)^3/d
```

Rubi [A]
time = 0.04, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4462, 12, 2686, 30}

$$\frac{a\sec^2(c+dx)}{2d}+\frac{b\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x]^2)/(2*d) + (b*Sec[c + d*x]^3)/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2686

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 4462

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx = a\int \sec^2(c+dx)\tan(c+dx)\,dx + \int b\sec^3(c+dx)\tan$$
$$= b\int \sec^3(c+dx)\tan(c+dx)\,dx + \frac{a\text{Subst}(\int x\,dx, x, \sec}{d}$$
$$= \frac{a\sec^2(c+dx)}{2d} + \frac{b\text{Subst}\left(\int x^2\,dx, x, \sec(c+dx)\right)}{d}$$
$$= \frac{a\sec^2(c+dx)}{2d} + \frac{b\sec^3(c+dx)}{3d}$$

Mathematica [A]
time = 0.03, size = 33, normalized size = 1.00

$$\frac{a\sec^2(c+dx)}{2d} + \frac{b\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x]^2)/(2*d) + (b*Sec[c + d*x]^3)/(3*d)
```

Maple [A]
time = 0.14, size = 28, normalized size = 0.85

method	result	size
derivativedivides	$\frac{\frac{(\sec^3(dx+c))b}{3}+\frac{(\sec^2(dx+c))a}{2}}{d}$	28
default	$\frac{\frac{(\sec^3(dx+c))b}{3}+\frac{(\sec^2(dx+c))a}{2}}{d}$	28
risch	$\frac{2a\,e^{4i(dx+c)}+\frac{8b\,e^{3i(dx+c)}}{3}+2a\,e^{2i(dx+c)}}{d\left(e^{2i(dx+c)}+1\right)^3}$	56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*sec(d*x+c)^3*b+1/2*sec(d*x+c)^2*a)
```

Maxima [A]
time = 0.27, size = 32, normalized size = 0.97

$$-\frac{\frac{3\,a}{\sin(dx+c)^2-1} - \frac{2\,b}{\cos(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(3*a/(sin(d*x + c)^2 - 1) - 2*b/cos(d*x + c)^3)/d
```

Fricas [A]
time = 2.04, size = 26, normalized size = 0.79

$$\frac{3\,a\cos\left(dx+c\right)+2\,b}{6\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*a*cos(d*x + c) + 2*b)/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))*sec(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(29) = 58$.
time = 0.49, size = 97, normalized size = 2.94

$$\frac{2\left(b-\frac{3\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{3\,a(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3\,b(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}\right)}{3\,d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 2/3*(b - 3*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 + 3*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(d
*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^3)
```

Mupad [B]
time = 0.69, size = 29, normalized size = 0.88

$$\frac{a}{2\,d\cos\left(c+d\,x\right)^{2}}+\frac{b}{3\,d\cos\left(c+d\,x\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b*tan(c + d*x))/cos(c + d*x)^3,x)
```

```
[Out] a/(2*d*cos(c + d*x)^2) + b/(3*d*cos(c + d*x)^3)
```

3.236 $\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=106

$$\frac{abx}{4}-\frac{ab\cos(c+dx)\sin(c+dx)}{4d}+\frac{(4a^2+b^2)\sin^3(c+dx)}{30d}+\frac{b(b+a\cos(c+dx))\sin^3(c+dx)}{10d}+\frac{(b+a\cos(c+$$

```
[Out] 1/4*a*b*x-1/4*a*b*cos(d*x+c)*sin(d*x+c)/d+1/30*(4*a^2+b^2)*sin(d*x+c)^3/d+1
/10*b*(b+a*cos(d*x+c))*sin(d*x+c)^3/d+1/5*(b+a*cos(d*x+c))^2*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.26, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {4482, 2941, 2748, 2715, 8}

$$\frac{(4a^2+b^2)\sin^3(c+dx)}{30d}+\frac{\sin^3(c+dx)(a\cos(c+dx)+b)^2}{5d}+\frac{b\sin^3(c+dx)(a\cos(c+dx)+b)}{10d}-\frac{ab\sin(c+dx)\cos(c+dx)}{4d}+\frac{abx}{4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (a*b*x)/4 - (a*b*Cos[c + d*x]*Sin[c + d*x])/(4*d) + ((4*a^2 + b^2)*Sin[c +
d*x]^3)/(30*d) + (b*(b + a*Cos[c + d*x])*Sin[c + d*x]^3)/(10*d) + ((b + a*C
os[c + d*x])^2*Sin[c + d*x]^3)/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2941

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-d)*
(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^m/(f*g*(m + p + 1))), x] + D
```

```
ist[1/(m + p + 1), Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m - 1)*Simp
[a*c*(m + p + 1) + b*d*m + (a*d*m + b*c*(m + p + 1))*Sin[e + f*x], x], x],
x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
&&  !LtQ[p, -1] && IntegerQ[2*m] &&  !(EqQ[m, 1] && NeQ[c^2 - d^2, 0] && Si
mplerQ[c + d*x, a + b*x])
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \int \cos(c+dx)(b+a\cos(c+dx))^2\sin^2(c+dx)\,dx$$
$$= \frac{(b+a\cos(c+dx))^2\sin^3(c+dx)}{5d} + \frac{1}{5}\int (b+a\cos(c+d$$
$$= \frac{b(b+a\cos(c+dx))\sin^3(c+dx)}{10d} + \frac{(b+a\cos(c+dx))^2}{5d}$$
$$= \frac{(4a^2+b^2)\sin^3(c+dx)}{30d} + \frac{b(b+a\cos(c+dx))\sin^3(c+}{10d}$$
$$= -\frac{ab\cos(c+dx)\sin(c+dx)}{4d} + \frac{(4a^2+b^2)\sin^3(c+dx)}{30d} +$$
$$= \frac{abx}{4} - \frac{ab\cos(c+dx)\sin(c+dx)}{4d} + \frac{(4a^2+b^2)\sin^3(c+}{30d}$$

Mathematica [A]
time = 0.45, size = 77, normalized size = 0.73

$$\frac{30(a^2+2b^2)\sin(c+dx)-5(a^2+4b^2)\sin(3(c+dx))-3a(-20b(c+dx)+5b\sin(4(c+dx))+a\sin(5(c+dx)))}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (30*(a^2 + 2*b^2)*Sin[c + d*x] - 5*(a^2 + 4*b^2)*Sin[3*(c + d*x)] - 3*a*(-2
0*b*(c + d*x) + 5*b*Sin[4*(c + d*x)] + a*Sin[5*(c + d*x)]))/(240*d)
```

Maple [A]
time = 0.12, size = 100, normalized size = 0.94

method	result

derivativedivides	$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)+2ab\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)}{d}$
default	$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^4(dx+c)\right)}{5}+\frac{\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{15}\right)+2ab\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)}{d}$
risch	$\frac{abx}{4}+\frac{a^2\sin(dx+c)}{8d}+\frac{b^2\sin(dx+c)}{4d}-\frac{\sin(5dx+5c)a^2}{80d}-\frac{ab\sin(4dx+4c)}{16d}-\frac{\sin(3dx+3c)a^2}{48d}-\frac{\sin(3dx+3c)b^2}{12d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/5*sin(d*x+c)*cos(d*x+c)^4+1/15*(2+cos(d*x+c)^2)*sin(d*x+c))+2*
a*b*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/8*c)+
1/3*b^2*sin(d*x+c)^3)
```

Maxima [A]
time = 0.27, size = 68, normalized size = 0.64

$$\frac{80\,b^2\sin\left(dx+c\right)^3-16\left(3\,\sin\left(dx+c\right)^5-5\,\sin\left(dx+c\right)^3\right)a^2+15\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)ab}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/240*(80*b^2*sin(d*x + c)^3 - 16*(3*sin(d*x + c)^5 - 5*sin(d*x + c)^3)*a^2
 + 15*(4*d*x + 4*c - sin(4*d*x + 4*c))*a*b)/d
```

Fricas [A]
time = 2.83, size = 85, normalized size = 0.80

$$\frac{15\,abdx-\left(12\,a^2\cos\left(dx+c\right)^4+30\,ab\cos\left(dx+c\right)^3-15\,ab\cos\left(dx+c\right)-4\left(a^2-5\,b^2\right)\cos\left(dx+c\right)^2-8\,a^2-20\,b^2\right)\sin\left(dx+c\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/60*(15*a*b*d*x - (12*a^2*cos(d*x + c)^4 + 30*a*b*cos(d*x + c)^3 - 15*a*b*
cos(d*x + c) - 4*(a^2 - 5*b^2)*cos(d*x + c)^2 - 8*a^2 - 20*b^2)*sin(d*x + c
))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)^2\cos^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**2*cos(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 43089 vs. $2(96) = 192$.

time = 138.27, size = 43089, normalized size = 406.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/80*a^2*sin(5*d*x + 5*c)/d - 1/48*a^2*sin(3*d*x + 3*c)/d + 1/8*a^2*sin(d*
x + c)/d + 1/96*(3*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*
tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(1/2*d*
x)^6*tan(1/2*c)^6*tan(c)^4 + 24*a*b*d*x*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c
)^6*tan(c)^4 + 3*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*
tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^4 + 6*pi
*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*t
an(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^6*tan
(c)^2 + 9*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) +
2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(1/2*d*x)^6*tan(
1/2*c)^4*tan(c)^4 + 9*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)
^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(1/2
*d*x)^4*tan(1/2*c)^6*tan(c)^4 + 6*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn
(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*
x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^4 + 6*a*b*arctan((tan(d*x) + tan(c)
)/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^4 -
6*a*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(1/2
*d*x)^6*tan(1/2*c)^6*tan(c)^4 + 48*a*b*d*x*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/
2*c)^6*tan(c)^2 + 6*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 +
 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 7
2*a*b*d*x*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^4 + 9*pi*a*b*sgn(-2
*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^
4*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^4 + 72*a*b*d*x*tan(d*x)^4*tan(1/2*d*x)
^4*tan(1/2*c)^6*tan(c)^4 + 9*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*t
an(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(1/2*d*x)^4*tan(1/2*c)^6*tan
(c)^4 + 48*a*b*d*x*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^4 + 6*pi*a
*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*
tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^4 + 3*pi*a*b*sgn(2*tan(d*x)^2
*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x)
- 2*tan(c))*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^6 + 18*pi*a*b*sgn(2*tan(d*
x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d
*x) - 2*tan(c))*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^2 + 18*pi*a*b
*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c
```

```
)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c)^
2 + 12*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*t
an(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2
*c)^6*tan(c)^2 + 12*a*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(c) - 1))*t
an(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 - 12*a*b*arctan(-(tan(d*x) -
 tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(
c)^2 + 24*a*b*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^3 + 9*pi*a*b*sg
n(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2
 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^2*tan(c)^4 +
 27*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(
d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(1/2*d*x)^4*tan(1/2*c)
^4*tan(c)^4 + 18*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*ta
n(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^2*tan(1/2*d*x)
^6*tan(1/2*c)^4*tan(c)^4 + 18*a*b*arctan((tan(d*x) + tan(c))/(tan(d*x)*tan(
c) - 1))*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^4 - 18*a*b*arctan(-(
tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2
*c)^4*tan(c)^4 + 9*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-2*tan(d*x)^2*
tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(1/2*d*
x)^2*tan(1/2*c)^6*tan(c)^4 + 18*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn(-
2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)
^2*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c)^4 + 18*a*b*arctan((tan(d*x) + tan(c))
/(tan(d*x)*tan(c) - 1))*tan(d*x)^4*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c)^4 - 1
8*a*b*arctan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^4*tan(1/2
*d*x)^4*tan(1/2*c)^6*tan(c)^4 + 3*pi*a*b*sgn(2*tan(d*x)^2*tan(c)^2 - 2)*sgn
(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(1/
2*d*x)^6*tan(1/2*c)^6*tan(c)^4 + 12*a*b*arctan((tan(d*x) + tan(c))/(tan(d*x
)*tan(c) - 1))*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^4 - 12*a*b*arc
tan(-(tan(d*x) - tan(c))/(tan(d*x)*tan(c) + 1))*tan(d*x)^2*tan(1/2*d*x)^6*t
an(1/2*c)^6*tan(c)^4 + 24*a*b*tan(d*x)^3*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)
^4 + 24*a*b*d*x*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*c)^6 + 3*pi*a*b*sgn(-2*ta
n(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 + 2*tan(d*x) - 2*tan(c))*tan(d*x)^4*t
an(1/2*d*x)^6*tan(1/2*c)^6 + 144*a*b*d*x*tan(d*x)^4*tan(1/2*d*x)^6*tan(1/2*
c)^4*tan(c)^2 + 18*pi*a*b*sgn(-2*tan(d*x)^2*tan(c) + 2*tan(d*x)*tan(c)^2 +
2*tan(d*x) - 2*tan(c))*tan(d*x)^4*tan(1/2*d*x)^...
```

Mupad [B]

time = 0.79, size = 101, normalized size = 0.95

$$\frac{a^2 \sin(c+dx)}{8d} + \frac{b^2 \sin(c+dx)}{4d} + \frac{abx}{4} - \frac{a^2 \sin(3c+3dx)}{48d} - \frac{a^2 \sin(5c+5dx)}{80d} - \frac{b^2 \sin(3c+3dx)}{12d} - \frac{ab \sin(4c+4dx)}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a*sin(c + d*x) + b*tan(c + d*x))^2,x)
[Out] (a^2*sin(c + d*x))/(8*d) + (b^2*sin(c + d*x))/(4*d) + (a*b*x)/4 - (a^2*sin(
3*c + 3*d*x))/(48*d) - (a^2*sin(5*c + 5*d*x))/(80*d) - (b^2*sin(3*c + 3*d*x
))/(12*d) - (a*b*sin(4*c + 4*d*x))/(16*d)
```

3.237 $\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=86

$$\frac{1}{8}\left(a^2+4b^2\right)x-\frac{\left(a^2+4b^2\right)\cos(c+dx)\sin(c+dx)}{8d}+\frac{5ab\sin^3(c+dx)}{12d}+\frac{a(b+a\cos(c+dx))\sin^3(c+dx)}{4d}$$

```
[Out] 1/8*(a^2+4*b^2)*x-1/8*(a^2+4*b^2)*cos(d*x+c)*sin(d*x+c)/d+5/12*a*b*sin(d*x+
c)^3/d+1/4*a*(b+a*cos(d*x+c))*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.14, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {4482, 2771, 2748, 2715, 8}

$$-\frac{\left(a^2+4b^2\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(a^2+4b^2\right)+\frac{5ab\sin^3(c+dx)}{12d}+\frac{a\sin^3(c+dx)(a\cos(c+dx)+b)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^2 + 4*b^2)*x)/8 - ((a^2 + 4*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (5*
a*b*Sin[c + d*x]^3)/(12*d) + (a*(b + a*Cos[c + d*x])*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2748

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[(-b)*((g*Cos[e + f*x])^(p + 1)/(f*g*(p + 1))), x] +
 Dist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] &&
(IntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2771

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m - 1)/(f*g*(m + p))), x] + Dist[1/(m + p), Int[(g*Cos[e + f*x])^p*(
```

```
a + b*Sin[e + f*x])^(m - 2)*(b^2*(m - 1) + a^2*(m + p) + a*b*(2*m + p - 1)*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& GtQ[m, 1] && NeQ[m + p, 0] && (IntegersQ[2*m, 2*p] || IntegerQ[m])
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \int (b+a\cos(c+dx))^2\sin^2(c+dx)\,dx$$
$$= \frac{a(b+a\cos(c+dx))\sin^3(c+dx)}{4d} + \frac{1}{4}\int \left(a^2+4b^2+5\right.$$
$$= \frac{5ab\sin^3(c+dx)}{12d} + \frac{a(b+a\cos(c+dx))\sin^3(c+dx)}{4d} +$$
$$= -\frac{(a^2+4b^2)\cos(c+dx)\sin(c+dx)}{8d} + \frac{5ab\sin^3(c+dx}{12d}$$
$$= \frac{1}{8}\left(a^2+4b^2\right)x - \frac{(a^2+4b^2)\cos(c+dx)\sin(c+dx)}{8d} +$$

Mathematica [A]
time = 0.25, size = 82, normalized size = 0.95

$$\frac{12a^2c+48b^2c+12a^2dx+48b^2dx+48ab\sin(c+dx)-24b^2\sin(2(c+dx))-16ab\sin(3(c+dx))-3a^2\sin(4(c+dx))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (12*a^2*c + 48*b^2*c + 12*a^2*d*x + 48*b^2*d*x + 48*a*b*Sin[c + d*x] - 24*b
^2*Sin[2*(c + d*x)] - 16*a*b*Sin[3*(c + d*x)] - 3*a^2*Sin[4*(c + d*x)])/(96
*d)
```

Maple [A]
time = 0.12, size = 86, normalized size = 1.00

method	result	si
risch	$\frac{a^2x}{8}+\frac{b^2x}{2}+\frac{ab\sin(dx+c)}{2d}-\frac{\sin(4dx+4c)a^2}{32d}-\frac{ab\sin(3dx+3c)}{6d}-\frac{\sin(2dx+2c)b^2}{4d}$	77
derivativedivides	$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+\frac{2ab\left(\sin^3(dx+c)\right)}{3}+b^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	86

default	$\frac{a^2\left(-\frac{\sin(dx+c)\left(\cos^3(dx+c)\right)}{4}+\frac{\cos(dx+c)\sin(dx+c)}{8}+\frac{dx}{8}+\frac{c}{8}\right)+\frac{2ab\left(\sin^3(dx+c)\right)}{3}+b^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/4*sin(d*x+c)*cos(d*x+c)^3+1/8*cos(d*x+c)*sin(d*x+c)+1/8*d*x+1/
8*c)+2/3*a*b*sin(d*x+c)^3+b^2*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.26, size = 66, normalized size = 0.77

$$\frac{64\,ab\sin\left(dx+c\right)^3+3\left(4\,dx+4\,c-\sin\left(4\,dx+4\,c\right)\right)a^2+24\left(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right)\right)b^2}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/96*(64*a*b*sin(d*x + c)^3 + 3*(4*d*x + 4*c - sin(4*d*x + 4*c))*a^2 + 24*(
2*d*x + 2*c - sin(2*d*x + 2*c))*b^2)/d
```

Fricas [A]
time = 3.56, size = 74, normalized size = 0.86

$$\frac{3\left(a^2+4\,b^2\right)dx-\left(6\,a^2\cos\left(dx+c\right)^3+16\,ab\cos\left(dx+c\right)^2-16\,ab-3\left(a^2-4\,b^2\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(a^2 + 4*b^2)*d*x - (6*a^2*cos(d*x + c)^3 + 16*a*b*cos(d*x + c)^2 -
 16*a*b - 3*(a^2 - 4*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)^2\cos^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**2*cos(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 5161 vs. 2(78) = 156.
time = 2.90, size = 5161, normalized size = 60.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")

[Out] 1/8*a^2*x - 1/32*a^2*sin(4*d*x + 4*c)/d + 1/6*(3*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 3*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^2 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c)^2 + 3*b^2*d*x*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 3*b^2*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c) + 3*b^2*tan(d*x)*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c)^2 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^6 + 3*b^2*d*x*tan(1/2*d*x)^6*tan(1/2*c)^6 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^2*tan(c)^2 + 27*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 + 9*b^2*d*x*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^2 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^6*tan(c)^2 + 9*b^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c)^2 - 3*b^2*tan(d*x)*tan(1/2*d*x)^6*tan(1/2*c)^6 + 9*b^2*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c) + 9*b^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c) - 3*b^2*tan(1/2*d*x)^6*tan(1/2*c)^6*tan(c) - 32*a*b*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^3*tan(c)^2 - 96*a*b*tan(d*x)^2*tan(1/2*d*x)^5*tan(1/2*c)^4*tan(c)^2 + 9*b^2*tan(d*x)*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c)^2 - 96*a*b*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^5*tan(c)^2 - 32*a*b*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^6*tan(c)^2 + 9*b^2*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c)^2 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^2 + 27*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4 + 9*b^2*d*x*tan(1/2*d*x)^6*tan(1/2*c)^4 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^6 + 9*b^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^6 + 3*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^6*tan(c)^2 + 27*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^2*tan(c)^2 + 9*b^2*d*x*tan(1/2*d*x)^6*tan(1/2*c)^2*tan(c)^2 + 27*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(c)^2 + 27*b^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 + 3*b^2*d*x*tan(d*x)^2*tan(1/2*c)^6*tan(c)^2 + 9*b^2*d*x*tan(1/2*d*x)^2*tan(1/2*c)^6*tan(c)^2 - 32*a*b*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^3 - 96*a*b*tan(d*x)^2*tan(1/2*d*x)^5*tan(1/2*c)^4 - 9*b^2*tan(d*x)*tan(1/2*d*x)^6*tan(1/2*c)^4 - 96*a*b*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^5 - 32*a*b*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^6 - 9*b^2*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^6 + 9*b^2*tan(d*x)^2*tan(1/2*d*x)^6*tan(1/2*c)^2*tan(c) + 27*b^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c) - 9*b^2*tan(1/2*d*x)^6*tan(1/2*c)^4*tan(c) + 9*b^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^6*tan(c) - 9*b^2*tan(1/2*d*x)^4*tan(1/2*c)^6*tan(c) + 96*a*b*tan(d*x)^2*tan(1/2*d*x)^5*tan(1/2*c)^2*tan(c)^2 + 9*b^2*tan(d*x)*tan(1/2*d*x)^6*tan(1/2*c)^2*tan(c)^2 + 288*a*b*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c)^3*tan(c)^2 - 32*a*b*tan(1/2*d*x)^6*tan(1/2*c)^3*tan(c)^2 + 288*a*b*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^4*tan(c)^2 + 27*b^2*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(c)^2 - 96*a*b*tan(1/2*d*x)^5*tan(1/2*c)^4*tan(c)^2 + 96*a*b*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^5*tan(c)^2 - 96*a*b*tan(1/2*d*x)^4*tan(1/2*c)^5*tan(c)^2 + 9*b^2*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^6*tan(c)^2 - 32*a*b*tan(1/2*d*x)^3*tan(1/2

```
*c)^6*tan(c)^2 + 3*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^6 + 27*b^2*d*x*tan(d*x)^
2*tan(1/2*d*x)^4*tan(1/2*c)^2 + 9*b^2*d*x*tan(1/2*d*x)^6*tan(1/2*c)^2 + 27*
b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^4 + 27*b^2*d*x*tan(1/2*d*x)^4*
tan(1/2*c)^4 + 3*b^2*d*x*tan(d*x)^2*tan(1/2*c)^6 + 9*b^2*d*x*tan(1/2*d*x)^2
*tan(1/2*c)^6 + 9*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^4*tan(c)^2 + 3*b^2*d*x*ta
n(1/2*d*x)^6*tan(c)^2 + 27*b^2*d*x*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*t
an(c)^2 + 27*b^2*d*x*tan(1/2*d*x)^4*tan(1/2*c)^2*tan(c)^2 + 9*b^2*d*x*tan(d
*x)^2*tan(1/2*c)^4*tan(c)^2 + 27*b^2*d*x*tan(1/2*d*x)^2*tan(1/2*c)^4*tan(c)
^2 + 3*b^2*d*x*tan(1/2*c)^6*tan(c)^2 + 96*a*b*tan(d*x)^2*tan(1/2*d*x)^5*tan
(1/2*c)^2 - 9*b^2*tan(d*x)*tan(1/2*d*x)^6*tan(1/2*c)^2 + 288*a*b*tan(d*x)^2
*tan(1/2*d*x)^4*tan(1/2*c)^3 - 32*a*b*tan(1/2*d*x)^6*tan(1/2*c)^3 + 288*a*b
*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*c)^4 - 27*b^2*tan(d*x)*tan(1/2*d*x)^4*ta
n(1/2*c)^4 - 96*a*b*tan(1/2*d*x)^5*tan(1/2*c)^4 + 96*a*b*tan(d*x)^2*tan(1/2
*d*x)^2*tan(1/2*c)^5 - 96*a*b*tan(1/2*d*x)^4*tan(1/2*c)^5 - 9*b^2*tan(d*x)*
tan(1/2*d*x)^2*tan(1/2*c)^6 - 32*a*b*tan(1/2*d*x)^3*tan(1/2*c)^6 + 3*b^2*ta
n(d*x)^2*tan(1/2*d*x)^6*tan(c) + 27*b^2*tan(d*x)^2*tan(1/2*d*x)^4*tan(1/2*c
)^2*tan(c) - 9*b^2*tan(1/2*d*x)^6*tan(1/2*c)^2*tan(c) + 27*b^2*tan(d*x)^2*t
an(1/2*d*x)^2*tan(1/2*c)^4*tan(c) - 27*b^2*tan(1/2*d*x)^4*tan(1/2*c)^4*tan(
c) + 3*b^2*tan(d*x)^2*tan(1/2*c)^6*tan(c) - 9*b^2*tan(1/2*d*x)^2*tan(1/2*c)
^6*tan(c) + 3*b^2*tan(d*x)*tan(1/2*d*x)^6*tan(c)^2 - 96*a*b*tan(d*x)^2*tan(
1/2*d*x)^4*tan(1/2*c)*tan(c)^2 - 288*a*b*tan(d*x)^2*tan(1/2*d*x)^3*tan(1/2*
c)^2*tan(c)^2 + 27*b^2*tan(d*x)*tan(1/2*d*x)^4*tan(1/2*c)^2*tan(c)^2 + 96*a
*b*tan(1/2*d*x)^5*tan(1/2*c)^2*tan(c)^2 - 288*a*b*tan(d*x)^2*tan(1/2*d*x)^2
*tan(1/2*c)^3*tan(c)^2 + 288*a*b*tan(1/2*d*x)^4*tan(1/2*c)^3*tan(c)^2 - 96*
a*b*tan(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^4*tan(c)^2 + 27*b^2*tan(d*x)*tan(1/2
*d*x)^2*tan(1/2*c)^4*tan(c)^2 + 288*a*b*tan(1/2...
```

Mupad [**B**]
time = 0.71, size = 76, normalized size = 0.88

$$\frac{a^2\,x}{8}+\frac{b^2\,x}{2}-\frac{a^2\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}-\frac{b^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{a\,b\,\sin\left(c+d\,x\right)}{2\,d}-\frac{a\,b\,\sin\left(3\,c+3\,d\,x\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a*sin(c + d*x) + b*tan(c + d*x))^2,x)
```

```
[Out] (a^2*x)/8 + (b^2*x)/2 - (a^2*sin(4*c + 4*d*x))/(32*d) - (b^2*sin(2*c + 2*d*
x))/(4*d) + (a*b*sin(c + d*x))/(2*d) - (a*b*sin(3*c + 3*d*x))/(6*d)
```

3.238 $\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=87

$$abx+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{(a^2-2b^2)\sin(c+dx)}{3d}-\frac{ab\cos(c+dx)\sin(c+dx)}{3d}-\frac{(b+a\cos(c+dx))^2\sin(c+dx)}{3d}$$

```
[Out] a*b*x+b^2*arctanh(sin(d*x+c))/d+1/3*(a^2-2*b^2)*sin(d*x+c)/d-1/3*a*b*cos(d*
x+c)*sin(d*x+c)/d-1/3*(b+a*cos(d*x+c))^2*sin(d*x+c)/d
```

Rubi [A]
time = 0.22, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4482, 2968, 3129, 3112, 3102, 2814, 3855}

$$\frac{(a^2-2b^2)\sin(c+dx)}{3d}-\frac{ab\sin(c+dx)\cos(c+dx)}{3d}-\frac{\sin(c+dx)(a\cos(c+dx)+b)^2}{3d}+abx+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] a*b*x + (b^2*ArcTanh[Sin[c + d*x]])/d + ((a^2 - 2*b^2)*Sin[c + d*x])/(3*d)
- (a*b*Cos[c + d*x]*Sin[c + d*x])/(3*d) - ((b + a*Cos[c + d*x])^2*Sin[c + d
*x])/(3*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3129

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n +
1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x]
)^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n
 + 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(
a*d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0
])))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \int (b+a\cos(c+dx))^2\sin(c+dx)\tan(c+dx)\,dx$$
$$= \int (b+a\cos(c+dx))^2\left(1-\cos^2(c+dx)\right)\sec(c+dx)\,dx$$
$$= -\frac{(b+a\cos(c+dx))^2\sin(c+dx)}{3d}+\frac{1}{3}\int (b+a\cos(c+$$
$$= -\frac{ab\cos(c+dx)\sin(c+dx)}{3d}-\frac{(b+a\cos(c+dx))^2\sin($$
$$= \frac{(a^2-2b^2)\sin(c+dx)}{3d}-\frac{ab\cos(c+dx)\sin(c+dx)}{3d}-$$
$$= abx+\frac{(a^2-2b^2)\sin(c+dx)}{3d}-\frac{ab\cos(c+dx)\sin(c+d}{3d}$$
$$= abx+\frac{b^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{(a^2-2b^2)\sin(c+dx)}{3d}$$

Mathematica [A]
time = 0.17, size = 117, normalized size = 1.34

$$\frac{12abc+12abdx-12b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+3(a^2-4b^2)\sin(c+dx)-6ab\sin(2(c+dx))-a^2\sin(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (12*a*b*c + 12*a*b*d*x - 12*b^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
12*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 3*(a^2 - 4*b^2)*Sin[c + d
*x] - 6*a*b*Sin[2*(c + d*x)] - a^2*Sin[3*(c + d*x)])/(12*d)
```

Maple [A]
time = 0.11, size = 72, normalized size = 0.83

method	result
derivativedivides	$\frac{\frac{a^2\left(\sin^3(dx+c)\right)}{3}+2ab\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
default	$\frac{\frac{a^2\left(\sin^3(dx+c)\right)}{3}+2ab\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+b^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))}{d}$
risch	$abx-\frac{i e^{i(dx+c)}a^2}{8d}+\frac{i e^{i(dx+c)}b^2}{2d}+\frac{i e^{-i(dx+c)}a^2}{8d}-\frac{i e^{-i(dx+c)}b^2}{2d}+\frac{b^2\ln\left(e^{i(dx+c)}+i\right)}{d}-\frac{b^2\ln\left(e^{i(dx+c)}-i\right)}{d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

[Out] 1/d*(1/3*a^2*sin(d*x+c)^3+2*a*b*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+b^2*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c))))

Maxima [A]
time = 0.28, size = 76, normalized size = 0.87

$$\frac{2\,a^2\sin\left(dx+c\right)^3+3\left(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right)\right)ab+3\,b^2(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)-2\,\sin\left(dx+c\right))}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")

[Out] 1/6*(2*a^2*sin(d*x + c)^3 + 3*(2*d*x + 2*c - sin(2*d*x + 2*c))*a*b + 3*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)))/d

Fricas [A]
time = 2.25, size = 83, normalized size = 0.95

$$\frac{6\,abdx+3\,b^2\log\left(\sin\left(dx+c\right)+1\right)-3\,b^2\log\left(-\sin\left(dx+c\right)+1\right)-2\left(a^2\cos\left(dx+c\right)^2+3\,ab\cos\left(dx+c\right)-a^2+3\,b^2\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")

[Out] 1/6*(6*a*b*d*x + 3*b^2*log(sin(d*x + c) + 1) - 3*b^2*log(-sin(d*x + c) + 1) - 2*(a^2*cos(d*x + c)^2 + 3*a*b*cos(d*x + c) - a^2 + 3*b^2)*sin(d*x + c))/d

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)^2\cos\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))**2,x)

[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**2*cos(c + d*x), x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 5713 vs. 2(81) = 162.
time = 2.22, size = 5713, normalized size = 65.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")

[Out] -1/12*a^2*sin(3*d*x + 3*c)/d + 1/4*a^2*sin(d*x + c)/d + 1/2*(2*a*b*d*x*tan(
d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 - b^2*log(2*(tan(1/2*d*x)^4*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + t
an(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/
2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*
tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*
tan(c)^2 + b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/
2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))
*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 + 2*a*b*d*x*tan(d*x)^2*tan
(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*b*d*x*tan(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 + 2
*a*b*d*x*tan(d*x)^2*tan(1/2*c)^2*tan(c)^2 + 2*a*b*d*x*tan(1/2*d*x)^2*tan(1/
2*c)^2*tan(c)^2 - b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4
*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*
x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(
1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^
2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2 + b^2*log(2*(tan(1/2*d*x)^4*
tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2
+ tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan
(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) +
 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)
^2 + 2*a*b*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - b^2*log(2*(tan(1
/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan
(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)
^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(
1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*d*x)^2*
tan(c)^2 + b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/
2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*ta
n(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))
*tan(d*x)^2*tan(1/2*d*x)^2*tan(c)^2 + 4*b^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(1
/2*c)*tan(c)^2 - b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*
tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x
)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1
/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2
 + 1))*tan(d*x)^2*tan(1/2*c)^2*tan(c)^2 + b^2*log(2*(tan(1/2*d*x)^4*tan(1/2
*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1
/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*
x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(
1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*tan(1/2*c)^2*tan(c)^2 + 4*b^2*ta
n(d*x)^2*tan(1/2*d*x)*tan(1/2*c)^2*tan(c)^2 - b^2*log(2*(tan(1/2*d*x)^4*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + t
an(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/
2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*

```
tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c)^2 +
b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*ta
n(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2
+ 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1
/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*
x)^2*tan(1/2*c)^2*tan(c)^2 + 2*a*b*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan
(c)^2 + 2*a*b*d*x*tan(d*x)^2*tan(1/2*d*x)^2 + 2*a*b*d*x*tan(d*x)^2*tan(1/2*
c)^2 + 2*a*b*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*a*b*d*x*tan(d*x)^2*tan(c)^
2 + 2*a*b*d*x*tan(1/2*d*x)^2*tan(c)^2 + 2*a*b*d*x*tan(1/2*c)^2*tan(c)^2 - b
^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan
(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 -
 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/
2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)^2*
tan(1/2*d*x)^2 + b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*
tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x
)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1
/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2
 + 1))*tan(d*x)^2*tan(1/2*d*x)^2 + 4*b^2*tan(d*x)^2*tan(1/2*d*x)^2*tan(1/2*
c) - b^2*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) +
 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*
c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + ...
```

Mupad [B]
time = 0.84, size = 121, normalized size = 1.39

$$\frac{a^2\,\sin\left(c+d\,x\right)}{4\,d}-\frac{b^2\,\sin\left(c+d\,x\right)}{d}+\frac{2\,b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a^2\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{2\,a\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a\,b\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(a*sin(c + d*x) + b*tan(c + d*x))^2,x)
```

```
[Out] (a^2*sin(c + d*x))/(4*d) - (b^2*sin(c + d*x))/d + (2*b^2*atanh(sin(c/2 + (d
*x)/2)/cos(c/2 + (d*x)/2)))/d - (a^2*sin(3*c + 3*d*x))/(12*d) + (2*a*b*atan
(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a*b*sin(2*c + 2*d*x))/(2*d)
```

3.239 $\int (a\sin(c+dx)+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=77

$$\frac{a^2x}{2}-b^2x+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ab\sin(c+dx)}{d}-\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}+\frac{b^2\tan(c+dx)}{d}$$

```
[Out] 1/2*a^2*x-b^2*x+2*a*b*arctanh(sin(d*x+c))/d-2*a*b*sin(d*x+c)/d-1/2*a^2*cos(
d*x+c)*sin(d*x+c)/d+b^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {4482, 2801, 2715, 8, 2672, 327, 212, 3554}

$$-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{a^2x}{2}-\frac{2ab\sin(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}-b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (a^2*x)/2 - b^2*x + (2*a*b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Sin[c + d*x])/
d - (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (b^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2672

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, a*(Sin[e + f*x]/ff)], x]
```

```
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2801

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 3554

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int (a\sin(c+dx)+b\tan(c+dx))^2\,dx = \int (b+a\cos(c+dx))^2\tan^2(c+dx)\,dx$$

$$= \int \left(a^2\sin^2(c+dx)+2ab\sin(c+dx)\tan(c+dx)+b^2\tan^2(c+dx)\right)$$

$$= a^2\int \sin^2(c+dx)\,dx+(2ab)\int \sin(c+dx)\tan(c+dx)\,dx+b^2\int \text{ta}$$

$$= -\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}+\frac{b^2\tan(c+dx)}{d}+\frac{1}{2}a^2\int 1\,dx-b^2\int 1$$

$$= \frac{a^2x}{2}-b^2x-\frac{2ab\sin(c+dx)}{d}-\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}+\frac{b^2\tan(c}{d}$$

$$= \frac{a^2x}{2}-b^2x+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ab\sin(c+dx)}{d}-\frac{a^2\cos(c+}{}$$

Mathematica [A]
time = 0.67, size = 116, normalized size = 1.51

$$-\frac{-2(a^2-2b^2)(c+dx)+8ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-8ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+8ab\sin(c+dx)+(a^2-4b^2+a^2\cos(2(c+dx)))\tan(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/4*(-2*(a^2 - 2*b^2)*(c + d*x) + 8*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] - 8*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 8*a*b*Sin[c + d*x
] + (a^2 - 4*b^2 + a^2*Cos[2*(c + d*x)])*Tan[c + d*x])/d
```

Maple [A]
time = 0.13, size = 77, normalized size = 1.00

method	result
derivativedivides	$\frac{a^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+b^2(\tan(dx+c)-dx-c)}{d}$
default	$\frac{a^2\left(-\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+b^2(\tan(dx+c)-dx-c)}{d}$
risch	$\frac{a^2x}{2}-b^2x+\frac{ia^2e^{2i(dx+c)}}{8d}+\frac{iab\,e^{i(dx+c)}}{d}-\frac{iab\,e^{-i(dx+c)}}{d}-\frac{ia^2e^{-2i(dx+c)}}{8d}+\frac{2ib^2}{d(e^{2i(dx+c)}+1)}+\frac{2ab\ln(e^{i(dx}}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(d*x+c)+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+2*a*b*(-sin(d*x+c)+ln(s
ec(d*x+c)+tan(d*x+c)))+b^2*(tan(d*x+c)-d*x-c))
```

Maxima [A]
time = 0.48, size = 84, normalized size = 1.09

$$\frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))a^2}{4\,d}-\frac{(dx+c-\tan(dx+c))b^2}{d}+\frac{ab(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\,\sin(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*d*x + 2*c - sin(2*d*x + 2*c))*a^2/d - (d*x + c - tan(d*x + c))*b^2/d
 + a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c))/d
```

Fricas [A]
time = 1.21, size = 108, normalized size = 1.40

$$\frac{(a^2-2\,b^2)dx\cos(dx+c)+2\,ab\cos(dx+c)\log(\sin(dx+c)+1)-2\,ab\cos(dx+c)\log(-\sin(dx+c)+1)-(a^2\cos(dx+c)^2+4\,ab\cos(dx+c)-2\,b^2)\sin(dx+c)}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 - 2*b^2)*d*x*cos(d*x + c) + 2*a*b*cos(d*x + c)*log(sin(d*x + c) +
 1) - 2*a*b*cos(d*x + c)*log(-sin(d*x + c) + 1) - (a^2*cos(d*x + c)^2 + 4*a
*b*cos(d*x + c) - 2*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(c+dx)+b\tan(c+dx))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2752 vs. 2(73) = 146.
time = 0.85, size = 2752, normalized size = 35.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*x - 1/4*a^2*sin(2*d*x + 2*c)/d - (b^2*d*x*tan(d*x)*tan(1/2*d*x)^2*t
an(1/2*c)^2*tan(c) + a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x
)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2
*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*t
an(1/2*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*
c)^2 + 1))*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - a*b*log(2*(tan(1/2
*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1
/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3
 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/
2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)*tan(1/2*d*x)^2*tan(
1/2*c)^2*tan(c) - b^2*d*x*tan(1/2*d*x)^2*tan(1/2*c)^2 + b^2*d*x*tan(d*x)*ta
n(1/2*d*x)^2*tan(c) + b^2*d*x*tan(d*x)*tan(1/2*c)^2*tan(c) - a*b*log(2*(tan
(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*d*x)^3*t
an(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*tan(1/2*d*
x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 - 2*ta
n(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^2*tan(1/2*c
)^2 + a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c)
- 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2
*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
 tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(
```

1/2*d*x)^2*tan(1/2*c)^2 + b^2*tan(d*x)*tan(1/2*d*x)^2*tan(1/2*c)^2 + a*b*lo
g(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1/2*
d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2*ta
n(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^
2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)*tan(1/2
*d*x)^2*tan(c) - a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*
tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x
)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1
/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2
 + 1))*tan(d*x)*tan(1/2*d*x)^2*tan(c) - 4*a*b*tan(d*x)*tan(1/2*d*x)^2*tan(1
/2*c)*tan(c) + a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*ta
n(1/2*c) + 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^
2*tan(1/2*c)^2 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2
*d*x)^2 + tan(1/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 +
 1))*tan(d*x)*tan(1/2*c)^2*tan(c) - a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2
 - 2*tan(1/2*d*x)^4*tan(1/2*c) - 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x
)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan
(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c)
 + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)*tan(1/2*c)^2*tan(c) - 4*a*b*tan(d*x)*tan
(1/2*d*x)*tan(1/2*c)^2*tan(c) + b^2*tan(1/2*d*x)^2*tan(1/2*c)^2*tan(c) - b^
2*d*x*tan(1/2*d*x)^2 - b^2*d*x*tan(1/2*c)^2 + b^2*d*x*tan(d*x)*tan(c) - a*b
log(2(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1
/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2
*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*d*x)^
2 + a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) -
2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c
)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + t
an(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/
2*d*x)^2 + b^2*tan(d*x)*tan(1/2*d*x)^2 + 4*a*b*tan(1/2*d*x)^2*tan(1/2*c) -
a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*ta
n(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2
 - 2*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1
/2*c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1/2*c)
^2 + a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) -
 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*
c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(1
/2*c)^2 + b^2*tan(d*x)*tan(1/2*c)^2 + 4*a*b*tan(1/2*d*x)*tan(1/2*c)^2 + a*b
log(2(tan(1/2*d*x)^4*tan(1/2*c)^2 + 2*tan(1/2*d*x)^4*tan(1/2*c) + 2*tan(1
/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*c)^2 - 2
*tan(1/2*d*x)^3 + 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 + tan(1/2*
c)^2 - 2*tan(1/2*d*x) - 2*tan(1/2*c) + 1)/(tan(1/2*c)^2 + 1))*tan(d*x)*tan(
c) - a*b*log(2*(tan(1/2*d*x)^4*tan(1/2*c)^2 - 2*tan(1/2*d*x)^4*tan(1/2*c) -
 2*tan(1/2*d*x)^3*tan(1/2*c)^2 + tan(1/2*d*x)^4 + 2*tan(1/2*d*x)^2*tan(1/2*

```
c)^2 + 2*tan(1/2*d*x)^3 - 2*tan(1/2*d*x)*tan(1/2*c)^2 + 2*tan(1/2*d*x)^2 +
tan(1/2*c)^2 + 2*tan(1/2*d*x) + 2*tan(1/2*c) + ...
```

Mupad [B]
time = 0.80, size = 143, normalized size = 1.86

$$\frac{a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{2\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{b^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}-\frac{2\,a\,b\,\sin\left(c+d\,x\right)}{d}+\frac{4\,a\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b*tan(c + d*x))^2,x)
```

```
[Out] (a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (2*b^2*atan(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^2*sin(c + d*x))/(d*cos(c + d*x)) - (2
*a*b*sin(c + d*x))/d + (4*a*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))
/d - (a^2*cos(c + d*x)*sin(c + d*x))/(2*d)
```

3.240 $\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=90

$$-2abx+\frac{(2a^2-b^2)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{3a^2\sin(c+dx)}{2d}+\frac{ab\tan(c+dx)}{d}+\frac{(b+a\cos(c+dx))^2\sec(c+dx)}{2d}$$

```
[Out] -2*a*b*x+1/2*(2*a^2-b^2)*arctanh(sin(d*x+c))/d-3/2*a^2*sin(d*x+c)/d+a*b*tan
(d*x+c)/d+1/2*(b+a*cos(d*x+c))^2*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A]
time = 0.30, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4482, 2968, 3127, 3110, 3102, 2814, 3855}

$$\frac{(2a^2-b^2)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{3a^2\sin(c+dx)}{2d}+\frac{ab\tan(c+dx)}{d}+\frac{\tan(c+dx)\sec(c+dx)(a\cos(c+dx)+b)^2}{2d}-2abx$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] -2*a*b*x + ((2*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(2*d) - (3*a^2*Sin[c + d*x
])/(2*d) + (a*b*Tan[c + d*x])/d + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]*Tan[
c + d*x])/(2*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \int (b+a\cos(c+dx))^2\sec(c+dx)\tan^2(c+dx)\,dx$$
$$= \int (b+a\cos(c+dx))^2\left(1-\cos^2(c+dx)\right)\sec^3(c+dx)\,a$$
$$= \frac{(b+a\cos(c+dx))^2\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\int (b+$$
$$= \frac{ab\tan(c+dx)}{d} + \frac{(b+a\cos(c+dx))^2\sec(c+dx)\tan(c}{2d}$$
$$= -\frac{3a^2\sin(c+dx)}{2d} + \frac{ab\tan(c+dx)}{d} + \frac{(b+a\cos(c+dx}{}$$
$$= -2abx - \frac{3a^2\sin(c+dx)}{2d} + \frac{ab\tan(c+dx)}{d} + \frac{(b+a\cos}{}$$
$$= -2abx + \frac{(2a^2-b^2)\tanh^{-1}(\sin(c+dx))}{2d} - \frac{3a^2\sin(c+}{2d}$$

Mathematica [A]
time = 0.22, size = 75, normalized size = 0.83

$$\frac{-4ab\text{ArcTan}(\tan(c+dx)) + (2a^2-b^2)\tanh^{-1}(\sin(c+dx)) - 2a^2\sin(c+dx) + 4ab\tan(c+dx) + b^2\sec(c+dx)\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
[Out] (-4*a*b*ArcTan[Tan[c + d*x]] + (2*a^2 - b^2)*ArcTanh[Sin[c + d*x]] - 2*a^2*
Sin[c + d*x] + 4*a*b*Tan[c + d*x] + b^2*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Maple [A]
time = 0.13, size = 98, normalized size = 1.09

method	result
derivativedivides	$\frac{a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+2ab(\tan(dx+c)-dx-c)+b^2\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(d}{2}\right.}{d}$
default	$\frac{a^2(-\sin(dx+c)+\ln(\sec(dx+c)+\tan(dx+c)))+2ab(\tan(dx+c)-dx-c)+b^2\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(d}{2}\right.}{d}$
risch	$-2abx+\frac{i e^{i(dx+c)}a^2}{2d}-\frac{i e^{-i(dx+c)}a^2}{2d}-\frac{ib\left(b\,e^{3i(dx+c)}-4a\,e^{2i(dx+c)}-b\,e^{i(dx+c)}-4a\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{\ln\left(e^{i(dx+c)}+i\right)a^2}{d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(-sin(d*x+c)+ln(sec(d*x+c)+tan(d*x+c)))+2*a*b*(tan(d*x+c)-d*x-c)+b
^2*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+tan(d*x+
c))))
```

Maxima [A]
time = 0.47, size = 102, normalized size = 1.13

$$-\frac{8\,(dx+c-\tan\left(dx+c\right))ab+b^2\left(\frac{2\,\sin\left(dx+c\right)}{\sin\left(dx+c\right)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)-2\,a^2(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)-2\,\sin\left(dx+c\right))}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(8*(d*x + c - tan(d*x + c))*a*b + b^2*(2*sin(d*x + c)/(sin(d*x + c)^2
- 1) + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 2*a^2*(log(sin(d*x
+ c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)))/d
```

Fricas [A]
time = 2.24, size = 126, normalized size = 1.40

$$-\frac{8\,abdx\cos\left(dx+c\right)^2-\left(2\,a^2-b^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)+\left(2\,a^2-b^2\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(2\,a^2\cos\left(dx+c\right)^2-4\,ab\cos\left(dx+c\right)-b^2\right)\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(8*a*b*d*x*cos(d*x + c)^2 - (2*a^2 - b^2)*cos(d*x + c)^2*log(sin(d*x +
 c) + 1) + (2*a^2 - b^2)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*a^2*c
os(d*x + c)^2 - 4*a*b*cos(d*x + c) - b^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)^2\sec\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**2*sec(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 171 vs. 2(84) = 168.
time = 0.99, size = 171, normalized size = 1.90

$$-\frac{4\,(dx+c)ab-\left(2\,a^2-b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+\left(2\,a^2-b^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{4\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+\frac{2\,\left(4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*(d*x + c)*a*b - (2*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) +
(2*a^2 - b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*a^2*tan(1/2*d*x + 1/2*
c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(4*a*b*tan(1/2*d*x + 1/2*c)^3 - b^2*tan
(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x + 1/2*c) - b^2*tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B]
time = 0.79, size = 147, normalized size = 1.63

$$\frac{2\,a^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a^2\,\sin\left(c+d\,x\right)}{d}-\frac{b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{b^2\,\sin\left(c+d\,x\right)}{2\,d\,\cos\left(c+d\,x\right)^2}-\frac{4\,a\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,a\,b\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b*tan(c + d*x))^2/cos(c + d*x),x)
```

```
[Out] (2*a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a^2*sin(c + d*x))
/d - (b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^2*sin(c + d*
x))/(2*d*cos(c + d*x)^2) - (4*a*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
)))/d + (2*a*b*sin(c + d*x))/(d*cos(c + d*x))
```

3.241 $\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=99

$$-a^2x-\frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{(2a^2-b^2)\tan(c+dx)}{3d}+\frac{ab\sec(c+dx)\tan(c+dx)}{3d}+\frac{(b+a\cos(c+dx))^2\sec$$

```
[Out] -a^2*x-a*b*arctanh(sin(d*x+c))/d+1/3*(2*a^2-b^2)*tan(d*x+c)/d+1/3*a*b*sec(d
*x+c)*tan(d*x+c)/d+1/3*(b+a*cos(d*x+c))^2*sec(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.33, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4482, 2968, 3127, 3110, 3100, 2814, 3855}

$$\frac{(2a^2-b^2)\tan(c+dx)}{3d}+a^2(-x)-\frac{ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{ab\tan(c+dx)\sec(c+dx)}{3d}+\frac{\tan(c+dx)\sec^2(c+dx)(a\cos(c+dx)+b)^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] -(a^2*x) - (a*b*ArcTanh[Sin[c + d*x]])/d + ((2*a^2 - b^2)*Tan[c + d*x])/(3*
d) + (a*b*Sec[c + d*x]*Tan[c + d*x])/(3*d) + ((b + a*Cos[c + d*x])^2*Sec[c
+ d*x]^2*Tan[c + d*x])/(3*d)
```

Rule 2814

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \int (b+a\cos(c+dx))^2\sec^2(c+dx)\tan^2(c+dx)\,dx$$

$$= \int (b+a\cos(c+dx))^2\left(1-\cos^2(c+dx)\right)\sec^4(c+dx)\,d$$

$$= \frac{(b+a\cos(c+dx))^2\sec^2(c+dx)\tan(c+dx)}{3d} + \frac{1}{3}\int (b+$$

$$= \frac{ab\sec(c+dx)\tan(c+dx)}{3d} + \frac{(b+a\cos(c+dx))^2\sec^2(c}{3d}$$

$$= \frac{(2a^2-b^2)\tan(c+dx)}{3d} + \frac{ab\sec(c+dx)\tan(c+dx)}{3d} + $$

$$= -a^2x + \frac{(2a^2-b^2)\tan(c+dx)}{3d} + \frac{ab\sec(c+dx)\tan(c+}{3d}$$

$$= -a^2x - \frac{ab\tanh^{-1}(\sin(c+dx))}{d} + \frac{(2a^2-b^2)\tan(c+dx}{3d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 201 vs. 2(99) = 198.
time = 1.25, size = 201, normalized size = 2.03

$$\frac{\sec^3(c+dx)\left(-9a\cos(c+dx)\left(a(c+dx)-b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-3a\cos(3(c+dx))\left(a(c+dx)-b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+2(3a^2+b^2+6ab\cos(c+dx)+(3a^2-b^2)\cos(2(c+dx)))\sin(c+dx)\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^3*(-9*a*Cos[c + d*x]*(a*(c + d*x) - b*Log[Cos[(c + d*x)/2] -
Sin[(c + d*x)/2]] + b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 3*a*Cos[3
*(c + d*x)]*(a*(c + d*x) - b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + b*L
og[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + 2*(3*a^2 + b^2 + 6*a*b*Cos[c + d
*x] + (3*a^2 - b^2)*Cos[2*(c + d*x)])*Sin[c + d*x]))/(12*d)
```

Maple [A]
time = 0.15, size = 92, normalized size = 0.93

method	result
derivativedivides	$\frac{a^2(\tan(dx+c)-dx-c)+2ab\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{b^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}}{d}$
default	$\frac{a^2(\tan(dx+c)-dx-c)+2ab\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{b^2\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}}{d}$
risch	$-a^2x-\frac{2i\left(3ab\,e^{5i(dx+c)}-3a^2e^{4i(dx+c)}+3b^2e^{4i(dx+c)}-6a^2e^{2i(dx+c)}-3ab\,e^{i(dx+c)}-3a^2+b^2\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}+\frac{ab\ln\left(e^{i(dx+c)}-i\right)}{d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(tan(d*x+c)-d*x-c)+2*a*b*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*
x+c)-1/2*ln(sec(d*x+c)+tan(d*x+c)))+1/3*b^2*sin(d*x+c)^3/cos(d*x+c)^3)
```

Maxima [A]
time = 0.47, size = 82, normalized size = 0.83

$$\frac{2\,b^2\tan\left(dx+c\right)^3-6\left(dx+c-\tan\left(dx+c\right)\right)a^2-3\,ab\left(\frac{2\,\sin\left(dx+c\right)}{\sin\left(dx+c\right)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*b^2*tan(d*x + c)^3 - 6*(d*x + c - tan(d*x + c))*a^2 - 3*a*b*(2*sin(d
*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
)))/d
```

Fricas [A]
time = 2.71, size = 115, normalized size = 1.16

$$-\frac{6\,a^2dx\cos\left(dx+c\right)^3+3\,ab\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,ab\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)-2\left(3\,ab\cos\left(dx+c\right)+\left(3\,a^2-b^2\right)\cos\left(dx+c\right)^2+b^2\right)\sin\left(dx+c\right)}{6\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/6*(6*a^2*d*x*cos(d*x + c)^3 + 3*a*b*cos(d*x + c)^3*log(sin(d*x + c) + 1)
 - 3*a*b*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(3*a*b*cos(d*x + c) + (3
*a^2 - b^2)*cos(d*x + c)^2 + b^2)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)^2\sec^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**2*sec(c + d*x)**2, x)
```

Giac [A]
time = 1.02, size = 158, normalized size = 1.60

$$-\frac{3\left(dx+c\right)a^2+3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3*(3*(d*x + c)*a^2 + 3*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*a*b*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*a*b*
tan(1/2*d*x + 1/2*c)^5 - 6*a^2*tan(1/2*d*x + 1/2*c)^3 + 4*b^2*tan(1/2*d*x +
 1/2*c)^3 + 3*a^2*tan(1/2*d*x + 1/2*c) + 3*a*b*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B]
time = 1.03, size = 227, normalized size = 2.29

$$-\frac{\frac{b^2\sin(3c+3dx)}{12}-\frac{b^2\sin(c+dx)}{4}-\frac{a^2\sin(3c+3dx)}{4}-\frac{a^2\sin(c+dx)}{4}+\frac{3a^2\cos(c+dx)\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{2}+\frac{a^2\operatorname{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos(3c+3dx)}{2}-\frac{ab\sin(2c+2dx)}{2}+\frac{3ab\cos(c+dx)\operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{2}+\frac{ab\operatorname{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\cos(3c+3dx)}{2}}{d\left(\frac{3\cos(c+dx)}{4}+\frac{\cos(3c+3dx)}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b*tan(c + d*x))^2/cos(c + d*x)^2,x)
```

```
[Out] -((b^2*sin(3*c + 3*d*x))/12 - (b^2*sin(c + d*x))/4 - (a^2*sin(3*c + 3*d*x))
/4 - (a^2*sin(c + d*x))/4 + (3*a^2*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2)))/2 + (a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(
3*c + 3*d*x))/2 - (a*b*sin(2*c + 2*d*x))/2 + (3*a*b*cos(c + d*x)*atanh(sin(
c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + (a*b*atanh(sin(c/2 + (d*x)/2)/cos(c
/2 + (d*x)/2))*cos(3*c + 3*d*x))/2)/(d*((3*cos(c + d*x))/4 + cos(3*c + 3*d*
x)/4))
```

3.242 $\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=125

$$-\frac{(4a^2+b^2)\tanh^{-1}(\sin(c+dx))}{8d}-\frac{2ab\tan(c+dx)}{3d}+\frac{(2a^2-b^2)\sec(c+dx)\tan(c+dx)}{8d}+\frac{ab\sec^2(c+dx)\tan}{6d}$$

```
[Out] -1/8*(4*a^2+b^2)*arctanh(sin(d*x+c))/d-2/3*a*b*tan(d*x+c)/d+1/8*(2*a^2-b^2)
*sec(d*x+c)*tan(d*x+c)/d+1/6*a*b*sec(d*x+c)^2*tan(d*x+c)/d+1/4*(b+a*cos(d*x
+c))^2*sec(d*x+c)^3*tan(d*x+c)/d
```

Rubi [A]
time = 0.31, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {4482, 2968, 3127, 3110, 3100, 2827, 3852, 8, 3855}

$$-\frac{(4a^2+b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(2a^2-b^2)\tan(c+dx)\sec(c+dx)}{8d}-\frac{2ab\tan(c+dx)}{3d}+\frac{ab\tan(c+dx)\sec^2(c+dx)}{6d}+\frac{\tan(c+dx)\sec^3(c+dx)(a\cos(c+dx)+b)^2}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/8*((4*a^2 + b^2)*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Tan[c + d*x])/(3*d) +
 ((2*a^2 - b^2)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*b*Sec[c + d*x]^2*Tan[
c + d*x])/(6*d) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^3*Tan[c + d*x])/(4*d
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
```

```
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 3110

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-(b*c - a*d))*(A*b^2 - a*b*B + a^2*C)*Cos[
e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - D
ist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(m
 + 1)*((b*B - a*C)*(b*c - a*d) - A*b*(a*c - b*d)) + (b*B*(a^2*d + b^2*d*(m
+ 1) - a*b*c*(m + 2)) + (b*c - a*d)*(A*b^2*(m + 2) + C*(a^2 + b^2*(m + 1)))
)*Sin[e + f*x] - b*C*d*(m + 1)*(a^2 - b^2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
0] && LtQ[m, -1]
```

Rule 3127

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(c^2*C + A*d^2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e +
 f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^
2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d
*(b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*
c*(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A
*d^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b
^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$
\begin{aligned}
\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx &= \int (b+a\cos(c+dx))^2\sec^3(c+dx)\tan^2(c+dx)\,dx \\
&= \int (b+a\cos(c+dx))^2\left(1-\cos^2(c+dx)\right)\sec^5(c+dx) \\
&= \frac{(b+a\cos(c+dx))^2\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int (b \\
&= \frac{ab\sec^2(c+dx)\tan(c+dx)}{6d}+\frac{(b+a\cos(c+dx))^2\sec^3}{4d} \\
&= \frac{(2a^2-b^2)\sec(c+dx)\tan(c+dx)}{8d}+\frac{ab\sec^2(c+dx)\,\mathrm{ta}}{6d} \\
&= \frac{(2a^2-b^2)\sec(c+dx)\tan(c+dx)}{8d}+\frac{ab\sec^2(c+dx)\,\mathrm{ta}}{6d} \\
&= -\frac{(4a^2+b^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(2a^2-b^2)\sec(c+}{8d} \\
&= -\frac{(4a^2+b^2)\tanh^{-1}(\sin(c+dx))}{8d}-\frac{2ab\tan(c+dx)}{3d}+
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 336 vs. 2(125) = 250.
time = 0.68, size = 336, normalized size = 2.69

$$
\frac{\sec^4(c+dx)\left(36a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+9b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12(4a^2+b^2)\cos(2(c+dx))\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+3(4a^2+b^2)\cos(4(c+dx))\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-36a^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-9b^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+24a^2\sin(c+dx)+42b^2\sin(c+dx)+32ab\sin(2(c+dx))+24a^2\sin(3(c+dx))-6b^2\sin(3(c+dx))-16ab\sin(4(c+dx))\right)}{192d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (Sec[c + d*x]^4*(36*a^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 9*b^2*Lo
g[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*(4*a^2 + b^2)*Cos[2*(c + d*x)]*
(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]]) + 3*(4*a^2 + b^2)*Cos[4*(c + d*x)]*(Log[Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 36*a^2*Log[Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]] - 9*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]] + 24*a^2*Sin[c + d*x] + 42*b^2*Sin[c + d*x] + 32*a*b*Sin[2*(c + d*x)]
 + 24*a^2*Sin[3*(c + d*x)] - 6*b^2*Sin[3*(c + d*x)] - 16*a*b*Sin[4*(c + d*x
)]))/(192*d)
```

Maple [**A**]
time = 0.17, size = 138, normalized size = 1.10

method	result
derivativedivides	$\frac{a^2\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{2ab\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+b^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\ln(\right.}{d}$
default	$\frac{a^2\left(\frac{\sin^3(dx+c)}{2\cos(dx+c)^2}+\frac{\sin(dx+c)}{2}-\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+\frac{2ab\left(\sin^3(dx+c)\right)}{3\cos(dx+c)^3}+b^2\left(\frac{\sin^3(dx+c)}{4\cos(dx+c)^4}+\frac{\sin^3(dx+c)}{8\cos(dx+c)^2}+\frac{\sin(dx+c)}{8}-\ln(\right.}{d}$
risch	$-\frac{i(12a^2e^{7i(dx+c)}-3b^2e^{7i(dx+c)}+48ab\,e^{6i(dx+c)}+12a^2e^{5i(dx+c)}+21b^2e^{5i(dx+c)}+48ab\,e^{4i(dx+c)}-12a^2e^{3i(dx+c)}-21b^2e^{3}}{12d\left(e^{2i(dx+c)}+1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*(1/2*sin(d*x+c)^3/cos(d*x+c)^2+1/2*sin(d*x+c)-1/2*ln(sec(d*x+c)+ta
n(d*x+c)))+2/3*a*b*sin(d*x+c)^3/cos(d*x+c)^3+b^2*(1/4*sin(d*x+c)^3/cos(d*x+
c)^4+1/8*sin(d*x+c)^3/cos(d*x+c)^2+1/8*sin(d*x+c)-1/8*ln(sec(d*x+c)+tan(d*x
+c))))
```

Maxima [A]
time = 0.28, size = 129, normalized size = 1.03

$$\frac{32\,ab\tan\left(dx+c\right)^3+3\,b^2\left(\frac{2\left(\sin(dx+c)^3+\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-12\,a^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/48*(32*a*b*tan(d*x + c)^3 + 3*b^2*(2*(sin(d*x + c)^3 + sin(d*x + c))/(sin
(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
 c) - 1)) - 12*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c)
+ 1) - log(sin(d*x + c) - 1)))/d
```

Fricas [A]
time = 2.18, size = 129, normalized size = 1.03

$$-\frac{3\left(4\,a^2+b^2\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(4\,a^2+b^2\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\,ab\cos\left(dx+c\right)^3-16\,ab\cos\left(dx+c\right)-3\left(4\,a^2-b^2\right)\cos\left(dx+c\right)^2-6\,b^2\right)\sin\left(dx+c\right)}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/48*(3*(4*a^2 + b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*a^2 + b^
2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(16*a*b*cos(d*x + c)^3 - 16*a*
b*cos(d*x + c) - 3*(4*a^2 - b^2)*cos(d*x + c)^2 - 6*b^2)*sin(d*x + c))/(d*c
os(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(c+dx)+b\tan(c+dx))^2\sec^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**2*sec(c + d*x)**3, x)
```

Giac [A]
time = 1.05, size = 226, normalized size = 1.81

$$-\frac{3\left(4a^2+b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3\left(4a^2+b^2\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(12a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-12a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-64ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+21b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-12a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+64ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+21b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+12a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^4}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(3*(4*a^2 + b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*a^2 + b^2)
*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(12*a^2*tan(1/2*d*x + 1/2*c)^7 + 3*
b^2*tan(1/2*d*x + 1/2*c)^7 - 12*a^2*tan(1/2*d*x + 1/2*c)^5 - 64*a*b*tan(1/2
*d*x + 1/2*c)^5 + 21*b^2*tan(1/2*d*x + 1/2*c)^5 - 12*a^2*tan(1/2*d*x + 1/2*
c)^3 + 64*a*b*tan(1/2*d*x + 1/2*c)^3 + 21*b^2*tan(1/2*d*x + 1/2*c)^3 + 12*a
^2*tan(1/2*d*x + 1/2*c) + 3*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)
^2 - 1)^4)/d
```

Mupad [B]
time = 3.27, size = 177, normalized size = 1.42

$$\frac{\left(a^2+\frac{b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(-a^2-\frac{16ab}{3}+\frac{7b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(-a^2+\frac{16ab}{3}+\frac{7b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\left(a^2+\frac{b^2}{4}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}-\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\left(a^2+\frac{b^2}{4}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b*tan(c + d*x))^2/cos(c + d*x)^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*((16*a*b)/3 - a^2 + (7*b^2)/4) + tan(c/2 + (d*x)/2)*(
a^2 + b^2/4) + tan(c/2 + (d*x)/2)^7*(a^2 + b^2/4) - tan(c/2 + (d*x)/2)^5*((
16*a*b)/3 + a^2 - (7*b^2)/4))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x
)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) - (atanh(tan(c
/2 + (d*x)/2))*(a^2 + b^2/4))/d
```

3.243 $\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=77

$$-\frac{(a^2-b^2)\,(b+a\cos(c+dx))^4}{4a^3d}-\frac{2b(b+a\cos(c+dx))^5}{5a^3d}+\frac{(b+a\cos(c+dx))^6}{6a^3d}$$

```
[Out] -1/4*(a^2-b^2)*(b+a*cos(d*x+c))^4/a^3/d-2/5*b*(b+a*cos(d*x+c))^5/a^3/d+1/6*
(b+a*cos(d*x+c))^6/a^3/d
```

Rubi [A]
time = 0.14, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {4482, 2747, 711}

$$\frac{(a\cos(c+dx)+b)^6}{6a^3d}-\frac{2b(a\cos(c+dx)+b)^5}{5a^3d}-\frac{(a^2-b^2)\,(a\cos(c+dx)+b)^4}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/4*((a^2 - b^2)*(b + a*Cos[c + d*x])^4)/(a^3*d) - (2*b*(b + a*Cos[c + d*x
])^5)/(5*a^3*d) + (b + a*Cos[c + d*x])^6/(6*a^3*d)
```

Rule 711

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, m},
 x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \cos^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int (b+a\cos(c+dx))^3\sin^3(c+dx)\,dx$$
$$= -\frac{\text{Subst}\left(\int (b+x)^3\left(a^2-x^2\right)\,dx, x, a\cos(c+dx)\right)}{a^3d}$$
$$= -\frac{\text{Subst}\left(\int \left(\left(a^2-b^2\right)(b+x)^3+2b(b+x)^4-(b+x)^5\right)$$
$$= -\frac{\left(a^2-b^2\right)(b+a\cos(c+dx))^4}{4a^3d} - \frac{2b(b+a\cos(c+dx)}{5a^3d}$$

Mathematica [A]

time = 0.26, size = 114, normalized size = 1.48

$$\frac{-360b(a^2+2b^2)\cos(c+dx)-45(a^3+8ab^2)\cos(2(c+dx))-60a^2b\cos(3(c+dx))+80b^3\cos(3(c+dx))+90ab^2\cos(4(c+dx))+36a^2b\cos(5(c+dx))+5a^3\cos(6(c+dx))}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (-360*b*(a^2 + 2*b^2)*Cos[c + d*x] - 45*(a^3 + 8*a*b^2)*Cos[2*(c + d*x)] -
60*a^2*b*Cos[3*(c + d*x)] + 80*b^3*Cos[3*(c + d*x)] + 90*a*b^2*Cos[4*(c + d
*x)] + 36*a^2*b*Cos[5*(c + d*x)] + 5*a^3*Cos[6*(c + d*x)])/(960*d)
```

Maple [A]

time = 0.15, size = 109, normalized size = 1.42

method	result
derivativedivides	$\frac{a^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^4(dx+c)\right)}{6}-\frac{\left(\cos^4(dx+c)\right)}{12}\right)+3a^2b\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+\frac{3a\,b^2\left(\sin^4(dx+c\right.}{4}}{d}$
default	$\frac{a^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^4(dx+c)\right)}{6}-\frac{\left(\cos^4(dx+c)\right)}{12}\right)+3a^2b\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+\frac{3a\,b^2\left(\sin^4(dx+c\right.}{4}}{d}$
risch	$-\frac{3a^2b\cos(dx+c)}{8d}-\frac{3b^3\cos(dx+c)}{4d}+\frac{a^3\cos(6dx+6c)}{192d}+\frac{3b\cos(5dx+5c)a^2}{80d}+\frac{3a\,b^2\cos(4dx+4c)}{32d}-\frac{b\cos(3dx+3}{16d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/6*sin(d*x+c)^2*cos(d*x+c)^4-1/12*cos(d*x+c)^4)+3*a^2*b*(-1/5*s
in(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+3/4*a*b^2*sin(d*x+c)^4-1/3*b^3*
(2+sin(d*x+c)^2)*cos(d*x+c))
```

Maxima [A]

time = 0.27, size = 95, normalized size = 1.23

$$\frac{45\,ab^2\sin(dx+c)^4-5\left(2\,\sin(dx+c)^6-3\,\sin(dx+c)^4\right)a^3+12\left(3\,\cos(dx+c)^5-5\,\cos(dx+c)^3\right)a^2b+20\left(\cos(dx+c)^3-3\,\cos(dx+c)\right)b^3}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(45*a*b^2*sin(d*x + c)^4 - 5*(2*sin(d*x + c)^6 - 3*sin(d*x + c)^4)*a^3
 + 12*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^2*b + 20*(cos(d*x + c)^3 - 3*
cos(d*x + c))*b^3)/d
```

Fricas [A]
time = 2.83, size = 100, normalized size = 1.30

$$\frac{10\,a^3\cos\left(dx+c\right)^6+36\,a^2b\cos\left(dx+c\right)^5-90\,ab^2\cos\left(dx+c\right)^2-15\,(a^3-3\,ab^2)\cos\left(dx+c\right)^4-60\,b^3\cos\left(dx+c\right)-20\,(3\,a^2b-b^3)\cos\left(dx+c\right)^3}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(10*a^3*cos(d*x + c)^6 + 36*a^2*b*cos(d*x + c)^5 - 90*a*b^2*cos(d*x +
c)^2 - 15*(a^3 - 3*a*b^2)*cos(d*x + c)^4 - 60*b^3*cos(d*x + c) - 20*(3*a^2*
b - b^3)*cos(d*x + c)^3)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)^3\cos^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**3*cos(c + d*x)**3, x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [B]
time = 0.80, size = 149, normalized size = 1.94

$$\frac{32\,a^3}{3\,d\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^6}+\frac{4\,{\left(a-b\right)}^3}{d\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^2}-\frac{32\,a^2\,\left(5\,a-3\,b\right)}{5\,d\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^5}-\frac{8\,{\left(a-b\right)}^2\,\left(7\,a-b\right)}{3\,d\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^3}+\frac{12\,a\,\left(3\,a^2-4\,a\,b+b^2\right)}{d\,{\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(a*sin(c + d*x) + b*tan(c + d*x))^3,x)
```

```
[Out] (32*a^3)/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^6) + (4*(a - b)^3)/(d*(tan(c/2 + (
d*x)/2)^2 + 1)^2) - (32*a^2*(5*a - 3*b))/(5*d*(tan(c/2 + (d*x)/2)^2 + 1)^5)
 - (8*(a - b)^2*(7*a - b))/(3*d*(tan(c/2 + (d*x)/2)^2 + 1)^3) + (12*a*(3*a^
2 - 4*a*b + b^2))/(d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

3.244 $\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=120

$$-\frac{3ab^2\cos(c+dx)}{d}-\frac{b(3a^2-b^2)\cos^2(c+dx)}{2d}-\frac{a(a^2-3b^2)\cos^3(c+dx)}{3d}+\frac{3a^2b\cos^4(c+dx)}{4d}+\frac{a^3\cos^5(c+dx)}{5d}-$$

```
[Out] -3*a*b^2*cos(d*x+c)/d-1/2*b*(3*a^2-b^2)*cos(d*x+c)^2/d-1/3*a*(a^2-3*b^2)*co
s(d*x+c)^3/d+3/4*a^2*b*cos(d*x+c)^4/d+1/5*a^3*cos(d*x+c)^5/d-b^3*ln(cos(d*x
+c))/d
```

Rubi [A]

time = 0.13, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4482, 2916, 12, 908}

$$\frac{a^3\cos^5(c+dx)}{5d}-\frac{a(a^2-3b^2)\cos^3(c+dx)}{3d}-\frac{b(3a^2-b^2)\cos^2(c+dx)}{2d}+\frac{3a^2b\cos^4(c+dx)}{4d}-\frac{3ab^2\cos(c+dx)}{d}-\frac{b^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (-3*a*b^2*Cos[c + d*x])/d - (b*(3*a^2 - b^2)*Cos[c + d*x]^2)/(2*d) - (a*(a^
2 - 3*b^2)*Cos[c + d*x]^3)/(3*d) + (3*a^2*b*Cos[c + d*x]^4)/(4*d) + (a^3*Co
s[c + d*x]^5)/(5*d) - (b^3*Log[Cos[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \cos^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int (b+a\cos(c+dx))^3\sin^2(c+dx)\tan(c+dx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{a(b+x)^3(a^2-x^2)}{x}\,dx, x, a\cos(c+dx)\right)}{a^3d}$$
$$= -\frac{\text{Subst}\left(\int \frac{(b+x)^3(a^2-x^2)}{x}\,dx, x, a\cos(c+dx)\right)}{a^2d}$$
$$= -\frac{\text{Subst}\left(\int \left(3a^2b^2+\frac{a^2b^3}{x}+b(3a^2-b^2)\,x+(a^2-3b^2)\right.}{a^2d}$$
$$= -\frac{3ab^2\cos(c+dx)}{d}-\frac{b(3a^2-b^2)\cos^2(c+dx)}{2d}-\frac{a(a^2}{}$$

Mathematica [A]
time = 0.21, size = 106, normalized size = 0.88

$$-\frac{3ab^2\cos(c+dx)+\frac{1}{2}b(3a^2-b^2)\cos^2(c+dx)+\frac{1}{3}a(a^2-3b^2)\cos^3(c+dx)-\frac{3}{4}a^2b\cos^4(c+dx)-\frac{1}{5}a^3\cos^5(c+dx)+b^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] -((3*a*b^2*Cos[c + d*x] + (b*(3*a^2 - b^2)*Cos[c + d*x]^2)/2 + (a*(a^2 - 3*
b^2)*Cos[c + d*x]^3)/3 - (3*a^2*b*Cos[c + d*x]^4)/4 - (a^3*Cos[c + d*x]^5)/
5 + b^3*Log[Cos[c + d*x]])/d)
```

Maple [A]
time = 0.15, size = 99, normalized size = 0.82

method	result
derivativedivides	$\frac{a^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+\frac{3a^2b\left(\sin^4(dx+c)\right)}{4}-a\,b^2\left(2+\sin^2(dx+c)\right)\cos(dx+c)+b^3\left(-\frac{\left(\sin^2(dx+c\right.}{2}\right.}{d}$
default	$\frac{a^3\left(-\frac{\left(\sin^2(dx+c)\right)\left(\cos^3(dx+c)\right)}{5}-\frac{2\left(\cos^3(dx+c)\right)}{15}\right)+\frac{3a^2b\left(\sin^4(dx+c)\right)}{4}-a\,b^2\left(2+\sin^2(dx+c)\right)\cos(dx+c)+b^3\left(-\frac{\left(\sin^2(dx+c\right.}{2}\right.}{d}$
risch	$ix\,b^3-\frac{3\,e^{2i(dx+c)}a^2b}{16d}+\frac{e^{2i(dx+c)}b^3}{8d}-\frac{3\,e^{-2i(dx+c)}a^2b}{16d}+\frac{e^{-2i(dx+c)}b^3}{8d}+\frac{2ib^3c}{d}-\frac{b^3\ln\left(e^{2i(dx+c)}+1\right)}{d}-a^3$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/5*sin(d*x+c)^2*cos(d*x+c)^3-2/15*cos(d*x+c)^3)+3/4*a^2*b*sin(d
*x+c)^4-a*b^2*(2+sin(d*x+c)^2)*cos(d*x+c)+b^3*(-1/2*sin(d*x+c)^2-ln(cos(d*x
+c))))
```

Maxima [A]
time = 0.27, size = 94, normalized size = 0.78

$$\frac{45\,a^2b\sin\left(dx+c\right)^4+4\left(3\,\cos\left(dx+c\right)^5-5\,\cos\left(dx+c\right)^3\right)a^3+60\left(\cos\left(dx+c\right)^3-3\,\cos\left(dx+c\right)\right)ab^2-30\left(\sin\left(dx+c\right)^2+\log\left(\sin\left(dx+c\right)^2-1\right)\right)b^3}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(45*a^2*b*sin(d*x + c)^4 + 4*(3*cos(d*x + c)^5 - 5*cos(d*x + c)^3)*a^3
 + 60*(cos(d*x + c)^3 - 3*cos(d*x + c))*a*b^2 - 30*(sin(d*x + c)^2 + log(si
n(d*x + c)^2 - 1))*b^3)/d
```

Fricas [A]
time = 3.15, size = 101, normalized size = 0.84

$$\frac{12\,a^3\cos\left(dx+c\right)^5+45\,a^2b\cos\left(dx+c\right)^4-180\,ab^2\cos\left(dx+c\right)-20\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^3-60\,b^3\log\left(-\cos\left(dx+c\right)\right)-30\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^2}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(12*a^3*cos(d*x + c)^5 + 45*a^2*b*cos(d*x + c)^4 - 180*a*b^2*cos(d*x +
 c) - 20*(a^3 - 3*a*b^2)*cos(d*x + c)^3 - 60*b^3*log(-cos(d*x + c)) - 30*(3
*a^2*b - b^3)*cos(d*x + c)^2)/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)^3\cos^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**3*cos(c + d*x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Modgcd: no suitable evaluation pointi
ndex.cc index_m operator + Error: Bad Argument ValueEvaluation time: 195.2D
one
```

Mupad [B]
time = 0.79, size = 237, normalized size = 1.98

$$\frac{40\,a^3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3\,d}-\frac{4\,a^3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{d}-\frac{16\,a^3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{d}+\frac{32\,a^3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{5\,d}-\frac{2\,b^3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{d}+\frac{2\,b^3\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{d}+\frac{2\,b^3\,\mathrm{atanh}\left(\frac{1}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}-1\right)}{d}-\frac{12\,a\,b^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{d}+\frac{12\,a^2\,b\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{d}+\frac{8\,a\,b^2\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{d}-\frac{24\,a^2\,b\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{d}+\frac{12\,a^2\,b\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(a*sin(c + d*x) + b*tan(c + d*x))^3,x)
```

```
[Out] (40*a^3*cos(c/2 + (d*x)/2)^6)/(3*d) - (4*a^3*cos(c/2 + (d*x)/2)^4)/d - (16*
a^3*cos(c/2 + (d*x)/2)^8)/d + (32*a^3*cos(c/2 + (d*x)/2)^10)/(5*d) - (2*b^3
*cos(c/2 + (d*x)/2)^2)/d + (2*b^3*cos(c/2 + (d*x)/2)^4)/d + (2*b^3*atanh(1/
cos(c/2 + (d*x)/2)^2 - 1))/d - (12*a*b^2*cos(c/2 + (d*x)/2)^4)/d + (12*a^2*
b*cos(c/2 + (d*x)/2)^4)/d + (8*a*b^2*cos(c/2 + (d*x)/2)^6)/d - (24*a^2*b*co
s(c/2 + (d*x)/2)^6)/d + (12*a^2*b*cos(c/2 + (d*x)/2)^8)/d
```

3.245 $\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=112

$$-\frac{b(3a^2-b^2)\cos(c+dx)}{d}-\frac{a(a^2-3b^2)\cos^2(c+dx)}{2d}+\frac{a^2b\cos^3(c+dx)}{d}+\frac{a^3\cos^4(c+dx)}{4d}-\frac{3ab^2\log(\cos(c+dx$$

```
[Out] -b*(3*a^2-b^2)*cos(d*x+c)/d-1/2*a*(a^2-3*b^2)*cos(d*x+c)^2/d+a^2*b*cos(d*x+
c)^3/d+1/4*a^3*cos(d*x+c)^4/d-3*a*b^2*ln(cos(d*x+c))/d+b^3*sec(d*x+c)/d
```

Rubi [A]
time = 0.12, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4482, 2916, 12, 908}

$$\frac{a^3\cos^4(c+dx)}{4d}-\frac{a(a^2-3b^2)\cos^2(c+dx)}{2d}-\frac{b(3a^2-b^2)\cos(c+dx)}{d}+\frac{a^2b\cos^3(c+dx)}{d}-\frac{3ab^2\log(\cos(c+dx))}{d}+\frac{b^3\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] -((b*(3*a^2 - b^2)*Cos[c + d*x])/d) - (a*(a^2 - 3*b^2)*Cos[c + d*x]^2)/(2*d
) + (a^2*b*Cos[c + d*x]^3)/d + (a^3*Cos[c + d*x]^4)/(4*d) - (3*a*b^2*Log[Co
s[c + d*x]])/d + (b^3*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \cos(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int (b+a\cos(c+dx))^3\sin(c+dx)\tan^2(c+dx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{a^2(b+x)^3(a^2-x^2)}{x^2}\,dx, x, a\cos(c+dx)\right)}{a^3d}$$
$$= -\frac{\text{Subst}\left(\int \frac{(b+x)^3(a^2-x^2)}{x^2}\,dx, x, a\cos(c+dx)\right)}{ad}$$
$$= -\frac{\text{Subst}\left(\int \left(3a^2b\left(1-\frac{b^2}{3a^2}\right)+\frac{a^2b^3}{x^2}+\frac{3a^2b^2}{x}+(a^2-3b^2)\right.\right.}{ad}$$
$$= -\frac{b(3a^2-b^2)\cos(c+dx)}{d}-\frac{a(a^2-3b^2)\cos^2(c+dx)}{2d}+$$

Mathematica [A]
time = 0.30, size = 98, normalized size = 0.88

$$\frac{8b(-9a^2+4b^2)\cos(c+dx)-4(a^3-6ab^2)\cos(2(c+dx))+8a^2b\cos(3(c+dx))+a^3\cos(4(c+dx))-96ab^2\log(\cos(c+dx))+32b^3\sec(c+dx)}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (8*b*(-9*a^2 + 4*b^2)*Cos[c + d*x] - 4*(a^3 - 6*a*b^2)*Cos[2*(c + d*x)] + 8
*a^2*b*Cos[3*(c + d*x)] + a^3*Cos[4*(c + d*x)] - 96*a*b^2*Log[Cos[c + d*x]]
 + 32*b^3*Sec[c + d*x])/(32*d)
```

Maple [A]
time = 0.12, size = 106, normalized size = 0.95

method	result
derivativedivides	$\frac{\frac{a^3\left(\sin^4(dx+c)\right)}{4}-a^2b(2+\sin^2(dx+c))\cos(dx+c)+3ab^2\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+b^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx\right.}{d}$
default	$\frac{\frac{a^3\left(\sin^4(dx+c)\right)}{4}-a^2b(2+\sin^2(dx+c))\cos(dx+c)+3ab^2\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+b^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx\right.}{d}$
risch	$3iab^2x-\frac{a^3e^{2i(dx+c)}}{16d}+\frac{3a\,e^{2i(dx+c)}b^2}{8d}-\frac{9\,e^{i(dx+c)}a^2b}{8d}+\frac{e^{i(dx+c)}b^3}{2d}-\frac{9\,e^{-i(dx+c)}a^2b}{8d}+\frac{e^{-i(dx+c)}b^3}{2d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/4*a^3*sin(d*x+c)^4-a^2*b*(2+sin(d*x+c)^2)*cos(d*x+c)+3*a*b^2*(-1/2*s
in(d*x+c)^2-ln(cos(d*x+c)))+b^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*c
os(d*x+c)))
```

Maxima [A]
time = 0.26, size = 87, normalized size = 0.78

$$\frac{a^3 \sin(dx+c)^4 + 4\left(\cos(dx+c)^3 - 3\cos(dx+c)\right)a^2b - 6\left(\sin(dx+c)^2 + \log\left(\sin(dx+c)^2 - 1\right)\right)ab^2 + 4b^3\left(\frac{1}{\cos(dx+c)} + \cos(dx+c)\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(a^3*sin(d*x + c)^4 + 4*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^2*b - 6*(si
n(d*x + c)^2 + log(sin(d*x + c)^2 - 1))*a*b^2 + 4*b^3*(1/cos(d*x + c) + cos
(d*x + c)))/d
```

Fricas [A]
time = 2.14, size = 128, normalized size = 1.14

$$\frac{8a^3\cos(dx+c)^5 + 32a^2b\cos(dx+c)^4 - 96ab^2\cos(dx+c)\log(-\cos(dx+c)) - 16(a^3 - 3ab^2)\cos(dx+c)^3 + 32b^3 - 32(3a^2b - b^3)\cos(dx+c)^2 + (5a^3 - 24ab^2)\cos(dx+c)}{32d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/32*(8*a^3*cos(d*x + c)^5 + 32*a^2*b*cos(d*x + c)^4 - 96*a*b^2*cos(d*x + c
)*log(-cos(d*x + c)) - 16*(a^3 - 3*a*b^2)*cos(d*x + c)^3 + 32*b^3 - 32*(3*a
^2*b - b^3)*cos(d*x + c)^2 + (5*a^3 - 24*a*b^2)*cos(d*x + c))/(d*cos(d*x +
c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin(c+dx) + b\tan(c+dx))^3 \cos(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**3*cos(c + d*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:Modgcd: no suitable evaluation pointindex.cc index_m operator + Error: Bad Argument ValueEvaluation time: 46.3Done

Mupad [B]
time = 4.18, size = 225, normalized size = 2.01

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4(4a^3+4a^2b-6ab^2+12b^3)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(12a^2b+6ab^2-12b^3)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6(-4a^3+12a^2b+6ab^2+4b^3)-4a^2b+4b^3+6ab^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{d\left(-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}+\frac{6ab^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)*(a*sin(c + d*x) + b*tan(c + d*x))^3,x)

[Out] (tan(c/2 + (d*x)/2)^4*(4*a^2*b - 6*a*b^2 + 4*a^3 + 12*b^3) - tan(c/2 + (d*x)/2)^2*(6*a*b^2 + 12*a^2*b - 12*b^3) + tan(c/2 + (d*x)/2)^6*(6*a*b^2 + 12*a^2*b - 4*a^3 + 4*b^3) - 4*a^2*b + 4*b^3 + 6*a*b^2*tan(c/2 + (d*x)/2)^8)/(d*(3*tan(c/2 + (d*x)/2)^2 + 2*tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^6 - 3*tan(c/2 + (d*x)/2)^8 - tan(c/2 + (d*x)/2)^10 + 1)) + (6*a*b^2*atanh(tan(c/2 + (d*x)/2)^2))/d

3.246 $\int (a\sin(c+dx)+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=116

$$-\frac{a(a^2-3b^2)\cos(c+dx)}{d}+\frac{3a^2b\cos^2(c+dx)}{2d}+\frac{a^3\cos^3(c+dx)}{3d}-\frac{b(3a^2-b^2)\log(\cos(c+dx))}{d}+\frac{3ab^2\sec(c+d$$

```
[Out] -a*(a^2-3*b^2)*cos(d*x+c)/d+3/2*a^2*b*cos(d*x+c)^2/d+1/3*a^3*cos(d*x+c)^3/d
-b*(3*a^2-b^2)*ln(cos(d*x+c))/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec(d*x+c)^2/d
```

Rubi [A]
time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4482, 2800, 908}

$$\frac{a^3\cos^3(c+dx)}{3d}-\frac{a(a^2-3b^2)\cos(c+dx)}{d}-\frac{b(3a^2-b^2)\log(\cos(c+dx))}{d}+\frac{3a^2b\cos^2(c+dx)}{2d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] -((a*(a^2 - 3*b^2)*Cos[c + d*x])/d) + (3*a^2*b*Cos[c + d*x]^2)/(2*d) + (a^3
*Cos[c + d*x]^3)/(3*d) - (b*(3*a^2 - b^2)*Log[Cos[c + d*x]])/d + (3*a*b^2*S
ec[c + d*x])/d + (b^3*Sec[c + d*x]^2)/(2*d)
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int (a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int (b+a\cos(c+dx))^3\tan^3(c+dx)\,dx$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{(b+x)^3(a^2-x^2)}{x^3}\,dx, x, a\cos(c+dx)\right)}{d}$$
$$= -\frac{\operatorname{Subst}\left(\int \left(a^2\left(1-\frac{3b^2}{a^2}\right)+\frac{a^2b^3}{x^3}+\frac{3a^2b^2}{x^2}+\frac{3a^2b-b^3}{x}-3bx-x^2\right)\,dx, x\right.}{d}$$
$$= -\frac{a(a^2-3b^2)\cos(c+dx)}{d}+\frac{3a^2b\cos^2(c+dx)}{2d}+\frac{a^3\cos^3(c+dx)}{3d}-$$

Mathematica [A]
time = 0.35, size = 102, normalized size = 0.88

$$\frac{-9a(a^2-4b^2)\cos(c+dx)+9a^2b\cos(2(c+dx))+a^3\cos(3(c+dx))-36a^2b\log(\cos(c+dx))+12b^3\log(\cos(c+dx))+36ab^2\sec(c+dx)+6b^3\sec^2(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (-9*a*(a^2 - 4*b^2)*Cos[c + d*x] + 9*a^2*b*Cos[2*(c + d*x)] + a^3*Cos[3*(c
+ d*x)] - 36*a^2*b*Log[Cos[c + d*x]] + 12*b^3*Log[Cos[c + d*x]] + 36*a*b^2*
Sec[c + d*x] + 6*b^3*Sec[c + d*x]^2)/(12*d)
```

Maple [A]
time = 0.14, size = 116, normalized size = 1.00

method	result
derivativedivides	$\frac{-\frac{a^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+3a^2b\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+3a\,b^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx+c)\right.}{d}$
default	$\frac{-\frac{a^3\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}+3a^2b\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+3a\,b^2\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+\left(2+\sin^2(dx+c)\right)\cos(dx+c)\right.}{d}$
risch	$3ia^2bx-\frac{2ib^3c}{d}+\frac{a^3e^{3i(dx+c)}}{24d}+\frac{3\,e^{2i(dx+c)}a^2b}{8d}-\frac{3a^3e^{i(dx+c)}}{8d}+\frac{3a\,e^{i(dx+c)}b^2}{2d}-\frac{3a^3e^{-i(dx+c)}}{8d}+\frac{3a\,e^{-i(d}}{2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(d*x+c)+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*a^3*(2+sin(d*x+c)^2)*cos(d*x+c)+3*a^2*b*(-1/2*sin(d*x+c)^2-ln(cos
(d*x+c)))+3*a*b^2*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+b^3
*(1/2*tan(d*x+c)^2+ln(cos(d*x+c))))
```

Maxima [A]
time = 0.27, size = 113, normalized size = 0.97

$$\frac{\left(\cos\left(dx+c\right)^{3}-3\,\cos\left(dx+c\right)\right)a^{3}}{3\,d}-\frac{3\,\left(\sin\left(dx+c\right)^{2}+\log\left(\sin\left(dx+c\right)^{2}-1\right)\right)a^{2}b}{2\,d}-\frac{b^{3}\left(\frac{1}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)^{2}-1\right)\right)}{2\,d}+\frac{3\,ab^{2}\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*(cos(d*x + c)^3 - 3*cos(d*x + c))*a^3/d - 3/2*(sin(d*x + c)^2 + log(sin
(d*x + c)^2 - 1))*a^2*b/d - 1/2*b^3*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x +
 c)^2 - 1))/d + 3*a*b^2*(1/cos(d*x + c) + cos(d*x + c))/d
```

Fricas [A]
time = 3.79, size = 123, normalized size = 1.06

$$\frac{4\,a^{3}\cos\left(dx+c\right)^{5}+18\,a^{2}b\cos\left(dx+c\right)^{4}-9\,a^{2}b\cos\left(dx+c\right)^{2}+36\,ab^{2}\cos\left(dx+c\right)-12\,\left(a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{3}-12\,\left(3\,a^{2}b-b^{3}\right)\cos\left(dx+c\right)^{2}\log\left(-\cos\left(dx+c\right)\right)+6\,b^{3}}{12\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*a^3*cos(d*x + c)^5 + 18*a^2*b*cos(d*x + c)^4 - 9*a^2*b*cos(d*x + c)
^2 + 36*a*b^2*cos(d*x + c) - 12*(a^3 - 3*a*b^2)*cos(d*x + c)^3 - 12*(3*a^2*
b - b^3)*cos(d*x + c)^2*log(-cos(d*x + c)) + 6*b^3)/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Modgcd: no suitable evaluation pointi
```

```
ndex.cc index_m operator + Error: Bad Argument ValueEvaluation time: 29.49D
one
```

Mupad [B]

time = 4.52, size = 219, normalized size = 1.89

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(-\frac{4a^3}{3}-6a^2b+12ab^2+2b^3\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6(4a^3-6a^2b+12ab^2-6b^3)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(\frac{20a^3}{3}+6a^2b-12ab^2+6b^3\right)+12ab^2-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8(6a^2b-2b^3)-\frac{4a^3}{3}}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)^2\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)^3}-\frac{2b^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)-6a^2b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b*tan(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*(12*a*b^2 - 6*a^2*b - (4*a^3)/3 + 2*b^3) - tan(c/2 +
(d*x)/2)^6*(12*a*b^2 - 6*a^2*b + 4*a^3 - 6*b^3) + tan(c/2 + (d*x)/2)^4*(6*a
^2*b - 12*a*b^2 + (20*a^3)/3 + 6*b^3) + 12*a*b^2 - tan(c/2 + (d*x)/2)^8*(6*
a^2*b - 2*b^3) - (4*a^3)/3)/(d*(tan(c/2 + (d*x)/2)^2 - 1)^2*(tan(c/2 + (d*x
)/2)^2 + 1)^3) - (2*b^3*atanh(tan(c/2 + (d*x)/2)^2) - 6*a^2*b*atanh(tan(c/2
 + (d*x)/2)^2))/d
```

3.247 $\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=115

$$\frac{3a^2b\cos(c+dx)}{d}+\frac{a^3\cos^2(c+dx)}{2d}-\frac{a(a^2-3b^2)\log(\cos(c+dx))}{d}+\frac{b(3a^2-b^2)\sec(c+dx)}{d}+\frac{3ab^2\sec^2(c+dx)}{2d}$$

```
[Out] 3*a^2*b*cos(d*x+c)/d+1/2*a^3*cos(d*x+c)^2/d-a*(a^2-3*b^2)*ln(cos(d*x+c))/d+
b*(3*a^2-b^2)*sec(d*x+c)/d+3/2*a*b^2*sec(d*x+c)^2/d+1/3*b^3*sec(d*x+c)^3/d
```

Rubi [A]
time = 0.17, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {4482, 2916, 12, 908}

$$\frac{a^3\cos^2(c+dx)}{2d}+\frac{b(3a^2-b^2)\sec(c+dx)}{d}-\frac{a(a^2-3b^2)\log(\cos(c+dx))}{d}+\frac{3a^2b\cos(c+dx)}{d}+\frac{3ab^2\sec^2(c+dx)}{2d}+\frac{b^3\sec^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (3*a^2*b*Cos[c + d*x])/d + (a^3*Cos[c + d*x]^2)/(2*d) - (a*(a^2 - 3*b^2)*Lo
g[Cos[c + d*x]])/d + (b*(3*a^2 - b^2)*Sec[c + d*x])/d + (3*a*b^2*Sec[c + d*
x]^2)/(2*d) + (b^3*Sec[c + d*x]^3)/(3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \sec(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int (b+a\cos(c+dx))^3\sec(c+dx)\tan^3(c+dx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{a^4(b+x)^3(a^2-x^2)}{x^4}\,dx, x, a\cos(c+dx)\right)}{a^3d}$$
$$= -\frac{a\text{Subst}\left(\int \frac{(b+x)^3(a^2-x^2)}{x^4}\,dx, x, a\cos(c+dx)\right)}{d}$$
$$= -\frac{a\text{Subst}\left(\int \left(-3b+\frac{a^2b^3}{x^4}+\frac{3a^2b^2}{x^3}+\frac{3a^2b-b^3}{x^2}+\frac{a^2-3b^2}{x}-x\right.\right.}{d}$$
$$= \frac{3a^2b\cos(c+dx)}{d}+\frac{a^3\cos^2(c+dx)}{2d}-\frac{a(a^2-3b^2)\log(c}{d}$$

Mathematica [A]
time = 0.56, size = 100, normalized size = 0.87

$$\frac{36a^2b\cos(c+dx)+3a^3\cos(2(c+dx))+2(-6a(a^2-3b^2)\log(\cos(c+dx))-6b(-3a^2+b^2)\sec(c+dx)+9ab^2\sec^2(c+dx)+2b^3\sec^3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
[Out] (36*a^2*b*Cos[c + d*x] + 3*a^3*Cos[2*(c + d*x)] + 2*(-6*a*(a^2 - 3*b^2)*Log
[Cos[c + d*x]] - 6*b*(-3*a^2 + b^2)*Sec[c + d*x] + 9*a*b^2*Sec[c + d*x]^2 +
 2*b^3*Sec[c + d*x]^3))/(12*d)
```

Maple [A]
time = 0.19, size = 154, normalized size = 1.34

method	result
derivativedivides	$\frac{a^3\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+3a^2b\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+3a\,b^2\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(d\right.}{d}$
default	$\frac{a^3\left(-\frac{\left(\sin^2(dx+c)\right)}{2}-\ln(\cos(dx+c))\right)+3a^2b\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+3a\,b^2\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(d\right.}{d}$
risch	$ia^3x-3ia\,b^2x+\frac{a^3e^{2i(dx+c)}}{8d}+\frac{3\,e^{i(dx+c)}a^2b}{2d}+\frac{3\,e^{-i(dx+c)}a^2b}{2d}+\frac{a^3e^{-2i(dx+c)}}{8d}+\frac{2ia^3c}{d}-\frac{6ia\,b^2c}{d}+\frac{2b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(-1/2*sin(d*x+c)^2-ln(cos(d*x+c)))+3*a^2*b*(sin(d*x+c)^4/cos(d*x+c
)+(2+sin(d*x+c)^2)*cos(d*x+c))+3*a*b^2*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))+b^
3*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x
+c)^2)*cos(d*x+c)))
```

Maxima [A]
time = 0.27, size = 109, normalized size = 0.95

$$-\frac{3\left(\sin\left(dx+c\right)^{2}+\log\left(\sin\left(dx+c\right)^{2}-1\right)\right)a^{3}+9\,ab^{2}\left(\frac{1}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)^{2}-1\right)\right)-18\,a^{2}b\left(\frac{1}{\cos(dx+c)}+\cos\left(dx+c\right)\right)+\frac{2\left(3\,\cos(dx+c)^{2}-1\right)b^{3}}{\cos(dx+c)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/6*(3*(sin(d*x + c)^2 + log(sin(d*x + c)^2 - 1))*a^3 + 9*a*b^2*(1/(sin(d*
x + c)^2 - 1) - log(sin(d*x + c)^2 - 1)) - 18*a^2*b*(1/cos(d*x + c) + cos(d
*x + c)) + 2*(3*cos(d*x + c)^2 - 1)*b^3/cos(d*x + c)^3)/d
```

Fricas [A]
time = 2.35, size = 122, normalized size = 1.06

$$\frac{6\,a^{3}\cos\left(dx+c\right)^{5}+36\,a^{2}b\cos\left(dx+c\right)^{4}-3\,a^{3}\cos\left(dx+c\right)^{3}-12\left(a^{3}-3\,ab^{2}\right)\cos\left(dx+c\right)^{3}\log\left(-\cos\left(dx+c\right)\right)+18\,ab^{2}\cos\left(dx+c\right)+4\,b^{3}+12\left(3\,a^{2}b-b^{3}\right)\cos\left(dx+c\right)^{2}}{12\,d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(6*a^3*cos(d*x + c)^5 + 36*a^2*b*cos(d*x + c)^4 - 3*a^3*cos(d*x + c)^3
 - 12*(a^3 - 3*a*b^2)*cos(d*x + c)^3*log(-cos(d*x + c)) + 18*a*b^2*cos(d*x
+ c) + 4*b^3 + 12*(3*a^2*b - b^3)*cos(d*x + c)^2)/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)^{3}\sec\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**3*sec(c + d*x), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Modgcd: no suitable evaluation pointi
ndex.cc index_m operator + Error: Bad Argument ValueEvaluation time: 30.42D
one
```

Mupad [B]
time = 4.45, size = 219, normalized size = 1.90

$$\frac{2\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)-6\,a\,b^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(-2\,a^3-12\,a^2\,b+6\,a\,b^2+\frac{4\,b^3}{3}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\left(6\,a^3-12\,a^2\,b+6\,a\,b^2-4\,b^3\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(6\,a^3-12\,a^2\,b+6\,a\,b^2+\frac{20\,b^3}{3}\right)+12\,a^2\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\left(6\,a\,b^2-2\,a^3\right)-\frac{4\,b^3}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)^3\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b*tan(c + d*x))^3/cos(c + d*x),x)
```

```
[Out] (2*a^3*atanh(tan(c/2 + (d*x)/2)^2) - 6*a*b^2*atanh(tan(c/2 + (d*x)/2)^2))/d
 - (tan(c/2 + (d*x)/2)^2*(6*a*b^2 - 12*a^2*b - 2*a^3 + (4*b^3)/3) - tan(c/2
 + (d*x)/2)^6*(6*a*b^2 - 12*a^2*b + 6*a^3 - 4*b^3) + tan(c/2 + (d*x)/2)^4*(
6*a*b^2 - 12*a^2*b + 6*a^3 + (20*b^3)/3) + 12*a^2*b - tan(c/2 + (d*x)/2)^8*
(6*a*b^2 - 2*a^3) - (4*b^3)/3)/(d*(tan(c/2 + (d*x)/2)^2 - 1)^3*(tan(c/2 + (
d*x)/2)^2 + 1)^2)
```

3.248 $\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=111

$$\frac{a^3\cos(c+dx)}{d}+\frac{3a^2b\log(\cos(c+dx))}{d}+\frac{a(a^2-3b^2)\sec(c+dx)}{d}+\frac{b(3a^2-b^2)\sec^2(c+dx)}{2d}+\frac{ab^2\sec^3(c+dx)}{d}+$$

[Out] a^3*cos(d*x+c)/d+3*a^2*b*ln(cos(d*x+c))/d+a*(a^2-3*b^2)*sec(d*x+c)/d+1/2*b*(3*a^2-b^2)*sec(d*x+c)^2/d+a*b^2*sec(d*x+c)^3/d+1/4*b^3*sec(d*x+c)^4/d

Rubi [A]
time = 0.17, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4482, 2916, 12, 908}

$$\frac{a^3\cos(c+dx)}{d}+\frac{b(3a^2-b^2)\sec^2(c+dx)}{2d}+\frac{a(a^2-3b^2)\sec(c+dx)}{d}+\frac{3a^2b\log(\cos(c+dx))}{d}+\frac{ab^2\sec^3(c+dx)}{d}+\frac{b^3\sec^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]

[Out] (a^3*Cos[c + d*x])/d + (3*a^2*b*Log[Cos[c + d*x]])/d + (a*(a^2 - 3*b^2)*Sec[c + d*x])/d + (b*(3*a^2 - b^2)*Sec[c + d*x]^2)/(2*d) + (a*b^2*Sec[c + d*x]^3)/d + (b^3*Sec[c + d*x]^4)/(4*d)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \sec^2(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int (b+a\cos(c+dx))^3\sec^2(c+dx)\tan^3(c+dx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{a^5(b+x)^3(a^2-x^2)}{x^5}\,dx, x, a\cos(c+dx)\right)}{a^3d}$$
$$= -\frac{a^2\text{Subst}\left(\int \frac{(b+x)^3(a^2-x^2)}{x^5}\,dx, x, a\cos(c+dx)\right)}{d}$$
$$= -\frac{a^2\text{Subst}\left(\int \left(-1+\frac{a^2b^3}{x^5}+\frac{3a^2b^2}{x^4}+\frac{3a^2b-b^3}{x^3}+\frac{a^2-3b^2}{x^2}-\right.\right.}{d}$$
$$= \frac{a^3\cos(c+dx)}{d}+\frac{3a^2b\log(\cos(c+dx))}{d}+\frac{a(a^2-3b^2)}{d}$$

Mathematica [A]
time = 1.98, size = 97, normalized size = 0.87

$$\frac{4a^3\cos(c+dx)+12a^2b\log(\cos(c+dx))+4a(a^2-3b^2)\sec(c+dx)+(6a^2b-2b^3)\sec^2(c+dx)+4ab^2\sec^3(c+dx)+b^3\sec^4(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
[Out] (4*a^3*Cos[c + d*x] + 12*a^2*b*Log[Cos[c + d*x]] + 4*a*(a^2 - 3*b^2)*Sec[c
+ d*x] + (6*a^2*b - 2*b^3)*Sec[c + d*x]^2 + 4*a*b^2*Sec[c + d*x]^3 + b^3*Se
c[c + d*x]^4)/(4*d)
```

Maple [A]
time = 0.23, size = 151, normalized size = 1.36

method	result
derivativedivides	$\frac{a^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+3a^2b\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)+3a\,b^2\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}\right.}{d}$
default	$\frac{a^3\left(\frac{\sin^4(dx+c)}{\cos(dx+c)}+(2+\sin^2(dx+c))\cos(dx+c)\right)+3a^2b\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)+3a\,b^2\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}\right.}{d}$
risch	$-3ia^2bx+\frac{a^3e^{i(dx+c)}}{2d}+\frac{a^3e^{-i(dx+c)}}{2d}-\frac{6ib\,a^2c}{d}+\frac{2a^3e^{7i(dx+c)}-6a\,b^2e^{7i(dx+c)}+6a^2b\,e^{6i(dx+c)}-2b^3e^{6i(dx+c)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(sin(d*x+c)^4/cos(d*x+c)+(2+sin(d*x+c)^2)*cos(d*x+c))+3*a^2*b*(1/2
*tan(d*x+c)^2+ln(cos(d*x+c)))+3*a*b^2*(1/3*sin(d*x+c)^4/cos(d*x+c)^3-1/3*si
n(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+c))+1/4*b^3*sin(d*x+c)^4
/cos(d*x+c)^4)
```

Maxima [A]
time = 0.28, size = 96, normalized size = 0.86

$$\frac{b^3\tan\left(dx+c\right)^4-6\,a^2b\left(\frac{1}{\sin\left(dx+c\right)^2-1}-\log\left(\sin\left(dx+c\right)^2-1\right)\right)+4\,a^3\left(\frac{1}{\cos\left(dx+c\right)}+\cos\left(dx+c\right)\right)-\frac{4\left(3\,\cos\left(dx+c\right)^2-1\right)ab^2}{\cos\left(dx+c\right)^3}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(b^3*tan(d*x + c)^4 - 6*a^2*b*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c
)^2 - 1)) + 4*a^3*(1/cos(d*x + c) + cos(d*x + c)) - 4*(3*cos(d*x + c)^2 - 1
)*a*b^2/cos(d*x + c)^3)/d
```

Fricas [A]
time = 3.33, size = 107, normalized size = 0.96

$$\frac{4\,a^3\cos\left(dx+c\right)^5+12\,a^2b\cos\left(dx+c\right)^4\log\left(-\cos\left(dx+c\right)\right)+4\,ab^2\cos\left(dx+c\right)+4\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^3+b^3+2\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^2}{4\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a^3*cos(d*x + c)^5 + 12*a^2*b*cos(d*x + c)^4*log(-cos(d*x + c)) + 4*
a*b^2*cos(d*x + c) + 4*(a^3 - 3*a*b^2)*cos(d*x + c)^3 + b^3 + 2*(3*a^2*b -
b^3)*cos(d*x + c)^2)/(d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)^3\sec^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**3*sec(c + d*x)**2, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Modgcd: no suitable evaluation pointi
ndex.cc index_m operator + Error: Bad Argument ValueEvaluation time: 31.16D
one
```

Mupad [B]
time = 4.22, size = 223, normalized size = 2.01

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(-12\,a^3+6\,a^2\,b+12\,a\,b^2\right)+\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(12\,a^3-6\,a^2\,b+4\,a\,b^2+4\,b^3\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\left(4\,a^3+6\,a^2\,b+12\,a\,b^2-4\,b^3\right)-4\,a\,b^2+4\,a^3+6\,a^2\,b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}-\frac{6\,a^2\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b*tan(c + d*x))^3/cos(c + d*x)^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^2*(12*a*b^2 + 6*a^2*b - 12*a^3) + tan(c/2 + (d*x)/2)^4*
(4*a*b^2 - 6*a^2*b + 12*a^3 + 4*b^3) - tan(c/2 + (d*x)/2)^6*(12*a*b^2 + 6*a
^2*b + 4*a^3 - 4*b^3) - 4*a*b^2 + 4*a^3 + 6*a^2*b*tan(c/2 + (d*x)/2)^8)/(d*
(2*tan(c/2 + (d*x)/2)^4 - 3*tan(c/2 + (d*x)/2)^2 + 2*tan(c/2 + (d*x)/2)^6 -
 3*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)) - (6*a^2*b*atanh(tan(
c/2 + (d*x)/2)^2))/d
```

3.249 $\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=119

$$\frac{a^3\log(\cos(c+dx))}{d}-\frac{3a^2b\sec(c+dx)}{d}+\frac{a(a^2-3b^2)\sec^2(c+dx)}{2d}+\frac{b(3a^2-b^2)\sec^3(c+dx)}{3d}+\frac{3ab^2\sec^4(c+dx)}{4d}$$

```
[Out] a^3*ln(cos(d*x+c))/d-3*a^2*b*sec(d*x+c)/d+1/2*a*(a^2-3*b^2)*sec(d*x+c)^2/d+
1/3*b*(3*a^2-b^2)*sec(d*x+c)^3/d+3/4*a*b^2*sec(d*x+c)^4/d+1/5*b^3*sec(d*x+c
)^5/d
```

Rubi [A]
time = 0.15, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4482, 2916, 12, 908}

$$\frac{a^3\log(\cos(c+dx))}{d}+\frac{b(3a^2-b^2)\sec^3(c+dx)}{3d}+\frac{a(a^2-3b^2)\sec^2(c+dx)}{2d}-\frac{3a^2b\sec(c+dx)}{d}+\frac{3ab^2\sec^4(c+dx)}{4d}+\frac{b^3\sec^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (a^3*Log[Cos[c + d*x]])/d - (3*a^2*b*Sec[c + d*x])/d + (a*(a^2 - 3*b^2)*Sec
[c + d*x]^2)/(2*d) + (b*(3*a^2 - b^2)*Sec[c + d*x]^3)/(3*d) + (3*a*b^2*Sec[
c + d*x]^4)/(4*d) + (b^3*Sec[c + d*x]^5)/(5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \sec^3(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int (b+a\cos(c+dx))^3\sec^3(c+dx)\tan^3(c+dx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{a^6(b+x)^3(a^2-x^2)}{x^6}\,dx, x, a\cos(c+dx)\right)}{a^3d}$$
$$= -\frac{a^3\text{Subst}\left(\int \frac{(b+x)^3(a^2-x^2)}{x^6}\,dx, x, a\cos(c+dx)\right)}{d}$$
$$= -\frac{a^3\text{Subst}\left(\int \left(\frac{a^2b^3}{x^6}+\frac{3a^2b^2}{x^5}+\frac{3a^2b-b^3}{x^4}+\frac{a^2-3b^2}{x^3}-\frac{3b}{x^2}-\frac{1}{x}\right.\right.}{d}$$
$$= \frac{a^3\log(\cos(c+dx))}{d}-\frac{3a^2b\sec(c+dx)}{d}+\frac{a(a^2-3b^2)\,\mathrm{s}}{2d}$$

Mathematica [A]
time = 0.36, size = 99, normalized size = 0.83

$$\frac{60a^3\log(\cos(c+dx))-180a^2b\sec(c+dx)+30a(a^2-3b^2)\sec^2(c+dx)-20b(-3a^2+b^2)\sec^3(c+dx)+45ab^2\sec^4(c+dx)+12b^3\sec^5(c+dx)}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
[Out] (60*a^3*Log[Cos[c + d*x]] - 180*a^2*b*Sec[c + d*x] + 30*a*(a^2 - 3*b^2)*Sec
[c + d*x]^2 - 20*b*(-3*a^2 + b^2)*Sec[c + d*x]^3 + 45*a*b^2*Sec[c + d*x]^4
+ 12*b^3*Sec[c + d*x]^5)/(60*d)
```

Maple [A]
time = 0.25, size = 188, normalized size = 1.58

method	result
derivativedivides	$\frac{a^3\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)+3a^2b\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+\frac{3a\,b^2\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}+}{d}$
default	$\frac{a^3\left(\frac{\left(\tan^2(dx+c)\right)}{2}+\ln(\cos(dx+c))\right)+3a^2b\left(\frac{\sin^4(dx+c)}{3\cos(dx+c)^3}-\frac{\sin^4(dx+c)}{3\cos(dx+c)}-\frac{\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3}\right)+\frac{3a\,b^2\left(\sin^4(dx+c)\right)}{4\cos(dx+c)^4}+}{d}$
risch	$-ia^3x-\frac{2ia^3c}{d}+\frac{-6a^2b\,e^{9i(dx+c)}+2a^3e^{8i(dx+c)}-6a\,b^2e^{8i(dx+c)}-16a^2b\,e^{7i(dx+c)}-\frac{8b^3e^{7i(dx+c)}}{3}+6a^3e^{6i(dx+c)}-}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^3*(1/2*tan(d*x+c)^2+ln(cos(d*x+c)))+3*a^2*b*(1/3*sin(d*x+c)^4/cos(d*
x+c)^3-1/3*sin(d*x+c)^4/cos(d*x+c)-1/3*(2+sin(d*x+c)^2)*cos(d*x+c))+3/4*a*b
^2*sin(d*x+c)^4/cos(d*x+c)^4+b^3*(1/5*sin(d*x+c)^4/cos(d*x+c)^5+1/15*sin(d*
x+c)^4/cos(d*x+c)^3-1/15*sin(d*x+c)^4/cos(d*x+c)-1/15*(2+sin(d*x+c)^2)*cos(
d*x+c)))
```

Maxima [A]
time = 0.28, size = 128, normalized size = 1.08

$$-\frac{30\,a^3\left(\frac{1}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)^2-1\right)\right)-\frac{45\left(2\,\sin(dx+c)^2-1\right)ab^2}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}+\frac{60\left(3\,\cos(dx+c)^2-1\right)a^2b}{\cos(dx+c)^3}+\frac{4\left(5\,\cos(dx+c)^2-3\right)b^3}{\cos(dx+c)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(30*a^3*(1/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)^2 - 1)) - 45*(2*si
n(d*x + c)^2 - 1)*a*b^2/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) + 60*(3*cos
(d*x + c)^2 - 1)*a^2*b/cos(d*x + c)^3 + 4*(5*cos(d*x + c)^2 - 3)*b^3/cos(d*
x + c)^5)/d
```

Fricas [A]
time = 2.50, size = 109, normalized size = 0.92

$$\frac{60\,a^3\cos\left(dx+c\right)^5\log\left(-\cos\left(dx+c\right)\right)-180\,a^2b\cos\left(dx+c\right)^4+45\,ab^2\cos\left(dx+c\right)+30\,\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^3+12\,b^3+20\,\left(3\,a^2b-b^3\right)\cos\left(dx+c\right)^2}{60\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/60*(60*a^3*cos(d*x + c)^5*log(-cos(d*x + c)) - 180*a^2*b*cos(d*x + c)^4 +
 45*a*b^2*cos(d*x + c) + 30*(a^3 - 3*a*b^2)*cos(d*x + c)^3 + 12*b^3 + 20*(3
*a^2*b - b^3)*cos(d*x + c)^2)/(d*cos(d*x + c)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)^3\sec^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**3*sec(c + d*x)**3, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Modgcd: no suitable evaluation pointi
ndex.cc index_m operator + Error: Bad Argument ValueEvaluation time: 30.99D
one
```

Mupad [B]
time = 4.15, size = 220, normalized size = 1.85

$$-\frac{2\,a^3\,\mathrm{atanh}\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d}-\frac{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(6\,a^3+12\,a^2\,b-12\,a\,b^2+4\,b^3\right)+\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(-6\,a^3-28\,a^2\,b+12\,a\,b^2+\frac{4\,b^3}{3}\right)-2\,a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\,a^2\,b+\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a^3+20\,a^2\,b+\frac{4\,b^3}{3}\right)-\frac{4\,b^3}{15}}{d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-5\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+10\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-10\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+5\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(c + d*x) + b*tan(c + d*x))^3/cos(c + d*x)^3,x)
```

```
[Out] - (2*a^3*atanh(tan(c/2 + (d*x)/2)^2))/d - (tan(c/2 + (d*x)/2)^6*(12*a^2*b -
 12*a*b^2 + 6*a^3 + 4*b^3) + tan(c/2 + (d*x)/2)^4*(12*a*b^2 - 28*a^2*b - 6*
a^3 + (4*b^3)/3) - 2*a^3*tan(c/2 + (d*x)/2)^8 - 4*a^2*b + tan(c/2 + (d*x)/2
)^2*(20*a^2*b + 2*a^3 + (4*b^3)/3) - (4*b^3)/15)/(d*(5*tan(c/2 + (d*x)/2)^2
 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)
^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.250 $\int \frac{\cos^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

Optimal. Leaf size=113

$$-\frac{b\cos(c+dx)}{a^2d}+\frac{\cos^2(c+dx)}{2ad}+\frac{\log(1-\cos(c+dx))}{2(a+b)d}+\frac{\log(1+\cos(c+dx))}{2(a-b)d}-\frac{b^4\log(b+a\cos(c+dx))}{a^3\left(a^2-b^2\right)d}$$

```
[Out] -b*cos(d*x+c)/a^2/d+1/2*cos(d*x+c)^2/a/d+1/2*ln(1-cos(d*x+c))/(a+b)/d+1/2*l
n(1+cos(d*x+c))/(a-b)/d-b^4*ln(b+a*cos(d*x+c))/a^3/(a^2-b^2)/d
```

Rubi [A]
time = 0.24, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4482, 2916, 12, 1643}

$$-\frac{b\cos(c+dx)}{a^2d}-\frac{b^4\log(a\cos(c+dx)+b)}{a^3d\left(a^2-b^2\right)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}+\frac{\log(\cos(c+dx)+1)}{2d(a-b)}+\frac{\cos^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] -((b*Cos[c + d*x])/(a^2*d)) + Cos[c + d*x]^2/(2*a*d) + Log[1 - Cos[c + d*x]
]/(2*(a + b)*d) + Log[1 + Cos[c + d*x]]/(2*(a - b)*d) - (b^4*Log[b + a*Cos[
c + d*x]])/(a^3*(a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \int \frac{\cos^3(c+dx)\cot(c+dx)}{b+a\cos(c+dx)}\,dx$$
$$= -\frac{a\mathrm{Subst}\left(\int \frac{x^4}{a^4(b+x)(a^2-x^2)}\,dx, x, a\cos(c+dx)\right)}{d}$$
$$= -\frac{\mathrm{Subst}\left(\int \frac{x^4}{(b+x)(a^2-x^2)}\,dx, x, a\cos(c+dx)\right)}{a^3d}$$
$$= -\frac{\mathrm{Subst}\left(\int \left(b+\frac{a^3}{2(a+b)(a-x)}-x-\frac{a^3}{2(a-b)(a+x)}+\frac{b^4}{(a-b)(a+b)(b+x)}\right)\,dx, x, a\right.}{a^3d}$$
$$= -\frac{b\cos(c+dx)}{a^2d}+\frac{\cos^2(c+dx)}{2ad}+\frac{\log(1-\cos(c+dx))}{2(a+b)d}+\frac{\log(1+\cos}{2(a-}$$

Mathematica [A]
time = 0.41, size = 100, normalized size = 0.88

$$\frac{-\frac{4b\cos(c+dx)}{a^2}+\frac{\cos(2(c+dx))}{a}+4\left(\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a-b}+\frac{b^4\log(b+a\cos(c+dx))}{a^3(-a^2+b^2)}+\frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a+b}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
[Out] ((-4*b*Cos[c + d*x])/a^2 + Cos[2*(c + d*x)]/a + 4*(Log[Cos[(c + d*x)/2]]/(a
 - b) + (b^4*Log[b + a*Cos[c + d*x]])/(a^3*(-a^2 + b^2)) + Log[Sin[(c + d*x
)/2]]/(a + b)))/(4*d)
```

Maple [A]
time = 0.34, size = 100, normalized size = 0.88

method	result
derivativedivides	$\frac{\frac{\frac{\left(\cos^2(dx+c)\right)a}{2}-b\cos(dx+c)}{a^2}-\frac{b^4\ln(b+a\cos(dx+c))}{a^3(a+b)(a-b)}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}+\frac{\ln(1+\cos(dx+c))}{2a-2b}}{d}$
default	$\frac{\frac{\frac{\left(\cos^2(dx+c)\right)a}{2}-b\cos(dx+c)}{a^2}-\frac{b^4\ln(b+a\cos(dx+c))}{a^3(a+b)(a-b)}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}+\frac{\ln(1+\cos(dx+c))}{2a-2b}}{d}$
risch	$\frac{ix}{a}+\frac{ix\,b^2}{a^3}+\frac{e^{2i(dx+c)}}{8ad}-\frac{b\,e^{i(dx+c)}}{2a^2d}-\frac{b\,e^{-i(dx+c)}}{2a^2d}+\frac{e^{-2i(dx+c)}}{8ad}-\frac{ix}{a+b}-\frac{ic}{d(a+b)}-\frac{ix}{a-b}-\frac{ic}{d(a-b)}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a^2*(1/2*cos(d*x+c)^2*a-b*cos(d*x+c))-1/a^3*b^4/(a+b)/(a-b)*ln(b+a*c
os(d*x+c))+1/(2*a+2*b)*ln(cos(d*x+c)-1)+1/(2*a-2*b)*ln(1+cos(d*x+c)))
```

Maxima [A]
time = 0.48, size = 188, normalized size = 1.66

$$-\frac{\frac{b^4\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^5-a^3b^2}+\frac{2\left(b+\frac{(a+b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^2+\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a+b}+\frac{(a^2+b^2)\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(b^4*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^5 - a^3*b
^2) + 2*(b + (a + b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^2 + 2*a^2*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)
- log(sin(d*x + c)/(cos(d*x + c) + 1))/(a + b) + (a^2 + b^2)*log(sin(d*x +
c)^2/(cos(d*x + c) + 1)^2 + 1)/a^3)/d
```

Fricas [A]
time = 3.27, size = 123, normalized size = 1.09

$$-\frac{2\,b^4\log\left(a\cos\left(dx+c\right)+b\right)-(a^4-a^2b^2)\cos\left(dx+c\right)^2+2\,(a^3b-ab^3)\cos\left(dx+c\right)-(a^4+a^3b)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-(a^4-a^3b)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,(a^5-a^3b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^4*log(a*cos(d*x + c) + b) - (a^4 - a^2*b^2)*cos(d*x + c)^2 + 2*(a
^3*b - a*b^3)*cos(d*x + c) - (a^4 + a^3*b)*log(1/2*cos(d*x + c) + 1/2) - (a
^4 - a^3*b)*log(-1/2*cos(d*x + c) + 1/2))/((a^5 - a^3*b^2)*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 303 vs. 2(107) = 214.

time = 0.52, size = 303, normalized size = 2.68

$$-\frac{\frac{2\,b^4\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^5-a^3b^2}-\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}+\frac{2\,(a^2+b^2)\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3}-\frac{3\,a^2-4\,ab+3\,b^2-\frac{2\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{4\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{6\,b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{3\,b^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^3\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^4*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^5 - a^3*b^2) - log(abs(-cos(d*x +
c) + 1)/abs(cos(d*x + c) + 1))/(a + b) + 2*(a^2 + b^2)*log(abs(-(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 - (3*a^2 - 4*a*b + 3*b^2 - 2*a^2*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) + 4*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) - 6*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a^2*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2 + 3*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)
/(a^3*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^2))/d
```

Mupad [B]

time = 1.16, size = 161, normalized size = 1.42

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d\,(a+b)}-\frac{\frac{2\,b}{a^2}+\frac{2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2(a+b)}{a^2}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}-\frac{b^4\,\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d\,(a^5-a^3\,b^2)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)\,(a^2+b^2)}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a*sin(c + d*x) + b*tan(c + d*x)),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(d*(a + b)) - ((2*b)/a^2 + (2*tan(c/2 + (d*x)/2)^2*
(a + b))/a^2)/(d*(2*tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 + 1)) - (b^
4*log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2))/(d*(a^5 - a
^3*b^2)) - (log(tan(c/2 + (d*x)/2)^2 + 1)*(a^2 + b^2))/(a^3*d)
```

3.251 $\int \frac{\cos^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

Optimal. Leaf size=92

$$\frac{\cos(c+dx)}{ad}+\frac{\log(1-\cos(c+dx))}{2(a+b)d}-\frac{\log(1+\cos(c+dx))}{2(a-b)d}+\frac{b^3\log(b+a\cos(c+dx))}{a^2\left(a^2-b^2\right)d}$$

```
[Out] cos(d*x+c)/a/d+1/2*ln(1-cos(d*x+c))/(a+b)/d-1/2*ln(1+cos(d*x+c))/(a-b)/d+b^
3*ln(b+a*cos(d*x+c))/a^2/(a^2-b^2)/d
```

Rubi [A]
time = 0.19, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4482, 2916, 12, 1643}

$$\frac{b^3\log(a\cos(c+dx)+b)}{a^2d\left(a^2-b^2\right)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}-\frac{\log(\cos(c+dx)+1)}{2d(a-b)}+\frac{\cos(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] Cos[c + d*x]/(a*d) + Log[1 - Cos[c + d*x]]/(2*(a + b)*d) - Log[1 + Cos[c +
d*x]]/(2*(a - b)*d) + (b^3*Log[b + a*Cos[c + d*x]])/(a^2*(a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \int \frac{\cos^2(c+dx)\cot(c+dx)}{b+a\cos(c+dx)}\,dx$$
$$= -\frac{a\text{Subst}\left(\int \frac{x^3}{a^3(b+x)(a^2-x^2)}\,dx, x, a\cos(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{x^3}{(b+x)(a^2-x^2)}\,dx, x, a\cos(c+dx)\right)}{a^2d}$$
$$= -\frac{\text{Subst}\left(\int \left(-1+\frac{a^2}{2(a+b)(a-x)}+\frac{a^2}{2(a-b)(a+x)}+\frac{b^3}{(-a+b)(a+b)(b+x)}\right)\,dx, x, a}{a^2d}$$
$$= \frac{\cos(c+dx)}{ad}+\frac{\log(1-\cos(c+dx))}{2(a+b)d}-\frac{\log(1+\cos(c+dx))}{2(a-b)d}+\frac{b^3\log(}{a}$$

Mathematica [A]
time = 0.25, size = 80, normalized size = 0.87

$$\frac{\frac{\cos(c+dx)}{a}+\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{-a+b}+\frac{b^3\log(b+a\cos(c+dx))}{a^4-a^2b^2}+\frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a+b}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]/a + Log[Cos[(c + d*x)/2]]/(-a + b) + (b^3*Log[b + a*Cos[c + d
*x]])/(a^4 - a^2*b^2) + Log[Sin[(c + d*x)/2]]/(a + b))/d
```

Maple [A]
time = 0.30, size = 85, normalized size = 0.92

method	result
derivativedivides	$\frac{\frac{\cos(dx+c)}{a}+\frac{b^3\ln(b+a\cos(dx+c))}{a^2(a+b)(a-b)}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}-\frac{\ln(1+\cos(dx+c))}{2a-2b}}{d}$
default	$\frac{\frac{\cos(dx+c)}{a}+\frac{b^3\ln(b+a\cos(dx+c))}{a^2(a+b)(a-b)}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}-\frac{\ln(1+\cos(dx+c))}{2a-2b}}{d}$
risch	$-\frac{ibx}{a^2}+\frac{e^{i(dx+c)}}{2ad}+\frac{e^{-i(dx+c)}}{2ad}+\frac{ix}{a-b}+\frac{ic}{d(a-b)}-\frac{ix}{a+b}-\frac{ic}{d(a+b)}-\frac{2ib^3x}{a^2(a^2-b^2)}-\frac{2ib^3c}{a^2d(a^2-b^2)}-\frac{\ln(e^{i(dx}}{d(a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/a*cos(d*x+c)+1/a^2*b^3/(a+b)/(a-b)*ln(b+a*cos(d*x+c))+1/(2*a+2*b)*ln
(cos(d*x+c)-1)-1/(2*a-2*b)*ln(1+cos(d*x+c)))
```

Maxima [A]
time = 0.47, size = 129, normalized size = 1.40

$$\frac{\frac{b^3\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^4-a^2b^2}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a+b}+\frac{b\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a^2}+\frac{2}{a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b^3*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^4 - a^2*b^
2) + log(sin(d*x + c)/(cos(d*x + c) + 1))/(a + b) + b*log(sin(d*x + c)^2/(c
os(d*x + c) + 1)^2 + 1)/a^2 + 2/(a + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
)/d
```

Fricas [A]
time = 3.03, size = 98, normalized size = 1.07

$$\frac{2\,b^3\log\left(a\cos\left(dx+c\right)+b\right)+2\left(a^3-ab^2\right)\cos\left(dx+c\right)-\left(a^3+a^2b\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(a^3-a^2b\right)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\left(a^4-a^2b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b^3*log(a*cos(d*x + c) + b) + 2*(a^3 - a*b^2)*cos(d*x + c) - (a^3 +
a^2*b)*log(1/2*cos(d*x + c) + 1/2) + (a^3 - a^2*b)*log(-1/2*cos(d*x + c) +
1/2))/((a^4 - a^2*b^2)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos^2\left(c+dx\right)}{a\sin\left(c+dx\right)+b\tan\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**2/(a*sin(c + d*x) + b*tan(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(88) = 176.
time = 0.54, size = 190, normalized size = 2.07

$$\frac{\frac{2\,b^3\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^4-a^2b^2}+\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}+\frac{2\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^2}-\frac{2\left(2\,a-b+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b^3*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^4 - a^2*b^2) + log(abs(-cos(d*x + c
) + 1)/abs(cos(d*x + c) + 1))/(a + b) + 2*b*log(abs(-(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) + 1))/a^2 - 2*(2*a - b + b*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1))/(a^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)))/d
```

Mupad [B]
time = 0.82, size = 117, normalized size = 1.27

$$\frac{2}{a\,d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d\,\left(a+b\right)}+\frac{b\,\ln\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{a^2\,d}+\frac{b^3\,\ln\left(a+b-a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}{a^2\,d\,\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a*sin(c + d*x) + b*tan(c + d*x)),x)
```

```
[Out] 2/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)) + log(tan(c/2 + (d*x)/2))/(d*(a + b)) +
(b*log(tan(c/2 + (d*x)/2)^2 + 1))/(a^2*d) + (b^3*log(a + b - a*tan(c/2 + (d
*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2))/(a^2*d*(a^2 - b^2))
```

3.252 $\int \frac{\cos(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

Optimal. Leaf size=80

$$\frac{\log(1-\cos(c+dx))}{2(a+b)d}+\frac{\log(1+\cos(c+dx))}{2(a-b)d}-\frac{b^2\log(b+a\cos(c+dx))}{a\left(a^2-b^2\right)d}$$

```
[Out] 1/2*ln(1-cos(d*x+c))/(a+b)/d+1/2*ln(1+cos(d*x+c))/(a-b)/d-b^2*ln(b+a*cos(d*
x+c))/a/(a^2-b^2)/d
```

Rubi [A]
time = 0.17, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {4482, 2916, 12, 1643}

$$-\frac{b^2\log(a\cos(c+dx)+b)}{ad\left(a^2-b^2\right)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}+\frac{\log(\cos(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] Log[1 - Cos[c + d*x]]/(2*(a + b)*d) + Log[1 + Cos[c + d*x]]/(2*(a - b)*d) -
 (b^2*Log[b + a*Cos[c + d*x]])/(a*(a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx &= \int \frac{\cos(c+dx)\cot(c+dx)}{b+a\cos(c+dx)}\,dx \\
&= -\frac{a\text{Subst}\left(\int \frac{x^2}{a^2(b+x)(a^2-x^2)}\,dx,x,a\cos(c+dx)\right)}{d} \\
&= -\frac{\text{Subst}\left(\int \frac{x^2}{(b+x)(a^2-x^2)}\,dx,x,a\cos(c+dx)\right)}{ad} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{a}{2(a+b)(a-x)}-\frac{a}{2(a-b)(a+x)}+\frac{b^2}{(a-b)(a+b)(b+x)}\right)\,dx,x,a\cos(c+\right.}{ad} \\
&= \frac{\log(1-\cos(c+dx))}{2(a+b)d}+\frac{\log(1+\cos(c+dx))}{2(a-b)d}-\frac{b^2\log(b+a\cos(c+d}{a\left(a^2-b^2\right)d}
\end{aligned}
$$

Mathematica [A]
time = 0.12, size = 70, normalized size = 0.88

$$
\frac{a(a+b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-b^2\log(b+a\cos(c+dx))+a(a-b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a(a-b)(a+b)d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] (a*(a + b)*Log[Cos[(c + d*x)/2]] - b^2*Log[b + a*Cos[c + d*x]] + a*(a - b)*
Log[Sin[(c + d*x)/2]])/(a*(a - b)*(a + b)*d)
```

Maple [A]
time = 0.24, size = 75, normalized size = 0.94

method	result
derivativedivides	$\frac{-\frac{b^2\ln(b+a\cos(dx+c))}{(a+b)(a-b)a}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}+\frac{\ln(1+\cos(dx+c))}{2a-2b}}{d}$
default	$\frac{-\frac{b^2\ln(b+a\cos(dx+c))}{(a+b)(a-b)a}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}+\frac{\ln(1+\cos(dx+c))}{2a-2b}}{d}$
risch	$\frac{ix}{a}-\frac{ix}{a+b}-\frac{ic}{d(a+b)}-\frac{ix}{a-b}-\frac{ic}{d(a-b)}+\frac{2ib^2x}{a(a^2-b^2)}+\frac{2ib^2c}{ad(a^2-b^2)}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{d(a+b)}+\frac{\ln\left(e^{i(dx+c)}+1\right)}{d(a-b)}-\frac{b}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-b^2/(a+b)/(a-b)/a*ln(b+a*cos(d*x+c))+1/(2*a+2*b)*ln(cos(d*x+c)-1)+1/(
2*a-2*b)*ln(1+cos(d*x+c)))
```

Maxima [A]
time = 0.46, size = 102, normalized size = 1.28

$$-\frac{\frac{b^2\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^3-ab^2}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a+b}+\frac{\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(b^2*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^3 - a*b^2
) - log(sin(d*x + c)/(cos(d*x + c) + 1))/(a + b) + log(sin(d*x + c)^2/(cos(
d*x + c) + 1)^2 + 1)/a)/d
```

Fricas [A]
time = 2.97, size = 75, normalized size = 0.94

$$-\frac{2\,b^2\log\left(a\cos\left(dx+c\right)+b\right)-\left(a^2+ab\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(a^2-ab\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\left(a^3-ab^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b^2*log(a*cos(d*x + c) + b) - (a^2 + a*b)*log(1/2*cos(d*x + c) + 1/
2) - (a^2 - a*b)*log(-1/2*cos(d*x + c) + 1/2))/((a^3 - a*b^2)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(c+dx\right)}{a\sin\left(c+dx\right)+b\tan\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(76) = 152.
time = 0.55, size = 257, normalized size = 3.21

$$-\frac{\frac{a\log\left(\left|-a-b+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right|\right)}{a^2-b^2}-\frac{(a^2-2\,b^2)\log\left(\frac{\left|-2\,b-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-2\,|a|\right|}{\left|-2\,b-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+2\,|a|\right|}\right)}{(a^2-b^2)|a|}-\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(a*log(abs(-a - b + 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2))/(a^2 - b^2) - (a^2 - 2*b^2)*log(abs(-2*b - 2*a*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*abs(a)
)/abs(-2*b - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*b*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) + 2*abs(a)))/((a^2 - b^2)*abs(a)) - log(abs(-cos(d*
x + c) + 1)/abs(cos(d*x + c) + 1))/(a + b))/d
```

Mupad [B]
time = 0.87, size = 93, normalized size = 1.16

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d\,\left(a+b\right)}-\frac{\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{a\,d}+\frac{b^2\,\ln\left(a+b-a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}{d\,\left(a\,b^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x)),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(d*(a + b)) - log(tan(c/2 + (d*x)/2)^2 + 1)/(a*d) +
 (b^2*log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2))/(d*(a*b
^2 - a^3))
```

3.253 $\int \frac{1}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

Optimal. Leaf size=74

$$\frac{\log(1-\cos(c+dx))}{2(a+b)d}-\frac{\log(1+\cos(c+dx))}{2(a-b)d}+\frac{b\log(b+a\cos(c+dx))}{(a^2-b^2)\,d}$$

```
[Out] 1/2*ln(1-cos(d*x+c))/(a+b)/d-1/2*ln(1+cos(d*x+c))/(a-b)/d+b*ln(b+a*cos(d*x+
c))/(a^2-b^2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {4482, 2800, 815}

$$\frac{b\log(a\cos(c+dx)+b)}{d\,(a^2-b^2)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}-\frac{\log(\cos(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[c + d*x] + b*Tan[c + d*x])^(-1),x]
```

```
[Out] Log[1 - Cos[c + d*x]]/(2*(a + b)*d) - Log[1 + Cos[c + d*x]]/(2*(a - b)*d) +
 (b*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)*d)
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{1}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \int \frac{\cot(c+dx)}{b+a\cos(c+dx)}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{x}{(b+x)(a^2-x^2)}\,dx, x, a\cos(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{2(a+b)(a-x)}+\frac{1}{2(a-b)(a+x)}+\frac{b}{(-a+b)(a+b)(b+x)}\right)\,dx, x, a\cos(c$$
$$= \frac{\log(1-\cos(c+dx))}{2(a+b)d} - \frac{\log(1+\cos(c+dx))}{2(a-b)d} + \frac{b\log(b+a\cos(c+dx}{(a^2-b^2)\,d}$$

Mathematica [A]
time = 0.10, size = 63, normalized size = 0.85

$$\frac{-\left((a+b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+b\log(b+a\cos(c+dx))+(a-b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a-b)(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[c + d*x] + b*Tan[c + d*x])^(-1),x]
[Out] (-((a + b)*Log[Cos[(c + d*x)/2]]) + b*Log[b + a*Cos[c + d*x]] + (a - b)*Log
[Sin[(c + d*x)/2]])/((a - b)*(a + b)*d)
```

Maple [A]
time = 0.22, size = 70, normalized size = 0.95

method	result
derivativedivides	$\frac{\frac{b\ln(b+a\cos(dx+c))}{(a+b)(a-b)}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}-\frac{\ln(1+\cos(dx+c))}{2a-2b}}{d}$
default	$\frac{\frac{b\ln(b+a\cos(dx+c))}{(a+b)(a-b)}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}-\frac{\ln(1+\cos(dx+c))}{2a-2b}}{d}$
risch	$\frac{ix}{a-b}+\frac{ic}{d(a-b)}-\frac{ix}{a+b}-\frac{ic}{d(a+b)}-\frac{2ibx}{a^2-b^2}-\frac{2ibc}{d(a^2-b^2)}-\frac{\ln(e^{i(dx+c)}+1)}{d(a-b)}+\frac{\ln(e^{i(dx+c)}-1)}{d(a+b)}+\frac{b\ln\left(e^{2i(dx+}\right.}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(d*x+c)+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(b/(a+b)/(a-b)*ln(b+a*cos(d*x+c))+1/(2*a+2*b)*ln(cos(d*x+c)-1)-1/(2*a-2
*b)*ln(1+cos(d*x+c)))
```

Maxima [A]
time = 0.27, size = 71, normalized size = 0.96

$$\frac{\frac{b\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^2-b^2}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a+b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (b*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^2 - b^2) + l
og(sin(d*x + c)/(cos(d*x + c) + 1))/(a + b))/d
```

Fricas [A]
time = 2.76, size = 64, normalized size = 0.86

$$\frac{2\,b\log\left(a\cos\left(dx+c\right)+b\right)-\left(a+b\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(a-b\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\left(a^2-b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*log(a*cos(d*x + c) + b) - (a + b)*log(1/2*cos(d*x + c) + 1/2) + (a
 - b)*log(-1/2*cos(d*x + c) + 1/2))/((a^2 - b^2)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{a\sin\left(c+dx\right)+b\tan\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral(1/(a*sin(c + d*x) + b*tan(c + d*x)), x)
```

Giac [A]
time = 0.48, size = 100, normalized size = 1.35

$$\frac{\frac{2\,b\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^2-b^2}+\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^2 - b^2) + log(abs(-cos(d*x + c) + 1)
/abs(cos(d*x + c) + 1))/(a + b))/d
```

Mupad [B]
time = 0.90, size = 67, normalized size = 0.91

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d\,\left(a+b\right)}+\frac{b\,\ln\left(a+b-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d\,\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(c + d*x) + b*tan(c + d*x)),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(d*(a + b)) + (b*log(a + b - a*tan(c/2 + (d*x)/2)^2
 + b*tan(c/2 + (d*x)/2)^2))/(d*(a^2 - b^2))
```

3.254 $\int \frac{\sec(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

Optimal. Leaf size=75

$$\frac{\log(1-\cos(c+dx))}{2(a+b)d}+\frac{\log(1+\cos(c+dx))}{2(a-b)d}-\frac{a\log(b+a\cos(c+dx))}{(a^2-b^2)\,d}$$

```
[Out] 1/2*ln(1-cos(d*x+c))/(a+b)/d+1/2*ln(1+cos(d*x+c))/(a-b)/d-a*ln(b+a*cos(d*x+
c))/(a^2-b^2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {4482, 2747, 720, 31, 647}

$$-\frac{a\log(a\cos(c+dx)+b)}{d\,(a^2-b^2)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}+\frac{\log(\cos(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] Log[1 - Cos[c + d*x]]/(2*(a + b)*d) + Log[1 + Cos[c + d*x]]/(2*(a - b)*d) -
 (a*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 647

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Rt[(-
a)*c, 2]}, Dist[e/2 + c*(d/(2*q)), Int[1/(-q + c*x), x], x] + Dist[e/2 - c*
(d/(2*q)), Int[1/(q + c*x), x], x]] /; FreeQ[{a, c, d, e}, x] && NiceSqrtQ[
(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
```

```
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \int \frac{\csc(c+dx)}{b+a\cos(c+dx)}\,dx$$
$$= -\frac{a\text{Subst}\left(\int \frac{1}{(b+x)(a^2-x^2)}\,dx, x, a\cos(c+dx)\right)}{d}$$
$$= -\frac{a\text{Subst}\left(\int \frac{1}{b+x}\,dx, x, a\cos(c+dx)\right)}{(a^2-b^2)\,d} - \frac{a\text{Subst}\left(\int \frac{-b+x}{a^2-x^2}\,dx, x, a\cos(c\right.}{(a^2-b^2)\,d}$$
$$= -\frac{a\log(b+a\cos(c+dx))}{(a^2-b^2)\,d} - \frac{\text{Subst}\left(\int \frac{1}{-a-x}\,dx, x, a\cos(c+dx)\right)}{2(a-b)d} - \text{Su}$$
$$= \frac{\log(1-\cos(c+dx))}{2(a+b)d} + \frac{\log(1+\cos(c+dx))}{2(a-b)d} - \frac{a\log(b+a\cos(c+dx}{(a^2-b^2)\,d}$$

Mathematica [A]
time = 0.07, size = 64, normalized size = 0.85

$$\frac{(a-b)\log(1-\cos(c+dx))+(a+b)\log(1+\cos(c+dx))-2a\log(b+a\cos(c+dx))}{2(a-b)(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
[Out] ((a - b)*Log[1 - Cos[c + d*x]] + (a + b)*Log[1 + Cos[c + d*x]] - 2*a*Log[b
+ a*Cos[c + d*x]])/(2*(a - b)*(a + b)*d)
```

Maple [A]
time = 0.28, size = 70, normalized size = 0.93

method	result
derivativedivides	$\frac{-\frac{a\ln(b+a\cos(dx+c))}{(a+b)(a-b)}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}+\frac{\ln(1+\cos(dx+c))}{2a-2b}}{d}$
default	$\frac{-\frac{a\ln(b+a\cos(dx+c))}{(a+b)(a-b)}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}+\frac{\ln(1+\cos(dx+c))}{2a-2b}}{d}$

risch	$-\frac{ix}{a+b}-\frac{ic}{d(a+b)}-\frac{ix}{a-b}-\frac{ic}{d(a-b)}+\frac{2iax}{a^2-b^2}+\frac{2iac}{d(a^2-b^2)}+\frac{\ln(e^{i(dx+c)}-1)}{d(a+b)}+\frac{\ln(e^{i(dx+c)}+1)}{d(a-b)}-\frac{a\ln\left(e^{2i(dx\cdots\right.}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a/(a+b)/(a-b)*ln(b+a*cos(d*x+c))+1/(2*a+2*b)*ln(cos(d*x+c)-1)+1/(2*a-
2*b)*ln(1+cos(d*x+c)))
```

Maxima [A]
time = 0.27, size = 73, normalized size = 0.97

$$-\frac{\frac{a\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^2-b^2}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a+b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(a*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^2 - b^2) -
log(sin(d*x + c)/(cos(d*x + c) + 1))/(a + b))/d
```

Fricas [A]
time = 2.42, size = 65, normalized size = 0.87

$$-\frac{2\,a\log\left(a\cos\left(dx+c\right)+b\right)-\left(a+b\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(a-b\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,(a^2-b^2)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*log(a*cos(d*x + c) + b) - (a + b)*log(1/2*cos(d*x + c) + 1/2) - (
a - b)*log(-1/2*cos(d*x + c) + 1/2))/((a^2 - b^2)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec\left(c+dx\right)}{a\sin\left(c+dx\right)+b\tan\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x)), x)
```

Giac [A]
time = 0.49, size = 101, normalized size = 1.35

$$-\frac{\frac{2\,a\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^2-b^2}-\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^2 - b^2) - log(abs(-cos(d*x + c) + 1
)/abs(cos(d*x + c) + 1))/(a + b))/d
```

Mupad [B]
time = 0.71, size = 68, normalized size = 0.91

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d\,\left(a+b\right)}-\frac{a\,\ln\left(a+b-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}{d\,\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a*sin(c + d*x) + b*tan(c + d*x))),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(d*(a + b)) - (a*log(a + b - a*tan(c/2 + (d*x)/2)^2
 + b*tan(c/2 + (d*x)/2)^2))/(d*(a^2 - b^2))
```

3.255 $\int \frac{\sec^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

Optimal. Leaf size=94

$$\frac{\log(1-\cos(c+dx))}{2(a+b)d}-\frac{\log(\cos(c+dx))}{bd}-\frac{\log(1+\cos(c+dx))}{2(a-b)d}+\frac{a^2\log(b+a\cos(c+dx))}{b\,(a^2-b^2)\,d}$$

```
[Out] 1/2*ln(1-cos(d*x+c))/(a+b)/d-ln(cos(d*x+c))/b/d-1/2*ln(1+cos(d*x+c))/(a-b)/
d+a^2*ln(b+a*cos(d*x+c))/b/(a^2-b^2)/d
```

Rubi [A]
time = 0.17, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}}=0.143$, Rules used = {4482, 2916, 12, 908}

$$\frac{a^2\log(a\cos(c+dx)+b)}{bd\,(a^2-b^2)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}-\frac{\log(\cos(c+dx)+1)}{2d(a-b)}-\frac{\log(\cos(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] Log[1 - Cos[c + d*x]]/(2*(a + b)*d) - Log[Cos[c + d*x]]/(b*d) - Log[1 + Cos
[c + d*x]]/(2*(a - b)*d) + (a^2*Log[b + a*Cos[c + d*x]])/(b*(a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx &= \int \frac{\csc(c+dx)\sec(c+dx)}{b+a\cos(c+dx)}\,dx\\ &= -\frac{a\text{Subst}\left(\int \frac{a}{x(b+x)(a^2-x^2)}\,dx,x,a\cos(c+dx)\right)}{d}\\ &= -\frac{a^2\text{Subst}\left(\int \frac{1}{x(b+x)(a^2-x^2)}\,dx,x,a\cos(c+dx)\right)}{d}\\ &= -\frac{a^2\text{Subst}\left(\int \left(\frac{1}{2a^2(a+b)(a-x)}+\frac{1}{a^2bx}+\frac{1}{2a^2(a-b)(a+x)}+\frac{1}{b(-a+b)(a+b)(b+x)}\right)\right.}{d}\\ &= \frac{\log(1-\cos(c+dx))}{2(a+b)d}-\frac{\log(\cos(c+dx))}{bd}-\frac{\log(1+\cos(c+dx))}{2(a-b)d}+\frac{a}{}\end{aligned}$$

Mathematica [A]
time = 0.18, size = 103, normalized size = 1.10

$$2\left(\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2(-a+b)d}-\frac{\log(\cos(c+dx))}{2bd}-\frac{a^2\log(b+a\cos(c+dx))}{2b\left(-a^2+b^2\right)d}+\frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2(a+b)d}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] 2*(Log[Cos[(c + d*x)/2]]/(2*(-a + b)*d) - Log[Cos[c + d*x]]/(2*b*d) - (a^2*
Log[b + a*Cos[c + d*x]])/(2*b*(-a^2 + b^2)*d) + Log[Sin[(c + d*x)/2]]/(2*(a
 + b)*d))
```

Maple [A]
time = 0.34, size = 87, normalized size = 0.93

method	result
derivativedivides	$\frac{\frac{a^2\ln(b+a\cos(dx+c))}{(a+b)(a-b)b}-\frac{\ln(\cos(dx+c))}{b}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}-\frac{\ln(1+\cos(dx+c))}{2a-2b}}{d}$
default	$\frac{\frac{a^2\ln(b+a\cos(dx+c))}{(a+b)(a-b)b}-\frac{\ln(\cos(dx+c))}{b}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}-\frac{\ln(1+\cos(dx+c))}{2a-2b}}{d}$
risch	$-\frac{ix}{a+b}-\frac{ic}{d(a+b)}+\frac{ix}{a-b}+\frac{ic}{d(a-b)}-\frac{2ia^2x}{b(a^2-b^2)}-\frac{2ia^2c}{bd(a^2-b^2)}+\frac{2ix}{b}+\frac{2ic}{bd}+\frac{\ln\left(e^{i(dx+c)}-1\right)}{d(a+b)}-\frac{\ln\left(e^{i(dx+c}\right.}{d(a-b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2/(a+b)/(a-b)/b*ln(b+a*cos(d*x+c))-1/b*ln(cos(d*x+c))+1/(2*a+2*b)*ln
(cos(d*x+c)-1)-1/(2*a-2*b)*ln(1+cos(d*x+c)))
```

Maxima [A]
time = 0.28, size = 125, normalized size = 1.33

$$\frac{\frac{a^2\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^2b-b^3}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b}+\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a+b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a^2*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^2*b - b^3)
 - log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b - log(sin(d*x + c)/(cos(d*x +
 c) + 1) - 1)/b + log(sin(d*x + c)/(cos(d*x + c) + 1))/(a + b))/d
```

Fricas [A]
time = 2.11, size = 96, normalized size = 1.02

$$\frac{2\,a^2\log\left(a\cos\left(dx+c\right)+b\right)-2\,(a^2-b^2)\log\left(-\cos\left(dx+c\right)\right)-(ab+b^2)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+(ab-b^2)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,(a^2b-b^3)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*log(a*cos(d*x + c) + b) - 2*(a^2 - b^2)*log(-cos(d*x + c)) - (a*
b + b^2)*log(1/2*cos(d*x + c) + 1/2) + (a*b - b^2)*log(-1/2*cos(d*x + c) +
1/2))/((a^2*b - b^3)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^2\left(c+dx\right)}{a\sin\left(c+dx\right)+b\tan\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*sin(c + d*x) + b*tan(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(90) = 180.
time = 0.56, size = 253, normalized size = 2.69

$$\frac{\frac{b\log\left(\left|a+b+\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right|\right)}{a^2-b^2}+\frac{(2\,a^2-b^2)\log\left(\frac{\left|-2\,a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-2\,|b|\right|}{\left|-2\,a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+2\,|b|\right|}\right)}{(a^2-b^2)|b|}+\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(b*log(abs(a + b + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d
*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2))/(a^2 - b^2) + (2*a^2 - b^2)*log(abs(-2*a - 2*a*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1) + 2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*abs(b))/
abs(-2*a - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*b*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) + 2*abs(b)))/((a^2 - b^2)*abs(b)) + log(abs(-cos(d*x
+ c) + 1)/abs(cos(d*x + c) + 1))/(a + b))/d
```

Mupad [B]
time = 0.84, size = 93, normalized size = 0.99

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d\,\left(a+b\right)}-\frac{\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}{b\,d}+\frac{a^2\,\ln\left(a+b-a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}{d\,\left(a^2\,b-b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a*sin(c + d*x) + b*tan(c + d*x))),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(d*(a + b)) - log(tan(c/2 + (d*x)/2)^2 - 1)/(b*d) +
 (a^2*log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2))/(d*(a^2
*b - b^3))
```

3.256 $\int \frac{\sec^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

Optimal. Leaf size=108

$$\frac{\log(1-\cos(c+dx))}{2(a+b)d}+\frac{a\log(\cos(c+dx))}{b^2d}+\frac{\log(1+\cos(c+dx))}{2(a-b)d}-\frac{a^3\log(b+a\cos(c+dx))}{b^2\left(a^2-b^2\right)d}+\frac{\sec(c+dx)}{bd}$$

```
[Out] 1/2*ln(1-cos(d*x+c))/(a+b)/d+a*ln(cos(d*x+c))/b^2/d+1/2*ln(1+cos(d*x+c))/(a
-b)/d-a^3*ln(b+a*cos(d*x+c))/b^2/(a^2-b^2)/d+sec(d*x+c)/b/d
```

Rubi [A]
time = 0.20, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4482, 2916, 12, 908}

$$-\frac{a^3\log(a\cos(c+dx)+b)}{b^2d\left(a^2-b^2\right)}+\frac{a\log(\cos(c+dx))}{b^2d}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}+\frac{\log(\cos(c+dx)+1)}{2d(a-b)}+\frac{\sec(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] Log[1 - Cos[c + d*x]]/(2*(a + b)*d) + (a*Log[Cos[c + d*x]])/(b^2*d) + Log[1
 + Cos[c + d*x]]/(2*(a - b)*d) - (a^3*Log[b + a*Cos[c + d*x]])/(b^2*(a^2 -
b^2)*d) + Sec[c + d*x]/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx = \int \frac{\csc(c+dx)\sec^2(c+dx)}{b+a\cos(c+dx)}\,dx$$

$$= -\frac{a\text{Subst}\left(\int \frac{a^2}{x^2(b+x)(a^2-x^2)}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= -\frac{a^3\text{Subst}\left(\int \frac{1}{x^2(b+x)(a^2-x^2)}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= -\frac{a^3\text{Subst}\left(\int \left(\frac{1}{2a^3(a+b)(a-x)}+\frac{1}{a^2bx^2}-\frac{1}{a^2b^2x}-\frac{1}{2a^3(a-b)(a+x)}-\frac{1}{b^2(-a+b)(a}\right.\right.}{d}$$

$$= \frac{\log(1-\cos(c+dx))}{2(a+b)d}+\frac{a\log(\cos(c+dx))}{b^2d}+\frac{\log(1+\cos(c+dx))}{2(a-b)d}-$$

Mathematica [A]
time = 0.32, size = 92, normalized size = 0.85

$$\frac{\frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{a-b}+\frac{a\log(\cos(c+dx))}{b^2}+\frac{a^3\log(b+a\cos(c+dx))}{-a^2b^2+b^4}+\frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{a+b}+\frac{\sec(c+dx)}{b}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] (Log[Cos[(c + d*x)/2]]/(a - b) + (a*Log[Cos[c + d*x]])/b^2 + (a^3*Log[b + a
*Cos[c + d*x]])/(-(a^2*b^2) + b^4) + Log[Sin[(c + d*x)/2]]/(a + b) + Sec[c
+ d*x]/b)/d
```

Maple [A]
time = 0.39, size = 99, normalized size = 0.92

method	result
derivativedivides	$\frac{-\frac{a^3\ln(b+a\cos(dx+c))}{(a+b)(a-b)b^2}+\frac{a\ln(\cos(dx+c))}{b^2}+\frac{1}{b\cos(dx+c)}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}+\frac{\ln(1+\cos(dx+c))}{2a-2b}}{d}$
default	$\frac{-\frac{a^3\ln(b+a\cos(dx+c))}{(a+b)(a-b)b^2}+\frac{a\ln(\cos(dx+c))}{b^2}+\frac{1}{b\cos(dx+c)}+\frac{\ln(\cos(dx+c)-1)}{2a+2b}+\frac{\ln(1+\cos(dx+c))}{2a-2b}}{d}$

risch	$-\frac{ix}{a+b}-\frac{ic}{d(a+b)}-\frac{ix}{a-b}-\frac{ic}{d(a-b)}-\frac{2iax}{b^2}-\frac{2iac}{b^2d}+\frac{2ia^3x}{b^2(a^2-b^2)}+\frac{2ia^3c}{b^2d(a^2-b^2)}+\frac{2\,\mathrm{e}^{i(dx+c)}}{db(\mathrm{e}^{2i(dx+c)}+1)}+\frac{\ln(\mathrm{e}^{i(c}}{d(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a^3/(a+b)/(a-b)/b^2*ln(b+a*cos(d*x+c))+a/b^2*ln(cos(d*x+c))+1/b/cos(d
*x+c)+1/(2*a+2*b)*ln(cos(d*x+c)-1)+1/(2*a-2*b)*ln(1+cos(d*x+c)))
```

Maxima [A]
time = 0.27, size = 158, normalized size = 1.46

$$-\frac{\frac{a^3\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^2b^2-b^4}-\frac{a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{b^2}-\frac{a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{b^2}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a+b}-\frac{2}{b-\frac{b\sin(dx+c)^2}{(\cos(dx+c)+1)^2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(a^3*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^2*b^2 - b
^4) - a*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/b^2 - a*log(sin(d*x + c)/(
cos(d*x + c) + 1) - 1)/b^2 - log(sin(d*x + c)/(cos(d*x + c) + 1))/(a + b) -
 2/(b - b*sin(d*x + c)^2/(cos(d*x + c) + 1)^2))/d
```

Fricas [A]
time = 3.56, size = 147, normalized size = 1.36

$$-\frac{2\,a^3\cos(dx+c)\log(a\cos(dx+c)+b)-2\,a^2b+2\,b^3-2\,(a^3-ab^2)\cos(dx+c)\log(-\cos(dx+c))-(ab^2+b^3)\cos(dx+c)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-(ab^2-b^3)\cos(dx+c)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{2\,(a^2b^2-b^4)d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*cos(d*x + c)*log(a*cos(d*x + c) + b) - 2*a^2*b + 2*b^3 - 2*(a^3
 - a*b^2)*cos(d*x + c)*log(-cos(d*x + c)) - (a*b^2 + b^3)*cos(d*x + c)*log(
1/2*cos(d*x + c) + 1/2) - (a*b^2 - b^3)*cos(d*x + c)*log(-1/2*cos(d*x + c)
+ 1/2))/((a^2*b^2 - b^4)*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^3(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*sin(c + d*x) + b*tan(c + d*x)), x)
```

Giac [A]
time = 0.55, size = 190, normalized size = 1.76

$$-\frac{\frac{2\,a^3\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^2b^2-b^4}-\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}-\frac{2\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{b^2}+\frac{2\left(a-2\,b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{b^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(2*a^3*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^2*b^2 - b^4) - log(abs(-cos(d*x +
c) + 1)/abs(cos(d*x + c) + 1))/(a + b) - 2*a*log(abs(-(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - 1))/b^2 + 2*(a - 2*b + a*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1))/(b^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

Mupad [B]
time = 0.83, size = 118, normalized size = 1.09

$$\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d\,\left(a+b\right)}-\frac{2}{b\,d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}+\frac{a\,\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}{b^2\,d}-\frac{a^3\,\ln\left(a+b-a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}{b^2\,d\,\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a*sin(c + d*x) + b*tan(c + d*x))),x)
```

```
[Out] log(tan(c/2 + (d*x)/2))/(d*(a + b)) - 2/(b*d*(tan(c/2 + (d*x)/2)^2 - 1)) +
(a*log(tan(c/2 + (d*x)/2)^2 - 1))/(b^2*d) - (a^3*log(a + b - a*tan(c/2 + (d
*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2))/(b^2*d*(a^2 - b^2))
```

3.257 $\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=243

$$\frac{2bx}{a^3}+\frac{2b^6\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d}+\frac{2b^4(5a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d}-\frac{\sin(c+dx)}{a^2d}-\frac{}{2(a+}$$

```
[Out] 2*b*x/a^3+2*b^6*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^3/(a-
b)^(5/2)/(a+b)^(5/2)/d+2*b^4*(5*a^2-3*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+
1/2*c)/(a+b)^(1/2))/a^3/(a-b)^(5/2)/(a+b)^(5/2)/d-sin(d*x+c)/a^2/d-1/2*sin(
d*x+c)/(a+b)^2/d/(1-cos(d*x+c))-1/2*sin(d*x+c)/(a-b)^2/d/(1+cos(d*x+c))-b^5
*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(b+a*cos(d*x+c))
```

Rubi [A]

time = 0.46, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4482, 2976, 2727, 2717, 2743, 12, 2738, 214}

$$\frac{2b^6\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{2bx}{a^3}-\frac{b^5\sin(c+dx)}{a^2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{\sin(c+dx)}{a^2d}+\frac{2b^4\left(5a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\sin(c+dx)}{2d(a+b)^2(1-\cos(c+dx))}-\frac{\sin(c+dx)}{2d(a-b)^2(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (2*b*x)/a^3 + (2*b^6*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(
a^3*(a - b)^(5/2)*(a + b)^(5/2)*d) + (2*b^4*(5*a^2 - 3*b^2)*ArcTanh[(Sqrt[a
 - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)^(5/2)*d) -
 Sin[c + d*x]/(a^2*d) - Sin[c + d*x]/(2*(a + b)^2*d*(1 - Cos[c + d*x])) - S
in[c + d*x]/(2*(a - b)^2*d*(1 + Cos[c + d*x])) - (b^5*Sin[c + d*x])/(a^2*(a
^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \int \frac{\cos^3(c+dx)\cot^2(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= -\int \left(-\frac{2b}{a^3} - \frac{1}{2(a-b)^2(-1-\cos(c+dx))} - \frac{1}{2(a+b)^2(1-\cos(c+$$

$$= \frac{2bx}{a^3} - \frac{\int \cos(c+dx)\,dx}{a^2} + \frac{\int \frac{1}{-1-\cos(c+dx)}\,dx}{2(a-b)^2} + \frac{\int \frac{1}{1-\cos(c+dx)}\,dx}{2(a+b)^2} - \frac{(b$$

$$= \frac{2bx}{a^3} - \frac{\sin(c+dx)}{a^2d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} - \frac{\sin(c+}{2(a-b)^2d(1+}$$

$$= \frac{2bx}{a^3} + \frac{2b^4(5a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{a^2d}$$

$$= \frac{2bx}{a^3} + \frac{2b^4(5a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{a^2d}$$

$$= \frac{2bx}{a^3} + \frac{2b^6\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d} + \frac{2b^4(5a^2-3b^2)\tanh^{-1}\left(\right.}{a^3(a-b)^{5/2}}$$

Mathematica [A]
time = 3.13, size = 164, normalized size = 0.67

$$-\frac{-\frac{4b(c+dx)}{a^3} + \frac{4b^4(5a^2-2b^2)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^3(a^2-b^2)^{5/2}} + \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2} + \frac{2\sin(c+dx)}{a^2} + \frac{2b^5\sin(c+dx)}{a^2(a-b)^2(a+b)^2(b+a\cos(c+dx))} + \frac{\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/2*((-4*b*(c + d*x))/a^3 + (4*b^4*(5*a^2 - 2*b^2)*ArcTanh[((-a + b)*Tan[(
c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^3*(a^2 - b^2)^(5/2)) + Cot[(c + d*x)/2]/(
a + b)^2 + (2*Sin[c + d*x])/a^2 + (2*b^5*Sin[c + d*x])/(a^2*(a - b)^2*(a +
b)^2*(b + a*Cos[c + d*x])) + Tan[(c + d*x)/2]/(a - b)^2)/d
```

Maple [A]
time = 0.43, size = 215, normalized size = 0.88

method	result

derivativedivides	$\dfrac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^2-2ab+b^2\right)}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b^4\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}-\frac{\left(5a^2-2b^2\right)\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\right.}{\sqrt{(a+b}}\right.}{\sqrt{(a+b)\,(a-}}\right.}{(a+b)^2(a-b)^2a^3}}{d}$
default	$\dfrac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^2-2ab+b^2\right)}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b^4\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}-\frac{\left(5a^2-2b^2\right)\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\right.}{\sqrt{(a+b}}\right.}{\sqrt{(a+b)\,(a-}}\right.}{(a+b)^2(a-b)^2a^3}}{d}$
risch	$\frac{2bx}{a^3}+\frac{ie^{i(dx+c)}}{2a^2d}-\frac{ie^{-i(dx+c)}}{2a^2d}-\frac{2i\left(a^6e^{3i(dx+c)}+a^4b^2e^{3i(dx+c)}+b^6e^{3i(dx+c)}+2a^3b^3e^{2i(dx+c)}+a\,b^5e^{2i(dx+c)}+a^6e^{i\ldots}\right.}{d(a^2-b^2)^2a^3\left(a\,e^{4i(dx+c)}+2b\,e^{3i(dx+c)}-2b\,e^{i\ldots}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-1/2/(a+b)^2/tan(1/2*d*x+1/2*c)
-2*b^4/(a+b)^2/(a-b)^2/a^3*(-a*b*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2
-b*tan(1/2*d*x+1/2*c)^2-a-b)-(5*a^2-2*b^2)/((a+b)*(a-b))^(1/2)*arctanh(tan(
1/2*d*x+1/2*c)*(a-b)/((a+b)*(a-b))^(1/2)))+2/a^3*(-a*tan(1/2*d*x+1/2*c)/(1+
tan(1/2*d*x+1/2*c)^2)+2*b*arctan(tan(1/2*d*x+1/2*c))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.21, size = 857, normalized size = 3.53

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(4*a^7*b - 6*a^5*b^3 + 6*a^3*b^5 - 4*a*b^7 - 2*(a^8 - 3*a^6*b^2 + 3*a
^4*b^4 - a^2*b^6)*cos(d*x + c)^3 + (5*a^2*b^5 - 2*b^7 + (5*a^3*b^4 - 2*a*b^
6)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*co
s(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2
- b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(3
*a^7*b - 5*a^5*b^3 + 4*a^3*b^5 - 2*a*b^7)*cos(d*x + c)^2 + 2*(2*a^8 - 5*a^6
*b^2 + 4*a^4*b^4 - a^2*b^6)*cos(d*x + c) - 4*((a^7*b - 3*a^5*b^3 + 3*a^3*b^
5 - a*b^7)*d*x*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d*x)*
sin(d*x + c))/(((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^4*b^6)*d*cos(d*x + c) + (
a^9*b - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*d)*sin(d*x + c)), -(2*a^7*b - 3*a^
5*b^3 + 3*a^3*b^5 - 2*a*b^7 - (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*cos(d
*x + c)^3 - (5*a^2*b^5 - 2*b^7 + (5*a^3*b^4 - 2*a*b^6)*cos(d*x + c))*sqrt(-
a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d
*x + c)))*sin(d*x + c) - (3*a^7*b - 5*a^5*b^3 + 4*a^3*b^5 - 2*a*b^7)*cos(d*
x + c)^2 + (2*a^8 - 5*a^6*b^2 + 4*a^4*b^4 - a^2*b^6)*cos(d*x + c) - 2*((a^7
*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*x*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4
 + 3*a^2*b^6 - b^8)*d*x)*sin(d*x + c))/(((a^10 - 3*a^8*b^2 + 3*a^6*b^4 - a^
4*b^6)*d*cos(d*x + c) + (a^9*b - 3*a^7*b^3 + 3*a^5*b^5 - a^3*b^7)*d)*sin(d*
x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1362 vs. $2(219) = 438$.
time = 0.85, size = 1362, normalized size = 5.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*((2*a^4*b - 2*a^3*b^2 - 4*a^2*b^3 - a*b^4 + 2*b^5)*sqrt(-a^2 + b^2)
*abs(a^7 - 2*a^5*b^2 + a^3*b^4)*abs(a - b) - (2*a^11*b - 2*a^10*b^2 - 8*a^9
*b^3 + 13*a^8*b^4 + 12*a^7*b^5 - 24*a^6*b^6 - 8*a^5*b^7 + 17*a^4*b^8 + 2*a^
3*b^9 - 4*a^2*b^10)*sqrt(-a^2 + b^2)*abs(a - b))*(pi*floor(1/2*(d*x + c)/pi
 + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(a^6*b - 2*a^4*b^3 + a^2*b^5 +
```

```
sqrt((a^7 + a^6*b - 2*a^5*b^2 - 2*a^4*b^3 + a^3*b^4 + a^2*b^5)*(a^7 - a^6*b
 - 2*a^5*b^2 + 2*a^4*b^3 + a^3*b^4 - a^2*b^5) + (a^6*b - 2*a^4*b^3 + a^2*b^
5)^2))/(a^7 - a^6*b - 2*a^5*b^2 + 2*a^4*b^3 + a^3*b^4 - a^2*b^5))))/((a^7 -
 2*a^5*b^2 + a^3*b^4)^2*(a^2 - 2*a*b + b^2) + (a^8*b - 2*a^7*b^2 - a^6*b^3
+ 4*a^5*b^4 - a^4*b^5 - 2*a^3*b^6 + a^2*b^7)*abs(a^7 - 2*a^5*b^2 + a^3*b^4)
) + 2*(2*a^11*b - 2*a^10*b^2 - 8*a^9*b^3 + 13*a^8*b^4 + 12*a^7*b^5 - 24*a^6
*b^6 - 8*a^5*b^7 + 17*a^4*b^8 + 2*a^3*b^9 - 4*a^2*b^10 + 2*a^4*b*abs(a^7 -
2*a^5*b^2 + a^3*b^4) - 2*a^3*b^2*abs(a^7 - 2*a^5*b^2 + a^3*b^4) - 4*a^2*b^3
*abs(a^7 - 2*a^5*b^2 + a^3*b^4) - a*b^4*abs(a^7 - 2*a^5*b^2 + a^3*b^4) + 2*
b^5*abs(a^7 - 2*a^5*b^2 + a^3*b^4))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arc
tan(tan(1/2*d*x + 1/2*c)/sqrt(-(a^6*b - 2*a^4*b^3 + a^2*b^5 - sqrt((a^7 + a
^6*b - 2*a^5*b^2 - 2*a^4*b^3 + a^3*b^4 + a^2*b^5)*(a^7 - a^6*b - 2*a^5*b^2
+ 2*a^4*b^3 + a^3*b^4 - a^2*b^5) + (a^6*b - 2*a^4*b^3 + a^2*b^5)^2))/(a^7 -
 a^6*b - 2*a^5*b^2 + 2*a^4*b^3 + a^3*b^4 - a^2*b^5))))/(a^6*b*abs(a^7 - 2*a
^5*b^2 + a^3*b^4) - 2*a^4*b^3*abs(a^7 - 2*a^5*b^2 + a^3*b^4) + a^2*b^5*abs(
a^7 - 2*a^5*b^2 + a^3*b^4) - (a^7 - 2*a^5*b^2 + a^3*b^4)^2) + tan(1/2*d*x +
 1/2*c)/(a^2 - 2*a*b + b^2) + (5*a^5*tan(1/2*d*x + 1/2*c)^4 - 7*a^4*b*tan(1
/2*d*x + 1/2*c)^4 - 5*a^3*b^2*tan(1/2*d*x + 1/2*c)^4 + 7*a^2*b^3*tan(1/2*d*
x + 1/2*c)^4 + 4*a*b^4*tan(1/2*d*x + 1/2*c)^4 - 8*b^5*tan(1/2*d*x + 1/2*c)^
4 - 4*a^5*tan(1/2*d*x + 1/2*c)^2 - 6*a^4*b*tan(1/2*d*x + 1/2*c)^2 + 12*a^3*
b^2*tan(1/2*d*x + 1/2*c)^2 + 6*a^2*b^3*tan(1/2*d*x + 1/2*c)^2 - 4*a*b^4*tan
(1/2*d*x + 1/2*c)^2 - 8*b^5*tan(1/2*d*x + 1/2*c)^2 - a^5 + a^4*b + a^3*b^2
- a^2*b^3)/((a^6 - 2*a^4*b^2 + a^2*b^4)*(a*tan(1/2*d*x + 1/2*c)^5 - b*tan(1
/2*d*x + 1/2*c)^5 - 2*b*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - b
*tan(1/2*d*x + 1/2*c))))/d
```

Mupad [B]
time = 5.44, size = 2500, normalized size = 10.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a*sin(c + d*x) + b*tan(c + d*x))^2,x)
```

```
[Out] ((a^2 - 2*a*b + b^2)/(a + b) + (2*tan(c/2 + (d*x)/2)^2*(2*a*b^4 + 3*a^4*b +
 2*a^5 + 4*b^5 - 3*a^2*b^3 - 6*a^3*b^2))/(a^2*(a + b)^2) - (tan(c/2 + (d*x)
/2)^4*(4*a*b^4 - 7*a^4*b + 5*a^5 - 8*b^5 + 7*a^2*b^3 - 5*a^3*b^2))/(a^2*(a
+ b)^2))/(d*(tan(c/2 + (d*x)/2)^5*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) - tan
(c/2 + (d*x)/2)^3*(4*a^2*b - 8*a*b^2 + 4*b^3) + tan(c/2 + (d*x)/2)*(2*a*b^2
 + 2*a^2*b - 2*a^3 - 2*b^3))) - tan(c/2 + (d*x)/2)/(2*d*(a - b)^2) + (4*b*a
tan((1920*a^7*b^22*tan(c/2 + (d*x)/2))/(1920*a^7*b^22 - 1920*a^8*b^21 - 166
40*a^9*b^20 + 16640*a^10*b^19 + 62080*a^11*b^18 - 62080*a^12*b^17 - 131072*
a^13*b^16 + 131072*a^14*b^15 + 172672*a^15*b^14 - 172672*a^16*b^13 - 147200
*a^17*b^12 + 147200*a^18*b^11 + 81280*a^19*b^10 - 81280*a^20*b^9 - 28160*a^
21*b^8 + 28160*a^22*b^7 + 5632*a^23*b^6 - 5632*a^24*b^5 - 512*a^25*b^4 + 51
```

2*a^26*b^3) - (1920*a^8*b^21*tan(c/2 + (d*x)/2))/(1920*a^7*b^22 - 1920*a^8*b^21 - 16640*a^9*b^20 + 16640*a^10*b^19 + 62080*a^11*b^18 - 62080*a^12*b^17 - 131072*a^13*b^16 + 131072*a^14*b^15 + 172672*a^15*b^14 - 172672*a^16*b^13 - 147200*a^17*b^12 + 147200*a^18*b^11 + 81280*a^19*b^10 - 81280*a^20*b^9 - 28160*a^21*b^8 + 28160*a^22*b^7 + 5632*a^23*b^6 - 5632*a^24*b^5 - 512*a^25*b^4 + 512*a^26*b^3) - (16640*a^9*b^20*tan(c/2 + (d*x)/2))/(1920*a^7*b^22 - 1920*a^8*b^21 - 16640*a^9*b^20 + 16640*a^10*b^19 + 62080*a^11*b^18 - 62080*a^12*b^17 - 131072*a^13*b^16 + 131072*a^14*b^15 + 172672*a^15*b^14 - 172672*a^16*b^13 - 147200*a^17*b^12 + 147200*a^18*b^11 + 81280*a^19*b^10 - 81280*a^20*b^9 - 28160*a^21*b^8 + 28160*a^22*b^7 + 5632*a^23*b^6 - 5632*a^24*b^5 - 512*a^25*b^4 + 512*a^26*b^3) + (16640*a^10*b^19*tan(c/2 + (d*x)/2))/(1920*a^7*b^22 - 1920*a^8*b^21 - 16640*a^9*b^20 + 16640*a^10*b^19 + 62080*a^11*b^18 - 62080*a^12*b^17 - 131072*a^13*b^16 + 131072*a^14*b^15 + 172672*a^15*b^14 - 172672*a^16*b^13 - 147200*a^17*b^12 + 147200*a^18*b^11 + 81280*a^19*b^10 - 81280*a^20*b^9 - 28160*a^21*b^8 + 28160*a^22*b^7 + 5632*a^23*b^6 - 5632*a^24*b^5 - 512*a^25*b^4 + 512*a^26*b^3) + (62080*a^11*b^18*tan(c/2 + (d*x)/2))/(1920*a^7*b^22 - 1920*a^8*b^21 - 16640*a^9*b^20 + 16640*a^10*b^19 + 62080*a^11*b^18 - 62080*a^12*b^17 - 131072*a^13*b^16 + 131072*a^14*b^15 + 172672*a^15*b^14 - 172672*a^16*b^13 - 147200*a^17*b^12 + 147200*a^18*b^11 + 81280*a^19*b^10 - 81280*a^20*b^9 - 28160*a^21*b^8 + 28160*a^22*b^7 + 5632*a^23*b^6 - 5632*a^24*b^5 - 512*a^25*b^4 + 512*a^26*b^3) - (62080*a^12*b^17*tan(c/2 + (d*x)/2))/(1920*a^7*b^22 - 1920*a^8*b^21 - 16640*a^9*b^20 + 16640*a^10*b^19 + 62080*a^11*b^18 - 62080*a^12*b^17 - 131072*a^13*b^16 + 131072*a^14*b^15 + 172672*a^15*b^14 - 172672*a^16*b^13 - 147200*a^17*b^12 + 147200*a^18*b^11 + 81280*a^19*b^10 - 81280*a^20*b^9 - 28160*a^21*b^8 + 28160*a^22*b^7 + 5632*a^23*b^6 - 5632*a^24*b^5 - 512*a^25*b^4 + 512*a^26*b^3) - (131072*a^13*b^16*tan(c/2 + (d*x)/2))/(1920*a^7*b^22 - 1920*a^8*b^21 - 16640*a^9*b^20 + 16640*a^10*b^19 + 62080*a^11*b^18 - 62080*a^12*b^17 - 131072*a^13*b^16 + 131072*a^14*b^15 + 172672*a^15*b^14 - 172672*a^16*b^13 - 147200*a^17*b^12 + 147200*a^18*b^11 + 81280*a^19*b^10 - 81280*a^20*b^9 - 28160*a^21*b^8 + 28160*a^22*b^7 + 5632*a^23*b^6 - 5632*a^24*b^5 - 512*a^25*b^4 + 512*a^26*b^3) + (131072*a^14*b^15*tan(c/2 + (d*x)/2))/(1920*a^7*b^22 - 1920*a^8*b^21 - 16640*a^9*b^20 + 16640*a^10*b^19 + 62080*a^11*b^18 - 62080*a^12*b^17 - 131072*a^13*b^16 + 131072*a^14*b^15 + 172672*a^15*b^14 - 172672*a^16*b^13 - 147200*a^17*b^12 + 147200*a^18*b^11 + 81280*a^19*b^10 - 81280*a^20*b^9 - 28160*a^21*b^8 + 28160*a^22*b^7 + 5632*a^23*b^6 - 5632*a^24*b^5 - 512*a^25*b^4 + 512*a^26*b^3) + (172672*a^15*b^14*tan(c/2 + (d*x)/2))/(1920*a^7*b^22 - 1920*a^8*b^21 - 16640*a^9*b^20 + 16640*a^10*b^19 + 62080*a^11*b^18 - 62080*a^12*b^17 - 131072*a^13*b^16 + 131072*a^14*b^15 + 172672*a^15*b^14 - 172672*a^16*b^13 - 147200*a^17*b^12 + 147200*a^18*b^11 + 81280*a^19*b^10 - 81280*a^20*b^9 - 28160*a^21*b^8 + 28160*a^22*b^7 + 5632*a^23*b^6 - 5632*a^24*b^5 - 512*a^25*b^4 + 512*a^26*b^3) - (172672*a^16*b^13*tan(c/2 + (d*x)/2))/(1920*a^7*b^22 - 1920*a^8*b^21 - 16640*a^9*b^20 + 16640*a^10*b^19 + 62080*a^11*b^18 - 62080*a^12*b^17 - 131072*a^13*b^16 + 131072*a^14*b^15 + 172672*a^15*b^14 - 172672*a^16*b^13 - 147200*a^17*b^12 + 147200*a^18*b^11 + 81280*a^1

9*b^10 - 81280*a^20*b^9 - 28160*a^21*b^8 + 28160*a^22*b^7 + 5632*a^23*b^6 - 5632*a^24*b^5 - 512*a^25*b^4 + 512*a^26*b^3) - (147200*a^17*b^12*tan(c/2 + (d*x)/2))/(1920*a^7*b^22 - 1920*a^8*b^21 - 16640*a^9*b^20 + 16640*a^10*b^19 + 62080*a^11*b^18 - 62080*a^12*b^17 - 131072*a^13*b^16 + 131072*a^14*b^15 + 172672*a^15*b^14 - 172672*a^16*b^13 - 147200*a^17*b^12 + 147200*a^18*b^11 + 81280*a^19*b^10 - 81280*a^20*b^9 - 28160*a^21*b^8 + 28160*a^22*b^7 + 5632*a^23*b^6 - 5632*a^24*b^5 - 512*a^25*b^4 + 512*a^26*b^3) + (147200*a^18*b^11*tan(c/2 + (d*x)/2))/(1920*a^7*b^22 - 1920*a^8*b^21 - 16640*a^9*b^20 + 16640*a^10*b^19 + 62080*a^11*b^18 - 62080*a^12*b^17 - 131072*a^13*b^16 + 131072*a^14*b^15 + 172672*a^15*b^14 - 172672*a^16*b^13 - 147200*a^17*b^12 + 147200*a^18*b^11 + 81280*a^19*b^10 - 81280*a^20*b...

3.258 $\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=227

$$-\frac{x}{a^2}-\frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d}-\frac{4b^3(2a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d}-\frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(}$$

```
[Out] -x/a^2-2*b^5*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^2/(a-b)^
(5/2)/(a+b)^(5/2)/d-4*b^3*(2*a^2-b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c
)/(a+b)^(1/2))/a^2/(a-b)^(5/2)/(a+b)^(5/2)/d-1/2*sin(d*x+c)/(a+b)^2/d/(1-co
s(d*x+c))+1/2*sin(d*x+c)/(a-b)^2/d/(1+cos(d*x+c))+b^4*sin(d*x+c)/a/(a^2-b^2
)^2/d/(b+a*cos(d*x+c))
```

Rubi [A]

time = 0.37, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4482, 2976, 2727, 2743, 12, 2738, 214}

$$-\frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{5/2}(a+b)^{5/2}}+\frac{b^4\sin(c+dx)}{ad\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{4b^3(2a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{5/2}(a+b)^{5/2}}-\frac{x}{a^2}-\frac{\sin(c+dx)}{2d(a+b)^2(1-\cos(c+dx))}+\frac{\sin(c+dx)}{2d(a-b)^2(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] -(x/a^2) - (2*b^5*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2
*(a - b)^(5/2)*(a + b)^(5/2)*d) - (4*b^3*(2*a^2 - b^2)*ArcTanh[(Sqrt[a - b]
*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^2*(a - b)^(5/2)*(a + b)^(5/2)*d) - Sin[
c + d*x]/(2*(a + b)^2*d*(1 - Cos[c + d*x])) + Sin[c + d*x]/(2*(a - b)^2*d*(
1 + Cos[c + d*x])) + (b^4*Sin[c + d*x])/(a*(a^2 - b^2)^2*d*(b + a*Cos[c + d
*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\cot^2(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= \int \left(-\frac{1}{a^2} - \frac{1}{2(a-b)^2(-1-\cos(c+dx))} + \frac{1}{2(a+b)^2(1-\cos(c+d$$

$$= -\frac{x}{a^2} - \frac{\int \frac{1}{-1-\cos(c+dx)}\,dx}{2(a-b)^2} + \frac{\int \frac{1}{1-\cos(c+dx)}\,dx}{2(a+b)^2} + \frac{b^4\int \frac{1}{(-b-a\cos(c+dx))^2}\,dx}{a^2(a^2-b^2)}$$

$$= -\frac{x}{a^2} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))}$$

$$= -\frac{x}{a^2} - \frac{4b^3(2a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c}{2(a+b)^2d(1}$$

$$= -\frac{x}{a^2} - \frac{4b^3(2a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c}{2(a+b)^2d(1}$$

$$= -\frac{x}{a^2} - \frac{2b^5\tanh^{-1}\left(\frac{\sqrt{a-b}\tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{a^2(a-b)^{5/2}(a+b)^{5/2}d} - \frac{4b^3(2a^2-b^2)\tanh^{-1}\left(\right.}{a^2(a-b)^{5/2}}$$

Mathematica [A]
time = 2.20, size = 151, normalized size = 0.67

$$\frac{-\frac{2(c+dx)}{a^2} - \frac{4b^3(-4a^2+b^2)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{a^2(a^2-b^2)^{5/2}} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2} + \frac{2b^4\sin(c+dx)}{a(a-b)^2(a+b)^2(b+a\cos(c+dx))} + \frac{\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] ((-2*(c + d*x))/a^2 - (4*b^3*(-4*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)
/2])/Sqrt[a^2 - b^2]])/(a^2*(a^2 - b^2)^(5/2)) - Cot[(c + d*x)/2]/(a + b)^2
 + (2*b^4*Sin[c + d*x])/(a*(a - b)^2*(a + b)^2*(b + a*Cos[c + d*x])) + Tan[
(c + d*x)/2]/(a - b)^2)/(2*d)
```

Maple [A]
time = 0.42, size = 184, normalized size = 0.81

method	result

derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-4ab+2b^2}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b^3\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}-\frac{(4a^2-b^2)\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(}{\sqrt{(a+b)(a}}\right.}{\sqrt{(a+b)(a-b)}}\right.}{(a-b)^2(a+b)^2a^2}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-4ab+2b^2}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b^3\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}-\frac{(4a^2-b^2)\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(}{\sqrt{(a+b)(a}}\right.}{\sqrt{(a+b)(a-b)}}\right.}{(a-b)^2(a+b)^2a^2}}{d}$
risch	$-\frac{x}{a^2}-\frac{2i\left(-2b\,a^4e^{3i(dx+c)}-b^5e^{3i(dx+c)}+a^5e^{2i(dx+c)}-3a^3b^2e^{2i(dx+c)}-a\,b^4e^{2i(dx+c)}+2a^2b^3e^{i(dx+c)}+b^5e^{i(dx+c)}+a^5\right.}{\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)(a^2-b^2)^2a^2\left(e^{2i(dx+c)}-1\right)d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-1/2/(a+b)^2/tan(1/2*d*x+1/2*c)+
2*b^3/(a-b)^2/(a+b)^2/a^2*(-a*b*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-
b*tan(1/2*d*x+1/2*c)^2-a-b)-(4*a^2-b^2)/((a+b)*(a-b))^(1/2)*arctanh(tan(1/2
*d*x+1/2*c)*(a-b)/((a+b)*(a-b))^(1/2)))-2/a^2*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 1.73, size = 705, normalized size = 3.11

$$\left[\frac{4a^5b^2-2a^3b^4-2ab^6-\left(4a^2b^4-b^6+(4a^3b^3-ab^5)\cos(dx+c)\right)\sqrt{a^2-b^2}\log\left(\frac{2ab\cos(dx+c)-(a^2-2b^2)\cos(dx+c)^2+2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2a^2-b^2}{a^2\cos(dx+c)^2+2ab\cos(dx+c)+b^2}\right)\sin(dx+c)-2(a^7-ab^6)\cos(dx+c)^2+2(a^6b-2a^4b^3+a^2b^5)\cos(dx+c)-2\left((a^7-3a^5b^2+3a^3b^4-ab^6)dx\cos(dx+c)+(a^6b-3a^4b^3+3a^2b^5-b^7)dx\right)\sin(dx+c)}{2\left((a^9-3a^7b^2+3a^5b^4-a^3b^6)d\cos(dx+c)+(a^8b-3a^6b^3+3a^4b^5-a^2b^7)d\right)\sin(dx+c)},\ \frac{2a^5b^2-a^3b^4-ab^6-\left(4a^2b^4-b^6+(4a^3b^3-ab^5)\cos(dx+c)\right)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)\sin(dx+c)-(a^7-ab^6)\cos(dx+c)^2+(a^6b-2a^4b^3+a^2b^5)\cos(dx+c)-\left((a^7-3a^5b^2+3a^3b^4-ab^6)dx\cos(dx+c)+(a^6b-3a^4b^3+3a^2b^5-b^7)dx\right)\sin(dx+c)}{\left((a^9-3a^7b^2+3a^5b^4-a^3b^6)d\cos(dx+c)+(a^8b-3a^6b^3+3a^4b^5-a^2b^7)d\right)\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(4*a^5*b^2 - 2*a^3*b^4 - 2*a*b^6 - (4*a^2*b^4 - b^6 + (4*a^3*b^3 - a*b
^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*c
os(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2
 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(
a^7 - a*b^6)*cos(d*x + c)^2 + 2*(a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c)
- 2*((a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*x*cos(d*x + c) + (a^6*b - 3*a^
4*b^3 + 3*a^2*b^5 - b^7)*d*x)*sin(d*x + c))/(((a^9 - 3*a^7*b^2 + 3*a^5*b^4
- a^3*b^6)*d*cos(d*x + c) + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d)*si
n(d*x + c)), (2*a^5*b^2 - a^3*b^4 - a*b^6 - (4*a^2*b^4 - b^6 + (4*a^3*b^3 -
 a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x
+ c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - (a^7 - a*b^6)*cos(d*x
+ c)^2 + (a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c) - ((a^7 - 3*a^5*b^2 + 3
*a^3*b^4 - a*b^6)*d*x*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*
d*x)*sin(d*x + c))/(((a^9 - 3*a^7*b^2 + 3*a^5*b^4 - a^3*b^6)*d*cos(d*x + c)
 + (a^8*b - 3*a^6*b^3 + 3*a^4*b^5 - a^2*b^7)*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)**2/(a*sin(c + d*x) + b*tan(c + d*x))**2, x)
```

Giac [A]
time = 0.69, size = 331, normalized size = 1.46

$$\frac{\frac{4\,(4\,a^2b^3-b^5)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^6-2\,a^4b^2+a^2b^4)\sqrt{-a^2+b^2}}+\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2-2\,ab+b^2}-\frac{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+4\,b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^4+a^3b+a^2b^2-ab^3}{(a^5-2\,a^3b^2+ab^4)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}-\frac{2\,(dx+c)}{a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(4*a^2*b^3 - b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) +
 arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2))
)/((a^6 - 2*a^4*b^2 + a^2*b^4)*sqrt(-a^2 + b^2)) + tan(1/2*d*x + 1/2*c)/(a^
2 - 2*a*b + b^2) - (a^4*tan(1/2*d*x + 1/2*c)^2 - 3*a^3*b*tan(1/2*d*x + 1/2*
c)^2 + 3*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - a*b^3*tan(1/2*d*x + 1/2*c)^2 + 4*
b^4*tan(1/2*d*x + 1/2*c)^2 - a^4 + a^3*b + a^2*b^2 - a*b^3)/((a^5 - 2*a^3*b
^2 + a*b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/
2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))) - 2*(d*x + c)/a^2)/d
```

Mupad [B]
time = 5.25, size = 2500, normalized size = 11.01

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a*sin(c + d*x) + b*tan(c + d*x))^2,x)
```

```
[Out] ((a^2 - 2*a*b + b^2)/(a + b) - (tan(c/2 + (d*x)/2)^2*(a^4 - 3*a^3*b - a*b^3
 + 4*b^4 + 3*a^2*b^2))/(a*(a + b)^2))/(d*(tan(c/2 + (d*x)/2)^3*(6*a*b^2 - 6
*a^2*b + 2*a^3 - 2*b^3) + tan(c/2 + (d*x)/2)*(2*a*b^2 + 2*a^2*b - 2*a^3 - 2
*b^3))) - (2*atan(-((tan(c/2 + (d*x)/2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23
+ 128*a^4*b^22 + 672*a^5*b^21 - 1376*a^6*b^20 - 3008*a^7*b^19 + 6528*a^8*b^
18 + 7072*a^9*b^17 - 17632*a^10*b^16 - 8480*a^11*b^15 + 29600*a^12*b^14 + 2
176*a^13*b^13 - 31744*a^14*b^12 + 8224*a^15*b^11 + 21344*a^16*b^10 - 12992*
a^17*b^9 - 8128*a^18*b^8 + 9568*a^19*b^7 + 992*a^20*b^6 - 4000*a^21*b^5 + 4
80*a^22*b^4 + 928*a^23*b^3 - 224*a^24*b^2) - ((32*a^28 - 32*a^27*b + 32*a^6
*b^22 - 416*a^8*b^20 + 224*a^9*b^19 + 2080*a^10*b^18 - 1824*a^11*b^17 - 547
2*a^12*b^16 + 6528*a^13*b^15 + 8256*a^14*b^14 - 13440*a^15*b^13 - 6720*a^16
*b^12 + 17472*a^17*b^11 + 1344*a^18*b^10 - 14784*a^19*b^9 + 2880*a^20*b^8 +
 8064*a^21*b^7 - 3168*a^22*b^6 - 2688*a^23*b^5 + 1504*a^24*b^4 + 480*a^25*b
^3 - 352*a^26*b^2 - (tan(c/2 + (d*x)/2)*(128*a^8*b^22 - 64*a^7*b^23 - 64*a^
29*b + 576*a^9*b^21 - 1280*a^10*b^20 - 2240*a^11*b^19 + 5760*a^12*b^18 + 48
00*a^13*b^17 - 15360*a^14*b^16 - 5760*a^15*b^15 + 26880*a^16*b^14 + 2688*a^
17*b^13 - 32256*a^18*b^12 + 2688*a^19*b^11 + 26880*a^20*b^10 - 5760*a^21*b^
9 - 15360*a^22*b^8 + 4800*a^23*b^7 + 5760*a^24*b^6 - 2240*a^25*b^5 - 1280*a
^26*b^4 + 576*a^27*b^3 + 128*a^28*b^2)*1i)/a^2)*1i)/a^2)/a^2 + (tan(c/2 + (
d*x)/2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23 + 128*a^4*b^22 + 672*a^5*b^21 -
1376*a^6*b^20 - 3008*a^7*b^19 + 6528*a^8*b^18 + 7072*a^9*b^17 - 17632*a^10*
b^16 - 8480*a^11*b^15 + 29600*a^12*b^14 + 2176*a^13*b^13 - 31744*a^14*b^12
+ 8224*a^15*b^11 + 21344*a^16*b^10 - 12992*a^17*b^9 - 8128*a^18*b^8 + 9568*
a^19*b^7 + 992*a^20*b^6 - 4000*a^21*b^5 + 480*a^22*b^4 + 928*a^23*b^3 - 224
*a^24*b^2) + ((32*a^28 - 32*a^27*b + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b
^19 + 2080*a^10*b^18 - 1824*a^11*b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8
256*a^14*b^14 - 13440*a^15*b^13 - 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a
^18*b^10 - 14784*a^19*b^9 + 2880*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 -
 2688*a^23*b^5 + 1504*a^24*b^4 + 480*a^25*b^3 - 352*a^26*b^2 + (tan(c/2 + (
d*x)/2)*(128*a^8*b^22 - 64*a^7*b^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*
b^20 - 2240*a^11*b^19 + 5760*a^12*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 -
 5760*a^15*b^15 + 26880*a^16*b^14 + 2688*a^17*b^13 - 32256*a^18*b^12 + 2688
*a^19*b^11 + 26880*a^20*b^10 - 5760*a^21*b^9 - 15360*a^22*b^8 + 4800*a^23*b
^7 + 5760*a^24*b^6 - 2240*a^25*b^5 - 1280*a^26*b^4 + 576*a^27*b^3 + 128*a^2
8*b^2)*1i)/a^2)*1i)/a^2)/a^2)/(64*a^2*b^22 - 192*a^3*b^21 - 640*a^4*b^20 +
1984*a^5*b^19 + 2624*a^6*b^18 - 8192*a^7*b^17 - 6400*a^8*b^16 + 18496*a^9*b
^15 + 11072*a^10*b^14 - 25856*a^11*b^13 - 14464*a^12*b^12 + 23872*a^13*b^11
 + 13760*a^14*b^10 - 15104*a^15*b^9 - 8704*a^16*b^8 + 6592*a^17*b^7 + 3200*
a^18*b^6 - 1856*a^19*b^5 - 512*a^20*b^4 + 256*a^21*b^3 - ((tan(c/2 + (d*x)/
2)*(32*a^26 - 96*a^25*b - 64*a^3*b^23 + 128*a^4*b^22 + 672*a^5*b^21 - 1376*
a^6*b^20 - 3008*a^7*b^19 + 6528*a^8*b^18 + 7072*a^9*b^17 - 17632*a^10*b^16
```

- 8480*a^11*b^15 + 29600*a^12*b^14 + 2176*a^13*b^13 - 31744*a^14*b^12 + 822
4*a^15*b^11 + 21344*a^16*b^10 - 12992*a^17*b^9 - 8128*a^18*b^8 + 9568*a^19*
b^7 + 992*a^20*b^6 - 4000*a^21*b^5 + 480*a^22*b^4 + 928*a^23*b^3 - 224*a^24
*b^2) - ((32*a^28 - 32*a^27*b + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b^19 +
 2080*a^10*b^18 - 1824*a^11*b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8256*a
^14*b^14 - 13440*a^15*b^13 - 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a^18*b
^10 - 14784*a^19*b^9 + 2880*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 - 2688
*a^23*b^5 + 1504*a^24*b^4 + 480*a^25*b^3 - 352*a^26*b^2 - (tan(c/2 + (d*x)/
2)*(128*a^8*b^22 - 64*a^7*b^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*b^20
- 2240*a^11*b^19 + 5760*a^12*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 - 5760
*a^15*b^15 + 26880*a^16*b^14 + 2688*a^17*b^13 - 32256*a^18*b^12 + 2688*a^19
*b^11 + 26880*a^20*b^10 - 5760*a^21*b^9 - 15360*a^22*b^8 + 4800*a^23*b^7 +
5760*a^24*b^6 - 2240*a^25*b^5 - 1280*a^26*b^4 + 576*a^27*b^3 + 128*a^28*b^2
)*1i)/a^2)*1i)/a^2)*1i)/a^2 + ((tan(c/2 + (d*x)/2)*(32*a^26 - 96*a^25*b - 6
4*a^3*b^23 + 128*a^4*b^22 + 672*a^5*b^21 - 1376*a^6*b^20 - 3008*a^7*b^19 +
6528*a^8*b^18 + 7072*a^9*b^17 - 17632*a^10*b^16 - 8480*a^11*b^15 + 29600*a^
12*b^14 + 2176*a^13*b^13 - 31744*a^14*b^12 + 8224*a^15*b^11 + 21344*a^16*b^
10 - 12992*a^17*b^9 - 8128*a^18*b^8 + 9568*a^19*b^7 + 992*a^20*b^6 - 4000*a
^21*b^5 + 480*a^22*b^4 + 928*a^23*b^3 - 224*a^24*b^2) + ((32*a^28 - 32*a^27
*b + 32*a^6*b^22 - 416*a^8*b^20 + 224*a^9*b^19 + 2080*a^10*b^18 - 1824*a^11
*b^17 - 5472*a^12*b^16 + 6528*a^13*b^15 + 8256*a^14*b^14 - 13440*a^15*b^13
- 6720*a^16*b^12 + 17472*a^17*b^11 + 1344*a^18*b^10 - 14784*a^19*b^9 + 2880
*a^20*b^8 + 8064*a^21*b^7 - 3168*a^22*b^6 - 2688*a^23*b^5 + 1504*a^24*b^4 +
 480*a^25*b^3 - 352*a^26*b^2 + (tan(c/2 + (d*x)/2)*(128*a^8*b^22 - 64*a^7*b
^23 - 64*a^29*b + 576*a^9*b^21 - 1280*a^10*b^20 - 2240*a^11*b^19 + 5760*a^1
2*b^18 + 4800*a^13*b^17 - 15360*a^14*b^16 - 576...

3.259 $\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=219

$$\frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{5/2}(a+b)^{5/2}d}+\frac{2b^2(3a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{5/2}(a+b)^{5/2}d}-\frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+d}$$

```
[Out] 2*b^4*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/(a-b)^(5/2)/(a+
b)^(5/2)/d+2*b^2*(3*a^2-b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(
1/2))/a/(a-b)^(5/2)/(a+b)^(5/2)/d-1/2*sin(d*x+c)/(a+b)^2/d/(1-cos(d*x+c))-1
/2*sin(d*x+c)/(a-b)^2/d/(1+cos(d*x+c))-b^3*sin(d*x+c)/(a^2-b^2)^2/d/(b+a*co
s(d*x+c))
```

Rubi [A]

time = 0.29, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {4482, 2976, 2727, 2738, 214, 2743, 12}

$$\frac{2b^2(3a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad(a-b)^{5/2}(a+b)^{5/2}}-\frac{b^3\sin(c+dx)}{d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}+\frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad(a-b)^{5/2}(a+b)^{5/2}}-\frac{\sin(c+dx)}{2d(a+b)^2(1-\cos(c+dx))}-\frac{\sin(c+dx)}{2d(a-b)^2(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (2*b^4*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*(a - b)^(5/2
)*(a + b)^(5/2)*d) + (2*b^2*(3*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/(a*(a - b)^(5/2)*(a + b)^(5/2)*d) - Sin[c + d*x]/(2*(a
+ b)^2*d*(1 - Cos[c + d*x])) - Sin[c + d*x]/(2*(a - b)^2*d*(1 + Cos[c + d*x
])) - (b^3*Sin[c + d*x])/((a^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \int \frac{\cos(c+dx)\cot^2(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= -\int \left(-\frac{1}{2(a-b)^2(-1-\cos(c+dx))} - \frac{1}{2(a+b)^2(1-\cos(c+dx)}\right.$$

$$= \frac{\int \frac{1}{-1-\cos(c+dx)}\,dx}{2(a-b)^2} + \frac{\int \frac{1}{1-\cos(c+dx)}\,dx}{2(a+b)^2} - \frac{b^3\int \frac{1}{(b+a\cos(c+dx))^2}\,dx}{a\,(a^2-b^2)} - \frac{(b^2(}{}$$

$$= -\frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} - \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))} - \frac{}{(a}$$

$$= \frac{2b^2(3a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx}{2(a+b)^2d(1-\cos}$$

$$= \frac{2b^2(3a^2-b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx}{2(a+b)^2d(1-\cos}$$

$$= \frac{2b^4\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{5/2}(a+b)^{5/2}d} + \frac{2b^2(3a^2-b^2)\tanh^{-1}\left(\sqrt{a-}\right.}{a(a-b)^{5/2}(a+b)}$$

Mathematica [A]
time = 1.41, size = 131, normalized size = 0.60

$$\frac{-\frac{12ab^2\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{(-3b^3-2a(a^2-b^2)\cos(c+dx)+(2a^2b+b^3)\cos(2(c+dx)))\csc(c+dx)}{b+a\cos(c+dx)}}{2(a-b)^2(a+b)^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] ((-12*a*b^2*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2
- b^2] + ((-3*b^3 - 2*a*(a^2 - b^2)*Cos[c + d*x] + (2*a^2*b + b^3)*Cos[2*(c
 + d*x)])*Csc[c + d*x])/(b + a*Cos[c + d*x]))/(2*(a - b)^2*(a + b)^2*d)
```

Maple [A]
time = 0.32, size = 155, normalized size = 0.71

method	result

derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^{2}-2ab+b^{2}\right)}-\frac{1}{2(a+b)^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b^{2}\left(-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}-\frac{3a\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a+b)(a-b}\cdots}\right)}{\sqrt{(a+b)(a-b)}}\cdots\right)}{(a-b)^{2}(a+b)^{2}}}{d}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^{2}-2ab+b^{2}\right)}-\frac{1}{2(a+b)^{2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b^{2}\left(-\frac{b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}-\frac{3a\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a+b)(a-b}\cdots}\right)}{\sqrt{(a+b)(a-b)}}\cdots\right)}{(a-b)^{2}(a+b)^{2}}}{d}$
risch	$-\frac{2i\left(a^{4}e^{3i(dx+c)}+a^{2}b^{2}e^{3i(dx+c)}+b^{4}e^{3i(dx+c)}+3a\,b^{3}e^{2i(dx+c)}+a^{4}e^{i(dx+c)}-3a^{2}b^{2}e^{i(dx+c)}-b^{4}e^{i(dx+c)}-2a^{3}b-a\,b^{3}\right)}{\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)\left(a^{2}-b^{2}\right)^{2}a\left(e^{2i(dx+c)}-1\right)d}+\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-1/2/(a+b)^2/tan(1/2*d*x+1/2*c)
-2*b^2/(a-b)^2/(a+b)^2*(-b*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-b*tan
(1/2*d*x+1/2*c)^2-a-b)-3*a/((a+b)*(a-b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(
a-b)/((a+b)*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.59, size = 518, normalized size = 2.37

$$\left[-\frac{2a^{4}b+2a^{2}b^{3}-4b^{5}-3\left(a^{2}b^{2}\cos(dx+c)+ab^{3}\right)\sqrt{a^{2}-b^{2}}\log\left(\frac{2ab\cos(dx+c)-(a^{2}-2b^{2})\cos(dx+c)^{2}+2\sqrt{a^{2}-b^{2}}(b\cos(dx+c)+a)\sin(dx+c)+2a^{2}-b^{2}}{a^{2}\cos(dx+c)^{2}+2ab\cos(dx+c)+b^{2}}\right)\sin(dx+c)-2\left(2a^{4}b-a^{2}b^{3}-b^{5}\right)\cos(dx+c)^{2}+2\left(a^{5}-2a^{3}b^{2}+ab^{4}\right)\cos(dx+c)}{2\left(\left(a^{7}-3a^{5}b^{2}+3a^{3}b^{4}-ab^{6}\right)d\cos(dx+c)+\left(a^{6}b-3a^{4}b^{3}+3a^{2}b^{5}-b^{7}\right)d\right)\sin(dx+c)},\ -\frac{a^{4}b+a^{2}b^{3}-2b^{5}-3\left(a^{2}b^{2}\cos(dx+c)+ab^{3}\right)\sqrt{-a^{2}+b^{2}}\arctan\left(-\frac{\sqrt{-a^{2}+b^{2}}(b\cos(dx+c)+a)}{(a^{2}-b^{2})\sin(dx+c)}\right)\sin(dx+c)-\left(2a^{4}b-a^{2}b^{3}-b^{5}\right)\cos(dx+c)^{2}+\left(a^{5}-2a^{3}b^{2}+ab^{4}\right)\cos(dx+c)}{\left(\left(a^{7}-3a^{5}b^{2}+3a^{3}b^{4}-ab^{6}\right)d\cos(dx+c)+\left(a^{6}b-3a^{4}b^{3}+3a^{2}b^{5}-b^{7}\right)d\right)\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a^4*b + 2*a^2*b^3 - 4*b^5 - 3*(a^2*b^2*cos(d*x + c) + a*b^3)*sqrt(
a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(
a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x +
c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(2*a^4*b - a^2*b^3 - b^5
)*cos(d*x + c)^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))/(((a^7 - 3*a^5
*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 -
 b^7)*d)*sin(d*x + c)), -(a^4*b + a^2*b^3 - 2*b^5 - 3*(a^2*b^2*cos(d*x + c)
 + a*b^3)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((
a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - (2*a^4*b - a^2*b^3 - b^5)*cos(d*x
+ c)^2 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))/(((a^7 - 3*a^5*b^2 + 3*a^3
*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d)*sin
(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(cos(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x))**2, x)
```

Giac [A]
time = 0.69, size = 282, normalized size = 1.29

$$-\frac{\frac{12\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)ab^2}{(a^4-2a^2b^2+b^4)\sqrt{-a^2+b^2}}+\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2-2ab+b^2}+\frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-5b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a^3+a^2b+ab^2-b^3}{(a^4-2a^2b^2+b^4)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1
/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a*b^2/((a^4 -
2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + tan(1/2*d*x + 1/2*c)/(a^2 - 2*a*b + b^
2) + (a^3*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^2 + 3*a*b^2
*tan(1/2*d*x + 1/2*c)^2 - 5*b^3*tan(1/2*d*x + 1/2*c)^2 - a^3 + a^2*b + a*b^
2 - b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x
 + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))))/d
```

Mupad [B]
time = 1.13, size = 213, normalized size = 0.97

$$\frac{\frac{a^2-2ab+b^2}{a+b}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2(a^3-3a^2b+3ab^2-5b^3)}{(a+b)^2}}{d\left((2a^3-6a^2b+6ab^2-2b^3)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+(-2a^3+2a^2b+2ab^2-2b^3)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d(a-b)^2}+\frac{6ab^2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(a^4-2a^2b^2+b^4)}{(a+b)^{5/2}(a-b)^{3/2}}\right)}{d(a+b)^{5/2}(a-b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x))^2,x)
```

```
[Out] ((a^2 - 2*a*b + b^2)/(a + b) - (tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b + a
^3 - 5*b^3))/(a + b)^2)/(d*(tan(c/2 + (d*x)/2)^3*(6*a*b^2 - 6*a^2*b + 2*a^3
 - 2*b^3) + tan(c/2 + (d*x)/2)*(2*a*b^2 + 2*a^2*b - 2*a^3 - 2*b^3))) - tan(
c/2 + (d*x)/2)/(2*d*(a - b)^2) + (6*a*b^2*atanh((tan(c/2 + (d*x)/2)*(a^4 +
b^4 - 2*a^2*b^2))/((a + b)^(5/2)*(a - b)^(3/2))))/(d*(a + b)^(5/2)*(a - b)^
(5/2))
```

3.260 $\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=203

$$-\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d}-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d}-\frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))}+\frac{}{2(}$$

```
[Out] -4*a^2*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/(a
+b)^(5/2)/d-2*b^3*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)
^(5/2)/(a+b)^(5/2)/d-1/2*sin(d*x+c)/(a+b)^2/d/(1-cos(d*x+c))+1/2*sin(d*x+c)
/(a-b)^2/d/(1+cos(d*x+c))+a*b^2*sin(d*x+c)/(a^2-b^2)^2/d/(b+a*cos(d*x+c))
```

Rubi [A]
time = 0.32, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {4482, 2810, 2727, 2743, 12, 2738, 214}

$$\frac{ab^2\sin(c+dx)}{d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\sin(c+dx)}{2d(a+b)^2(1-\cos(c+dx))}+\frac{\sin(c+dx)}{2d(a-b)^2(\cos(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[c + d*x] + b*Tan[c + d*x])^(-2),x]
```

```
[Out] (-4*a^2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/
2)*(a + b)^(5/2)*d) - (2*b^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
+ b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - Sin[c + d*x]/(2*(a + b)^2*d*(1 - C
os[c + d*x])) + Sin[c + d*x]/(2*(a - b)^2*d*(1 + Cos[c + d*x])) + (a*b^2*Si
n[c + d*x])/((a^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2810

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[Sin[e + f*x]^p*((a + b*Sin[e + f*x])^m/
(1 - Sin[e + f*x]^2)^(p/2)), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= \int \left(-\frac{1}{2(a+b)^2(-1+\cos(c+dx))} + \frac{1}{2(a-b)^2(1+\cos(c+dx))}\right.$$

$$= \frac{\int \frac{1}{1+\cos(c+dx)}\,dx}{2(a-b)^2} - \frac{\int \frac{1}{-1+\cos(c+dx)}\,dx}{2(a+b)^2} - \frac{(2a^2b)\int \frac{1}{b+a\cos(c+dx)}\,dx}{(a^2-b^2)^2} + \frac{b^2}{}$$

$$= -\frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))} + \frac{}{(a}$$

$$= -\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx}$$

$$= -\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx}$$

$$= -\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}\right.}{\sqrt{a+b}}\right.}{(a-b)^{5/2}(a+b)^{5/2}}$$

Mathematica [**A**]
time = 1.33, size = 128, normalized size = 0.63

$$\frac{\frac{4b(2a^2+b^2)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2} + \frac{\frac{2ab^2\sin(c+dx)}{(a+b)^2(b+a\cos(c+dx))}+\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[c + d*x] + b*Tan[c + d*x])^(-2),x]
```

```
[Out] ((4*b*(2*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^2 - b^2)^(5/2) - Cot[(c + d*x)/2]/(a + b)^2 + ((2*a*b^2*Sin[c + d*x])/((a
 + b)^2*(b + a*Cos[c + d*x])) + Tan[(c + d*x)/2])/(a - b)^2)/(2*d)
```

Maple [**A**]
time = 0.38, size = 162, normalized size = 0.80

method	result

derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-4ab+2b^2}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}-\frac{\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b}{\sqrt{(a+b)(a-}}\right.}{\sqrt{(a+b)(a-b)}}\right.}{(a-b)^2(a+b)^2}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-4ab+2b^2}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}-\frac{\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b}{\sqrt{(a+b)(a-}}\right.}{\sqrt{(a+b)(a-b)}}\right.}{(a-b)^2(a+b)^2}}{d}$
risch	$-\frac{2i\left(-2a^2b\,e^{3i(dx+c)}-b^3e^{3i(dx+c)}+a^3e^{2i(dx+c)}-4a\,b^2e^{2i(dx+c)}+3b^3e^{i(dx+c)}+a^3+2a\,b^2\right)}{\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)\left(a^2-b^2\right)^2\left(e^{2i(dx+c)}-1\right)d}+\frac{2b\ln\left(e^{i(dx+c)}+\frac{-ia^2+ib^2}{a\sqrt{}}\right.}{\sqrt{a^2-b^2}\,(a+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(d*x+c)+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-1/2/(a+b)^2/tan(1/2*d*x+1/2*c)+
2*b/(a-b)^2/(a+b)^2*(-a*b*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-b*tan(
1/2*d*x+1/2*c)^2-a-b)-(2*a^2+b^2)/((a+b)*(a-b))^(1/2)*arctanh(tan(1/2*d*x+1
/2*c)*(a-b)/((a+b)*(a-b))^(1/2))))
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]

time = 2.37, size = 526, normalized size = 2.59

$$\left[\frac{6\,a^3b^2-6\,ab^4+\left(2\,a^2b^2+b^4+\left(2\,a^3b+ab^3\right)\cos\left(dx+c\right)\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)\sin\left(dx+c\right)-2\left(a^5+a^3b^2-2\,ab^4\right)\cos\left(dx+c\right)^2+2\left(a^4b-2\,a^2b^3+b^5\right)\cos\left(dx+c\right)}{2\left(\left(a^7-3\,a^5b^2+3\,a^3b^4-ab^6\right)d\cos\left(dx+c\right)+\left(a^6b-3\,a^4b^3+3\,a^2b^5-b^7\right)d\right)\sin\left(dx+c\right)},\frac{3\,a^3b^2-3\,ab^4-\left(2\,a^2b^2+b^4+\left(2\,a^3b+ab^3\right)\cos\left(dx+c\right)\right)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)\sin\left(dx+c\right)-\left(a^5+a^3b^2-2\,ab^4\right)\cos\left(dx+c\right)^2+\left(a^4b-2\,a^2b^3+b^5\right)\cos\left(dx+c\right)}{\left(\left(a^7-3\,a^5b^2+3\,a^3b^4-ab^6\right)d\cos\left(dx+c\right)+\left(a^6b-3\,a^4b^3+3\,a^2b^5-b^7\right)d\right)\sin\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(6*a^3*b^2 - 6*a*b^4 + (2*a^2*b^2 + b^4 + (2*a^3*b + a*b^3)*cos(d*x +
c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2
- 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*c
os(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(a^5 + a^3*b^2
- 2*a*b^4)*cos(d*x + c)^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c))/(((a^
7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*
a^2*b^5 - b^7)*d)*sin(d*x + c)), (3*a^3*b^2 - 3*a*b^4 - (2*a^2*b^2 + b^4 +
(2*a^3*b + a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(
b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - (a^5 + a^3*b
^2 - 2*a*b^4)*cos(d*x + c)^2 + (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c))/(((a
^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3
*a^2*b^5 - b^7)*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**(-2), x)
```

Giac [A]
time = 0.51, size = 289, normalized size = 1.42

$$\frac{\frac{4\,(2\,a^2b+b^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{-a^2+b^2}}+\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2-2\,ab+b^2}-\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+7\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^3+a^2b+ab^2-b^3}{(a^4-2\,a^2b^2+b^4)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(2*a^2*b + b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + a
rctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/
((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + tan(1/2*d*x + 1/2*c)/(a^2 - 2*
a*b + b^2) - (a^3*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^2 +
 7*a*b^2*tan(1/2*d*x + 1/2*c)^2 - b^3*tan(1/2*d*x + 1/2*c)^2 - a^3 + a^2*b
+ a*b^2 - b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1
/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))))/d
```

Mupad [B]
time = 1.19, size = 245, normalized size = 1.21

$$\frac{\frac{a^2-2\,a\,b+b^2}{a+b}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,(a^3-3\,a^2\,b+7\,a\,b^2-b^3)}{{(a+b)}^2}}{d\,\left(\left(2\,a^3-6\,a^2\,b+6\,a\,b^2-2\,b^3\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\left(-2\,a^3+2\,a^2\,b+2\,a\,b^2-2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d\,{(a-b)}^2}+\frac{b\,\mathrm{atan}\left(\frac{1\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^4-2\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^2+1\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^4}{{(a+b)}^{5/2}\,{(a-b)}^{3/2}}\right)\,\left(2\,a^2+b^2\right)\,2\mathrm{i}}{d\,{(a+b)}^{5/2}\,{(a-b)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(c + d*x) + b*tan(c + d*x))^2,x)
```

```
[Out] ((a^2 - 2*a*b + b^2)/(a + b) - (tan(c/2 + (d*x)/2)^2*(7*a*b^2 - 3*a^2*b + a
^3 - b^3))/(a + b)^2)/(d*(tan(c/2 + (d*x)/2)^3*(6*a*b^2 - 6*a^2*b + 2*a^3 -
 2*b^3) + tan(c/2 + (d*x)/2)*(2*a*b^2 + 2*a^2*b - 2*a^3 - 2*b^3))) + tan(c/
2 + (d*x)/2)/(2*d*(a - b)^2) + (b*atan((a^4*tan(c/2 + (d*x)/2)*1i + b^4*tan
(c/2 + (d*x)/2)*1i - a^2*b^2*tan(c/2 + (d*x)/2)*2i)/((a + b)^(5/2)*(a - b)^
(3/2)))*(2*a^2 + b^2)*2i)/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

3.261 $\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=136

$$\frac{2a(a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d}-\frac{b\csc(c+dx)}{(a^2-b^2)\,d(b+a\cos(c+dx))}-\frac{(a^2+2b^2-3ab\cos(c+dx))\,c}{(a^2-b^2)^2\,d}$$

```
[Out] 2*a*(a^2+2*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(
5/2)/(a+b)^(5/2)/d-b*csc(d*x+c)/(a^2-b^2)/d/(b+a*cos(d*x+c))-(a^2+2*b^2-3*a
*b*cos(d*x+c))*csc(d*x+c)/(a^2-b^2)^2/d
```

Rubi [A]
time = 0.22, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {4482, 2943, 2945, 12, 2738, 214}

$$-\frac{\csc(c+dx)\left(a^2-3ab\cos(c+dx)+2b^2\right)}{d\left(a^2-b^2\right)^2}-\frac{b\csc(c+dx)}{d\left(a^2-b^2\right)\left(a\cos(c+dx)+b\right)}+\frac{2a(a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (2*a*(a^2 + 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a
 - b)^(5/2)*(a + b)^(5/2)*d) - (b*Csc[c + d*x])/((a^2 - b^2)*d*(b + a*Cos[c
 + d*x])) - ((a^2 + 2*b^2 - 3*a*b*Cos[c + d*x])*Csc[c + d*x])/((a^2 - b^2)^
2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2943

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c
 - a*d))*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x])^(m + 1)/(f*g*(a^2 -
 b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1)), Int[(g*Cos[e + f*x])^p*
(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m + p
+ 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[
a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx &= \int \frac{\cot(c+dx)\csc(c+dx)}{(b+a\cos(c+dx))^2}\,dx \\
&= -\frac{b\csc(c+dx)}{(a^2-b^2)\,d(b+a\cos(c+dx))} - \frac{\int \frac{(-a+2b\cos(c+dx))\csc^2(c+dx)}{b+a\cos(c+dx)}\,dx}{a^2-b^2} \\
&= -\frac{b\csc(c+dx)}{(a^2-b^2)\,d(b+a\cos(c+dx))} - \frac{(a^2+2b^2-3ab\cos(c+dx))\csc(}{(a^2-b^2)^2\,d} \\
&= -\frac{b\csc(c+dx)}{(a^2-b^2)\,d(b+a\cos(c+dx))} - \frac{(a^2+2b^2-3ab\cos(c+dx))\csc(}{(a^2-b^2)^2\,d} \\
&= -\frac{b\csc(c+dx)}{(a^2-b^2)\,d(b+a\cos(c+dx))} - \frac{(a^2+2b^2-3ab\cos(c+dx))\csc(}{(a^2-b^2)^2\,d} \\
&= \frac{2a(a^2+2b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{b\csc(c+dx}{(a^2-b^2)\,d(b+a\cos(}
\end{aligned}$$

Mathematica [A]
time = 1.22, size = 127, normalized size = 0.93

$$-\frac{\frac{4a(a^2+2b^2)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}+\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2}+\frac{\frac{2a^2b\sin(c+dx)}{(a+b)^2(b+a\cos(c+dx))}+\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/2*((4*a*(a^2 + 2*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2
]])/(a^2 - b^2)^(5/2) + Cot[(c + d*x)/2]/(a + b)^2 + ((2*a^2*b*Sin[c + d*x]
)/((a + b)^2*(b + a*Cos[c + d*x])) + Tan[(c + d*x)/2])/(a - b)^2)/d
```

Maple [A]
time = 0.38, size = 162, normalized size = 1.19

method	result
derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^2-2ab+b^2\right)}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2a\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}-\frac{\left(a^2+2b^2\right)\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{\sqrt{(a+b)}}\right.}{\sqrt{(a+b)\,(a-b}}\right.}{(a-b)^2(a+b)^2}}{d}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^2-2ab+b^2\right)}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2a\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}-\frac{\left(a^2+2b^2\right)\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}{\sqrt{(a+b)}}\right.}{\sqrt{(a+b)\,(a-b}}\right.}{(a-b)^2(a+b)^2}}{d}$
risch	$-\frac{2i\left(a^3e^{3i(dx+c)}+2a\,b^2e^{3i(dx+c)}+a^2b\,e^{2i(dx+c)}+2b^3e^{2i(dx+c)}+a^3e^{i(dx+c)}-4a\,b^2e^{i(dx+c)}-3a^2b\right)}{\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)\left(-a^2+b^2\right)^2\left(e^{2i(dx+c)}-1\right)d}+\frac{a^3\ln\left(e^{i(dx+c)}+\right.}{\sqrt{a^2-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-1/2/(a+b)^2/tan(1/2*d*x+1/2*c)
-2*a/(a-b)^2/(a+b)^2*(-a*b*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-b*tan
(1/2*d*x+1/2*c)^2-a-b)-(a^2+2*b^2)/((a+b)*(a-b))^(1/2)*arctanh(tan(1/2*d*x+
1/2*c)*(a-b)/((a+b)*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 1.89, size = 532, normalized size = 3.91

$$\left[-\frac{4\,a^4b-2\,a^2b^3-2\,b^5-(a^3b+2\,ab^3+(a^4+2\,a^2b^2)\cos(dx+c))\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)\sin(dx+c)-6\,(a^4b-a^2b^3)\cos(dx+c)^2+2\,(a^5-2\,a^3b^2+ab^4)\cos(dx+c)}{2\,((a^7-3\,a^5b^2+3\,a^3b^4-ab^6)d\cos(dx+c)+(a^6b-3\,a^4b^3+3\,a^2b^5-b^7)d)\sin(dx+c)},-\frac{2\,a^4b-a^2b^3-b^5-(a^3b+2\,ab^3+(a^4+2\,a^2b^2)\cos(dx+c))\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)\sin(dx+c)-3\,(a^4b-a^2b^3)\cos(dx+c)^2+(a^5-2\,a^3b^2+ab^4)\cos(dx+c)}{((a^7-3\,a^5b^2+3\,a^3b^4-ab^6)d\cos(dx+c)+(a^6b-3\,a^4b^3+3\,a^2b^5-b^7)d)\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(4*a^4*b - 2*a^2*b^3 - 2*b^5 - (a^3*b + 2*a*b^3 + (a^4 + 2*a^2*b^2)*c
os(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*
x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^
2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 6*(a^4*b
 - a^2*b^3)*cos(d*x + c)^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))/(((a
^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3
*a^2*b^5 - b^7)*d)*sin(d*x + c)), -(2*a^4*b - a^2*b^3 - b^5 - (a^3*b + 2*a*
b^3 + (a^4 + 2*a^2*b^2)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 +
b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - 3*(a^4
*b - a^2*b^3)*cos(d*x + c)^2 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c))/(((a
^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3
*a^2*b^5 - b^7)*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(128) = 256.
time = 0.62, size = 288, normalized size = 2.12

$$-\frac{\frac{4\,(a^3+2\,ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{-a^2+b^2}}+\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2-2\,ab+b^2}+\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-7\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^3+a^2b+ab^2-b^3}{(a^4-2\,a^2b^2+b^4)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*(a^3 + 2*a*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) +
arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))
/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + tan(1/2*d*x + 1/2*c)/(a^2 - 2
*a*b + b^2) + (a^3*tan(1/2*d*x + 1/2*c)^2 - 7*a^2*b*tan(1/2*d*x + 1/2*c)^2
+ 3*a*b^2*tan(1/2*d*x + 1/2*c)^2 - b^3*tan(1/2*d*x + 1/2*c)^2 - a^3 + a^2*b
 + a*b^2 - b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(
1/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))))/d
```

Mupad [B]
time = 1.21, size = 245, normalized size = 1.80

$$\frac{\frac{a^2-2\,a\,b+b^2}{a+b}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(a^3-7\,a^2\,b+3\,a\,b^2-b^3\right)}{{\left(a+b\right)}^2}}{d\,\left(\left(2\,a^3-6\,a^2\,b+6\,a\,b^2-2\,b^3\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\left(-2\,a^3+2\,a^2\,b+2\,a\,b^2-2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d\,{\left(a-b\right)}^2}-\frac{a\,\mathrm{atan}\left(\frac{1\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^4-2\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^2+1\mathrm{i}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,b^4}{{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{3/2}}\right)\,\left(a^2+2\,b^2\right)\,2\mathrm{i}}{d\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a*sin(c + d*x) + b*tan(c + d*x))^2),x)
```

```
[Out] ((a^2 - 2*a*b + b^2)/(a + b) - (tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 7*a^2*b + a
^3 - b^3))/(a + b)^2)/(d*(tan(c/2 + (d*x)/2)^3*(6*a*b^2 - 6*a^2*b + 2*a^3 -
 2*b^3) + tan(c/2 + (d*x)/2)*(2*a*b^2 + 2*a^2*b - 2*a^3 - 2*b^3))) - tan(c/
2 + (d*x)/2)/(2*d*(a - b)^2) - (a*atan((a^4*tan(c/2 + (d*x)/2)*1i + b^4*tan
(c/2 + (d*x)/2)*1i - a^2*b^2*tan(c/2 + (d*x)/2)*2i)/((a + b)^(5/2)*(a - b)^
(3/2)))*(a^2 + 2*b^2)*2i)/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

3.262 $\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=131

$$-\frac{6a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d}+\frac{a\csc(c+dx)}{(a^2-b^2)\,d(b+a\cos(c+dx))}+\frac{(3ab-(2a^2+b^2)\cos(c+dx))\csc(c+}{(a^2-b^2)^2\,d}$$

```
[Out] -6*a^2*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/(a
+b)^(5/2)/d+a*csc(d*x+c)/(a^2-b^2)/d/(b+a*cos(d*x+c))+(3*a*b-(2*a^2+b^2)*co
s(d*x+c))*csc(d*x+c)/(a^2-b^2)^2/d
```

Rubi [A]
time = 0.24, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {4482, 2773, 2945, 12, 2738, 214}

$$\frac{a\csc(c+dx)}{d(a^2-b^2)(a\cos(c+dx)+b)}+\frac{\csc(c+dx)(3ab-(2a^2+b^2)\cos(c+dx))}{d(a^2-b^2)^2}-\frac{6a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] (-6*a^2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/
2)*(a + b)^(5/2)*d) + (a*Csc[c + d*x])/((a^2 - b^2)*d*(b + a*Cos[c + d*x]))
 + ((3*a*b - (2*a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x])/((a^2 - b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2773

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(-b)*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e +
f*x])^(m + 1)/(f*g*(a^2 - b^2)*(m + 1))), x] + Dist[1/((a^2 - b^2)*(m + 1))
, Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p
 + 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^
2, 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2945

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \int \frac{\csc^2(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= \frac{a\csc(c+dx)}{(a^2-b^2)\,d(b+a\cos(c+dx))} + \frac{\int \frac{(-b+2a\cos(c+dx))\csc^2(c+dx)}{b+a\cos(c+dx)}\,dx}{a^2-b^2}$$

$$= \frac{a\csc(c+dx)}{(a^2-b^2)\,d(b+a\cos(c+dx))} + \frac{(3ab-(2a^2+b^2)\cos(c+dx))\,\mathrm{csc}}{(a^2-b^2)^2\,d}$$

$$= \frac{a\csc(c+dx)}{(a^2-b^2)\,d(b+a\cos(c+dx))} + \frac{(3ab-(2a^2+b^2)\cos(c+dx))\,\mathrm{csc}}{(a^2-b^2)^2\,d}$$

$$= \frac{a\csc(c+dx)}{(a^2-b^2)\,d(b+a\cos(c+dx))} + \frac{(3ab-(2a^2+b^2)\cos(c+dx))\,\mathrm{csc}}{(a^2-b^2)^2\,d}$$

$$= -\frac{6a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{a\csc(c+dx)}{(a^2-b^2)\,d(b+a\cos(c+a}$$

Mathematica [A]

time = 0.84, size = 121, normalized size = 0.92

$$\frac{\frac{12a^2b\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}-\frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2}+\frac{\frac{2a^3\sin(c+dx)}{(a+b)^2(b+a\cos(c+dx))}+\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] ((12*a^2*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2
)^(5/2) - Cot[(c + d*x)/2]/(a + b)^2 + ((2*a^3*Sin[c + d*x])/((a + b)^2*(b
+ a*Cos[c + d*x])) + Tan[(c + d*x)/2])/(a - b)^2)/(2*d)
```

Maple [A]
time = 0.38, size = 155, normalized size = 1.18

method	result
derivativedivides	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-4ab+2b^2}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2a^2\left(-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}-\frac{3b\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)^2(a+b)^2}}{d}$
default	$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-4ab+2b^2}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2a^2\left(-\frac{a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b}-\frac{3b\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)^2(a+b)^2}}{d}$
risch	$-\frac{2i\left(-3a^2b\,e^{3i(dx+c)}-3a\,b^2e^{2i(dx+c)}+a^2b\,e^{i(dx+c)}+2b^3e^{i(dx+c)}+2a^3+a\,b^2\right)}{\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)(a^2-b^2)^2\left(e^{2i(dx+c)}-1\right)d}+\frac{3b\ln\left(e^{i(dx+c)}+\frac{-ia^2+ib^2+\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right.}{\sqrt{a^2-b^2}\,(a+b)^2(a-b)^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-1/2/(a+b)^2/tan(1/2*d*x+1/2*c)+
2*a^2/(a-b)^2/(a+b)^2*(-a*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-b*tan(
1/2*d*x+1/2*c)^2-a-b)-3*b/((a+b)*(a-b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a
-b)/((a+b)*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [**A**]
time = 1.42, size = 516, normalized size = 3.94

$$\left[\frac{2\,a^{5}+2\,a^{3}b^{2}-4\,ab^{4}+3\,(a^{3}b\cos\left(dx+c\right)+a^{2}b^{2})\sqrt{a^{2}-b^{2}}\log\left(\frac{2\,ab\cos(dx+c)-(a^{2}-2\,b^{2})\cos(dx+c)^{2}-2\,\sqrt{a^{2}-b^{2}}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^{2}-b^{2}}{a^{2}\cos(dx+c)^{2}+2\,ab\cos(dx+c)+b^{2}}\right)\sin\left(dx+c\right)-2\,(2\,a^{5}-a^{3}b^{2}-ab^{4})\cos\left(dx+c\right)^{2}+2\,(a^{4}b-2\,a^{2}b^{3}+b^{5})\cos\left(dx+c\right)}{2\,((a^{7}-3\,a^{5}b^{2}+3\,a^{3}b^{4}-ab^{6})d\cos\left(dx+c\right)+(a^{6}b-3\,a^{4}b^{3}+3\,a^{2}b^{5}-b^{7})d)\sin\left(dx+c\right)},\ \frac{a^{5}+a^{3}b^{2}-2\,ab^{4}-3\,(a^{3}b\cos\left(dx+c\right)+a^{2}b^{2})\sqrt{-a^{2}+b^{2}}\arctan\left(-\frac{\sqrt{-a^{2}+b^{2}}(b\cos(dx+c)+a)}{(a^{2}-b^{2})\sin(dx+c)}\right)\sin\left(dx+c\right)-(2\,a^{5}-a^{3}b^{2}-ab^{4})\cos\left(dx+c\right)^{2}+(a^{4}b-2\,a^{2}b^{3}+b^{5})\cos\left(dx+c\right)}{((a^{7}-3\,a^{5}b^{2}+3\,a^{3}b^{4}-ab^{6})d\cos\left(dx+c\right)+(a^{6}b-3\,a^{4}b^{3}+3\,a^{2}b^{5}-b^{7})d)\sin\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a^5 + 2*a^3*b^2 - 4*a*b^4 + 3*(a^3*b*cos(d*x + c) + a^2*b^2)*sqrt(a
^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a
^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c
)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(2*a^5 - a^3*b^2 - a*b^4)
*cos(d*x + c)^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c))/(((a^7 - 3*a^5*
b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 -
b^7)*d)*sin(d*x + c)), (a^5 + a^3*b^2 - 2*a*b^4 - 3*(a^3*b*cos(d*x + c) + a
^2*b^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^
2 - b^2)*sin(d*x + c)))*sin(d*x + c) - (2*a^5 - a^3*b^2 - a*b^4)*cos(d*x +
c)^2 + (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c))/(((a^7 - 3*a^5*b^2 + 3*a^3*b
^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d)*sin(d
*x + c))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{2}\left(c+dx\right)}{\left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*sin(c + d*x) + b*tan(c + d*x))**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. 2(122) = 244.
time = 0.60, size = 284, normalized size = 2.17

$$\frac{\frac{12\,\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^{2}+b^{2}}}\right)\right)a^{2}b}{(a^{4}-2\,a^{2}b^{2}+b^{4})\sqrt{-a^{2}+b^{2}}}+\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{2}-2\,ab+b^{2}}-\frac{5\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-3\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+3\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a^{3}+a^{2}b+ab^{2}-b^{3}}{(a^{4}-2\,a^{2}b^{2}+b^{4})\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(12*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/
2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^2*b/((a^4 - 2
*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + tan(1/2*d*x + 1/2*c)/(a^2 - 2*a*b + b^2
) - (5*a^3*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^2 + 3*a*b^
2*tan(1/2*d*x + 1/2*c)^2 - b^3*tan(1/2*d*x + 1/2*c)^2 - a^3 + a^2*b + a*b^2
 - b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x
+ 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))))/d
```

Mupad [B]
time = 1.08, size = 215, normalized size = 1.64

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d\,{\left(a-b\right)}^2}+\frac{\frac{a^2-2\,a\,b+b^2}{a+b}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(5\,a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{{\left(a+b\right)}^2}}{d\,\left(\left(2\,a^3-6\,a^2\,b+6\,a\,b^2-2\,b^3\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\left(-2\,a^3+2\,a^2\,b+2\,a\,b^2-2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}-\frac{6\,a^2\,b\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}{{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{3/2}}\right)}{d\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a*sin(c + d*x) + b*tan(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*d*(a - b)^2) + ((a^2 - 2*a*b + b^2)/(a + b) - (tan(c/
2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b + 5*a^3 - b^3))/(a + b)^2)/(d*(tan(c/2 +
(d*x)/2)^3*(6*a*b^2 - 6*a^2*b + 2*a^3 - 2*b^3) + tan(c/2 + (d*x)/2)*(2*a*b^
2 + 2*a^2*b - 2*a^3 - 2*b^3))) - (6*a^2*b*atanh((tan(c/2 + (d*x)/2)*(a^4 +
b^4 - 2*a^2*b^2))/((a + b)^(5/2)*(a - b)^(3/2))))/(d*(a + b)^(5/2)*(a - b)^
(5/2))
```

3.263 $\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$

Optimal. Leaf size=231

$$\frac{\tanh^{-1}(\sin(c+dx))}{b^2d}+\frac{2a^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d}-\frac{2a^3(a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^2(a+b)^{5/2}d}-\frac{}{2(}$$

```
[Out] arctanh(sin(d*x+c))/b^2/d+2*a^3*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b
)^(1/2))/(a-b)^(5/2)/(a+b)^(5/2)/d-2*a^3*(a^2-3*b^2)*arctanh((a-b)^(1/2)*ta
n(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/b^2/(a+b)^(5/2)/d-1/2*sin(d*x+c)/
(a+b)^2/d/(1-cos(d*x+c))-1/2*sin(d*x+c)/(a-b)^2/d/(1+cos(d*x+c))-a^4*sin(d*
x+c)/b/(a^2-b^2)^2/d/(b+a*cos(d*x+c))
```

Rubi [A]
time = 0.33, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4482, 2976, 2727, 2743, 12, 2738, 214, 3855}

$$\frac{2a^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{a^4\sin(c+dx)}{bd\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{2a^3(a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\sin(c+dx)}{2d(a+b)^2(1-\cos(c+dx))}-\frac{\sin(c+dx)}{2d(a-b)^2(\cos(c+dx)+1)}+\frac{\tanh^{-1}(\sin(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] ArcTanh[Sin[c + d*x]]/(b^2*d) + (2*a^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2
])/Sqrt[a + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - (2*a^3*(a^2 - 3*b^2)*Arc
Tanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^2*(a + b
)^(5/2)*d) - Sin[c + d*x]/(2*(a + b)^2*d*(1 - Cos[c + d*x])) - Sin[c + d*x]
/(2*(a - b)^2*d*(1 + Cos[c + d*x])) - (a^4*Sin[c + d*x])/(b*(a^2 - b^2)^2*d
*(b + a*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2727

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2743

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Dist
[1/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) -
 b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2976

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx = \int \frac{\csc^2(c+dx)\sec(c+dx)}{(b+a\cos(c+dx))^2}\,dx$$

$$= -\int \left(-\frac{1}{2(a-b)^2(-1-\cos(c+dx))} - \frac{1}{2(a+b)^2(1-\cos(c+dx)}\right.$$

$$= \frac{\int \frac{1}{-1-\cos(c+dx)}\,dx}{2(a-b)^2} + \frac{\int \sec(c+dx)\,dx}{b^2} + \frac{\int \frac{1}{1-\cos(c+dx)}\,dx}{2(a+b)^2} + \frac{(a^3(a^2 -}{}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^2d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} - \frac{\sin}{2(a-b)^2d(}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^2d} - \frac{2a^3(a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right.}{(a-b)^{5/2}b^2(a+b)^{5/2}d}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^2d} - \frac{2a^3(a^2-3b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right.}{(a-b)^{5/2}b^2(a+b)^{5/2}d}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{b^2d} + \frac{2a^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{2a^3(a}{}$$

Mathematica [A]

time = 2.09, size = 196, normalized size = 0.85

$$-\frac{-\frac{4(a^5-4a^3b^2)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^2(a^2-b^2)^{5/2}} + \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2} + \frac{2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{b^2} - \frac{2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{b^2} + \frac{2a^4\sin(c+dx)}{(a-b)^2b(a+b)^2(b+a\cos(c+dx))} + \frac{\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] -1/2*((-4*(a^5 - 4*a^3*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 -
b^2]])/(b^2*(a^2 - b^2)^(5/2)) + Cot[(c + d*x)/2]/(a + b)^2 + (2*Log[Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]])/b^2 - (2*Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]])/b^2 + (2*a^4*Sin[c + d*x])/((a - b)^2*b*(a + b)^2*(b + a*Cos[c + d*
x])) + Tan[(c + d*x)/2]/(a - b)^2)/d
```

Maple [A]

time = 0.64, size = 199, normalized size = 0.86

method	result

derivativedivides	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^2-2ab+b^2\right)}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}+\frac{2a^3\left(\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}\right.}{(}}{d}$
default	$\frac{-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2\left(a^2-2ab+b^2\right)}-\frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}+\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}+\frac{2a^3\left(\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}\right.}{(}}{d}$
risch	$-\frac{2i\left(2a^3b\,e^{3i(dx+c)}+a\,b^3e^{3i(dx+c)}+a^4e^{2i(dx+c)}+2b^4e^{2i(dx+c)}-3a\,b^3e^{i(dx+c)}-a^4-2a^2b^2\right)}{\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)b\left(-a^2+b^2\right)^2\left(e^{2i(dx+c)}-1\right)d}+\frac{a^5\ln\left(e^{i(dx+c)}+\frac{-ia^2+ib^2}{a\sqrt{}}\right.}{\sqrt{a^2-b^2}\,(a+b}$ [illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)-1/2/(a+b)^2/tan(1/2*d*x+1/2*c)
-1/b^2*ln(tan(1/2*d*x+1/2*c)-1)+1/b^2*ln(tan(1/2*d*x+1/2*c)+1)+2*a^3/(a-b)^
2/b^2/(a+b)^2*(a*b*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x
+1/2*c)^2-a-b)-(a^2-4*b^2)/((a+b)*(a-b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(
a-b)/((a+b)*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 4.10, size = 864, normalized size = 3.74

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a^6*b - 2*b^7 + (a^5*b - 4*a^3*b^3 + (a^6 - 4*a^4*b^2)*cos(d*x + c
))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 +
 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*co
s(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(a^6*b + a^4*b^3
 - 2*a^2*b^5)*cos(d*x + c)^2 - (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7 + (a^7
- 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c))*log(sin(d*x + c) + 1)*sin(d*
x + c) + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7 + (a^7 - 3*a^5*b^2 + 3*a^3*b^
4 - a*b^6)*cos(d*x + c))*log(-sin(d*x + c) + 1)*sin(d*x + c) + 2*(a^5*b^2 -
 2*a^3*b^4 + a*b^6)*cos(d*x + c))/(((a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^
8)*d*cos(d*x + c) + (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*d)*sin(d*x + c)
), -1/2*(2*a^6*b - 2*b^7 + 2*(a^5*b - 4*a^3*b^3 + (a^6 - 4*a^4*b^2)*cos(d*x
 + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2
 - b^2)*sin(d*x + c)))*sin(d*x + c) - 2*(a^6*b + a^4*b^3 - 2*a^2*b^5)*cos(d
*x + c)^2 - (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7 + (a^7 - 3*a^5*b^2 + 3*a^3
*b^4 - a*b^6)*cos(d*x + c))*log(sin(d*x + c) + 1)*sin(d*x + c) + (a^6*b - 3
*a^4*b^3 + 3*a^2*b^5 - b^7 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x
+ c))*log(-sin(d*x + c) + 1)*sin(d*x + c) + 2*(a^5*b^2 - 2*a^3*b^4 + a*b^6)
*cos(d*x + c))/(((a^7*b^2 - 3*a^5*b^4 + 3*a^3*b^6 - a*b^8)*d*cos(d*x + c) +
 (a^6*b^3 - 3*a^4*b^5 + 3*a^2*b^7 - b^9)*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*sin(c + d*x) + b*tan(c + d*x))**2, x)
```

Giac [A]
time = 0.68, size = 354, normalized size = 1.53

$$\frac{\frac{4\,(a^5-4\,a^3b^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^4b^2-2\,a^2b^4+b^6)\sqrt{-a^2+b^2}}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2-2\,ab+b^2}+\frac{4\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+3\,a^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3\,ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^3b-a^2b^2-ab^3+b^4}{(a^4b-2\,a^2b^3+b^5)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}+\frac{2\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^2}-\frac{2\,\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(a^5 - 4*a^3*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) +
 arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2))
)/((a^4*b^2 - 2*a^2*b^4 + b^6)*sqrt(-a^2 + b^2)) - tan(1/2*d*x + 1/2*c)/(a^
2 - 2*a*b + b^2) + (4*a^4*tan(1/2*d*x + 1/2*c)^2 - a^3*b*tan(1/2*d*x + 1/2*
c)^2 + 3*a^2*b^2*tan(1/2*d*x + 1/2*c)^2 - 3*a*b^3*tan(1/2*d*x + 1/2*c)^2 +
```

```
b^4*tan(1/2*d*x + 1/2*c)^2 + a^3*b - a^2*b^2 - a*b^3 + b^4)/((a^4*b - 2*a^2
*b^3 + b^5)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/
2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))) + 2*log(abs(tan(1/2*d*x + 1/2*c)
+ 1))/b^2 - 2*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^2)/d
```

Mupad [B]
time = 5.21, size = 2500, normalized size = 10.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a*sin(c + d*x) + b*tan(c + d*x))^2),x)
```

```
[Out] ((a^2 - 2*a*b + b^2)/(a + b) + (tan(c/2 + (d*x)/2)^2*(4*a^4 - a^3*b - 3*a*b
^3 + b^4 + 3*a^2*b^2))/(b*(a + b)^2))/(d*(tan(c/2 + (d*x)/2)^3*(6*a*b^2 - 6
*a^2*b + 2*a^3 - 2*b^3) + tan(c/2 + (d*x)/2)*(2*a*b^2 + 2*a^2*b - 2*a^3 - 2
*b^3))) - (atan(-(((tan(c/2 + (d*x)/2)*(32*b^26 - 96*a*b^25 - 224*a^2*b^24
+ 928*a^3*b^23 + 480*a^4*b^22 - 4000*a^5*b^21 + 992*a^6*b^20 + 9568*a^7*b^1
9 - 8128*a^8*b^18 - 12992*a^9*b^17 + 21344*a^10*b^16 + 8224*a^11*b^15 - 317
44*a^12*b^14 + 2176*a^13*b^13 + 29600*a^14*b^12 - 8480*a^15*b^11 - 17632*a^
16*b^10 + 7072*a^17*b^9 + 6528*a^18*b^8 - 3008*a^19*b^7 - 1376*a^20*b^6 + 6
72*a^21*b^5 + 128*a^22*b^4 - 64*a^23*b^3) + (32*b^28 - 32*a*b^27 - 352*a^2*
b^26 + 480*a^3*b^25 + 1504*a^4*b^24 - 2688*a^5*b^23 - 3168*a^6*b^22 + 8064*
a^7*b^21 + 2880*a^8*b^20 - 14784*a^9*b^19 + 1344*a^10*b^18 + 17472*a^11*b^1
7 - 6720*a^12*b^16 - 13440*a^13*b^15 + 8256*a^14*b^14 + 6528*a^15*b^13 - 54
72*a^16*b^12 - 1824*a^17*b^11 + 2080*a^18*b^10 + 224*a^19*b^9 - 416*a^20*b^
8 + 32*a^22*b^6 - (tan(c/2 + (d*x)/2)*(128*a^2*b^28 - 64*a*b^29 + 576*a^3*b
^27 - 1280*a^4*b^26 - 2240*a^5*b^25 + 5760*a^6*b^24 + 4800*a^7*b^23 - 15360
*a^8*b^22 - 5760*a^9*b^21 + 26880*a^10*b^20 + 2688*a^11*b^19 - 32256*a^12*b
^18 + 2688*a^13*b^17 + 26880*a^14*b^16 - 5760*a^15*b^15 - 15360*a^16*b^14 +
 4800*a^17*b^13 + 5760*a^18*b^12 - 2240*a^19*b^11 - 1280*a^20*b^10 + 576*a^
21*b^9 + 128*a^22*b^8 - 64*a^23*b^7))/b^2)/b^2)*1i)/b^2 + ((tan(c/2 + (d*x)
/2)*(32*b^26 - 96*a*b^25 - 224*a^2*b^24 + 928*a^3*b^23 + 480*a^4*b^22 - 400
0*a^5*b^21 + 992*a^6*b^20 + 9568*a^7*b^19 - 8128*a^8*b^18 - 12992*a^9*b^17
+ 21344*a^10*b^16 + 8224*a^11*b^15 - 31744*a^12*b^14 + 2176*a^13*b^13 + 296
00*a^14*b^12 - 8480*a^15*b^11 - 17632*a^16*b^10 + 7072*a^17*b^9 + 6528*a^18
*b^8 - 3008*a^19*b^7 - 1376*a^20*b^6 + 672*a^21*b^5 + 128*a^22*b^4 - 64*a^2
3*b^3) - (32*b^28 - 32*a*b^27 - 352*a^2*b^26 + 480*a^3*b^25 + 1504*a^4*b^24
 - 2688*a^5*b^23 - 3168*a^6*b^22 + 8064*a^7*b^21 + 2880*a^8*b^20 - 14784*a^
9*b^19 + 1344*a^10*b^18 + 17472*a^11*b^17 - 6720*a^12*b^16 - 13440*a^13*b^1
5 + 8256*a^14*b^14 + 6528*a^15*b^13 - 5472*a^16*b^12 - 1824*a^17*b^11 + 208
0*a^18*b^10 + 224*a^19*b^9 - 416*a^20*b^8 + 32*a^22*b^6 + (tan(c/2 + (d*x)/
2)*(128*a^2*b^28 - 64*a*b^29 + 576*a^3*b^27 - 1280*a^4*b^26 - 2240*a^5*b^25
 + 5760*a^6*b^24 + 4800*a^7*b^23 - 15360*a^8*b^22 - 5760*a^9*b^21 + 26880*a
^10*b^20 + 2688*a^11*b^19 - 32256*a^12*b^18 + 2688*a^13*b^17 + 26880*a^14*b
```

```
^16 - 5760*a^15*b^15 - 15360*a^16*b^14 + 4800*a^17*b^13 + 5760*a^18*b^12 -
2240*a^19*b^11 - 1280*a^20*b^10 + 576*a^21*b^9 + 128*a^22*b^8 - 64*a^23*b^7
))/b^2)/b^2)*1i)/b^2)/(256*a^3*b^21 - 512*a^4*b^20 - 1856*a^5*b^19 + 3200*a
^6*b^18 + 6592*a^7*b^17 - 8704*a^8*b^16 - 15104*a^9*b^15 + 13760*a^10*b^14
+ 23872*a^11*b^13 - 14464*a^12*b^12 - 25856*a^13*b^11 + 11072*a^14*b^10 + 1
8496*a^15*b^9 - 6400*a^16*b^8 - 8192*a^17*b^7 + 2624*a^18*b^6 + 1984*a^19*b
^5 - 640*a^20*b^4 - 192*a^21*b^3 + 64*a^22*b^2 - (tan(c/2 + (d*x)/2)*(32*b^
26 - 96*a*b^25 - 224*a^2*b^24 + 928*a^3*b^23 + 480*a^4*b^22 - 4000*a^5*b^21
 + 992*a^6*b^20 + 9568*a^7*b^19 - 8128*a^8*b^18 - 12992*a^9*b^17 + 21344*a^
10*b^16 + 8224*a^11*b^15 - 31744*a^12*b^14 + 2176*a^13*b^13 + 29600*a^14*b^
12 - 8480*a^15*b^11 - 17632*a^16*b^10 + 7072*a^17*b^9 + 6528*a^18*b^8 - 300
8*a^19*b^7 - 1376*a^20*b^6 + 672*a^21*b^5 + 128*a^22*b^4 - 64*a^23*b^3) + (
32*b^28 - 32*a*b^27 - 352*a^2*b^26 + 480*a^3*b^25 + 1504*a^4*b^24 - 2688*a^
5*b^23 - 3168*a^6*b^22 + 8064*a^7*b^21 + 2880*a^8*b^20 - 14784*a^9*b^19 + 1
344*a^10*b^18 + 17472*a^11*b^17 - 6720*a^12*b^16 - 13440*a^13*b^15 + 8256*a
^14*b^14 + 6528*a^15*b^13 - 5472*a^16*b^12 - 1824*a^17*b^11 + 2080*a^18*b^1
0 + 224*a^19*b^9 - 416*a^20*b^8 + 32*a^22*b^6 - (tan(c/2 + (d*x)/2)*(128*a^
2*b^28 - 64*a*b^29 + 576*a^3*b^27 - 1280*a^4*b^26 - 2240*a^5*b^25 + 5760*a^
6*b^24 + 4800*a^7*b^23 - 15360*a^8*b^22 - 5760*a^9*b^21 + 26880*a^10*b^20 +
 2688*a^11*b^19 - 32256*a^12*b^18 + 2688*a^13*b^17 + 26880*a^14*b^16 - 5760
*a^15*b^15 - 15360*a^16*b^14 + 4800*a^17*b^13 + 5760*a^18*b^12 - 2240*a^19*
b^11 - 1280*a^20*b^10 + 576*a^21*b^9 + 128*a^22*b^8 - 64*a^23*b^7))/b^2)/b^
2)/b^2 + (tan(c/2 + (d*x)/2)*(32*b^26 - 96*a*b^25 - 224*a^2*b^24 + 928*a^3*
b^23 + 480*a^4*b^22 - 4000*a^5*b^21 + 992*a^6*b^20 + 9568*a^7*b^19 - 8128*a
^8*b^18 - 12992*a^9*b^17 + 21344*a^10*b^16 + 8224*a^11*b^15 - 31744*a^12*b^
14 + 2176*a^13*b^13 + 29600*a^14*b^12 - 8480*a^15*b^11 - 17632*a^16*b^10 +
7072*a^17*b^9 + 6528*a^18*b^8 - 3008*a^19*b^7 - 1376*a^20*b^6 + 672*a^21*b^
5 + 128*a^22*b^4 - 64*a^23*b^3) - (32*b^28 - 32*a*b^27 - 352*a^2*b^26 + 480
*a^3*b^25 + 1504*a^4*b^24 - 2688*a^5*b^23 - 3168*a^6*b^22 + 8064*a^7*b^21 +
 2880*a^8*b^20 - 14784*a^9*b^19 + 1344*a^10*b^18 + 17472*a^11*b^17 - 6720*a
^12*b^16 - 13440*a^13*b^15 + 8256*a^14*b^14 + 6528*a^15*b^13 - 5472*a^16*b^
12 - 1824*a^17*b^11 + 2080*a^18*b^10 + 224*a^19*b^9 - 416*a^20*b^8 + 32*a^2
2*b^6 + (tan(c/2 + (d*x)/2)*(128*a^2*b^28 - 64*a*b^29 + 576*a^3*b^27 - 1280
*a^4*b^26 - 2240*a^5*b^25 + 5760*a^6*b^24 + 4800*a^7*b^23 - 15360*a^8*b^22
- 5760*a^9*b^21 + 26880*a^10*b^20 + 2688*a^11*b...
```

3.264 $\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=248

$$\frac{b^6}{2a^3\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2}-\frac{2b^5(3a^2-b^2)}{a^3\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))}-\frac{(a(a^2+3b^2)-b(3a^2+b^2)\cos(c+d}{2\left(a^2-b^2\right)^3 d}$$

```
[Out] 1/2*b^6/a^3/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2-2*b^5*(3*a^2-b^2)/a^3/(a^2-b^2
)^3/d/(b+a*cos(d*x+c))-1/2*(a*(a^2+3*b^2)-b*(3*a^2+b^2)*cos(d*x+c))*csc(d*x
+c)^2/(a^2-b^2)^3/d-1/4*(2*a+5*b)*ln(1-cos(d*x+c))/(a+b)^4/d-1/4*(2*a-5*b)*
ln(1+cos(d*x+c))/(a-b)^4/d-b^4*(15*a^4-4*a^2*b^2+b^4)*ln(b+a*cos(d*x+c))/a^
3/(a^2-b^2)^4/d
```

Rubi [A]
time = 0.75, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {4482, 2916, 12, 1661, 1643}

$$-\frac{\csc^2(c+dx)\left(a(a^2+3b^2)-b(3a^2+b^2)\cos(c+dx)\right)}{2d\left(a^2-b^2\right)^3}+\frac{b^6}{2a^3d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}-\frac{2b^5(3a^2-b^2)}{a^3d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{b^4(15a^4-4a^2b^2+b^4)\log(a\cos(c+dx)+b)}{a^3d\left(a^2-b^2\right)^4}-\frac{(2a+5b)\log(1-\cos(c+dx))}{4d(a+b)^4}-\frac{(2a-5b)\log(\cos(c+dx)+1)}{4d(a-b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] b^6/(2*a^3*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) - (2*b^5*(3*a^2 - b^2))/
(a^3*(a^2 - b^2)^3*d*(b + a*Cos[c + d*x])) - ((a*(a^2 + 3*b^2) - b*(3*a^2 +
 b^2)*Cos[c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)^3*d) - ((2*a + 5*b)*Log[
1 - Cos[c + d*x]])/(4*(a + b)^4*d) - ((2*a - 5*b)*Log[1 + Cos[c + d*x]])/(4
*(a - b)^4*d) - (b^4*(15*a^4 - 4*a^2*b^2 + b^4)*Log[b + a*Cos[c + d*x]])/(a
^3*(a^2 - b^2)^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
```

```
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \int \frac{\cos^3(c+dx)\cot^3(c+dx)}{(b+a\cos(c+dx))^3}\,dx$$

$$= -\frac{a^3\text{Subst}\left(\int \frac{x^6}{a^6(b+x)^3(a^2-x^2)^2}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^6}{(b+x)^3(a^2-x^2)^2}\,dx, x, a\cos(c+dx)\right)}{a^3 d}$$

$$= -\frac{(a(a^2+3b^2)-b(3a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)^3 d} - \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{(a(a^2+3b^2)-b(3a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)^3 d} - \frac{\text{Subst}\left(\int \right.}{}$$

$$= \frac{b^6}{2a^3(a^2-b^2)^2 d(b+a\cos(c+dx))^2} - \frac{2b^5(3a^2-b^2)}{a^3(a^2-b^2)^3 d(b+a\cos(c+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.37, size = 713, normalized size = 2.88

$$\frac{b^6(b+a\cos(c+dx))\tan^3(c+dx)}{2a^3(-a+b)^2(a+b)^2d(a\sin(c+dx)+b\tan(c+dx))^3} - \frac{2b^5(-3a^2+b^2)(b+a\cos(c+dx))^2\tan^3(c+dx)}{a^3(-a+b)^3(a+b)^3d(a\sin(c+dx)+b\tan(c+dx))^3} - \frac{2i(a^5-4a^3b^2-9ab^4)(c+dx)(b+a\cos(c+dx))^3\tan^3(c+dx)}{(a-b)^4(a+b)^4d(a\sin(c+dx)+b\tan(c+dx))^3} - \frac{i(-2a-5b)\text{ArcTan}(\tan(c+dx))(b+a\cos(c+dx))^3\tan^3(c+dx)}{2(a+b)^4d(a\sin(c+dx)+b\tan(c+dx))^3} - \frac{i(-2a+5b)\text{ArcTan}(\tan(c+dx))(b+a\cos(c+dx))^3\tan^3(c+dx)}{2(-a+b)^4d(a\sin(c+dx)+b\tan(c+dx))^3} - \frac{(b+a\cos(c+dx))^3\csc^2\left(\frac{1}{2}(c+dx)\right)\tan^3(c+dx)}{8(a+b)^3d(a\sin(c+dx)+b\tan(c+dx))^3} + \frac{(-2a+5b)(b+a\cos(c+dx))^3\log\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\right)\tan^3(c+dx)}{4(-a+b)^4d(a\sin(c+dx)+b\tan(c+dx))^3} + \frac{(-15a^4b^4+4a^2b^6-b^8)(b+a\cos(c+dx))^3\log(b+a\cos(c+dx))\tan^3(c+dx)}{a^3(-a^2+b^2)^4d(a\sin(c+dx)+b\tan(c+dx))^3} + \frac{(-2a-5b)(b+a\cos(c+dx))^3\log\left(\sin^2\left(\frac{1}{2}(c+dx)\right)\right)\tan^3(c+dx)}{4(a+b)^4d(a\sin(c+dx)+b\tan(c+dx))^3} + \frac{(b+a\cos(c+dx))^3\sec^2\left(\frac{1}{2}(c+dx)\right)\tan^3(c+dx)}{8(-a+b)^3d(a\sin(c+dx)+b\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (b^6*(b + a*Cos[c + d*x])*Tan[c + d*x]^3)/(2*a^3*(-a + b)^2*(a + b)^2*d*(a*
Sin[c + d*x] + b*Tan[c + d*x])^3) - (2*b^5*(-3*a^2 + b^2)*(b + a*Cos[c + d*
x])^2*Tan[c + d*x]^3)/(a^3*(-a + b)^3*(a + b)^3*d*(a*Sin[c + d*x] + b*Tan[c
 + d*x])^3) - ((2*I)*(a^5 - 4*a^3*b^2 - 9*a*b^4)*(c + d*x)*(b + a*Cos[c + d
*x])^3*Tan[c + d*x]^3)/((a - b)^4*(a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d
*x])^3) - ((I/2)*(-2*a - 5*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])^3*T
an[c + d*x]^3)/((a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) - ((I/2)*(
-2*a + 5*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])^3*Tan[c + d*x]^3)/((-
a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) - ((b + a*Cos[c + d*x])^3*C
sc[(c + d*x)/2]^2*Tan[c + d*x]^3)/(8*(a + b)^3*d*(a*Sin[c + d*x] + b*Tan[c
+ d*x])^3) + ((-2*a + 5*b)*(b + a*Cos[c + d*x])^3*Log[Cos[(c + d*x)/2]^2]*T
an[c + d*x]^3)/(4*(-a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) + ((-15
*a^4*b^4 + 4*a^2*b^6 - b^8)*(b + a*Cos[c + d*x])^3*Log[b + a*Cos[c + d*x]]*
Tan[c + d*x]^3)/(a^3*(-a^2 + b^2)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3)
+ ((-2*a - 5*b)*(b + a*Cos[c + d*x])^3*Log[Sin[(c + d*x)/2]^2]*Tan[c + d*x]
^3)/(4*(a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) + ((b + a*Cos[c + d
*x])^3*Sec[(c + d*x)/2]^2*Tan[c + d*x]^3)/(8*(-a + b)^3*d*(a*Sin[c + d*x] +
 b*Tan[c + d*x])^3)
```

Maple [A]
time = 0.71, size = 213, normalized size = 0.86

method	result
derivativedivides	$\frac{\frac{b^6}{2a^3(a+b)^2(a-b)^2(b+a\cos(dx+c))^2}-\frac{2b^5(3a^2-b^2)}{a^3(a+b)^3(a-b)^3(b+a\cos(dx+c))}-\frac{b^4(15a^4-4a^2b^2+b^4)\ln(b+a\cos(dx+c))}{(a+b)^4(a-b)^4a^3}+\frac{1}{4(a+b)^3(\cos(dx+}}{d}$
default	$\frac{\frac{b^6}{2a^3(a+b)^2(a-b)^2(b+a\cos(dx+c))^2}-\frac{2b^5(3a^2-b^2)}{a^3(a+b)^3(a-b)^3(b+a\cos(dx+c))}-\frac{b^4(15a^4-4a^2b^2+b^4)\ln(b+a\cos(dx+c))}{(a+b)^4(a-b)^4a^3}+\frac{1}{4(a+b)^3(\cos(dx+}}{d}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2/a^3*b^6/(a+b)^2/(a-b)^2/(b+a*cos(d*x+c))^2-2/a^3*b^5*(3*a^2-b^2)/(
a+b)^3/(a-b)^3/(b+a*cos(d*x+c))-b^4*(15*a^4-4*a^2*b^2+b^4)/(a+b)^4/(a-b)^4/
a^3*ln(b+a*cos(d*x+c))+1/4/(a+b)^3/(cos(d*x+c)-1)+1/4/(a+b)^4*(-2*a-5*b)*ln
(cos(d*x+c)-1)-1/4/(a-b)^3/(1+cos(d*x+c))+1/4/(a-b)^4*(-2*a+5*b)*ln(1+cos(d
*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(240) = 480$.
time = 0.55, size = 684, normalized size = 2.76

$$-\frac{\frac{8\,(15\,a^4b^4-4\,a^2b^6+b^8)\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^{11}-4\,a^9b^2+6\,a^7b^4-4\,a^5b^6+a^3b^8}+\frac{4\,(2\,a+5\,b)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{a^8-2\,a^7b-a^6b^2+4\,a^5b^3-a^4b^4-2\,a^3b^5+a^2b^6-\frac{2\,(a^8-4\,a^7b+5\,a^6b^2-5\,a^4b^4-44\,a^3b^5-49\,a^2b^6+8\,ab^7+8\,b^8)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{(a^8-6\,a^7b+15\,a^6b^2-20\,a^5b^3+15\,a^4b^4-102\,a^3b^5+81\,a^2b^6+32\,ab^7-16\,b^8)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{\frac{(a^{11}+a^{10}b-4\,a^9b^2-4\,a^8b^3+6\,a^7b^4+6\,a^6b^5-4\,a^5b^6-4\,a^4b^7+a^3b^8+a^2b^9)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2\,(a^{11}-a^{10}b-4\,a^9b^2+4\,a^8b^3+6\,a^7b^4-6\,a^6b^5-4\,a^5b^6+4\,a^4b^7+a^3b^8-a^2b^9)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{(a^{11}-3\,a^{10}b+8\,a^8b^3-6\,a^7b^4-6\,a^6b^5+8\,a^5b^6-3\,a^3b^8+a^2b^9)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\sin(dx+c)^2}{(a^3-3\,a^2b+3\,ab^2-b^3)(\cos(dx+c)+1)^2}-\frac{8\,\log\left(\frac{\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)}{a^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/8*(8*(15*a^4*b^4 - 4*a^2*b^6 + b^8)*log(a + b - (a - b)*sin(d*x + c)^2/(
cos(d*x + c) + 1)^2)/(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8) +
 4*(2*a + 5*b)*log(sin(d*x + c)/(cos(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^2*
b^2 + 4*a*b^3 + b^4) + (a^8 - 2*a^7*b - a^6*b^2 + 4*a^5*b^3 - a^4*b^4 - 2*a
^3*b^5 + a^2*b^6 - 2*(a^8 - 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 44*a^3*b^5 -
49*a^2*b^6 + 8*a*b^7 + 8*b^8)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + (a^8 -
6*a^7*b + 15*a^6*b^2 - 20*a^5*b^3 + 15*a^4*b^4 - 102*a^3*b^5 + 81*a^2*b^6 +
 32*a*b^7 - 16*b^8)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/((a^11 + a^10*b -
4*a^9*b^2 - 4*a^8*b^3 + 6*a^7*b^4 + 6*a^6*b^5 - 4*a^5*b^6 - 4*a^4*b^7 + a^3
*b^8 + a^2*b^9)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(a^11 - a^10*b - 4*
a^9*b^2 + 4*a^8*b^3 + 6*a^7*b^4 - 6*a^6*b^5 - 4*a^5*b^6 + 4*a^4*b^7 + a^3*b
^8 - a^2*b^9)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^11 - 3*a^10*b + 8*a^
8*b^3 - 6*a^7*b^4 - 6*a^6*b^5 + 8*a^5*b^6 - 3*a^3*b^8 + a^2*b^9)*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6) + sin(d*x + c)^2/((a^3 - 3*a^2*b + 3*a*b^2 - b^3
)*(cos(d*x + c) + 1)^2) - 8*log(sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)/a^
3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1180 vs. $2(240) = 480$.
time = 3.97, size = 1180, normalized size = 4.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^8*b^2 + 4*a^6*b^4 + 16*a^4*b^6 - 28*a^2*b^8 + 6*b^10 - 2*(3*a^9*b
- 2*a^7*b^3 + 11*a^5*b^5 - 16*a^3*b^7 + 4*a*b^9)*cos(d*x + c)^3 + 2*(a^10 -
 4*a^8*b^2 + a^6*b^4 - 9*a^4*b^6 + 14*a^2*b^8 - 3*b^10)*cos(d*x + c)^2 + 2*
(2*a^9*b + a^7*b^3 + 8*a^5*b^5 - 15*a^3*b^7 + 4*a*b^9)*cos(d*x + c) + 4*(15
*a^4*b^6 - 4*a^2*b^8 + b^10 - (15*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*cos(d*x +
c)^4 - 2*(15*a^5*b^5 - 4*a^3*b^7 + a*b^9)*cos(d*x + c)^3 + (15*a^6*b^4 - 19
*a^4*b^6 + 5*a^2*b^8 - b^10)*cos(d*x + c)^2 + 2*(15*a^5*b^5 - 4*a^3*b^7 + a
*b^9)*cos(d*x + c))*log(a*cos(d*x + c) + b) + (2*a^8*b^2 + 3*a^7*b^3 - 8*a^
6*b^4 - 22*a^5*b^5 - 18*a^4*b^6 - 5*a^3*b^7 - (2*a^10 + 3*a^9*b - 8*a^8*b^2
 - 22*a^7*b^3 - 18*a^6*b^4 - 5*a^5*b^5)*cos(d*x + c)^4 - 2*(2*a^9*b + 3*a^8
*b^2 - 8*a^7*b^3 - 22*a^6*b^4 - 18*a^5*b^5 - 5*a^4*b^6)*cos(d*x + c)^3 + (2
*a^10 + 3*a^9*b - 10*a^8*b^2 - 25*a^7*b^3 - 10*a^6*b^4 + 17*a^5*b^5 + 18*a^
4*b^6 + 5*a^3*b^7)*cos(d*x + c)^2 + 2*(2*a^9*b + 3*a^8*b^2 - 8*a^7*b^3 - 22
*a^6*b^4 - 18*a^5*b^5 - 5*a^4*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2
) + (2*a^8*b^2 - 3*a^7*b^3 - 8*a^6*b^4 + 22*a^5*b^5 - 18*a^4*b^6 + 5*a^3*b^
7 - (2*a^10 - 3*a^9*b - 8*a^8*b^2 + 22*a^7*b^3 - 18*a^6*b^4 + 5*a^5*b^5)*co
```

```
s(d*x + c)^4 - 2*(2*a^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 22*a^6*b^4 - 18*a^5*b^5
 + 5*a^4*b^6)*cos(d*x + c)^3 + (2*a^10 - 3*a^9*b - 10*a^8*b^2 + 25*a^7*b^3
- 10*a^6*b^4 - 17*a^5*b^5 + 18*a^4*b^6 - 5*a^3*b^7)*cos(d*x + c)^2 + 2*(2*a
^9*b - 3*a^8*b^2 - 8*a^7*b^3 + 22*a^6*b^4 - 18*a^5*b^5 + 5*a^4*b^6)*cos(d*x
 + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^13 - 4*a^11*b^2 + 6*a^9*b^4 - 4*a^
7*b^6 + a^5*b^8)*d*cos(d*x + c)^4 + 2*(a^12*b - 4*a^10*b^3 + 6*a^8*b^5 - 4*
a^6*b^7 + a^4*b^9)*d*cos(d*x + c)^3 - (a^13 - 5*a^11*b^2 + 10*a^9*b^4 - 10*
a^7*b^6 + 5*a^5*b^8 - a^3*b^10)*d*cos(d*x + c)^2 - 2*(a^12*b - 4*a^10*b^3 +
 6*a^8*b^5 - 4*a^6*b^7 + a^4*b^9)*d*cos(d*x + c) - (a^11*b^2 - 4*a^9*b^4 +
6*a^7*b^6 - 4*a^5*b^8 + a^3*b^10)*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 848 vs. 2(240) = 480.
time = 1.05, size = 848, normalized size = 3.42

$$-\frac{\frac{2\,(2\,a+5\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{8\,(15\,a^4b^4-4\,a^2b^6+b^8)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^{11}-4\,a^9b^2+6\,a^7b^4-4\,a^5b^6+a^3b^8}-\frac{\left(a+b+\frac{4\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{10\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)(\cos(dx+c)-1)}-\frac{\cos(dx+c)-1}{(a^3-3\,a^2b+3\,ab^2-b^3)(\cos(dx+c)+1)}-\frac{4\left(45\,a^6b^4+66\,a^5b^5-15\,a^4b^6-44\,a^3b^7-a^2b^8+10\,ab^9+3\,b^{10}+\frac{90\,a^6b^4(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{24\,a^5b^5(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{118\,a^4b^6(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{28\,a^3b^7(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{34\,a^2b^8(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,ab^9(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{6\,b^{10}(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{45\,a^6b^4(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{90\,a^5b^5(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{33\,a^4b^6(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{24\,a^3b^7(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{9\,a^2b^8(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{6\,ab^9(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{3\,b^{10}(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{(a^{11}-4\,a^9b^2+6\,a^7b^4-4\,a^5b^6+a^3b^8)\left(a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)^2}-\frac{8\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(2*(2*a + 5*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4
+ 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 8*(15*a^4*b^4 - 4*a^2*b^6 + b^8)*l
og(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1)))/(a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^
8) - (a + b + 4*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 10*b*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 +
 4*a*b^3 + b^4)*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/((a^3 - 3*a^2*b +
3*a*b^2 - b^3)*(cos(d*x + c) + 1)) - 4*(45*a^6*b^4 + 66*a^5*b^5 - 15*a^4*b^
6 - 44*a^3*b^7 - a^2*b^8 + 10*a*b^9 + 3*b^10 + 90*a^6*b^4*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) - 24*a^5*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1
18*a^4*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 28*a^3*b^7*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + 34*a^2*b^8*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 - 4*a*b^9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 6*b^10*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 45*a^6*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
 - 90*a^5*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 33*a^4*b^6*(cos(d
*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 24*a^3*b^7*(cos(d*x + c) - 1)^2/(cos(
```

```
d*x + c) + 1)^2 - 9*a^2*b^8*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6*a
*b^9*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 3*b^10*(cos(d*x + c) - 1)^
2/(cos(d*x + c) + 1)^2)/((a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^
8)*(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1))^2) - 8*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 1))/a^3)/d
```

Mupad [B]
time = 2.05, size = 527, normalized size = 2.12

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(-a^7+5\,a^6\,b-10\,a^5\,b^2+10\,a^4\,b^3-5\,a^3\,b^4+97\,a^2\,b^5+16\,a\,b^6-16\,b^7\right)}{2\,a^2\,\left(a+b\right)\,\left(a^2+2\,a\,b+b^2\right)}-\frac{a^3-3\,a^2\,b+3\,a\,b^2-b^3}{2\,\left(a+b\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(a^7-5\,a^6\,b+10\,a^5\,b^2-10\,a^4\,b^3+5\,a^3\,b^4-49\,a^2\,b^5+8\,b^7\right)}{a^2\,{\left(a+b\right)}^2\,\left(a-b\right)}}{d\,\left(\left(4\,a^5-20\,a^4\,b+40\,a^3\,b^2-40\,a^2\,b^3+20\,a\,b^4-4\,b^5\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+\left(-8\,a^5+24\,a^4\,b-16\,a^3\,b^2-16\,a^2\,b^3+24\,a\,b^4-8\,b^5\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+\left(4\,a^5-4\,a^4\,b-8\,a^3\,b^2+8\,a^2\,b^3+4\,a\,b^4-4\,b^5\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{8\,d\,{\left(a-b\right)}^3}+\frac{\ln\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}{a^3\,d}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,a+5\,b\right)}{d\,\left(2\,a^4+8\,a^3\,b+12\,a^2\,b^2+8\,a\,b^3+2\,b^4\right)}-\frac{b^4\,\ln\left(a+b-a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+b\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)\,\left(15\,a^4-4\,a^2\,b^2+b^4\right)}{a^3\,d\,{\left(a^2-b^2\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a*sin(c + d*x) + b*tan(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^4*(16*a*b^6 + 5*a^6*b - a^7 - 16*b^7 + 97*a^2*b^5 - 5*
a^3*b^4 + 10*a^4*b^3 - 10*a^5*b^2))/(2*a^2*(a + b)*(2*a*b + a^2 + b^2)) - (
3*a*b^2 - 3*a^2*b + a^3 - b^3)/(2*(a + b)) + (tan(c/2 + (d*x)/2)^2*(a^7 - 5
*a^6*b + 8*b^7 - 49*a^2*b^5 + 5*a^3*b^4 - 10*a^4*b^3 + 10*a^5*b^2))/(a^2*(a
 + b)^2*(a - b)))/(d*(tan(c/2 + (d*x)/2)^2*(4*a*b^4 - 4*a^4*b + 4*a^5 - 4*b
^5 + 8*a^2*b^3 - 8*a^3*b^2) - tan(c/2 + (d*x)/2)^4*(8*a^5 - 24*a^4*b - 24*a
*b^4 + 8*b^5 + 16*a^2*b^3 + 16*a^3*b^2) + tan(c/2 + (d*x)/2)^6*(20*a*b^4 -
20*a^4*b + 4*a^5 - 4*b^5 - 40*a^2*b^3 + 40*a^3*b^2))) - tan(c/2 + (d*x)/2)^
2/(8*d*(a - b)^3) + log(tan(c/2 + (d*x)/2)^2 + 1)/(a^3*d) - (log(tan(c/2 +
(d*x)/2))*(2*a + 5*b))/(d*(8*a*b^3 + 8*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2))
 - (b^4*log(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(15*a^
4 + b^4 - 4*a^2*b^2))/(a^3*d*(a^2 - b^2)^4)
```

3.265 $\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=232

$$-\frac{b^5}{2a^2\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2}+\frac{b^4(5a^2-b^2)}{a^2\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))}+\frac{(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+}{2\left(a^2-b^2\right)^3 d}$$

```
[Out] -1/2*b^5/a^2/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2+b^4*(5*a^2-b^2)/a^2/(a^2-b^2)
^3/d/(b+a*cos(d*x+c))+1/2*(b*(3*a^2+b^2)-a*(a^2+3*b^2)*cos(d*x+c))*csc(d*x+
c)^2/(a^2-b^2)^3/d-1/4*(a+4*b)*ln(1-cos(d*x+c))/(a+b)^4/d+1/4*(a-4*b)*ln(1+
cos(d*x+c))/(a-b)^4/d+2*b^3*(5*a^2+b^2)*ln(b+a*cos(d*x+c))/(a^2-b^2)^4/d
```

Rubi [A]
time = 0.62, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {4482, 2916, 12, 1661, 1643}

$$\frac{\csc^2(c+dx)\left(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx)\right)}{2d\left(a^2-b^2\right)^3}-\frac{b^5}{2a^2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}+\frac{b^4(5a^2-b^2)}{a^2d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}+\frac{2b^3(5a^2+b^2)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^4}-\frac{(a+4b)\log(1-\cos(c+dx))}{4d(a+b)^4}+\frac{(a-4b)\log(\cos(c+dx)+1)}{4d(a-b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] -1/2*b^5/(a^2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) + (b^4*(5*a^2 - b^2))
/(a^2*(a^2 - b^2)^3*d*(b + a*Cos[c + d*x])) + ((b*(3*a^2 + b^2) - a*(a^2 +
3*b^2)*Cos[c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)^3*d) - ((a + 4*b)*Log[1
 - Cos[c + d*x]])/(4*(a + b)^4*d) + ((a - 4*b)*Log[1 + Cos[c + d*x]])/(4*(a
 - b)^4*d) + (2*b^3*(5*a^2 + b^2)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
```

```
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \int \frac{\cos^2(c+dx)\cot^3(c+dx)}{(b+a\cos(c+dx))^3}\,dx$$

$$= -\frac{a^3\text{Subst}\left(\int \frac{x^5}{a^5(b+x)^3(a^2-x^2)^2}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^5}{(b+x)^3(a^2-x^2)^2}\,dx, x, a\cos(c+dx)\right)}{a^2d}$$

$$= \frac{(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx))\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} - \frac{\text{Subst}\left(\int -\right.}{}$$

$$= \frac{(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx))\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} - \frac{\text{Subst}\left(\int \left(\right.\right.}{}$$

$$= -\frac{b^5}{2a^2\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2} + \frac{b^4(5a^2-b^2)}{a^2\left(a^2-b^2\right)^3 d(b+a\cos(c}$$

Mathematica [A]

time = 6.20, size = 204, normalized size = 0.88

$$\frac{-\frac{4b^5}{a^2(a-b)^2(a+b)^2(b+a\cos(c+dx))^2}+\frac{8b^4(-5a^2+b^2)}{a^2(-a+b)^3(a+b)^3(b+a\cos(c+dx))}-\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3}+\frac{4(a-4b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{(a-b)^4}+\frac{16b^3(5a^2+b^2)\log(b+a\cos(c+dx))}{(a^2-b^2)^4}-\frac{4(a+4b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a+b)^4}+\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{(a-b)^3}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] ((-4*b^5)/(a^2*(a - b)^2*(a + b)^2*(b + a*Cos[c + d*x])^2) + (8*b^4*(-5*a^2
 + b^2))/(a^2*(-a + b)^3*(a + b)^3*(b + a*Cos[c + d*x])) - Csc[(c + d*x)/2]
^2/(a + b)^3 + (4*(a - 4*b)*Log[Cos[(c + d*x)/2]])/(a - b)^4 + (16*b^3*(5*a
^2 + b^2)*Log[b + a*Cos[c + d*x]])/(a^2 - b^2)^4 - (4*(a + 4*b)*Log[Sin[(c
+ d*x)/2]])/(a + b)^4 + Sec[(c + d*x)/2]^2/(a - b)^3)/(8*d)
```

Maple [A]
time = 0.74, size = 199, normalized size = 0.86

method	result
derivativedivides	$\frac{-\frac{b^{5}}{2a^{2}(a+b)^{2}(a-b)^{2}(b+a\cos(dx+c))^{2}}+\frac{2b^{3}\left(5a^{2}+b^{2}\right)\ln(b+a\cos(dx+c))}{(a+b)^{4}(a-b)^{4}}+\frac{b^{4}\left(5a^{2}-b^{2}\right)}{(a+b)^{3}(a-b)^{3}a^{2}(b+a\cos(dx+c))}+\frac{1}{4(a+b)^{3}(\cos(dx+c)-1)}}{d}$
default	$\frac{-\frac{b^{5}}{2a^{2}(a+b)^{2}(a-b)^{2}(b+a\cos(dx+c))^{2}}+\frac{2b^{3}\left(5a^{2}+b^{2}\right)\ln(b+a\cos(dx+c))}{(a+b)^{4}(a-b)^{4}}+\frac{b^{4}\left(5a^{2}-b^{2}\right)}{(a+b)^{3}(a-b)^{3}a^{2}(b+a\cos(dx+c))}+\frac{1}{4(a+b)^{3}(\cos(dx+c)-1)}}{d}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2/a^2*b^5/(a+b)^2/(a-b)^2/(b+a*cos(d*x+c))^2+2*b^3*(5*a^2+b^2)/(a+b
)^4/(a-b)^4*ln(b+a*cos(d*x+c))+b^4*(5*a^2-b^2)/(a+b)^3/(a-b)^3/a^2/(b+a*cos
(d*x+c))+1/4/(a+b)^3/(cos(d*x+c)-1)+1/4/(a+b)^4*(-a-4*b)*ln(cos(d*x+c)-1)+1
/4/(a-b)^3/(1+cos(d*x+c))+1/4*(a-4*b)/(a-b)^4*ln(1+cos(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 589 vs. $2(224) = 448$.
time = 0.32, size = 589, normalized size = 2.54

$$\frac{\frac{16\,(5\,a^2b^3+b^5)\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}-\frac{4\,(a+4\,b)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{a^6-2\,a^5b-a^4b^2+4\,a^3b^3-a^2b^4-2\,ab^5+b^6-\frac{2\left(a^6-4\,a^5b+5\,a^4b^2+35\,a^2b^4+44\,ab^5-b^6\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\left(a^6-6\,a^5b+15\,a^4b^2-20\,a^3b^3+95\,a^2b^4-70\,ab^5-15\,b^6\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{\frac{\left(a^9+a^8b-4\,a^7b^2-4\,a^6b^3+6\,a^5b^4+6\,a^4b^5-4\,a^3b^6-4\,a^2b^7+ab^8+b^9\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2\left(a^9-a^8b-4\,a^7b^2+4\,a^6b^3+6\,a^5b^4-6\,a^4b^5-4\,a^3b^6+4\,a^2b^7+ab^8-b^9\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\left(a^9-3\,a^8b+8\,a^6b^3-6\,a^5b^4-6\,a^4b^5+8\,a^3b^6-3\,ab^8+b^9\right)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\sin(dx+c)^2}{(a^3-3\,a^2b+3\,ab^2-b^3)(\cos(dx+c)+1)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(16*(5*a^2*b^3 + b^5)*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c)
+ 1)^2)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - 4*(a + 4*b)*log(s
in(d*x + c)/(cos(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)
 - (a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*(a^6
- 4*a^5*b + 5*a^4*b^2 + 35*a^2*b^4 + 44*a*b^5 - b^6)*sin(d*x + c)^2/(cos(d*
x + c) + 1)^2 + (a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 95*a^2*b^4 - 70*
a*b^5 - 15*b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/((a^9 + a^8*b - 4*a^7*
b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b
```

```
^9)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(a^9 - a^8*b - 4*a^7*b^2 + 4*a^
6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4 + (a^9 - 3*a^8*b + 8*a^6*b^3 - 6*a^5*b^4 - 6*
a^4*b^5 + 8*a^3*b^6 - 3*a*b^8 + b^9)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) +
 sin(d*x + c)^2/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(cos(d*x + c) + 1)^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1045 vs. $2(224) = 448$.
time = 3.52, size = 1045, normalized size = 4.50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(6*a^6*b^3 + 14*a^4*b^5 - 22*a^2*b^7 + 2*b^9 - 2*(a^9 + 2*a^7*b^2 + 7*
a^5*b^4 - 12*a^3*b^6 + 2*a*b^8)*cos(d*x + c)^3 + 2*(a^8*b - 6*a^6*b^3 - 4*a
^4*b^5 + 10*a^2*b^7 - b^9)*cos(d*x + c)^2 + 2*(5*a^7*b^2 + 4*a^5*b^4 - 11*a
^3*b^6 + 2*a*b^8)*cos(d*x + c) + 8*(5*a^4*b^5 + a^2*b^7 - (5*a^6*b^3 + a^4*
b^5)*cos(d*x + c)^4 - 2*(5*a^5*b^4 + a^3*b^6)*cos(d*x + c)^3 + (5*a^6*b^3 -
 4*a^4*b^5 - a^2*b^7)*cos(d*x + c)^2 + 2*(5*a^5*b^4 + a^3*b^6)*cos(d*x + c)
)*log(a*cos(d*x + c) + b) + (a^7*b^2 - 10*a^5*b^4 - 20*a^4*b^5 - 15*a^3*b^6
 - 4*a^2*b^7 - (a^9 - 10*a^7*b^2 - 20*a^6*b^3 - 15*a^5*b^4 - 4*a^4*b^5)*cos
(d*x + c)^4 - 2*(a^8*b - 10*a^6*b^3 - 20*a^5*b^4 - 15*a^4*b^5 - 4*a^3*b^6)*
cos(d*x + c)^3 + (a^9 - 11*a^7*b^2 - 20*a^6*b^3 - 5*a^5*b^4 + 16*a^4*b^5 +
15*a^3*b^6 + 4*a^2*b^7)*cos(d*x + c)^2 + 2*(a^8*b - 10*a^6*b^3 - 20*a^5*b^4
 - 15*a^4*b^5 - 4*a^3*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) - (a^7
*b^2 - 10*a^5*b^4 + 20*a^4*b^5 - 15*a^3*b^6 + 4*a^2*b^7 - (a^9 - 10*a^7*b^2
 + 20*a^6*b^3 - 15*a^5*b^4 + 4*a^4*b^5)*cos(d*x + c)^4 - 2*(a^8*b - 10*a^6*
b^3 + 20*a^5*b^4 - 15*a^4*b^5 + 4*a^3*b^6)*cos(d*x + c)^3 + (a^9 - 11*a^7*b
^2 + 20*a^6*b^3 - 5*a^5*b^4 - 16*a^4*b^5 + 15*a^3*b^6 - 4*a^2*b^7)*cos(d*x
+ c)^2 + 2*(a^8*b - 10*a^6*b^3 + 20*a^5*b^4 - 15*a^4*b^5 + 4*a^3*b^6)*cos(d
*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^12 - 4*a^10*b^2 + 6*a^8*b^4 - 4*
a^6*b^6 + a^4*b^8)*d*cos(d*x + c)^4 + 2*(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4
*a^5*b^7 + a^3*b^9)*d*cos(d*x + c)^3 - (a^12 - 5*a^10*b^2 + 10*a^8*b^4 - 10
*a^6*b^6 + 5*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^2 - 2*(a^11*b - 4*a^9*b^3 +
 6*a^7*b^5 - 4*a^5*b^7 + a^3*b^9)*d*cos(d*x + c) - (a^10*b^2 - 4*a^8*b^4 +
6*a^6*b^6 - 4*a^4*b^8 + a^2*b^10)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)**2/(a*sin(c + d*x) + b*tan(c + d*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 676 vs. 2(224) = 448.
time = 1.03, size = 676, normalized size = 2.91

$$-\frac{\frac{2\,(a+4\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{16\,(5\,a^2b^3+b^5)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}-\frac{\left(a+b+\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{8\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)(\cos(dx+c)-1)}+\frac{\cos(dx+c)-1}{(a^3-3\,a^2b+3\,ab^2-b^3)(\cos(dx+c)+1)}+\frac{8\left(15\,a^4b^3+20\,a^3b^4-2\,a^2b^5-4\,ab^6+3\,b^7+\frac{30\,a^4b^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{10\,a^3b^4(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{26\,a^2b^5(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{10\,ab^6(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b^7(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{15\,a^4b^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{30\,a^3b^4(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{18\,a^2b^5(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{6\,ab^6(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{3\,b^7(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)\left(a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(2*(a + 4*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 +
4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 16*(5*a^2*b^3 + b^5)*log(abs(-a - b
- a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1)))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - (a + b + 2*a*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*b*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(cos
(d*x + c) - 1)) + (cos(d*x + c) - 1)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(cos(
d*x + c) + 1)) + 8*(15*a^4*b^3 + 20*a^3*b^4 - 2*a^2*b^5 - 4*a*b^6 + 3*b^7 +
 30*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 10*a^3*b^4*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) - 26*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1
) + 10*a*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b^7*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) + 15*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^
2 - 30*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 18*a^2*b^5*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6*a*b^6*(cos(d*x + c) - 1)^2/(cos(d*
x + c) + 1)^2 + 3*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((a^8 - 4*
a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b + a*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2))/d
```

Mupad [B]
time = 1.21, size = 491, normalized size = 2.12

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d\,(a-b)^3}-\frac{\frac{a^3-3\,a^2\,b+3\,a\,b^2-b^3}{2\,(a+b)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,(a^5-5\,a^4\,b+10\,a^3\,b^2-10\,a^2\,b^3+85\,a\,b^4+15\,b^5)}{2\,(a+b)\,(a^2+2\,a\,b+b^2)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(a^5-5\,a^4\,b+10\,a^3\,b^2-10\,a^2\,b^3+45\,a\,b^4-b^5)}{(a-b)\,(a^2+2\,a\,b+b^2)}}{d\,\left((4\,a^5-20\,a^4\,b+40\,a^3\,b^2-40\,a^2\,b^3+20\,a\,b^4-4\,b^5)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+(-8\,a^5+24\,a^4\,b-16\,a^3\,b^2-16\,a^2\,b^3+24\,a\,b^4-8\,b^5)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+(4\,a^5-4\,a^4\,b-8\,a^3\,b^2+8\,a^2\,b^3+4\,a\,b^4-4\,b^5)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,(a+4\,b)}{d\,(2\,a^4+8\,a^3\,b+12\,a^2\,b^2+8\,a\,b^3+2\,b^4)}+\frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)\,(10\,a^2\,b^3+2\,b^5)}{d\,(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a*sin(c + d*x) + b*tan(c + d*x))^3,x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*d*(a - b)^3) - ((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(2*
(a + b)) + (tan(c/2 + (d*x)/2)^4*(85*a*b^4 - 5*a^4*b + a^5 + 15*b^5 - 10*a^
2*b^3 + 10*a^3*b^2))/(2*(a + b)*(2*a*b + a^2 + b^2)) - (tan(c/2 + (d*x)/2)^
2*(45*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2))/((a - b)*(2*a
*b + a^2 + b^2)))/(d*(tan(c/2 + (d*x)/2)^2*(4*a*b^4 - 4*a^4*b + 4*a^5 - 4*b
^5 + 8*a^2*b^3 - 8*a^3*b^2) - tan(c/2 + (d*x)/2)^4*(8*a^5 - 24*a^4*b - 24*a
*b^4 + 8*b^5 + 16*a^2*b^3 + 16*a^3*b^2) + tan(c/2 + (d*x)/2)^6*(20*a*b^4 -
```

```
20*a^4*b + 4*a^5 - 4*b^5 - 40*a^2*b^3 + 40*a^3*b^2))) - (log(tan(c/2 + (d*x
)/2))*(a + 4*b))/(d*(8*a*b^3 + 8*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2)) + (lo
g(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(2*b^5 + 10*a^2*
b^3))/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2))
```

3.266 $\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=211

$$\frac{b^4}{2a\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2}-\frac{4ab^3}{\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))}-\frac{\left(a(a^2+3b^2)-b(3a^2+b^2)\cos(c+dx)\right)}{2\left(a^2-b^2\right)^3 d}$$

```
[Out] 1/2*b^4/a/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2-4*a*b^3/(a^2-b^2)^3/d/(b+a*cos(d
*x+c))-1/2*(a*(a^2+3*b^2)-b*(3*a^2+b^2)*cos(d*x+c))*csc(d*x+c)^2/(a^2-b^2)^
3/d-3/4*b*ln(1-cos(d*x+c))/(a+b)^4/d+3/4*b*ln(1+cos(d*x+c))/(a-b)^4/d-6*a*b
^2*(a^2+b^2)*ln(b+a*cos(d*x+c))/(a^2-b^2)^4/d
```

Rubi [A]
time = 0.53, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {4482, 2916, 12, 1661, 1643}

$$-\frac{6ab^2(a^2+b^2)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^4}-\frac{\csc^2(c+dx)\left(a(a^2+3b^2)-b(3a^2+b^2)\cos(c+dx)\right)}{2d\left(a^2-b^2\right)^3}+\frac{b^4}{2ad\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}-\frac{4ab^3}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{3b\log(1-\cos(c+dx))}{4d(a+b)^4}+\frac{3b\log(\cos(c+dx)+1)}{4d(a-b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] b^4/(2*a*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) - (4*a*b^3)/((a^2 - b^2)^3
*d*(b + a*Cos[c + d*x])) - ((a*(a^2 + 3*b^2) - b*(3*a^2 + b^2)*Cos[c + d*x]
)*Csc[c + d*x]^2)/(2*(a^2 - b^2)^3*d) - (3*b*Log[1 - Cos[c + d*x]])/(4*(a +
 b)^4*d) + (3*b*Log[1 + Cos[c + d*x]])/(4*(a - b)^4*d) - (6*a*b^2*(a^2 + b^
2)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
```

```
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \int \frac{\cos(c+dx)\cot^3(c+dx)}{(b+a\cos(c+dx))^3}\,dx$$

$$= -\frac{a^3\text{Subst}\left(\int \frac{x^4}{a^4(b+x)^3(a^2-x^2)^2}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{x^4}{(b+x)^3(a^2-x^2)^2}\,dx, x, a\cos(c+dx)\right)}{ad}$$

$$= -\frac{(a(a^2+3b^2)-b(3a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{2\,(a^2-b^2)^3\,d} - \frac{\text{Subst}\Bigg(\int}{}$$

$$= -\frac{(a(a^2+3b^2)-b(3a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{2\,(a^2-b^2)^3\,d} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{b^4}{2a\,(a^2-b^2)^2\,d(b+a\cos(c+dx))^2} - \frac{4ab^3}{(a^2-b^2)^3\,d(b+a\cos(c+dx}$$

Mathematica [A]
time = 5.62, size = 184, normalized size = 0.87

$$\frac{\frac{4b^4}{a(a-b)^2(a+b)^2(b+a\cos(c+dx))^2}+\frac{32ab^3}{(-a+b)^3(a+b)^3(b+a\cos(c+dx))}-\frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3}+\frac{12b\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{(a-b)^4}-\frac{48ab^2(a^2+b^2)\log(b+a\cos(c+dx))}{(a^2-b^2)^4}-\frac{12b\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{(a+b)^4}+\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{(-a+b)^3}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] ((4*b^4)/(a*(a - b)^2*(a + b)^2*(b + a*Cos[c + d*x])^2) + (32*a*b^3)/((-a +
 b)^3*(a + b)^3*(b + a*Cos[c + d*x])) - Csc[(c + d*x)/2]^2/(a + b)^3 + (12*
b*Log[Cos[(c + d*x)/2]])/(a - b)^4 - (48*a*b^2*(a^2 + b^2)*Log[b + a*Cos[c
+ d*x]])/(a^2 - b^2)^4 - (12*b*Log[Sin[(c + d*x)/2]])/(a + b)^4 + Sec[(c +
d*x)/2]^2/(-a + b)^3)/(8*d)
```

Maple [A]
time = 0.69, size = 176, normalized size = 0.83

method	result
derivativedivides	$\frac{\frac{b^4}{2(a+b)^2(a-b)^2a(b+a\cos(dx+c))^2}-\frac{4a\,b^3}{(a+b)^3(a-b)^3(b+a\cos(dx+c))}-\frac{6a\,b^2\left(a^2+b^2\right)\ln(b+a\cos(dx+c))}{(a+b)^4(a-b)^4}+\frac{1}{4(a+b)^3(\cos(dx+c)-1)}-\frac{3b\,\mathrm{l}}{}}{d}$
default	$\frac{\frac{b^4}{2(a+b)^2(a-b)^2a(b+a\cos(dx+c))^2}-\frac{4a\,b^3}{(a+b)^3(a-b)^3(b+a\cos(dx+c))}-\frac{6a\,b^2\left(a^2+b^2\right)\ln(b+a\cos(dx+c))}{(a+b)^4(a-b)^4}+\frac{1}{4(a+b)^3(\cos(dx+c)-1)}-\frac{3b\,\mathrm{l}}{}}{d}$
risch	$\frac{3ibx}{2(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4)}+\frac{3ibc}{2d(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4)}-\frac{3ibx}{2(a^4-4a^3b+6a^2b^2-4a\,b^3+b^4)}-\frac{3}{2d(a^4-4a^3b+6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*b^4/(a+b)^2/(a-b)^2/a/(b+a*cos(d*x+c))^2-4*a*b^3/(a+b)^3/(a-b)^3/(
b+a*cos(d*x+c))-6*a*b^2*(a^2+b^2)/(a+b)^4/(a-b)^4*ln(b+a*cos(d*x+c))+1/4/(a
+b)^3/(cos(d*x+c)-1)-3/4*b/(a+b)^4*ln(cos(d*x+c)-1)-1/4/(a-b)^3/(1+cos(d*x+
c))+3/4*b/(a-b)^4*ln(1+cos(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(203) = 406$.
time = 0.32, size = 593, normalized size = 2.81

$$-\frac{\frac{48\,(a^3b^2+ab^4)\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}+\frac{12\,b\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{a^6-2\,a^5b-a^4b^2+4\,a^3b^3-a^2b^4-2\,ab^5+b^6-\frac{2\left(a^6-4\,a^5b+5\,a^4b^2-32\,a^3b^3-37\,a^2b^4-4\,ab^5-9\,b^6\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\left(a^6-6\,a^5b+15\,a^4b^2-84\,a^3b^3+63\,a^2b^4-6\,ab^5+17\,b^6\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{\frac{\left(a^9+a^8b-4\,a^7b^2-4\,a^6b^3+6\,a^5b^4+6\,a^4b^5-4\,a^3b^6-4\,a^2b^7+ab^8+b^9\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2\left(a^9-a^8b-4\,a^7b^2+4\,a^6b^3+6\,a^5b^4-6\,a^4b^5-4\,a^3b^6+4\,a^2b^7+ab^8-b^9\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\left(a^9-3\,a^8b+8\,a^6b^3-6\,a^5b^4-6\,a^4b^5+8\,a^3b^6-3\,ab^8+b^9\right)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\sin(dx+c)^2}{(a^3-3\,a^2b+3\,ab^2-b^3)(\cos(dx+c)+1)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/8*(48*(a^3*b^2 + a*b^4)*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + 12*b*log(sin(d*x
 + c)/(cos(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + (a^
6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*(a^6 - 4*a^
5*b + 5*a^4*b^2 - 32*a^3*b^3 - 37*a^2*b^4 - 4*a*b^5 - 9*b^6)*sin(d*x + c)^2
/(cos(d*x + c) + 1)^2 + (a^6 - 6*a^5*b + 15*a^4*b^2 - 84*a^3*b^3 + 63*a^2*b
^4 - 6*a*b^5 + 17*b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/((a^9 + a^8*b -
 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*
```

```
b^8 + b^9)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(a^9 - a^8*b - 4*a^7*b^2
 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)
*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^9 - 3*a^8*b + 8*a^6*b^3 - 6*a^5*b
^4 - 6*a^4*b^5 + 8*a^3*b^6 - 3*a*b^8 + b^9)*sin(d*x + c)^6/(cos(d*x + c) +
1)^6) + sin(d*x + c)^2/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(cos(d*x + c) + 1)^
2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 994 vs. $2(203) = 406$.
time = 3.21, size = 994, normalized size = 4.71

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^6*b^2 + 18*a^4*b^4 - 18*a^2*b^6 - 2*b^8 - 6*(a^7*b + 2*a^5*b^3 - 3
*a^3*b^5)*cos(d*x + c)^3 + 2*(a^8 - 4*a^6*b^2 - 6*a^4*b^4 + 8*a^2*b^6 + b^8
)*cos(d*x + c)^2 + 2*(2*a^7*b + 9*a^5*b^3 - 12*a^3*b^5 + a*b^7)*cos(d*x + c
) + 24*(a^4*b^4 + a^2*b^6 - (a^6*b^2 + a^4*b^4)*cos(d*x + c)^4 - 2*(a^5*b^3
 + a^3*b^5)*cos(d*x + c)^3 + (a^6*b^2 - a^2*b^6)*cos(d*x + c)^2 + 2*(a^5*b^
3 + a^3*b^5)*cos(d*x + c))*log(a*cos(d*x + c) + b) - 3*(a^5*b^3 + 4*a^4*b^4
 + 6*a^3*b^5 + 4*a^2*b^6 + a*b^7 - (a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4*a^4*b
^4 + a^3*b^5)*cos(d*x + c)^4 - 2*(a^6*b^2 + 4*a^5*b^3 + 6*a^4*b^4 + 4*a^3*b
^5 + a^2*b^6)*cos(d*x + c)^3 + (a^7*b + 4*a^6*b^2 + 5*a^5*b^3 - 5*a^3*b^5 -
 4*a^2*b^6 - a*b^7)*cos(d*x + c)^2 + 2*(a^6*b^2 + 4*a^5*b^3 + 6*a^4*b^4 + 4
*a^3*b^5 + a^2*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 3*(a^5*b^3
- 4*a^4*b^4 + 6*a^3*b^5 - 4*a^2*b^6 + a*b^7 - (a^7*b - 4*a^6*b^2 + 6*a^5*b^
3 - 4*a^4*b^4 + a^3*b^5)*cos(d*x + c)^4 - 2*(a^6*b^2 - 4*a^5*b^3 + 6*a^4*b^
4 - 4*a^3*b^5 + a^2*b^6)*cos(d*x + c)^3 + (a^7*b - 4*a^6*b^2 + 5*a^5*b^3 -
5*a^3*b^5 + 4*a^2*b^6 - a*b^7)*cos(d*x + c)^2 + 2*(a^6*b^2 - 4*a^5*b^3 + 6*
a^4*b^4 - 4*a^3*b^5 + a^2*b^6)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/
((a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^4 + 2*
(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^3 - (
a^11 - 5*a^9*b^2 + 10*a^7*b^4 - 10*a^5*b^6 + 5*a^3*b^8 - a*b^10)*d*cos(d*x
+ c)^2 - 2*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x
 + c) - (a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(cos(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 690 vs. 2(203) = 406.

time = 0.97, size = 690, normalized size = 3.27

$$-\frac{\frac{6\,b\log\left(\frac{\left|-\cos(dx+c)+1\right|}{\left|\cos(dx+c)+1\right|}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{48\,(a^3b^2+ab^4)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}-\frac{\left(a+b+\frac{6\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)(\cos(dx+c)-1)}-\frac{\cos(dx+c)-1}{(a^3-3\,a^2b+3\,ab^2-b^3)(\cos(dx+c)+1)}-\frac{8\left(9\,a^5b^2+10\,a^4b^3+2\,a^3b^4+8\,a^2b^5+5\,ab^6-2\,b^7+\frac{18\,a^5b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{8\,a^4b^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,a^3b^4(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{6\,a^2b^5(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{16\,ab^6(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2\,b^7(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9\,a^5b^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{18\,a^4b^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{18\,a^3b^4(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{18\,a^2b^5(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{9\,ab^6(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)\left(a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(6*b*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 + 4*a^3*b
+ 6*a^2*b^2 + 4*a*b^3 + b^4) + 48*(a^3*b^2 + a*b^4)*log(abs(-a - b - a*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - (a + b + 6*b*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^
2 + 4*a*b^3 + b^4)*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/((a^3 - 3*a^2*b
 + 3*a*b^2 - b^3)*(cos(d*x + c) + 1)) - 8*(9*a^5*b^2 + 10*a^4*b^3 + 2*a^3*b
^4 + 8*a^2*b^5 + 5*a*b^6 - 2*b^7 + 18*a^5*b^2*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) - 8*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a^3*b^4*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + 6*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) - 16*a*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*b^7*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + 9*a^5*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 - 18*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 18*a^3*b^4
*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 18*a^2*b^5*(cos(d*x + c) - 1)^
2/(cos(d*x + c) + 1)^2 + 9*a*b^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)
/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b + a*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2))/d
```

Mupad [B]

time = 1.11, size = 490, normalized size = 2.32

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(a^5-5\,a^4\,b+10\,a^3\,b^2-42\,a^2\,b^3+5\,a\,b^4-9\,b^5)}{(a-b)\,(a^2+2\,a\,b+b^2)}-\frac{a^3-3\,a^2\,b+3\,a\,b^2-b^3}{2\,(a+b)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,(-a^5+5\,a^4\,b-10\,a^3\,b^2+74\,a^2\,b^3+11\,a\,b^4+17\,b^5)}{2\,(a+b)\,(a^2+2\,a\,b+b^2)}}{d\,\left((4\,a^5-20\,a^4\,b+40\,a^3\,b^2-40\,a^2\,b^3+20\,a\,b^4-4\,b^5)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+(-8\,a^5+24\,a^4\,b-16\,a^3\,b^2-16\,a^2\,b^3+24\,a\,b^4-8\,b^5)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+(4\,a^5-4\,a^4\,b-8\,a^3\,b^2+8\,a^2\,b^3+4\,a\,b^4-4\,b^5)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d\,(a-b)^3}-\frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)\,(6\,a^3\,b^2+6\,a\,b^4)}{d\,(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8)}-\frac{3\,b\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d\,(2\,a^4+8\,a^3\,b+12\,a^2\,b^2+8\,a\,b^3+2\,b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^2*(5*a*b^4 - 5*a^4*b + a^5 - 9*b^5 - 42*a^2*b^3 + 10*a
^3*b^2))/((a - b)*(2*a*b + a^2 + b^2)) - (3*a*b^2 - 3*a^2*b + a^3 - b^3)/(2
*(a + b)) + (tan(c/2 + (d*x)/2)^4*(11*a*b^4 + 5*a^4*b - a^5 + 17*b^5 + 74*a
^2*b^3 - 10*a^3*b^2))/(2*(a + b)*(2*a*b + a^2 + b^2)))/(d*(tan(c/2 + (d*x)/
2)^2*(4*a*b^4 - 4*a^4*b + 4*a^5 - 4*b^5 + 8*a^2*b^3 - 8*a^3*b^2) - tan(c/2
+ (d*x)/2)^4*(8*a^5 - 24*a^4*b - 24*a*b^4 + 8*b^5 + 16*a^2*b^3 + 16*a^3*b^2
) + tan(c/2 + (d*x)/2)^6*(20*a*b^4 - 20*a^4*b + 4*a^5 - 4*b^5 - 40*a^2*b^3
+ 40*a^3*b^2))) - tan(c/2 + (d*x)/2)^2/(8*d*(a - b)^3) - (log(a + b - a*tan
(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(6*a*b^4 + 6*a^3*b^2))/(d*(a^8
+ b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (3*b*log(tan(c/2 + (d*x)/2)))
/(d*(8*a*b^3 + 8*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2))
```

3.267 $\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=229

$$-\frac{b^3}{2\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2}+\frac{b^2(3a^2+b^2)}{\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))}+\frac{(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx}{2\left(a^2-b^2\right)^3 d}$$

```
[Out] -1/2*b^3/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2+b^2*(3*a^2+b^2)/(a^2-b^2)^3/d/(b+
a*cos(d*x+c))+1/2*(b*(3*a^2+b^2)-a*(a^2+3*b^2)*cos(d*x+c))*csc(d*x+c)^2/(a^
2-b^2)^3/d+1/4*(a-2*b)*ln(1-cos(d*x+c))/(a+b)^4/d-1/4*(a+2*b)*ln(1+cos(d*x+
c))/(a-b)^4/d+b*(3*a^4+8*a^2*b^2+b^4)*ln(b+a*cos(d*x+c))/(a^2-b^2)^4/d
```

Rubi [A]
time = 0.38, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {4482, 2800, 1661, 1643}

$$\frac{b^2(3a^2+b^2)}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}+\frac{\csc^2(c+dx)\left(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx)\right)}{2d\left(a^2-b^2\right)^3}-\frac{b^3}{2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}+\frac{b(3a^4+8a^2b^2+b^4)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^4}+\frac{(a-2b)\log(1-\cos(c+dx))}{4d(a+b)^4}-\frac{(a+2b)\log(\cos(c+dx)+1)}{4d(a-b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[c + d*x] + b*Tan[c + d*x])^(-3),x]
```

```
[Out] -1/2*b^3/((a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) + (b^2*(3*a^2 + b^2))/((a
^2 - b^2)^3*d*(b + a*Cos[c + d*x])) + ((b*(3*a^2 + b^2) - a*(a^2 + 3*b^2)*C
os[c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)^3*d) + ((a - 2*b)*Log[1 - Cos[c
 + d*x]])/(4*(a + b)^4*d) - ((a + 2*b)*Log[1 + Cos[c + d*x]])/(4*(a - b)^4*
d) + (b*(3*a^4 + 8*a^2*b^2 + b^4)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d
)
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2800

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \int \frac{\cot^3(c+dx)}{(b+a\cos(c+dx))^3}\,dx$$

$$= -\frac{\text{Subst}\left(\int \frac{x^3}{(b+x)^3(a^2-x^2)^2}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= \frac{\left(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} - \frac{\text{Subst}\left(\int -\ldots\right.}{\ldots}$$

$$= \frac{\left(b(3a^2+b^2)-a(a^2+3b^2)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} - \frac{\text{Subst}\left(\int \left(\frac{\ldots}{2(\ldots}\right.\right.}{\ldots}$$

$$= -\frac{b^3}{2\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2} + \frac{b^2(3a^2+b^2)}{\left(a^2-b^2\right)^3 d(b+a\cos(c+dx)\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.34, size = 696, normalized size = 3.04

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[c + d*x] + b*Tan[c + d*x])^(-3),x]
```

```
[Out] -1/2*(b^3*(b + a*Cos[c + d*x])*Tan[c + d*x]^3)/((-a + b)^2*(a + b)^2*d*(a*S
in[c + d*x] + b*Tan[c + d*x])^3) - (b^2*(3*a^2 + b^2)*(b + a*Cos[c + d*x])^
2*Tan[c + d*x]^3)/((-a + b)^3*(a + b)^3*d*(a*Sin[c + d*x] + b*Tan[c + d*x])
^3) - ((2*I)*(3*a^4*b + 8*a^2*b^3 + b^5)*(c + d*x)*(b + a*Cos[c + d*x])^3*T
an[c + d*x]^3)/((a - b)^4*(a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3)
- ((I/2)*(-a - 2*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])^3*Tan[c + d*x
```

```
]^3)/((-a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) - ((I/2)*(a - 2*b)*
ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])^3*Tan[c + d*x]^3)/((a + b)^4*d*(a
*Sin[c + d*x] + b*Tan[c + d*x])^3) - ((b + a*Cos[c + d*x])^3*Csc[(c + d*x)/
2]^2*Tan[c + d*x]^3)/(8*(a + b)^3*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) +
((-a - 2*b)*(b + a*Cos[c + d*x])^3*Log[Cos[(c + d*x)/2]^2]*Tan[c + d*x]^3)/
(4*(-a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) + ((3*a^4*b + 8*a^2*b^
3 + b^5)*(b + a*Cos[c + d*x])^3*Log[b + a*Cos[c + d*x]]*Tan[c + d*x]^3)/((-
a^2 + b^2)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) + ((a - 2*b)*(b + a*Cos
[c + d*x])^3*Log[Sin[(c + d*x)/2]^2]*Tan[c + d*x]^3)/(4*(a + b)^4*d*(a*Sin[
c + d*x] + b*Tan[c + d*x])^3) - ((b + a*Cos[c + d*x])^3*Sec[(c + d*x)/2]^2*
Tan[c + d*x]^3)/(8*(-a + b)^3*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3)
```

Maple [A]
time = 0.69, size = 196, normalized size = 0.86 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(d*x+c)+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/2*b^3/(a+b)^2/(a-b)^2/(b+a*cos(d*x+c))^2+b*(3*a^4+8*a^2*b^2+b^4)/(a
+b)^4/(a-b)^4*ln(b+a*cos(d*x+c))+b^2*(3*a^2+b^2)/(a+b)^3/(a-b)^3/(b+a*cos(d
*x+c))+1/4/(a+b)^3/(cos(d*x+c)-1)+1/4*(a-2*b)/(a+b)^4*ln(cos(d*x+c)-1)+1/4/
(a-b)^3/(1+cos(d*x+c))+1/4/(a-b)^4*(-a-2*b)*ln(1+cos(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(221) = 442$.
time = 0.31, size = 601, normalized size = 2.62

$$\frac{\frac{8\,(3\,a^4b+8\,a^2b^3+b^5)\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}+\frac{4\,(a-2\,b)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{a^6-2\,a^5b-a^4b^2+4\,a^3b^3-a^2b^4-2\,ab^5+b^6-\frac{2\,(a^6-4\,a^5b+29\,a^4b^2+24\,a^3b^3+11\,a^2b^4+20\,ab^5-b^6)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{(a^6-6\,a^5b+63\,a^4b^2-52\,a^3b^3+31\,a^2b^4-38\,ab^5+b^6)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{\frac{(a^9+a^8b-4\,a^7b^2-4\,a^6b^3+6\,a^5b^4+6\,a^4b^5-4\,a^3b^6-4\,a^2b^7+ab^8+b^9)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2\,(a^9-a^8b-4\,a^7b^2+4\,a^6b^3+6\,a^5b^4-6\,a^4b^5-4\,a^3b^6+4\,a^2b^7+ab^8-b^9)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{(a^9-3\,a^8b+8\,a^6b^3-6\,a^5b^4-6\,a^4b^5+8\,a^3b^6-3\,ab^8+b^9)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\sin(dx+c)^2}{(a^3-3\,a^2b+3\,ab^2-b^3)(\cos(dx+c)+1)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/8*(8*(3*a^4*b + 8*a^2*b^3 + b^5)*log(a + b - (a - b)*sin(d*x + c)^2/(cos(
d*x + c) + 1)^2)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + 4*(a - 2
*b)*log(sin(d*x + c)/(cos(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b
^3 + b^4) - (a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6
- 2*(a^6 - 4*a^5*b + 29*a^4*b^2 + 24*a^3*b^3 + 11*a^2*b^4 + 20*a*b^5 - b^6)
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + (a^6 - 6*a^5*b + 63*a^4*b^2 - 52*a^3
*b^3 + 31*a^2*b^4 - 38*a*b^5 + b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/((
a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4
*a^2*b^7 + a*b^8 + b^9)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(a^9 - a^8*
b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 +
 a*b^8 - b^9)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^9 - 3*a^8*b + 8*a^6*
b^3 - 6*a^5*b^4 - 6*a^4*b^5 + 8*a^3*b^6 - 3*a*b^8 + b^9)*sin(d*x + c)^6/(co
s(d*x + c) + 1)^6) + sin(d*x + c)^2/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(cos(d
*x + c) + 1)^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. $2(221) = 442$.
time = 4.46, size = 1071, normalized size = 4.68

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(16*a^4*b^3 - 8*a^2*b^5 - 8*b^7 - 2*(a^7 + 8*a^5*b^2 - 7*a^3*b^4 - 2*a
*b^6)*cos(d*x + c)^3 + 2*(a^6*b - 11*a^4*b^3 + 7*a^2*b^5 + 3*b^7)*cos(d*x +
 c)^2 + 2*(11*a^5*b^2 - 10*a^3*b^4 - a*b^6)*cos(d*x + c) + 4*(3*a^4*b^3 + 8
*a^2*b^5 + b^7 - (3*a^6*b + 8*a^4*b^3 + a^2*b^5)*cos(d*x + c)^4 - 2*(3*a^5*
b^2 + 8*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (3*a^6*b + 5*a^4*b^3 - 7*a^2*b^5
- b^7)*cos(d*x + c)^2 + 2*(3*a^5*b^2 + 8*a^3*b^4 + a*b^6)*cos(d*x + c))*log
(a*cos(d*x + c) + b) - (a^5*b^2 + 6*a^4*b^3 + 14*a^3*b^4 + 16*a^2*b^5 + 9*a
*b^6 + 2*b^7 - (a^7 + 6*a^6*b + 14*a^5*b^2 + 16*a^4*b^3 + 9*a^3*b^4 + 2*a^2
*b^5)*cos(d*x + c)^4 - 2*(a^6*b + 6*a^5*b^2 + 14*a^4*b^3 + 16*a^3*b^4 + 9*a
^2*b^5 + 2*a*b^6)*cos(d*x + c)^3 + (a^7 + 6*a^6*b + 13*a^5*b^2 + 10*a^4*b^3
 - 5*a^3*b^4 - 14*a^2*b^5 - 9*a*b^6 - 2*b^7)*cos(d*x + c)^2 + 2*(a^6*b + 6*
a^5*b^2 + 14*a^4*b^3 + 16*a^3*b^4 + 9*a^2*b^5 + 2*a*b^6)*cos(d*x + c))*log(
1/2*cos(d*x + c) + 1/2) + (a^5*b^2 - 6*a^4*b^3 + 14*a^3*b^4 - 16*a^2*b^5 +
9*a*b^6 - 2*b^7 - (a^7 - 6*a^6*b + 14*a^5*b^2 - 16*a^4*b^3 + 9*a^3*b^4 - 2*
a^2*b^5)*cos(d*x + c)^4 - 2*(a^6*b - 6*a^5*b^2 + 14*a^4*b^3 - 16*a^3*b^4 +
9*a^2*b^5 - 2*a*b^6)*cos(d*x + c)^3 + (a^7 - 6*a^6*b + 13*a^5*b^2 - 10*a^4*
b^3 - 5*a^3*b^4 + 14*a^2*b^5 - 9*a*b^6 + 2*b^7)*cos(d*x + c)^2 + 2*(a^6*b -
 6*a^5*b^2 + 14*a^4*b^3 - 16*a^3*b^4 + 9*a^2*b^5 - 2*a*b^6)*cos(d*x + c))*l
og(-1/2*cos(d*x + c) + 1/2))/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a
^2*b^8)*d*cos(d*x + c)^4 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a
*b^9)*d*cos(d*x + c)^3 - (a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^
2*b^8 - b^10)*d*cos(d*x + c)^2 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b
^7 + a*b^9)*d*cos(d*x + c) - (a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 +
 b^10)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**(-3), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 800 vs. 2(221) = 442.

time = 0.54, size = 800, normalized size = 3.49

$$\frac{\frac{2\,(a-2\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{8\,(3\,a^4b+8\,a^2b^3+b^5)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}+\frac{\left(a+b-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)(\cos(dx+c)-1)}-\frac{\cos(dx+c)-1}{(a^3-3\,a^2b+3\,ab^2-b^3)(\cos(dx+c)+1)}-\frac{4\left(9\,a^6b+6\,a^5b^2+9\,a^4b^3+28\,a^3b^4+11\,a^2b^5-2\,ab^6+3\,b^7+\frac{18\,a^6b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{12\,a^5b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{26\,a^4b^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{4\,a^3b^4(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{38\,a^2b^5(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{8\,ab^6(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{6\,b^7(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9\,a^6b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{18\,a^5b^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{33\,a^4b^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{48\,a^3b^4(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{27\,a^2b^5(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{6\,ab^6(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{3\,b^7(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)\left(a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(2*(a - 2*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 + 4
*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 8*(3*a^4*b + 8*a^2*b^3 + b^5)*log(abs
(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1)))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + (a + b
 - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4*b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b
^4)*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/((a^3 - 3*a^2*b + 3*a*b^2 - b^
3)*(cos(d*x + c) + 1)) - 4*(9*a^6*b + 6*a^5*b^2 + 9*a^4*b^3 + 28*a^3*b^4 +
11*a^2*b^5 - 2*a*b^6 + 3*b^7 + 18*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) - 12*a^5*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 26*a^4*b^3*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 4*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) - 38*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*a*b^6*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) - 6*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
+ 9*a^6*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 18*a^5*b^2*(cos(d*x +
 c) - 1)^2/(cos(d*x + c) + 1)^2 + 33*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x
+ c) + 1)^2 - 48*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 27*a^2
*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6*a*b^6*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 + 3*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/
((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b + a*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2))/d
```

Mupad [B]

time = 1.13, size = 494, normalized size = 2.16

$$\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2}{8\,d\,{\left(a-b\right)}^3}-\frac{\frac{a^3-3\,a^2\,b+3\,a\,b^2-b^3}{2\,\left(a+b\right)}+\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(a^5-5\,a^4\,b+58\,a^3\,b^2+6\,a^2\,b^3+37\,a\,b^4-b^5\right)}{2\,\left(a+b\right)\,\left(a^2+2\,a\,b+b^2\right)}-\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(a^5-5\,a^4\,b+34\,a^3\,b^2-10\,a^2\,b^3+21\,a\,b^4-b^5\right)}{\left(a-b\right)\,\left(a^2+2\,a\,b+b^2\right)}}{d\,\left(\left(4\,a^5-20\,a^4\,b+40\,a^3\,b^2-40\,a^2\,b^3+20\,a\,b^4-4\,b^5\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+\left(-8\,a^5+24\,a^4\,b-16\,a^3\,b^2-16\,a^2\,b^3+24\,a\,b^4-8\,b^5\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+\left(4\,a^5-4\,a^4\,b-8\,a^3\,b^2+8\,a^2\,b^3+4\,a\,b^4-4\,b^5\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)}+\frac{\ln\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(a-2\,b\right)}{d\,\left(2\,a^4+8\,a^3\,b+12\,a^2\,b^2+8\,a\,b^3+2\,b^4\right)}+\frac{\ln\left(a+b-a\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+b\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\right)\,\left(3\,a^4\,b+8\,a^2\,b^3+b^5\right)}{d\,\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(c + d*x) + b*tan(c + d*x))^3,x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*d*(a - b)^3) - ((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(2*
(a + b)) + (tan(c/2 + (d*x)/2)^4*(37*a*b^4 - 5*a^4*b + a^5 - b^5 + 6*a^2*b^
3 + 58*a^3*b^2))/(2*(a + b)*(2*a*b + a^2 + b^2)) - (tan(c/2 + (d*x)/2)^2*(2
1*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 34*a^3*b^2))/((a - b)*(2*a*b +
 a^2 + b^2)))/(d*(tan(c/2 + (d*x)/2)^2*(4*a*b^4 - 4*a^4*b + 4*a^5 - 4*b^5 +
 8*a^2*b^3 - 8*a^3*b^2) - tan(c/2 + (d*x)/2)^4*(8*a^5 - 24*a^4*b - 24*a*b^4
 + 8*b^5 + 16*a^2*b^3 + 16*a^3*b^2) + tan(c/2 + (d*x)/2)^6*(20*a*b^4 - 20*a
```

```
^4*b + 4*a^5 - 4*b^5 - 40*a^2*b^3 + 40*a^3*b^2))) + (log(tan(c/2 + (d*x)/2)
)*(a - 2*b))/(d*(8*a*b^3 + 8*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2)) + (log(a
+ b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(3*a^4*b + b^5 + 8*a
^2*b^3))/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2))
```

3.268 $\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=231

$$\frac{ab^2}{2\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2}-\frac{2ab(a^2+b^2)}{\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))}-\frac{(a(a^2+3b^2)-b(3a^2+b^2)\cos(c+dx))}{2\left(a^2-b^2\right)^3 d}$$

```
[Out] 1/2*a*b^2/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2-2*a*b*(a^2+b^2)/(a^2-b^2)^3/d/(b
+a*cos(d*x+c))-1/2*(a*(a^2+3*b^2)-b*(3*a^2+b^2)*cos(d*x+c))*csc(d*x+c)^2/(a
^2-b^2)^3/d+1/4*(2*a-b)*ln(1-cos(d*x+c))/(a+b)^4/d+1/4*(2*a+b)*ln(1+cos(d*x
+c))/(a-b)^4/d-a*(a^4+8*a^2*b^2+3*b^4)*ln(b+a*cos(d*x+c))/(a^2-b^2)^4/d
```

Rubi [A]
time = 0.49, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {4482, 2916, 12, 1661, 1643}

$$\frac{ab^2}{2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}-\frac{2ab(a^2+b^2)}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{\csc^2(c+dx)\left(a(a^2+3b^2)-b(3a^2+b^2)\cos(c+dx)\right)}{2d\left(a^2-b^2\right)^3}-\frac{a(a^4+8a^2b^2+3b^4)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^4}+\frac{(2a-b)\log(1-\cos(c+dx))}{4d(a+b)^4}+\frac{(2a+b)\log(\cos(c+dx)+1)}{4d(a-b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*b^2)/(2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) - (2*a*b*(a^2 + b^2))/((
a^2 - b^2)^3*d*(b + a*Cos[c + d*x])) - ((a*(a^2 + 3*b^2) - b*(3*a^2 + b^2)*
Cos[c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)^3*d) + ((2*a - b)*Log[1 - Cos[
c + d*x]])/(4*(a + b)^4*d) + ((2*a + b)*Log[1 + Cos[c + d*x]])/(4*(a - b)^4
*d) - (a*(a^4 + 8*a^2*b^2 + 3*b^4)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*
d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
```

```
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \int \frac{\cot^2(c+dx)\csc(c+dx)}{(b+a\cos(c+dx))^3}\,dx$$

$$= -\frac{a^3\text{Subst}\left(\int \frac{x^2}{a^2(b+x)^3(a^2-x^2)^2}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= -\frac{a\text{Subst}\left(\int \frac{x^2}{(b+x)^3(a^2-x^2)^2}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= -\frac{(a(a^2+3b^2)-b(3a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} - \frac{\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= -\frac{(a(a^2+3b^2)-b(3a^2+b^2)\cos(c+dx))\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} - \frac{\text{Subst}\left(\int \left(\cdots\right.\right.}{\cdots}$$

$$= \frac{ab^2}{2\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2} - \frac{2ab(a^2+b^2)}{\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.33, size = 703, normalized size = 3.04

$$\frac{ab^2(b+a\cos(c+dx))\tan^3(c+dx)}{2(-a+b)^2(a+b)^2d(a\sin(c+dx)+b\tan(c+dx))^3}+\frac{2ab(-ia+b)(ia+b)(b+a\cos(c+dx))^2\tan^3(c+dx)}{(-a+b)^3(a+b)^3d(a\sin(c+dx)+b\tan(c+dx))^3}+\frac{2i(a^5+8a^3b^2+3ab^4)(c+dx)(b+a\cos(c+dx))^3\tan^3(c+dx)}{(a-b)^4(a+b)^4d(a\sin(c+dx)+b\tan(c+dx))^3}-\frac{i(2a-b)\text{ArcTan}(\tan(c+dx))(b+a\cos(c+dx))^3\tan^3(c+dx)}{2(a+b)^4d(a\sin(c+dx)+b\tan(c+dx))^3}-\frac{i(2a+b)\text{ArcTan}(\tan(c+dx))(b+a\cos(c+dx))^3\tan^3(c+dx)}{2(-a+b)^4d(a\sin(c+dx)+b\tan(c+dx))^3}-\frac{(b+a\cos(c+dx))^3\csc^2\left(\frac{1}{2}(c+dx)\right)\tan^3(c+dx)}{8(a+b)^3d(a\sin(c+dx)+b\tan(c+dx))^3}+\frac{(2a+b)(b+a\cos(c+dx))^3\log\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\right)\tan^3(c+dx)}{4(-a+b)^4d(a\sin(c+dx)+b\tan(c+dx))^3}+\frac{(-a^5-8a^3b^2-3ab^4)(b+a\cos(c+dx))^3\log(b+a\cos(c+dx))\tan^3(c+dx)}{(-a^2+b^2)^4d(a\sin(c+dx)+b\tan(c+dx))^3}+\frac{(2a-b)(b+a\cos(c+dx))^3\log\left(\sin^2\left(\frac{1}{2}(c+dx)\right)\right)\tan^3(c+dx)}{4(a+b)^4d(a\sin(c+dx)+b\tan(c+dx))^3}+\frac{(b+a\cos(c+dx))^3\sec^2\left(\frac{1}{2}(c+dx)\right)\tan^3(c+dx)}{8(-a+b)^3d(a\sin(c+dx)+b\tan(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*b^2*(b + a*Cos[c + d*x])*Tan[c + d*x]^3)/(2*(-a + b)^2*(a + b)^2*d*(a*Si
n[c + d*x] + b*Tan[c + d*x])^3) + (2*a*b*((-I)*a + b)*(I*a + b)*(b + a*Cos[
c + d*x])^2*Tan[c + d*x]^3)/((-a + b)^3*(a + b)^3*d*(a*Sin[c + d*x] + b*Tan
[c + d*x])^3) + ((2*I)*(a^5 + 8*a^3*b^2 + 3*a*b^4)*(c + d*x)*(b + a*Cos[c +
 d*x])^3*Tan[c + d*x]^3)/((a - b)^4*(a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c +
 d*x])^3) - ((I/2)*(2*a - b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])^3*Ta
n[c + d*x]^3)/((a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) - ((I/2)*(2
*a + b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])^3*Tan[c + d*x]^3)/((-a +
b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) - ((b + a*Cos[c + d*x])^3*Csc[(
c + d*x)/2]^2*Tan[c + d*x]^3)/(8*(a + b)^3*d*(a*Sin[c + d*x] + b*Tan[c + d*
x])^3) + ((2*a + b)*(b + a*Cos[c + d*x])^3*Log[Cos[(c + d*x)/2]^2]*Tan[c +
d*x]^3)/(4*(-a + b)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) + ((-a^5 - 8*a
^3*b^2 - 3*a*b^4)*(b + a*Cos[c + d*x])^3*Log[b + a*Cos[c + d*x]]*Tan[c + d*
x]^3)/((-a^2 + b^2)^4*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) + ((2*a - b)*(
b + a*Cos[c + d*x])^3*Log[Sin[(c + d*x)/2]^2]*Tan[c + d*x]^3)/(4*(a + b)^4*
d*(a*Sin[c + d*x] + b*Tan[c + d*x])^3) + ((b + a*Cos[c + d*x])^3*Sec[(c + d
*x)/2]^2*Tan[c + d*x]^3)/(8*(-a + b)^3*d*(a*Sin[c + d*x] + b*Tan[c + d*x])^
3)
```

Maple [A]
time = 0.62, size = 196, normalized size = 0.85 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-a*(a^4+8*a^2*b^2+3*b^4)/(a+b)^4/(a-b)^4*ln(b+a*cos(d*x+c))+1/2*b^2/(a
+b)^2*a/(a-b)^2/(b+a*cos(d*x+c))^2-2*a*b*(a^2+b^2)/(a+b)^3/(a-b)^3/(b+a*cos
(d*x+c))+1/4/(a+b)^3/(cos(d*x+c)-1)+1/4*(2*a-b)/(a+b)^4*ln(cos(d*x+c)-1)-1/
4/(a-b)^3/(1+cos(d*x+c))+1/4*(2*a+b)/(a-b)^4*ln(1+cos(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 602 vs. 2(223) = 446.
time = 0.31, size = 602, normalized size = 2.61

$$-\frac{\frac{8\,(a^5+8\,a^3b^2+3\,ab^4)\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}-\frac{4\,(2\,a-b)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{a^6-2\,a^5b-a^4b^2+4\,a^3b^3-a^2b^4-2\,ab^5+b^6-\frac{2\,(a^6-20\,a^5b-11\,a^4b^2-24\,a^3b^3-29\,a^2b^4+4\,ab^5-b^6)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{(a^6-38\,a^5b+31\,a^4b^2-52\,a^3b^3+63\,a^2b^4-6\,ab^5+b^6)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{\frac{(a^9+a^8b-4\,a^7b^2-4\,a^6b^3+6\,a^5b^4+6\,a^4b^5-4\,a^3b^6-4\,a^2b^7+ab^8+b^9)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2\,(a^9-a^8b-4\,a^7b^2+4\,a^6b^3+6\,a^5b^4-6\,a^4b^5-4\,a^3b^6+4\,a^2b^7+ab^8-b^9)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{(a^9-3\,a^8b+8\,a^6b^3-6\,a^5b^4-6\,a^4b^5+8\,a^3b^6-3\,ab^8+b^9)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\sin(dx+c)^2}{(a^3-3\,a^2b+3\,ab^2-b^3)(\cos(dx+c)+1)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/8*(8*(a^5 + 8*a^3*b^2 + 3*a*b^4)*log(a + b - (a - b)*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - 4*(2*a
- b)*log(sin(d*x + c)/(cos(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*
b^3 + b^4) + (a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6
 - 2*(a^6 - 20*a^5*b - 11*a^4*b^2 - 24*a^3*b^3 - 29*a^2*b^4 + 4*a*b^5 - b^6
```

```
)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + (a^6 - 38*a^5*b + 31*a^4*b^2 - 52*a
^3*b^3 + 63*a^2*b^4 - 6*a*b^5 + b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/(
(a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 -
4*a^2*b^7 + a*b^8 + b^9)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(a^9 - a^8
*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7
+ a*b^8 - b^9)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^9 - 3*a^8*b + 8*a^6
*b^3 - 6*a^5*b^4 - 6*a^4*b^5 + 8*a^3*b^6 - 3*a*b^8 + b^9)*sin(d*x + c)^6/(c
os(d*x + c) + 1)^6) + sin(d*x + c)^2/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(cos(
d*x + c) + 1)^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1076 vs. $2(223) = 446$.
time = 3.83, size = 1076, normalized size = 4.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(8*a^5*b^2 + 8*a^3*b^4 - 16*a*b^6 - 2*(7*a^6*b - 2*a^4*b^3 - 5*a^2*b^5)
*cos(d*x + c)^3 + 2*(a^7 - 7*a^5*b^2 - a^3*b^4 + 7*a*b^6)*cos(d*x + c)^2 +
2*(6*a^6*b + a^4*b^3 - 8*a^2*b^5 + b^7)*cos(d*x + c) + 4*(a^5*b^2 + 8*a^3*b
^4 + 3*a*b^6 - (a^7 + 8*a^5*b^2 + 3*a^3*b^4)*cos(d*x + c)^4 - 2*(a^6*b + 8*
a^4*b^3 + 3*a^2*b^5)*cos(d*x + c)^3 + (a^7 + 7*a^5*b^2 - 5*a^3*b^4 - 3*a*b^
6)*cos(d*x + c)^2 + 2*(a^6*b + 8*a^4*b^3 + 3*a^2*b^5)*cos(d*x + c))*log(a*c
os(d*x + c) + b) - (2*a^5*b^2 + 9*a^4*b^3 + 16*a^3*b^4 + 14*a^2*b^5 + 6*a*b
^6 + b^7 - (2*a^7 + 9*a^6*b + 16*a^5*b^2 + 14*a^4*b^3 + 6*a^3*b^4 + a^2*b^5
)*cos(d*x + c)^4 - 2*(2*a^6*b + 9*a^5*b^2 + 16*a^4*b^3 + 14*a^3*b^4 + 6*a^2
*b^5 + a*b^6)*cos(d*x + c)^3 + (2*a^7 + 9*a^6*b + 14*a^5*b^2 + 5*a^4*b^3 -
10*a^3*b^4 - 13*a^2*b^5 - 6*a*b^6 - b^7)*cos(d*x + c)^2 + 2*(2*a^6*b + 9*a^
5*b^2 + 16*a^4*b^3 + 14*a^3*b^4 + 6*a^2*b^5 + a*b^6)*cos(d*x + c))*log(1/2*
cos(d*x + c) + 1/2) - (2*a^5*b^2 - 9*a^4*b^3 + 16*a^3*b^4 - 14*a^2*b^5 + 6*
a*b^6 - b^7 - (2*a^7 - 9*a^6*b + 16*a^5*b^2 - 14*a^4*b^3 + 6*a^3*b^4 - a^2*
b^5)*cos(d*x + c)^4 - 2*(2*a^6*b - 9*a^5*b^2 + 16*a^4*b^3 - 14*a^3*b^4 + 6*
a^2*b^5 - a*b^6)*cos(d*x + c)^3 + (2*a^7 - 9*a^6*b + 14*a^5*b^2 - 5*a^4*b^3
 - 10*a^3*b^4 + 13*a^2*b^5 - 6*a*b^6 + b^7)*cos(d*x + c)^2 + 2*(2*a^6*b - 9
*a^5*b^2 + 16*a^4*b^3 - 14*a^3*b^4 + 6*a^2*b^5 - a*b^6)*cos(d*x + c))*log(-
1/2*cos(d*x + c) + 1/2))/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b
^8)*d*cos(d*x + c)^4 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9
)*d*cos(d*x + c)^3 - (a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^
8 - b^10)*d*cos(d*x + c)^2 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 +
 a*b^9)*d*cos(d*x + c) - (a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^1
0)*d)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec (c+dx)}{(a\sin (c+dx)+b\tan (c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)/(a*sin(c + d*x) + b*tan(c + d*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 801 vs. 2(223) = 446.
time = 0.78, size = 801, normalized size = 3.47

$$\frac{\frac{2\,(2\,a-b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{8\,(a^5+8\,a^3b^2+3\,ab^4)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}+\frac{\left(a+b-\frac{4\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)(\cos(dx+c)-1)}+\frac{\cos(dx+c)-1}{(a^3-3\,a^2b+3\,ab^2-b^3)(\cos(dx+c)+1)}+\frac{4\left(3\,a^7-2\,a^6b+11\,a^5b^2+28\,a^4b^3+9\,a^3b^4+6\,a^2b^5+9\,ab^6+\frac{6\,a^7(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{8\,a^6b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{38\,a^5b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,a^4b^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{26\,a^3b^4(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{12\,a^2b^5(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{18\,ab^6(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a^7(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{6\,a^6b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{27\,a^5b^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{48\,a^4b^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{33\,a^3b^4(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{18\,a^2b^5(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{9\,ab^6(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)\left(a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(2*(2*a - b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 + 4
*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 8*(a^5 + 8*a^3*b^2 + 3*a*b^4)*log(abs
(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1)))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + (a + b
 - 4*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b
^4)*(cos(d*x + c) - 1)) + (cos(d*x + c) - 1)/((a^3 - 3*a^2*b + 3*a*b^2 - b^
3)*(cos(d*x + c) + 1)) + 4*(3*a^7 - 2*a^6*b + 11*a^5*b^2 + 28*a^4*b^3 + 9*a
^3*b^4 + 6*a^2*b^5 + 9*a*b^6 + 6*a^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
- 8*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 38*a^5*b^2*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - 4*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) -
 26*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 12*a^2*b^5*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) - 18*a*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
+ 3*a^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6*a^6*b*(cos(d*x + c) -
 1)^2/(cos(d*x + c) + 1)^2 + 27*a^5*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 - 48*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 33*a^3*b^4*
(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 18*a^2*b^5*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 + 9*a*b^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/
((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b + a*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2))/d
```

Mupad [B]
time = 1.12, size = 496, normalized size = 2.15

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\,(-a^5+37\,a^4\,b+6\,a^3\,b^2+58\,a^2\,b^3-5\,a\,b^4+b^5)}{2\,(a+b)\,(a^2+2\,a\,b+b^2)}-\frac{a^3-3\,a^2\,b+3\,a\,b^2-b^3}{2\,(a+b)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\,(a^5-21\,a^4\,b+10\,a^3\,b^2-34\,a^2\,b^3+5\,a\,b^4-b^5)}{(a-b)\,(a^2+2\,a\,b+b^2)}}{d\,\left((4\,a^5-20\,a^4\,b+40\,a^3\,b^2-40\,a^2\,b^3+20\,a\,b^4-4\,b^5)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+(-8\,a^5+24\,a^4\,b-16\,a^3\,b^2-16\,a^2\,b^3+24\,a\,b^4-8\,b^5)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+(4\,a^5-4\,a^4\,b-8\,a^3\,b^2+8\,a^2\,b^3+4\,a\,b^4-4\,b^5)\,\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{8\,d\,(a-b)^3}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,(2\,a-b)}{d\,(2\,a^4+8\,a^3\,b+12\,a^2\,b^2+8\,a\,b^3+2\,b^4)}-\frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\right)\,(a^5+8\,a^3\,b^2+3\,a\,b^4)}{d\,(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a*sin(c + d*x) + b*tan(c + d*x))^3),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^4*(37*a^4*b - 5*a*b^4 - a^5 + b^5 + 58*a^2*b^3 + 6*a^3
*b^2))/(2*(a + b)*(2*a*b + a^2 + b^2)) - (3*a*b^2 - 3*a^2*b + a^3 - b^3)/(2
*(a + b)) + (tan(c/2 + (d*x)/2)^2*(5*a*b^4 - 21*a^4*b + a^5 - b^5 - 34*a^2*
b^3 + 10*a^3*b^2))/((a - b)*(2*a*b + a^2 + b^2)))/(d*(tan(c/2 + (d*x)/2)^2*
(4*a*b^4 - 4*a^4*b + 4*a^5 - 4*b^5 + 8*a^2*b^3 - 8*a^3*b^2) - tan(c/2 + (d*
x)/2)^4*(8*a^5 - 24*a^4*b - 24*a*b^4 + 8*b^5 + 16*a^2*b^3 + 16*a^3*b^2) + t
an(c/2 + (d*x)/2)^6*(20*a*b^4 - 20*a^4*b + 4*a^5 - 4*b^5 - 40*a^2*b^3 + 40*
a^3*b^2))) - tan(c/2 + (d*x)/2)^2/(8*d*(a - b)^3) + (log(tan(c/2 + (d*x)/2)
)*(2*a - b))/(d*(8*a*b^3 + 8*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2)) - (log(a
+ b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(3*a*b^4 + a^5 + 8*a
^3*b^2))/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2))
```

3.269 $\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=212

$$-\frac{3a^2b}{2\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2}+\frac{3a^2(a^2+3b^2)}{2\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))}+\frac{(b-a\cos(c+dx))\csc^2(c+dx)}{2\left(a^2-b^2\right) d(b+a\cos(c+dx))^2}+3$$

```
[Out] -3/2*a^2*b/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2+3/2*a^2*(a^2+3*b^2)/(a^2-b^2)^3
/d/(b+a*cos(d*x+c))+1/2*(b-a*cos(d*x+c))*csc(d*x+c)^2/(a^2-b^2)/d/(b+a*cos(
d*x+c))^2+3/4*a*ln(1-cos(d*x+c))/(a+b)^4/d-3/4*a*ln(1+cos(d*x+c))/(a-b)^4/d
+6*a^2*b*(a^2+b^2)*ln(b+a*cos(d*x+c))/(a^2-b^2)^4/d
```

Rubi [A]
time = 0.28, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {4482, 2916, 12, 837, 815}

$$\frac{3a^2(a^2+3b^2)}{2d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{3a^2b}{2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}+\frac{6a^2b(a^2+b^2)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^4}+\frac{\csc^2(c+dx)(b-a\cos(c+dx))}{2d\left(a^2-b^2\right)(a\cos(c+dx)+b)^2}+\frac{3a\log(1-\cos(c+dx))}{4d(a+b)^4}-\frac{3a\log(\cos(c+dx)+1)}{4d(a-b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (-3*a^2*b)/(2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) + (3*a^2*(a^2 + 3*b^2
))/(2*(a^2 - b^2)^3*d*(b + a*Cos[c + d*x])) + ((b - a*Cos[c + d*x])*Csc[c +
 d*x]^2)/(2*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) + (3*a*Log[1 - Cos[c + d*
x]])/(4*(a + b)^4*d) - (3*a*Log[1 + Cos[c + d*x]])/(4*(a - b)^4*d) + (6*a^2
*b*(a^2 + b^2)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
```

```
a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \int \frac{\cot(c+dx)\csc^2(c+dx)}{(b+a\cos(c+dx))^3}\,dx$$

$$= -\frac{a^3\text{Subst}\left(\int \frac{x}{a(b+x)^3(a^2-x^2)^2}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= -\frac{a^2\text{Subst}\left(\int \frac{x}{(b+x)^3(a^2-x^2)^2}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= \frac{(b-a\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)\,d(b+a\cos(c+dx))^2} - \frac{\text{Subst}\left(\int \frac{-3a^2b+3a^2x}{(b+x)^3(a^2-x^2)}\,dx, x, a\cos\right.}{2(a^2-b^2)\,d}$$

$$= \frac{(b-a\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)\,d(b+a\cos(c+dx))^2} - \frac{\text{Subst}\left(\int \left(\frac{3a(a-b)}{2(a+b)^3(a-x)} + \frac{3a(a+b}{2(a-b)^3(a}\right.\right.}{}$$

$$= -\frac{3a^2b}{2(a^2-b^2)^2\,d(b+a\cos(c+dx))^2} + \frac{3a^2(a^2+3b^2)}{2(a^2-b^2)^3\,d(b+a\cos(c+d}$$

Mathematica [A]
time = 6.30, size = 217, normalized size = 1.02

$$-\frac{a^2b}{2(-a+b)^2(a+b)^2d(b+a\cos(c+dx))^2} - \frac{a^2(a^2+3b^2)}{(-a+b)^3(a+b)^3d(b+a\cos(c+dx))} - \frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{8(a+b)^3d} - \frac{3a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2(-a+b)^4d} + \frac{6(a^4b+a^2b^3)\log(b+a\cos(c+dx))}{(-a^2+b^2)^4\,d} + \frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2(a+b)^4d} - \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{8(-a+b)^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

[Out] -1/2*(a^2*b)/((-a + b)^2*(a + b)^2*d*(b + a*Cos[c + d*x])^2) - (a^2*(a^2 + 3*b^2))/((-a + b)^3*(a + b)^3*d*(b + a*Cos[c + d*x])) - Csc[(c + d*x)/2]^2/(8*(a + b)^3*d) - (3*a*Log[Cos[(c + d*x)/2]])/(2*(-a + b)^4*d) + (6*(a^4*b + a^2*b^3)*Log[b + a*Cos[c + d*x]])/((-a^2 + b^2)^4*d) + (3*a*Log[Sin[(c + d*x)/2]])/(2*(a + b)^4*d) - Sec[(c + d*x)/2]^2/(8*(-a + b)^3*d)

Maple [A]
time = 0.60, size = 181, normalized size = 0.85

method	result
derivativedivides	$\frac{-\frac{a^2 b}{2(a+b)^2(a-b)^2(b+a\cos(dx+c))^2}+\frac{a^2\left(a^2+3b^2\right)}{(a+b)^3(a-b)^3(b+a\cos(dx+c))}+\frac{6a^2b\left(a^2+b^2\right)\ln(b+a\cos(dx+c))}{(a+b)^4(a-b)^4}+\frac{1}{4(a+b)^3(\cos(dx+c)-1)}+}{d}$
default	$\frac{-\frac{a^2 b}{2(a+b)^2(a-b)^2(b+a\cos(dx+c))^2}+\frac{a^2\left(a^2+3b^2\right)}{(a+b)^3(a-b)^3(b+a\cos(dx+c))}+\frac{6a^2b\left(a^2+b^2\right)\ln(b+a\cos(dx+c))}{(a+b)^4(a-b)^4}+\frac{1}{4(a+b)^3(\cos(dx+c)-1)}+}{d}$
risch	$\frac{3iax}{2(a^4-4a^3b+6a^2b^2-4a\,b^3+b^4)}+\frac{3iac}{2d(a^4-4a^3b+6a^2b^2-4a\,b^3+b^4)}-\frac{3iax}{2(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4)}-\frac{}{2d(a^4+4a^3b}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/d*(-1/2*a^2*b/(a+b)^2/(a-b)^2/(b+a*cos(d*x+c))^2+a^2*(a^2+3*b^2)/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c))+6*a^2*b*(a^2+b^2)/(a+b)^4/(a-b)^4*ln(b+a*cos(d*x+c))+1/4/(a+b)^3/(cos(d*x+c)-1)+3/4/(a+b)^4*a*ln(cos(d*x+c)-1)+1/4/(a-b)^3/(1+cos(d*x+c))-3/4/(a-b)^4*a*ln(1+cos(d*x+c)))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 596 vs. $2(203) = 406$.
time = 0.30, size = 596, normalized size = 2.81

$$\frac{\frac{48\left(a^4b+a^2b^3\right)\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}+\frac{12a\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}-\frac{a^6-2a^5b-a^4b^2+4a^3b^3-a^2b^4-2ab^5+b^6-\frac{2\left(9a^6+4a^5b+37a^4b^2+32a^3b^3-5a^2b^4+4ab^5-b^6\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{\left(17a^6-6a^5b+63a^4b^2-84a^3b^3+15a^2b^4-6ab^5+b^6\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{\frac{\left(a^9+a^8b-4a^7b^2-4a^6b^3+6a^5b^4+6a^4b^5-4a^3b^6-4a^2b^7+ab^8+b^9\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2\left(a^9-a^8b-4a^7b^2+4a^6b^3+6a^5b^4-6a^4b^5-4a^3b^6+4a^2b^7+ab^8-b^9\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\left(a^9-3a^8b+8a^6b^3-6a^5b^4-6a^4b^5+8a^3b^6-3ab^8+b^9\right)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\sin(dx+c)^2}{(a^3-3a^2b+3ab^2-b^3)(\cos(dx+c)+1)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")

[Out] 1/8*(48*(a^4*b + a^2*b^3)*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + 12*a*log(sin(d*x + c)/(cos(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - (a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*(9*a^6 + 4*a^5*b + 37*a^4*b^2 + 32*a^3*b^3 - 5*a^2*b^4 + 4*a*b^5 - b^6)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + (17*a^6 - 6*a^5*b + 63*a^4*b^2 - 84*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/((a^9 + a^8*b - 4*a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 + b^9)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(a^9 - a^8*b - 4*a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*

```
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + (a^9 - 3*a^8*b + 8*a^6*b^3 - 6*a^5*b^
4 - 6*a^4*b^5 + 8*a^3*b^6 - 3*a*b^8 + b^9)*sin(d*x + c)^6/(cos(d*x + c) + 1
)^6) + sin(d*x + c)^2/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(cos(d*x + c) + 1)^2
))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 939 vs. $2(203) = 406$.
time = 2.88, size = 939, normalized size = 4.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^6*b + 18*a^4*b^3 - 18*a^2*b^5 - 2*b^7 - 6*(a^7 + 2*a^5*b^2 - 3*a^
3*b^4)*cos(d*x + c)^3 - 24*(a^4*b^3 - a^2*b^5)*cos(d*x + c)^2 + 2*(2*a^7 +
9*a^5*b^2 - 12*a^3*b^4 + a*b^6)*cos(d*x + c) + 24*(a^4*b^3 + a^2*b^5 - (a^6
*b + a^4*b^3)*cos(d*x + c)^4 - 2*(a^5*b^2 + a^3*b^4)*cos(d*x + c)^3 + (a^6*
b - a^2*b^5)*cos(d*x + c)^2 + 2*(a^5*b^2 + a^3*b^4)*cos(d*x + c))*log(a*cos
(d*x + c) + b) - 3*(a^5*b^2 + 4*a^4*b^3 + 6*a^3*b^4 + 4*a^2*b^5 + a*b^6 - (
a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*cos(d*x + c)^4 - 2*(a^6*b
+ 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*cos(d*x + c)^3 + (a^7 + 4*a^
6*b + 5*a^5*b^2 - 5*a^3*b^4 - 4*a^2*b^5 - a*b^6)*cos(d*x + c)^2 + 2*(a^6*b
+ 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*cos(d*x + c))*log(1/2*cos(d*
x + c) + 1/2) + 3*(a^5*b^2 - 4*a^4*b^3 + 6*a^3*b^4 - 4*a^2*b^5 + a*b^6 - (a
^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*cos(d*x + c)^4 - 2*(a^6*b -
 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*cos(d*x + c)^3 + (a^7 - 4*a^6
*b + 5*a^5*b^2 - 5*a^3*b^4 + 4*a^2*b^5 - a*b^6)*cos(d*x + c)^2 + 2*(a^6*b -
 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*cos(d*x + c))*log(-1/2*cos(d*
x + c) + 1/2))/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*d*cos(
d*x + c)^4 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*
x + c)^3 - (a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10)*
d*cos(d*x + c)^2 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*
cos(d*x + c) - (a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**2/(a*sin(c + d*x) + b*tan(c + d*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 689 vs. $2(203) = 406$.

time = 0.86, size = 689, normalized size = 3.25

$$\frac{\frac{6\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{48\,(a^4b+a^2b^3)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}+\frac{\left(a+b-\frac{6\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)(\cos(dx+c)-1)}-\frac{\cos(dx+c)-1}{(a^3-3\,a^2b+3\,ab^2-b^3)(\cos(dx+c)+1)}+\frac{8\left(2\,a^7-5\,a^6b-8\,a^5b^2-2\,a^4b^3-10\,a^3b^4-9\,a^2b^5+\frac{2\,a^7(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{16\,a^6b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{6\,a^5b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,a^4b^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{8\,a^3b^4(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{18\,a^2b^5(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{9\,a^6b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{18\,a^5b^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{18\,a^4b^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{18\,a^3b^4(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{9\,a^2b^5(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{(a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8)\left(a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(6*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 + 4*a^3*b +
 6*a^2*b^2 + 4*a*b^3 + b^4) + 48*(a^4*b + a^2*b^3)*log(abs(-a - b - a*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))
)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + (a + b - 6*a*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2
 + 4*a*b^3 + b^4)*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/((a^3 - 3*a^2*b
+ 3*a*b^2 - b^3)*(cos(d*x + c) + 1)) + 8*(2*a^7 - 5*a^6*b - 8*a^5*b^2 - 2*a
^4*b^3 - 10*a^3*b^4 - 9*a^2*b^5 + 2*a^7*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) - 16*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 6*a^5*b^2*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 2*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1
) - 8*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 18*a^2*b^5*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) - 9*a^6*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
1)^2 + 18*a^5*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 18*a^4*b^3*(c
os(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 18*a^3*b^4*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 - 9*a^2*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/
((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b + a*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2))/d
```

Mupad [B]

time = 1.13, size = 492, normalized size = 2.32

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d\,(a-b)^3}-\frac{\frac{a^3-3\,a^2\,b+3\,a\,b^2-b^3}{2\,(a+b)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(9\,a^5-5\,a^4\,b+42\,a^3\,b^2-10\,a^2\,b^3+5\,a\,b^4-b^5)}{(a-b)\,(a^2+2\,a\,b+b^2)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,(17\,a^5+11\,a^4\,b+74\,a^3\,b^2-10\,a^2\,b^3+5\,a\,b^4-b^5)}{2\,(a+b)\,(a^2+2\,a\,b+b^2)}}{d\,\left((4\,a^5-20\,a^4\,b+40\,a^3\,b^2-40\,a^2\,b^3+20\,a\,b^4-4\,b^5)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+(-8\,a^5+24\,a^4\,b-16\,a^3\,b^2-16\,a^2\,b^3+24\,a\,b^4-8\,b^5)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+(4\,a^5-4\,a^4\,b-8\,a^3\,b^2+8\,a^2\,b^3+4\,a\,b^4-4\,b^5)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}+\frac{\ln\left(a+b-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)\,(6\,a^4\,b+6\,a^2\,b^3)}{d\,(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8)}+\frac{3\,a\,\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d\,(2\,a^4+8\,a^3\,b+12\,a^2\,b^2+8\,a\,b^3+2\,b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a*sin(c + d*x) + b*tan(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^2/(8*d*(a - b)^3) - ((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(2*
(a + b)) - (tan(c/2 + (d*x)/2)^2*(5*a*b^4 - 5*a^4*b + 9*a^5 - b^5 - 10*a^2*
b^3 + 42*a^3*b^2))/((a - b)*(2*a*b + a^2 + b^2)) + (tan(c/2 + (d*x)/2)^4*(5
*a*b^4 + 11*a^4*b + 17*a^5 - b^5 - 10*a^2*b^3 + 74*a^3*b^2))/(2*(a + b)*(2*
a*b + a^2 + b^2)))/(d*(tan(c/2 + (d*x)/2)^2*(4*a*b^4 - 4*a^4*b + 4*a^5 - 4*
b^5 + 8*a^2*b^3 - 8*a^3*b^2) - tan(c/2 + (d*x)/2)^4*(8*a^5 - 24*a^4*b - 24*
a*b^4 + 8*b^5 + 16*a^2*b^3 + 16*a^3*b^2) + tan(c/2 + (d*x)/2)^6*(20*a*b^4 -
 20*a^4*b + 4*a^5 - 4*b^5 - 40*a^2*b^3 + 40*a^3*b^2))) + (log(a + b - a*tan
(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(6*a^4*b + 6*a^2*b^3))/(d*(a^8
+ b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (3*a*log(tan(c/2 + (d*x)/2)))
/(d*(8*a*b^3 + 8*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2))
```

3.270 $\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$

Optimal. Leaf size=228

$$\frac{a(2a^2+b^2)}{2\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2}-\frac{ab(11a^2+b^2)}{2\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))}-\frac{(a-b\cos(c+dx))\csc^2(c+dx)}{2\left(a^2-b^2\right)d(b+a\cos(c+dx))^2}+\frac{(4a+\cdots}{\cdots}$$

```
[Out] 1/2*a*(2*a^2+b^2)/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2-1/2*a*b*(11*a^2+b^2)/(a^
2-b^2)^3/d/(b+a*cos(d*x+c))-1/2*(a-b*cos(d*x+c))*csc(d*x+c)^2/(a^2-b^2)/d/(
b+a*cos(d*x+c))^2+1/4*(4*a+b)*ln(1-cos(d*x+c))/(a+b)^4/d+1/4*(4*a-b)*ln(1+c
os(d*x+c))/(a-b)^4/d-2*a^3*(a^2+5*b^2)*ln(b+a*cos(d*x+c))/(a^2-b^2)^4/d
```

Rubi [A]

time = 0.29, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4482, 2747, 755, 815}

$$-\frac{ab(11a^2+b^2)}{2d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}+\frac{a(2a^2+b^2)}{2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}-\frac{\csc^2(c+dx)(a-b\cos(c+dx))}{2d\left(a^2-b^2\right)(a\cos(c+dx)+b)^2}-\frac{2a^3(a^2+5b^2)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^4}+\frac{(4a+b)\log(1-\cos(c+dx))}{4d(a+b)^4}+\frac{(4a-b)\log(\cos(c+dx)+1)}{4d(a-b)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (a*(2*a^2 + b^2))/(2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) - (a*b*(11*a^2
 + b^2))/(2*(a^2 - b^2)^3*d*(b + a*Cos[c + d*x])) - ((a - b*Cos[c + d*x])*C
sc[c + d*x]^2)/(2*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) + ((4*a + b)*Log[1
- Cos[c + d*x]])/(4*(a + b)^4*d) + ((4*a - b)*Log[1 + Cos[c + d*x]])/(4*(a
- b)^4*d) - (2*a^3*(a^2 + 5*b^2)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d)
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
```

```
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx = \int \frac{\csc^3(c+dx)}{(b+a\cos(c+dx))^3}\,dx$$

$$= -\frac{a^3\text{Subst}\left(\int \frac{1}{(b+x)^3(a^2-x^2)^2}\,dx, x, a\cos(c+dx)\right)}{d}$$

$$= -\frac{(a-b\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)\,d(b+a\cos(c+dx))^2} + \frac{a\text{Subst}\left(\int \frac{-4a^2+b^2+3bx}{(b+x)^3(a^2-x^2)}\,dx, x, \right.}{2(a^2-b^2)\,d}$$

$$= -\frac{(a-b\cos(c+dx))\csc^2(c+dx)}{2(a^2-b^2)\,d(b+a\cos(c+dx))^2} + \frac{a\text{Subst}\left(\int \left(\frac{(-4a-b)(a-b)}{2a(a+b)^3(a-x)} + \frac{(}{2a}\right.\right.}{}$$

$$= \frac{a(2a^2+b^2)}{2(a^2-b^2)^2\,d(b+a\cos(c+dx))^2} - \frac{ab(11a^2+b^2)}{2(a^2-b^2)^3\,d(b+a\cos(c+dx}$$

Mathematica [A]
time = 6.25, size = 217, normalized size = 0.95

$$\frac{a^3}{2(-a+b)^2(a+b)^2d(b+a\cos(c+dx))^2} + \frac{4a^3b}{(-a+b)^3(a+b)^3d(b+a\cos(c+dx))} - \frac{\csc^2\left(\frac{1}{2}(c+dx)\right)}{8(a+b)^3d} + \frac{(4a-b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2(-a+b)^4d} - \frac{2(a^5+5a^3b^2)\log(b+a\cos(c+dx))}{(-a^2+b^2)^4\,d} + \frac{(4a+b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2(a+b)^4d} + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)}{8(-a+b)^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] a^3/(2*(-a + b)^2*(a + b)^2*d*(b + a*Cos[c + d*x])^2) + (4*a^3*b)/((-a + b)
^3*(a + b)^3*d*(b + a*Cos[c + d*x])) - Csc[(c + d*x)/2]^2/(8*(a + b)^3*d) +
 ((4*a - b)*Log[Cos[(c + d*x)/2]])/(2*(-a + b)^4*d) - (2*(a^5 + 5*a^3*b^2)*
Log[b + a*Cos[c + d*x]])/((-a^2 + b^2)^4*d) + ((4*a + b)*Log[Sin[(c + d*x)/
2]])/(2*(a + b)^4*d) + Sec[(c + d*x)/2]^2/(8*(-a + b)^3*d)
```

Maple [A]
time = 0.59, size = 184, normalized size = 0.81

method	result

derivativedivides	$\frac{\frac{a^3}{2(a+b)^2(a-b)^2(b+a\cos(dx+c))^2}-\frac{4a^3b}{(a+b)^3(a-b)^3(b+a\cos(dx+c))}-\frac{2a^3\left(a^2+5b^2\right)\ln(b+a\cos(dx+c))}{(a+b)^4(a-b)^4}+\frac{1}{4(a+b)^3(\cos(dx+c)-1)}+\frac{(4a+}{}}{d}$
default	$\frac{\frac{a^3}{2(a+b)^2(a-b)^2(b+a\cos(dx+c))^2}-\frac{4a^3b}{(a+b)^3(a-b)^3(b+a\cos(dx+c))}-\frac{2a^3\left(a^2+5b^2\right)\ln(b+a\cos(dx+c))}{(a+b)^4(a-b)^4}+\frac{1}{4(a+b)^3(\cos(dx+c)-1)}+\frac{(4a+}{}}{d}$
risch	$\frac{20ib^2a^3x}{a^8-4b^2a^6+6a^4b^4-4a^2b^6+b^8}+\frac{20ib^2a^3c}{d(a^8-4b^2a^6+6a^4b^4-4a^2b^6+b^8)}-\frac{2iac}{d(a^4+4a^3b+6a^2b^2+4a\,b^3+b^4)}+\frac{i}{2d(a^4-4a^3b+6}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/2*a^3/(a+b)^2/(a-b)^2/(b+a*cos(d*x+c))^2-4*a^3*b/(a+b)^3/(a-b)^3/(b+
a*cos(d*x+c))-2*a^3*(a^2+5*b^2)/(a+b)^4/(a-b)^4*ln(b+a*cos(d*x+c))+1/4/(a+b
)^3/(cos(d*x+c)-1)+1/4*(4*a+b)/(a+b)^4*ln(cos(d*x+c)-1)-1/4/(a-b)^3/(1+cos(
d*x+c))+1/4*(4*a-b)/(a-b)^4*ln(1+cos(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 591 vs. $2(219) = 438$.
time = 0.33, size = 591, normalized size = 2.59

$$-\frac{\frac{16\left(a^5+5a^3b^2\right)\log\left(a+b-\frac{(a-b)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}-\frac{4(4a+b)\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}+\frac{a^6-2a^5b-a^4b^2+4a^3b^3-a^2b^4-2ab^5+b^6-\frac{2\left(a^6-44a^5b-35a^4b^2-5a^2b^4+4ab^5-b^6\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{\left(15a^6+70a^5b-95a^4b^2+20a^3b^3-15a^2b^4+6ab^5-b^6\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}{\frac{\left(a^9+a^8b-4a^7b^2-4a^6b^3+6a^5b^4+6a^4b^5-4a^3b^6-4a^2b^7+ab^8+b^9\right)\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{2\left(a^9-a^8b-4a^7b^2+4a^6b^3+6a^5b^4-6a^4b^5-4a^3b^6+4a^2b^7+ab^8-b^9\right)\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{\left(a^9-3a^8b+8a^6b^3-6a^5b^4-6a^4b^5+8a^3b^6-3ab^8+b^9\right)\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\sin(dx+c)^2}{(a^3-3a^2b+3ab^2-b^3)(\cos(dx+c)+1)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/8*(16*(a^5 + 5*a^3*b^2)*log(a + b - (a - b)*sin(d*x + c)^2/(cos(d*x + c)
 + 1)^2)/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - 4*(4*a + b)*log(
sin(d*x + c)/(cos(d*x + c) + 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4
) + (a^6 - 2*a^5*b - a^4*b^2 + 4*a^3*b^3 - a^2*b^4 - 2*a*b^5 + b^6 - 2*(a^6
 - 44*a^5*b - 35*a^4*b^2 - 5*a^2*b^4 + 4*a*b^5 - b^6)*sin(d*x + c)^2/(cos(d
*x + c) + 1)^2 - (15*a^6 + 70*a^5*b - 95*a^4*b^2 + 20*a^3*b^3 - 15*a^2*b^4
+ 6*a*b^5 - b^6)*sin(d*x + c)^4/(cos(d*x + c) + 1)^4)/((a^9 + a^8*b - 4*a^7
*b^2 - 4*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 - 4*a^3*b^6 - 4*a^2*b^7 + a*b^8 +
b^9)*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 2*(a^9 - a^8*b - 4*a^7*b^2 + 4*a
^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 - b^9)*sin(d
*x + c)^4/(cos(d*x + c) + 1)^4 + (a^9 - 3*a^8*b + 8*a^6*b^3 - 6*a^5*b^4 - 6
*a^4*b^5 + 8*a^3*b^6 - 3*a*b^8 + b^9)*sin(d*x + c)^6/(cos(d*x + c) + 1)^6)
+ sin(d*x + c)^2/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(cos(d*x + c) + 1)^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 971 vs. $2(219) = 438$.
time = 2.82, size = 971, normalized size = 4.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^7 - 22*a^5*b^2 + 14*a^3*b^4 + 6*a*b^6 + 2*(11*a^6*b - 10*a^4*b^3
- a^2*b^5)*cos(d*x + c)^3 - 4*(a^7 - 7*a^5*b^2 + 5*a^3*b^4 + a*b^6)*cos(d*x
 + c)^2 - 2*(10*a^6*b - 7*a^4*b^3 - 4*a^2*b^5 + b^7)*cos(d*x + c) - 8*(a^5*
b^2 + 5*a^3*b^4 - (a^7 + 5*a^5*b^2)*cos(d*x + c)^4 - 2*(a^6*b + 5*a^4*b^3)*
cos(d*x + c)^3 + (a^7 + 4*a^5*b^2 - 5*a^3*b^4)*cos(d*x + c)^2 + 2*(a^6*b +
5*a^4*b^3)*cos(d*x + c))*log(a*cos(d*x + c) + b) + (4*a^5*b^2 + 15*a^4*b^3
+ 20*a^3*b^4 + 10*a^2*b^5 - b^7 - (4*a^7 + 15*a^6*b + 20*a^5*b^2 + 10*a^4*b
^3 - a^2*b^5)*cos(d*x + c)^4 - 2*(4*a^6*b + 15*a^5*b^2 + 20*a^4*b^3 + 10*a^
3*b^4 - a*b^6)*cos(d*x + c)^3 + (4*a^7 + 15*a^6*b + 16*a^5*b^2 - 5*a^4*b^3
- 20*a^3*b^4 - 11*a^2*b^5 + b^7)*cos(d*x + c)^2 + 2*(4*a^6*b + 15*a^5*b^2 +
 20*a^4*b^3 + 10*a^3*b^4 - a*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2)
 + (4*a^5*b^2 - 15*a^4*b^3 + 20*a^3*b^4 - 10*a^2*b^5 + b^7 - (4*a^7 - 15*a^
6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*cos(d*x + c)^4 - 2*(4*a^6*b - 15*a
^5*b^2 + 20*a^4*b^3 - 10*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (4*a^7 - 15*a^6*
b + 16*a^5*b^2 + 5*a^4*b^3 - 20*a^3*b^4 + 11*a^2*b^5 - b^7)*cos(d*x + c)^2
+ 2*(4*a^6*b - 15*a^5*b^2 + 20*a^4*b^3 - 10*a^3*b^4 + a*b^6)*cos(d*x + c))*
log(-1/2*cos(d*x + c) + 1/2))/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 +
a^2*b^8)*d*cos(d*x + c)^4 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 +
a*b^9)*d*cos(d*x + c)^3 - (a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a
^2*b^8 - b^10)*d*cos(d*x + c)^2 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*
b^7 + a*b^9)*d*cos(d*x + c) - (a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8
+ b^10)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{(a\sin(c+dx)+b\tan(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral(sec(c + d*x)**3/(a*sin(c + d*x) + b*tan(c + d*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 675 vs. 2(219) = 438.
time = 0.94, size = 675, normalized size = 2.96

$$\frac{\frac{2(4a+b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}-\frac{16(a^5+5a^3b^2)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}+\frac{\left(a+b-\frac{8a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{(a^4+4a^3b+6a^2b^2+4ab^3+b^4)(\cos(dx+c)-1)}+\frac{\cos(dx+c)-1}{(a^3-3a^2b+3ab^2-b^3)(\cos(dx+c)+1)}+\frac{8\left(3a^7-4a^6b-2a^5b^2+20a^4b^3+15a^3b^4+\frac{4a^7(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{10a^6b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{26a^5b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{10a^4b^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{30a^3b^4(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3a^7(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{6a^6b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{18a^5b^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{30a^4b^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{15a^3b^4(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8)\left(a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(2*(4*a + b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 + 4
*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 16*(a^5 + 5*a^3*b^2)*log(abs(-a - b -
 a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1)))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + (a + b - 8*a*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(cos(
d*x + c) - 1)) + (cos(d*x + c) - 1)/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*(cos(d
*x + c) + 1)) + 8*(3*a^7 - 4*a^6*b - 2*a^5*b^2 + 20*a^4*b^3 + 15*a^3*b^4 +
4*a^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 10*a^6*b*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) + 26*a^5*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 10*a
^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 30*a^3*b^4*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 3*a^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6*
a^6*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 18*a^5*b^2*(cos(d*x + c)
- 1)^2/(cos(d*x + c) + 1)^2 - 30*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 + 15*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((a^8 - 4*a
^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b + a*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2))/d
```

Mupad [B]
time = 1.13, size = 490, normalized size = 2.15

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(15\,a^5+85\,a^4\,b-10\,a^3\,b^2+10\,a^2\,b^3-5\,a\,b^4+b^5\right)}{2\,\left(a+b\right)\,\left(a^2+2\,a\,b+b^2\right)}-\frac{a^3-3\,a^2\,b+3\,a\,b^2-b^3}{2\,\left(a+b\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(a^5-45\,a^4\,b+10\,a^3\,b^2-10\,a^2\,b^3+5\,a\,b^4-b^5\right)}{\left(a-b\right)\,\left(a^2+2\,a\,b+b^2\right)}}{d\,\left(\left(4\,a^5-20\,a^4\,b+40\,a^3\,b^2-40\,a^2\,b^3+20\,a\,b^4-4\,b^5\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(-8\,a^5+24\,a^4\,b-16\,a^3\,b^2-16\,a^2\,b^3+24\,a\,b^4-8\,b^5\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(4\,a^5-4\,a^4\,b-8\,a^3\,b^2+8\,a^2\,b^3+4\,a\,b^4-4\,b^5\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{8\,d\,{\left(a-b\right)}^3}+\frac{\ln\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(4\,a+b\right)}{d\,\left(2\,a^4+8\,a^3\,b+12\,a^2\,b^2+8\,a\,b^3+2\,b^4\right)}-\frac{\ln\left(a+b-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+b\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)\,\left(2\,a^5+10\,a^3\,b^2\right)}{d\,\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a*sin(c + d*x) + b*tan(c + d*x))^3),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^4*(85*a^4*b - 5*a*b^4 + 15*a^5 + b^5 + 10*a^2*b^3 - 10
*a^3*b^2))/(2*(a + b)*(2*a*b + a^2 + b^2)) - (3*a*b^2 - 3*a^2*b + a^3 - b^3
)/(2*(a + b)) + (tan(c/2 + (d*x)/2)^2*(5*a*b^4 - 45*a^4*b + a^5 - b^5 - 10*
a^2*b^3 + 10*a^3*b^2))/((a - b)*(2*a*b + a^2 + b^2)))/(d*(tan(c/2 + (d*x)/2
)^2*(4*a*b^4 - 4*a^4*b + 4*a^5 - 4*b^5 + 8*a^2*b^3 - 8*a^3*b^2) - tan(c/2 +
 (d*x)/2)^4*(8*a^5 - 24*a^4*b - 24*a*b^4 + 8*b^5 + 16*a^2*b^3 + 16*a^3*b^2)
 + tan(c/2 + (d*x)/2)^6*(20*a*b^4 - 20*a^4*b + 4*a^5 - 4*b^5 - 40*a^2*b^3 +
 40*a^3*b^2))) - tan(c/2 + (d*x)/2)^2/(8*d*(a - b)^3) + (log(tan(c/2 + (d*x
)/2))*(4*a + b))/(d*(8*a*b^3 + 8*a^3*b + 2*a^4 + 2*b^4 + 12*a^2*b^2)) - (lo
g(a + b - a*tan(c/2 + (d*x)/2)^2 + b*tan(c/2 + (d*x)/2)^2)*(2*a^5 + 10*a^3*
b^2))/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2))
```

3.271 $\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx$

Optimal. Leaf size=155

$$\frac{b^3\cos^{-2+m}(c+dx)}{d(2-m)}+\frac{3ab^2\cos^{-1+m}(c+dx)}{d(1-m)}-\frac{b(3a^2-b^2)\cos^m(c+dx)}{dm}-\frac{a(a^2-3b^2)\cos^{1+m}(c+dx)}{d(1+m)}+\frac{3a^2b\,c}{}$$

```
[Out] b^3*cos(d*x+c)^(-2+m)/d/(2-m)+3*a*b^2*cos(d*x+c)^(-1+m)/d/(1-m)-b*(3*a^2-b^
2)*cos(d*x+c)^m/d/m-a*(a^2-3*b^2)*cos(d*x+c)^(1+m)/d/(1+m)+3*a^2*b*cos(d*x+
c)^(2+m)/d/(2+m)+a^3*cos(d*x+c)^(3+m)/d/(3+m)
```

Rubi [A]

time = 0.29, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {4482, 2916, 962}

$$\frac{a^3\cos^{m+3}(c+dx)}{d(m+3)}-\frac{a(a^2-3b^2)\cos^{m+1}(c+dx)}{d(m+1)}-\frac{b(3a^2-b^2)\cos^m(c+dx)}{dm}+\frac{3a^2b\cos^{m+2}(c+dx)}{d(m+2)}+\frac{3ab^2\cos^{m-1}(c+dx)}{d(1-m)}+\frac{b^3\cos^{m-2}(c+dx)}{d(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (b^3*Cos[c + d*x]^(-2 + m))/(d*(2 - m)) + (3*a*b^2*Cos[c + d*x]^(-1 + m))/(
d*(1 - m)) - (b*(3*a^2 - b^2)*Cos[c + d*x]^m)/(d*m) - (a*(a^2 - 3*b^2)*Cos[
c + d*x]^(1 + m))/(d*(1 + m)) + (3*a^2*b*Cos[c + d*x]^(2 + m))/(d*(2 + m))
+ (a^3*Cos[c + d*x]^(3 + m))/(d*(3 + m))
```

Rule 962

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^3\,dx = \int \cos^{-3+m}(c+dx)(b+a\cos(c+dx))^3\sin^3(c+dx)\,dx$$

$$= -\frac{\operatorname{Subst}\left(\int \left(\frac{x}{a}\right)^{-3+m}(b+x)^3\left(a^2-x^2\right)\,dx, x, a\cos(c+\right.}{a^3d}$$

$$= -\frac{\operatorname{Subst}\left(\int \left(a^2b^3\left(\frac{x}{a}\right)^{-3+m}+3a^3b^2\left(\frac{x}{a}\right)^{-2+m}+a^2b(3a^2-\right.\right.}{}$$

$$= \frac{b^3\cos^{-2+m}(c+dx)}{d(2-m)}+\frac{3ab^2\cos^{-1+m}(c+dx)}{d(1-m)}-\frac{b(3a^2-}{}$$

Mathematica [A]
time = 1.61, size = 246, normalized size = 1.59

$$\frac{\cos^{1+m}(c+dx)\left(-4b^3m(-6-5m+5m^2+5m^3+m^4)-12ab^2m(-12-16m-m^2+4m^3+m^4)\cos(c+dx)-am(4-4m-m^2+m^3)\left(-12b^2(3+m)+a^2(9+m)\right)\cos^3(c+dx)+(2-m-2m^2+m^3)\cos^2(c+dx)\left(2b(3+m)\left(2b^2(2+m)-3a^2(4+m)\right)+6a^2bm(3+m)\cos(2(c+dx))+a^3m(2+m)\cos(3(c+dx))\right)\right)(a+b\sec(c+dx))^3}{4d(-2+m)(-1+m)m(1+m)(2+m)(3+m)(b+a\cos(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(a*Sin[c + d*x] + b*Tan[c + d*x])^3,x]
```

```
[Out] (Cos[c + d*x]^(1 + m)*(-4*b^3*m*(-6 - 5*m + 5*m^2 + 5*m^3 + m^4) - 12*a*b^2
*m*(-12 - 16*m - m^2 + 4*m^3 + m^4)*Cos[c + d*x] - a*m*(4 - 4*m - m^2 + m^3
)*(-12*b^2*(3 + m) + a^2*(9 + m))*Cos[c + d*x]^3 + (2 - m - 2*m^2 + m^3)*Co
s[c + d*x]^2*(2*b*(3 + m)*(2*b^2*(2 + m) - 3*a^2*(4 + m)) + 6*a^2*b*m*(3 +
m)*Cos[2*(c + d*x)] + a^3*m*(2 + m)*Cos[3*(c + d*x)]))*(a + b*Sec[c + d*x])
^3)/(4*d*(-2 + m)*(-1 + m)*m*(1 + m)*(2 + m)*(3 + m)*(b + a*Cos[c + d*x])^3
)
```

Maple [F]
time = 0.62, size = 0, normalized size = 0.00

$$\int \left(\cos^m(dx+c)\right)\left(a\sin(dx+c)+b\tan(dx+c)\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

```
[Out] int(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^3,x)
```

Maxima [A]
time = 0.28, size = 180, normalized size = 1.16

$$\frac{\frac{\left((m+1)\cos(dx+c)^3-(m+3)\cos(dx+c)\right)a^3\cos(dx+c)^m}{m^2+4m+3}+\frac{3\left(m\cos(dx+c)^2-m-2\right)a^2b\cos(dx+c)^m}{m^2+2m}+\frac{3\left((m-1)\cos(dx+c)^2-m-1\right)ab^2\cos(dx+c)^m}{(m^2-1)\cos(dx+c)}+\frac{\left((m-2)\cos(dx+c)^2-m\right)b^3\cos(dx+c)^m}{(m^2-2m)\cos(dx+c)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] (((m + 1)*cos(d*x + c)^3 - (m + 3)*cos(d*x + c))*a^3*cos(d*x + c)^m/(m^2 +
4*m + 3) + 3*(m*cos(d*x + c)^2 - m - 2)*a^2*b*cos(d*x + c)^m/(m^2 + 2*m) +
3*((m - 1)*cos(d*x + c)^2 - m - 1)*a*b^2*cos(d*x + c)^m/((m^2 - 1)*cos(d*x
+ c)) + ((m - 2)*cos(d*x + c)^2 - m)*b^3*cos(d*x + c)^m/((m^2 - 2*m)*cos(d*
x + c)^2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(152) = 304$.
time = 2.77, size = 412, normalized size = 2.66

$$-\frac{\left(b^3m^5+5\,b^3m^4+5\,b^3m^3-(a^3m^5-5\,a^3m^3+4\,a^3m)\cos\left(dx+c\right)^5-5\,b^3m^2-3\,(a^2bm^5+a^2bm^4-7\,a^2bm^3-a^2bm^2+6\,a^2bm)\cos\left(dx+c\right)^4-6\,b^3m+((a^3-3\,ab^2)m^5+2\,(a^3-3\,ab^2)m^4-7\,(a^3-3\,ab^2)m^3-8\,(a^3-3\,ab^2)m^2+12\,(a^3-3\,ab^2)m)\cos\left(dx+c\right)^3+((3\,a^2b-b^3)m^5+3\,(3\,a^2b-b^3)m^4-5\,(3\,a^2b-b^3)m^3+36\,a^2b-12\,b^3-15\,(3\,a^2b-b^3)m^2+4\,(3\,a^2b-b^3)m)\cos\left(dx+c\right)^2+3\,(ab^2m^5+4\,ab^2m^4-ab^2m^3-16\,ab^2m^2-12\,ab^2m)\cos\left(dx+c\right)\right)\cos\left(dx+c\right)^m}{(dm^6+3\,dm^5-5\,dm^4-15\,dm^3+4\,dm^2+12\,dm)\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -(b^3*m^5 + 5*b^3*m^4 + 5*b^3*m^3 - (a^3*m^5 - 5*a^3*m^3 + 4*a^3*m)*cos(d*x
 + c)^5 - 5*b^3*m^2 - 3*(a^2*b*m^5 + a^2*b*m^4 - 7*a^2*b*m^3 - a^2*b*m^2 +
6*a^2*b*m)*cos(d*x + c)^4 - 6*b^3*m + ((a^3 - 3*a*b^2)*m^5 + 2*(a^3 - 3*a*b
^2)*m^4 - 7*(a^3 - 3*a*b^2)*m^3 - 8*(a^3 - 3*a*b^2)*m^2 + 12*(a^3 - 3*a*b^2
)*m)*cos(d*x + c)^3 + ((3*a^2*b - b^3)*m^5 + 3*(3*a^2*b - b^3)*m^4 - 5*(3*a
^2*b - b^3)*m^3 + 36*a^2*b - 12*b^3 - 15*(3*a^2*b - b^3)*m^2 + 4*(3*a^2*b -
 b^3)*m)*cos(d*x + c)^2 + 3*(a*b^2*m^5 + 4*a*b^2*m^4 - a*b^2*m^3 - 16*a*b^2
*m^2 - 12*a*b^2*m)*cos(d*x + c))*cos(d*x + c)^m/((d*m^6 + 3*d*m^5 - 5*d*m^4
 - 15*d*m^3 + 4*d*m^2 + 12*d*m)*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sin{(c+dx)}+b\tan{(c+dx)})^3\cos^m{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a*sin(d*x+c)+b*tan(d*x+c))**3,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**3*cos(c + d*x)**m, x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Modgcd: no suitable evaluation pointi
ndex.cc index_m operator + Error: Bad Argument ValueEvaluation time: 31.29D
one
```

Mupad [B]
time = 7.51, size = 861, normalized size = 5.55

$$\frac{\left(\frac{1}{2}\right)^m\left(\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}+\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\right)^m\left(\frac{a^3\left(\frac{m^4}{8}-\frac{5\,m^2}{8}+\frac{1}{2}\right)}{d\,(m^5+3\,m^4-5\,m^3-15\,m^2+4\,m+12)}+\frac{a^3\,\mathrm{e}^{c\,10\mathrm{i}+d\,x\,10\mathrm{i}}\,(m^4-5\,m^2+4)}{8\,d\,(m^5+3\,m^4-5\,m^3-15\,m^2+4\,m+12)}-\frac{a\,\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,(-m^3+m^2+4\,m-4)\,(a^2\,m+12\,b^2\,m-7\,a^2+36\,b^2)}{8\,d\,(m^5+3\,m^4-5\,m^3-15\,m^2+4\,m+12)}-\frac{a\,\mathrm{e}^{c\,8\mathrm{i}+d\,x\,8\mathrm{i}}\,(-m^3+m^2+4\,m-4)\,(a^2\,m+12\,b^2\,m-7\,a^2+36\,b^2)}{8\,d\,(m^5+3\,m^4-5\,m^3-15\,m^2+4\,m+12)}+\frac{3\,a^2\,b\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,(m^4+m^3-7\,m^2-m+6)}{4\,d\,(m^5+3\,m^4-5\,m^3-15\,m^2+4\,m+12)}+\frac{3\,a^2\,b\,\mathrm{e}^{c\,9\mathrm{i}+d\,x\,9\mathrm{i}}\,(m^4+m^3-7\,m^2-m+6)}{4\,d\,(m^5+3\,m^4-5\,m^3-15\,m^2+4\,m+12)}-\frac{a\,\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,(m^2-4)\,(a^2\,m^2+12\,a^2\,m-13\,a^2+6\,b^2\,m^2+60\,b^2\,m+126\,b^2)}{4\,d\,(m^5+3\,m^4-5\,m^3-15\,m^2+4\,m+12)}-\frac{a\,\mathrm{e}^{c\,6\mathrm{i}+d\,x\,6\mathrm{i}}\,(m^2-4)\,(a^2\,m^2+12\,a^2\,m-13\,a^2+6\,b^2\,m^2+60\,b^2\,m+126\,b^2)}{4\,d\,(m^5+3\,m^4-5\,m^3-15\,m^2+4\,m+12)}+\frac{b\,\mathrm{e}^{c\,3\mathrm{i}+d\,x\,3\mathrm{i}}\,(b^2\,m-6\,a^2+2\,b^2)\,(m^4+m^3-7\,m^2-m+6)}{d\,m\,(m^5+3\,m^4-5\,m^3-15\,m^2+4\,m+12)}+\frac{b\,\mathrm{e}^{c\,7\mathrm{i}+d\,x\,7\mathrm{i}}\,(b^2\,m-6\,a^2+2\,b^2)\,(m^4+m^3-7\,m^2-m+6)}{d\,m\,(m^5+3\,m^4-5\,m^3-15\,m^2+4\,m+12)}+\frac{b\,\mathrm{e}^{c\,5\mathrm{i}+d\,x\,5\mathrm{i}}\,(-m^3-3\,m^2+m+3)\,(3\,a^2\,m^2+18\,a^2\,m-48\,a^2+4\,b^2\,m^2+16\,b^2\,m+16\,b^2)}{2\,d\,m\,(m^5+3\,m^4-5\,m^3-15\,m^2+4\,m+12)}\right)}{\mathrm{e}^{c\,3\mathrm{i}+d\,x\,3\mathrm{i}}+2\,\mathrm{e}^{c\,5\mathrm{i}+d\,x\,5\mathrm{i}}+\mathrm{e}^{c\,7\mathrm{i}+d\,x\,7\mathrm{i}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(a*sin(c + d*x) + b*tan(c + d*x))^3,x)
```

```
[Out] ((1/2)^m*(exp(- c*1i - d*x*1i) + exp(c*1i + d*x*1i))^m*((a^3*(m^4/8 - (5*m^
2)/8 + 1/2))/(d*(4*m - 15*m^2 - 5*m^3 + 3*m^4 + m^5 + 12)) + (a^3*exp(c*10i
 + d*x*10i)*(m^4 - 5*m^2 + 4))/(8*d*(4*m - 15*m^2 - 5*m^3 + 3*m^4 + m^5 + 1
2)) - (a*exp(c*2i + d*x*2i)*(4*m + m^2 - m^3 - 4)*(a^2*m + 12*b^2*m - 7*a^2
 + 36*b^2))/(8*d*(4*m - 15*m^2 - 5*m^3 + 3*m^4 + m^5 + 12)) - (a*exp(c*8i +
 d*x*8i)*(4*m + m^2 - m^3 - 4)*(a^2*m + 12*b^2*m - 7*a^2 + 36*b^2))/(8*d*(4
*m - 15*m^2 - 5*m^3 + 3*m^4 + m^5 + 12)) + (3*a^2*b*exp(c*1i + d*x*1i)*(m^3
 - 7*m^2 - m + m^4 + 6))/(4*d*(4*m - 15*m^2 - 5*m^3 + 3*m^4 + m^5 + 12)) +
(3*a^2*b*exp(c*9i + d*x*9i)*(m^3 - 7*m^2 - m + m^4 + 6))/(4*d*(4*m - 15*m^2
 - 5*m^3 + 3*m^4 + m^5 + 12)) - (a*exp(c*4i + d*x*4i)*(m^2 - 4)*(12*a^2*m +
 60*b^2*m - 13*a^2 + 126*b^2 + a^2*m^2 + 6*b^2*m^2))/(4*d*(4*m - 15*m^2 - 5
*m^3 + 3*m^4 + m^5 + 12)) - (a*exp(c*6i + d*x*6i)*(m^2 - 4)*(12*a^2*m + 60*
b^2*m - 13*a^2 + 126*b^2 + a^2*m^2 + 6*b^2*m^2))/(4*d*(4*m - 15*m^2 - 5*m^3
 + 3*m^4 + m^5 + 12)) + (b*exp(c*3i + d*x*3i)*(b^2*m - 6*a^2 + 2*b^2)*(m^3
- 7*m^2 - m + m^4 + 6))/(d*m*(4*m - 15*m^2 - 5*m^3 + 3*m^4 + m^5 + 12)) + (
b*exp(c*7i + d*x*7i)*(b^2*m - 6*a^2 + 2*b^2)*(m^3 - 7*m^2 - m + m^4 + 6))/(
d*m*(4*m - 15*m^2 - 5*m^3 + 3*m^4 + m^5 + 12)) + (b*exp(c*5i + d*x*5i)*(m -
 3*m^2 - m^3 + 3)*(18*a^2*m + 16*b^2*m - 48*a^2 + 16*b^2 + 3*a^2*m^2 + 4*b^
2*m^2))/(2*d*m*(4*m - 15*m^2 - 5*m^3 + 3*m^4 + m^5 + 12))))/(exp(c*3i + d*x
*3i) + 2*exp(c*5i + d*x*5i) + exp(c*7i + d*x*7i))
```

3.272 $\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx$

Optimal. Leaf size=264

$$\frac{(a^2-2b^2)\cos^{-1+m}(c+dx)\sin(c+dx)}{dm(2+m)}-\frac{2ab\cos^m(c+dx)\sin(c+dx)}{d\left(2+3m+m^2\right)}-\frac{\cos^{-1+m}(c+dx)(b+a\cos(c+dx))}{d(2+m)}$$

```
[Out] (a^2-2*b^2)*cos(d*x+c)^(-1+m)*sin(d*x+c)/d/m/(2+m)-2*a*b*cos(d*x+c)^m*sin(d
*x+c)/d/(m^2+3*m+2)-cos(d*x+c)^(-1+m)*(b+a*cos(d*x+c))^2*sin(d*x+c)/d/(2+m)
-(a^2*(1-m)-b^2*(2+m))*cos(d*x+c)^(-1+m)*hypergeom([1/2, -1/2+1/2*m],[1/2+1
/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/(1-m)/m/(2+m)/(sin(d*x+c)^2)^(1/2)-2*a*b*c
os(d*x+c)^m*hypergeom([1/2, 1/2*m],[1+1/2*m],cos(d*x+c)^2)*sin(d*x+c)/d/m/(
1+m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.52, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4482, 2968, 3129, 3112, 3102, 2827, 2722}

$$-\frac{(a^2(1-m)-b^2(m+2))\sin(c+dx)\cos^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m-1}{2};\frac{m+1}{2};\cos^2(c+dx)\right)}{d(1-m)m(m+2)\sqrt{\sin^2(c+dx)}}+\frac{(a^2-2b^2)\sin(c+dx)\cos^{m-1}(c+dx)}{dm(m+2)}-\frac{2ab\sin(c+dx)\cos^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{m+2}{2};\cos^2(c+dx)\right)}{dm(m+1)\sqrt{\sin^2(c+dx)}}-\frac{2ab\sin(c+dx)\cos^m(c+dx)}{d\left(m^2+3m+2\right)}-\frac{\sin(c+dx)\cos^{m-1}(c+dx)(a\cos(c+dx)+b)^2}{d(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] ((a^2 - 2*b^2)*Cos[c + d*x]^(-1 + m)*Sin[c + d*x])/(d*m*(2 + m)) - (2*a*b*C
os[c + d*x]^m*Sin[c + d*x])/(d*(2 + 3*m + m^2)) - (Cos[c + d*x]^(-1 + m)*(b
 + a*Cos[c + d*x])^2*Sin[c + d*x])/(d*(2 + m)) - ((a^2*(1 - m) - b^2*(2 + m
))*Cos[c + d*x]^(-1 + m)*Hypergeometric2F1[1/2, (-1 + m)/2, (1 + m)/2, Cos[
c + d*x]^2]*Sin[c + d*x])/(d*(1 - m)*m*(2 + m)*Sqrt[Sin[c + d*x]^2]) - (2*a
*b*Cos[c + d*x]^m*Hypergeometric2F1[1/2, m/2, (2 + m)/2, Cos[c + d*x]^2]*Si
n[c + d*x])/(d*m*(1 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2968

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rule 3112

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*d*Cos[e + f*x]*Sin[e + f*x]*((a + b*Si
n[e + f*x])^(m + 1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin
[e + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A
*(m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3129

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n +
1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x]
)^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n
 + 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(
a*d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0
] && GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0
])))
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))^2\,dx = \int \cos^{-2+m}(c+dx)(b+a\cos(c+dx))^2\sin^2(c+dx)\,d$$

$$= \int \cos^{-2+m}(c+dx)(b+a\cos(c+dx))^2\left(1-\cos^2(c+\right.$$

$$= -\frac{\cos^{-1+m}(c+dx)(b+a\cos(c+dx))^2\sin(c+dx)}{d(2+m)} +$$

$$= -\frac{2ab\cos^m(c+dx)\sin(c+dx)}{d\left(2+3m+m^2\right)} - \frac{\cos^{-1+m}(c+dx)(b}{}$$

$$= \frac{\left(a^2-2b^2\right)\cos^{-1+m}(c+dx)\sin(c+dx)}{dm(2+m)} - \frac{2ab\cos^m(c}{d(2+}$$

$$= \frac{\left(a^2-2b^2\right)\cos^{-1+m}(c+dx)\sin(c+dx)}{dm(2+m)} - \frac{2ab\cos^m(c}{d(2+}$$

$$= \frac{\left(a^2-2b^2\right)\cos^{-1+m}(c+dx)\sin(c+dx)}{dm(2+m)} - \frac{2ab\cos^m(c}{d(2+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 29.92, size = 6669, normalized size = 25.26

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^m*(a*Sin[c + d*x] + b*Tan[c + d*x])^2,x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.42, size = 0, normalized size = 0.00

$$\int \left(\cos^m(dx+c)\right)\left(a\sin(dx+c)+b\tan(dx+c)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

```
[Out] int(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sin(d*x + c) + b*tan(d*x + c))^2*cos(d*x + c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(d*x + c)^2 - 2*a*b*sin(d*x + c)*tan(d*x + c) - b^2*tan(d
*x + c)^2 - a^2)*cos(d*x + c)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin\left(c+dx\right)+b\tan\left(c+dx\right)\right)^{2}\cos^{m}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a*sin(d*x+c)+b*tan(d*x+c))**2,x)
```

```
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))**2*cos(c + d*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((a*sin(d*x + c) + b*tan(d*x + c))^2*cos(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\cos\left(c+d\,x\right)}^m\,{\left(a\,\sin\left(c+d\,x\right)+b\,\mathrm{tan}\left(c+d\,x\right)\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(a*sin(c + d*x) + b*tan(c + d*x))^2,x)
```

```
[Out] int(cos(c + d*x)^m*(a*sin(c + d*x) + b*tan(c + d*x))^2, x)
```

3.273 $\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx$

Optimal. Leaf size=39

$$-\frac{b\cos^m(c+dx)}{dm}-\frac{a\cos^{1+m}(c+dx)}{d(1+m)}$$

```
[Out] -b*cos(d*x+c)^m/d/m-a*cos(d*x+c)^(1+m)/d/(1+m)
```

Rubi [A]
time = 0.05, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.154$, Rules used = {4462, 12, 2645, 30}

$$-\frac{a\cos^{m+1}(c+dx)}{d(m+1)}-\frac{b\cos^m(c+dx)}{dm}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] -((b*Cos[c + d*x]^m)/(d*m)) - (a*Cos[c + d*x]^(1 + m))/(d*(1 + m))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 4462

```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Dist[d, Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[c
*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Integer
Q[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Rubi steps

$$\begin{aligned}\int \cos^m(c+dx)(a\sin(c+dx)+b\tan(c+dx))\,dx &= a\int \cos^m(c+dx)\sin(c+dx)\,dx+\int b\cos^{-1+m}(c+dx) \\ &= b\int \cos^{-1+m}(c+dx)\sin(c+dx)\,dx-\frac{a\mathrm{Subst}\left(\int x^m\,dx,\right.}{d} \\ &= -\frac{a\cos^{1+m}(c+dx)}{d(1+m)}-\frac{b\mathrm{Subst}\left(\int x^{-1+m}\,dx,x,\cos(c+dx\right.}{d} \\ &= -\frac{b\cos^m(c+dx)}{dm}-\frac{a\cos^{1+m}(c+dx)}{d(1+m)}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 35, normalized size = 0.90

$$-\frac{\cos^m(c+dx)(b+bm+am\cos(c+dx))}{dm(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m*(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
[Out] -((Cos[c + d*x]^m*(b + b*m + a*m*Cos[c + d*x]))/(d*m*(1 + m)))
```

Maple [A]
time = 1.30, size = 40, normalized size = 1.03

method	result
default	$-\frac{b(\cos^m(dx+c))}{dm}-\frac{a\left(\cos^{1+m}(dx+c)\right)}{d(1+m)}$
risch	$-\frac{a\left(\frac{1}{2}\right)^m\left(\mathrm{e}^{i(dx+c)}\right)^{-m}\left(\mathrm{e}^{2i(dx+c)}+1\right)^m \mathrm{e}^{-\frac{i\left(-m\,\mathrm{csgn}\left(i\left(\mathrm{e}^{2i(dx+c)}+1\right)\right)\mathrm{csgn}(i\cos(dx+c))^2\pi+m\,\mathrm{csgn}\left(i\left(\mathrm{e}^{2i(dx+c)}+1\right)\right)\mathrm{csgn}(i\cos(dx+c))\pi\,\mathrm{csgn}\left(\right.}{2}}}{2d(1+m)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c)),x,method=_RETURNVERBOSE)
[Out] -b*cos(d*x+c)^m/d/m-a*cos(d*x+c)^(1+m)/d/(1+m)
```

Maxima [A]
time = 0.28, size = 36, normalized size = 0.92

$$-\frac{\frac{a\cos(dx+c)^{m+1}}{m+1}+\frac{b\cos(dx+c)^m}{m}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
[Out] -(a*cos(d*x + c)^(m + 1)/(m + 1) + b*cos(d*x + c)^m/m)/d
```

Fricas [A]
time = 4.66, size = 35, normalized size = 0.90

$$-\frac{(am\cos(dx+c)+bm+b)\cos(dx+c)^m}{dm^2+dm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
[Out] -(a*m*cos(d*x + c) + b*m + b)*cos(d*x + c)^m/(d*m^2 + d*m)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(a\sin(c+dx)+b\tan(c+dx))\cos^m(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(a*sin(d*x+c)+b*tan(d*x+c)),x)
[Out] Integral((a*sin(c + d*x) + b*tan(c + d*x))*cos(c + d*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
[Out] integrate((a*sin(d*x + c) + b*tan(d*x + c))*cos(d*x + c)^m, x)
```

Mupad [B]
time = 0.98, size = 35, normalized size = 0.90

$$-\frac{\cos(c+d\,x)^m\,(b+b\,m+a\,m\,\cos(c+d\,x))}{d\,m\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m*(a*sin(c + d*x) + b*tan(c + d*x)),x)
[Out] -(cos(c + d*x)^m*(b + b*m + a*m*cos(c + d*x)))/(d*m*(m + 1))
```

3.274 $\int \frac{\cos^m(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$

Optimal. Leaf size=144

$$\frac{\cos^{2+m}(c+dx)\,{}_2F_1(1,2+m;3+m;-\cos(c+dx))}{2(a-b)d(2+m)}-\frac{\cos^{2+m}(c+dx)\,{}_2F_1(1,2+m;3+m;\cos(c+dx))}{2(a+b)d(2+m)}-\frac{a^2\,\mathrm{c}}{}$$

```
[Out] 1/2*cos(d*x+c)^(2+m)*hypergeom([1, 2+m],[3+m],-cos(d*x+c))/(a-b)/d/(2+m)-1/
2*cos(d*x+c)^(2+m)*hypergeom([1, 2+m],[3+m],cos(d*x+c))/(a+b)/d/(2+m)-a^2*c
os(d*x+c)^(2+m)*hypergeom([1, 2+m],[3+m],-a*cos(d*x+c)/b)/b/(a^2-b^2)/d/(2+
m)
```

Rubi [A]
time = 0.30, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4482, 2916, 975, 66}

$$-\frac{a^2\cos^{m+2}(c+dx)\,{}_2F_1\left(1,m+2;m+3;-\frac{a\cos(c+dx)}{b}\right)}{bd(m+2)\left(a^2-b^2\right)}+\frac{\cos^{m+2}(c+dx)\,{}_2F_1(1,m+2;m+3;-\cos(c+dx))}{2d(m+2)(a-b)}-\frac{\cos^{m+2}(c+dx)\,{}_2F_1(1,m+2;m+3;\cos(c+dx))}{2d(m+2)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 + m, -Cos[c + d*x]])/(2
*(a - b)*d*(2 + m)) - (Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1, 2 + m, 3 +
 m, Cos[c + d*x]])/(2*(a + b)*d*(2 + m)) - (a^2*Cos[c + d*x]^(2 + m)*Hyperg
eometric2F1[1, 2 + m, 3 + m, -((a*Cos[c + d*x])/b)])/(b*(a^2 - b^2)*d*(2 +
m))
```

Rule 66

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

Rule 975

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && (IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[
m, 0] || IGtQ[n, 0])
```

Rule 2916

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d/b)*x)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 4482

```
Int[u_, x_Symbol] :> Int[TrigSimplify[u], x] /; TrigSimplifyQ[u]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^m(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx &= \int \frac{\cos^{1+m}(c+dx)\csc(c+dx)}{b+a\cos(c+dx)}\,dx\\ &= -\frac{a\text{Subst}\left(\int \frac{\left(\frac{x}{a}\right)^{1+m}}{(b+x)(a^2-x^2)}\,dx, x, a\cos(c+dx)\right)}{d}\\ &= -\frac{a\text{Subst}\left(\int \left(\frac{\left(\frac{x}{a}\right)^{1+m}}{2a(a+b)(a-x)}-\frac{\left(\frac{x}{a}\right)^{1+m}}{2a(a-b)(a+x)}+\frac{\left(\frac{x}{a}\right)^{1+m}}{(a-b)(a+b)(b+x)}\right)dx, x, a\cos\right.}{d}\\ &= \frac{\text{Subst}\left(\int \frac{\left(\frac{x}{a}\right)^{1+m}}{a+x}\,dx, x, a\cos(c+dx)\right)}{2(a-b)d}-\frac{\text{Subst}\left(\int \frac{\left(\frac{x}{a}\right)^{1+m}}{a-x}\,dx, x, a\cos\right.}{2(a+b)d}\\ &= \frac{\cos^{2+m}(c+dx)\,{}_2F_1(1,2+m;3+m;-\cos(c+dx))}{2(a-b)d(2+m)}-\frac{\cos^{2+m}(c+dx}{}\end{aligned}$$

Mathematica [A]
time = 0.66, size = 106, normalized size = 0.74

$$\frac{\cos^{2+m}(c+dx)\left(b(a+b)\,{}_2F_1(1,2+m;3+m;-\cos(c+dx))-(a-b)b\,{}_2F_1(1,2+m;3+m;\cos(c+dx))-2a^2\,{}_2F_1\left(1,2+m;3+m;-\frac{a\cos(c+dx)}{b}\right)\right)}{2(a-b)b(a+b)d(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^m/(a*Sin[c + d*x] + b*Tan[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^(2 + m)*(b*(a + b)*Hypergeometric2F1[1, 2 + m, 3 + m, -Cos[c
 + d*x]] - (a - b)*b*Hypergeometric2F1[1, 2 + m, 3 + m, Cos[c + d*x]] - 2*a^
2*Hypergeometric2F1[1, 2 + m, 3 + m, -((a*Cos[c + d*x])/b)]))/(2*(a - b)*b*
(a + b)*d*(2 + m))
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int \frac{\cos^m(dx+c)}{a\sin(dx+c)+b\tan(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^m/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^m/(a*sin(d*x + c) + b*tan(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(cos(d*x + c)^m/(a*sin(d*x + c) + b*tan(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{m}(c+dx)}{a\sin(c+dx)+b\tan(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m/(a*sin(d*x+c)+b*tan(d*x+c)),x)
```

```
[Out] Integral(cos(c + d*x)**m/(a*sin(c + d*x) + b*tan(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m/(a*sin(d*x+c)+b*tan(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^m/(a*sin(d*x + c) + b*tan(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos (c+d\,x)\,\cos (c+d\,x)^m}{\sin (c+d\,x)\,\left(b+a\,\cos (c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^m/(a*sin(c + d*x) + b*tan(c + d*x)),x)
[Out] int((cos(c + d*x)*cos(c + d*x)^m)/(sin(c + d*x)*(b + a*cos(c + d*x))), x)
```

3.275 $\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=65

$$\frac{ab\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{a\cos(x)}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}$$

```
[Out] a*b*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(3/2)-a*cos(x)/(
a^2+b^2)+b*sin(x)/(a^2+b^2)
```

Rubi [A]
time = 0.05, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3188, 2717, 2718, 3153, 212}

$$\frac{b\sin(x)}{a^2+b^2}-\frac{a\cos(x)}{a^2+b^2}+\frac{ab\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x])/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (a*b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) - (a
*Cos[x])/(a^2 + b^2) + (b*Sin[x])/(a^2 + b^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
```

```
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx &= \frac{a\int \sin(x)\,dx}{a^2+b^2}+\frac{b\int \cos(x)\,dx}{a^2+b^2}-\frac{(ab)\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}\\ &= -\frac{a\cos(x)}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}+\frac{(ab)\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,b\cos(x)-a\sin(x)\right)}{a^2+b^2}\\ &= \frac{ab\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}-\frac{a\cos(x)}{a^2+b^2}+\frac{b\sin(x)}{a^2+b^2}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 61, normalized size = 0.94

$$-\frac{2ab\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{3/2}}+\frac{-a\cos(x)+b\sin(x)}{a^2+b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (-2*a*b*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(3/2) + (-(
a*Cos[x]) + b*Sin[x])/(a^2 + b^2)
```

Maple [A]
time = 0.20, size = 82, normalized size = 1.26

method	result	size

default	$-\frac{4ab\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(2a^2+2b^2)\sqrt{a^2+b^2}}+\frac{2b\tan\left(\frac{x}{2}\right)-2a}{(a^2+b^2)\left(1+\tan^2\left(\frac{x}{2}\right)\right)}$	82
risch	$-\frac{e^{ix}}{2(-ib+a)}-\frac{e^{-ix}}{2(ib+a)}+\frac{iba\ln\left(e^{ix}+\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,(a^2+b^2)}-\frac{iba\ln\left(e^{ix}-\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,(a^2+b^2)}$	141

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -4*a*b/(2*a^2+2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+
b^2)^(1/2))+2/(a^2+b^2)*(b*tan(1/2*x)-a)/(1+tan(1/2*x)^2)
```

Maxima [A]
time = 0.47, size = 105, normalized size = 1.62

$$\frac{ab\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{\frac{3}{2}}}-\frac{2\left(a-\frac{b\sin(x)}{\cos(x)+1}\right)}{a^2+b^2+\frac{(a^2+b^2)\sin(x)^2}{(\cos(x)+1)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] a*b*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(cos(x)
 + 1) - sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) - 2*(a - b*sin(x)/(cos(x) + 1))
/(a^2 + b^2 + (a^2 + b^2)*sin(x)^2/(cos(x) + 1)^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(61) = 122$.
time = 2.08, size = 142, normalized size = 2.18

$$\frac{\sqrt{a^2+b^2}\,ab\log\left(\frac{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2-2\,a^2-b^2-2\,\sqrt{a^2+b^2}\,(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2}\right)-2\,(a^3+ab^2)\cos(x)+2\,(a^2b+b^3)\sin(x)}{2\,(a^4+2\,a^2b^2+b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + b^2)*a*b*log((2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 -
2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x)
 + (a^2 - b^2)*cos(x)^2 + b^2)) - 2*(a^3 + a*b^2)*cos(x) + 2*(a^2*b + b^3)*
sin(x))/(a^4 + 2*a^2*b^2 + b^4)
```

Sympy [C] Result contains complex when optimal does not.
time = 108.26, size = 699, normalized size = 10.75

$$\begin{cases} \tilde{\infty}\sin(x) & \text{for } a = 0 \wedge b = 0 \\ \frac{\sin(x)}{b} & \text{for } a = 0 \\ \frac{i\sin^2(x)}{3ib\sin(x)+3b\cos(x)} + \frac{\sin(x)\cos(x)}{3ib\sin(x)+3b\cos(x)} - \frac{i\cos^2(x)}{3ib\sin(x)+3b\cos(x)} & \text{for } a = -ib \\ -\frac{i\sin^2(x)}{-3ib\sin(x)+3b\cos(x)} + \frac{\sin(x)\cos(x)}{-3ib\sin(x)+3b\cos(x)} + \frac{i\cos^2(x)}{-3ib\sin(x)+3b\cos(x)} & \text{for } a = ib \\ \frac{ab\log\left(\tan\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)\tan^2\left(\frac{x}{2}\right)}{a^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+a^2\sqrt{a^2+b^2}+b^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+b^2\sqrt{a^2+b^2}} + \frac{ab\log\left(\tan\left(\frac{x}{2}\right)-\frac{b}{a}-\frac{\sqrt{a^2+b^2}}{a}\right)}{a^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+a^2\sqrt{a^2+b^2}+b^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+b^2\sqrt{a^2+b^2}} - \frac{ab\log\left(\tan\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)\tan^2\left(\frac{x}{2}\right)}{a^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+a^2\sqrt{a^2+b^2}+b^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+b^2\sqrt{a^2+b^2}} - \frac{ab\log\left(\tan\left(\frac{x}{2}\right)-\frac{b}{a}+\frac{\sqrt{a^2+b^2}}{a}\right)}{a^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+a^2\sqrt{a^2+b^2}+b^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+b^2\sqrt{a^2+b^2}} - \frac{2a\sqrt{a^2+b^2}}{a^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+a^2\sqrt{a^2+b^2}+b^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+b^2\sqrt{a^2+b^2}} + \frac{2b\sqrt{a^2+b^2}\tan\left(\frac{x}{2}\right)}{a^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+a^2\sqrt{a^2+b^2}+b^2\sqrt{a^2+b^2}\tan^2\left(\frac{x}{2}\right)+b^2\sqrt{a^2+b^2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Piecewise((zoo*sin(x), Eq(a, 0) & Eq(b, 0)), (sin(x)/b, Eq(a, 0)), (I*sin(x
)**2/(3*I*b*sin(x) + 3*b*cos(x)) + sin(x)*cos(x)/(3*I*b*sin(x) + 3*b*cos(x)
) - I*cos(x)**2/(3*I*b*sin(x) + 3*b*cos(x)), Eq(a, -I*b)), (-I*sin(x)**2/(-
3*I*b*sin(x) + 3*b*cos(x)) + sin(x)*cos(x)/(-3*I*b*sin(x) + 3*b*cos(x)) + I
*cos(x)**2/(-3*I*b*sin(x) + 3*b*cos(x)), Eq(a, I*b)), (a*b*log(tan(x/2) - b
/a - sqrt(a**2 + b**2)/a)*tan(x/2)**2/(a**2*sqrt(a**2 + b**2)*tan(x/2)**2 +
 a**2*sqrt(a**2 + b**2) + b**2*sqrt(a**2 + b**2)*tan(x/2)**2 + b**2*sqrt(a*
*2 + b**2)) + a*b*log(tan(x/2) - b/a - sqrt(a**2 + b**2)/a)/(a**2*sqrt(a**2
 + b**2)*tan(x/2)**2 + a**2*sqrt(a**2 + b**2) + b**2*sqrt(a**2 + b**2)*tan(
x/2)**2 + b**2*sqrt(a**2 + b**2)) - a*b*log(tan(x/2) - b/a + sqrt(a**2 + b*
*2)/a)*tan(x/2)**2/(a**2*sqrt(a**2 + b**2)*tan(x/2)**2 + a**2*sqrt(a**2 + b
**2) + b**2*sqrt(a**2 + b**2)*tan(x/2)**2 + b**2*sqrt(a**2 + b**2)) - a*b*l
og(tan(x/2) - b/a + sqrt(a**2 + b**2)/a)/(a**2*sqrt(a**2 + b**2)*tan(x/2)**
2 + a**2*sqrt(a**2 + b**2) + b**2*sqrt(a**2 + b**2)*tan(x/2)**2 + b**2*sqrt
(a**2 + b**2)) - 2*a*sqrt(a**2 + b**2)/(a**2*sqrt(a**2 + b**2)*tan(x/2)**2
+ a**2*sqrt(a**2 + b**2) + b**2*sqrt(a**2 + b**2)*tan(x/2)**2 + b**2*sqrt(a
**2 + b**2)) + 2*b*sqrt(a**2 + b**2)*tan(x/2)/(a**2*sqrt(a**2 + b**2)*tan(x
/2)**2 + a**2*sqrt(a**2 + b**2) + b**2*sqrt(a**2 + b**2)*tan(x/2)**2 + b**2
*sqrt(a**2 + b**2)), True))
```

Giac [A]
time = 0.46, size = 94, normalized size = 1.45

$$\frac{ab\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^2+b^2\right)^{\frac{3}{2}}} + \frac{2\,\left(b\tan\left(\frac{1}{2}\,x\right)-a\right)}{\left(a^2+b^2\right)\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] a*b*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x) -
2*b + 2*sqrt(a^2 + b^2)))/(a^2 + b^2)^(3/2) + 2*(b*tan(1/2*x) - a)/((a^2 +
b^2)*(tan(1/2*x)^2 + 1))
```

Mupad [B]
time = 1.38, size = 93, normalized size = 1.43

$$\frac{2\,a\,b\,\mathrm{atanh}\left(\frac{2\,a^2\,b+2\,b^3-2\,a\,\tan\left(\frac{x}{2}\right)\,\left(a^2+b^2\right)}{2\,{\left(a^2+b^2\right)}^{3/2}}\right)}{{\left(a^2+b^2\right)}^{3/2}}-\frac{\frac{2\,a}{a^2+b^2}-\frac{2\,b\,\tan\left(\frac{x}{2}\right)}{a^2+b^2}}{{\tan\left(\frac{x}{2}\right)}^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*sin(x))/(a*cos(x) + b*sin(x)),x)
```

```
[Out] (2*a*b*atanh((2*a^2*b + 2*b^3 - 2*a*tan(x/2)*(a^2 + b^2))/(2*(a^2 + b^2)^(3
/2))))/(a^2 + b^2)^(3/2) - ((2*a)/(a^2 + b^2) - (2*b*tan(x/2))/(a^2 + b^2))
/(tan(x/2)^2 + 1)
```

3.276 $\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=92

$$-\frac{ab^2x}{(a^2+b^2)^2}+\frac{ax}{2(a^2+b^2)}+\frac{a^2b\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^2}-\frac{a\cos(x)\sin(x)}{2(a^2+b^2)}+\frac{b\sin^2(x)}{2(a^2+b^2)}$$

```
[Out] -a*b^2*x/(a^2+b^2)^2+1/2*a*x/(a^2+b^2)+a^2*b*ln(a*cos(x)+b*sin(x))/(a^2+b^2
)^2-1/2*a*cos(x)*sin(x)/(a^2+b^2)+1/2*b*sin(x)^2/(a^2+b^2)
```

Rubi [A]
time = 0.09, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3188, 2644, 30, 2715, 8, 3176, 3212}

$$\frac{ax}{2(a^2+b^2)}-\frac{ab^2x}{(a^2+b^2)^2}+\frac{b\sin^2(x)}{2(a^2+b^2)}-\frac{a\sin(x)\cos(x)}{2(a^2+b^2)}+\frac{a^2b\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x]^2)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -((a*b^2*x)/(a^2 + b^2)^2) + (a*x)/(2*(a^2 + b^2)) + (a^2*b*Log[a*Cos[x] +
b*Sin[x]])/(a^2 + b^2)^2 - (a*Cos[x]*Sin[x])/(2*(a^2 + b^2)) + (b*Sin[x]^2)
/(2*(a^2 + b^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
```

```
*n]
```

Rule 3176

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx &= \frac{a\int \sin^2(x)\,dx}{a^2+b^2}+\frac{b\int \cos(x)\sin(x)\,dx}{a^2+b^2}-\frac{(ab)\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}\\ &= -\frac{ab^2x}{(a^2+b^2)^2}-\frac{a\cos(x)\sin(x)}{2(a^2+b^2)}+\frac{(a^2b)\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{(a^2+b^2)^2}+\frac{a\int 1\,dx}{2(a^2+b^2)}+\frac{b\text{Subst}(\cdots}{\cdots}\\ &= -\frac{ab^2x}{(a^2+b^2)^2}+\frac{ax}{2(a^2+b^2)}+\frac{a^2b\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^2}-\frac{a\cos(x)\sin(x)}{2(a^2+b^2)}+\frac{b\sin\cdots}{2(a\cdots}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.34, size = 153, normalized size = 1.66

$$-\frac{-2a^3x-6ia^2bx+6ab^2x+2ib^3x-2ib(-3a^2+b^2)\,\text{ArcTan}(\tan(x))+2b(a^2+b^2)\cos(2x)-2(a^2+b^2)\,(ax+b\log(a\cos(x)+b\sin(x)))-3a^2b\log\left((a\cos(x)+b\sin(x))^2\right)+b^3\log\left((a\cos(x)+b\sin(x))^2\right)+2a^3\sin(2x)+2ab^2\sin(2x)}{8\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x]^2)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -1/8*(-2*a^3*x - (6*I)*a^2*b*x + 6*a*b^2*x + (2*I)*b^3*x - (2*I)*b*(-3*a^2
+ b^2)*ArcTan[Tan[x]] + 2*b*(a^2 + b^2)*Cos[2*x] - 2*(a^2 + b^2)*(a*x + b*L
og[a*Cos[x] + b*Sin[x]]) - 3*a^2*b*Log[(a*Cos[x] + b*Sin[x])^2] + b^3*Log[(
a*Cos[x] + b*Sin[x])^2] + 2*a^3*Sin[2*x] + 2*a*b^2*Sin[2*x])/(a^2 + b^2)^2
```

Maple [A]
time = 0.17, size = 98, normalized size = 1.07

method	result
default	$\frac{b\,a^{2}\ln(a+b\tan(x))}{(a^{2}+b^{2})^{2}}+\frac{\frac{\left(-\frac{1}{2}a^{3}-\frac{1}{2}a\,b^{2}\right)\tan(x)-\frac{a^{2}b}{2}-\frac{b^{3}}{2}}{\tan^{2}(x)+1}+\frac{a\left(-ab\ln\left(\tan^{2}(x)+1\right)+\left(a^{2}-b^{2}\right)\arctan(\tan(x))\right)}{2}}{(a^{2}+b^{2})^{2}}$
risch	$-\frac{ax}{2(2iab-a^{2}+b^{2})}+\frac{i\mathrm{e}^{2ix}}{-8ib+8a}-\frac{i\mathrm{e}^{-2ix}}{8(ib+a)}-\frac{2ia^{2}bx}{a^{4}+2a^{2}b^{2}+b^{4}}+\frac{a^{2}b\ln\left(\mathrm{e}^{2ix}-\frac{ib+a}{ib-a}\right)}{a^{4}+2a^{2}b^{2}+b^{4}}$
norman	$\frac{\frac{a\left(\tan^{5}\left(\frac{x}{2}\right)\right)}{a^{2}+b^{2}}+\frac{2b\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{a^{2}+b^{2}}+\frac{2b\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{a^{2}+b^{2}}-\frac{a\tan\left(\frac{x}{2}\right)}{a^{2}+b^{2}}+\frac{a\left(a^{2}-b^{2}\right)x}{2a^{4}+4a^{2}b^{2}+2b^{4}}+\frac{3a\left(a^{2}-b^{2}\right)x\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}+\frac{3a\left(a^{2}-b^{2}\right)x\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{2\left(a^{4}+2a^{2}b^{2}+b^{4}\right)}+\frac{a\left(a^{2}-b^{2}\right)\cdots}{2a^{4}+4a\cdots}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] b*a^2/(a^2+b^2)^2*ln(a+b*tan(x))+1/(a^2+b^2)^2*(((-1/2*a^3-1/2*a*b^2)*tan(x
)-1/2*a^2*b-1/2*b^3)/(tan(x)^2+1)+1/2*a*(-a*b*ln(tan(x)^2+1)+(a^2-b^2)*arct
an(tan(x))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. 2(86) = 172.
time = 0.49, size = 211, normalized size = 2.29

$$\frac{a^{2}b\log\left(-a-\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^{2}}{(\cos(x)+1)^{2}}\right)}{a^{4}+2\,a^{2}b^{2}+b^{4}}-\frac{a^{2}b\log\left(\frac{\sin(x)^{2}}{(\cos(x)+1)^{2}}+1\right)}{a^{4}+2\,a^{2}b^{2}+b^{4}}+\frac{(a^{3}-ab^{2})\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{a^{4}+2\,a^{2}b^{2}+b^{4}}-\frac{\frac{a\sin(x)}{\cos(x)+1}-\frac{2\,b\sin(x)^{2}}{(\cos(x)+1)^{2}}-\frac{a\sin(x)^{3}}{(\cos(x)+1)^{3}}}{a^{2}+b^{2}+\frac{2\,(a^{2}+b^{2})\sin(x)^{2}}{(\cos(x)+1)^{2}}+\frac{(a^{2}+b^{2})\sin(x)^{4}}{(\cos(x)+1)^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] a^2*b*log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/(a^4 +
2*a^2*b^2 + b^4) - a^2*b*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(a^4 + 2*a^2*b^2
+ b^4) + (a^3 - a*b^2)*arctan(sin(x)/(cos(x) + 1))/(a^4 + 2*a^2*b^2 + b^4)
- (a*sin(x)/(cos(x) + 1) - 2*b*sin(x)^2/(cos(x) + 1)^2 - a*sin(x)^3/(cos(x)
 + 1)^3)/(a^2 + b^2 + 2*(a^2 + b^2)*sin(x)^2/(cos(x) + 1)^2 + (a^2 + b^2)*s
in(x)^4/(cos(x) + 1)^4)
```

Fricas [A]
time = 1.90, size = 94, normalized size = 1.02

$$\frac{a^2b\log\left(2\,ab\cos\left(x\right)\sin\left(x\right)+\left(a^2-b^2\right)\cos\left(x\right)^2+b^2\right)-\left(a^2b+b^3\right)\cos\left(x\right)^2-\left(a^3+ab^2\right)\cos\left(x\right)\sin\left(x\right)+\left(a^3-ab^2\right)x}{2\left(a^4+2\,a^2b^2+b^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(a^2*b*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - (a^2*b +
 b^3)*cos(x)^2 - (a^3 + a*b^2)*cos(x)*sin(x) + (a^3 - a*b^2)*x)/(a^4 + 2*a^
2*b^2 + b^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)**2/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.43, size = 152, normalized size = 1.65

$$\frac{a^2b^2\log\left(|b\tan\left(x\right)+a|\right)}{a^4b+2\,a^2b^3+b^5}-\frac{a^2b\log\left(\tan\left(x\right)^2+1\right)}{2\left(a^4+2\,a^2b^2+b^4\right)}+\frac{\left(a^3-ab^2\right)x}{2\left(a^4+2\,a^2b^2+b^4\right)}+\frac{a^2b\tan\left(x\right)^2-a^3\tan\left(x\right)-ab^2\tan\left(x\right)-b^3}{2\left(a^4+2\,a^2b^2+b^4\right)\left(\tan\left(x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] a^2*b^2*log(abs(b*tan(x) + a))/(a^4*b + 2*a^2*b^3 + b^5) - 1/2*a^2*b*log(ta
n(x)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) + 1/2*(a^3 - a*b^2)*x/(a^4 + 2*a^2*b^2
+ b^4) + 1/2*(a^2*b*tan(x)^2 - a^3*tan(x) - a*b^2*tan(x) - b^3)/((a^4 + 2*a
^2*b^2 + b^4)*(tan(x)^2 + 1))
```

Mupad [B]
time = 6.23, size = 2500, normalized size = 27.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*sin(x)^2)/(a*cos(x) + b*sin(x)),x)
```

```
[Out] ((a*tan(x/2)^3)/(a^2 + b^2) - (a*tan(x/2))/(a^2 + b^2) + (2*b*tan(x/2)^2)/(
a^2 + b^2))/(2*tan(x/2)^2 + tan(x/2)^4 + 1) + (a^2*b*log(a + 2*b*tan(x/2) -
```

```
a*tan(x/2)^2))/(a^4 + b^4 + 2*a^2*b^2) - (4*a^2*b*log(1/(cos(x) + 1)))/(4*
a^4 + 4*b^4 + 8*a^2*b^2) - (a*atan((tan(x/2)*((((4*a^2*b*((a*(a + b)*((8*(1
2*a^9*b + 12*a^5*b^5 + 24*a^7*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (
32*a^2*b*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/(
(4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a - b))/
(2*(a^4 + b^4 + 2*a^2*b^2)) - (16*a^3*b*(a + b)*(a - b)*(12*a*b^10 + 48*a^3
*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*
(a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b
^4 + 8*a^2*b^2) - (a*((8*(a^9 + 2*a^3*b^6 - 7*a^5*b^4 - 8*a^7*b^2))/(a^6 +
b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (4*a^2*b*((8*(12*a^9*b + 12*a^5*b^5 + 24*a^7
*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*a^2*b*(12*a*b^10 + 48*a^3*
b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(
a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2))*(a + b)*
(a - b))/(2*(a^4 + b^4 + 2*a^2*b^2)) + (a^3*(a + b)^3*(a - b)^3*(12*a*b^10
+ 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((a^4 + b^4 + 2*a^2*b
^2)^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a^6 - b^6 + 35*a^2*b^4 - 35*a^
4*b^2))/(a^6 + b^6 + 15*a^2*b^4 + 15*a^4*b^2)^2 - (2*a*b*(5*a^4 + 5*b^4 - 2
6*a^2*b^2)*((8*(a^7*b + 2*a^5*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (
4*a^2*b*((8*(a^9 + 2*a^3*b^6 - 7*a^5*b^4 - 8*a^7*b^2))/(a^6 + b^6 + 3*a^2*b
^4 + 3*a^4*b^2) - (4*a^2*b*((8*(12*a^9*b + 12*a^5*b^5 + 24*a^7*b^3))/(a^6 +
 b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*a^2*b*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*
b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3
*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2)))/(4*a^4 + 4*b^4 + 8*a
^2*b^2) + (a*(a + b)*(a - b)*((a*(a + b)*((8*(12*a^9*b + 12*a^5*b^5 + 24*a^
7*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*a^2*b*(12*a*b^10 + 48*a^3
*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*
(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a - b))/(2*(a^4 + b^4 + 2*a^2*b^2))
- (16*a^3*b*(a + b)*(a - b)*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b
^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)*(a^6
 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(2*(a^4 + b^4 + 2*a^2*b^2)) - (8*a^4*b*(
a + b)^2*(a - b)^2*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a
^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)^2*(a^6 + b^6
+ 3*a^2*b^4 + 3*a^4*b^2))))/(a^6 + b^6 + 15*a^2*b^4 + 15*a^4*b^2)^2)*(a^10
+ b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2))/(4*a^6 - 4*a^4*b
^2) + (((a*(a + b)*((8*(3*a^8*b + 3*a^4*b^5 + 6*a^6*b^3))/(a^6 + b^6 + 3*a^
2*b^4 + 3*a^4*b^2) - (4*a^2*b*((8*(4*a^4*b^6 - 2*a^2*b^8 - 2*a^10 + 12*a^6*
b^4 + 4*a^8*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (32*a^2*b*(12*a^10*
b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 + 8
*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2
))*(a - b))/(2*(a^4 + b^4 + 2*a^2*b^2)) - (4*a^2*b*((a*(a + b)*(a - b)*((8*
(4*a^4*b^6 - 2*a^2*b^8 - 2*a^10 + 12*a^6*b^4 + 4*a^8*b^2))/(a^6 + b^6 + 3*a
^2*b^4 + 3*a^4*b^2) + (32*a^2*b*(12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a
^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 +
 3*a^4*b^2))))/(2*(a^4 + b^4 + 2*a^2*b^2)) + (16*a^3*b*(a + b)*(a - b)*(12*
a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^
```

```
4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))
))/(4*a^4 + 4*b^4 + 8*a^2*b^2) + (a^3*(a + b)^3*(a - b)^3*(12*a^10*b + 12*a
^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((a^4 + b^4 + 2*a^2*b^2)^3*
(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a^6 - b^6 + 35*a^2*b^4 - 35*a^4*b^2)
*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2))/((4*a^6 -
 4*a^4*b^2)*(a^6 + b^6 + 15*a^2*b^4 + 15*a^4*b^2)^2) + (2*a*b*(5*a^4 + 5*b^
4 - 26*a^2*b^2)*((8*a^6*b^2)/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (4*a^2*b
*((8*(3*a^8*b + 3*a^4*b^5 + 6*a^6*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)
 - (4*a^2*b*((8*(4*a^4*b^6 - 2*a^2*b^8 - 2*a^10 + 12*a^6*b^4 + 4*a^8*b^2))/
(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (32*a^2*b*(12*a^10*b + 12*a^2*b^9 + 4
8*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b
^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2)))/(4*a^4 + 4*b^4
 + 8*a^2*b^2) + (a*((a*(a + b)*(a - b)*((8*(4*a^4*b^6 - 2*a^2*b^8 - 2*a^10
+ 12*a^6*b^4 + 4*a^8*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (32*a^2*b*
(12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 +
4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(2*(a^4 + b^4 + 2
*a^2*b^2)) + (16*a^3*b*(a + b)*(a - b)*(12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7
 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^
2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))*(a + b)*(a - b))/(2*(a^4 + b^4
 + 2*a^2*b^2)) + (8*a^4*b*(a + b)^2*(a - b)^2*(...
```

3.277 $\int \frac{\cos(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=122

$$\frac{a^3b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}+\frac{ab^2\cos(x)}{(a^2+b^2)^2}-\frac{a\cos(x)}{a^2+b^2}+\frac{a\cos^3(x)}{3(a^2+b^2)}+\frac{a^2b\sin(x)}{(a^2+b^2)^2}+\frac{b\sin^3(x)}{3(a^2+b^2)}$$

```
[Out] a^3*b*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)+a*b^2*co
s(x)/(a^2+b^2)^2-a*cos(x)/(a^2+b^2)+1/3*a*cos(x)^3/(a^2+b^2)+a^2*b*sin(x)/(
a^2+b^2)^2+1/3*b*sin(x)^3/(a^2+b^2)
```

Rubi [A]
time = 0.12, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3188, 2644, 30, 2713, 3178, 3153, 212, 2718}

$$\frac{b\sin^3(x)}{3(a^2+b^2)}+\frac{a^2b\sin(x)}{(a^2+b^2)^2}+\frac{a\cos^3(x)}{3(a^2+b^2)}-\frac{a\cos(x)}{a^2+b^2}+\frac{ab^2\cos(x)}{(a^2+b^2)^2}+\frac{a^3b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x]^3)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (a^3*b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) +
(a*b^2*Cos[x])/(a^2 + b^2)^2 - (a*Cos[x])/(a^2 + b^2) + (a*Cos[x]^3)/(3*(a^
2 + b^2)) + (a^2*b*Sin[x])/(a^2 + b^2)^2 + (b*Sin[x]^3)/(3*(a^2 + b^2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3178

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*
Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*
x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m
, 1]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a\int \sin^3(x)\,dx}{a^2+b^2} + \frac{b\int \cos(x)\sin^2(x)\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}$$
$$= \frac{a^2b\sin(x)}{(a^2+b^2)^2} - \frac{(a^3b)\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{(a^2+b^2)^2} - \frac{(ab^2)\int \sin(x)\,dx}{(a^2+b^2)^2} - \frac{a\text{Subst}\left(\int (1-x^2)\right.}{a^2+b^2}$$
$$= \frac{ab^2\cos(x)}{(a^2+b^2)^2} - \frac{a\cos(x)}{a^2+b^2} + \frac{a\cos^3(x)}{3(a^2+b^2)} + \frac{a^2b\sin(x)}{(a^2+b^2)^2} + \frac{b\sin^3(x)}{3(a^2+b^2)} + \frac{(a^3b)\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{a^3b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} + \frac{ab^2\cos(x)}{(a^2+b^2)^2} - \frac{a\cos(x)}{a^2+b^2} + \frac{a\cos^3(x)}{3(a^2+b^2)} + \frac{a^2b\sin(x)}{(a^2+b^2)^2}$$

Mathematica [A]
time = 1.05, size = 113, normalized size = 0.93

$$-\frac{2a^3b\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} + \frac{(-9a^3+3ab^2)\cos(x)+a(a^2+b^2)\cos(3x)-2b(-7a^2-b^2+(a^2+b^2)\cos(2x))\sin(x)}{12(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x]^3)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (-2*a^3*b*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) + (
(-9*a^3 + 3*a*b^2)*Cos[x] + a*(a^2 + b^2)*Cos[3*x] - 2*b*(-7*a^2 - b^2 + (a
^2 + b^2)*Cos[2*x])*Sin[x])/(12*(a^2 + b^2)^2)
```

Maple [A]
time = 0.29, size = 163, normalized size = 1.34

method	result
default	$-\frac{16a^3b\,\text{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(8a^4+16a^2b^2+8b^4)\sqrt{a^2+b^2}} + \frac{2a^2b\left(\tan^5\left(\frac{x}{2}\right)\right)+2a\,b^2\left(\tan^4\left(\frac{x}{2}\right)\right)+2\left(\frac{10}{3}a^2b+\frac{4}{3}b^3\right)\left(\tan^3\left(\frac{x}{2}\right)\right)-4a^3\left(\tan^2\left(\frac{x}{2}\right)\right)+2a^2b\tan\left(\frac{x}{2}\right)-}{(a^4+2a^2b^2+b^4)\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}$
risch	$\frac{ie^{ix}b}{-16iab+8a^2-8b^2} - \frac{3\,e^{ix}a}{8(-2iab+a^2-b^2)} - \frac{ie^{-ix}b}{8(ib+a)^2} - \frac{3\,e^{-ix}a}{8(ib+a)^2} - \frac{ia^3b\ln\left(e^{ix}-\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,(a^2+b^2)^2} + \frac{ia^3b\ln\left(e^{ix}+\frac{ib+}{\sqrt{-a^2}}\right.}{\sqrt{-a^2-b^2}\,(a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -16*a^3*b/(8*a^4+16*a^2*b^2+8*b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2
*x)-2*b)/(a^2+b^2)^(1/2))+2/(a^4+2*a^2*b^2+b^4)*(a^2*b*tan(1/2*x)^5+a*b^2*t
```

```
an(1/2*x)^4+(10/3*a^2*b+4/3*b^3)*tan(1/2*x)^3-2*a^3*tan(1/2*x)^2+a^2*b*tan(
1/2*x)-2/3*a^3+1/3*a*b^2)/(1+tan(1/2*x)^2)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(114) = 228$.
time = 0.48, size = 278, normalized size = 2.28

$$\frac{a^3 b \log\left(\frac{b - \frac{a\sin(x)}{\cos(x)+1} + \sqrt{a^2+b^2}}{b - \frac{a\sin(x)}{\cos(x)+1} - \sqrt{a^2+b^2}}\right)}{(a^4 + 2a^2b^2 + b^4)\sqrt{a^2+b^2}} - \frac{2\left(2a^3 - ab^2 - \frac{3a^2b\sin(x)}{\cos(x)+1} + \frac{6a^3\sin(x)^2}{(\cos(x)+1)^2} - \frac{3ab^2\sin(x)^4}{(\cos(x)+1)^4} - \frac{3a^2b\sin(x)^5}{(\cos(x)+1)^5} - \frac{2(5a^2b+2b^3)\sin(x)^3}{(\cos(x)+1)^3}\right)}{3\left(a^4 + 2a^2b^2 + b^4 + \frac{3(a^4+2a^2b^2+b^4)\sin(x)^2}{(\cos(x)+1)^2} + \frac{3(a^4+2a^2b^2+b^4)\sin(x)^4}{(\cos(x)+1)^4} + \frac{(a^4+2a^2b^2+b^4)\sin(x)^6}{(\cos(x)+1)^6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] a^3*b*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(cos(
x) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2/3
*(2*a^3 - a*b^2 - 3*a^2*b*sin(x)/(cos(x) + 1) + 6*a^3*sin(x)^2/(cos(x) + 1)
^2 - 3*a*b^2*sin(x)^4/(cos(x) + 1)^4 - 3*a^2*b*sin(x)^5/(cos(x) + 1)^5 - 2*
(5*a^2*b + 2*b^3)*sin(x)^3/(cos(x) + 1)^3)/(a^4 + 2*a^2*b^2 + b^4 + 3*(a^4
+ 2*a^2*b^2 + b^4)*sin(x)^2/(cos(x) + 1)^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*sin(
x)^4/(cos(x) + 1)^4 + (a^4 + 2*a^2*b^2 + b^4)*sin(x)^6/(cos(x) + 1)^6)
```

Fricas [A]
time = 1.77, size = 210, normalized size = 1.72

$$\frac{3\sqrt{a^2+b^2}\,a^3 b \log\left(\frac{2ab\cos(x)\sin(x) + (a^2-b^2)\cos(x)^2 - 2a^2 - b^2 - 2\sqrt{a^2+b^2}\,(b\cos(x) - a\sin(x))}{2ab\cos(x)\sin(x) + (a^2-b^2)\cos(x)^2 + b^2}\right) + 2(a^5 + 2a^3b^2 + ab^4)\cos(x)^3 - 6(a^5 + a^3b^2)\cos(x) + 2\left(4a^4b + 5a^2b^3 + b^5 - (a^4b + 2a^2b^3 + b^5)\cos(x)^2\right)\sin(x)}{6(a^6 + 3a^4b^2 + 3a^2b^4 + b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(a^2 + b^2)*a^3*b*log((2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^
2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*si
n(x) + (a^2 - b^2)*cos(x)^2 + b^2)) + 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^3
- 6*(a^5 + a^3*b^2)*cos(x) + 2*(4*a^4*b + 5*a^2*b^3 + b^5 - (a^4*b + 2*a^2*
b^3 + b^5)*cos(x)^2)*sin(x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)**3/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.45, size = 190, normalized size = 1.56

$$\frac{a^3b\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^4+2\,a^2b^2+b^4\right)\sqrt{a^2+b^2}}+\frac{2\left(3\,a^2b\tan\left(\frac{1}{2}\,x\right)^5+3\,ab^2\tan\left(\frac{1}{2}\,x\right)^4+10\,a^2b\tan\left(\frac{1}{2}\,x\right)^3+4\,b^3\tan\left(\frac{1}{2}\,x\right)^3-6\,a^3\tan\left(\frac{1}{2}\,x\right)^2+3\,a^2b\tan\left(\frac{1}{2}\,x\right)-2\,a^3+ab^2\right)}{3\left(a^4+2\,a^2b^2+b^4\right)\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] a^3*b*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x)
- 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2/3
*(3*a^2*b*tan(1/2*x)^5 + 3*a*b^2*tan(1/2*x)^4 + 10*a^2*b*tan(1/2*x)^3 + 4*b
^3*tan(1/2*x)^3 - 6*a^3*tan(1/2*x)^2 + 3*a^2*b*tan(1/2*x) - 2*a^3 + a*b^2)/
((a^4 + 2*a^2*b^2 + b^4)*(tan(1/2*x)^2 + 1)^3)
```

Mupad [B]
time = 1.20, size = 286, normalized size = 2.34

$$\frac{\frac{2\,\left(a\,b^2-2\,a^3\right)}{3\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{4\,{\mathrm{tan}\left(\frac{x}{2}\right)}^3\,\left(5\,a^2\,b+2\,b^3\right)}{3\,\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{4\,a^3\,{\mathrm{tan}\left(\frac{x}{2}\right)}^2}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,a\,b^2\,{\mathrm{tan}\left(\frac{x}{2}\right)}^4}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,a^2\,b\,{\mathrm{tan}\left(\frac{x}{2}\right)}^5}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,a^2\,b\,\mathrm{tan}\left(\frac{x}{2}\right)}{a^4+2\,a^2\,b^2+b^4}}{{\mathrm{tan}\left(\frac{x}{2}\right)}^6+3\,{\mathrm{tan}\left(\frac{x}{2}\right)}^4+3\,{\mathrm{tan}\left(\frac{x}{2}\right)}^2+1}+\frac{2\,a^3\,b\,\mathrm{atanh}\left(\frac{2\,a^4\,b+2\,b^5+4\,a^2\,b^3-2\,a\,\mathrm{tan}\left(\frac{x}{2}\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}{2\,{\left(a^2+b^2\right)}^{5/2}}\right)}{{\left(a^2+b^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*sin(x)^3)/(a*cos(x) + b*sin(x)),x)
```

```
[Out] ((2*(a*b^2 - 2*a^3))/(3*(a^4 + b^4 + 2*a^2*b^2)) + (4*tan(x/2)^3*(5*a^2*b +
 2*b^3))/(3*(a^4 + b^4 + 2*a^2*b^2)) - (4*a^3*tan(x/2)^2)/(a^4 + b^4 + 2*a^
2*b^2) + (2*a*b^2*tan(x/2)^4)/(a^4 + b^4 + 2*a^2*b^2) + (2*a^2*b*tan(x/2)^5
)/(a^4 + b^4 + 2*a^2*b^2) + (2*a^2*b*tan(x/2))/(a^4 + b^4 + 2*a^2*b^2))/(3*
tan(x/2)^2 + 3*tan(x/2)^4 + tan(x/2)^6 + 1) + (2*a^3*b*atanh((2*a^4*b + 2*b
^5 + 4*a^2*b^3 - 2*a*tan(x/2)*(a^4 + b^4 + 2*a^2*b^2))/(2*(a^2 + b^2)^(5/2)
)))/(a^2 + b^2)^(5/2)
```

3.278 $\int \frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=93

$$-\frac{a^2bx}{(a^2+b^2)^2}+\frac{bx}{2(a^2+b^2)}-\frac{ab^2\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^2}+\frac{b\cos(x)\sin(x)}{2(a^2+b^2)}+\frac{a\sin^2(x)}{2(a^2+b^2)}$$

```
[Out] -a^2*b*x/(a^2+b^2)^2+1/2*b*x/(a^2+b^2)-a*b^2*ln(a*cos(x)+b*sin(x))/(a^2+b^2
)^2+1/2*b*cos(x)*sin(x)/(a^2+b^2)+1/2*a*sin(x)^2/(a^2+b^2)
```

Rubi [A]
time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3188, 2715, 8, 2644, 30, 3177, 3212}

$$-\frac{a^2bx}{(a^2+b^2)^2}+\frac{bx}{2(a^2+b^2)}+\frac{a\sin^2(x)}{2(a^2+b^2)}+\frac{b\sin(x)\cos(x)}{2(a^2+b^2)}-\frac{ab^2\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x])/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -((a^2*b*x)/(a^2 + b^2)^2) + (b*x)/(2*(a^2 + b^2)) - (a*b^2*Log[a*Cos[x] +
b*Sin[x]])/(a^2 + b^2)^2 + (b*Cos[x]*Sin[x])/(2*(a^2 + b^2)) + (a*Sin[x]^2)
/(2*(a^2 + b^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
```

```
*n]
```

Rule 3177

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx &= \frac{a\int\cos(x)\sin(x)\,dx}{a^2+b^2}+\frac{b\int\cos^2(x)\,dx}{a^2+b^2}-\frac{(ab)\int\frac{\cos(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}\\ &= -\frac{a^2bx}{\left(a^2+b^2\right)^2}+\frac{b\cos(x)\sin(x)}{2\left(a^2+b^2\right)}-\frac{\left(ab^2\right)\int\frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2}+\frac{a\mathrm{Subst}(\int x\,dx,x,\sin\\ &= -\frac{a^2bx}{\left(a^2+b^2\right)^2}+\frac{bx}{2\left(a^2+b^2\right)}-\frac{ab^2\log(a\cos(x)+b\sin(x))}{\left(a^2+b^2\right)^2}+\frac{b\cos(x)\sin(x)}{2\left(a^2+b^2\right)}+\frac{a}{2(\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.34, size = 82, normalized size = 0.88

$$\frac{4iab^2\mathrm{ArcTan}(\tan(x))-a(a^2+b^2)\cos(2x)-2b((a+ib)^2x+ab\log\left((a\cos(x)+b\sin(x))^2\right))+b(a^2+b^2)\sin(2x)}{4\left(a^2+b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x])/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] ((4*I)*a*b^2*ArcTan[Tan[x]] - a*(a^2 + b^2)*Cos[2*x] - 2*b*((a + I*b)^2*x +
 a*b*Log[(a*Cos[x] + b*Sin[x])^2]) + b*(a^2 + b^2)*Sin[2*x])/(4*(a^2 + b^2)
^2)
```

Maple [A]
time = 0.18, size = 98, normalized size = 1.05

method	result
default	$-\frac{b^{2} a \ln \left(a +b \tan \left(x \right)\right)}{\left(a^{2}+b^{2}\right)^{2}}+\frac{\frac{\left(\frac{1}{2} a^{2} b +\frac{1}{2} b^{3}\right) \tan \left(x \right)-\frac{a^{3}}{2}-\frac{a \,b^{2}}{2}}{\tan^{2}\left(x \right)+1}+\frac{b \left(a b \ln \left(\tan^{2}\left(x \right)+1\right)+\left(-a^{2}+b^{2}\right) \arctan \left(\tan \left(x \right)\right)\right)}{2}}{\left(a^{2}+b^{2}\right)^{2}}$
risch	$\frac{b x}{4 i a b -2 a^{2}+2 b^{2}}-\frac{{\mathrm e}^{2 i x}}{8 \left(-i b +a \right)}-\frac{{\mathrm e}^{-2 i x}}{8 \left(i b +a \right)}+\frac{2 i a \,b^{2} x}{a^{4}+2 a^{2} b^{2}+b^{4}}-\frac{a \,b^{2} \ln \left({\mathrm e}^{2 i x}-\frac{i b +a}{i b -a}\right)}{a^{4}+2 a^{2} b^{2}+b^{4}}$
norman	$\frac{\frac{b \tan \left(\frac{x}{2}\right)}{a^{2}+b^{2}}+\frac{2 a \left(\tan^{2}\left(\frac{x}{2}\right)\right)}{a^{2}+b^{2}}+\frac{2 a \left(\tan^{4}\left(\frac{x}{2}\right)\right)}{a^{2}+b^{2}}-\frac{b \left(\tan^{5}\left(\frac{x}{2}\right)\right)}{a^{2}+b^{2}}-\frac{\left(a^{2}-b^{2}\right) b x}{2 \left(a^{4}+2 a^{2} b^{2}+b^{4}\right)}-\frac{3 \left(a^{2}-b^{2}\right) b x \left(\tan^{2}\left(\frac{x}{2}\right)\right)}{2 \left(a^{4}+2 a^{2} b^{2}+b^{4}\right)}-\frac{3 \left(a^{2}-b^{2}\right) b x \left(\tan^{4}\left(\frac{x}{2}\right)\right)}{2 \left(a^{4}+2 a^{2} b^{2}+b^{4}\right)}-\frac{\left(a^{2}-b^{2}\right) b x \left(\cdots\right.}{2 \left(a^{4}+2 a^{2}\cdots\right.}}{\left(1+\tan^{2}\left(\frac{x}{2}\right)\right)^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -b^2*a/(a^2+b^2)^2*ln(a+b*tan(x))+1/(a^2+b^2)^2*(((1/2*a^2*b+1/2*b^3)*tan(x
)-1/2*a^3-1/2*a*b^2)/(tan(x)^2+1)+1/2*b*(a*b*ln(tan(x)^2+1)+(-a^2+b^2)*arct
an(tan(x))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(87) = 174.
time = 0.48, size = 212, normalized size = 2.28

$$-\frac{ab^{2} \log\left(-a - \frac{2\,b \sin\left(x\right)}{\cos\left(x\right) + 1} + \frac{a \sin\left(x\right)^{2}}{{\left(\cos\left(x\right) + 1\right)}^{2}}\right)}{a^{4} + 2\,a^{2} b^{2} + b^{4}} + \frac{ab^{2} \log\left(\frac{\sin\left(x\right)^{2}}{{\left(\cos\left(x\right) + 1\right)}^{2}} + 1\right)}{a^{4} + 2\,a^{2} b^{2} + b^{4}} - \frac{{\left(a^{2} b - b^{3}\right)} \arctan\left(\frac{\sin\left(x\right)}{\cos\left(x\right) + 1}\right)}{a^{4} + 2\,a^{2} b^{2} + b^{4}} + \frac{\frac{b \sin\left(x\right)}{\cos\left(x\right) + 1} + \frac{2\,a \sin\left(x\right)^{2}}{{\left(\cos\left(x\right) + 1\right)}^{2}} - \frac{b \sin\left(x\right)^{3}}{{\left(\cos\left(x\right) + 1\right)}^{3}}}{a^{2} + b^{2} + \frac{2\,{\left(a^{2} + b^{2}\right)} \sin\left(x\right)^{2}}{{\left(\cos\left(x\right) + 1\right)}^{2}} + \frac{{\left(a^{2} + b^{2}\right)} \sin\left(x\right)^{4}}{{\left(\cos\left(x\right) + 1\right)}^{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] -a*b^2*log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/(a^4 +
 2*a^2*b^2 + b^4) + a*b^2*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(a^4 + 2*a^2*b^2
 + b^4) - (a^2*b - b^3)*arctan(sin(x)/(cos(x) + 1))/(a^4 + 2*a^2*b^2 + b^4)
 + (b*sin(x)/(cos(x) + 1) + 2*a*sin(x)^2/(cos(x) + 1)^2 - b*sin(x)^3/(cos(x
) + 1)^3)/(a^2 + b^2 + 2*(a^2 + b^2)*sin(x)^2/(cos(x) + 1)^2 + (a^2 + b^2)*
sin(x)^4/(cos(x) + 1)^4)
```

Fricas [A]
time = 1.62, size = 94, normalized size = 1.01

$$-\frac{ab^{2} \log\left(2\,ab \cos\left(x\right) \sin\left(x\right) + {\left(a^{2} - b^{2}\right)} \cos\left(x\right)^{2} + b^{2}\right) + {\left(a^{3} + ab^{2}\right)} \cos\left(x\right)^{2} - {\left(a^{2} b + b^{3}\right)} \cos\left(x\right) \sin\left(x\right) + {\left(a^{2} b - b^{3}\right)} x}{2\,{\left(a^{4} + 2\,a^{2} b^{2} + b^{4}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/2*(a*b^2*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) + (a^3 +
a*b^2)*cos(x)^2 - (a^2*b + b^3)*cos(x)*sin(x) + (a^2*b - b^3)*x)/(a^4 + 2*a
^2*b^2 + b^4)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.45, size = 156, normalized size = 1.68

$$-\frac{ab^3\log\left(|b\tan\left(x\right)+a|\right)}{a^4b+2\,a^2b^3+b^5}+\frac{ab^2\log\left(\tan\left(x\right)^2+1\right)}{2\left(a^4+2\,a^2b^2+b^4\right)}-\frac{\left(a^2b-b^3\right)x}{2\left(a^4+2\,a^2b^2+b^4\right)}-\frac{ab^2\tan\left(x\right)^2-a^2b\tan\left(x\right)-b^3\tan\left(x\right)+a^3+2\,ab^2}{2\left(a^4+2\,a^2b^2+b^4\right)\left(\tan\left(x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] -a*b^3*log(abs(b*tan(x) + a))/(a^4*b + 2*a^2*b^3 + b^5) + 1/2*a*b^2*log(tan
(x)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) - 1/2*(a^2*b - b^3)*x/(a^4 + 2*a^2*b^2 +
 b^4) - 1/2*(a*b^2*tan(x)^2 - a^2*b*tan(x) - b^3*tan(x) + a^3 + 2*a*b^2)/((
a^4 + 2*a^2*b^2 + b^4)*(tan(x)^2 + 1))
```

Mupad [**B**]
time = 6.19, size = 2500, normalized size = 26.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2*sin(x))/(a*cos(x) + b*sin(x)),x)
```

```
[Out] ((b*tan(x/2))/(a^2 + b^2) + (2*a*tan(x/2)^2)/(a^2 + b^2) - (b*tan(x/2)^3)/(
a^2 + b^2))/(2*tan(x/2)^2 + tan(x/2)^4 + 1) - (a*b^2*log(a + 2*b*tan(x/2) -
 a*tan(x/2)^2))/(a^4 + b^4 + 2*a^2*b^2) + (4*a*b^2*log(1/(cos(x) + 1)))/(4*
a^4 + 4*b^4 + 8*a^2*b^2) - (b*atan((tan(x/2)*((((4*a*b^2*((b*(a + b)*(a - b
)*((8*(12*a^4*b^6 + 24*a^6*b^4 + 12*a^8*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^
4*b^2) - (32*a*b^2*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a
^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))
```

```
)/(2*(a^4 + b^4 + 2*a^2*b^2)) - (16*a*b^3*(a + b)*(a - b)*(12*a*b^10 + 48*a
^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2
)*(a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4
*b^4 + 8*a^2*b^2) - (b*(a + b)*((8*(2*a*b^8 - 7*a^3*b^6 - 8*a^5*b^4 + a^7*b
^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (4*a*b^2*((8*(12*a^4*b^6 + 24*a^
6*b^4 + 12*a^8*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*a*b^2*(12*a*
b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4
+ 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*
b^2))*(a - b))/(2*(a^4 + b^4 + 2*a^2*b^2)) + (b^3*(a + b)^3*(a - b)^3*(12*a
*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((a^4 + b^4 + 2
*a^2*b^2)^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a^6 - b^6 + 35*a^2*b^4 -
 35*a^4*b^2))/(a^6 + b^6 + 15*a^2*b^4 + 15*a^4*b^2)^2 - (2*a*b*(5*a^4 + 5*b
^4 - 26*a^2*b^2)*((8*(2*a^2*b^6 + a^4*b^4))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*
b^2) + (4*a*b^2*((8*(2*a*b^8 - 7*a^3*b^6 - 8*a^5*b^4 + a^7*b^2))/(a^6 + b^6
 + 3*a^2*b^4 + 3*a^4*b^2) - (4*a*b^2*((8*(12*a^4*b^6 + 24*a^6*b^4 + 12*a^8*
b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*a*b^2*(12*a*b^10 + 48*a^3*b
^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a
^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2)))/(4*a^4 +
 4*b^4 + 8*a^2*b^2) + (b*((b*(a + b)*(a - b)*((8*(12*a^4*b^6 + 24*a^6*b^4 +
 12*a^8*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*a*b^2*(12*a*b^10 +
48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2
*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(2*(a^4 + b^4 + 2*a^2*b^2)) -
(16*a*b^3*(a + b)*(a - b)*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*a^7*b^4
 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)*(a^6 +
 b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a + b)*(a - b))/(2*(a^4 + b^4 + 2*a^2*b^2)
) - (8*a*b^4*(a + b)^2*(a - b)^2*(12*a*b^10 + 48*a^3*b^8 + 72*a^5*b^6 + 48*
a^7*b^4 + 12*a^9*b^2))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)
^2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(a^6 + b^6 + 15*a^2*b^4 + 15*a^4*
b^2)^2)*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8*b^2))/(4
*a*b^5 - 4*a^3*b^3) + (((b*(a + b)*(a - b)*((8*(3*a^2*b^7 + 6*a^4*b^5 + 3*a
^6*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (4*a*b^2*((8*(4*a^3*b^7 - 2*
a^9*b - 2*a*b^9 + 12*a^5*b^5 + 4*a^7*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b
^2) + (32*a*b^2*(12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*
b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(
4*a^4 + 4*b^4 + 8*a^2*b^2)))/(2*(a^4 + b^4 + 2*a^2*b^2)) - (4*a*b^2*((b*(a
+ b)*(a - b)*((8*(4*a^3*b^7 - 2*a^9*b - 2*a*b^9 + 12*a^5*b^5 + 4*a^7*b^3))/
(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (32*a*b^2*(12*a^10*b + 12*a^2*b^9 + 4
8*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b
^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(2*(a^4 + b^4 + 2*a^2*b^2)) + (16*a*b^3*(a +
 b)*(a - b)*(12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3)
)/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b
^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^2) + (b^3*(a + b)^3*(a - b)^3*(
12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((a^4 + b^4
 + 2*a^2*b^2)^3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a^6 - b^6 + 35*a^2*b
^4 - 35*a^4*b^2)*(a^10 + b^10 + 5*a^2*b^8 + 10*a^4*b^6 + 10*a^6*b^4 + 5*a^8
```

```
*b^2))/((4*a*b^5 - 4*a^3*b^3)*(a^6 + b^6 + 15*a^2*b^4 + 15*a^4*b^2)^2) + (2
*a*b*(5*a^4 + 5*b^4 - 26*a^2*b^2)*((8*a^3*b^5)/(a^6 + b^6 + 3*a^2*b^4 + 3*a
^4*b^2) + (4*a*b^2*((8*(3*a^2*b^7 + 6*a^4*b^5 + 3*a^6*b^3))/(a^6 + b^6 + 3*
a^2*b^4 + 3*a^4*b^2) - (4*a*b^2*((8*(4*a^3*b^7 - 2*a^9*b - 2*a*b^9 + 12*a^5
*b^5 + 4*a^7*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (32*a*b^2*(12*a^10
*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 +
8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(4*a^4 + 4*b^4 + 8*a^2*b^
2)))/(4*a^4 + 4*b^4 + 8*a^2*b^2) + (b*((b*(a + b)*(a - b)*((8*(4*a^3*b^7 -
2*a^9*b - 2*a*b^9 + 12*a^5*b^5 + 4*a^7*b^3))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4
*b^2) + (32*a*b^2*(12*a^10*b + 12*a^2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^
8*b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))
/(2*(a^4 + b^4 + 2*a^2*b^2)) + (16*a*b^3*(a + b)*(a - b)*(12*a^10*b + 12*a^
2*b^9 + 48*a^4*b^7 + 72*a^6*b^5 + 48*a^8*b^3))/((4*a^4 + 4*b^4 + 8*a^2*b^2)
*(a^4 + b^4 + 2*a^2*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(a + b)*(a -
 b))/(2*(a^4 + b^4 + 2*a^2*b^2)) + (8*a*b^4*(a ...
```

3.279 $\int \frac{\cos^2(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=112

$$-\frac{a^2b^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}+\frac{a^2b\cos(x)}{(a^2+b^2)^2}-\frac{b\cos^3(x)}{3(a^2+b^2)}-\frac{ab^2\sin(x)}{(a^2+b^2)^2}+\frac{a\sin^3(x)}{3(a^2+b^2)}$$

```
[Out] -a^2*b^2*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)+a^2*b
*cos(x)/(a^2+b^2)^2-1/3*b*cos(x)^3/(a^2+b^2)-a*b^2*sin(x)/(a^2+b^2)^2+1/3*a
*sin(x)^3/(a^2+b^2)
```

Rubi [A]
time = 0.14, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3188, 2645, 30, 2644, 2717, 2718, 3153, 212}

$$\frac{a\sin^3(x)}{3(a^2+b^2)}-\frac{ab^2\sin(x)}{(a^2+b^2)^2}-\frac{b\cos^3(x)}{3(a^2+b^2)}+\frac{a^2b\cos(x)}{(a^2+b^2)^2}-\frac{a^2b^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x]^2)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -((a^2*b^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2
)) + (a^2*b*Cos[x])/(a^2 + b^2)^2 - (b*Cos[x]^3)/(3*(a^2 + b^2)) - (a*b^2*S
in[x])/(a^2 + b^2)^2 + (a*Sin[x]^3)/(3*(a^2 + b^2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^2(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a\int \cos(x)\sin^2(x)\,dx}{a^2+b^2} + \frac{b\int \cos^2(x)\sin(x)\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}$$

$$= -\frac{(a^2b)\int \sin(x)\,dx}{(a^2+b^2)^2} - \frac{(ab^2)\int \cos(x)\,dx}{(a^2+b^2)^2} + \frac{(a^2b^2)\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{(a^2+b^2)^2} + \frac{a\text{Subst}\left(\int x\right.}{a^2}$$

$$= \frac{a^2b\cos(x)}{(a^2+b^2)^2} - \frac{b\cos^3(x)}{3(a^2+b^2)} - \frac{ab^2\sin(x)}{(a^2+b^2)^2} + \frac{a\sin^3(x)}{3(a^2+b^2)} - \frac{(a^2b^2)\,\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx\right.}{(a^2+b}$$

$$= -\frac{a^2b^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} + \frac{a^2b\cos(x)}{(a^2+b^2)^2} - \frac{b\cos^3(x)}{3(a^2+b^2)} - \frac{ab^2\sin(x)}{(a^2+b^2)^2} + \frac{a\sin^3}{3(a^2+}$$

Mathematica [A]
time = 0.68, size = 115, normalized size = 1.03

$$\frac{2a^2b^2\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{(-9a^2b+3b^3)\cos(x)+b(a^2+b^2)\cos(3x)+2a(-a^2+5b^2+(a^2+b^2)\cos(2x))\sin(x)}{12\,(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x]^2)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (2*a^2*b^2*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) -
((-9*a^2*b + 3*b^3)*Cos[x] + b*(a^2 + b^2)*Cos[3*x] + 2*a*(-a^2 + 5*b^2 + (
a^2 + b^2)*Cos[2*x])*Sin[x])/(12*(a^2 + b^2)^2)
```

Maple [A]
time = 0.29, size = 165, normalized size = 1.47

method	result
default	$\frac{8a^2b^2\,\text{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(4a^4+8a^2b^2+4b^4)\sqrt{a^2+b^2}} - \frac{2\left(a\,b^2\left(\tan^5\left(\frac{x}{2}\right)\right)+b^3\left(\tan^4\left(\frac{x}{2}\right)\right)+\left(-\frac{4}{3}a^3+\frac{2}{3}a\,b^2\right)\left(\tan^3\left(\frac{x}{2}\right)\right)-2a^2b\left(\tan^2\left(\frac{x}{2}\right)\right)+a\,b^2\tan\left(\frac{x}{2}\right)-\frac{2a^2}{3}\right.}{(a^4+2a^2b^2+b^4)\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}$
risch	$\frac{e^{ix}b}{-16iab+8a^2-8b^2} - \frac{ie^{ix}a}{8(-2iab+a^2-b^2)} + \frac{e^{-ix}b}{8(ib+a)^2} + \frac{ie^{-ix}a}{8(ib+a)^2} - \frac{b^2a^2\ln\left(e^{ix}-\frac{ia^5+2ia^3b^2+ia\,b^4-b\,a^4-2b^3a^2-b^5}{(a^2+b^2)^{\frac{5}{2}}}\right)}{(a^2+b^2)^{\frac{5}{2}}} + \frac{b^2a^2\ln\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 8*a^2*b^2/(4*a^4+8*a^2*b^2+4*b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*
x)-2*b)/(a^2+b^2)^(1/2))-2/(a^4+2*a^2*b^2+b^4)*(a*b^2*tan(1/2*x)^5+b^3*tan(
```

```
1/2*x)^4+(-4/3*a^3+2/3*a*b^2)*tan(1/2*x)^3-2*a^2*b*tan(1/2*x)^2+a*b^2*tan(1
/2*x)-2/3*a^2*b+1/3*b^3)/(1+tan(1/2*x)^2)^3
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(104) = 208$.
time = 0.49, size = 281, normalized size = 2.51

$$-\frac{a^2b^2\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}} + \frac{2\left(2a^2b-b^3-\frac{3ab^2\sin(x)}{\cos(x)+1}+\frac{6a^2b\sin(x)^2}{(\cos(x)+1)^2}-\frac{3b^3\sin(x)^4}{(\cos(x)+1)^4}-\frac{3ab^2\sin(x)^5}{(\cos(x)+1)^5}+\frac{2(2a^3-ab^2)\sin(x)^3}{(\cos(x)+1)^3}\right)}{3\left(a^4+2a^2b^2+b^4+\frac{3(a^4+2a^2b^2+b^4)\sin(x)^2}{(\cos(x)+1)^2}+\frac{3(a^4+2a^2b^2+b^4)\sin(x)^4}{(\cos(x)+1)^4}+\frac{(a^4+2a^2b^2+b^4)\sin(x)^6}{(\cos(x)+1)^6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] -a^2*b^2*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(c
os(x) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) +
2/3*(2*a^2*b - b^3 - 3*a*b^2*sin(x)/(cos(x) + 1) + 6*a^2*b*sin(x)^2/(cos(x)
 + 1)^2 - 3*b^3*sin(x)^4/(cos(x) + 1)^4 - 3*a*b^2*sin(x)^5/(cos(x) + 1)^5 +
 2*(2*a^3 - a*b^2)*sin(x)^3/(cos(x) + 1)^3)/(a^4 + 2*a^2*b^2 + b^4 + 3*(a^4
 + 2*a^2*b^2 + b^4)*sin(x)^2/(cos(x) + 1)^2 + 3*(a^4 + 2*a^2*b^2 + b^4)*sin
(x)^4/(cos(x) + 1)^4 + (a^4 + 2*a^2*b^2 + b^4)*sin(x)^6/(cos(x) + 1)^6)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(104) = 208$.
time = 1.48, size = 215, normalized size = 1.92

$$\frac{3\sqrt{a^2+b^2}\,a^2b^2\log\left(-\frac{2ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2-2a^2-b^2+2\sqrt{a^2+b^2}\,(b\cos(x)-a\sin(x))}{2ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2}\right)-2(a^4b+2a^2b^3+b^5)\cos(x)^3+6(a^4b+a^2b^3)\cos(x)+2\left(a^5-a^3b^2-2ab^4-(a^5+2a^3b^2+ab^4)\cos(x)^2\right)\sin(x)}{6(a^6+3a^4b^2+3a^2b^4+b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(a^2 + b^2)*a^2*b^2*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(
x)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)
*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)) - 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(x)
^3 + 6*(a^4*b + a^2*b^3)*cos(x) + 2*(a^5 - a^3*b^2 - 2*a*b^4 - (a^5 + 2*a^3
*b^2 + a*b^4)*cos(x)^2)*sin(x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)**2/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 192, normalized size = 1.71

$$-\frac{a^{2}b^{2}\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}+b^{2}}}-\frac{2\left(3\,ab^{2}\tan\left(\frac{1}{2}\,x\right)^{5}+3\,b^{3}\tan\left(\frac{1}{2}\,x\right)^{4}-4\,a^{3}\tan\left(\frac{1}{2}\,x\right)^{3}+2\,ab^{2}\tan\left(\frac{1}{2}\,x\right)^{3}-6\,a^{2}b\tan\left(\frac{1}{2}\,x\right)^{2}+3\,ab^{2}\tan\left(\frac{1}{2}\,x\right)-2\,a^{2}b+b^{3}\right)}{3\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\left(\tan\left(\frac{1}{2}\,x\right)^{2}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] -a^2*b^2*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*
x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) -
2/3*(3*a*b^2*tan(1/2*x)^5 + 3*b^3*tan(1/2*x)^4 - 4*a^3*tan(1/2*x)^3 + 2*a*b
^2*tan(1/2*x)^3 - 6*a^2*b*tan(1/2*x)^2 + 3*a*b^2*tan(1/2*x) - 2*a^2*b + b^3
)/((a^4 + 2*a^2*b^2 + b^4)*(tan(1/2*x)^2 + 1)^3)
```

Mupad [B]
time = 1.00, size = 277, normalized size = 2.47

$$-\frac{\frac{4\tan\left(\frac{x}{2}\right)^{3}\left(a\,b^{2}-2\,a^{3}\right)}{3\left(a^{4}+2\,a^{2}\,b^{2}+b^{4}\right)}-\frac{2\,b\left(2\,a^{2}-b^{2}\right)}{3\left(a^{2}+b^{2}\right)^{2}}+\frac{2\,b^{3}\tan\left(\frac{x}{2}\right)^{4}}{a^{4}+2\,a^{2}\,b^{2}+b^{4}}-\frac{4\,a^{2}\,b\tan\left(\frac{x}{2}\right)^{2}}{a^{4}+2\,a^{2}\,b^{2}+b^{4}}+\frac{2\,a\,b^{2}\tan\left(\frac{x}{2}\right)^{5}}{a^{4}+2\,a^{2}\,b^{2}+b^{4}}+\frac{2\,a\,b^{2}\tan\left(\frac{x}{2}\right)}{a^{4}+2\,a^{2}\,b^{2}+b^{4}}}{\tan\left(\frac{x}{2}\right)^{6}+3\tan\left(\frac{x}{2}\right)^{4}+3\tan\left(\frac{x}{2}\right)^{2}+1}-\frac{2\,a^{2}\,b^{2}\,\mathrm{atanh}\left(\frac{2\,a^{4}\,b+2\,b^{5}+4\,a^{2}\,b^{3}-2\,a\tan\left(\frac{x}{2}\right)\left(a^{4}+2\,a^{2}\,b^{2}+b^{4}\right)}{2\left(a^{2}+b^{2}\right)^{5/2}}\right)}{\left(a^{2}+b^{2}\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2*sin(x)^2)/(a*cos(x) + b*sin(x)),x)
```

```
[Out] - ((4*tan(x/2)^3*(a*b^2 - 2*a^3))/(3*(a^4 + b^4 + 2*a^2*b^2)) - (2*b*(2*a^2
 - b^2))/(3*(a^2 + b^2)^2) + (2*b^3*tan(x/2)^4)/(a^4 + b^4 + 2*a^2*b^2) - (
4*a^2*b*tan(x/2)^2)/(a^4 + b^4 + 2*a^2*b^2) + (2*a*b^2*tan(x/2)^5)/(a^4 + b
^4 + 2*a^2*b^2) + (2*a*b^2*tan(x/2))/(a^4 + b^4 + 2*a^2*b^2))/(3*tan(x/2)^2
 + 3*tan(x/2)^4 + tan(x/2)^6 + 1) - (2*a^2*b^2*atanh((2*a^4*b + 2*b^5 + 4*a
^2*b^3 - 2*a*tan(x/2)*(a^4 + b^4 + 2*a^2*b^2))/(2*(a^2 + b^2)^(5/2))))/(a^2
 + b^2)^(5/2)
```

3.280 $\int \frac{\cos^2(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=176

$$\frac{a^2b^3x}{(a^2+b^2)^3}-\frac{a^2bx}{2\,(a^2+b^2)^2}+\frac{bx}{8\,(a^2+b^2)}-\frac{a^3b^2\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^3}+\frac{a^2b\cos(x)\sin(x)}{2\,(a^2+b^2)^2}+\frac{b\cos(x)\sin(x)}{8\,(a^2+b^2)}-\frac{b\,\mathrm{co}}{4}$$

```
[Out] a^2*b^3*x/(a^2+b^2)^3-1/2*a^2*b*x/(a^2+b^2)^2+1/8*b*x/(a^2+b^2)-a^3*b^2*ln(
a*cos(x)+b*sin(x))/(a^2+b^2)^3+1/2*a^2*b*cos(x)*sin(x)/(a^2+b^2)^2+1/8*b*co
s(x)*sin(x)/(a^2+b^2)-1/4*b*cos(x)^3*sin(x)/(a^2+b^2)-1/2*a*b^2*sin(x)^2/(a
^2+b^2)^2+1/4*a*sin(x)^4/(a^2+b^2)
```

Rubi [A]
time = 0.19, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3188, 2648, 2715, 8, 2644, 30, 3176, 3212}

$$\frac{bx}{8\,(a^2+b^2)}-\frac{a^2bx}{2\,(a^2+b^2)^2}+\frac{a\sin^4(x)}{4\,(a^2+b^2)}-\frac{ab^2\sin^2(x)}{2\,(a^2+b^2)^2}-\frac{b\sin(x)\cos^3(x)}{4\,(a^2+b^2)}+\frac{b\sin(x)\cos(x)}{8\,(a^2+b^2)}+\frac{a^2b\sin(x)\cos(x)}{2\,(a^2+b^2)^2}+\frac{a^2b^3x}{(a^2+b^2)^3}-\frac{a^3b^2\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x]^3)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (a^2*b^3*x)/(a^2 + b^2)^3 - (a^2*b*x)/(2*(a^2 + b^2)^2) + (b*x)/(8*(a^2 + b
^2)) - (a^3*b^2*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^3 + (a^2*b*Cos[x]*Sin
[x])/(2*(a^2 + b^2)^2) + (b*Cos[x]*Sin[x])/(8*(a^2 + b^2)) - (b*Cos[x]^3*Si
n[x])/(4*(a^2 + b^2)) - (a*b^2*Sin[x]^2)/(2*(a^2 + b^2)^2) + (a*Sin[x]^4)/(
4*(a^2 + b^2))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3176

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cos^2(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a\int \cos(x)\sin^3(x)\,dx}{a^2+b^2} + \frac{b\int \cos^2(x)\sin^2(x)\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}$$

$$= -\frac{b\cos^3(x)\sin(x)}{4(a^2+b^2)} - \frac{(a^2b)\int \sin^2(x)\,dx}{(a^2+b^2)^2} - \frac{(ab^2)\int \cos(x)\sin(x)\,dx}{(a^2+b^2)^2} + \frac{(a^2b^2)\int \frac{\cdots}{a\,\mathrm{co}\cdots}}{(a^2\cdots}$$

$$= \frac{a^2b^3x}{(a^2+b^2)^3} + \frac{a^2b\cos(x)\sin(x)}{2(a^2+b^2)^2} + \frac{b\cos(x)\sin(x)}{8(a^2+b^2)} - \frac{b\cos^3(x)\sin(x)}{4(a^2+b^2)} + \frac{a\sin^4(x)}{4(a^2+b^2)\cdots}$$

$$= \frac{a^2b^3x}{(a^2+b^2)^3} - \frac{a^2bx}{2(a^2+b^2)^2} + \frac{bx}{8(a^2+b^2)} - \frac{a^3b^2\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^3} + \frac{a^2b\cos\cdots}{2(a\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.56, size = 178, normalized size = 1.01

$$\frac{-12a^4bx-32ia^3b^2x+24a^2b^3x+4b^5x+32ia^3b^2\mathrm{ArcTan}(\tan(x))-4a(a^4-b^4)\cos(2x)+a^5\cos(4x)+2a^3b^2\cos(4x)+ab^4\cos(4x)-16a^3b^2\log\left((a\cos(x)+b\sin(x))^2\right)+8a^4b\sin(2x)+8a^2b^3\sin(2x)-a^4b\sin(4x)-2a^2b^3\sin(4x)-b^5\sin(4x)}{32(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x]^3)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (-12*a^4*b*x - (32*I)*a^3*b^2*x + 24*a^2*b^3*x + 4*b^5*x + (32*I)*a^3*b^2*A
rcTan[Tan[x]] - 4*a*(a^4 - b^4)*Cos[2*x] + a^5*Cos[4*x] + 2*a^3*b^2*Cos[4*x
] + a*b^4*Cos[4*x] - 16*a^3*b^2*Log[(a*Cos[x] + b*Sin[x])^2] + 8*a^4*b*Sin[
2*x] + 8*a^2*b^3*Sin[2*x] - a^4*b*Sin[4*x] - 2*a^2*b^3*Sin[4*x] - b^5*Sin[4
*x])/(32*(a^2 + b^2)^3)
```

Maple [A]
time = 0.26, size = 163, normalized size = 0.93

method	result
default	$-\frac{a^3b^2\ln(a+b\tan(x))}{(a^2+b^2)^3} + \frac{\frac{\left(\frac{3}{4}b^3a^2+\frac{1}{8}b^5+\frac{5}{8}ba^4\right)\left(\tan^3(x)\right)+\left(-\frac{1}{2}a^5-\frac{1}{2}a^3b^2\right)\left(\tan^2(x)\right)+\left(\frac{3}{8}ba^4+\frac{1}{4}b^3a^2-\frac{1}{8}b^5\right)\tan(x)-\frac{a^5}{4}+\frac{ab^4}{4}}{\left(\tan^2(x)+1\right)^2}+\frac{b\left(4a^3b\ln\cdots\right.}{\cdots}}{(a^2+b^2)^3}$
risch	$-\frac{3ibxa}{8(ia^3-3iab^2+3a^2b-b^3)} - \frac{b^2x}{8(ia^3-3iab^2+3a^2b-b^3)} - \frac{a\,\mathrm{e}^{2ix}}{16(-2iab+a^2-b^2)} - \frac{a\,\mathrm{e}^{-2ix}}{16(ib+a)^2} + \frac{2ia^3b^2x}{a^6+3a^4b^2+3a^2b^4+b^6} - \frac{a^3b^2\ln\cdots}{a^6+3a\cdots}$
norman	$\frac{-\frac{2ab^2\left(\tan^2\left(\frac{x}{2}\right)\right)}{a^4+2a^2b^2+b^4}-\frac{2ab^2\left(\tan^8\left(\frac{x}{2}\right)\right)}{a^4+2a^2b^2+b^4}+\frac{2\left(2a^3-ab^2\right)\left(\tan^4\left(\frac{x}{2}\right)\right)}{a^4+2a^2b^2+b^4}+\frac{2\left(2a^3-ab^2\right)\left(\tan^6\left(\frac{x}{2}\right)\right)}{a^4+2a^2b^2+b^4}+\frac{\left(3a^2-b^2\right)b\tan\left(\frac{x}{2}\right)}{4a^4+8a^2b^2+4b^4}-\frac{\left(3a^2-b^2\right)b\left(\tan^9\left(\frac{x}{2}\right)\right)}{4\left(a^4+2a^2b^2+b^4\right)}+\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -a^3*b^2/(a^2+b^2)^3*ln(a+b*tan(x))+1/(a^2+b^2)^3*(((3/4*b^3*a^2+1/8*b^5+5/
8*b*a^4)*tan(x)^3+(-1/2*a^5-1/2*a^3*b^2)*tan(x)^2+(3/8*b*a^4+1/4*b^3*a^2-1/
8*b^5)*tan(x)-1/4*a^5+1/4*a*b^4)/(tan(x)^2+1)^2+1/8*b*(4*a^3*b*ln(tan(x)^2+
1)+(-3*a^4+6*a^2*b^2+b^4)*arctan(tan(x))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(162) = 324$.
time = 0.50, size = 431, normalized size = 2.45

$$-\frac{a^3b^2\log\left(-a-\frac{2b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^6+3a^4b^2+3a^2b^4+b^6}+\frac{a^3b^2\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{(3a^4b-6a^2b^3-b^5)\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{4(a^6+3a^4b^2+3a^2b^4+b^6)}-\frac{\frac{8ab^2\sin(x)^2}{(\cos(x)+1)^2}-\frac{16a^3\sin(x)^4}{(\cos(x)+1)^4}+\frac{8ab^2\sin(x)^6}{(\cos(x)+1)^6}-\frac{(3a^2b-b^3)\sin(x)}{\cos(x)+1}-\frac{(11a^2b+7b^3)\sin(x)^3}{(\cos(x)+1)^3}+\frac{(11a^2b+7b^3)\sin(x)^5}{(\cos(x)+1)^5}+\frac{(3a^2b-b^3)\sin(x)^7}{(\cos(x)+1)^7}}{4\left(a^4+2a^2b^2+b^4+\frac{4(a^4+2a^2b^2+b^4)\sin(x)^2}{(\cos(x)+1)^2}+\frac{6(a^4+2a^2b^2+b^4)\sin(x)^4}{(\cos(x)+1)^4}+\frac{4(a^4+2a^2b^2+b^4)\sin(x)^6}{(\cos(x)+1)^6}+\frac{(a^4+2a^2b^2+b^4)\sin(x)^8}{(\cos(x)+1)^8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] -a^3*b^2*log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/(a^6
 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + a^3*b^2*log(sin(x)^2/(cos(x) + 1)^2 + 1)/
(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/4*(3*a^4*b - 6*a^2*b^3 - b^5)*arcta
n(sin(x)/(cos(x) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/4*(8*a*b^2*s
in(x)^2/(cos(x) + 1)^2 - 16*a^3*sin(x)^4/(cos(x) + 1)^4 + 8*a*b^2*sin(x)^6/
(cos(x) + 1)^6 - (3*a^2*b - b^3)*sin(x)/(cos(x) + 1) - (11*a^2*b + 7*b^3)*s
in(x)^3/(cos(x) + 1)^3 + (11*a^2*b + 7*b^3)*sin(x)^5/(cos(x) + 1)^5 + (3*a^
2*b - b^3)*sin(x)^7/(cos(x) + 1)^7)/(a^4 + 2*a^2*b^2 + b^4 + 4*(a^4 + 2*a^2
*b^2 + b^4)*sin(x)^2/(cos(x) + 1)^2 + 6*(a^4 + 2*a^2*b^2 + b^4)*sin(x)^4/(c
os(x) + 1)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*sin(x)^6/(cos(x) + 1)^6 + (a^4 + 2
*a^2*b^2 + b^4)*sin(x)^8/(cos(x) + 1)^8)
```

Fricas [A]
time = 1.73, size = 174, normalized size = 0.99

$$-\frac{4a^3b^2\log\left(2ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2\right)-2(a^5+2a^3b^2+ab^4)\cos(x)^4+4(a^5+a^3b^2)\cos(x)^2+(3a^4b-6a^2b^3-b^5)x+\left(2(a^4b+2a^2b^3+b^5)\cos(x)^3-(5a^4b+6a^2b^3+b^5)\cos(x)\right)\sin(x)}{8(a^6+3a^4b^2+3a^2b^4+b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] -1/8*(4*a^3*b^2*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - 2*(
a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^4 + 4*(a^5 + a^3*b^2)*cos(x)^2 + (3*a^4*b -
 6*a^2*b^3 - b^5)*x + (2*(a^4*b + 2*a^2*b^3 + b^5)*cos(x)^3 - (5*a^4*b + 6*
a^2*b^3 + b^5)*cos(x))*sin(x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)**3/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.43, size = 275, normalized size = 1.56

$$-\frac{a^{3} b^{3} \log\left(\left|b \tan\left(x\right) + a\right|\right)}{a^{6} b + 3 \, a^{4} b^{3} + 3 \, a^{2} b^{5} + b^{7}} + \frac{a^{3} b^{2} \log\left(\tan\left(x\right)^{2} + 1\right)}{2 \, {\left(a^{6} + 3 \, a^{4} b^{2} + 3 \, a^{2} b^{4} + b^{6}\right)}} - \frac{{\left(3 \, a^{4} b - 6 \, a^{2} b^{3} - b^{5}\right)} x}{8 \, {\left(a^{6} + 3 \, a^{4} b^{2} + 3 \, a^{2} b^{4} + b^{6}\right)}} - \frac{6 \, a^{3} b^{2} \tan\left(x\right)^{4} - 5 \, a^{4} b \tan\left(x\right)^{3} - 6 \, a^{2} b^{3} \tan\left(x\right)^{3} - b^{5} \tan\left(x\right)^{3} + 4 \, a^{5} \tan\left(x\right)^{2} + 16 \, a^{3} b^{2} \tan\left(x\right)^{2} - 3 \, a^{4} b \tan\left(x\right) - 2 \, a^{2} b^{3} \tan\left(x\right) + b^{5} \tan\left(x\right) + 2 \, a^{5} + 6 \, a^{3} b^{2} - 2 \, a b^{4}}{8 \, {\left(a^{6} + 3 \, a^{4} b^{2} + 3 \, a^{2} b^{4} + b^{6}\right)} {\left(\tan\left(x\right)^{2} + 1\right)}^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] -a^3*b^3*log(abs(b*tan(x) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) + 1/2
*a^3*b^2*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/8*(3*a^4
*b - 6*a^2*b^3 - b^5)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/8*(6*a^3*b^
2*tan(x)^4 - 5*a^4*b*tan(x)^3 - 6*a^2*b^3*tan(x)^3 - b^5*tan(x)^3 + 4*a^5*t
an(x)^2 + 16*a^3*b^2*tan(x)^2 - 3*a^4*b*tan(x) - 2*a^2*b^3*tan(x) + b^5*tan
(x) + 2*a^5 + 6*a^3*b^2 - 2*a*b^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(ta
n(x)^2 + 1)^2)
```

Mupad [B]
time = 11.95, size = 2500, normalized size = 14.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2*sin(x)^3)/(a*cos(x) + b*sin(x)),x)
```

```
[Out] (64*a^3*b^2*log(1/(cos(x) + 1)))/(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b
^2) - (b*atan((tan(x/2)*((((64*a^3*b^2*((b*((448*a^8*b^8 - 96*a^4*b^12 - 48
*a^6*b^10 - 16*a^2*b^14 + 912*a^10*b^6 + 672*a^12*b^4 + 176*a^14*b^2)/(2*(a
^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2
)) - (32*a^3*b^2*(192*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^1
0 + 6720*a^9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/((64*a^6
+ 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^
8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(b^4 - 3*a^4 + 6*a^2*b^2))/(8*(
a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (4*a^3*b^3*(b^4 - 3*a^4 + 6*a^2*b^2)*
(192*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8
+ 4032*a^11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^
2*b^4 + 192*a^4*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a
^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(64*a^6 + 6
4*b^6 + 192*a^2*b^4 + 192*a^4*b^2) - (b*((2*a*b^14 + 27*a^3*b^12 + 129*a^5*
b^10 + 62*a^7*b^8 - 156*a^9*b^6 - 105*a^11*b^4 + 9*a^13*b^2)/(2*(a^12 + b^1
2 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (64*
a^3*b^2*((448*a^8*b^8 - 96*a^4*b^12 - 48*a^6*b^10 - 16*a^2*b^14 + 912*a^10*
b^6 + 672*a^12*b^4 + 176*a^14*b^2)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^
8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (32*a^3*b^2*(192*a*b^16 + 1344
*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6 +
```

1344*a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^
2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^1
0*b^2))))/(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2))*(b^4 - 3*a^4 + 6*a
^2*b^2))/(8*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (b^3*(b^4 - 3*a^4 + 6*a^
2*b^2)^3*(192*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720
*a^9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/(1024*(a^6 + b^6
+ 3*a^2*b^4 + 3*a^4*b^2)^3*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*
b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(9*a^10 - b^10 - 11*a^2*b^8 + 46*a^4*b^6 +
706*a^6*b^4 - 493*a^8*b^2))/(9*a^10 + b^10 + 13*a^2*b^8 + 42*a^4*b^6 + 250
*a^6*b^4 + 229*a^8*b^2)^2 + (2*a*b*((2*a^4*b^10 + 21*a^6*b^8 + 44*a^8*b^6 +
9*a^10*b^4)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^
8*b^4 + 6*a^10*b^2)) + (64*a^3*b^2*((2*a*b^14 + 27*a^3*b^12 + 129*a^5*b^10
+ 62*a^7*b^8 - 156*a^9*b^6 - 105*a^11*b^4 + 9*a^13*b^2)/(2*(a^12 + b^12 + 6
*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (64*a^3*b
^2*((448*a^8*b^8 - 96*a^4*b^12 - 48*a^6*b^10 - 16*a^2*b^14 + 912*a^10*b^6 +
672*a^12*b^4 + 176*a^14*b^2)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 2
0*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (32*a^3*b^2*(192*a*b^16 + 1344*a^3*
b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6 + 1344*
a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a
^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2
))))/(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)))/(64*a^6 + 64*b^6 + 192
*a^2*b^4 + 192*a^4*b^2) + (b*((b*((448*a^8*b^8 - 96*a^4*b^12 - 48*a^6*b^10
- 16*a^2*b^14 + 912*a^10*b^6 + 672*a^12*b^4 + 176*a^14*b^2)/(2*(a^12 + b^12
+ 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (32*a
^3*b^2*(192*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a
^9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 +
192*a^2*b^4 + 192*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6
*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(b^4 - 3*a^4 + 6*a^2*b^2))/(8*(a^6 + b^6
+ 3*a^2*b^4 + 3*a^4*b^2)) - (4*a^3*b^3*(b^4 - 3*a^4 + 6*a^2*b^2)*(192*a*b^1
6 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^1
1*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 19
2*a^4*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 +
15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(b^4 - 3*a^4 + 6*a^2*b
^2))/(8*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (a^3*b^4*(b^4 - 3*a^4 + 6*a^
2*b^2)^2*(192*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720
*a^9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/(2*(64*a^6 + 64*b
^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^2*(a^12
+ b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))
*(57*a^8 + b^8 + 28*a^2*b^6 + 110*a^4*b^4 - 436*a^6*b^2))/(9*a^10 + b^10 +
13*a^2*b^8 + 42*a^4*b^6 + 250*a^6*b^4 + 229*a^8*b^2)^2)*(16*a^16 + 16*b^16
+ 128*a^2*b^14 + 448*a^4*b^12 + 896*a^6*b^10 + 1120*a^8*b^8 + 896*a^10*b^6
+ 448*a^12*b^4 + 128*a^14*b^2))/(a*b^7 + 6*a^3*b^5 - 3*a^5*b^3) + (((64*a^3
b^2((b*((8*a*b^15 + 24*a^15*b + 72*a^3*b^13 + 72*a^5*b^11 - 248*a^7*b^9 -
552*a^9*b^7 - 360*a^11*b^5 - 40*a^13*b^3)/(2*(a^12 + b^12 + 6*a^2*b^10 + 1
5*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (32*a^3*b^2*(192*a^16*

```
b + 192*a^2*b^15 + 1344*a^4*b^13 + 4032*a^6*b^11 + 6720*a^8*b^9 + 6720*a^10
*b^7 + 4032*a^12*b^5 + 1344*a^14*b^3))/((64*a^6...
```

3.281 $\int \frac{\cos^3(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=123

$$\frac{ab^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}-\frac{ab^2\cos(x)}{(a^2+b^2)^2}-\frac{a\cos^3(x)}{3(a^2+b^2)}-\frac{a^2b\sin(x)}{(a^2+b^2)^2}+\frac{b\sin(x)}{a^2+b^2}-\frac{b\sin^3(x)}{3(a^2+b^2)}$$

```
[Out] a*b^3*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)-a*b^2*co
s(x)/(a^2+b^2)^2-1/3*a*cos(x)^3/(a^2+b^2)-a^2*b*sin(x)/(a^2+b^2)^2+b*sin(x)
/(a^2+b^2)-1/3*b*sin(x)^3/(a^2+b^2)
```

Rubi [A]

time = 0.11, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3188, 2713, 2645, 30, 3179, 2717, 3153, 212}

$$-\frac{b\sin^3(x)}{3(a^2+b^2)}+\frac{b\sin(x)}{a^2+b^2}-\frac{a^2b\sin(x)}{(a^2+b^2)^2}-\frac{a\cos^3(x)}{3(a^2+b^2)}-\frac{ab^2\cos(x)}{(a^2+b^2)^2}+\frac{ab^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^3*Sin[x])/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (a*b^3*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) -
(a*b^2*Cos[x])/(a^2 + b^2)^2 - (a*Cos[x]^3)/(3*(a^2 + b^2)) - (a^2*b*Sin[x]
)/(a^2 + b^2)^2 + (b*Sin[x])/(a^2 + b^2) - (b*Sin[x]^3)/(3*(a^2 + b^2))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3179

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^3(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a\int \cos^2(x)\sin(x)\,dx}{a^2+b^2}+\frac{b\int \cos^3(x)\,dx}{a^2+b^2}-\frac{(ab)\int \frac{\cos^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}$$

$$= -\frac{ab^2\cos(x)}{(a^2+b^2)^2}-\frac{(a^2b)\int \cos(x)\,dx}{(a^2+b^2)^2}-\frac{(ab^3)\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{(a^2+b^2)^2}-\frac{a\text{Subst}\left(\int x^2\,dx,x,\right.}{a^2+b^2}$$

$$= -\frac{ab^2\cos(x)}{(a^2+b^2)^2}-\frac{a\cos^3(x)}{3(a^2+b^2)}-\frac{a^2b\sin(x)}{(a^2+b^2)^2}+\frac{b\sin(x)}{a^2+b^2}-\frac{b\sin^3(x)}{3(a^2+b^2)}+\frac{(ab^3)\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{ab^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}-\frac{ab^2\cos(x)}{(a^2+b^2)^2}-\frac{a\cos^3(x)}{3(a^2+b^2)}-\frac{a^2b\sin(x)}{(a^2+b^2)^2}+\frac{b\sin(x)}{a^2+b^2}-$$

Mathematica [A]
time = 0.56, size = 112, normalized size = 0.91

$$-\frac{2ab^3\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}-\frac{3a(a^2+5b^2)\cos(x)+a(a^2+b^2)\cos(3x)-2b(-a^2+5b^2+(a^2+b^2)\cos(2x))\sin(x)}{12(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^3*Sin[x])/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (-2*a*b^3*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) - (
3*a*(a^2 + 5*b^2)*Cos[x] + a*(a^2 + b^2)*Cos[3*x] - 2*b*(-a^2 + 5*b^2 + (a^
2 + b^2)*Cos[2*x])*Sin[x])/(12*(a^2 + b^2)^2)
```

Maple [A]
time = 0.29, size = 170, normalized size = 1.38

method	result
default	$-\frac{4a\,b^3\,\text{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(2a^4+4a^2b^2+2b^4)\sqrt{a^2+b^2}}+\frac{2b^3\left(\tan^5\left(\frac{x}{2}\right)\right)+2\left(-a^3-2a\,b^2\right)\left(\tan^4\left(\frac{x}{2}\right)\right)+2\left(-\frac{4}{3}a^2b+\frac{2}{3}b^3\right)\left(\tan^3\left(\frac{x}{2}\right)\right)-4a\,b^2\left(\tan^2\left(\frac{x}{2}\right)\right)+2b^3\,\text{ta}}{(a^4+2a^2b^2+b^4)\left(1+\tan^2\left(\frac{x}{2}\right)\right)^3}$
risch	$\frac{3i\mathrm{e}^{ix}b}{8(-2iab+a^2-b^2)}-\frac{\mathrm{e}^{ix}a}{8(-2iab+a^2-b^2)}-\frac{3i\mathrm{e}^{-ix}b}{8(ib+a)^2}-\frac{\mathrm{e}^{-ix}a}{8(ib+a)^2}+\frac{ib^3a\ln\left(\mathrm{e}^{ix}+\frac{ib+a}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,(a^2+b^2)^2}-\frac{ib^3a\ln\left(\mathrm{e}^{ix}-\frac{ib+a}{\sqrt{-a^2-}}\right.}{\sqrt{-a^2-b^2}\,(a^2+}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -4*a*b^3/(2*a^4+4*a^2*b^2+2*b^4)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x
)-2*b)/(a^2+b^2)^(1/2))+2/(a^4+2*a^2*b^2+b^4)*(b^3*tan(1/2*x)^5+(-a^3-2*a*b
```

```
^2)*tan(1/2*x)^4+(-4/3*a^2*b+2/3*b^3)*tan(1/2*x)^3-2*a*b^2*tan(1/2*x)^2+b^3
*tan(1/2*x)-1/3*a^3-4/3*a*b^2)/(1+tan(1/2*x)^2)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(115) = 230$.
time = 0.48, size = 281, normalized size = 2.28

$$\frac{ab^3\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}}-\frac{2\left(a^3+4ab^2-\frac{3b^3\sin(x)}{\cos(x)+1}+\frac{6ab^2\sin(x)^2}{(\cos(x)+1)^2}-\frac{3b^3\sin(x)^5}{(\cos(x)+1)^5}+\frac{2(2a^2b-b^3)\sin(x)^3}{(\cos(x)+1)^3}+\frac{3(a^3+2ab^2)\sin(x)^4}{(\cos(x)+1)^4}\right)}{3\left(a^4+2a^2b^2+b^4+\frac{3(a^4+2a^2b^2+b^4)\sin(x)^2}{(\cos(x)+1)^2}+\frac{3(a^4+2a^2b^2+b^4)\sin(x)^4}{(\cos(x)+1)^4}+\frac{(a^4+2a^2b^2+b^4)\sin(x)^6}{(\cos(x)+1)^6}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] a*b^3*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(cos(
x) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) - 2/3
*(a^3 + 4*a*b^2 - 3*b^3*sin(x)/(cos(x) + 1) + 6*a*b^2*sin(x)^2/(cos(x) + 1)
^2 - 3*b^3*sin(x)^5/(cos(x) + 1)^5 + 2*(2*a^2*b - b^3)*sin(x)^3/(cos(x) + 1
)^3 + 3*(a^3 + 2*a*b^2)*sin(x)^4/(cos(x) + 1)^4)/(a^4 + 2*a^2*b^2 + b^4 + 3
*(a^4 + 2*a^2*b^2 + b^4)*sin(x)^2/(cos(x) + 1)^2 + 3*(a^4 + 2*a^2*b^2 + b^4
)*sin(x)^4/(cos(x) + 1)^4 + (a^4 + 2*a^2*b^2 + b^4)*sin(x)^6/(cos(x) + 1)^6
)
```

Fricas [A]
time = 1.43, size = 213, normalized size = 1.73

$$\frac{3\sqrt{a^2+b^2}\,ab^3\log\left(\frac{2ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2-2a^2-b^2-2\sqrt{a^2+b^2}\,(b\cos(x)-a\sin(x))}{2ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2}\right)-2(a^5+2a^3b^2+ab^4)\cos(x)^3-6(a^3b^2+ab^4)\cos(x)-2\left(a^4b-a^2b^3-2b^5-(a^4b+2a^2b^3+b^5)\cos(x)^2\right)\sin(x)}{6(a^6+3a^4b^2+3a^2b^4+b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(a^2 + b^2)*a*b^3*log((2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^
2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*si
n(x) + (a^2 - b^2)*cos(x)^2 + b^2)) - 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^3
- 6*(a^3*b^2 + a*b^4)*cos(x) - 2*(a^4*b - a^2*b^3 - 2*b^5 - (a^4*b + 2*a^2*
b^3 + b^5)*cos(x)^2)*sin(x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 201, normalized size = 1.63

$$\frac{ab^3\log\left(\frac{\left|2a\tan\left(\frac{1}{2}x\right)-2b-2\sqrt{a^2+b^2}\right|}{\left|2a\tan\left(\frac{1}{2}x\right)-2b+2\sqrt{a^2+b^2}\right|}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}}+\frac{2\left(3b^3\tan\left(\frac{1}{2}x\right)^5-3a^3\tan\left(\frac{1}{2}x\right)^4-6ab^2\tan\left(\frac{1}{2}x\right)^4-4a^2b\tan\left(\frac{1}{2}x\right)^3+2b^3\tan\left(\frac{1}{2}x\right)^3-6ab^2\tan\left(\frac{1}{2}x\right)^2+3b^3\tan\left(\frac{1}{2}x\right)-a^3-4ab^2\right)}{3(a^4+2a^2b^2+b^4)\left(\tan\left(\frac{1}{2}x\right)^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] a*b^3*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x)
- 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b^2)) + 2/3
*(3*b^3*tan(1/2*x)^5 - 3*a^3*tan(1/2*x)^4 - 6*a*b^2*tan(1/2*x)^4 - 4*a^2*b*
tan(1/2*x)^3 + 2*b^3*tan(1/2*x)^3 - 6*a*b^2*tan(1/2*x)^2 + 3*b^3*tan(1/2*x)
 - a^3 - 4*a*b^2)/((a^4 + 2*a^2*b^2 + b^4)*(tan(1/2*x)^2 + 1)^3)
```

Mupad [B]
time = 1.26, size = 291, normalized size = 2.37

$$\frac{2\,a\,b^3\,\mathrm{atanh}\left(\frac{2\,a^4\,b+2\,b^5+4\,a^2\,b^3-2\,a\,\tan\left(\frac{x}{2}\right)\,\left(a^4+2\,a^2\,b^2+b^4\right)}{2\,{\left(a^2+b^2\right)}^{5/2}}\right)}{{\left(a^2+b^2\right)}^{5/2}}-\frac{\frac{2\,\left(a^3+4\,a\,b^2\right)}{3\,\left(a^4+2\,a^2\,b^2+b^4\right)}+\frac{4\,\tan\left(\frac{x}{2}\right)^3\,\left(2\,a^2\,b-b^3\right)}{3\,\left(a^4+2\,a^2\,b^2+b^4\right)}-\frac{2\,b^3\,\tan\left(\frac{x}{2}\right)}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,\tan\left(\frac{x}{2}\right)^4\,\left(a^3+2\,a\,b^2\right)}{a^4+2\,a^2\,b^2+b^4}-\frac{2\,b^3\,\tan\left(\frac{x}{2}\right)^5}{a^4+2\,a^2\,b^2+b^4}+\frac{4\,a\,b^2\,\tan\left(\frac{x}{2}\right)^2}{a^4+2\,a^2\,b^2+b^4}}{\tan\left(\frac{x}{2}\right)^6+3\,\tan\left(\frac{x}{2}\right)^4+3\,\tan\left(\frac{x}{2}\right)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^3*sin(x))/(a*cos(x) + b*sin(x)),x)
```

```
[Out] (2*a*b^3*atanh((2*a^4*b + 2*b^5 + 4*a^2*b^3 - 2*a*tan(x/2)*(a^4 + b^4 + 2*a
^2*b^2))/(2*(a^2 + b^2)^(5/2))))/(a^2 + b^2)^(5/2) - ((2*(4*a*b^2 + a^3))/(
3*(a^4 + b^4 + 2*a^2*b^2)) + (4*tan(x/2)^3*(2*a^2*b - b^3))/(3*(a^4 + b^4 +
 2*a^2*b^2)) - (2*b^3*tan(x/2))/(a^4 + b^4 + 2*a^2*b^2) + (2*tan(x/2)^4*(2*
a*b^2 + a^3))/(a^4 + b^4 + 2*a^2*b^2) - (2*b^3*tan(x/2)^5)/(a^4 + b^4 + 2*a
^2*b^2) + (4*a*b^2*tan(x/2)^2)/(a^4 + b^4 + 2*a^2*b^2))/(3*tan(x/2)^2 + 3*t
an(x/2)^4 + tan(x/2)^6 + 1)
```

3.282 $\int \frac{\cos^3(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=175

$$\frac{a^3b^2x}{(a^2+b^2)^3}-\frac{ab^2x}{2\,(a^2+b^2)^2}+\frac{ax}{8\,(a^2+b^2)}-\frac{b\cos^4(x)}{4\,(a^2+b^2)}+\frac{a^2b^3\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^3}-\frac{ab^2\cos(x)\sin(x)}{2\,(a^2+b^2)^2}+\frac{a\cos(x}{8\,(a^2}$$

```
[Out] a^3*b^2*x/(a^2+b^2)^3-1/2*a*b^2*x/(a^2+b^2)^2+1/8*a*x/(a^2+b^2)-1/4*b*cos(x
)^4/(a^2+b^2)+a^2*b^3*ln(a*cos(x)+b*sin(x))/(a^2+b^2)^3-1/2*a*b^2*cos(x)*si
n(x)/(a^2+b^2)^2+1/8*a*cos(x)*sin(x)/(a^2+b^2)-1/4*a*cos(x)^3*sin(x)/(a^2+b
^2)-1/2*a^2*b*sin(x)^2/(a^2+b^2)^2
```

Rubi [A]
time = 0.19, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3188, 2645, 30, 2648, 2715, 8, 2644, 3177, 3212}

$$\frac{ax}{8\,(a^2+b^2)}-\frac{ab^2x}{2\,(a^2+b^2)^2}-\frac{a^2b\sin^2(x)}{2\,(a^2+b^2)^2}-\frac{b\cos^4(x)}{4\,(a^2+b^2)}-\frac{a\sin(x)\cos^3(x)}{4\,(a^2+b^2)}+\frac{a\sin(x)\cos(x)}{8\,(a^2+b^2)}-\frac{ab^2\sin(x)\cos(x)}{2\,(a^2+b^2)^2}+\frac{a^2b^3\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^3}+\frac{a^3b^2x}{(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^3*Sin[x]^2)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (a^3*b^2*x)/(a^2 + b^2)^3 - (a*b^2*x)/(2*(a^2 + b^2)^2) + (a*x)/(8*(a^2 + b
^2)) - (b*Cos[x]^4)/(4*(a^2 + b^2)) + (a^2*b^3*Log[a*Cos[x] + b*Sin[x]])/(a
^2 + b^2)^3 - (a*b^2*Cos[x]*Sin[x])/(2*(a^2 + b^2)^2) + (a*Cos[x]*Sin[x])/(
8*(a^2 + b^2)) - (a*Cos[x]^3*Sin[x])/(4*(a^2 + b^2)) - (a^2*b*Sin[x]^2)/(2*
(a^2 + b^2)^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3177

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cos^3(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a\int \cos^2(x)\sin^2(x)\,dx}{a^2+b^2} + \frac{b\int \cos^3(x)\sin(x)\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}$$

$$= -\frac{a\cos^3(x)\sin(x)}{4(a^2+b^2)} - \frac{(a^2b)\int \cos(x)\sin(x)\,dx}{(a^2+b^2)^2} - \frac{(ab^2)\int \cos^2(x)\,dx}{(a^2+b^2)^2} + \frac{(a^2b^2)\int \frac{}{a\,\mathrm{co}}}{(a^2}$$

$$= \frac{a^3b^2x}{(a^2+b^2)^3} - \frac{b\cos^4(x)}{4(a^2+b^2)} - \frac{ab^2\cos(x)\sin(x)}{2(a^2+b^2)^2} + \frac{a\cos(x)\sin(x)}{8(a^2+b^2)} - \frac{a\cos^3(x)\sin(x}{4(a^2+b^2)}$$

$$= \frac{a^3b^2x}{(a^2+b^2)^3} - \frac{ab^2x}{2(a^2+b^2)^2} + \frac{ax}{8(a^2+b^2)} - \frac{b\cos^4(x)}{4(a^2+b^2)} + \frac{a^2b^3\log(a\cos(x)+b\sin}{(a^2+b^2)^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.81, size = 287, normalized size = 1.64

$$-\frac{-4a^5x+4ia^4bx-24a^3b^2x-24ia^2b^3x+12ab^4x+4ib^5x-4ib(a^4-6a^2b^2+b^4)\,\mathrm{ArcTan}(\tan(x))+4b(-a^4+b^4)\cos(2x)+a^4b\cos(4x)+2a^2b^3\cos(4x)+b^5\cos(4x)-4a^4b\log(a\cos(x)+b\sin(x))-8a^2b^3\log(a\cos(x)+b\sin(x))-4b^5\log(a\cos(x)+b\sin(x))+2a^4b\log\left((a\cos(x)+b\sin(x))^2\right)-12a^2b^3\log\left((a\cos(x)+b\sin(x))^2\right)+2b^5\log\left((a\cos(x)+b\sin(x))^2\right)+8a^3b^2\sin(2x)+8ab^4\sin(2x)+a^5\sin(4x)+2a^3b^2\sin(4x)+ab^4\sin(4x)}{32(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^3*Sin[x]^2)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] -1/32*(-4*a^5*x + (4*I)*a^4*b*x - 24*a^3*b^2*x - (24*I)*a^2*b^3*x + 12*a*b^
4*x + (4*I)*b^5*x - (4*I)*b*(a^4 - 6*a^2*b^2 + b^4)*ArcTan[Tan[x]] + 4*b*(-
a^4 + b^4)*Cos[2*x] + a^4*b*Cos[4*x] + 2*a^2*b^3*Cos[4*x] + b^5*Cos[4*x] -
4*a^4*b*Log[a*Cos[x] + b*Sin[x]] - 8*a^2*b^3*Log[a*Cos[x] + b*Sin[x]] - 4*b
^5*Log[a*Cos[x] + b*Sin[x]] + 2*a^4*b*Log[(a*Cos[x] + b*Sin[x])^2] - 12*a^2
*b^3*Log[(a*Cos[x] + b*Sin[x])^2] + 2*b^5*Log[(a*Cos[x] + b*Sin[x])^2] + 8*
a^3*b^2*Sin[2*x] + 8*a*b^4*Sin[2*x] + a^5*Sin[4*x] + 2*a^3*b^2*Sin[4*x] + a
*b^4*Sin[4*x])/(a^2 + b^2)^3
```

Maple [A]
time = 0.25, size = 163, normalized size = 0.93

method	result
default	$\frac{a^2b^3\ln(a+b\tan(x))}{(a^2+b^2)^3} + \frac{\frac{\left(\frac{1}{8}a^5-\frac{1}{4}a^3b^2-\frac{3}{8}a\,b^4\right)\left(\tan^3(x)\right)+\left(\frac{1}{2}b\,a^4+\frac{1}{2}b^3a^2\right)\left(\tan^2(x)\right)+\left(-\frac{3}{4}a^3b^2-\frac{5}{8}a\,b^4-\frac{1}{8}a^5\right)\tan(x)+\frac{b\,a^4}{4}-\frac{b^5}{4}}{\left(\tan^2(x)+1\right)^2}+\frac{a\left(-4a\,b^3\right.}{}}{(a^2+b^2)^3}$
risch	$\frac{3iaxb}{4(6ia^2b-2ib^3-2a^3+6a\,b^2)} - \frac{a^2x}{4(6ia^2b-2ib^3-2a^3+6a\,b^2)} - \frac{b\,\mathrm{e}^{2ix}}{16(2iab-a^2+b^2)} - \frac{b\,\mathrm{e}^{-2ix}}{16(-ia+b)^2} - \frac{2ib^3a^2x}{a^6+3a^4b^2+3a^2b^4+b^6} + \frac{b^3a^2}{a^6+3}$
norman	$\frac{\frac{2b^3\left(\tan^2\left(\frac{x}{2}\right)\right)}{a^4+2a^2b^2+b^4}+\frac{2b^3\left(\tan^8\left(\frac{x}{2}\right)\right)}{a^4+2a^2b^2+b^4}+\frac{2\left(-2a^2b+b^3\right)\left(\tan^4\left(\frac{x}{2}\right)\right)}{a^4+2a^2b^2+b^4}+\frac{2\left(-2a^2b+b^3\right)\left(\tan^6\left(\frac{x}{2}\right)\right)}{a^4+2a^2b^2+b^4}+\frac{a\left(a^4+6a^2b^2-3b^4\right)x}{8a^6+24a^4b^2+24a^2b^4+8b^6}-\frac{\left(a^2+5b^2\right)a\tan\left(\frac{x}{2}\right)}{4\left(a^4+2a^2b^2+b^4\right)}+\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] a^2*b^3/(a^2+b^2)^3*ln(a+b*tan(x))+1/(a^2+b^2)^3*(((1/8*a^5-1/4*a^3*b^2-3/8
*a*b^4)*tan(x)^3+(1/2*b*a^4+1/2*b^3*a^2)*tan(x)^2+(-3/4*a^3*b^2-5/8*a*b^4-1
/8*a^5)*tan(x)+1/4*b*a^4-1/4*b^5)/(tan(x)^2+1)^2+1/8*a*(-4*a*b^3*ln(tan(x)^
2+1)+(a^4+6*a^2*b^2-3*b^4)*arctan(tan(x))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(161) = 322$.
time = 0.52, size = 424, normalized size = 2.42

$$\frac{a^2b^3\log\left(-a-\frac{2\,b\sin(x)}{\cos(x)+1}+\frac{a\sin(x)^2}{(\cos(x)+1)^2}\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{a^2b^3\log\left(\frac{\sin(x)^2}{(\cos(x)+1)^2}+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{(a^5+6\,a^3b^2-3\,ab^4)\arctan\left(\frac{\sin(x)}{\cos(x)+1}\right)}{4\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)}+\frac{\frac{8\,b^3\sin(x)^2}{(\cos(x)+1)^2}-\frac{16\,a^2b\sin(x)^4}{(\cos(x)+1)^4}+\frac{8\,b^3\sin(x)^6}{(\cos(x)+1)^6}-\frac{(a^3+5\,ab^2)\sin(x)}{\cos(x)+1}+\frac{(7\,a^3+3\,ab^2)\sin(x)^3}{(\cos(x)+1)^3}-\frac{(7\,a^3+3\,ab^2)\sin(x)^5}{(\cos(x)+1)^5}+\frac{(a^3+5\,ab^2)\sin(x)^7}{(\cos(x)+1)^7}}{4\left(a^4+2\,a^2b^2+b^4+\frac{4\,(a^4+2\,a^2b^2+b^4)\sin(x)^2}{(\cos(x)+1)^2}+\frac{6\,(a^4+2\,a^2b^2+b^4)\sin(x)^4}{(\cos(x)+1)^4}+\frac{4\,(a^4+2\,a^2b^2+b^4)\sin(x)^6}{(\cos(x)+1)^6}+\frac{(a^4+2\,a^2b^2+b^4)\sin(x)^8}{(\cos(x)+1)^8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] a^2*b^3*log(-a - 2*b*sin(x)/(cos(x) + 1) + a*sin(x)^2/(cos(x) + 1)^2)/(a^6
+ 3*a^4*b^2 + 3*a^2*b^4 + b^6) - a^2*b^3*log(sin(x)^2/(cos(x) + 1)^2 + 1)/(
a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/4*(a^5 + 6*a^3*b^2 - 3*a*b^4)*arctan
(sin(x)/(cos(x) + 1))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/4*(8*b^3*sin(
x)^2/(cos(x) + 1)^2 - 16*a^2*b*sin(x)^4/(cos(x) + 1)^4 + 8*b^3*sin(x)^6/(co
s(x) + 1)^6 - (a^3 + 5*a*b^2)*sin(x)/(cos(x) + 1) + (7*a^3 + 3*a*b^2)*sin(x
)^3/(cos(x) + 1)^3 - (7*a^3 + 3*a*b^2)*sin(x)^5/(cos(x) + 1)^5 + (a^3 + 5*a
*b^2)*sin(x)^7/(cos(x) + 1)^7)/(a^4 + 2*a^2*b^2 + b^4 + 4*(a^4 + 2*a^2*b^2
+ b^4)*sin(x)^2/(cos(x) + 1)^2 + 6*(a^4 + 2*a^2*b^2 + b^4)*sin(x)^4/(cos(x)
 + 1)^4 + 4*(a^4 + 2*a^2*b^2 + b^4)*sin(x)^6/(cos(x) + 1)^6 + (a^4 + 2*a^2*
b^2 + b^4)*sin(x)^8/(cos(x) + 1)^8)
```

Fricas [A]
time = 1.99, size = 175, normalized size = 1.00

$$\frac{4\,a^2b^3\log\left(2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2\right)-2\,(a^4b+2\,a^2b^3+b^5)\cos(x)^4+4\,(a^4b+a^2b^3)\cos(x)^2+(a^5+6\,a^3b^2-3\,ab^4)x-\left(2\,(a^5+2\,a^3b^2+ab^4)\cos(x)^3-(a^5-2\,a^3b^2-3\,ab^4)\cos(x)\right)\sin(x)}{8\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/8*(4*a^2*b^3*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - 2*(a
^4*b + 2*a^2*b^3 + b^5)*cos(x)^4 + 4*(a^4*b + a^2*b^3)*cos(x)^2 + (a^5 + 6*
a^3*b^2 - 3*a*b^4)*x - (2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^3 - (a^5 - 2*a^3
*b^2 - 3*a*b^4)*cos(x))*sin(x))/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)**2/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.46, size = 273, normalized size = 1.56

$$\frac{a^{2}b^{4}\log\left(|b\tan\left(x\right)+a|\right)}{a^{6}b+3\,a^{4}b^{3}+3\,a^{2}b^{5}+b^{7}}-\frac{a^{2}b^{3}\log\left(\tan\left(x\right)^{2}+1\right)}{2\left(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}\right)}+\frac{\left(a^{5}+6\,a^{3}b^{2}-3\,ab^{4}\right)x}{8\left(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}\right)}+\frac{6\,a^{2}b^{3}\tan\left(x\right)^{4}+a^{5}\tan\left(x\right)^{3}-2\,a^{3}b^{2}\tan\left(x\right)^{3}-3\,ab^{4}\tan\left(x\right)^{3}+4\,a^{4}b\tan\left(x\right)^{2}+16\,a^{2}b^{3}\tan\left(x\right)^{2}-a^{5}\tan\left(x\right)-6\,a^{3}b^{2}\tan\left(x\right)-5\,ab^{4}\tan\left(x\right)+2\,a^{4}b+6\,a^{2}b^{3}-2\,b^{5}}{8\left(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}\right)\left(\tan\left(x\right)^{2}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] a^2*b^4*log(abs(b*tan(x) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) - 1/2*
a^2*b^3*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/8*(a^5 +
6*a^3*b^2 - 3*a*b^4)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/8*(6*a^2*b^3
*tan(x)^4 + a^5*tan(x)^3 - 2*a^3*b^2*tan(x)^3 - 3*a*b^4*tan(x)^3 + 4*a^4*b*
tan(x)^2 + 16*a^2*b^3*tan(x)^2 - a^5*tan(x) - 6*a^3*b^2*tan(x) - 5*a*b^4*ta
n(x) + 2*a^4*b + 6*a^2*b^3 - 2*b^5)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(t
an(x)^2 + 1)^2)
```

Mupad [B]
time = 11.14, size = 2500, normalized size = 14.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^3*sin(x)^2)/(a*cos(x) + b*sin(x)),x)
```

```
[Out] ((tan(x/2)^3*(3*a*b^2 + 7*a^3))/(4*(a^4 + b^4 + 2*a^2*b^2)) - (tan(x/2)^5*(
3*a*b^2 + 7*a^3))/(4*(a^4 + b^4 + 2*a^2*b^2)) - (tan(x/2)*(5*a*b^2 + a^3))/
(4*(a^4 + b^4 + 2*a^2*b^2)) + (tan(x/2)^7*(5*a*b^2 + a^3))/(4*(a^4 + b^4 +
2*a^2*b^2)) + (2*b^3*tan(x/2)^2)/(a^4 + b^4 + 2*a^2*b^2) + (2*b^3*tan(x/2)^
6)/(a^4 + b^4 + 2*a^2*b^2) - (4*a^2*b*tan(x/2)^4)/(a^4 + b^4 + 2*a^2*b^2))/
(4*tan(x/2)^2 + 6*tan(x/2)^4 + 4*tan(x/2)^6 + tan(x/2)^8 + 1) - (a*atan((ta
n(x/2)*((((64*a^2*b^3*((a*((16*a^15*b + 16*a^3*b^13 + 288*a^5*b^11 + 1008*a
^7*b^9 + 1472*a^9*b^7 + 1008*a^11*b^5 + 288*a^13*b^3)/(2*(a^12 + b^12 + 6*a
^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (32*a^2*b^3
*(192*a*b^16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8
 + 4032*a^11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a
^2*b^4 + 192*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 +
 15*a^8*b^4 + 6*a^10*b^2)))*(a^4 - 3*b^4 + 6*a^2*b^2))/(8*(a^6 + b^6 + 3*a^
2*b^4 + 3*a^4*b^2)) - (4*a^3*b^3*(a^4 - 3*b^4 + 6*a^2*b^2)*(192*a*b^16 + 13
44*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6
+ 1344*a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*
```

b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4
*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(64*a^6 + 64*b^6 + 192*a^2*
b^4 + 192*a^4*b^2) - (a*((a^15 + 18*a^3*b^12 - 141*a^5*b^10 - 327*a^7*b^8 -
146*a^9*b^6 + 36*a^11*b^4 + 15*a^13*b^2)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15
*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (64*a^2*b^3*((16*a^15*b
+ 16*a^3*b^13 + 288*a^5*b^11 + 1008*a^7*b^9 + 1472*a^9*b^7 + 1008*a^11*b^5
+ 288*a^13*b^3)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 1
5*a^8*b^4 + 6*a^10*b^2)) - (32*a^2*b^3*(192*a*b^16 + 1344*a^3*b^14 + 4032*a
^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^4 + 19
2*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^12 + b^12 +
6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(64*a^6
+ 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2))*(a^4 - 3*b^4 + 6*a^2*b^2))/(8*(a^6 +
b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (a^3*(a^4 - 3*b^4 + 6*a^2*b^2)^3*(192*a*b^
16 + 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^
11*b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/(1024*(a^6 + b^6 + 3*a^2*b^4 + 3*a^
4*b^2)^3*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 +
6*a^10*b^2)))*(a^10 - 9*b^10 + 493*a^2*b^8 - 706*a^4*b^6 - 46*a^6*b^4 + 11
*a^8*b^2))/(a^10 + 9*b^10 + 229*a^2*b^8 + 250*a^4*b^6 + 42*a^6*b^4 + 13*a^8
*b^2)^2 - (2*a*b*((18*a^5*b^9 + 13*a^7*b^7 + 12*a^9*b^5 + a^11*b^3)/(2*(a^1
2 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))
+ (a*((a*((16*a^15*b + 16*a^3*b^13 + 288*a^5*b^11 + 1008*a^7*b^9 + 1472*a^
9*b^7 + 1008*a^11*b^5 + 288*a^13*b^3)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4
*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (32*a^2*b^3*(192*a*b^16 + 1
344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6
+ 1344*a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4
b^2)(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*
a^10*b^2)))*(a^4 - 3*b^4 + 6*a^2*b^2))/(8*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^
2)) - (4*a^3*b^3*(a^4 - 3*b^4 + 6*a^2*b^2)*(192*a*b^16 + 1344*a^3*b^14 + 40
32*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6 + 1344*a^13*b^4
+ 192*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a^6 + b^6
+ 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^
6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^4 - 3*b^4 + 6*a^2*b^2))/(8*(a^6 + b^6 + 3
*a^2*b^4 + 3*a^4*b^2)) + (64*a^2*b^3*((a^15 + 18*a^3*b^12 - 141*a^5*b^10 -
327*a^7*b^8 - 146*a^9*b^6 + 36*a^11*b^4 + 15*a^13*b^2)/(2*(a^12 + b^12 + 6*
a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (64*a^2*b^
3*((16*a^15*b + 16*a^3*b^13 + 288*a^5*b^11 + 1008*a^7*b^9 + 1472*a^9*b^7 +
1008*a^11*b^5 + 288*a^13*b^3)/(2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 2
0*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)) - (32*a^2*b^3*(192*a*b^16 + 1344*a^3*
b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*b^6 + 1344*
a^13*b^4 + 192*a^15*b^2))/((64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)*(a
^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2
))))/(64*a^6 + 64*b^6 + 192*a^2*b^4 + 192*a^4*b^2)))/(64*a^6 + 64*b^6 + 192
*a^2*b^4 + 192*a^4*b^2) - (a^4*b^3*(a^4 - 3*b^4 + 6*a^2*b^2)^2*(192*a*b^16
+ 1344*a^3*b^14 + 4032*a^5*b^12 + 6720*a^7*b^10 + 6720*a^9*b^8 + 4032*a^11*
b^6 + 1344*a^13*b^4 + 192*a^15*b^2))/(2*(64*a^6 + 64*b^6 + 192*a^2*b^4 + 19

```
2*a^4*b^2)*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^2*(a^12 + b^12 + 6*a^2*b^10
+ 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^8 + 57*b^8 - 436*
a^2*b^6 + 110*a^4*b^4 + 28*a^6*b^2))/(a^10 + 9*b^10 + 229*a^2*b^8 + 250*a^4
*b^6 + 42*a^6*b^4 + 13*a^8*b^2)^2)*(16*a^16 + 16*b^16 + 128*a^2*b^14 + 448*
a^4*b^12 + 896*a^6*b^10 + 1120*a^8*b^8 + 896*a^10*b^6 + 448*a^12*b^4 + 128*
a^14*b^2))/(a^8 - 3*a^4*b^4 + 6*a^6*b^2) + (((a...
```

3.283 $\int \frac{\cos^3(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx$

Optimal. Leaf size=193

$$\frac{a^3b^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}}-\frac{a^3b^2\cos(x)}{(a^2+b^2)^3}+\frac{ab^2\cos^3(x)}{3(a^2+b^2)^2}-\frac{a\cos^3(x)}{3(a^2+b^2)}+\frac{a\cos^5(x)}{5(a^2+b^2)}+\frac{a^2b^3\sin(x)}{(a^2+b^2)^3}-\frac{a^2b\sin^3(x)}{3(a^2+b^2)^2}$$

```
[Out] a^3*b^3*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(7/2)-a^3*b^
2*cos(x)/(a^2+b^2)^3+1/3*a*b^2*cos(x)^3/(a^2+b^2)^2-1/3*a*cos(x)^3/(a^2+b^2
)+1/5*a*cos(x)^5/(a^2+b^2)+a^2*b^3*sin(x)/(a^2+b^2)^3-1/3*a^2*b*sin(x)^3/(a
^2+b^2)^2+1/3*b*sin(x)^3/(a^2+b^2)-1/5*b*sin(x)^5/(a^2+b^2)
```

Rubi [A]
time = 0.25, antiderivative size = 193, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3188, 2644, 14, 2645, 30, 2717, 2718, 3153, 212}

$$-\frac{b\sin^5(x)}{5(a^2+b^2)}+\frac{b\sin^3(x)}{3(a^2+b^2)}-\frac{a^2b\sin^3(x)}{3(a^2+b^2)^2}+\frac{a\cos^5(x)}{5(a^2+b^2)}-\frac{a\cos^3(x)}{3(a^2+b^2)}+\frac{ab^2\cos^3(x)}{3(a^2+b^2)^2}+\frac{a^2b^3\sin(x)}{(a^2+b^2)^3}-\frac{a^3b^2\cos(x)}{(a^2+b^2)^3}+\frac{a^3b^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^3*Sin[x]^3)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (a^3*b^3*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2)
- (a^3*b^2*Cos[x])/(a^2 + b^2)^3 + (a*b^2*Cos[x]^3)/(3*(a^2 + b^2)^2) - (a*
Cos[x]^3)/(3*(a^2 + b^2)) + (a*Cos[x]^5)/(5*(a^2 + b^2)) + (a^2*b^3*Sin[x])
/(a^2 + b^2)^3 - (a^2*b*Sin[x]^3)/(3*(a^2 + b^2)^2) + (b*Sin[x]^3)/(3*(a^2
+ b^2)) - (b*Sin[x]^5)/(5*(a^2 + b^2))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^3(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx = \frac{a\int \cos^2(x)\sin^3(x)\,dx}{a^2+b^2} + \frac{b\int \cos^3(x)\sin^2(x)\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\cos^2(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}$$

$$= -\frac{(a^2b)\int \cos(x)\sin^2(x)\,dx}{(a^2+b^2)^2} - \frac{(ab^2)\int \cos^2(x)\sin(x)\,dx}{(a^2+b^2)^2} + \frac{(a^2b^2)\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{(a^2+b^2)^2}$$

$$= \frac{(a^3b^2)\int \sin(x)\,dx}{(a^2+b^2)^3} + \frac{(a^2b^3)\int \cos(x)\,dx}{(a^2+b^2)^3} - \frac{(a^3b^3)\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{(a^2+b^2)^3} - \frac{(a^2b)\,\mathrm{Subst}}{(}$$

$$= -\frac{a^3b^2\cos(x)}{(a^2+b^2)^3} + \frac{ab^2\cos^3(x)}{3(a^2+b^2)^2} - \frac{a\cos^3(x)}{3(a^2+b^2)} + \frac{a\cos^5(x)}{5(a^2+b^2)} + \frac{a^2b^3\sin(x)}{(a^2+b^2)^3} - \frac{a^2b\sin^3}{3(a^2+}$$

$$= \frac{a^3b^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} - \frac{a^3b^2\cos(x)}{(a^2+b^2)^3} + \frac{ab^2\cos^3(x)}{3(a^2+b^2)^2} - \frac{a\cos^3(x)}{3(a^2+b^2)} + \frac{a\cos^5}{5(a^2}$$

Mathematica [A]
time = 1.63, size = 223, normalized size = 1.16

$$-\frac{2a^3b^3\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} + \frac{-30a(a^4+8a^2b^2-b^4)\cos(x)-5a(a^4-2a^2b^2-3b^4)\cos(3x)+3a^5\cos(5x)+6a^3b^2\cos(5x)+3ab^4\cos(5x)-30a^4b\sin(x)+240a^2b^3\sin(x)+30b^5\sin(x)+15a^4b\sin(3x)+10a^2b^3\sin(3x)-5b^5\sin(3x)-3a^4b\sin(5x)-6a^2b^3\sin(5x)-3b^5\sin(5x)}{240(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^3*Sin[x]^3)/(a*Cos[x] + b*Sin[x]),x]
```

```
[Out] (-2*a^3*b^3*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2) +
 (-30*a*(a^4 + 8*a^2*b^2 - b^4)*Cos[x] - 5*a*(a^4 - 2*a^2*b^2 - 3*b^4)*Cos[
3*x] + 3*a^5*Cos[5*x] + 6*a^3*b^2*Cos[5*x] + 3*a*b^4*Cos[5*x] - 30*a^4*b*Si
n[x] + 240*a^2*b^3*Sin[x] + 30*b^5*Sin[x] + 15*a^4*b*Sin[3*x] + 10*a^2*b^3*
Sin[3*x] - 5*b^5*Sin[3*x] - 3*a^4*b*Sin[5*x] - 6*a^2*b^3*Sin[5*x] - 3*b^5*S
in[5*x])/(240*(a^2 + b^2)^3)
```

Maple [A]
time = 0.41, size = 302, normalized size = 1.56

method	result
default	$-\frac{16a^3b^3\,\mathrm{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{(8a^6+24a^4b^2+24a^2b^4+8b^6)\sqrt{a^2+b^2}} + \frac{2b^3a^2\left(\tan^9\left(\frac{x}{2}\right)\right)+2a\,b^4\left(\tan^8\left(\frac{x}{2}\right)\right)+2\left(\frac{16}{3}b^3a^2+\frac{4}{3}b^5\right)\left(\tan^7\left(\frac{x}{2}\right)\right)+2\left(-2a^5-6a^3b^2\right)(\tan}{}$
risch	$-\frac{i e^{3ix}b}{96(-2iab+a^2-b^2)} - \frac{e^{3ix}a}{96(-2iab+a^2-b^2)} + \frac{i e^{ix}ab}{-12ia^2b+4ib^3+4a^3-12a\,b^2} - \frac{e^{ix}a^2}{16(-3ia^2b+ib^3+a^3-3a\,b^2)} + \frac{e^{ix}b^2}{-48ia^2b+16ib^3+16a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] -16*a^3*b^3/(8*a^6+24*a^4*b^2+24*a^2*b^4+8*b^6)/(a^2+b^2)^(1/2)*arctanh(1/2
*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2))+2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*(b^3
*a^2*tan(1/2*x)^9+a*b^4*tan(1/2*x)^8+(16/3*b^3*a^2+4/3*b^5)*tan(1/2*x)^7+(-
2*a^5-6*a^3*b^2)*tan(1/2*x)^6+(-16/5*b*a^4+34/15*b^3*a^2-8/15*b^5)*tan(1/2*
x)^5+(2/3*a^5-10/3*a^3*b^2+2*a*b^4)*tan(1/2*x)^4+(16/3*b^3*a^2+4/3*b^5)*tan
(1/2*x)^3+(-2/3*a^5-14/3*a^3*b^2)*tan(1/2*x)^2+b^3*a^2*tan(1/2*x)-2/15*a^5-
14/15*a^3*b^2+1/5*a*b^4)/(1+tan(1/2*x)^2)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(177) = 354$.
time = 0.49, size = 521, normalized size = 2.70

$$\frac{a^3b^3\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{(a^6+3a^4b^2+3a^2b^4+b^6)\sqrt{a^2+b^2}}-\frac{2\left(2a^5+14a^3b^2-3ab^4-\frac{15a^2b^3\sin(x)}{\cos(x)+1}-\frac{15ab^4\sin(x)^8}{(\cos(x)+1)^8}-\frac{15a^2b^3\sin(x)^9}{(\cos(x)+1)^9}+\frac{10(a^5+7a^3b^2)\sin(x)^2}{(\cos(x)+1)^2}-\frac{20(4a^2b^3+b^5)\sin(x)^3}{(\cos(x)+1)^3}-\frac{10(a^5-5a^3b^2+3ab^4)\sin(x)^4}{(\cos(x)+1)^4}+\frac{2(24a^4b-17a^2b^3+4b^5)\sin(x)^5}{(\cos(x)+1)^5}+\frac{30(a^5+3a^3b^2)\sin(x)^6}{(\cos(x)+1)^6}-\frac{20(4a^2b^3+b^5)\sin(x)^7}{(\cos(x)+1)^7}\right)}{15\left(a^6+3a^4b^2+3a^2b^4+b^6+\frac{5(a^6+3a^4b^2+3a^2b^4+b^6)\sin(x)^2}{(\cos(x)+1)^2}+\frac{10(a^6+3a^4b^2+3a^2b^4+b^6)\sin(x)^4}{(\cos(x)+1)^4}+\frac{10(a^6+3a^4b^2+3a^2b^4+b^6)\sin(x)^6}{(\cos(x)+1)^6}+\frac{5(a^6+3a^4b^2+3a^2b^4+b^6)\sin(x)^8}{(\cos(x)+1)^8}+\frac{(a^6+3a^4b^2+3a^2b^4+b^6)\sin(x)^{10}}{(\cos(x)+1)^{10}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="maxima")
```

```
[Out] a^3*b^3*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*sin(x)/(co
s(x) + 1) - sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2
 + b^2)) - 2/15*(2*a^5 + 14*a^3*b^2 - 3*a*b^4 - 15*a^2*b^3*sin(x)/(cos(x) +
 1) - 15*a*b^4*sin(x)^8/(cos(x) + 1)^8 - 15*a^2*b^3*sin(x)^9/(cos(x) + 1)^9
 + 10*(a^5 + 7*a^3*b^2)*sin(x)^2/(cos(x) + 1)^2 - 20*(4*a^2*b^3 + b^5)*sin(
x)^3/(cos(x) + 1)^3 - 10*(a^5 - 5*a^3*b^2 + 3*a*b^4)*sin(x)^4/(cos(x) + 1)^
4 + 2*(24*a^4*b - 17*a^2*b^3 + 4*b^5)*sin(x)^5/(cos(x) + 1)^5 + 30*(a^5 + 3
*a^3*b^2)*sin(x)^6/(cos(x) + 1)^6 - 20*(4*a^2*b^3 + b^5)*sin(x)^7/(cos(x) +
 1)^7)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4
+ b^6)*sin(x)^2/(cos(x) + 1)^2 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sin
(x)^4/(cos(x) + 1)^4 + 10*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sin(x)^6/(cos
(x) + 1)^6 + 5*(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sin(x)^8/(cos(x) + 1)^8
+ (a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sin(x)^10/(cos(x) + 1)^10)
```

Fricas [A]
time = 1.21, size = 307, normalized size = 1.59

$$\frac{15\sqrt{a^2+b^2}\,a^3b^3\log\left(\frac{2ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2-2a^2-b^2-2\sqrt{a^2+b^2}\,(b\cos(x)-a\sin(x))}{2ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2}\right)+6(a^7+3a^5b^2+3a^3b^4+ab^6)\cos(x)^5-10(a^7+2a^5b^2+a^3b^4)\cos(x)^3-30(a^5b^2+a^3b^4)\cos(x)-2\left(3a^6b-11a^4b^3-16a^2b^5-2b^7+3(a^6b+3a^4b^3+3a^2b^5+b^7)\cos(x)^4-(6a^6b+13a^4b^3+8a^2b^5+b^7)\cos(x)^2\right)\sin(x)}{30(a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/30*(15*sqrt(a^2 + b^2)*a^3*b^3*log((2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos
(x)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x
)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)) + 6*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 +
a*b^6)*cos(x)^5 - 10*(a^7 + 2*a^5*b^2 + a^3*b^4)*cos(x)^3 - 30*(a^5*b^2 + a
^3*b^4)*cos(x) - 2*(3*a^6*b - 11*a^4*b^3 - 16*a^2*b^5 - 2*b^7 + 3*(a^6*b +
```

```
3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(x)^4 - (6*a^6*b + 13*a^4*b^3 + 8*a^2*b^5 +
 b^7)*cos(x)^2)*sin(x))/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)**3/(a*cos(x)+b*sin(x)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 361 vs. 2(177) = 354.
time = 0.48, size = 361, normalized size = 1.87

$$\frac{a^3b^3\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\sqrt{a^2+b^2}}+\frac{2\left(15\,a^2b^3\tan\left(\frac{1}{2}\,x\right)^9+15\,ab^4\tan\left(\frac{1}{2}\,x\right)^8+80\,a^2b^3\tan\left(\frac{1}{2}\,x\right)^7+20\,b^5\tan\left(\frac{1}{2}\,x\right)^7-30\,a^5\tan\left(\frac{1}{2}\,x\right)^6-90\,a^3b^2\tan\left(\frac{1}{2}\,x\right)^6-48\,a^4b\tan\left(\frac{1}{2}\,x\right)^5+34\,a^2b^3\tan\left(\frac{1}{2}\,x\right)^5-8\,b^5\tan\left(\frac{1}{2}\,x\right)^5+10\,a^5\tan\left(\frac{1}{2}\,x\right)^4-50\,a^3b^2\tan\left(\frac{1}{2}\,x\right)^4+30\,ab^4\tan\left(\frac{1}{2}\,x\right)^4+80\,a^2b^3\tan\left(\frac{1}{2}\,x\right)^3+20\,b^5\tan\left(\frac{1}{2}\,x\right)^3-10\,a^5\tan\left(\frac{1}{2}\,x\right)^2-70\,a^3b^2\tan\left(\frac{1}{2}\,x\right)^2+15\,a^2b^3\tan\left(\frac{1}{2}\,x\right)-2\,a^5-14\,a^3b^2+3\,ab^4\right)}{15\left(a^6+3\,a^4b^2+3\,a^2b^4+b^6\right)\left(\tan\left(\frac{1}{2}\,x\right)^2+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x)),x, algorithm="giac")
```

```
[Out] a^3*b^3*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*tan(1/2*x
) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*sqrt(a^2
 + b^2)) + 2/15*(15*a^2*b^3*tan(1/2*x)^9 + 15*a*b^4*tan(1/2*x)^8 + 80*a^2*b
^3*tan(1/2*x)^7 + 20*b^5*tan(1/2*x)^7 - 30*a^5*tan(1/2*x)^6 - 90*a^3*b^2*ta
n(1/2*x)^6 - 48*a^4*b*tan(1/2*x)^5 + 34*a^2*b^3*tan(1/2*x)^5 - 8*b^5*tan(1/
2*x)^5 + 10*a^5*tan(1/2*x)^4 - 50*a^3*b^2*tan(1/2*x)^4 + 30*a*b^4*tan(1/2*x
)^4 + 80*a^2*b^3*tan(1/2*x)^3 + 20*b^5*tan(1/2*x)^3 - 10*a^5*tan(1/2*x)^2 -
 70*a^3*b^2*tan(1/2*x)^2 + 15*a^2*b^3*tan(1/2*x) - 2*a^5 - 14*a^3*b^2 + 3*a
*b^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(tan(1/2*x)^2 + 1)^5)
```

Mupad [**B**]
time = 1.64, size = 600, normalized size = 3.11

$$\frac{\frac{8\tan\left(\frac{x}{2}\right)^3\left(4\,a^2\,b^3+b^5\right)}{3\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}-\frac{4\tan\left(\frac{x}{2}\right)^2\left(a^5+7\,a^3\,b^2\right)}{3\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}-\frac{4\tan\left(\frac{x}{2}\right)^6\left(a^5+3\,a^3\,b^2\right)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}-\frac{2\left(2\,a^5+14\,a^3\,b^2-3\,a\,b^4\right)}{15\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{4\tan\left(\frac{x}{2}\right)^4\left(a^5-5\,a^3\,b^2+3\,a\,b^4\right)}{3\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{8\,b^3\tan\left(\frac{x}{2}\right)^7\left(4\,a^2+b^2\right)}{3\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}+\frac{2\,a^2\,b^3\tan\left(\frac{x}{2}\right)}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+\frac{2\,a\,b^4\tan\left(\frac{x}{2}\right)^8}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}+\frac{2\,a^2\,b^3\tan\left(\frac{x}{2}\right)^9}{a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6}-\frac{4\,b\tan\left(\frac{x}{2}\right)^5\left(24\,a^4-17\,a^2\,b^2+4\,b^4\right)}{15\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}}{\tan\left(\frac{x}{2}\right)^{10}+5\tan\left(\frac{x}{2}\right)^8+10\tan\left(\frac{x}{2}\right)^6+10\tan\left(\frac{x}{2}\right)^4+5\tan\left(\frac{x}{2}\right)^2+1}+\frac{2\,a^3\,b^3\,\mathrm{atanh}\left(\frac{2\,a^6\,b+2\,b^7+6\,a^2\,b^5+6\,a^4\,b^3-2\,a\tan\left(\frac{x}{2}\right)\left(a^6+3\,a^4\,b^2+3\,a^2\,b^4+b^6\right)}{2\left(a^2+b^2\right)^{7/2}}\right)}{\left(a^2+b^2\right)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^3*sin(x)^3)/(a*cos(x) + b*sin(x)),x)
```

```
[Out] ((8*tan(x/2)^3*(b^5 + 4*a^2*b^3))/(3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) -
 (4*tan(x/2)^2*(a^5 + 7*a^3*b^2))/(3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) -
 (4*tan(x/2)^6*(a^5 + 3*a^3*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (2*
(2*a^5 - 3*a*b^4 + 14*a^3*b^2))/(15*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) +
(4*tan(x/2)^4*(3*a*b^4 + a^5 - 5*a^3*b^2))/(3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^
4*b^2)) + (8*b^3*tan(x/2)^7*(4*a^2 + b^2))/(3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^
```

```
4*b^2)) + (2*a^2*b^3*tan(x/2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (2*a*b
^4*tan(x/2)^8)/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (2*a^2*b^3*tan(x/2)^9)
/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (4*b*tan(x/2)^5*(24*a^4 + 4*b^4 - 17
*a^2*b^2))/(15*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))/(5*tan(x/2)^2 + 10*tan
(x/2)^4 + 10*tan(x/2)^6 + 5*tan(x/2)^8 + tan(x/2)^10 + 1) + (2*a^3*b^3*atan
h((2*a^6*b + 2*b^7 + 6*a^2*b^5 + 6*a^4*b^3 - 2*a*tan(x/2)*(a^6 + b^6 + 3*a^
2*b^4 + 3*a^4*b^2))/(2*(a^2 + b^2)^(7/2))))/(a^2 + b^2)^(7/2)
```

3.284 $\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=70

$$\frac{2abx}{(a^2+b^2)^2}-\frac{(a^2-b^2)\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^2}-\frac{b\sin(x)}{(a^2+b^2)\,(a\cos(x)+b\sin(x))}$$

```
[Out] 2*a*b*x/(a^2+b^2)^2-(a^2-b^2)*ln(a*cos(x)+b*sin(x))/(a^2+b^2)^2-b*sin(x)/(a
^2+b^2)/(a*cos(x)+b*sin(x))
```

Rubi [A]
time = 0.12, antiderivative size = 87, normalized size of antiderivative = 1.24, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.312$, Rules used = {3190, 3177, 3212, 3176, 3154}

$$\frac{2abx}{(a^2+b^2)^2}-\frac{b\sin(x)}{(a^2+b^2)\,(a\cos(x)+b\sin(x))}-\frac{a^2\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^2}+\frac{b^2\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x])/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (2*a*b*x)/(a^2 + b^2)^2 - (a^2*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^2 + (b
^2*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^2 - (b*Sin[x])/((a^2 + b^2)*(a*Cos
[x] + b*Sin[x]))
```

Rule 3154

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3176

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3177

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3190

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[a*(b/(a^2 + b^2)), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx &= \frac{a\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}+\frac{b\int \frac{\cos(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}-\frac{(ab)\int \frac{1}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2+b^2}\\ &= \frac{2abx}{(a^2+b^2)^2}-\frac{b\sin(x)}{(a^2+b^2)(a\cos(x)+b\sin(x))}-\frac{a^2\int \frac{b\cos(x)-a\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{(a^2+b^2)^2}+\frac{b^2\int \frac{b\,\mathrm{c}}{a\,\mathrm{c}}}{(}\\ &= \frac{2abx}{(a^2+b^2)^2}-\frac{a^2\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^2}+\frac{b^2\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^2}-\frac{}{(a^2+}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.27, size = 144, normalized size = 2.06

$$\frac{a\cos(x)\left(-2i(a+ib)^2x+(-a^2+b^2)\log\left((a\cos(x)+b\sin(x))^2\right)\right)+b(2(a+ib)(a(-1-ix)+b(i+x))+(-a^2+b^2)\log\left((a\cos(x)+b\sin(x))^2\right))\sin(x)+2i(a^2-b^2)\,\mathrm{ArcTan}(\tan(x))(a\cos(x)+b\sin(x))}{2\left(a^2+b^2\right)^2(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x])/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (a*Cos[x]*((-2*I)*(a + I*b)^2*x + (-a^2 + b^2)*Log[(a*Cos[x] + b*Sin[x])^2]
) + b*(2*(a + I*b)*(a*(-1 - I*x) + b*(I + x)) + (-a^2 + b^2)*Log[(a*Cos[x]
+ b*Sin[x])^2])*Sin[x] + (2*I)*(a^2 - b^2)*ArcTan[Tan[x]]*(a*Cos[x] + b*Sin
[x]))/(2*(a^2 + b^2)^2*(a*Cos[x] + b*Sin[x]))
```

Maple [A]
time = 0.22, size = 84, normalized size = 1.20

method	result
default	$\frac{a}{(a^2+b^2)(a+b\tan(x))}-\frac{(a^2-b^2)\ln(a+b\tan(x))}{(a^2+b^2)^2}+\frac{\frac{(a^2-b^2)\ln(\tan^2(x)+1)}{2}+2ab\arctan(\tan(x))}{(a^2+b^2)^2}$
risch	$\frac{ix}{2iab-a^2+b^2}+\frac{2ix\,a^2}{a^4+2a^2b^2+b^4}-\frac{2ix\,b^2}{a^4+2a^2b^2+b^4}-\frac{2iab}{(ib+a)(-ib+a)^2(-ib\,e^{2ix}+a\,e^{2ix}+ib+a)}-\frac{\ln\left(e^{2ix}-\frac{ib+a}{ib-a}\right)a^2}{a^4+2a^2b^2+b^4}+\frac{\ln\left(e^{2ix}-\frac{ib}{ib}\right.}{a^4+2a^2b}$ [cut off]
norman	$\frac{-\frac{2a^2bx}{a^4+2a^2b^2+b^4}-\frac{4a\,b^2x\tan(\frac{x}{2})}{a^4+2a^2b^2+b^4}-\frac{8a\,b^2x(\tan^3(\frac{x}{2}))}{a^4+2a^2b^2+b^4}-\frac{4a\,b^2x(\tan^5(\frac{x}{2}))}{a^4+2a^2b^2+b^4}-\frac{2a^2bx(\tan^2(\frac{x}{2}))}{a^4+2a^2b^2+b^4}+\frac{2a^2bx(\tan^4(\frac{x}{2}))}{a^4+2a^2b^2+b^4}+\frac{2a^2bx(\tan^6(\frac{x}{2}))}{a^4+2a^2b^2+b^4}+\frac{2b\tan(\frac{x}{2}}{a^2+b^2}}{(1+\tan^2(\frac{x}{2}))^2(a(\tan^2(\frac{x}{2}))-2b\tan(\frac{x}{2})-a)}$ [cut off]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a/(a^2+b^2)/(a+b*tan(x))-(a^2-b^2)/(a^2+b^2)^2*ln(a+b*tan(x))+1/(a^2+b^2)^2
*(1/2*(a^2-b^2)*ln(tan(x)^2+1)+2*a*b*arctan(tan(x)))
```

Maxima [A]
time = 0.48, size = 118, normalized size = 1.69

$$\frac{2\,abx}{a^4+2\,a^2b^2+b^4}-\frac{(a^2-b^2)\log(b\tan(x)+a)}{a^4+2\,a^2b^2+b^4}+\frac{(a^2-b^2)\log(\tan(x)^2+1)}{2\,(a^4+2\,a^2b^2+b^4)}+\frac{a}{a^3+ab^2+(a^2b+b^3)\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] 2*a*b*x/(a^4 + 2*a^2*b^2 + b^4) - (a^2 - b^2)*log(b*tan(x) + a)/(a^4 + 2*a^
2*b^2 + b^4) + 1/2*(a^2 - b^2)*log(tan(x)^2 + 1)/(a^4 + 2*a^2*b^2 + b^4) +
a/(a^3 + a*b^2 + (a^2*b + b^3)*tan(x))
```

Fricas [A]
time = 2.32, size = 138, normalized size = 1.97

$$\frac{2\,(2\,a^2bx+ab^2)\cos(x)-((a^3-ab^2)\cos(x)+(a^2b-b^3)\sin(x))\log\left(2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2\right)+2\,(2\,ab^2x-a^2b)\sin(x)}{2\,((a^5+2\,a^3b^2+ab^4)\cos(x)+(a^4b+2\,a^2b^3+b^5)\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(2*a^2*b*x + a*b^2)*cos(x) - ((a^3 - a*b^2)*cos(x) + (a^2*b - b^3)*s
in(x))*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) + 2*(2*a*b^2*x
 - a^2*b)*sin(x))/((a^5 + 2*a^3*b^2 + a*b^4)*cos(x) + (a^4*b + 2*a^2*b^3 +
b^5)*sin(x))
```

Sympy [C] Result contains complex when optimal does not.
time = 0.66, size = 991, normalized size = 14.16

$$\begin{cases} \tilde{\infty}\log(\sin(x)) & \text{for}\ a=0\wedge b=0 \\ -\frac{\log(\cos(x))}{a^2} & \text{for}\ b=0 \\ -\frac{2ix\sin^2(x)}{-8b^2\sin^2(x)+16ib^2\sin(x)\cos(x)+8b^2\cos^2(x)}-\frac{4x\sin(x)\cos(x)}{-8b^2\sin^2(x)+16ib^2\sin(x)\cos(x)+8b^2\cos^2(x)}+\frac{2ix\cos^2(x)}{-8b^2\sin^2(x)+16ib^2\sin(x)\cos(x)+8b^2\cos^2(x)}-\frac{\sin^2(x)}{-8b^2\sin^2(x)+16ib^2\sin(x)\cos(x)+8b^2\cos^2(x)}+\frac{\cos^2(x)}{-8b^2\sin^2(x)+16ib^2\sin(x)\cos(x)+8b^2\cos^2(x)} & \text{for}\ a=-ib \\ \frac{2ix\sin^2(x)}{-8b^2\sin^2(x)-16ib^2\sin(x)\cos(x)+8b^2\cos^2(x)}-\frac{4x\sin(x)\cos(x)}{-8b^2\sin^2(x)-16ib^2\sin(x)\cos(x)+8b^2\cos^2(x)}-\frac{2ix\cos^2(x)}{-8b^2\sin^2(x)-16ib^2\sin(x)\cos(x)+8b^2\cos^2(x)}-\frac{\sin^2(x)}{-8b^2\sin^2(x)-16ib^2\sin(x)\cos(x)+8b^2\cos^2(x)}+\frac{\cos^2(x)}{-8b^2\sin^2(x)-16ib^2\sin(x)\cos(x)+8b^2\cos^2(x)} & \text{for}\ a=ib \\ -\frac{a^3\log\left(\frac{a\cos(x)}{b}+\sin(x)\right)\cos(x)}{D}+\frac{2a^2bx\cos(x)}{D}-\frac{a^2b\log\left(\frac{a\cos(x)}{b}+\sin(x)\right)\sin(x)}{D}-\frac{a^2b\sin(x)}{D}+\frac{2ab^2x\sin(x)}{D}+\frac{ab^2\log\left(\frac{a\cos(x)}{b}+\sin(x)\right)\cos(x)}{D}+\frac{b^3\log\left(\frac{a\cos(x)}{b}+\sin(x)\right)\sin(x)}{D}-\frac{b^3\sin(x)}{D} & \text{otherwise} \end{cases}$$

where $D = a^5\cos(x)+a^4b\sin(x)+2a^3b^2\cos(x)+2a^2b^3\sin(x)+ab^4\cos(x)+b^5\sin(x)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Piecewise((zoo*log(sin(x)), Eq(a, 0) & Eq(b, 0)), (-log(cos(x))/a**2, Eq(b,
 0)), (-2*I*x*sin(x)**2/(-8*b**2*sin(x)**2 + 16*I*b**2*sin(x)*cos(x) + 8*b*
*2*cos(x)**2) - 4*x*sin(x)*cos(x)/(-8*b**2*sin(x)**2 + 16*I*b**2*sin(x)*cos
(x) + 8*b**2*cos(x)**2) + 2*I*x*cos(x)**2/(-8*b**2*sin(x)**2 + 16*I*b**2*si
n(x)*cos(x) + 8*b**2*cos(x)**2) - sin(x)**2/(-8*b**2*sin(x)**2 + 16*I*b**2*
sin(x)*cos(x) + 8*b**2*cos(x)**2) + cos(x)**2/(-8*b**2*sin(x)**2 + 16*I*b**
2*sin(x)*cos(x) + 8*b**2*cos(x)**2), Eq(a, -I*b)), (2*I*x*sin(x)**2/(-8*b**
2*sin(x)**2 - 16*I*b**2*sin(x)*cos(x) + 8*b**2*cos(x)**2) - 4*x*sin(x)*cos(
x)/(-8*b**2*sin(x)**2 - 16*I*b**2*sin(x)*cos(x) + 8*b**2*cos(x)**2) - 2*I*x
*cos(x)**2/(-8*b**2*sin(x)**2 - 16*I*b**2*sin(x)*cos(x) + 8*b**2*cos(x)**2)
 - sin(x)**2/(-8*b**2*sin(x)**2 - 16*I*b**2*sin(x)*cos(x) + 8*b**2*cos(x)**
2) + cos(x)**2/(-8*b**2*sin(x)**2 - 16*I*b**2*sin(x)*cos(x) + 8*b**2*cos(x)
**2), Eq(a, I*b)), (-a**3*log(a*cos(x)/b + sin(x))*cos(x)/(a**5*cos(x) + a*
*4*b*sin(x) + 2*a**3*b**2*cos(x) + 2*a**2*b**3*sin(x) + a*b**4*cos(x) + b**
5*sin(x)) + 2*a**2*b*x*cos(x)/(a**5*cos(x) + a**4*b*sin(x) + 2*a**3*b**2*co
s(x) + 2*a**2*b**3*sin(x) + a*b**4*cos(x) + b**5*sin(x)) - a**2*b*log(a*cos
(x)/b + sin(x))*sin(x)/(a**5*cos(x) + a**4*b*sin(x) + 2*a**3*b**2*cos(x) +
2*a**2*b**3*sin(x) + a*b**4*cos(x) + b**5*sin(x)) - a**2*b*sin(x)/(a**5*cos
(x) + a**4*b*sin(x) + 2*a**3*b**2*cos(x) + 2*a**2*b**3*sin(x) + a*b**4*cos(
x) + b**5*sin(x)) + 2*a*b**2*x*sin(x)/(a**5*cos(x) + a**4*b*sin(x) + 2*a**3
*b**2*cos(x) + 2*a**2*b**3*sin(x) + a*b**4*cos(x) + b**5*sin(x)) + a*b**2*l
og(a*cos(x)/b + sin(x))*cos(x)/(a**5*cos(x) + a**4*b*sin(x) + 2*a**3*b**2*c
os(x) + 2*a**2*b**3*sin(x) + a*b**4*cos(x) + b**5*sin(x)) + b**3*log(a*cos(
x)/b + sin(x))*sin(x)/(a**5*cos(x) + a**4*b*sin(x) + 2*a**3*b**2*cos(x) + 2
*a**2*b**3*sin(x) + a*b**4*cos(x) + b**5*sin(x)) - b**3*sin(x)/(a**5*cos(x)
 + a**4*b*sin(x) + 2*a**3*b**2*cos(x) + 2*a**2*b**3*sin(x) + a*b**4*cos(x)
+ b**5*sin(x)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. 2(70) = 140.
time = 0.43, size = 144, normalized size = 2.06

$$\frac{2\,abx}{a^4+2\,a^2b^2+b^4}+\frac{(a^2-b^2)\log\left(\tan\left(x\right)^2+1\right)}{2\,(a^4+2\,a^2b^2+b^4)}-\frac{(a^2b-b^3)\log\left(\left|b\tan\left(x\right)+a\right|\right)}{a^4b+2\,a^2b^3+b^5}+\frac{a^2b\tan\left(x\right)-b^3\tan\left(x\right)+2\,a^3}{(a^4+2\,a^2b^2+b^4)(b\tan\left(x\right)+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] 2*a*b*x/(a^4 + 2*a^2*b^2 + b^4) + 1/2*(a^2 - b^2)*log(tan(x)^2 + 1)/(a^4 +
2*a^2*b^2 + b^4) - (a^2*b - b^3)*log(abs(b*tan(x) + a))/(a^4*b + 2*a^2*b^3
+ b^5) + (a^2*b*tan(x) - b^3*tan(x) + 2*a^3)/((a^4 + 2*a^2*b^2 + b^4)*(b*ta
n(x) + a))
```

Mupad [B]
time = 5.16, size = 1017, normalized size = 14.53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*sin(x))/(a*cos(x) + b*sin(x))^2,x)
```

```
[Out] -(b^3*sin(x) + a^3*log((a*cos(x) + b*sin(x))/cos(x/2)^2)*cos(x) - b^3*log((
a*cos(x) + b*sin(x))/cos(x/2)^2)*sin(x) + a^2*b*sin(x) - a^3*log(-(65536*a^
4*b^10 - 131072*a^6*b^8 + 196608*a^8*b^6 - 131072*a^10*b^4 + 65536*a^12*b^2
)/(a^16/2 + b^16/2 + 4*a^2*b^14 + 14*a^4*b^12 + 28*a^6*b^10 + 35*a^8*b^8 +
28*a^10*b^6 + 14*a^12*b^4 + 4*a^14*b^2 + (a^16*cos(x))/2 + (b^16*cos(x))/2
+ 4*a^2*b^14*cos(x) + 14*a^4*b^12*cos(x) + 28*a^6*b^10*cos(x) + 35*a^8*b^8*
cos(x) + 28*a^10*b^6*cos(x) + 14*a^12*b^4*cos(x) + 4*a^14*b^2*cos(x)))*cos(
x) + b^3*log(-(65536*a^4*b^10 - 131072*a^6*b^8 + 196608*a^8*b^6 - 131072*a^
10*b^4 + 65536*a^12*b^2)/(a^16/2 + b^16/2 + 4*a^2*b^14 + 14*a^4*b^12 + 28*a
^6*b^10 + 35*a^8*b^8 + 28*a^10*b^6 + 14*a^12*b^4 + 4*a^14*b^2 + (a^16*cos(x
))/2 + (b^16*cos(x))/2 + 4*a^2*b^14*cos(x) + 14*a^4*b^12*cos(x) + 28*a^6*b^
10*cos(x) + 35*a^8*b^8*cos(x) + 28*a^10*b^6*cos(x) + 14*a^12*b^4*cos(x) + 4
*a^14*b^2*cos(x)))*sin(x) - 4*a^2*b*atan(sin(x/2)/cos(x/2))*cos(x) - 4*a*b^
2*atan(sin(x/2)/cos(x/2))*sin(x) + a*b^2*log(-(65536*a^4*b^10 - 131072*a^6*
b^8 + 196608*a^8*b^6 - 131072*a^10*b^4 + 65536*a^12*b^2)/(a^16/2 + b^16/2 +
 4*a^2*b^14 + 14*a^4*b^12 + 28*a^6*b^10 + 35*a^8*b^8 + 28*a^10*b^6 + 14*a^1
2*b^4 + 4*a^14*b^2 + (a^16*cos(x))/2 + (b^16*cos(x))/2 + 4*a^2*b^14*cos(x)
+ 14*a^4*b^12*cos(x) + 28*a^6*b^10*cos(x) + 35*a^8*b^8*cos(x) + 28*a^10*b^6
*cos(x) + 14*a^12*b^4*cos(x) + 4*a^14*b^2*cos(x)))*cos(x) - a^2*b*log(-(655
36*a^4*b^10 - 131072*a^6*b^8 + 196608*a^8*b^6 - 131072*a^10*b^4 + 65536*a^1
2*b^2)/(a^16/2 + b^16/2 + 4*a^2*b^14 + 14*a^4*b^12 + 28*a^6*b^10 + 35*a^8*b
^8 + 28*a^10*b^6 + 14*a^12*b^4 + 4*a^14*b^2 + (a^16*cos(x))/2 + (b^16*cos(x
))/2 + 4*a^2*b^14*cos(x) + 14*a^4*b^12*cos(x) + 28*a^6*b^10*cos(x) + 35*a^8
*b^8*cos(x) + 28*a^10*b^6*cos(x) + 14*a^12*b^4*cos(x) + 4*a^14*b^2*cos(x)))
*sin(x) - a*b^2*log((a*cos(x) + b*sin(x))/cos(x/2)^2)*cos(x) + a^2*b*log((a
*cos(x) + b*sin(x))/cos(x/2)^2)*sin(x))/(b^5*sin(x) + a^5*cos(x) + a*b^4*co
s(x) + a^4*b*sin(x) + 2*a^3*b^2*cos(x) + 2*a^2*b^3*sin(x))
```

3.285 $\int \frac{\cos(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=110

$$-\frac{a(a^2-2b^2)\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}-\frac{2ab\cos(x)}{(a^2+b^2)^2}-\frac{(a^2-b^2)\sin(x)}{(a^2+b^2)^2}-\frac{a^2b}{(a^2+b^2)^2\,(a\cos(x)+b\sin(x))}$$

```
[Out] -a*(a^2-2*b^2)*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2)
-2*a*b*cos(x)/(a^2+b^2)^2-(a^2-b^2)*sin(x)/(a^2+b^2)^2-a^2*b/(a^2+b^2)^2/(a
*cos(x)+b*sin(x))
```

Rubi [A]
time = 0.17, antiderivative size = 152, normalized size of antiderivative = 1.38, number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3190, 3188, 2717, 2718, 3153, 212, 3178, 3233}

$$-\frac{a^2\sin(x)}{(a^2+b^2)^2}+\frac{b^2\sin(x)}{(a^2+b^2)^2}-\frac{2ab\cos(x)}{(a^2+b^2)^2}-\frac{a^2b}{(a^2+b^2)^2\,(a\cos(x)+b\sin(x))}+\frac{2ab^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}-\frac{a^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x]^2)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] -((a^3*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2)) +
 (2*a*b^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2)
 - (2*a*b*Cos[x])/(a^2 + b^2)^2 - (a^2*Sin[x])/(a^2 + b^2)^2 + (b^2*Sin[x])
/(a^2 + b^2)^2 - (a^2*b)/((a^2 + b^2)^2*(a*Cos[x] + b*Sin[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3178

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*
Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*
x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m
, 1]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3190

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[a*(b/(a^2 + b^2)), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3233

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[-(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rubi steps

$$\int \frac{\cos(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{a\int \frac{\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} + \frac{b\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2+b^2}$$

$$= -\frac{a^2\sin(x)}{(a^2+b^2)^2} - \frac{a^2b}{(a^2+b^2)^2(a\cos(x)+b\sin(x))} + \frac{a^3\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{(a^2+b^2)^2} + 2\frac{(ab$$

$$= -\frac{2ab\cos(x)}{(a^2+b^2)^2} - \frac{a^2\sin(x)}{(a^2+b^2)^2} + \frac{b^2\sin(x)}{(a^2+b^2)^2} - \frac{a^2b}{(a^2+b^2)^2(a\cos(x)+b\sin(x))} - \frac{a}{}$$

$$= -\frac{a^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} + \frac{2ab^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{2ab\cos(x)}{(a^2+b^2)^2} -$$

Mathematica [A]
time = 0.64, size = 111, normalized size = 1.01

$$\frac{2a(a^2-2b^2)\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{5a^2b-b^3+b(a^2+b^2)\cos(2x)+a(a^2+b^2)\sin(2x)}{2(a^2+b^2)^2(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x]^2)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (2*a*(a^2 - 2*b^2)*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^
(5/2) - (5*a^2*b - b^3 + b*(a^2 + b^2)*Cos[2*x] + a*(a^2 + b^2)*Sin[2*x])/(
2*(a^2 + b^2)^2*(a*Cos[x] + b*Sin[x]))
```

Maple [A]
time = 0.42, size = 142, normalized size = 1.29

method	result
default	$-\frac{2a\left(\frac{-b^2\tan\left(\frac{x}{2}\right)-ab}{a\left(\tan^2\left(\frac{x}{2}\right)\right)-2b\tan\left(\frac{x}{2}\right)-a}-\frac{\left(a^2-2b^2\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{\left(a^2+b^2\right)^2}+\frac{2\left(-a^2+b^2\right)\tan\left(\frac{x}{2}\right)-4ab}{\left(a^4+2a^2b^2+b^4\right)\left(1+\tan^2\left(\frac{x}{2}\right)\right)}$
risch	$\frac{i e^{ix}}{-4iab+2a^2-2b^2}-\frac{i e^{-ix}}{2(2iab+a^2-b^2)}-\frac{2b\,a^2 e^{ix}}{(ib+a)^2(-ib+a)^2(-ib\,e^{2ix}+a\,e^{2ix}+ib+a)}-\frac{a^3\ln\left(e^{ix}-\frac{ia^5+2ia^3b^2+ia\,b^4-b\,a^4-2b^3a^2-b^5}{\left(a^2+b^2\right)^{\frac{5}{2}}}\right.}{\left(a^2+b^2\right)^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*a/(a^2+b^2)^2*((-b^2*tan(1/2*x)-a*b)/(a*tan(1/2*x)^2-2*b*tan(1/2*x)-a)-(
a^2-2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2)
))+2/(a^4+2*a^2*b^2+b^4)*((-a^2+b^2)*tan(1/2*x)-2*a*b)/(1+tan(1/2*x)^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(106) = 212$.
time = 0.50, size = 265, normalized size = 2.41

$$-\frac{(a^2-2b^2)a\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{(a^4+2a^2b^2+b^4)\sqrt{a^2+b^2}}-\frac{2\left(3a^2b+\frac{(a^3+4ab^2)\sin(x)}{\cos(x)+1}+\frac{(a^2b-2b^3)\sin(x)^2}{(\cos(x)+1)^2}-\frac{(a^3-2ab^2)\sin(x)^3}{(\cos(x)+1)^3}\right)}{a^5+2a^3b^2+ab^4+\frac{2(a^4b+2a^2b^3+b^5)\sin(x)}{\cos(x)+1}+\frac{2(a^4b+2a^2b^3+b^5)\sin(x)^3}{(\cos(x)+1)^3}-\frac{(a^5+2a^3b^2+ab^4)\sin(x)^4}{(\cos(x)+1)^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -(a^2 - 2*b^2)*a*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*s
in(x)/(cos(x) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 +
b^2)) - 2*(3*a^2*b + (a^3 + 4*a*b^2)*sin(x)/(cos(x) + 1) + (a^2*b - 2*b^3)*
sin(x)^2/(cos(x) + 1)^2 - (a^3 - 2*a*b^2)*sin(x)^3/(cos(x) + 1)^3)/(a^5 + 2
*a^3*b^2 + a*b^4 + 2*(a^4*b + 2*a^2*b^3 + b^5)*sin(x)/(cos(x) + 1) + 2*(a^4
*b + 2*a^2*b^3 + b^5)*sin(x)^3/(cos(x) + 1)^3 - (a^5 + 2*a^3*b^2 + a*b^4)*s
in(x)^4/(cos(x) + 1)^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(106) = 212$.
time = 2.34, size = 252, normalized size = 2.29

$$-\frac{4a^4b+2a^2b^3-2b^5+2(a^4b+2a^2b^3+b^5)\cos(x)^2+2(a^5+2a^3b^2+ab^4)\cos(x)\sin(x)+\sqrt{a^2+b^2}\left((a^4-2a^2b^2)\cos(x)+(a^3b-2ab^3)\sin(x)\right)\log\left(\frac{2ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2-2a^2-b^2-2\sqrt{a^2+b^2}(b\cos(x)-a\sin(x))}{2ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2}\right)}{2\left((a^7+3a^5b^2+3a^3b^4+ab^6)\cos(x)+(a^6b+3a^4b^3+3a^2b^5+b^7)\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a^4*b + 2*a^2*b^3 - 2*b^5 + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(x)^2 +
2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)*sin(x) + sqrt(a^2 + b^2)*((a^4 - 2*a^2*b
^2)*cos(x) + (a^3*b - 2*a*b^3)*sin(x))*log((2*a*b*cos(x)*sin(x) + (a^2 - b^
2)*cos(x)^2 - 2*a^2 - b^2 - 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b
*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)))/((a^7 + 3*a^5*b^2 + 3*a^3*b^
4 + a*b^6)*cos(x) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)**2/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.48, size = 209, normalized size = 1.90

$$-\frac{\left(a^{3}-2\,ab^{2}\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\sqrt{a^{2}+b^{2}}}-\frac{2\left(a^{3}\tan\left(\frac{1}{2}\,x\right)^{3}-2\,ab^{2}\tan\left(\frac{1}{2}\,x\right)^{3}-a^{2}b\tan\left(\frac{1}{2}\,x\right)^{2}+2\,b^{3}\tan\left(\frac{1}{2}\,x\right)^{2}-a^{3}\tan\left(\frac{1}{2}\,x\right)-4\,ab^{2}\tan\left(\frac{1}{2}\,x\right)-3\,a^{2}b\right)}{\left(a\tan\left(\frac{1}{2}\,x\right)^{4}-2\,b\tan\left(\frac{1}{2}\,x\right)^{3}-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -(a^3 - 2*a*b^2)*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*
tan(1/2*x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 +
b^2)) - 2*(a^3*tan(1/2*x)^3 - 2*a*b^2*tan(1/2*x)^3 - a^2*b*tan(1/2*x)^2 + 2
*b^3*tan(1/2*x)^2 - a^3*tan(1/2*x) - 4*a*b^2*tan(1/2*x) - 3*a^2*b)/((a*tan(
1/2*x)^4 - 2*b*tan(1/2*x)^3 - 2*b*tan(1/2*x) - a)*(a^4 + 2*a^2*b^2 + b^4))
```

Mupad [B]
time = 1.23, size = 249, normalized size = 2.26

$$-\frac{\frac{2\tan\left(\frac{x}{2}\right)\,\left(a^{3}+4\,a\,b^{2}\right)}{a^{4}+2\,a^{2}\,b^{2}+b^{4}}+\frac{6\,a^{2}\,b}{a^{4}+2\,a^{2}\,b^{2}+b^{4}}-\frac{2\,a\tan\left(\frac{x}{2}\right)^{3}\,\left(a^{2}-2\,b^{2}\right)}{a^{4}+2\,a^{2}\,b^{2}+b^{4}}+\frac{2\,b\tan\left(\frac{x}{2}\right)^{2}\,\left(a^{2}-2\,b^{2}\right)}{a^{4}+2\,a^{2}\,b^{2}+b^{4}}}{-a\tan\left(\frac{x}{2}\right)^{4}+2\,b\tan\left(\frac{x}{2}\right)^{3}+2\,b\tan\left(\frac{x}{2}\right)+a}-\frac{a\,\mathrm{atan}\left(\frac{1\mathrm{i}\tan\left(\frac{x}{2}\right)\,a^{5}-a^{4}\,b\,1\mathrm{i}+2\mathrm{i}\tan\left(\frac{x}{2}\right)\,a^{3}\,b^{2}-a^{2}\,b^{3}\,2\mathrm{i}+1\mathrm{i}\tan\left(\frac{x}{2}\right)\,a\,b^{4}-b^{5}\,1\mathrm{i}}{{\left(a^{2}+b^{2}\right)}^{5/2}}\right)\,\left(a^{2}-2\,b^{2}\right)\,2\mathrm{i}}{{\left(a^{2}+b^{2}\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*sin(x)^2)/(a*cos(x) + b*sin(x))^2,x)
```

```
[Out] - ((2*tan(x/2)*(4*a*b^2 + a^3))/(a^4 + b^4 + 2*a^2*b^2) + (6*a^2*b)/(a^4 +
b^4 + 2*a^2*b^2) - (2*a*tan(x/2)^3*(a^2 - 2*b^2))/(a^4 + b^4 + 2*a^2*b^2) +
 (2*b*tan(x/2)^2*(a^2 - 2*b^2))/(a^4 + b^4 + 2*a^2*b^2))/(a + 2*b*tan(x/2)
- a*tan(x/2)^4 + 2*b*tan(x/2)^3) - (a*atan((a^5*tan(x/2)*1i - a^4*b*1i - b^
5*1i - a^2*b^3*2i + a^3*b^2*tan(x/2)*2i + a*b^4*tan(x/2)*1i)/(a^2 + b^2)^(5
/2))*(a^2 - 2*b^2)*2i)/(a^2 + b^2)^(5/2)
```

3.286 $\int \frac{\cos(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=129

$$\frac{b(3a^3-ab^2)\,x}{(a^2+b^2)^3}-\frac{a^2(a^2-3b^2)\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^3}-\frac{ab\cos(x)\sin(x)}{(a^2+b^2)^2}-\frac{(a^2-b^2)\sin^2(x)}{2\,(a^2+b^2)^2}-\frac{a^2b\sin(x)}{(a^2+b^2)^2\,(a\cos(x)+b\sin(x))}$$

```
[Out] b*(3*a^3-a*b^2)*x/(a^2+b^2)^3-a^2*(a^2-3*b^2)*ln(a*cos(x)+b*sin(x))/(a^2+b^
2)^3-a*b*cos(x)*sin(x)/(a^2+b^2)^2-1/2*(a^2-b^2)*sin(x)^2/(a^2+b^2)^2-a^2*b
*sin(x)/(a^2+b^2)^2/(a*cos(x)+b*sin(x))
```

Rubi [A]
time = 0.30, antiderivative size = 198, normalized size of antiderivative = 1.53, number of steps used = 17, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {3190, 3188, 2644, 30, 2715, 8, 3176, 3212, 3178, 3164, 3564, 3612, 3611}

$$\frac{abx}{(a^2+b^2)^2}+\frac{abx(a^2-b^2)}{(a^2+b^2)^3}-\frac{a^2\sin^2(x)}{2\,(a^2+b^2)^2}+\frac{b^2\sin^2(x)}{2\,(a^2+b^2)^2}-\frac{a^2b}{(a^2+b^2)^2\,(a\cot(x)+b)}-\frac{ab\sin(x)\cos(x)}{(a^2+b^2)^2}+\frac{3a^2b^2\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^3}-\frac{ab^3x}{(a^2+b^2)^3}-\frac{a^4\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^3}+\frac{a^3bx}{(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]*Sin[x]^3)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (a^3*b*x)/(a^2 + b^2)^3 - (a*b^3*x)/(a^2 + b^2)^3 + (a*b*(a^2 - b^2)*x)/(a^
2 + b^2)^3 + (a*b*x)/(a^2 + b^2)^2 - (a^2*b)/((a^2 + b^2)^2*(b + a*Cot[x]))
 - (a^4*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^3 + (3*a^2*b^2*Log[a*Cos[x] +
 b*Sin[x]])/(a^2 + b^2)^3 - (a*b*Cos[x]*Sin[x])/(a^2 + b^2)^2 - (a^2*Sin[x]
^2)/(2*(a^2 + b^2)^2) + (b^2*Sin[x]^2)/(2*(a^2 + b^2)^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
```

```
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3164

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(b + a*Cot[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3176

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3178

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*
Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*
x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m
, 1]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3190

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[a*(b/(a^2 + b^2)), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
```

```
b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\cos(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{a\int \frac{\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} + \frac{b\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2+b^2}$$

$$= -\frac{a^2\sin^2(x)}{2\left(a^2+b^2\right)^2} + \frac{a^3\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{\left(a^2+b^2\right)^2} + 2\frac{(ab)\int \sin^2(x)\,dx}{\left(a^2+b^2\right)^2} + \frac{b^2\int \cos(x)\sin(\cdots}{\left(a^2+b^2\right)^2\cdots}$$

$$= \frac{a^3bx}{\left(a^2+b^2\right)^3} - \frac{ab^3x}{\left(a^2+b^2\right)^3} - \frac{a^2b}{\left(a^2+b^2\right)^2(b+a\cot(x))} - \frac{a^2\sin^2(x)}{2\left(a^2+b^2\right)^2} - \frac{a^4\int \frac{b\cos\cdots}{a\cos\cdots}}{(a^2\cdots}$$

$$= \frac{a^3bx}{\left(a^2+b^2\right)^3} - \frac{ab^3x}{\left(a^2+b^2\right)^3} + \frac{ab(a^2-b^2)\,x}{\left(a^2+b^2\right)^3} - \frac{a^2b}{\left(a^2+b^2\right)^2(b+a\cot(x))} - \frac{a^4\log(\cdots}{\cdots}$$

$$= \frac{a^3bx}{\left(a^2+b^2\right)^3} - \frac{ab^3x}{\left(a^2+b^2\right)^3} + \frac{ab(a^2-b^2)\,x}{\left(a^2+b^2\right)^3} - \frac{a^2b}{\left(a^2+b^2\right)^2(b+a\cot(x))} - \frac{a^4\log(\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.57, size = 226, normalized size = 1.75

$$\frac{4ia^2(a^2-3b^2)\,\text{ArcTan}(\tan(x))(a\cos(x)+b\sin(x)) + a\cos(x)\left((a^4-b^4)\cos(2x) + 2a(2(ia-b)^3x - a(a^2-3b^2)\log\left((a\cos(x)+b\sin(x))^2\right) - b(a^2+b^2)\sin(2x))\right) - b\sin(x)\left((-a^4+b^4)\cos(2x) + 2a(2(a^3(1+ix) + ab^2(1-3ix) - 3a^2bx + b^3x) + a(a^2-3b^2)\log\left((a\cos(x)+b\sin(x))^2\right) + b(a^2+b^2)\sin(2x))\right)}{4\left(a^2+b^2\right)^3(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]*Sin[x]^3)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] ((4*I)*a^2*(a^2 - 3*b^2)*ArcTan[Tan[x]]*(a*Cos[x] + b*Sin[x]) + a*Cos[x]*((
a^4 - b^4)*Cos[2*x] + 2*a*(2*(I*a - b)^3*x - a*(a^2 - 3*b^2)*Log[(a*Cos[x]
+ b*Sin[x])^2] - b*(a^2 + b^2)*Sin[2*x])) - b*Sin[x]*((-a^4 + b^4)*Cos[2*x]
 + 2*a*(2*(a^3*(1 + I*x) + a*b^2*(1 - (3*I)*x) - 3*a^2*b*x + b^3*x) + a*(a^
2 - 3*b^2)*Log[(a*Cos[x] + b*Sin[x])^2] + b*(a^2 + b^2)*Sin[2*x])))/(4*(a^2
 + b^2)^3*(a*Cos[x] + b*Sin[x]))
```

Maple [A]
time = 0.40, size = 138, normalized size = 1.07

method	result
default	$\frac{a^3}{(a^2+b^2)^2(a+b\tan(x))} - \frac{a^2(a^2-3b^2)\ln(a+b\tan(x))}{(a^2+b^2)^3} + \frac{\frac{\left(-a^3b-a\,b^3\right)\tan(x)+\frac{a^4}{2}-\frac{b^4}{2}}{\tan^2(x)+1} + a\left(\frac{\left(a^3-3a\,b^2\right)\ln\left(\tan^2(x)+1\right)}{2} + (3a^2b-b^3)\,\text{ar}\cdots\right.}{(a^2+b^2)^3}$
risch	$\frac{iax}{3ia^2b-ib^3-a^3+3a\,b^2} + \frac{e^{2ix}}{-16iab+8a^2-8b^2} + \frac{e^{-2ix}}{16iab+8a^2-8b^2} + \frac{2ia^4x}{a^6+3a^4b^2+3a^2b^4+b^6} - \frac{6ia^2x\,b^2}{a^6+3a^4b^2+3a^2b^4+b^6} - \frac{\cdots}{(ib+a)^2(-\cdots}$
norman	$\frac{-\frac{2a\left(\tan^8\left(\frac{x}{2}\right)\right)}{a^2+b^2} + \frac{\left(3a^2-b^2\right)a^2bx\left(\tan^{10}\left(\frac{x}{2}\right)\right)}{a^6+3a^4b^2+3a^2b^4+b^6} + \frac{2a\left(\tan^4\left(\frac{x}{2}\right)\right)}{a^2+b^2} - \frac{2a\left(\tan^6\left(\frac{x}{2}\right)\right)}{a^2+b^2} + \frac{2a\left(\tan^2\left(\frac{x}{2}\right)\right)}{a^2+b^2} + \frac{4a^2b\tan\left(\frac{x}{2}\right)}{a^4+2a^2b^2+b^4} + \frac{4a^2b\left(\tan^9\left(\frac{x}{2}\right)\right)}{a^4+2a^2b^2+b^4} - \frac{4b\left(-3a^3+\cdots\right.}{a\left(a^4\cdots\right.}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^3/(a^2+b^2)^2/(a+b*tan(x))-a^2*(a^2-3*b^2)/(a^2+b^2)^3*ln(a+b*tan(x))+1/(
a^2+b^2)^3*(((-a^3*b-a*b^3)*tan(x)+1/2*a^4-1/2*b^4)/(tan(x)^2+1)+a*(1/2*(a^
3-3*a*b^2)*ln(tan(x)^2+1)+(3*a^2*b-b^3)*arctan(tan(x))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(127) = 254$.
time = 0.47, size = 259, normalized size = 2.01

$$\frac{(3\,a^3b-ab^3)x}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{(a^4-3\,a^2b^2)\log\left(b\tan\left(x\right)+a\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{(a^4-3\,a^2b^2)\log\left(\tan\left(x\right)^2+1\right)}{2\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)}+\frac{3\,a^3-ab^2+2\,(a^3-ab^2)\tan\left(x\right)^2-(a^2b+b^3)\tan\left(x\right)}{2\left(a^5+2\,a^3b^2+ab^4+(a^4b+2\,a^2b^3+b^5)\tan\left(x\right)^3+(a^5+2\,a^3b^2+ab^4)\tan\left(x\right)^2+(a^4b+2\,a^2b^3+b^5)\tan\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] (3*a^3*b - a*b^3)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^4 - 3*a^2*b^2)
*log(b*tan(x) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(a^4 - 3*a^2*b
^2)*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(3*a^3 - a*
b^2 + 2*(a^3 - a*b^2)*tan(x)^2 - (a^2*b + b^3)*tan(x))/(a^5 + 2*a^3*b^2 + a
*b^4 + (a^4*b + 2*a^2*b^3 + b^5)*tan(x)^3 + (a^5 + 2*a^3*b^2 + a*b^4)*tan(x
)^2 + (a^4*b + 2*a^2*b^3 + b^5)*tan(x))
```

Fricas [A]
time = 1.97, size = 236, normalized size = 1.83

$$\frac{2\,(a^5+2\,a^3b^2+ab^4)\cos\left(x\right)^3-(a^5+3\,ab^4-4\,(3\,a^4b-a^2b^3)x)\cos\left(x\right)-2\left((a^5-3\,a^3b^2)\cos\left(x\right)+(a^4b-3\,a^2b^3)\sin\left(x\right)\right)\log\left(2\,ab\cos\left(x\right)\sin\left(x\right)+(a^2-b^2)\cos\left(x\right)^2+b^2\right)-\left(5\,a^4b-b^5+2\,(a^4b+2\,a^2b^3+b^5)\cos\left(x\right)^2-4\,(3\,a^3b^2-ab^4)x\right)\sin\left(x\right)}{4\left((a^7+3\,a^5b^2+3\,a^3b^4+ab^6)\cos\left(x\right)+(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^3 - (a^5 + 3*a*b^4 - 4*(3*a^4*b - a
^2*b^3)*x)*cos(x) - 2*((a^5 - 3*a^3*b^2)*cos(x) + (a^4*b - 3*a^2*b^3)*sin(x
))*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - (5*a^4*b - b^5 +
 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(x)^2 - 4*(3*a^3*b^2 - a*b^4)*x)*sin(x))/((
a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(x) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^
5 + b^7)*sin(x))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)**3/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.39, size = 223, normalized size = 1.73

$$\frac{(3\,a^3b-ab^3)x}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{(a^4-3\,a^2b^2)\log\left(\tan\left(x\right)^2+1\right)}{2\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)}-\frac{(a^4b-3\,a^2b^3)\log\left(|b\tan\left(x\right)+a|\right)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}+\frac{2\,a^3\tan\left(x\right)^2-2\,ab^2\tan\left(x\right)^2-a^2b\tan\left(x\right)-b^3\tan\left(x\right)+3\,a^3-ab^2}{2\,(a^4+2\,a^2b^2+b^4)\left(b\tan\left(x\right)^3+a\tan\left(x\right)^2+b\tan\left(x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] (3*a^3*b - a*b^3)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(a^4 - 3*a^2*
b^2)*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^4*b - 3*a^2
*b^3)*log(abs(b*tan(x) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) + 1/2*(2
*a^3*tan(x)^2 - 2*a*b^2*tan(x)^2 - a^2*b*tan(x) - b^3*tan(x) + 3*a^3 - a*b^
2)/((a^4 + 2*a^2*b^2 + b^4)*(b*tan(x)^3 + a*tan(x)^2 + b*tan(x) + a))
```

Mupad [B]
time = 7.68, size = 2500, normalized size = 19.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)*sin(x)^3)/(a*cos(x) + b*sin(x))^2,x)
```

```
[Out] (log(1/(cos(x) + 1))*(2*a^4 - 6*a^2*b^2))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4
*b^2)) - (log(a + 2*b*tan(x/2) - a*tan(x/2)^2)*(a^4 - 3*a^2*b^2))/(a^6 + b^
6 + 3*a^2*b^4 + 3*a^4*b^2) - ((2*a*tan(x/2)^2)/(a^2 + b^2) - (2*a*tan(x/2)^
4)/(a^2 + b^2) + (4*a^2*b*tan(x/2))/(a^2 + b^2)^2 + (4*a^2*b*tan(x/2)^5)/(a
^4 + b^4 + 2*a^2*b^2) + (4*b*tan(x/2)^3*(a^2 - b^2))/(a^2 + b^2)^2)/(a + 2*
b*tan(x/2) + a*tan(x/2)^2 - a*tan(x/2)^4 - a*tan(x/2)^6 + 4*b*tan(x/2)^3 +
2*b*tan(x/2)^5) + (2*a*b*atan((((((a*b*((32*(3*a^4*b^11 - a^2*b^13 - 4*a^14
*b + 18*a^6*b^9 + 22*a^8*b^7 + 3*a^10*b^5 - 9*a^12*b^3))/(a^12 + b^12 + 6*a
^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (16*(2*a^4 -
 6*a^2*b^2)*(3*a^16*b + 3*a^2*b^15 + 21*a^4*b^13 + 63*a^6*b^11 + 105*a^8*b^
9 + 105*a^10*b^7 + 63*a^12*b^5 + 21*a^14*b^3))/((a^6 + b^6 + 3*a^2*b^4 + 3*
a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 +
 6*a^10*b^2)))*(3*a^2 - b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (16*a*b
*(2*a^4 - 6*a^2*b^2)*(3*a^2 - b^2)*(3*a^16*b + 3*a^2*b^15 + 21*a^4*b^13 + 6
3*a^6*b^11 + 105*a^8*b^9 + 105*a^10*b^7 + 63*a^12*b^5 + 21*a^14*b^3))/((a^6
 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 +
20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(2*a^4 - 6*a^2*b^2))/(2*(a^6 + b^6
+ 3*a^2*b^4 + 3*a^4*b^2)) + (a*b*((32*(5*a^4*b^9 - 3*a^12*b + 12*a^6*b^7 +
6*a^8*b^5 - 4*a^10*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^
6 + 15*a^8*b^4 + 6*a^10*b^2) + (((32*(3*a^4*b^11 - a^2*b^13 - 4*a^14*b + 18
```

*a^6*b^9 + 22*a^8*b^7 + 3*a^10*b^5 - 9*a^12*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (16*(2*a^4 - 6*a^2*b^2)*(3*a^16*b + 3*a^2*b^15 + 21*a^4*b^13 + 63*a^6*b^11 + 105*a^8*b^9 + 105*a^10*b^7 + 63*a^12*b^5 + 21*a^14*b^3))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(2*a^4 - 6*a^2*b^2))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(3*a^2 - b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (32*a^3*b^3*(3*a^2 - b^2)^3*(3*a^16*b + 3*a^2*b^15 + 21*a^4*b^13 + 63*a^6*b^11 + 105*a^8*b^9 + 105*a^10*b^7 + 63*a^12*b^5 + 21*a^14*b^3))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^3*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(4*a^8 + b^8 - 67*a^2*b^6 + 155*a^4*b^4 - 61*a^6*b^2)*(a^16 + b^16 + 8*a^2*b^14 + 28*a^4*b^12 + 56*a^6*b^10 + 70*a^8*b^8 + 56*a^10*b^6 + 28*a^12*b^4 + 8*a^14*b^2))/((96*a^6*b - 32*a^4*b^3)*(4*a^8 + b^8 + 31*a^2*b^6 + 15*a^4*b^4 - 11*a^6*b^2)^2) - (tan(x/2)*(((((2*a^4 - 6*a^2*b^2)*((a*b*(3*a^2 - b^2)*((32*(a^3*b^12 - 2*a^15 + 15*a^5*b^10 + 48*a^7*b^8 + 62*a^9*b^6 + 33*a^11*b^4 + 3*a^13*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + (16*(2*a^4 - 6*a^2*b^2)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (16*a*b*(2*a^4 - 6*a^2*b^2)*(3*a^2 - b^2)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (a*b*((32*(2*a^3*b^10 - 24*a^5*b^8 - 36*a^7*b^6 + 8*a^9*b^4 + 18*a^11*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + ((2*a^4 - 6*a^2*b^2)*((32*(a^3*b^12 - 2*a^15 + 15*a^5*b^10 + 48*a^7*b^8 + 62*a^9*b^6 + 33*a^11*b^4 + 3*a^13*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + (16*(2*a^4 - 6*a^2*b^2)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)))*(3*a^2 - b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (32*a^3*b^3*(3*a^2 - b^2)^3*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^3*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(4*a^8 + b^8 - 67*a^2*b^6 + 155*a^4*b^4 - 61*a^6*b^2))/(4*a^8 + b^8 + 31*a^2*b^6 + 15*a^4*b^4 - 11*a^6*b^2)^2 - (2*a*b*((32*(2*a^11 - 6*a^5*b^6 + 2*a^7*b^4 - 6*a^9*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + ((2*a^4 - 6*a^2*b^2)*((32*(2*a^3*b^10 - 24*a^5*b^8 - 36*a^7*b^6 + 8*a^9*b^4 + 18*a^11*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + ((2*a^4 - 6*a^2*b^2)*((32*(a^3*b^12 - 2*a^15 + 15*a^5*b^10 + 48*a^7*b^8 + 62*a^9*b^6 + 33*a^11*b^4 + 3*a^13*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20

```
*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + (16*(2*a^4 - 6*a^2*b^2)*(3*a*b^16 + 2
1*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a...
```

3.287 $\int \frac{\cos^2(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=109

$$-\frac{b(-2a^2+b^2)\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}-\frac{(a^2-b^2)\cos(x)}{(a^2+b^2)^2}+\frac{2ab\sin(x)}{(a^2+b^2)^2}+\frac{ab^2}{(a^2+b^2)^2(a\cos(x)+b\sin(x))}$$

```
[Out] -b*(-2*a^2+b^2)*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^(5/2
)-(a^2-b^2)*cos(x)/(a^2+b^2)^2+2*a*b*sin(x)/(a^2+b^2)^2+a*b^2/(a^2+b^2)^2/(
a*cos(x)+b*sin(x))
```

Rubi [A]
time = 0.18, antiderivative size = 151, normalized size of antiderivative = 1.39, number of steps used = 13, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3190, 3179, 2717, 3153, 212, 3188, 2718, 3234}

$$\frac{2ab\sin(x)}{(a^2+b^2)^2}+\frac{b^2\cos(x)}{(a^2+b^2)^2}-\frac{a^2\cos(x)}{(a^2+b^2)^2}+\frac{ab^2}{(a^2+b^2)^2(a\cos(x)+b\sin(x))}+\frac{2a^2b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}-\frac{b^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x])/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (2*a^2*b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2)
 - (b^3*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(5/2) -
(a^2*Cos[x])/(a^2 + b^2)^2 + (b^2*Cos[x])/(a^2 + b^2)^2 + (2*a*b*Sin[x])/(a
^2 + b^2)^2 + (a*b^2)/((a^2 + b^2)^2*(a*Cos[x] + b*Sin[x]))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3179

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3190

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[a*(b/(a^2 + b^2)), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3234

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rubi steps

$$\int \frac{\cos^2(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{a\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} + \frac{b\int \frac{\cos^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\cos(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2+b^2}$$

$$= \frac{b^2\cos(x)}{(a^2+b^2)^2} + \frac{ab^2}{(a^2+b^2)^2\,(a\cos(x)+b\sin(x))} + \frac{a^2\int \sin(x)\,dx}{(a^2+b^2)^2} + 2\frac{(ab)\int \cos(x)}{(a^2+b^2)^2}$$

$$= -\frac{a^2\cos(x)}{(a^2+b^2)^2} + \frac{b^2\cos(x)}{(a^2+b^2)^2} + \frac{2ab\sin(x)}{(a^2+b^2)^2} + \frac{ab^2}{(a^2+b^2)^2\,(a\cos(x)+b\sin(x))} + 2\frac{(a}{}$$

$$= \frac{2a^2b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{b^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{a^2\cos(x)}{(a^2+b^2)^2} + \frac{b^2}{(a^2}$$

Mathematica [A]
time = 0.73, size = 110, normalized size = 1.01

$$\frac{2b(-2a^2+b^2)\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{5/2}} - \frac{a^3-5ab^2+a(a^2+b^2)\cos(2x)-b(a^2+b^2)\sin(2x)}{2\,(a^2+b^2)^2\,(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x])/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (2*b*(-2*a^2 + b^2)*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 + b^2)
^(5/2) - (a^3 - 5*a*b^2 + a*(a^2 + b^2)*Cos[2*x] - b*(a^2 + b^2)*Sin[2*x])/
(2*(a^2 + b^2)^2*(a*Cos[x] + b*Sin[x]))
```

Maple [A]
time = 0.42, size = 143, normalized size = 1.31

method	result
default	$\frac{4b\left(\frac{-\frac{b^2\tan\left(\frac{x}{2}\right)}{2}-\frac{ab}{2}}{a\left(\tan^2\left(\frac{x}{2}\right)\right)-2b\tan\left(\frac{x}{2}\right)-a}-\frac{\left(2a^2-b^2\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^2}+\frac{4ab\tan\left(\frac{x}{2}\right)-2a^2+2b^2}{(a^4+2a^2b^2+b^4)\left(1+\tan^2\left(\frac{x}{2}\right)\right)}$
risch	$-\frac{e^{ix}}{2(-2iab+a^2-b^2)}-\frac{e^{-ix}}{2(2iab+a^2-b^2)}+\frac{2ia\,b^2e^{ix}}{(-ia+b)^2(ia+b)^2(b\,e^{2ix}+ia\,e^{2ix}-b+ia)}+\frac{2ib\ln\left(e^{ix}+\frac{ib+a}{\sqrt{-a^2-b^2}}\right)a^2}{\sqrt{-a^2-b^2}\,(a^2+b^2)^2}-\frac{ib^3\ln\left(\right.}{\sqrt{-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4*b/(a^2+b^2)^2*((-1/2*b^2*tan(1/2*x)-1/2*a*b)/(a*tan(1/2*x)^2-2*b*tan(1/2*
x)-a)-1/2*(2*a^2-b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2*x)-2*b)/(a^2
+b^2)^(1/2)))+4/(a^4+2*a^2*b^2+b^4)*(a*b*tan(1/2*x)-1/2*a^2+1/2*b^2)/(1+tan
(1/2*x)^2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(106) = 212$.
time = 0.48, size = 264, normalized size = 2.42

$$\frac{(2\,a^2b - b^3)\log\left(\frac{b - \frac{a\sin(x)}{\cos(x)+1} + \sqrt{a^2+b^2}}{b - \frac{a\sin(x)}{\cos(x)+1} - \sqrt{a^2+b^2}}\right)}{(a^4 + 2\,a^2b^2 + b^4)\sqrt{a^2+b^2}} - \frac{2\left(a^3 - 2\,ab^2 - \frac{3\,b^3\sin(x)}{\cos(x)+1} - \frac{(a^3+4\,ab^2)\sin(x)^2}{(\cos(x)+1)^2} + \frac{(2\,a^2b-b^3)\sin(x)^3}{(\cos(x)+1)^3}\right)}{a^5 + 2\,a^3b^2 + ab^4 + \frac{2\,(a^4b+2\,a^2b^3+b^5)\sin(x)}{\cos(x)+1} + \frac{2\,(a^4b+2\,a^2b^3+b^5)\sin(x)^3}{(\cos(x)+1)^3} - \frac{(a^5+2\,a^3b^2+ab^4)\sin(x)^4}{(\cos(x)+1)^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] (2*a^2*b - b^3)*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b - a*si
n(x)/(cos(x) + 1) - sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b
^2)) - 2*(a^3 - 2*a*b^2 - 3*b^3*sin(x)/(cos(x) + 1) - (a^3 + 4*a*b^2)*sin(x
)^2/(cos(x) + 1)^2 + (2*a^2*b - b^3)*sin(x)^3/(cos(x) + 1)^3)/(a^5 + 2*a^3*
b^2 + a*b^4 + 2*(a^4*b + 2*a^2*b^3 + b^5)*sin(x)/(cos(x) + 1) + 2*(a^4*b +
2*a^2*b^3 + b^5)*sin(x)^3/(cos(x) + 1)^3 - (a^5 + 2*a^3*b^2 + a*b^4)*sin(x)
^4/(cos(x) + 1)^4)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(106) = 212$.
time = 1.92, size = 252, normalized size = 2.31

$$\frac{6\,a^3b^2 + 6\,ab^4 - 2\,(a^5 + 2\,a^3b^2 + ab^4)\cos(x)^2 + 2\,(a^4b + 2\,a^2b^3 + b^5)\cos(x)\sin(x) - \sqrt{a^2+b^2}\,((2\,a^3b - ab^3)\cos(x) + (2\,a^2b^2 - b^4)\sin(x))\log\left(-\frac{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\,(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2}\right)}{2\,((a^7 + 3\,a^5b^2 + 3\,a^3b^4 + ab^6)\cos(x) + (a^6b + 3\,a^4b^3 + 3\,a^2b^5 + b^7)\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a^3*b^2 + 6*a*b^4 - 2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^2 + 2*(a^4*b
+ 2*a^2*b^3 + b^5)*cos(x)*sin(x) - sqrt(a^2 + b^2)*((2*a^3*b - a*b^3)*cos(x
) + (2*a^2*b^2 - b^4)*sin(x))*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x
)^2 - 2*a^2 - b^2 + 2*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*
sin(x) + (a^2 - b^2)*cos(x)^2 + b^2)))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^
6)*cos(x) + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.47, size = 204, normalized size = 1.87

$$\frac{(2\,a^2b-b^3)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^2+b^2}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^2+b^2}\right|}\right)}{(a^4+2\,a^2b^2+b^4)\sqrt{a^2+b^2}}+\frac{2\left(2\,a^2b\tan\left(\frac{1}{2}\,x\right)^3-b^3\tan\left(\frac{1}{2}\,x\right)^3-a^3\tan\left(\frac{1}{2}\,x\right)^2-4\,ab^2\tan\left(\frac{1}{2}\,x\right)^2-3\,b^3\tan\left(\frac{1}{2}\,x\right)+a^3-2\,ab^2\right)}{\left(a\tan\left(\frac{1}{2}\,x\right)^4-2\,b\tan\left(\frac{1}{2}\,x\right)^3-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)(a^4+2\,a^2b^2+b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] (2*a^2*b - b^3)*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs(2*a*t
an(1/2*x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^4 + 2*a^2*b^2 + b^4)*sqrt(a^2 + b
^2)) + 2*(2*a^2*b*tan(1/2*x)^3 - b^3*tan(1/2*x)^3 - a^3*tan(1/2*x)^2 - 4*a*
b^2*tan(1/2*x)^2 - 3*b^3*tan(1/2*x) + a^3 - 2*a*b^2)/((a*tan(1/2*x)^4 - 2*b
*tan(1/2*x)^3 - 2*b*tan(1/2*x) - a)*(a^4 + 2*a^2*b^2 + b^4))
```

Mupad [B]
time = 1.16, size = 253, normalized size = 2.32

$$\frac{\frac{2\,\left(2\,a\,b^2-a^3\right)}{a^4+2\,a^2\,b^2+b^4}+\frac{6\,b^3\,\tan\left(\frac{x}{2}\right)}{a^4+2\,a^2\,b^2+b^4}+\frac{2\,\tan\left(\frac{x}{2}\right)^2\,\left(a^3+4\,a\,b^2\right)}{a^4+2\,a^2\,b^2+b^4}-\frac{2\,b\,\tan\left(\frac{x}{2}\right)^3\,\left(2\,a^2-b^2\right)}{a^4+2\,a^2\,b^2+b^4}}{-a\,\tan\left(\frac{x}{2}\right)^4+2\,b\,\tan\left(\frac{x}{2}\right)^3+2\,b\,\tan\left(\frac{x}{2}\right)+a}+\frac{b\,\mathrm{atan}\left(\frac{1\mathrm{i}\,\tan\left(\frac{x}{2}\right)\,a^5-a^4\,b\,1\mathrm{i}+2\mathrm{i}\,\tan\left(\frac{x}{2}\right)\,a^3\,b^2-a^2\,b^3\,2\mathrm{i}+1\mathrm{i}\,\tan\left(\frac{x}{2}\right)\,a\,b^4-b^5\,1\mathrm{i}}{{\left(a^2+b^2\right)}^{5/2}}\right)\,\left(2\,a^2-b^2\right)\,2\mathrm{i}}{{\left(a^2+b^2\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2*sin(x))/(a*cos(x) + b*sin(x))^2,x)
```

```
[Out] ((2*(2*a*b^2 - a^3))/(a^4 + b^4 + 2*a^2*b^2) + (6*b^3*tan(x/2))/(a^4 + b^4
+ 2*a^2*b^2) + (2*tan(x/2)^2*(4*a*b^2 + a^3))/(a^4 + b^4 + 2*a^2*b^2) - (2*
b*tan(x/2)^3*(2*a^2 - b^2))/(a^4 + b^4 + 2*a^2*b^2))/(a + 2*b*tan(x/2) - a*
tan(x/2)^4 + 2*b*tan(x/2)^3) + (b*atan((a^5*tan(x/2)*1i - a^4*b*1i - b^5*1i
 - a^2*b^3*2i + a^3*b^2*tan(x/2)*2i + a*b^4*tan(x/2)*1i)/(a^2 + b^2)^(5/2))
*(2*a^2 - b^2)*2i)/(a^2 + b^2)^(5/2)
```

3.288 $\int \frac{\cos^2(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=131

$$\frac{(a^4-6a^2b^2+b^4)\,x}{2\,(a^2+b^2)^3}+\frac{2ab(a^2-b^2)\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^3}+\frac{(-a^2+b^2)\cos(x)\sin(x)}{2\,(a^2+b^2)^2}+\frac{ab\sin^2(x)}{(a^2+b^2)^2}+\frac{}{(a^2+b^2)^2}$$

```
[Out] 1/2*(a^4-6*a^2*b^2+b^4)*x/(a^2+b^2)^3+2*a*b*(a^2-b^2)*ln(a*cos(x)+b*sin(x))
/(a^2+b^2)^3+1/2*(-a^2+b^2)*cos(x)*sin(x)/(a^2+b^2)^2+a*b*sin(x)^2/(a^2+b^2
)^2+a*b^2*sin(x)/(a^2+b^2)^2/(a*cos(x)+b*sin(x))
```

Rubi [A]
time = 0.37, antiderivative size = 186, normalized size of antiderivative = 1.42, number of steps used = 21, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3190, 3188, 2715, 8, 2644, 30, 3177, 3212, 3176, 3154}

$$\frac{a^2x}{2\,(a^2+b^2)^2}-\frac{4a^2b^2x}{(a^2+b^2)^3}+\frac{b^2x}{2\,(a^2+b^2)^2}+\frac{ab\sin^2(x)}{(a^2+b^2)^2}-\frac{a^2\sin(x)\cos(x)}{2\,(a^2+b^2)^2}+\frac{ab^2\sin(x)}{(a^2+b^2)^2\,(a\cos(x)+b\sin(x))}+\frac{b^2\sin(x)\cos(x)}{2\,(a^2+b^2)^2}-\frac{2ab^3\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^3}+\frac{2a^3b\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x]^2)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (-4*a^2*b^2*x)/(a^2 + b^2)^3 + (a^2*x)/(2*(a^2 + b^2)^2) + (b^2*x)/(2*(a^2
+ b^2)^2) + (2*a^3*b*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^3 - (2*a*b^3*Log
[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^3 - (a^2*Cos[x]*Sin[x])/(2*(a^2 + b^2)^2
) + (b^2*Cos[x]*Sin[x])/(2*(a^2 + b^2)^2) + (a*b*Sin[x]^2)/(a^2 + b^2)^2 +
(a*b^2*Sin[x])/((a^2 + b^2)^2*(a*Cos[x] + b*Sin[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
```

```
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3154

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3176

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3177

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3190

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[a*(b/(a^2 + b^2)), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\int \frac{\cos^2(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{a\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} + \frac{b\int \frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\cos(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2+b^2}$$

$$= \frac{a^2\int \sin^2(x)\,dx}{(a^2+b^2)^2} + 2\frac{(ab)\int \cos(x)\sin(x)\,dx}{(a^2+b^2)^2} - 2\frac{(a^2b)\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{(a^2+b^2)^2} + \frac{b^2\int}{(a}$$

$$= -\frac{a^2\cos(x)\sin(x)}{2(a^2+b^2)^2} + \frac{b^2\cos(x)\sin(x)}{2(a^2+b^2)^2} + \frac{ab^2\sin(x)}{(a^2+b^2)^2(a\cos(x)+b\sin(x))} - 2\Bigg($$

$$= \frac{a^2x}{2(a^2+b^2)^2} + \frac{b^2x}{2(a^2+b^2)^2} - 2\left(\frac{a^2b^2x}{(a^2+b^2)^3} - \frac{a^3b\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^3}\right) -$$

Mathematica [A]
time = 1.76, size = 145, normalized size = 1.11

$$\frac{\sin(x)}{8a(a\cos(x)+b\sin(x))} - \frac{-4(a^4-6a^2b^2+b^4)\,x+4ab(a^2+b^2)\cos(2x)-16ab(a^2-b^2)\log(a\cos(x)+b\sin(x))+\frac{(a^2+b^2)(a^4-6a^2b^2+b^4)\sin(x)}{a(a\cos(x)+b\sin(x))}+2(a^4-b^4)\sin(2x)}{8(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x]^2)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] Sin[x]/(8*a*(a*Cos[x] + b*Sin[x])) - (-4*(a^4 - 6*a^2*b^2 + b^4)*x + 4*a*b*
(a^2 + b^2)*Cos[2*x] - 16*a*b*(a^2 - b^2)*Log[a*Cos[x] + b*Sin[x]] + ((a^2
+ b^2)*(a^4 - 6*a^2*b^2 + b^4)*Sin[x])/(a*(a*Cos[x] + b*Sin[x])) + 2*(a^4 -
 b^4)*Sin[2*x])/(8*(a^2 + b^2)^3)
```

Maple [A]
time = 0.40, size = 143, normalized size = 1.09

method	result
default	$-\frac{b\,a^2}{(a^2+b^2)^2(a+b\tan(x))} + \frac{2ab(a^2-b^2)\ln(a+b\tan(x))}{(a^2+b^2)^3} + \frac{\frac{\left(-\frac{a^4}{2}+\frac{b^4}{2}\right)\tan(x)-a^3b-a\,b^3}{\tan^2(x)+1} + \frac{\left(-4a^3b+4a\,b^3\right)\ln\left(\tan^2(x)+1\right)}{4} + \frac{(a^4-6a^2b}{}}{(a^2+b^2)^3}$
risch	$-\frac{ixb}{2(3ia^2b-ib^3-a^3+3a\,b^2)} - \frac{xa}{2(3ia^2b-ib^3-a^3+3a\,b^2)} + \frac{ie^{2ix}}{-16iab+8a^2-8b^2} - \frac{ie^{-2ix}}{8(2iab+a^2-b^2)} - \frac{4ia^3bx}{a^6+3a^4b^2+3a^2b^4+b^6} + \frac{}{a^6}$

norman	$\frac{\frac{2b\left(\tan^{8}\left(\frac{x}{2}\right)\right)}{a^2+b^2}+\frac{\left(a^4-6a^2b^2+b^4\right)ax\left(\tan^{6}\left(\frac{x}{2}\right)\right)}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{2b\left(\tan^{4}\left(\frac{x}{2}\right)\right)}{a^2+b^2}+\frac{2b\left(\tan^{6}\left(\frac{x}{2}\right)\right)}{a^2+b^2}-\frac{2b\left(\tan^{2}\left(\frac{x}{2}\right)\right)}{a^2+b^2}-\frac{\left(-a^4+3a^2b^2\right)\tan\left(\frac{x}{2}\right)}{a\left(a^4+2a^2b^2+b^4\right)}-\frac{\left(-a^4+3a^2b^2\right)\left(\tan^{9}\left(\right.\right.}{a\left(a^4+2a^2b^2+b^4\right)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -b*a^2/(a^2+b^2)^2/(a+b*tan(x))+2*a*b*(a^2-b^2)/(a^2+b^2)^3*ln(a+b*tan(x))+
1/(a^2+b^2)^3*(((-1/2*a^4+1/2*b^4)*tan(x)-a^3*b-a*b^3)/(tan(x)^2+1)+1/4*(-4
*a^3*b+4*a*b^3)*ln(tan(x)^2+1)+1/2*(a^4-6*a^2*b^2+b^4)*arctan(tan(x)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(127) = 254$.
time = 0.47, size = 257, normalized size = 1.96

$$\frac{(a^4-6\,a^2b^2+b^4)x}{2\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)}+\frac{2\,(a^3b-ab^3)\log\left(b\tan\left(x\right)+a\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{(a^3b-ab^3)\log\left(\tan\left(x\right)^2+1\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{4\,a^2b+(3\,a^2b-b^3)\tan\left(x\right)^2+(a^3+ab^2)\tan\left(x\right)}{2\,\left(a^5+2\,a^3b^2+ab^4+(a^4b+2\,a^2b^3+b^5)\tan\left(x\right)^3+(a^5+2\,a^3b^2+ab^4)\tan\left(x\right)^2+(a^4b+2\,a^2b^3+b^5)\tan\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a^4 - 6*a^2*b^2 + b^4)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(a^3*
b - a*b^3)*log(b*tan(x) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^3*b -
 a*b^3)*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - 1/2*(4*a^2*
b + (3*a^2*b - b^3)*tan(x)^2 + (a^3 + a*b^2)*tan(x))/(a^5 + 2*a^3*b^2 + a*b
^4 + (a^4*b + 2*a^2*b^3 + b^5)*tan(x)^3 + (a^5 + 2*a^3*b^2 + a*b^4)*tan(x)^
2 + (a^4*b + 2*a^2*b^3 + b^5)*tan(x))
```

Fricas [A]
time = 1.93, size = 244, normalized size = 1.86

$$-\frac{(a^4b+2\,a^2b^3+b^5)\cos\left(x\right)^3+(a^2b^3-b^5-(a^5-6\,a^3b^2+ab^4)x)\cos\left(x\right)-2\,((a^4b-a^2b^3)\cos\left(x\right)+(a^3b^2-ab^4)\sin\left(x\right))\log\left(2\,ab\cos\left(x\right)\sin\left(x\right)+(a^2-b^2)\cos\left(x\right)^2+b^2\right)-(3\,a^3b^2+ab^4-(a^5+2\,a^3b^2+ab^4)\cos\left(x\right)^2+(a^4b-6\,a^2b^3+b^5)x)\sin\left(x\right)}{2\,((a^7+3\,a^5b^2+3\,a^3b^4+ab^6)\cos\left(x\right)+(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)\sin\left(x\right))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/2*((a^4*b + 2*a^2*b^3 + b^5)*cos(x)^3 + (a^2*b^3 - b^5 - (a^5 - 6*a^3*b^
2 + a*b^4)*x)*cos(x) - 2*((a^4*b - a^2*b^3)*cos(x) + (a^3*b^2 - a*b^4)*sin(
x))*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - (3*a^3*b^2 + a*
b^4 - (a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^2 + (a^4*b - 6*a^2*b^3 + b^5)*x)*sin
(x))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(x) + (a^6*b + 3*a^4*b^3 + 3
*a^2*b^5 + b^7)*sin(x))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)**2/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.45, size = 219, normalized size = 1.67

$$\frac{\left(a^{4}-6\,a^{2}b^{2}+b^{4}\right)x}{2\,\left(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}\right)}-\frac{\left(a^{3}b-ab^{3}\right)\log\left(\tan\left(x\right)^{2}+1\right)}{a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}}+\frac{2\,\left(a^{3}b^{2}-ab^{4}\right)\log\left(\left|b\tan\left(x\right)+a\right|\right)}{a^{6}b+3\,a^{4}b^{3}+3\,a^{2}b^{5}+b^{7}}-\frac{3\,a^{2}b\tan\left(x\right)^{2}-b^{3}\tan\left(x\right)^{2}+a^{3}\tan\left(x\right)+ab^{2}\tan\left(x\right)+4\,a^{2}b}{2\,\left(a^{4}+2\,a^{2}b^{2}+b^{4}\right)\left(b\tan\left(x\right)^{3}+a\tan\left(x\right)^{2}+b\tan\left(x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] 1/2*(a^4 - 6*a^2*b^2 + b^4)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (a^3*b
- a*b^3)*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 2*(a^3*b^2
 - a*b^4)*log(abs(b*tan(x) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) - 1/
2*(3*a^2*b*tan(x)^2 - b^3*tan(x)^2 + a^3*tan(x) + a*b^2*tan(x) + 4*a^2*b)/(
(a^4 + 2*a^2*b^2 + b^4)*(b*tan(x)^3 + a*tan(x)^2 + b*tan(x) + a))
```

Mupad [B]
time = 12.13, size = 2500, normalized size = 19.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2*sin(x)^2)/(a*cos(x) + b*sin(x))^2,x)
```

```
[Out] ((tan(x/2)^5*(3*a*b^2 - a^3))/(a^4 + b^4 + 2*a^2*b^2) + (2*b*tan(x/2)^2)/(a
^2 + b^2) - (2*b*tan(x/2)^4)/(a^2 + b^2) + (tan(x/2)*(3*a*b^2 - a^3))/(a^2
+ b^2)^2 + (2*tan(x/2)^3*(5*a*b^2 + a^3))/(a^2 + b^2)^2)/(a + 2*b*tan(x/2)
+ a*tan(x/2)^2 - a*tan(x/2)^4 - a*tan(x/2)^6 + 4*b*tan(x/2)^3 + 2*b*tan(x/2
)^5) - (log(a + 2*b*tan(x/2) - a*tan(x/2)^2)*(2*a*b^3 - 2*a^3*b))/(a^6 + b^
6 + 3*a^2*b^4 + 3*a^4*b^2) + (log(1/(cos(x) + 1))*(16*a*b^3 - 16*a^3*b))/(2
*(4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)) - (atan((tan(x/2)*((((((((8*(4*
a^2*b^13 - 20*a^14*b + 48*a^4*b^11 + 132*a^6*b^9 + 128*a^8*b^7 + 12*a^10*b^
5 - 48*a^12*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*
a^8*b^4 + 6*a^10*b^2) - (4*(16*a*b^3 - 16*a^3*b)*(12*a*b^16 + 84*a^3*b^14 +
 252*a^5*b^12 + 420*a^7*b^10 + 420*a^9*b^8 + 252*a^11*b^6 + 84*a^13*b^4 + 1
2*a^15*b^2))/((4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)*(a^12 + b^12 + 6*a^
2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(2*a*b - a^2
+ b^2)*(2*a*b + a^2 - b^2))/(2*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) - (2*(1
6*a*b^3 - 16*a^3*b)*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2)*(12*a*b^16 + 84
*a^3*b^14 + 252*a^5*b^12 + 420*a^7*b^10 + 420*a^9*b^8 + 252*a^11*b^6 + 84*a
^13*b^4 + 12*a^15*b^2))/((a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)*(4*a^6 + 4*b^6
 + 12*a^2*b^4 + 12*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6
```

*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(16*a*b^3 - 16*a^3*b))/(2*(4*a^6 + 4*b^6
+ 12*a^2*b^4 + 12*a^4*b^2)) - (((8*(2*a*b^12 + a^13 - 53*a^3*b^10 - 7*a^5*b
^8 + 126*a^7*b^6 + 52*a^9*b^4 - 25*a^11*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 1
5*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - ((16*a*b^3 - 16*a^3*b)*
((8*(4*a^2*b^13 - 20*a^14*b + 48*a^4*b^11 + 132*a^6*b^9 + 128*a^8*b^7 + 12*
a^10*b^5 - 48*a^12*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^
6 + 15*a^8*b^4 + 6*a^10*b^2) - (4*(16*a*b^3 - 16*a^3*b)*(12*a*b^16 + 84*a^3
*b^14 + 252*a^5*b^12 + 420*a^7*b^10 + 420*a^9*b^8 + 252*a^11*b^6 + 84*a^13*
b^4 + 12*a^15*b^2))/((4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)*(a^12 + b^12
 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(2*(4
*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)))*(2*a*b - a^2 + b^2)*(2*a*b + a^2
- b^2))/(2*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) + ((2*a*b - a^2 + b^2)^3*(2
*a*b + a^2 - b^2)^3*(12*a*b^16 + 84*a^3*b^14 + 252*a^5*b^12 + 420*a^7*b^10
+ 420*a^9*b^8 + 252*a^11*b^6 + 84*a^13*b^4 + 12*a^15*b^2))/((a^2 + b^2)^3*(
a^4 + b^4 + 2*a^2*b^2)^3*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^
6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^10 - b^10 + 109*a^2*b^8 - 466*a^4*b^6 + 4
66*a^6*b^4 - 109*a^8*b^2))/(a^10 + b^10 + 53*a^2*b^8 - 38*a^4*b^6 - 38*a^6*
b^4 + 53*a^8*b^2)^2 + (((8*(2*a^10*b - 4*a^2*b^9 + 10*a^4*b^7 - 30*a^6*b^5
+ 22*a^8*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8
*b^4 + 6*a^10*b^2) - ((16*a*b^3 - 16*a^3*b)*((8*(2*a*b^12 + a^13 - 53*a^3*b
^10 - 7*a^5*b^8 + 126*a^7*b^6 + 52*a^9*b^4 - 25*a^11*b^2))/(a^12 + b^12 + 6
*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - ((16*a*b^3
 - 16*a^3*b)*((8*(4*a^2*b^13 - 20*a^14*b + 48*a^4*b^11 + 132*a^6*b^9 + 128*
a^8*b^7 + 12*a^10*b^5 - 48*a^12*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^
8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (4*(16*a*b^3 - 16*a^3*b)*(12*a*
b^16 + 84*a^3*b^14 + 252*a^5*b^12 + 420*a^7*b^10 + 420*a^9*b^8 + 252*a^11*b
^6 + 84*a^13*b^4 + 12*a^15*b^2))/((4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)
*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*
b^2))))/(2*(4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2))))/(2*(4*a^6 + 4*b^6 +
 12*a^2*b^4 + 12*a^4*b^2)) - (((((8*(4*a^2*b^13 - 20*a^14*b + 48*a^4*b^11 +
 132*a^6*b^9 + 128*a^8*b^7 + 12*a^10*b^5 - 48*a^12*b^3))/(a^12 + b^12 + 6*a
^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (4*(16*a*b^3
 - 16*a^3*b)*(12*a*b^16 + 84*a^3*b^14 + 252*a^5*b^12 + 420*a^7*b^10 + 420*a
^9*b^8 + 252*a^11*b^6 + 84*a^13*b^4 + 12*a^15*b^2))/((4*a^6 + 4*b^6 + 12*a^
2*b^4 + 12*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 1
5*a^8*b^4 + 6*a^10*b^2)))*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2))/(2*(a^2
+ b^2)*(a^4 + b^4 + 2*a^2*b^2)) - (2*(16*a*b^3 - 16*a^3*b)*(2*a*b - a^2 + b
^2)*(2*a*b + a^2 - b^2)*(12*a*b^16 + 84*a^3*b^14 + 252*a^5*b^12 + 420*a^7*b
^10 + 420*a^9*b^8 + 252*a^11*b^6 + 84*a^13*b^4 + 12*a^15*b^2))/((a^2 + b^2)
*(a^4 + b^4 + 2*a^2*b^2)*(4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)*(a^12 +
b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(2
*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2))/(2*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^
2)) + ((16*a*b^3 - 16*a^3*b)*(2*a*b - a^2 + b^2)^2*(2*a*b + a^2 - b^2)^2*(1
2*a*b^16 + 84*a^3*b^14 + 252*a^5*b^12 + 420*a^7*b^10 + 420*a^9*b^8 + 252*a^
11*b^6 + 84*a^13*b^4 + 12*a^15*b^2))/((a^2 + b^2)^2*(a^4 + b^4 + 2*a^2*b^2)

```
^2*(4*a^6 + 4*b^6 + 12*a^2*b^4 + 12*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15
*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(18*a*b^9 + 18*a^9*b - 2
80*a^3*b^7 + 556*a^5*b^5 - 280*a^7*b^3))/(a^10 + b^10 + 53*a^2*b^8 - 38*a^4
*b^6 - 38*a^6*b^4 + 53*a^8*b^2)^2)*(a^16 + b^16...
```

3.289 $\int \frac{\cos^2(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=172

$$\frac{a^2b(2a^2-3b^2)\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}}-\frac{a^2(a^2-3b^2)\cos(x)}{(a^2+b^2)^3}+\frac{(a^2-b^2)\cos^3(x)}{3\,(a^2+b^2)^2}+\frac{2ab(a^2-b^2)\sin(x)}{(a^2+b^2)^3}+\frac{2ab\sin}{3\,(a^2-}$$

```
[Out] a^2*b*(2*a^2-3*b^2)*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)^
(7/2)-a^2*(a^2-3*b^2)*cos(x)/(a^2+b^2)^3+1/3*(a^2-b^2)*cos(x)^3/(a^2+b^2)^2
+2*a*b*(a^2-b^2)*sin(x)/(a^2+b^2)^3+2/3*a*b*sin(x)^3/(a^2+b^2)^2+a^3*b^2/(a
^2+b^2)^3/(a*cos(x)+b*sin(x))
```

Rubi [A]
time = 0.50, antiderivative size = 238, normalized size of antiderivative = 1.38, number of steps used = 33, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3190, 3188, 2645, 30, 2644, 2717, 2718, 3153, 212, 2713, 3178, 3233}

$$\frac{2ab\sin^3(x)}{3\,(a^2+b^2)^2}+\frac{a^2\cos^3(x)}{3\,(a^2+b^2)^2}-\frac{b^2\cos^3(x)}{3\,(a^2+b^2)^2}-\frac{a^2\cos(x)}{(a^2+b^2)^2}+\frac{4a^2b^2\cos(x)}{(a^2+b^2)^3}-\frac{2ab^3\sin(x)}{(a^2+b^2)^3}-\frac{3a^2b^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}}+\frac{2a^4b\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}}+\frac{2a^3b\sin(x)}{(a^2+b^2)^3}+\frac{a^3b^2}{(a^2+b^2)^3\,(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^2*Sin[x]^3)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (2*a^4*b*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/2)
- (3*a^2*b^3*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7
/2) + (4*a^2*b^2*Cos[x])/(a^2 + b^2)^3 - (a^2*Cos[x])/(a^2 + b^2)^2 + (a^2*
Cos[x]^3)/(3*(a^2 + b^2)^2) - (b^2*Cos[x]^3)/(3*(a^2 + b^2)^2) + (2*a^3*b*S
in[x])/(a^2 + b^2)^3 - (2*a*b^3*Sin[x])/(a^2 + b^2)^3 + (2*a*b*Sin[x]^3)/(3
*(a^2 + b^2)^2) + (a^3*b^2)/((a^2 + b^2)^3*(a*Cos[x] + b*Sin[x]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
```

```
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3178

```
Int[sin[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[(-a)*(Sin[c + d*x]^(m - 1)/(d*(a^2
 + b^2)*(m - 1))), x] + (Dist[a^2/(a^2 + b^2), Int[Sin[c + d*x]^(m - 2)/(a*
Cos[c + d*x] + b*Sin[c + d*x]), x], x] + Dist[b/(a^2 + b^2), Int[Sin[c + d*
x]^(m - 1), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m
, 1]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
```

```
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3190

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[a*(b/(a^2 + b^2)), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3233

```
Int[((A_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[-(b*C + (a*C
- c*A)*Cos[d + e*x] + b*A*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
 e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - c*C)/(a^2 - b^2 - c^2), Int[1/(a
 + b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, C},
 x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - c*C, 0]
```

Rubi steps

$$\int \frac{\cos^2(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{a\int \frac{\cos(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} + \frac{b\int \frac{\cos^2(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\cos(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2+b^2}$$

$$= \frac{a^2\int \sin^3(x)\,dx}{(a^2+b^2)^2} + 2\frac{(ab)\int \cos(x)\sin^2(x)\,dx}{(a^2+b^2)^2} - 2\frac{(a^2b)\int \frac{\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{(a^2+b^2)^2} + \frac{b^2\int$$

$$= \frac{a^3b^2}{(a^2+b^2)^3(a\cos(x)+b\sin(x))} - 2\left(-\frac{a^3b\sin(x)}{(a^2+b^2)^3} + \frac{(a^4b)\int \frac{1}{a\cos(x)+b\sin(x)}\,dx}{(a^2+b^2)^3} +\right.$$

$$= -\frac{a^2\cos(x)}{(a^2+b^2)^2} + \frac{a^2\cos^3(x)}{3(a^2+b^2)^2} - \frac{b^2\cos^3(x)}{3(a^2+b^2)^2} + \frac{2ab\sin^3(x)}{3(a^2+b^2)^2} + \frac{a^3b^2}{(a^2+b^2)^3(a\cos(x)}$$

$$= -\frac{a^2b^3\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}} - \frac{a^2\cos(x)}{(a^2+b^2)^2} + \frac{a^2\cos^3(x)}{3(a^2+b^2)^2} - \frac{b^2\cos^3(x)}{3(a^2+b^2)^2} + \frac{2}{3}$$

Mathematica [A]
time = 1.34, size = 200, normalized size = 1.16

$$-\frac{2a^2b(2a^2-3b^2)\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}}+\frac{-9a^5+90a^3b^2-21ab^4+(-8a^5+4a^3b^2+12ab^4)\cos(2x)+a(a^2+b^2)^2\cos(4x)+18a^4b\sin(2x)+16a^2b^3\sin(2x)-2b^5\sin(2x)-a^4b\sin(4x)-2a^2b^3\sin(4x)-b^5\sin(4x)}{24(a^2+b^2)^3(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^2*Sin[x]^3)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (-2*a^2*b*(2*a^2 - 3*b^2)*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2
+ b^2)^(7/2) + (-9*a^5 + 90*a^3*b^2 - 21*a*b^4 + (-8*a^5 + 4*a^3*b^2 + 12*a
*b^4)*Cos[2*x] + a*(a^2 + b^2)^2*Cos[4*x] + 18*a^4*b*Sin[2*x] + 16*a^2*b^3*
Sin[2*x] - 2*b^5*Sin[2*x] - a^4*b*Sin[4*x] - 2*a^2*b^3*Sin[4*x] - b^5*Sin[4
*x])/(24*(a^2 + b^2)^3*(a*Cos[x] + b*Sin[x]))
```

Maple [A]
time = 0.59, size = 271, normalized size = 1.58

method	result
default	$\frac{4a^2b\left(\frac{-\frac{b^2\tan\left(\frac{x}{2}\right)}{2}-\frac{ab}{2}}{a\left(\tan^2\left(\frac{x}{2}\right)\right)-2b\tan\left(\frac{x}{2}\right)-a}-\frac{\left(2a^2-3b^2\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{2\sqrt{a^2+b^2}}\right)}{(a^4+2a^2b^2+b^4)(a^2+b^2)}+\frac{4(a^3b-a\,b^3)\left(\tan^5\left(\frac{x}{2}\right)\right)+4\left(\frac{3}{2}a^2b^2-\frac{1}{2}b^4\right)\left(\tan^4\left(\frac{x}{2}\right)\right)+4}{}$
risch	$\frac{e^{3ix}}{-48iab+24a^2-24b^2}-\frac{ie^{ix}b}{8(-3ia^2b+ib^3+a^3-3a\,b^2)}-\frac{3\,e^{ix}a}{8(-3ia^2b+ib^3+a^3-3a\,b^2)}+\frac{ie^{-ix}b}{8(ib+a)^3}-\frac{3\,e^{-ix}a}{8(ib+a)^3}+\frac{e^{-3ix}}{24(ib+a)^2}+\frac{}{(-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 4*a^2*b/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*((-1/2*b^2*tan(1/2*x)-1/2*a*b)/(a*tan
(1/2*x)^2-2*b*tan(1/2*x)-a)-1/2*(2*a^2-3*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(
2*a*tan(1/2*x)-2*b)/(a^2+b^2)^(1/2)))+4/(a^2+b^2)/(a^4+2*a^2*b^2+b^4)*((a^3
*b-a*b^3)*tan(1/2*x)^5+(3/2*a^2*b^2-1/2*b^4)*tan(1/2*x)^4+(10/3*a^3*b-2/3*a
*b^3)*tan(1/2*x)^3+(-a^4+3*a^2*b^2)*tan(1/2*x)^2+(a^3*b-a*b^3)*tan(1/2*x)-1
/3*a^4+3/2*a^2*b^2-1/6*b^4)/(1+tan(1/2*x)^2)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 611 vs. 2(164) = 328.
time = 0.50, size = 611, normalized size = 3.55

$$\frac{(2\,a^2b-3\,b^3)a^2\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{(a^6+3\,a^4b^2+3\,a^2b^4+b^6)\sqrt{a^2+b^2}}-\frac{2\left(2\,a^5-12\,a^3b^2+ab^4-\frac{(2\,a^4b+15\,a^2b^3-2\,b^5)\sin(x)}{\cos(x)+1}+\frac{(4\,a^5-30\,a^3b^2+11\,ab^4)\sin(x)^2}{(\cos(x)+1)^2}-\frac{(2\,a^4b+47\,a^2b^3)\sin(x)^3}{(\cos(x)+1)^3}-\frac{(6\,a^5+40\,a^3b^2-11\,ab^4)\sin(x)^4}{(\cos(x)+1)^4}+\frac{(14\,a^4b-25\,a^2b^3+6\,b^5)\sin(x)^5}{(\cos(x)+1)^5}-\frac{3\,(2\,a^3b^2-3\,ab^4)\sin(x)^6}{(\cos(x)+1)^6}+\frac{3\,(2\,a^4b-3\,a^2b^3)\sin(x)^7}{(\cos(x)+1)^7}\right)}{3\left(a^7+3\,a^5b^2+3\,a^3b^4+ab^6+\frac{2\,(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)\sin(x)}{\cos(x)+1}+\frac{2\,(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)\sin(x)^2}{(\cos(x)+1)^2}+\frac{6\,(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)\sin(x)^3}{(\cos(x)+1)^3}+\frac{6\,(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)\sin(x)^5}{(\cos(x)+1)^5}-\frac{2\,(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)\sin(x)^6}{(\cos(x)+1)^6}+\frac{2\,(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)\sin(x)^7}{(\cos(x)+1)^7}-\frac{(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)\sin(x)^8}{(\cos(x)+1)^8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] (2*a^2*b - 3*b^3)*a^2*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b
- a*sin(x)/(cos(x) + 1) - sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
 b^6)*sqrt(a^2 + b^2)) - 2/3*(2*a^5 - 12*a^3*b^2 + a*b^4 - (2*a^4*b + 15*a^
2*b^3 - 2*b^5)*sin(x)/(cos(x) + 1) + (4*a^5 - 30*a^3*b^2 + 11*a*b^4)*sin(x)
^2/(cos(x) + 1)^2 - (2*a^4*b + 47*a^2*b^3)*sin(x)^3/(cos(x) + 1)^3 - (6*a^5
 + 40*a^3*b^2 - 11*a*b^4)*sin(x)^4/(cos(x) + 1)^4 + (14*a^4*b - 25*a^2*b^3
+ 6*b^5)*sin(x)^5/(cos(x) + 1)^5 - 3*(2*a^3*b^2 - 3*a*b^4)*sin(x)^6/(cos(x)
 + 1)^6 + 3*(2*a^4*b - 3*a^2*b^3)*sin(x)^7/(cos(x) + 1)^7)/(a^7 + 3*a^5*b^2
 + 3*a^3*b^4 + a*b^6 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x)/(cos(
x) + 1) + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*sin(x)^2/(cos(x) + 1)^2 +
 6*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x)^3/(cos(x) + 1)^3 + 6*(a^6*b
 + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x)^5/(cos(x) + 1)^5 - 2*(a^7 + 3*a^5*b^
2 + 3*a^3*b^4 + a*b^6)*sin(x)^6/(cos(x) + 1)^6 + 2*(a^6*b + 3*a^4*b^3 + 3*a
^2*b^5 + b^7)*sin(x)^7/(cos(x) + 1)^7 - (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^
6)*sin(x)^8/(cos(x) + 1)^8)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(164) = 328$.
time = 1.68, size = 360, normalized size = 2.09

$$\frac{22\,a^5b^2+14\,a^3b^4-8\,ab^6+2\,(a^7+3\,a^5b^2+3\,a^3b^4+ab^6)\cos(x)^4-2\,(3\,a^7+4\,a^5b^2-a^3b^4-2\,ab^6)\cos(x)^2-3\,\sqrt{a^2+b^2}\,((2\,a^5b-3\,a^3b^3)\cos(x)+(2\,a^4b^2-3\,a^2b^4)\sin(x))\log\left(-\frac{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2-2\,a^2-b^2+2\,\sqrt{a^2+b^2}\,(b\cos(x)-a\sin(x))}{2\,ab\cos(x)\sin(x)+(a^2-b^2)\cos(x)^2+b^2}\right)-2\,((a^6b+3\,a^4b^3+3\,a^2b^5+b^7)\cos(x)^3-5\,(a^6b+2\,a^4b^3+a^2b^5)\cos(x))\sin(x)}{6\,((a^9+4\,a^7b^2+6\,a^5b^4+4\,a^3b^6+ab^8)\cos(x)+(a^8b+4\,a^6b^3+6\,a^4b^5+4\,a^2b^7+b^9)\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(22*a^5*b^2 + 14*a^3*b^4 - 8*a*b^6 + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a
*b^6)*cos(x)^4 - 2*(3*a^7 + 4*a^5*b^2 - a^3*b^4 - 2*a*b^6)*cos(x)^2 - 3*sqr
t(a^2 + b^2)*((2*a^5*b - 3*a^3*b^3)*cos(x) + (2*a^4*b^2 - 3*a^2*b^4)*sin(x)
)*log(-(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2 + 2*sqrt(a
^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^
2 + b^2)) - 2*((a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(x)^3 - 5*(a^6*b +
2*a^4*b^3 + a^2*b^5)*cos(x))*sin(x))/((a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a^3*
b^6 + a*b^8)*cos(x) + (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*sin
(x))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2*sin(x)**3/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 342 vs. 2(164) = 328.
time = 0.45, size = 342, normalized size = 1.99

$$\frac{\left(2 a^{4} b-3 a^{2} b^{3}\right) \log \left(\frac{\left|2 a \tan \left(\frac{1}{2} x\right)-2 b-2 \sqrt{a^{2}+b^{2}}\right|}{\left|2 a \tan \left(\frac{1}{2} x\right)-2 b+2 \sqrt{a^{2}+b^{2}}\right|}\right)}{\left(a^{6}+3 a^{4} b^{2}+3 a^{2} b^{4}+b^{6}\right) \sqrt{a^{2}+b^{2}}}-\frac{2\left(a^{2} b^{3} \tan \left(\frac{1}{2} x\right)+a^{3} b^{2}\right)}{\left(a^{6}+3 a^{4} b^{2}+3 a^{2} b^{4}+b^{6}\right)\left(a \tan \left(\frac{1}{2} x\right)^{2}-2 b \tan \left(\frac{1}{2} x\right)-a\right)}+\frac{2\left(6 a^{3} b \tan \left(\frac{1}{2} x\right)^{5}-6 a b^{3} \tan \left(\frac{1}{2} x\right)^{5}+9 a^{2} b^{2} \tan \left(\frac{1}{2} x\right)^{4}-3 b^{4} \tan \left(\frac{1}{2} x\right)^{4}+20 a^{3} b \tan \left(\frac{1}{2} x\right)^{3}-4 a b^{3} \tan \left(\frac{1}{2} x\right)^{3}-6 a^{4} \tan \left(\frac{1}{2} x\right)^{2}+18 a^{2} b^{2} \tan \left(\frac{1}{2} x\right)^{2}+6 a^{3} b \tan \left(\frac{1}{2} x\right)-6 a b^{3} \tan \left(\frac{1}{2} x\right)-2 a^{4}+9 a^{2} b^{2}-b^{4}\right)}{3\left(a^{6}+3 a^{4} b^{2}+3 a^{2} b^{4}+b^{6}\right)\left(\tan \left(\frac{1}{2} x\right)^{2}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] (2*a^4*b - 3*a^2*b^3)*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/abs
(2*a*tan(1/2*x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 +
 b^6)*sqrt(a^2 + b^2)) - 2*(a^2*b^3*tan(1/2*x) + a^3*b^2)/((a^6 + 3*a^4*b^2
 + 3*a^2*b^4 + b^6)*(a*tan(1/2*x)^2 - 2*b*tan(1/2*x) - a)) + 2/3*(6*a^3*b*t
an(1/2*x)^5 - 6*a*b^3*tan(1/2*x)^5 + 9*a^2*b^2*tan(1/2*x)^4 - 3*b^4*tan(1/2
*x)^4 + 20*a^3*b*tan(1/2*x)^3 - 4*a*b^3*tan(1/2*x)^3 - 6*a^4*tan(1/2*x)^2 +
 18*a^2*b^2*tan(1/2*x)^2 + 6*a^3*b*tan(1/2*x) - 6*a*b^3*tan(1/2*x) - 2*a^4
+ 9*a^2*b^2 - b^4)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(tan(1/2*x)^2 + 1)^
3)
```

Mupad [B]
time = 2.92, size = 594, normalized size = 3.45

$$-\frac{\frac{2 \tan \left(\frac{x}{2}\right)^{6}\left(3 a b^{4}-2 a^{3} b^{2}\right)}{a^{6}+3 a^{4} b^{2}+3 a^{2} b^{4}+b^{6}}-\frac{2 \tan \left(\frac{x}{2}\right)^{4}\left(6 a^{5}+40 a^{3} b^{2}-11 a b^{4}\right)}{3\left(a^{6}+3 a^{4} b^{2}+3 a^{2} b^{4}+b^{6}\right)}-\frac{2 \tan \left(\frac{x}{2}\right)^{3}\left(2 a^{4} b+47 a^{2} b^{3}\right)}{3\left(a^{2}+b^{2}\right)\left(a^{4}+2 a^{2} b^{2}+b^{4}\right)}+\frac{2 a\left(2 a^{4}-12 a^{2} b^{2}+b^{4}\right)}{3\left(a^{2}+b^{2}\right)\left(a^{4}+2 a^{2} b^{2}+b^{4}\right)}+\frac{2 a \tan \left(\frac{x}{2}\right)^{2}\left(4 a^{4}-30 a^{2} b^{2}+11 b^{4}\right)}{3\left(a^{2}+b^{2}\right)\left(a^{4}+2 a^{2} b^{2}+b^{4}\right)}+\frac{2 b \tan \left(\frac{x}{2}\right)^{5}\left(14 a^{4}-25 a^{2} b^{2}+6 b^{4}\right)}{3\left(a^{2}+b^{2}\right)\left(a^{4}+2 a^{2} b^{2}+b^{4}\right)}+\frac{2 a^{2} b \tan \left(\frac{x}{2}\right)^{7}\left(2 a^{2}-3 b^{2}\right)}{a^{6}+3 a^{4} b^{2}+3 a^{2} b^{4}+b^{6}}-\frac{2 b \tan \left(\frac{x}{2}\right)\left(2 a^{4}+15 a^{2} b^{2}-2 b^{4}\right)}{3\left(a^{2}+b^{2}\right)\left(a^{4}+2 a^{2} b^{2}+b^{4}\right)}}{-a \tan \left(\frac{x}{2}\right)^{8}+2 b \tan \left(\frac{x}{2}\right)^{7}-2 a \tan \left(\frac{x}{2}\right)^{6}+6 b \tan \left(\frac{x}{2}\right)^{5}+6 b \tan \left(\frac{x}{2}\right)^{3}+2 a \tan \left(\frac{x}{2}\right)^{2}+2 b \tan \left(\frac{x}{2}\right)+a}+\frac{a^{2} b \operatorname{atan}\left(\frac{1 \mathrm{i} \tan \left(\frac{x}{2}\right) a^{7}-a^{6} b\, 1 \mathrm{i}+3 \mathrm{i} \tan \left(\frac{x}{2}\right) a^{5} b^{2}-a^{4} b^{3}\, 3 \mathrm{i}+3 \mathrm{i} \tan \left(\frac{x}{2}\right) a^{3} b^{4}-a^{2} b^{5}\, 3 \mathrm{i}+1 \mathrm{i} \tan \left(\frac{x}{2}\right) a b^{6}-b^{7}\, 1 \mathrm{i}}{\left(a^{2}+b^{2}\right)^{7/2}}\right)\left(2 a^{2}-3 b^{2}\right) 2 \mathrm{i}}{\left(a^{2}+b^{2}\right)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^2*sin(x)^3)/(a*cos(x) + b*sin(x))^2,x)
```

```
[Out] (a^2*b*atan((a^7*tan(x/2)*1i - a^6*b*1i - b^7*1i - a^2*b^5*3i - a^4*b^3*3i
+ a^3*b^4*tan(x/2)*3i + a^5*b^2*tan(x/2)*3i + a*b^6*tan(x/2)*1i)/(a^2 + b^2
)^(7/2))*(2*a^2 - 3*b^2)*2i)/(a^2 + b^2)^(7/2) - ((2*tan(x/2)^6*(3*a*b^4 -
2*a^3*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (2*tan(x/2)^4*(6*a^5 - 11
*a*b^4 + 40*a^3*b^2))/(3*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) - (2*tan(x/2)
^3*(2*a^4*b + 47*a^2*b^3))/(3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) + (2*a*(
2*a^4 + b^4 - 12*a^2*b^2))/(3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) + (2*a*t
an(x/2)^2*(4*a^4 + 11*b^4 - 30*a^2*b^2))/(3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*
b^2)) + (2*b*tan(x/2)^5*(14*a^4 + 6*b^4 - 25*a^2*b^2))/(3*(a^2 + b^2)*(a^4
+ b^4 + 2*a^2*b^2)) + (2*a^2*b*tan(x/2)^7*(2*a^2 - 3*b^2))/(a^6 + b^6 + 3*a
^2*b^4 + 3*a^4*b^2) - (2*b*tan(x/2)*(2*a^4 - 2*b^4 + 15*a^2*b^2))/(3*(a^2 +
 b^2)*(a^4 + b^4 + 2*a^2*b^2)))/(a + 2*b*tan(x/2) + 2*a*tan(x/2)^2 - 2*a*ta
n(x/2)^6 - a*tan(x/2)^8 + 6*b*tan(x/2)^3 + 6*b*tan(x/2)^5 + 2*b*tan(x/2)^7)
```

3.290 $\int \frac{\cos^3(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=128

$$-\frac{ab(a^2-3b^2)\,x}{(a^2+b^2)^3}-\frac{b^2(3a^2-b^2)\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^3}+\frac{ab\cos(x)\sin(x)}{(a^2+b^2)^2}+\frac{(a^2-b^2)\sin^2(x)}{2\,(a^2+b^2)^2}+\frac{ab^2\,\mathrm{co}}{(a^2+b^2)^2\,(a\cos}$$

```
[Out] -a*b*(a^2-3*b^2)*x/(a^2+b^2)^3-b^2*(3*a^2-b^2)*ln(a*cos(x)+b*sin(x))/(a^2+b
^2)^3+a*b*cos(x)*sin(x)/(a^2+b^2)^2+1/2*(a^2-b^2)*sin(x)^2/(a^2+b^2)^2+a*b^
2*cos(x)/(a^2+b^2)^2/(a*cos(x)+b*sin(x))
```

Rubi [A]
time = 0.30, antiderivative size = 196, normalized size of antiderivative = 1.53, number of steps used = 17, number of rules used = 13, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$,
Rules used = {3190, 3179, 2715, 8, 3177, 3212, 3188, 2644, 30, 3165, 3564, 3612, 3611}

$$\frac{abx}{(a^2+b^2)^2}-\frac{abx(a^2-b^2)}{(a^2+b^2)^3}+\frac{a^2\sin^2(x)}{2\,(a^2+b^2)^2}+\frac{b^2\cos^2(x)}{2\,(a^2+b^2)^2}+\frac{ab^2}{(a^2+b^2)^2\,(a+b\tan(x))}+\frac{ab\sin(x)\cos(x)}{(a^2+b^2)^2}-\frac{3a^2b^2\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^3}+\frac{b^4\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^3}+\frac{ab^3x}{(a^2+b^2)^3}-\frac{a^3bx}{(a^2+b^2)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^3*Sin[x])/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] -((a^3*b*x)/(a^2 + b^2)^3) + (a*b^3*x)/(a^2 + b^2)^3 - (a*b*(a^2 - b^2)*x)/
(a^2 + b^2)^3 + (a*b*x)/(a^2 + b^2)^2 + (b^2*Cos[x]^2)/(2*(a^2 + b^2)^2) -
(3*a^2*b^2*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^3 + (b^4*Log[a*Cos[x] + b*
Sin[x]])/(a^2 + b^2)^3 + (a*b*Cos[x]*Sin[x])/(a^2 + b^2)^2 + (a^2*Sin[x]^2)
/(2*(a^2 + b^2)^2) + (a*b^2)/((a^2 + b^2)^2*(a + b*Tan[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
```

```
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3165

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)*(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Int[(a + b*Tan[c + d*x])^n, x] /;
 FreeQ[{a, b, c, d}, x] && EqQ[m + n, 0] && IntegerQ[n] && NeQ[a^2 + b^2, 0
]
```

Rule 3177

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3179

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3190

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[a*(b/(a^2 + b^2)), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
```

```
b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rule 3564

```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((a +
b*Tan[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 + b^2))), x] + Dist[1/(a^2 + b^2),
Int[(a - b*Tan[c + d*x])*(a + b*Tan[c + d*x])^(n + 1), x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 + b^2, 0] && LtQ[n, -1]
```

Rule 3611

```
Int[((c_) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_) + (b_.)*tan[(e_.) + (f_.)*
(x_)]), x_Symbol] :> Simp[(c/(b*f))*Log[RemoveContent[a*Cos[e + f*x] + b*Si
n[e + f*x], x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &&
 NeQ[a^2 + b^2, 0] && EqQ[a*c + b*d, 0]
```

Rule 3612

```
Int[((c_.) + (d_.)*tan[(e_.) + (f_.)*(x_)])/((a_.) + (b_.)*tan[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(a*c + b*d)*(x/(a^2 + b^2)), x] + Dist[(b*c - a
*d)/(a^2 + b^2), Int[(b - a*Tan[e + f*x])/(a + b*Tan[e + f*x]), x], x] /; F
reeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 + b^2, 0] && Ne
Q[a*c + b*d, 0]
```

Rubi steps

$$\int \frac{\cos^3(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx = \frac{a\int \frac{\cos^2(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} + \frac{b\int \frac{\cos^3(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2} - \frac{(ab)\int \frac{\cos^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2+b^2}$$

$$= \frac{b^2\cos^2(x)}{2\left(a^2+b^2\right)^2} + \frac{a^2\int \cos(x)\sin(x)\,dx}{\left(a^2+b^2\right)^2} + 2\frac{(ab)\int \cos^2(x)\,dx}{\left(a^2+b^2\right)^2} - \frac{\left(a^2b\right)\int \frac{\cos(x)}{a\cos(x)+b\,\mathrm{s}}}{\left(a^2+b^2\right)^2}$$

$$= -\frac{a^3bx}{\left(a^2+b^2\right)^3} + \frac{ab^3x}{\left(a^2+b^2\right)^3} + \frac{b^2\cos^2(x)}{2\left(a^2+b^2\right)^2} + \frac{ab^2}{\left(a^2+b^2\right)^2(a+b\tan(x))} - \frac{\left(a^2b^2\right)}{}$$

$$= -\frac{a^3bx}{\left(a^2+b^2\right)^3} + \frac{ab^3x}{\left(a^2+b^2\right)^3} - \frac{ab(a^2-b^2)\,x}{\left(a^2+b^2\right)^3} + \frac{b^2\cos^2(x)}{2\left(a^2+b^2\right)^2} - \frac{a^2b^2\log(a\cos(x)}{(a^2+b^2}$$

$$= -\frac{a^3bx}{\left(a^2+b^2\right)^3} + \frac{ab^3x}{\left(a^2+b^2\right)^3} - \frac{ab(a^2-b^2)\,x}{\left(a^2+b^2\right)^3} + \frac{b^2\cos^2(x)}{2\left(a^2+b^2\right)^2} - \frac{3a^2b^2\log(a\cos(x)}{(a^2+b^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.38, size = 221, normalized size = 1.73

$$\frac{-4ib^2(-3a^2+b^2)\,\mathrm{ArcTan}(\tan(x))(a\cos(x)+b\sin(x)) - a\cos(x)\left((a^4-b^4)\cos(2x)+2b(2(a+ib)^3x-b(-3a^2+b^2)\log\left((a\cos(x)+b\sin(x))^2\right)-a(a^2+b^2)\sin(2x))\right)+b\sin(x)\left((-a^4+b^4)\cos(2x)+2b(-2(a+ib)\left(a^2x-b^2(i+x)+a(b+2ibx)\right)+(-3a^2b+b^3)\log\left((a\cos(x)+b\sin(x))^2\right)+a(a^2+b^2)\sin(2x))\right)}{4\left(a^2+b^2\right)^3(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^3*Sin[x])/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] ((-4*I)*b^2*(-3*a^2 + b^2)*ArcTan[Tan[x]]*(a*Cos[x] + b*Sin[x]) - a*Cos[x]*
((a^4 - b^4)*Cos[2*x] + 2*b*(2*(a + I*b)^3*x - b*(-3*a^2 + b^2)*Log[(a*Cos[
x] + b*Sin[x])^2] - a*(a^2 + b^2)*Sin[2*x])) + b*Sin[x]*((-a^4 + b^4)*Cos[2
*x] + 2*b*(-2*(a + I*b)*(a^2*x - b^2*(I + x) + a*(b + (2*I)*b*x)) + (-3*a^2
*b + b^3)*Log[(a*Cos[x] + b*Sin[x])^2] + a*(a^2 + b^2)*Sin[2*x])))/(4*(a^2
+ b^2)^3*(a*Cos[x] + b*Sin[x]))
```

Maple [A]
time = 0.41, size = 141, normalized size = 1.10

method	result
default	$\frac{a\,b^2}{(a^2+b^2)^2(a+b\tan(x))} - \frac{b^2\left(3a^2-b^2\right)\ln(a+b\tan(x))}{(a^2+b^2)^3} + \frac{\frac{\left(a^3b+a\,b^3\right)\tan(x)-\frac{a^4}{2}+\frac{b^4}{2}}{\tan^2(x)+1}+b\left(\frac{\left(3a^2b-b^3\right)\ln\left(\tan^2(x)+1\right)}{2}+\left(-a^3+3a\,b^2\right)\mathrm{ar}\right.}{(a^2+b^2)^3}$
risch	$-\frac{ibx}{ia^3-3ia\,b^2+3a^2b-b^3} - \frac{\mathrm{e}^{2ix}}{8(-2iab+a^2-b^2)} - \frac{\mathrm{e}^{-2ix}}{8(2iab+a^2-b^2)} + \frac{6ia^2x\,b^2}{a^6+3a^4b^2+3a^2b^4+b^6} - \frac{2ib^4x}{a^6+3a^4b^2+3a^2b^4+b^6} + \frac{}{(-ia+b)}$
norman	$\frac{\frac{2a\left(\tan^8\left(\frac{x}{2}\right)\right)}{a^2+b^2}+\frac{b\,a^2\left(a^2-3b^2\right)x}{a^6+3a^4b^2+3a^2b^4+b^6}-\frac{2a\left(\tan^4\left(\frac{x}{2}\right)\right)}{a^2+b^2}+\frac{2a\left(\tan^6\left(\frac{x}{2}\right)\right)}{a^2+b^2}-\frac{2a\left(\tan^2\left(\frac{x}{2}\right)\right)}{a^2+b^2}-\frac{2b\left(a^3-a\,b^2\right)\tan\left(\frac{x}{2}\right)}{a\left(a^4+2a^2b^2+b^4\right)}-\frac{2b\left(a^3-a\,b^2\right)\left(\tan^9\left(\frac{x}{2}\right)\right)}{a\left(a^4+2a^2b^2+b^4\right)}-\frac{4(}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a*b^2/(a^2+b^2)^2/(a+b*tan(x))-b^2*(3*a^2-b^2)/(a^2+b^2)^3*ln(a+b*tan(x))+1
/(a^2+b^2)^3*(((a^3*b+a*b^3)*tan(x)-1/2*a^4+1/2*b^4)/(tan(x)^2+1)+b*(1/2*(3
*a^2*b-b^3)*ln(tan(x)^2+1)+(-a^3+3*a*b^2)*arctan(tan(x))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(126) = 252$.
time = 0.49, size = 256, normalized size = 2.00

$$-\frac{(a^3b-3\,ab^3)x}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}-\frac{(3\,a^2b^2-b^4)\log\left(b\tan\left(x\right)+a\right)}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{(3\,a^2b^2-b^4)\log\left(\tan\left(x\right)^2+1\right)}{2\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)}+\frac{4\,ab^2\tan\left(x\right)^2-a^3+3\,ab^2+(a^2b+b^3)\tan\left(x\right)}{2\,(a^5+2\,a^3b^2+ab^4+(a^4b+2\,a^2b^3+b^5)\tan\left(x\right)^3+(a^5+2\,a^3b^2+ab^4)\tan\left(x\right)^2+(a^4b+2\,a^2b^3+b^5)\tan\left(x\right))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -(a^3*b - 3*a*b^3)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (3*a^2*b^2 - b^4
)*log(b*tan(x) + a)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(3*a^2*b^2 -
b^4)*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(4*a*b^2*t
an(x)^2 - a^3 + 3*a*b^2 + (a^2*b + b^3)*tan(x))/(a^5 + 2*a^3*b^2 + a*b^4 +
(a^4*b + 2*a^2*b^3 + b^5)*tan(x)^3 + (a^5 + 2*a^3*b^2 + a*b^4)*tan(x)^2 + (
a^4*b + 2*a^2*b^3 + b^5)*tan(x))
```

Fricas [A]
time = 2.01, size = 252, normalized size = 1.97

$$-\frac{2\,(a^5+2\,a^3b^2+ab^4)\cos\left(x\right)^3-(a^5+4\,a^3b^2+7\,ab^4-4\,(a^4b-3\,a^2b^3)x)\cos\left(x\right)+2\,((3\,a^3b^2-ab^4)\cos\left(x\right)+(3\,a^2b^3-b^5)\sin\left(x\right))\log\left(2\,ab\cos\left(x\right)\sin\left(x\right)+(a^2-b^2)\cos\left(x\right)^2+b^2\right)-(a^4b-4\,a^2b^3-b^5+2\,(a^4b+2\,a^2b^3+b^5)\cos\left(x\right)^2-4\,(a^3b^2-3\,ab^4)x)\sin\left(x\right)}{4\,((a^7+3\,a^5b^2+3\,a^3b^4+ab^6)\cos\left(x\right)+(a^6b+3\,a^4b^3+3\,a^2b^5+b^7)\sin\left(x\right))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(a^5 + 2*a^3*b^2 + a*b^4)*cos(x)^3 - (a^5 + 4*a^3*b^2 + 7*a*b^4 - 4
*(a^4*b - 3*a^2*b^3)*x)*cos(x) + 2*((3*a^3*b^2 - a*b^4)*cos(x) + (3*a^2*b^3
 - b^5)*sin(x))*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b^2) - (a^
4*b - 4*a^2*b^3 - b^5 + 2*(a^4*b + 2*a^2*b^3 + b^5)*cos(x)^2 - 4*(a^3*b^2 -
 3*a*b^4)*x)*sin(x))/((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(x) + (a^6*b
 + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.44, size = 214, normalized size = 1.67

$$-\frac{(a^3b-3\,ab^3)x}{a^6+3\,a^4b^2+3\,a^2b^4+b^6}+\frac{(3\,a^2b^2-b^4)\log\left(\tan\left(x\right)^2+1\right)}{2\,(a^6+3\,a^4b^2+3\,a^2b^4+b^6)}-\frac{(3\,a^2b^3-b^5)\log\left(|b\tan\left(x\right)+a|\right)}{a^6b+3\,a^4b^3+3\,a^2b^5+b^7}+\frac{4\,ab^2\tan\left(x\right)^2+a^2b\tan\left(x\right)+b^3\tan\left(x\right)-a^3+3\,ab^2}{2\,(a^4+2\,a^2b^2+b^4)\left(b\tan\left(x\right)^3+a\tan\left(x\right)^2+b\tan\left(x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -(a^3*b - 3*a*b^3)*x/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) + 1/2*(3*a^2*b^2 -
 b^4)*log(tan(x)^2 + 1)/(a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6) - (3*a^2*b^3 -
b^5)*log(abs(b*tan(x) + a))/(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7) + 1/2*(4*
a*b^2*tan(x)^2 + a^2*b*tan(x) + b^3*tan(x) - a^3 + 3*a*b^2)/((a^4 + 2*a^2*b
^2 + b^4)*(b*tan(x)^3 + a*tan(x)^2 + b*tan(x) + a))
```

Mupad [B]
time = 8.16, size = 2500, normalized size = 19.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^3*sin(x))/(a*cos(x) + b*sin(x))^2,x)
```

```
[Out] ((2*a*tan(x/2)^2)/(a^2 + b^2) - (8*b^3*tan(x/2)^3)/(a^2 + b^2)^2 - (2*a*tan
(x/2)^4)/(a^2 + b^2) + (2*b*tan(x/2)*(a^2 - b^2))/(a^2 + b^2)^2 + (2*b*tan(
x/2)^5*(a^2 - b^2))/(a^4 + b^4 + 2*a^2*b^2))/(a + 2*b*tan(x/2) + a*tan(x/2)
^2 - a*tan(x/2)^4 - a*tan(x/2)^6 + 4*b*tan(x/2)^3 + 2*b*tan(x/2)^5) + (log(
a + 2*b*tan(x/2) - a*tan(x/2)^2)*(b^4 - 3*a^2*b^2))/(a^6 + b^6 + 3*a^2*b^4
+ 3*a^4*b^2) - (log(1/(cos(x) + 1))*(2*b^4 - 6*a^2*b^2))/(2*(a^6 + b^6 + 3*
a^2*b^4 + 3*a^4*b^2)) - (2*a*b*atan((tan(x/2)*((((((a*b*((32*(a*b^14 + 9*a^
3*b^12 + 18*a^5*b^10 + 2*a^7*b^8 - 27*a^9*b^6 - 27*a^11*b^4 - 8*a^13*b^2))/
(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b
^2) - (16*(2*b^4 - 6*a^2*b^2)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a
^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^
6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*
b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^2 - 3*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*
a^4*b^2) - (16*a*b*(a^2 - 3*b^2)*(2*b^4 - 6*a^2*b^2)*(3*a*b^16 + 21*a^3*b^1
4 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 +
3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^2*(a^12 + b^12 + 6*a^2*b^
10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(2*b^4 - 6*a^2*b^
2))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (a*b*(a^2 - 3*b^2)*((32*(3*a*
b^12 - 21*a^3*b^10 - 34*a^5*b^8 + 6*a^7*b^6 + 15*a^9*b^4 - a^11*b^2))/(a^12
 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) +
```

(((32*(a*b^14 + 9*a^3*b^12 + 18*a^5*b^10 + 2*a^7*b^8 - 27*a^9*b^6 - 27*a^1
1*b^4 - 8*a^13*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 +
15*a^8*b^4 + 6*a^10*b^2) - (16*(2*b^4 - 6*a^2*b^2)*(3*a*b^16 + 21*a^3*b^14
+ 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*
a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 +
15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(2*b^4 - 6*a^2*b^2))/
(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^
2) + (32*a^3*b^3*(a^2 - 3*b^2)^3*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 10
5*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 +
b^6 + 3*a^2*b^4 + 3*a^4*b^2)^3*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20
*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(a^8 + 4*b^8 - 61*a^2*b^6 + 155*a^4*b
^4 - 67*a^6*b^2))/(a^8 + 4*b^8 - 11*a^2*b^6 + 15*a^4*b^4 + 31*a^6*b^2)^2 +
(2*a*b*(7*a^6 - 10*b^6 + 59*a^2*b^4 - 68*a^4*b^2)*((32*(a*b^10 + 3*a^3*b^8
- 17*a^5*b^6 - 3*a^7*b^4))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*
b^6 + 15*a^8*b^4 + 6*a^10*b^2) - ((2*b^4 - 6*a^2*b^2)*((32*(3*a*b^12 - 21*a
^3*b^10 - 34*a^5*b^8 + 6*a^7*b^6 + 15*a^9*b^4 - a^11*b^2))/(a^12 + b^12 + 6
*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) + (((32*(a*b
^14 + 9*a^3*b^12 + 18*a^5*b^10 + 2*a^7*b^8 - 27*a^9*b^6 - 27*a^11*b^4 - 8*a
^13*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4
+ 6*a^10*b^2) - (16*(2*b^4 - 6*a^2*b^2)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^
12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/
((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8
+ 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*(2*b^4 - 6*a^2*b^2))/(2*(a^6 + b
^6 + 3*a^2*b^4 + 3*a^4*b^2))))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (a
b(a^2 - 3*b^2)*((a*b*((32*(a*b^14 + 9*a^3*b^12 + 18*a^5*b^10 + 2*a^7*b^8
- 27*a^9*b^6 - 27*a^11*b^4 - 8*a^13*b^2))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^
4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (16*(2*b^4 - 6*a^2*b^2)*(3*
a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b
^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12
+ b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2)))*
(a^2 - 3*b^2))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (16*a*b*(a^2 - 3*b^2)*
(2*b^4 - 6*a^2*b^2)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12 + 105*a^7*b^10 +
105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a^6 + b^6 + 3*a^2*
b^4 + 3*a^4*b^2)^2*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15
*a^8*b^4 + 6*a^10*b^2))))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) - (16*a^2*b^2
*(a^2 - 3*b^2)^2*(2*b^4 - 6*a^2*b^2)*(3*a*b^16 + 21*a^3*b^14 + 63*a^5*b^12
+ 105*a^7*b^10 + 105*a^9*b^8 + 63*a^11*b^6 + 21*a^13*b^4 + 3*a^15*b^2))/((a
^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)^3*(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b^8
+ 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2))))/(a^8 + 4*b^8 - 11*a^2*b^6 + 15*a
^4*b^4 + 31*a^6*b^2)^2)*(a^16 + b^16 + 8*a^2*b^14 + 28*a^4*b^12 + 56*a^6*b^
10 + 70*a^8*b^8 + 56*a^10*b^6 + 28*a^12*b^4 + 8*a^14*b^2))/(96*a^2*b^5 - 32
*a^4*b^3) + (((((a*b*((32*(3*a^6*b^9 - 4*a^2*b^13 - 9*a^4*b^11 - a^14*b + 2
2*a^8*b^7 + 18*a^10*b^5 + 3*a^12*b^3))/(a^12 + b^12 + 6*a^2*b^10 + 15*a^4*b
^8 + 20*a^6*b^6 + 15*a^8*b^4 + 6*a^10*b^2) - (16*(2*b^4 - 6*a^2*b^2)*(3*a^1
6*b + 3*a^2*b^15 + 21*a^4*b^13 + 63*a^6*b^11 + 105*a^8*b^9 + 105*a^10*b^7 +

```
63*a^12*b^5 + 21*a^14*b^3))/((a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)*(a^12 + b
^12 + 6*a^2*b^10 + 15*a^4*b^8 + 20*a^6*b^6 + 15...
```

3.291 $\int \frac{\cos^3(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=176

$$-\frac{ab^2(3a^2-2b^2)\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}}+\frac{2ab(a^2-b^2)\cos(x)}{(a^2+b^2)^3}-\frac{2ab\cos^3(x)}{3(a^2+b^2)^2}-\frac{b^2(3a^2-b^2)\sin(x)}{(a^2+b^2)^3}+\frac{(a^2-b^2)}{3(a^2+}$$

```
[Out] -a*b^2*(3*a^2-2*b^2)*arctanh((b*cos(x)-a*sin(x))/(a^2+b^2)^(1/2))/(a^2+b^2)
^(7/2)+2*a*b*(a^2-b^2)*cos(x)/(a^2+b^2)^3-2/3*a*b*cos(x)^3/(a^2+b^2)^2-b^2*
(3*a^2-b^2)*sin(x)/(a^2+b^2)^3+1/3*(a^2-b^2)*sin(x)^3/(a^2+b^2)^2-a^2*b^3/(
a^2+b^2)^3/(a*cos(x)+b*sin(x))
```

Rubi [A]
time = 0.49, antiderivative size = 238, normalized size of antiderivative = 1.35, number of steps used = 33, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$,
Rules used = {3190, 3188, 2713, 2645, 30, 3179, 2717, 3153, 212, 2644, 2718, 3234}

$$-\frac{b^2\sin^3(x)}{3(a^2+b^2)^2}+\frac{a^2\sin^3(x)}{3(a^2+b^2)^2}+\frac{b^2\sin(x)}{(a^2+b^2)^2}-\frac{4a^2b^2\sin(x)}{(a^2+b^2)^3}-\frac{2ab\cos^3(x)}{3(a^2+b^2)^2}+\frac{2ab^4\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}}-\frac{2ab^3\cos(x)}{(a^2+b^2)^3}-\frac{a^2b^3}{(a^2+b^2)^3(a\cos(x)+b\sin(x))}+\frac{2a^3b\cos(x)}{(a^2+b^2)^3}-\frac{3a^3b^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^3*Sin[x]^2)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (-3*a^3*b^2*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(7/
2) + (2*a*b^4*ArcTanh[(b*Cos[x] - a*Sin[x])/Sqrt[a^2 + b^2]])/(a^2 + b^2)^(
7/2) + (2*a^3*b*Cos[x])/(a^2 + b^2)^3 - (2*a*b^3*Cos[x])/(a^2 + b^2)^3 - (2
*a*b*Cos[x]^3)/(3*(a^2 + b^2)^2) - (4*a^2*b^2*Sin[x])/(a^2 + b^2)^3 + (b^2*
Sin[x])/(a^2 + b^2)^2 + (a^2*Sin[x]^3)/(3*(a^2 + b^2)^2) - (b^2*Sin[x]^3)/(
3*(a^2 + b^2)^2) - (a^2*b^3)/((a^2 + b^2)^3*(a*Cos[x] + b*Sin[x]))
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
```

```
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3179

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_)/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin
[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(Cos[c + d*x]^(m - 1)/(d*(a^2 +
b^2)*(m - 1))), x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m - 1), x], x]
+ Dist[b^2/(a^2 + b^2), Int[Cos[c + d*x]^(m - 2)/(a*Cos[c + d*x] + b*Sin[c
+ d*x]), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] && GtQ[m, 1
]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
```

```
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3190

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[a*(b/(a^2 + b^2)), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3234

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.))/((a_.) + cos[(d_.) + (e_.)*(x_)
]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)])^2, x_Symbol] :> Simp[(c*B + c*A*Co
s[d + e*x] + (a*B - b*A)*Sin[d + e*x])/(e*(a^2 - b^2 - c^2)*(a + b*Cos[d +
e*x] + c*Sin[d + e*x])), x] + Dist[(a*A - b*B)/(a^2 - b^2 - c^2), Int[1/(a
+ b*Cos[d + e*x] + c*Sin[d + e*x]), x], x] /; FreeQ[{a, b, c, d, e, A, B},
x] && NeQ[a^2 - b^2 - c^2, 0] && NeQ[a*A - b*B, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx &= \frac{a\int \frac{\cos^2(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}+\frac{b\int \frac{\cos^3(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}-\frac{(ab)\int \frac{\cos^2(x)\sin(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2+b^2}\\ &= \frac{a^2\int \cos(x)\sin^2(x)\,dx}{(a^2+b^2)^2}+2\frac{(ab)\int \cos^2(x)\sin(x)\,dx}{(a^2+b^2)^2}-2\frac{(a^2b)\int \frac{\cos(x)\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{(a^2+b^2)^2}\\ &= -\frac{a^2b^3}{(a^2+b^2)^3(a\cos(x)+b\sin(x))}-2\left(\frac{(a^3b)\int \sin(x)\,dx}{(a^2+b^2)^3}+\frac{(a^2b^2)\int \cos(x)\,dx}{(a^2+b^2)^3}\right.\\ &= -\frac{2ab\cos^3(x)}{3(a^2+b^2)^2}+\frac{b^2\sin(x)}{(a^2+b^2)^2}+\frac{a^2\sin^3(x)}{3(a^2+b^2)^2}-\frac{b^2\sin^3(x)}{3(a^2+b^2)^2}-\frac{a^2b^3}{(a^2+b^2)^3(a\cos(x)}\\ &= -\frac{a^3b^2\tanh^{-1}\left(\frac{b\cos(x)-a\sin(x)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}}-\frac{2ab\cos^3(x)}{3(a^2+b^2)^2}+\frac{b^2\sin(x)}{(a^2+b^2)^2}+\frac{a^2\sin^3(x)}{3(a^2+b^2)^2}-\end{aligned}$$

Mathematica [A]
time = 1.34, size = 198, normalized size = 1.12

$$\frac{2ab^2(3a^2-2b^2)\tanh^{-1}\left(\frac{-b+a\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{(a^2+b^2)^{7/2}}-\frac{-21a^4b+90a^2b^3-9b^5-4b(3a^4+a^2b^2-2b^4)\cos(2x)+b(a^2+b^2)^2\cos(4x)-2a^5\sin(2x)+16a^3b^2\sin(2x)+18ab^4\sin(2x)+a^5\sin(4x)+2a^3b^2\sin(4x)+ab^4\sin(4x)}{24\left(a^2+b^2\right)^3(a\cos(x)+b\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^3*Sin[x]^2)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (2*a*b^2*(3*a^2 - 2*b^2)*ArcTanh[(-b + a*Tan[x/2])/Sqrt[a^2 + b^2]])/(a^2 +
 b^2)^(7/2) - (-21*a^4*b + 90*a^2*b^3 - 9*b^5 - 4*b*(3*a^4 + a^2*b^2 - 2*b^
4)*Cos[2*x] + b*(a^2 + b^2)^2*Cos[4*x] - 2*a^5*Sin[2*x] + 16*a^3*b^2*Sin[2*
x] + 18*a*b^4*Sin[2*x] + a^5*Sin[4*x] + 2*a^3*b^2*Sin[4*x] + a*b^4*Sin[4*x]
)/(24*(a^2 + b^2)^3*(a*Cos[x] + b*Sin[x]))
```

Maple [A]
time = 0.59, size = 265, normalized size = 1.51

method	result
default	$-\frac{2ab^2\left(\frac{-b^2\tan\left(\frac{x}{2}\right)-ab}{a\left(\tan^2\left(\frac{x}{2}\right)\right)-2b\tan\left(\frac{x}{2}\right)-a}-\frac{\left(3a^2-2b^2\right)\operatorname{arctanh}\left(\frac{2a\tan\left(\frac{x}{2}\right)-2b}{2\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}\right)}{(a^4+2a^2b^2+b^4)(a^2+b^2)}-\frac{2\left(\left(3a^2b^2-b^4\right)\left(\tan^5\left(\frac{x}{2}\right)\right)+4ab^3\left(\tan^4\left(\frac{x}{2}\right)\right)+\left(-\frac{4}{3}a^4\right.\right.}{}$
risch	$\frac{ie^{3ix}}{-48iab+24a^2-24b^2}+\frac{3\,e^{ix}b}{8(-3ia^2b+ib^3+a^3-3a\,b^2)}-\frac{ie^{ix}a}{8(-3ia^2b+ib^3+a^3-3a\,b^2)}+\frac{3\,e^{-ix}b}{8(ib+a)^3}+\frac{ie^{-ix}a}{8(ib+a)^3}-\frac{ie^{-3ix}}{24(ib+a)^2}-\frac{}{(ib}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*a*b^2/(a^4+2*a^2*b^2+b^4)/(a^2+b^2)*((-b^2*tan(1/2*x)-a*b)/(a*tan(1/2*x)
^2-2*b*tan(1/2*x)-a)-(3*a^2-2*b^2)/(a^2+b^2)^(1/2)*arctanh(1/2*(2*a*tan(1/2
*x)-2*b)/(a^2+b^2)^(1/2)))-2/(a^2+b^2)/(a^4+2*a^2*b^2+b^4)*((3*a^2*b^2-b^4)
*tan(1/2*x)^5+4*a*b^3*tan(1/2*x)^4+(-4/3*a^4+6*a^2*b^2-2/3*b^4)*tan(1/2*x)^
3+(-4*a^3*b+4*a*b^3)*tan(1/2*x)^2+(3*a^2*b^2-b^4)*tan(1/2*x)-4/3*a^3*b+8/3*
a*b^3)/(1+tan(1/2*x)^2)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 606 vs. 2(168) = 336.
time = 0.49, size = 606, normalized size = 3.44

$$-\frac{(3a^2b^2-2b^4)a\log\left(\frac{b-\frac{a\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{b-\frac{a\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{(a^6+3a^4b^2+3a^2b^4+b^6)\sqrt{a^2+b^2}}+\frac{2\left(4a^4b-11a^2b^3-\frac{(a^3b^2+16ab^4)\sin(x)}{\cos(x)+1}+\frac{(8a^4b-31a^2b^3+6b^5)\sin(x)^2}{(\cos(x)+1)^2}+\frac{(4a^5+15a^3b^2-34ab^4)\sin(x)^3}{(\cos(x)+1)^3}-\frac{(4a^4b+45a^2b^3-4b^5)\sin(x)^4}{(\cos(x)+1)^4}-\frac{(4a^5-9a^3b^2+32ab^4)\sin(x)^5}{(\cos(x)+1)^5}-\frac{3(3a^2b^3-2b^5)\sin(x)^6}{(\cos(x)+1)^6}+\frac{3(3a^3b^2-2ab^4)\sin(x)^7}{(\cos(x)+1)^7}\right)}{3\left(a^7+3a^5b^2+3a^3b^4+ab^6+\frac{2(a^6b+3a^4b^3+3a^2b^5+b^7)\sin(x)}{\cos(x)+1}+\frac{2(a^7+3a^5b^2+3a^3b^4+ab^6)\sin(x)^2}{(\cos(x)+1)^2}+\frac{6(a^6b+3a^4b^3+3a^2b^5+b^7)\sin(x)^3}{(\cos(x)+1)^3}+\frac{6(a^6b+3a^4b^3+3a^2b^5+b^7)\sin(x)^5}{(\cos(x)+1)^5}-\frac{2(a^7+3a^5b^2+3a^3b^4+ab^6)\sin(x)^6}{(\cos(x)+1)^6}+\frac{2(a^6b+3a^4b^3+3a^2b^5+b^7)\sin(x)^7}{(\cos(x)+1)^7}-\frac{(a^7+3a^5b^2+3a^3b^4+ab^6)\sin(x)^8}{(\cos(x)+1)^8}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -(3*a^2*b^2 - 2*b^4)*a*log((b - a*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(b
 - a*sin(x)/(cos(x) + 1) - sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4
+ b^6)*sqrt(a^2 + b^2)) + 2/3*(4*a^4*b - 11*a^2*b^3 - (a^3*b^2 + 16*a*b^4)*
sin(x)/(cos(x) + 1) + (8*a^4*b - 31*a^2*b^3 + 6*b^5)*sin(x)^2/(cos(x) + 1)^
2 + (4*a^5 + 15*a^3*b^2 - 34*a*b^4)*sin(x)^3/(cos(x) + 1)^3 - (4*a^4*b + 45
*a^2*b^3 - 4*b^5)*sin(x)^4/(cos(x) + 1)^4 - (4*a^5 - 9*a^3*b^2 + 32*a*b^4)*
sin(x)^5/(cos(x) + 1)^5 - 3*(3*a^2*b^3 - 2*b^5)*sin(x)^6/(cos(x) + 1)^6 + 3
*(3*a^3*b^2 - 2*a*b^4)*sin(x)^7/(cos(x) + 1)^7)/(a^7 + 3*a^5*b^2 + 3*a^3*b^
4 + a*b^6 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x)/(cos(x) + 1) + 2
*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*sin(x)^2/(cos(x) + 1)^2 + 6*(a^6*b +
 3*a^4*b^3 + 3*a^2*b^5 + b^7)*sin(x)^3/(cos(x) + 1)^3 + 6*(a^6*b + 3*a^4*b^
3 + 3*a^2*b^5 + b^7)*sin(x)^5/(cos(x) + 1)^5 - 2*(a^7 + 3*a^5*b^2 + 3*a^3*b
^4 + a*b^6)*sin(x)^6/(cos(x) + 1)^6 + 2*(a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^
7)*sin(x)^7/(cos(x) + 1)^7 - (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*sin(x)^8
/(cos(x) + 1)^8)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(168) = 336$.
time = 1.72, size = 369, normalized size = 2.10

$$\frac{2\,a^6b - 22\,a^4b^3 - 20\,a^2b^5 + 4\,b^7 - 2\,(a^6b + 3\,a^4b^3 + 3\,a^2b^5 + b^7)\cos(x)^4 + 2\,(4\,a^6b + 7\,a^4b^3 + 2\,a^2b^5 - b^7)\cos(x)^2 - 3\,\sqrt{a^2+b^2}\,((3\,a^4b^2 - 2\,a^2b^4)\cos(x) + (3\,a^3b^3 - 2\,ab^5)\sin(x))\log\left(\frac{2\,ab\cos(x)\sin(x) + (a^2-b^2)\cos(x)^2 - 2\,a^2 - b^2 - 2\,\sqrt{a^2+b^2}\,(b\cos(x) - a\sin(x))}{2\,ab\cos(x)\sin(x) + (a^2-b^2)\cos(x)^2 + b^2}\right) - 2\,((a^7 + 3\,a^5b^2 + 3\,a^3b^4 + ab^6)\cos(x)^3 - (a^7 - 2\,a^5b^2 - 7\,a^3b^4 - 4\,ab^6)\cos(x))\sin(x)}{6\,((a^9 + 4\,a^7b^2 + 6\,a^5b^4 + 4\,a^3b^6 + ab^8)\cos(x) + (a^8b + 4\,a^6b^3 + 6\,a^4b^5 + 4\,a^2b^7 + b^9)\sin(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^6*b - 22*a^4*b^3 - 20*a^2*b^5 + 4*b^7 - 2*(a^6*b + 3*a^4*b^3 + 3*a
^2*b^5 + b^7)*cos(x)^4 + 2*(4*a^6*b + 7*a^4*b^3 + 2*a^2*b^5 - b^7)*cos(x)^2
 - 3*sqrt(a^2 + b^2)*((3*a^4*b^2 - 2*a^2*b^4)*cos(x) + (3*a^3*b^3 - 2*a*b^5
)*sin(x))*log((2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 - 2*a^2 - b^2 - 2
*sqrt(a^2 + b^2)*(b*cos(x) - a*sin(x)))/(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*
cos(x)^2 + b^2)) - 2*((a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(x)^3 - (a^7
 - 2*a^5*b^2 - 7*a^3*b^4 - 4*a*b^6)*cos(x))*sin(x))/((a^9 + 4*a^7*b^2 + 6*a
^5*b^4 + 4*a^3*b^6 + a*b^8)*cos(x) + (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2
*b^7 + b^9)*sin(x))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)**2/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.46, size = 335, normalized size = 1.90

$$-\frac{\left(3\,a^{3}b^{2}-2\,ab^{4}\right)\log\left(\frac{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b-2\,\sqrt{a^{2}+b^{2}}\right|}{\left|2\,a\tan\left(\frac{1}{2}\,x\right)-2\,b+2\,\sqrt{a^{2}+b^{2}}\right|}\right)}{\left(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}\right)\sqrt{a^{2}+b^{2}}}+\frac{2\,\left(ab^{4}\tan\left(\frac{1}{2}\,x\right)+a^{2}b^{3}\right)}{\left(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}\right)\left(a\tan\left(\frac{1}{2}\,x\right)^{2}-2\,b\tan\left(\frac{1}{2}\,x\right)-a\right)}-\frac{2\,\left(9\,a^{2}b^{2}\tan\left(\frac{1}{2}\,x\right)^{5}-3\,b^{4}\tan\left(\frac{1}{2}\,x\right)^{5}+12\,ab^{3}\tan\left(\frac{1}{2}\,x\right)^{4}-4\,a^{4}\tan\left(\frac{1}{2}\,x\right)^{3}+18\,a^{2}b^{2}\tan\left(\frac{1}{2}\,x\right)^{3}-2\,b^{4}\tan\left(\frac{1}{2}\,x\right)^{3}-12\,a^{3}b\tan\left(\frac{1}{2}\,x\right)^{2}+12\,ab^{3}\tan\left(\frac{1}{2}\,x\right)^{2}+9\,a^{2}b^{2}\tan\left(\frac{1}{2}\,x\right)-3\,b^{4}\tan\left(\frac{1}{2}\,x\right)-4\,a^{3}b+8\,ab^{3}\right)}{3\,\left(a^{6}+3\,a^{4}b^{2}+3\,a^{2}b^{4}+b^{6}\right)\left(\tan\left(\frac{1}{2}\,x\right)^{2}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^2/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -(3*a^3*b^2 - 2*a*b^4)*log(abs(2*a*tan(1/2*x) - 2*b - 2*sqrt(a^2 + b^2))/ab
s(2*a*tan(1/2*x) - 2*b + 2*sqrt(a^2 + b^2)))/((a^6 + 3*a^4*b^2 + 3*a^2*b^4
+ b^6)*sqrt(a^2 + b^2)) + 2*(a*b^4*tan(1/2*x) + a^2*b^3)/((a^6 + 3*a^4*b^2
+ 3*a^2*b^4 + b^6)*(a*tan(1/2*x)^2 - 2*b*tan(1/2*x) - a)) - 2/3*(9*a^2*b^2*
tan(1/2*x)^5 - 3*b^4*tan(1/2*x)^5 + 12*a*b^3*tan(1/2*x)^4 - 4*a^4*tan(1/2*x
)^3 + 18*a^2*b^2*tan(1/2*x)^3 - 2*b^4*tan(1/2*x)^3 - 12*a^3*b*tan(1/2*x)^2
+ 12*a*b^3*tan(1/2*x)^2 + 9*a^2*b^2*tan(1/2*x) - 3*b^4*tan(1/2*x) - 4*a^3*b
 + 8*a*b^3)/((a^6 + 3*a^4*b^2 + 3*a^2*b^4 + b^6)*(tan(1/2*x)^2 + 1)^3)
```

Mupad [B]
time = 2.87, size = 586, normalized size = 3.33

$$-\frac{\frac{2\tan\left(\frac{x}{2}\right)^{4}\,\left(4\,a^{4}\,b+45\,a^{2}\,b^{3}-4\,b^{5}\right)}{3\,\left(a^{6}+3\,a^{4}\,b^{2}+3\,a^{2}\,b^{4}+b^{6}\right)}-\frac{2\tan\left(\frac{x}{2}\right)^{6}\,\left(2\,b^{5}-3\,a^{2}\,b^{3}\right)}{a^{6}+3\,a^{4}\,b^{2}+3\,a^{2}\,b^{4}+b^{6}}+\frac{2\tan\left(\frac{x}{2}\right)^{5}\,\left(4\,a^{5}-9\,a^{3}\,b^{2}+32\,a\,b^{4}\right)}{3\,\left(a^{2}+b^{2}\right)\,\left(a^{4}+2\,a^{2}\,b^{2}+b^{4}\right)}-\frac{2\tan\left(\frac{x}{2}\right)^{3}\,\left(4\,a^{5}+15\,a^{3}\,b^{2}-34\,a\,b^{4}\right)}{3\,\left(a^{2}+b^{2}\right)\,\left(a^{4}+2\,a^{2}\,b^{2}+b^{4}\right)}-\frac{2\tan\left(\frac{x}{2}\right)^{2}\,\left(8\,a^{4}\,b-31\,a^{2}\,b^{3}+6\,b^{5}\right)}{3\,\left(a^{2}+b^{2}\right)\,\left(a^{4}+2\,a^{2}\,b^{2}+b^{4}\right)}+\frac{2\,a\,\left(11\,a\,b^{3}-4\,a^{3}\,b\right)}{3\,\left(a^{2}+b^{2}\right)\,\left(a^{4}+2\,a^{2}\,b^{2}+b^{4}\right)}+\frac{2\,b\tan\left(\frac{x}{2}\right)^{7}\,\left(2\,a\,b^{3}-3\,a^{3}\,b\right)}{a^{6}+3\,a^{4}\,b^{2}+3\,a^{2}\,b^{4}+b^{6}}+\frac{2\,b\tan\left(\frac{x}{2}\right)\,\left(a^{3}\,b+16\,a\,b^{3}\right)}{3\,\left(a^{2}+b^{2}\right)\,\left(a^{4}+2\,a^{2}\,b^{2}+b^{4}\right)}}{-a\tan\left(\frac{x}{2}\right)^{8}+2\,b\tan\left(\frac{x}{2}\right)^{7}-2\,a\tan\left(\frac{x}{2}\right)^{6}+6\,b\tan\left(\frac{x}{2}\right)^{5}+6\,b\tan\left(\frac{x}{2}\right)^{3}+2\,a\tan\left(\frac{x}{2}\right)^{2}+2\,b\tan\left(\frac{x}{2}\right)+a}-\frac{a\,b^{2}\,\mathrm{atan}\left(\frac{1\mathrm{i}\tan\left(\frac{x}{2}\right)\,a^{7}-a^{6}\,b\,1\mathrm{i}+3\mathrm{i}\tan\left(\frac{x}{2}\right)\,a^{5}\,b^{2}-a^{4}\,b^{3}\,3\mathrm{i}+3\mathrm{i}\tan\left(\frac{x}{2}\right)\,a^{3}\,b^{4}-a^{2}\,b^{5}\,3\mathrm{i}+1\mathrm{i}\tan\left(\frac{x}{2}\right)\,a\,b^{6}-b^{7}\,1\mathrm{i}}{{\left(a^{2}+b^{2}\right)}^{7/2}}\right)\,\left(3\,a^{2}-2\,b^{2}\right)\,2\mathrm{i}}{{\left(a^{2}+b^{2}\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^3*sin(x)^2)/(a*cos(x) + b*sin(x))^2,x)
```

```
[Out] - ((2*tan(x/2)^4*(4*a^4*b - 4*b^5 + 45*a^2*b^3))/(3*(a^6 + b^6 + 3*a^2*b^4
+ 3*a^4*b^2)) - (2*tan(x/2)^6*(2*b^5 - 3*a^2*b^3))/(a^6 + b^6 + 3*a^2*b^4 +
 3*a^4*b^2) + (2*tan(x/2)^5*(32*a*b^4 + 4*a^5 - 9*a^3*b^2))/(3*(a^2 + b^2)*
(a^4 + b^4 + 2*a^2*b^2)) - (2*tan(x/2)^3*(4*a^5 - 34*a*b^4 + 15*a^3*b^2))/(
3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) - (2*tan(x/2)^2*(8*a^4*b + 6*b^5 - 3
1*a^2*b^3))/(3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) + (2*a*(11*a*b^3 - 4*a^
3*b))/(3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) + (2*b*tan(x/2)^7*(2*a*b^3 -
3*a^3*b))/(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2) + (2*b*tan(x/2)*(16*a*b^3 + a
^3*b))/(3*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)))/(a + 2*b*tan(x/2) + 2*a*tan
(x/2)^2 - 2*a*tan(x/2)^6 - a*tan(x/2)^8 + 6*b*tan(x/2)^3 + 6*b*tan(x/2)^5 +
 2*b*tan(x/2)^7) - (a*b^2*atan((a^7*tan(x/2)*1i - a^6*b*1i - b^7*1i - a^2*b
^5*3i - a^4*b^3*3i + a^3*b^4*tan(x/2)*3i + a^5*b^2*tan(x/2)*3i + a*b^6*tan(
x/2)*1i)/(a^2 + b^2)^(7/2))*(3*a^2 - 2*b^2)*2i)/(a^2 + b^2)^(7/2)
```

3.292 $\int \frac{\cos^3(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx$

Optimal. Leaf size=210

$$-\frac{3ab(a^4-6a^2b^2+b^4)\,x}{4\,(a^2+b^2)^4}-\frac{b^2\cos^4(x)}{4\,(a^2+b^2)^2}-\frac{3a^2b^2(a^2-b^2)\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^4}+\frac{ab(5a^2-3b^2)\cos(x)\sin(x)}{4\,(a^2+b^2)^3}-$$

```
[Out] -3/4*a*b*(a^4-6*a^2*b^2+b^4)*x/(a^2+b^2)^4-1/4*b^2*cos(x)^4/(a^2+b^2)^2-3*a
^2*b^2*(a^2-b^2)*ln(a*cos(x)+b*sin(x))/(a^2+b^2)^4+1/4*a*b*(5*a^2-3*b^2)*co
s(x)*sin(x)/(a^2+b^2)^3-1/2*a*b*cos(x)^3*sin(x)/(a^2+b^2)^2-2*a^2*b^2*sin(x
)^2/(a^2+b^2)^3+1/4*a^2*sin(x)^4/(a^2+b^2)^2-a^2*b^3*sin(x)/(a^2+b^2)^3/(a*
cos(x)+b*sin(x))
```

Rubi [A]
time = 0.86, antiderivative size = 289, normalized size of antiderivative = 1.38, number of steps used = 48, number of rules used = 12, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3190, 3188, 2645, 30, 2648, 2715, 8, 2644, 3177, 3212, 3176, 3154}

$$\frac{abx}{4\,(a^2+b^2)^2}+\frac{a^2\sin^4(x)}{4\,(a^2+b^2)^2}-\frac{2a^2b^2\sin^2(x)}{(a^2+b^2)^3}-\frac{b^2\cos^4(x)}{4\,(a^2+b^2)^2}-\frac{ab\sin(x)\cos^3(x)}{2\,(a^2+b^2)^2}+\frac{ab\sin(x)\cos(x)}{4\,(a^2+b^2)^2}+\frac{3a^2b^4\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^4}-\frac{ab^3x}{(a^2+b^2)^3}-\frac{a^2b^3\sin(x)}{(a^2+b^2)^3\,(a\cos(x)+b\sin(x))}-\frac{ab^3\sin(x)\cos(x)}{(a^2+b^2)^3}-\frac{3a^4b^2\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^4}-\frac{a^3bx}{(a^2+b^2)^3}+\frac{a^3b\sin(x)\cos(x)}{(a^2+b^2)^3}+\frac{6a^3b^3x}{(a^2+b^2)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[x]^3*Sin[x]^3)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (6*a^3*b^3*x)/(a^2 + b^2)^4 - (a^3*b*x)/(a^2 + b^2)^3 - (a*b^3*x)/(a^2 + b^
2)^3 + (a*b*x)/(4*(a^2 + b^2)^2) - (b^2*Cos[x]^4)/(4*(a^2 + b^2)^2) - (3*a^
4*b^2*Log[a*Cos[x] + b*Sin[x]])/(a^2 + b^2)^4 + (3*a^2*b^4*Log[a*Cos[x] + b
*Sin[x]])/(a^2 + b^2)^4 + (a^3*b*Cos[x]*Sin[x])/(a^2 + b^2)^3 - (a*b^3*Cos[
x]*Sin[x])/(a^2 + b^2)^3 + (a*b*Cos[x]*Sin[x])/(4*(a^2 + b^2)^2) - (a*b*Cos
[x]^3*Sin[x])/(2*(a^2 + b^2)^2) - (2*a^2*b^2*Sin[x]^2)/(a^2 + b^2)^3 + (a^2
*Sin[x]^4)/(4*(a^2 + b^2)^2) - (a^2*b^3*Sin[x])/((a^2 + b^2)^3*(a*Cos[x] +
b*Sin[x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2644

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2645

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> Dist[-(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2648

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(-a)*(b*Cos[e + f*x])^(n + 1)*((a*Sin[e + f*x])^(m -
1)/(b*f*(m + n))), x] + Dist[a^2*((m - 1)/(m + n)), Int[(b*Cos[e + f*x])^n*
(a*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1]
&& NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3154

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-2), x
_Symbol] :> Simp[Sin[c + d*x]/(a*d*(a*Cos[c + d*x] + b*Sin[c + d*x])), x] /
; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3176

```
Int[sin[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[b*(x/(a^2 + b^2)), x] - Dist[a/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3177

```
Int[cos[(c_.) + (d_.)*(x_)]/(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.
) + (d_.)*(x_)]), x_Symbol] :> Simp[a*(x/(a^2 + b^2)), x] + Dist[b/(a^2 + b
^2), Int[(b*Cos[c + d*x] - a*Sin[c + d*x])/(a*Cos[c + d*x] + b*Sin[c + d*x]
), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3188

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Dist[b
/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1), x], x] + (Dist[a/(a^
2 + b^2), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^n, x], x] - Dist[a*(b/(a^2
+ b^2)), Int[Cos[c + d*x]^(m - 1)*(Sin[c + d*x]^(n - 1)/(a*Cos[c + d*x] + b
*Sin[c + d*x])), x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0] &&
IGtQ[m, 0] && IGtQ[n, 0]
```

Rule 3190

```
Int[cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.)*(cos[(c_.)
+ (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(p_), x_Symbol] :> Dis
t[b/(a^2 + b^2), Int[Cos[c + d*x]^m*Sin[c + d*x]^(n - 1)*(a*Cos[c + d*x] +
b*Sin[c + d*x])^(p + 1), x], x] + (Dist[a/(a^2 + b^2), Int[Cos[c + d*x]^(m
- 1)*Sin[c + d*x]^n*(a*Cos[c + d*x] + b*Sin[c + d*x])^(p + 1), x], x] - Dis
t[a*(b/(a^2 + b^2)), Int[Cos[c + d*x]^(m - 1)*Sin[c + d*x]^(n - 1)*(a*Cos[c
 + d*x] + b*Sin[c + d*x])^p, x], x]) /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 +
 b^2, 0] && IGtQ[m, 0] && IGtQ[n, 0] && ILtQ[p, 0]
```

Rule 3212

```
Int[((A_.) + cos[(d_.) + (e_.)*(x_)]*(B_.) + (C_.)*sin[(d_.) + (e_.)*(x_)])
/((a_.) + cos[(d_.) + (e_.)*(x_)]*(b_.) + (c_.)*sin[(d_.) + (e_.)*(x_)]), x
_Symbol] :> Simp[(b*B + c*C)*(x/(b^2 + c^2)), x] + Simp[(c*B - b*C)*(Log[a
+ b*Cos[d + e*x] + c*Sin[d + e*x]]/(e*(b^2 + c^2))), x] /; FreeQ[{a, b, c,
d, e, A, B, C}, x] && NeQ[b^2 + c^2, 0] && EqQ[A*(b^2 + c^2) - a*(b*B + c*C
), 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(x)\sin^3(x)}{(a\cos(x)+b\sin(x))^2}\,dx &= \frac{a\int \frac{\cos^2(x)\sin^3(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}+\frac{b\int \frac{\cos^3(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{a^2+b^2}-\frac{(ab)\int \frac{\cos^2(x)\sin^2(x)}{(a\cos(x)+b\sin(x))^2}\,dx}{a^2+b^2}\\
&= \frac{a^2\int \cos(x)\sin^3(x)\,dx}{(a^2+b^2)^2}+2\frac{(ab)\int \cos^2(x)\sin^2(x)\,dx}{(a^2+b^2)^2}-2\frac{(a^2b)\int \frac{\cos(x)\sin^2(x)}{a\cos(x)+b\sin(x)}\,dx}{(a^2+b^2)^2}\\
&= -2\left(\frac{(a^3b)\int \sin^2(x)\,dx}{(a^2+b^2)^3}+\frac{(a^2b^2)\int \cos(x)\sin(x)\,dx}{(a^2+b^2)^3}-\frac{(a^3b^2)\int \frac{\sin(x)}{a\cos(x)+b\sin(x)}\,dx}{(a^2+b^2)^3}\right.\\
&= \frac{2a^3b^3x}{(a^2+b^2)^4}-\frac{b^2\cos^4(x)}{4(a^2+b^2)^2}+\frac{a^2\sin^4(x)}{4(a^2+b^2)^2}-\frac{a^2b^3\sin(x)}{(a^2+b^2)^3(a\cos(x)+b\sin(x))}-\\
&= \frac{2a^3b^3x}{(a^2+b^2)^4}-\frac{b^2\cos^4(x)}{4(a^2+b^2)^2}-\frac{a^4b^2\log(a\cos(x)+b\sin(x))}{(a^2+b^2)^4}+\frac{a^2b^4\log(a\cos(x)+}{(a^2+b^2)^4}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.02, size = 409, normalized size = 1.95

$$\frac{-12ab(a^2-3b^2)(3a^2-b^2)x+6i(a^6-15a^4b^2+15a^2b^4-b^6)x-6i(a^6-15a^4b^2+15a^2b^4-b^6)\operatorname{ArcTan}(\tan(x))-4(a^2+b^2)(a^4-6a^2b^2+b^4)\cos(2x)+(a^2-b^2)(a^2+b^2)^2\cos(4x)+3(a^6-15a^4b^2+15a^2b^4-b^6)\log((a\cos(x)+b\sin(x))^2)+\frac{2b(a^2+b^2)(3a^4-10a^2b^2+3b^4)\sin(x)}{a\cos(x)+b\sin(x)}+\frac{3(a^2+b^2)^2\left(a\cos(x)(-2i(a+ib)^2x+(-a^2+b^2)\log((a\cos(x)+b\sin(x))^2))+b(2(a+ib)(a(-1-ix)+b(i+x))+(-a^2+b^2)\log((a\cos(x)+b\sin(x))^2))\sin(x)+2i(a^2-b^2)\operatorname{ArcTan}(\tan(x))(a\cos(x)+b\sin(x))\right)}{a\cos(x)+b\sin(x)}+16ab(a^4-b^4)\sin(2x)-2ab(a^2+b^2)^2\sin(4x)}{32(a^2+b^2)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[x]^3*Sin[x]^3)/(a*Cos[x] + b*Sin[x])^2,x]
```

```
[Out] (-12*a*b*(a^2 - 3*b^2)*(3*a^2 - b^2)*x + (6*I)*(a^6 - 15*a^4*b^2 + 15*a^2*b
^4 - b^6)*x - (6*I)*(a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)*ArcTan[Tan[x]] -
4*(a^2 + b^2)*(a^4 - 6*a^2*b^2 + b^4)*Cos[2*x] + (a^2 - b^2)*(a^2 + b^2)^2*
Cos[4*x] + 3*(a^6 - 15*a^4*b^2 + 15*a^2*b^4 - b^6)*Log[(a*Cos[x] + b*Sin[x]
)^2] + (2*b*(a^2 + b^2)*(3*a^4 - 10*a^2*b^2 + 3*b^4)*Sin[x])/(a*Cos[x] + b*
Sin[x]) + (3*(a^2 + b^2)^2*(a*Cos[x]*((-2*I)*(a + I*b)^2*x + (-a^2 + b^2)*L
og[(a*Cos[x] + b*Sin[x])^2]) + b*(2*(a + I*b)*(a*(-1 - I*x) + b*(I + x)) +
(-a^2 + b^2)*Log[(a*Cos[x] + b*Sin[x])^2])*Sin[x] + (2*I)*(a^2 - b^2)*ArcTa
n[Tan[x]]*(a*Cos[x] + b*Sin[x])))/(a*Cos[x] + b*Sin[x]) + 16*a*b*(a^4 - b^4
)*Sin[2*x] - 2*a*b*(a^2 + b^2)^2*Sin[4*x])/(32*(a^2 + b^2)^4)
```

Maple [A]
time = 0.54, size = 232, normalized size = 1.10

method	result
default	$\frac{a^3b^2}{(a^2+b^2)^3(a+b\tan(x))}-\frac{3a^2b^2(a^2-b^2)\ln(a+b\tan(x))}{(a^2+b^2)^4}+\frac{\left(\frac{1}{2}a^3b^3-\frac{3}{4}ab^5+\frac{5}{4}a^5b\right)(\tan^3(x))+\left(-\frac{1}{2}a^6+a^4b^2+\frac{3}{2}a^2b^4\right)(\tan^2(x))+\left(\frac{3}{4}a^5b\right.}{(\tan^2(x)+1)^2}$
risch	$\frac{3abx}{4(4ia^3b-4iab^3-a^4+6a^2b^2-b^4)}+\frac{e^{4ix}}{-128iab+64a^2-64b^2}-\frac{ie^{2ix}b}{16(-3ia^2b+ib^3+a^3-3ab^2)}-\frac{e^{2ix}a}{16(-3ia^2b+ib^3+a^3-3ab^2)}+\frac{}{16(2i}$
norman	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*b^2/(a^2+b^2)^3/(a+b*tan(x))-3*a^2*b^2*(a^2-b^2)/(a^2+b^2)^4*ln(a+b*tan
(x))+1/(a^2+b^2)^4*(((1/2*a^3*b^3-3/4*a*b^5+5/4*a^5*b)*tan(x)^3+(-1/2*a^6+a
^4*b^2+3/2*a^2*b^4)*tan(x)^2+(3/4*a^5*b-1/2*a^3*b^3-5/4*a*b^5)*tan(x)-1/4*a
^6+5/4*a^4*b^2+5/4*a^2*b^4-1/4*b^6)/(tan(x)^2+1)^2+3/4*a*b*(1/2*(4*a^3*b-4*
a*b^3)*ln(tan(x)^2+1)+(-a^4+6*a^2*b^2-b^4)*arctan(tan(x))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 456 vs. 2(200) = 400.
time = 0.49, size = 456, normalized size = 2.17

$$-\frac{3(a^5b-6a^3b^3+ab^5)x}{4(a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8)}-\frac{3(a^4b^2-a^2b^4)\log(b\tan(x)+a)}{a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8}+\frac{3(a^4b^2-a^2b^4)\log(\tan(x)^2+1)}{2(a^8+4a^6b^2+6a^4b^4+4a^2b^6+b^8)}-\frac{a^5-10a^3b^2+ab^4-3(3a^3b^2-ab^4)\tan(x)^4-3(a^4b+a^2b^3)\tan(x)^3+(2a^5-17a^3b^2+5ab^4)\tan(x)^2-(2a^4b+a^2b^3-b^5)\tan(x)}{4(a^7+3a^5b^2+3a^3b^4+ab^6+(a^6b+3a^4b^3+3a^2b^5+b^7)\tan(x)^5+(a^7+3a^5b^2+3a^3b^4+ab^6)\tan(x)^4+2(a^6b+3a^4b^3+3a^2b^5+b^7)\tan(x)^3+2(a^7+3a^5b^2+3a^3b^4+ab^6)\tan(x)^2+(a^6b+3a^4b^3+3a^2b^5+b^7)\tan(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="maxima")
```

```
[Out] -3/4*(a^5*b - 6*a^3*b^3 + a*b^5)*x/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
 + b^8) - 3*(a^4*b^2 - a^2*b^4)*log(b*tan(x) + a)/(a^8 + 4*a^6*b^2 + 6*a^4*
b^4 + 4*a^2*b^6 + b^8) + 3/2*(a^4*b^2 - a^2*b^4)*log(tan(x)^2 + 1)/(a^8 + 4
*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8) - 1/4*(a^5 - 10*a^3*b^2 + a*b^4 - 3
*(3*a^3*b^2 - a*b^4)*tan(x)^4 - 3*(a^4*b + a^2*b^3)*tan(x)^3 + (2*a^5 - 17*
a^3*b^2 + 5*a*b^4)*tan(x)^2 - (2*a^4*b + a^2*b^3 - b^5)*tan(x))/(a^7 + 3*a^
5*b^2 + 3*a^3*b^4 + a*b^6 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*tan(x)^5
+ (a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*tan(x)^4 + 2*(a^6*b + 3*a^4*b^3 + 3
*a^2*b^5 + b^7)*tan(x)^3 + 2*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*tan(x)^2
 + (a^6*b + 3*a^4*b^3 + 3*a^2*b^5 + b^7)*tan(x))
```

Fricas [A]
time = 2.69, size = 371, normalized size = 1.77

$$\frac{8\left(a^{7}+3\,a^{5}b^{2}+3\,a^{3}b^{4}+ab^{6}\right)\cos\left(x\right)^{5}-8\left(2\,a^{7}+3\,a^{5}b^{2}-ab^{6}\right)\cos\left(x\right)^{3}+\left(5\,a^{7}+21\,a^{5}b^{2}+27\,a^{3}b^{4}-21\,ab^{6}-24\left(a^{6}b-6\,a^{4}b^{3}+a^{2}b^{5}\right)x\right)\cos\left(x\right)-48\left(\left(a^{5}b^{2}-a^{3}b^{4}\right)\cos\left(x\right)+\left(a^{4}b^{3}-a^{2}b^{5}\right)\sin\left(x\right)\right)\log\left(2\,ab\cos\left(x\right)\sin\left(x\right)+\left(a^{2}-b^{2}\right)\cos\left(x\right)^{2}+b^{2}\right)+\left(5\,a^{6}b-51\,a^{4}b^{3}-21\,a^{2}b^{5}+3\,b^{7}-8\left(a^{6}b+3\,a^{4}b^{3}+3\,a^{2}b^{5}+b^{7}\right)\cos\left(x\right)^{4}+24\left(a^{6}b+2\,a^{4}b^{3}+a^{2}b^{5}\right)\cos\left(x\right)^{2}-24\left(a^{5}b^{2}-6\,a^{3}b^{4}+ab^{6}\right)x\right)\sin\left(x\right)}{32\left(\left(a^{9}+4\,a^{7}b^{2}+6\,a^{5}b^{4}+4\,a^{3}b^{6}+ab^{8}\right)\cos\left(x\right)+\left(a^{8}b+4\,a^{6}b^{3}+6\,a^{4}b^{5}+4\,a^{2}b^{7}+b^{9}\right)\sin\left(x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="fricas")
```

```
[Out] 1/32*(8*(a^7 + 3*a^5*b^2 + 3*a^3*b^4 + a*b^6)*cos(x)^5 - 8*(2*a^7 + 3*a^5*b
^2 - a*b^6)*cos(x)^3 + (5*a^7 + 21*a^5*b^2 + 27*a^3*b^4 - 21*a*b^6 - 24*(a^
6*b - 6*a^4*b^3 + a^2*b^5)*x)*cos(x) - 48*((a^5*b^2 - a^3*b^4)*cos(x) + (a^
4*b^3 - a^2*b^5)*sin(x))*log(2*a*b*cos(x)*sin(x) + (a^2 - b^2)*cos(x)^2 + b
^2) + (5*a^6*b - 51*a^4*b^3 - 21*a^2*b^5 + 3*b^7 - 8*(a^6*b + 3*a^4*b^3 + 3
*a^2*b^5 + b^7)*cos(x)^4 + 24*(a^6*b + 2*a^4*b^3 + a^2*b^5)*cos(x)^2 - 24*(
a^5*b^2 - 6*a^3*b^4 + a*b^6)*x)*sin(x))/((a^9 + 4*a^7*b^2 + 6*a^5*b^4 + 4*a
^3*b^6 + a*b^8)*cos(x) + (a^8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9)*
sin(x))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3*sin(x)**3/(a*cos(x)+b*sin(x))**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 435 vs. 2(200) = 400.
time = 0.44, size = 435, normalized size = 2.07

$$-\frac{3\left(a^{5}b-6\,a^{3}b^{3}+ab^{5}\right)x}{4\left(a^{8}+4\,a^{6}b^{2}+6\,a^{4}b^{4}+4\,a^{2}b^{6}+b^{8}\right)}+\frac{3\left(a^{4}b^{2}-a^{2}b^{4}\right)\log\left(\tan\left(x\right)^{2}+1\right)}{2\left(a^{8}+4\,a^{6}b^{2}+6\,a^{4}b^{4}+4\,a^{2}b^{6}+b^{8}\right)}-\frac{3\left(a^{4}b^{3}-a^{2}b^{5}\right)\log\left(\left|b\tan\left(x\right)+a\right|\right)}{a^{8}b+4\,a^{6}b^{3}+6\,a^{4}b^{5}+4\,a^{2}b^{7}+b^{9}}+\frac{3\,a^{4}b^{3}\tan\left(x\right)-3\,a^{2}b^{5}\tan\left(x\right)+4\,a^{5}b^{2}-2\,a^{3}b^{4}}{\left(a^{8}+4\,a^{6}b^{2}+6\,a^{4}b^{4}+4\,a^{2}b^{6}+b^{8}\right)\left(b\tan\left(x\right)+a\right)}-\frac{9\,a^{4}b^{2}\tan\left(x\right)^{4}-9\,a^{2}b^{4}\tan\left(x\right)^{4}-5\,a^{5}b\tan\left(x\right)^{3}-2\,a^{3}b^{3}\tan\left(x\right)^{3}+3\,ab^{5}\tan\left(x\right)^{3}+2\,a^{6}\tan\left(x\right)^{2}+14\,a^{4}b^{2}\tan\left(x\right)^{2}-24\,a^{2}b^{4}\tan\left(x\right)^{2}-3\,a^{5}b\tan\left(x\right)+2\,a^{3}b^{3}\tan\left(x\right)+5\,ab^{5}\tan\left(x\right)+a^{6}+4\,a^{4}b^{2}-14\,a^{2}b^{4}+b^{6}}{4\left(a^{8}+4\,a^{6}b^{2}+6\,a^{4}b^{4}+4\,a^{2}b^{6}+b^{8}\right)\left(\tan\left(x\right)^{2}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3*sin(x)^3/(a*cos(x)+b*sin(x))^2,x, algorithm="giac")
```

```
[Out] -3/4*(a^5*b - 6*a^3*b^3 + a*b^5)*x/(a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b^6
 + b^8) + 3/2*(a^4*b^2 - a^2*b^4)*log(tan(x)^2 + 1)/(a^8 + 4*a^6*b^2 + 6*a^
4*b^4 + 4*a^2*b^6 + b^8) - 3*(a^4*b^3 - a^2*b^5)*log(abs(b*tan(x) + a))/(a^
8*b + 4*a^6*b^3 + 6*a^4*b^5 + 4*a^2*b^7 + b^9) + (3*a^4*b^3*tan(x) - 3*a^2*
b^5*tan(x) + 4*a^5*b^2 - 2*a^3*b^4)/((a^8 + 4*a^6*b^2 + 6*a^4*b^4 + 4*a^2*b
^6 + b^8)*(b*tan(x) + a)) - 1/4*(9*a^4*b^2*tan(x)^4 - 9*a^2*b^4*tan(x)^4 -
5*a^5*b*tan(x)^3 - 2*a^3*b^3*tan(x)^3 + 3*a*b^5*tan(x)^3 + 2*a^6*tan(x)^2 +
 14*a^4*b^2*tan(x)^2 - 24*a^2*b^4*tan(x)^2 - 3*a^5*b*tan(x) + 2*a^3*b^3*tan
(x) + 5*a*b^5*tan(x) + a^6 + 4*a^4*b^2 - 14*a^2*b^4 + b^6)/((a^8 + 4*a^6*b^
2 + 6*a^4*b^4 + 4*a^2*b^6 + b^8)*(tan(x)^2 + 1)^2)
```

Mupad [B]
time = 13.92, size = 2500, normalized size = 11.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(x)^3*sin(x)^3)/(a*cos(x) + b*sin(x))^2,x)
```

```
[Out] ((tan(x/2)^4*(a*b^2 + 4*a^3))/(a^4 + b^4 + 2*a^2*b^2) - (tan(x/2)^6*(a*b^2
+ 4*a^3))/(a^4 + b^4 + 2*a^2*b^2) - (3*a*b^2*tan(x/2)^2)/(a^4 + b^4 + 2*a^2
*b^2) + (3*a*b^2*tan(x/2)^8)/(a^4 + b^4 + 2*a^2*b^2) + (3*b*tan(x/2)^9*(a^4
 - 3*a^2*b^2))/(2*(a^6 + b^6 + 3*a^2*b^4 + 3*a^4*b^2)) + (3*b*tan(x/2)*(a^4
 - 3*a^2*b^2))/(2*(a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) + (4*b*tan(x/2)^3*(a
^4 + b^4 - 4*a^2*b^2))/((a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) + (4*b*tan(x/2
)^7*(a^4 + b^4 - 4*a^2*b^2))/((a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)) - (3*b*t
an(x/2)^5*(a^4 + 13*a^2*b^2))/((a^2 + b^2)*(a^4 + b^4 + 2*a^2*b^2)))/(a + 2
*b*tan(x/2) + 3*a*tan(x/2)^2 + 2*a*tan(x/2)^4 - 2*a*tan(x/2)^6 - 3*a*tan(x/
2)^8 - a*tan(x/2)^10 + 8*b*tan(x/2)^3 + 12*b*tan(x/2)^5 + 8*b*tan(x/2)^7 +
2*b*tan(x/2)^9) + (log(a + 2*b*tan(x/2) - a*tan(x/2)^2)*(3*a^2*b^4 - 3*a^4*
b^2))/(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2) - (log(1/(cos(x) + 1)
)*(96*a^2*b^4 - 96*a^4*b^2))/(2*(16*a^8 + 16*b^8 + 64*a^2*b^6 + 96*a^4*b^4
+ 64*a^6*b^2)) + (3*a*b*atan((tan(x/2)*((((6*(45*a^7*b^10 - 18*a^5*b^12 - 1
35*a^9*b^8 + 99*a^11*b^6 + 9*a^13*b^4))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*
b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^
4 + 9*a^16*b^2) - (((6*(6*a^3*b^16 - 153*a^5*b^14 - 180*a^7*b^12 + 357*a^9*
b^10 + 534*a^11*b^8 + 81*a^13*b^6 - 72*a^15*b^4 + 3*a^17*b^2))/(a^18 + b^18
 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 8
4*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2) - ((96*a^2*b^4 - 96*a^4*b^2)*((6*(8*
a^3*b^18 + 112*a^5*b^16 + 464*a^7*b^14 + 880*a^9*b^12 + 800*a^11*b^10 + 208
*a^13*b^8 - 208*a^15*b^6 - 176*a^17*b^4 - 40*a^19*b^2))/(a^18 + b^18 + 9*a^
2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*
```

b^6 + 36*a^14*b^4 + 9*a^16*b^2) - (3*(96*a^2*b^4 - 96*a^4*b^2)*(16*a*b^22 + 160*a^3*b^20 + 720*a^5*b^18 + 1920*a^7*b^16 + 3360*a^9*b^14 + 4032*a^11*b^12 + 3360*a^13*b^10 + 1920*a^15*b^8 + 720*a^17*b^6 + 160*a^19*b^4 + 16*a^21*b^2))/((16*a^8 + 16*b^8 + 64*a^2*b^6 + 96*a^4*b^4 + 64*a^6*b^2)*(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2))))/(2*(16*a^8 + 16*b^8 + 64*a^2*b^6 + 96*a^4*b^4 + 64*a^6*b^2)))*(96*a^2*b^4 - 96*a^4*b^2))/(2*(16*a^8 + 16*b^8 + 64*a^2*b^6 + 96*a^4*b^4 + 64*a^6*b^2)) - (3*a*b*((3*a*b*((6*(8*a^3*b^18 + 112*a^5*b^16 + 464*a^7*b^14 + 880*a^9*b^12 + 800*a^11*b^10 + 208*a^13*b^8 - 208*a^15*b^6 - 176*a^17*b^4 - 40*a^19*b^2))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2) - (3*(96*a^2*b^4 - 96*a^4*b^2)*(16*a*b^22 + 160*a^3*b^20 + 720*a^5*b^18 + 1920*a^7*b^16 + 3360*a^9*b^14 + 4032*a^11*b^12 + 3360*a^13*b^10 + 1920*a^15*b^8 + 720*a^17*b^6 + 160*a^19*b^4 + 16*a^21*b^2))/((16*a^8 + 16*b^8 + 64*a^2*b^6 + 96*a^4*b^4 + 64*a^6*b^2)*(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2)))*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2))/(4*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) - (9*a*b*(96*a^2*b^4 - 96*a^4*b^2)*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2)*(16*a*b^22 + 160*a^3*b^20 + 720*a^5*b^18 + 1920*a^7*b^16 + 3360*a^9*b^14 + 4032*a^11*b^12 + 3360*a^13*b^10 + 1920*a^15*b^8 + 720*a^17*b^6 + 160*a^19*b^4 + 16*a^21*b^2))/(4*(16*a^8 + 16*b^8 + 64*a^2*b^6 + 96*a^4*b^4 + 64*a^6*b^2)*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)*(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2)))*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2))/(4*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)) + (27*a^2*b^2*(96*a^2*b^4 - 96*a^4*b^2)*(2*a*b - a^2 + b^2)^2*(2*a*b + a^2 - b^2)^2*(16*a*b^22 + 160*a^3*b^20 + 720*a^5*b^18 + 1920*a^7*b^16 + 3360*a^9*b^14 + 4032*a^11*b^12 + 3360*a^13*b^10 + 1920*a^15*b^8 + 720*a^17*b^6 + 160*a^19*b^4 + 16*a^21*b^2))/(16*(16*a^8 + 16*b^8 + 64*a^2*b^6 + 96*a^4*b^4 + 64*a^6*b^2)*(a^8 + b^8 + 4*a^2*b^6 + 6*a^4*b^4 + 4*a^6*b^2)^2*(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2)))*(18*a*b^9 + 18*a^9*b - 280*a^3*b^7 + 556*a^5*b^5 - 280*a^7*b^3))/(a^10 + b^10 + 53*a^2*b^8 - 38*a^4*b^6 - 38*a^6*b^4 + 53*a^8*b^2)^2 + ((((96*a^2*b^4 - 96*a^4*b^2)*((3*a*b*((6*(8*a^3*b^18 + 112*a^5*b^16 + 464*a^7*b^14 + 880*a^9*b^12 + 800*a^11*b^10 + 208*a^13*b^8 - 208*a^15*b^6 - 176*a^17*b^4 - 40*a^19*b^2))/(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2) - (3*(96*a^2*b^4 - 96*a^4*b^2)*(16*a*b^22 + 160*a^3*b^20 + 720*a^5*b^18 + 1920*a^7*b^16 + 3360*a^9*b^14 + 4032*a^11*b^12 + 3360*a^13*b^10 + 1920*a^15*b^8 + 720*a^17*b^6 + 160*a^19*b^4 + 16*a^21*b^2))/((16*a^8 + 16*b^8 + 64*a^2*b^6 + 96*a^4*b^4 + 64*a^6*b^2)*(a^18 + b^18 + 9*a^2*b^16 + 36*a^4*b^14 + 84*a^6*b^12 + 126*a^8*b^10 + 126*a^10*b^8 + 84*a^12*b^6 + 36*a^14*b^4 + 9*a^16*b^2)))*(2*a*b - a^2 + b^2)*(2*a*b + a^2 - b^2))/(4*(a^8 + b^8 +...

3.293 $\int \frac{\tan(x)}{b\cos(x)+a\sin(x)}\,dx$

Optimal. Leaf size=47

$$\frac{\tanh^{-1}(\sin(x))}{a}+\frac{b\tanh^{-1}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$$

```
[Out] arctanh(sin(x))/a+b*arctanh((a*cos(x)-b*sin(x))/(a^2+b^2)^(1/2))/a/(a^2+b^2
)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3189, 3855, 3153, 212}

$$\frac{b\tanh^{-1}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}+\frac{\tanh^{-1}(\sin(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[x]/(b*Cos[x] + a*Sin[x]),x]
```

```
[Out] ArcTanh[Sin[x]]/a + (b*ArcTanh[(a*Cos[x] - b*Sin[x])/Sqrt[a^2 + b^2]])/(a*S
qrt[a^2 + b^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3189

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan(x)}{b\cos(x)+a\sin(x)}\,dx &= \int \left(\frac{\sec(x)}{a}-\frac{b}{a(b\cos(x)+a\sin(x))}\right)dx\\ &= \frac{\int \sec(x)\,dx}{a}-\frac{b\int \frac{1}{b\cos(x)+a\sin(x)}\,dx}{a}\\ &= \frac{\tanh^{-1}(\sin(x))}{a}+\frac{b\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,a\cos(x)-b\sin(x)\right)}{a}\\ &= \frac{\tanh^{-1}(\sin(x))}{a}+\frac{b\tanh^{-1}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 76, normalized size = 1.62

$$\frac{-\frac{2b\tanh^{-1}\left(\frac{-a+b\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\cos\left(\frac{x}{2}\right)+\sin\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[x]/(b*Cos[x] + a*Sin[x]),x]
[Out] ((-2*b*ArcTanh[(-a + b*Tan[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] - Log[Co
s[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])/a
```

Maple [A]
time = 0.27, size = 63, normalized size = 1.34

method	result	size
default	$\frac{\ln\left(\tan\left(\frac{x}{2}\right)+1\right)}{a}-\frac{\ln\left(\tan\left(\frac{x}{2}\right)-1\right)}{a}+\frac{2b\,\mathrm{arctanh}\left(\frac{-2b\tan\left(\frac{x}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{a\sqrt{a^2+b^2}}$	63
risch	$\frac{ib\ln\left(\mathrm{e}^{ix}+\frac{ia+b}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,a}-\frac{ib\ln\left(\mathrm{e}^{ix}-\frac{ia+b}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,a}+\frac{\ln\left(\mathrm{e}^{ix}+i\right)}{a}-\frac{\ln\left(\mathrm{e}^{ix}-i\right)}{a}$	124

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(b*cos(x)+a*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*ln(tan(1/2*x)+1)-1/a*ln(tan(1/2*x)-1)+2/a*b/(a^2+b^2)^(1/2)*arctanh(1/2
*(-2*b*tan(1/2*x)+2*a)/(a^2+b^2)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 98 vs. $2(43) = 86$.
time = 0.48, size = 98, normalized size = 2.09

$$\frac{b\log\left(\frac{a-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{a-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,a}+\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}-1\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(b*cos(x)+a*sin(x)),x, algorithm="maxima")
```

```
[Out] b*log((a - b*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(a - b*sin(x)/(cos(x) +
 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a) + log(sin(x)/(cos(x) + 1) + 1)/
a - log(sin(x)/(cos(x) + 1) - 1)/a
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(43) = 86$.
time = 2.79, size = 140, normalized size = 2.98

$$\frac{\sqrt{a^2+b^2}\,b\log\left(\frac{2\,ab\cos(x)\sin(x)-(a^2-b^2)\cos(x)^2-a^2-2\,b^2-2\,\sqrt{a^2+b^2}\,(a\cos(x)-b\sin(x))}{2\,ab\cos(x)\sin(x)-(a^2-b^2)\cos(x)^2+a^2}\right)+(a^2+b^2)\log\left(\sin\left(x\right)+1\right)-(a^2+b^2)\log\left(-\sin\left(x\right)+1\right)}{2\left(a^3+ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(b*cos(x)+a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + b^2)*b*log((2*a*b*cos(x)*sin(x) - (a^2 - b^2)*cos(x)^2 - a^
2 - 2*b^2 - 2*sqrt(a^2 + b^2)*(a*cos(x) - b*sin(x)))/(2*a*b*cos(x)*sin(x) -
 (a^2 - b^2)*cos(x)^2 + a^2)) + (a^2 + b^2)*log(sin(x) + 1) - (a^2 + b^2)*l
og(-sin(x) + 1))/(a^3 + a*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan\left(x\right)}{a\sin\left(x\right)+b\cos\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(b*cos(x)+a*sin(x)),x)
```

```
[Out] Integral(tan(x)/(a*sin(x) + b*cos(x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(43) = 86.
time = 0.45, size = 90, normalized size = 1.91

$$\frac{b\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,a}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)+1\right|\right)}{a}-\frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)-1\right|\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(x)/(b*cos(x)+a*sin(x)),x, algorithm="giac")
```

```
[Out] b*log(abs(2*b*tan(1/2*x) - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*tan(1/2*x) - 2*
a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*a) + log(abs(tan(1/2*x) + 1))/a -
log(abs(tan(1/2*x) - 1))/a
```

Mupad [B]
time = 1.48, size = 408, normalized size = 8.68

$$\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{x}{2}\right)\right)}{a}-\frac{2\,b\,\mathrm{atanh}\left(\frac{64\,b^3}{\sqrt{a^2+b^2}\,\left(128\,b^2\tan\left(\frac{x}{2}\right)-\frac{128\,b^4\tan\left(\frac{x}{2}\right)}{a^2+b^2}+\frac{64\,a\,b^3}{a^2+b^2}\right)}-\frac{64\,b^5}{{\left(a^2+b^2\right)}^{3/2}\,\left(128\,b^2\tan\left(\frac{x}{2}\right)-\frac{128\,b^4\tan\left(\frac{x}{2}\right)}{a^2+b^2}+\frac{64\,a\,b^3}{a^2+b^2}\right)}+\frac{128\,b^4\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}\,\left(\frac{64\,a^2\,b^3}{a^2+b^2}+128\,a\,b^2\tan\left(\frac{x}{2}\right)-\frac{128\,a\,b^4\tan\left(\frac{x}{2}\right)}{a^2+b^2}\right)}-\frac{128\,b^6\tan\left(\frac{x}{2}\right)}{{\left(a^2+b^2\right)}^{3/2}\,\left(\frac{64\,a^2\,b^3}{a^2+b^2}+128\,a\,b^2\tan\left(\frac{x}{2}\right)-\frac{128\,a\,b^4\tan\left(\frac{x}{2}\right)}{a^2+b^2}\right)}+\frac{128\,a\,b^2\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}\,\left(128\,b^2\tan\left(\frac{x}{2}\right)-\frac{128\,b^4\tan\left(\frac{x}{2}\right)}{a^2+b^2}+\frac{64\,a\,b^3}{a^2+b^2}\right)}-\frac{192\,a\,b^4\tan\left(\frac{x}{2}\right)}{{\left(a^2+b^2\right)}^{3/2}\,\left(128\,b^2\tan\left(\frac{x}{2}\right)-\frac{128\,b^4\tan\left(\frac{x}{2}\right)}{a^2+b^2}+\frac{64\,a\,b^3}{a^2+b^2}\right)}\right)}{a\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(x)/(b*cos(x) + a*sin(x)),x)
```

```
[Out] (2*atanh(tan(x/2)))/a - (2*b*atanh((64*b^3)/((a^2 + b^2)^(1/2)*(128*b^2*tan
(x/2) - (128*b^4*tan(x/2))/(a^2 + b^2) + (64*a*b^3)/(a^2 + b^2))) - (64*b^5
)/((a^2 + b^2)^(3/2)*(128*b^2*tan(x/2) - (128*b^4*tan(x/2))/(a^2 + b^2) + (
64*a*b^3)/(a^2 + b^2))) + (128*b^4*tan(x/2))/((a^2 + b^2)^(1/2)*((64*a^2*b^
3)/(a^2 + b^2) + 128*a*b^2*tan(x/2) - (128*a*b^4*tan(x/2))/(a^2 + b^2))) -
(128*b^6*tan(x/2))/((a^2 + b^2)^(3/2)*((64*a^2*b^3)/(a^2 + b^2) + 128*a*b^2
*tan(x/2) - (128*a*b^4*tan(x/2))/(a^2 + b^2))) + (128*a*b^2*tan(x/2))/((a^2
 + b^2)^(1/2)*(128*b^2*tan(x/2) - (128*b^4*tan(x/2))/(a^2 + b^2) + (64*a*b^
3)/(a^2 + b^2))) - (192*a*b^4*tan(x/2))/((a^2 + b^2)^(3/2)*(128*b^2*tan(x/2
) - (128*b^4*tan(x/2))/(a^2 + b^2) + (64*a*b^3)/(a^2 + b^2)))))/(a*(a^2 + b
^2)^(1/2))
```

3.294 $\int \frac{\cot(x)}{b\cos(x)+a\sin(x)}\,dx$

Optimal. Leaf size=48

$$-\frac{\tanh^{-1}(\cos(x))}{b}+\frac{a\tanh^{-1}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}$$

```
[Out] -arctanh(cos(x))/b+a*arctanh((a*cos(x)-b*sin(x))/(a^2+b^2)^(1/2))/b/(a^2+b^
2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3189, 3855, 3153, 212}

$$\frac{a\tanh^{-1}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}-\frac{\tanh^{-1}(\cos(x))}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/(b*Cos[x] + a*Sin[x]),x]
```

```
[Out] -(ArcTanh[Cos[x]]/b) + (a*ArcTanh[(a*Cos[x] - b*Sin[x])/Sqrt[a^2 + b^2]])/(
b*Sqrt[a^2 + b^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3153

```
Int[(cos[(c_.) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x
_Symbol] :> Dist[-d^(-1), Subst[Int[1/(a^2 + b^2 - x^2), x], x, b*Cos[c + d
*x] - a*Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 + b^2, 0]
```

Rule 3189

```
Int[(cos[(c_.) + (d_.)*(x_)]^(m_.)*sin[(c_.) + (d_.)*(x_)]^(n_.))/(cos[(c_.
) + (d_.)*(x_)]*(a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)]), x_Symbol] :> Int[Ex
pandTrig[cos[c + d*x]^m*(sin[c + d*x]^n/(a*cos[c + d*x] + b*sin[c + d*x])),
 x], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegersQ[m, n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(x)}{b\cos(x)+a\sin(x)}\,dx &= \int \left(\frac{\csc(x)}{b}-\frac{a}{b(b\cos(x)+a\sin(x))}\right)dx \\
&= \frac{\int \csc(x)\,dx}{b}-\frac{a\int \frac{1}{b\cos(x)+a\sin(x)}\,dx}{b} \\
&= -\frac{\tanh^{-1}(\cos(x))}{b}+\frac{a\text{Subst}\left(\int \frac{1}{a^2+b^2-x^2}\,dx,x,a\cos(x)-b\sin(x)\right)}{b} \\
&= -\frac{\tanh^{-1}(\cos(x))}{b}+\frac{a\tanh^{-1}\left(\frac{a\cos(x)-b\sin(x)}{\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 60, normalized size = 1.25

$$\frac{-\frac{2a\tanh^{-1}\left(\frac{-a+b\tan\left(\frac{x}{2}\right)}{\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}}-\log\left(\cos\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)\right)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/(b*Cos[x] + a*Sin[x]),x]
[Out] ((-2*a*ArcTanh[(-a + b*Tan[x/2])/Sqrt[a^2 + b^2]])/Sqrt[a^2 + b^2] - Log[Co
s[x/2]] + Log[Sin[x/2]])/b
```

Maple [A]
time = 0.27, size = 49, normalized size = 1.02

method	result	size
default	$\frac{\ln\left(\tan\left(\frac{x}{2}\right)\right)}{b}+\frac{2a\,\mathrm{arctanh}\left(\frac{-2b\tan\left(\frac{x}{2}\right)+2a}{2\sqrt{a^2+b^2}}\right)}{b\sqrt{a^2+b^2}}$	49
risch	$-\frac{ia\ln\left(\mathrm{e}^{ix}-\frac{ia+b}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,b}+\frac{ia\ln\left(\mathrm{e}^{ix}+\frac{ia+b}{\sqrt{-a^2-b^2}}\right)}{\sqrt{-a^2-b^2}\,b}+\frac{\ln\left(\mathrm{e}^{ix}-1\right)}{b}-\frac{\ln\left(\mathrm{e}^{ix}+1\right)}{b}$	122

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(b*cos(x)+a*sin(x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*ln(tan(1/2*x))+2*a/b/(a^2+b^2)^(1/2)*arctanh(1/2*(-2*b*tan(1/2*x)+2*a)/
(a^2+b^2)^(1/2))
```

Maxima [A]
time = 0.48, size = 79, normalized size = 1.65

$$\frac{a\log\left(\frac{a-\frac{b\sin(x)}{\cos(x)+1}+\sqrt{a^2+b^2}}{a-\frac{b\sin(x)}{\cos(x)+1}-\sqrt{a^2+b^2}}\right)}{\sqrt{a^2+b^2}\,b}+\frac{\log\left(\frac{\sin(x)}{\cos(x)+1}\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(b*cos(x)+a*sin(x)),x, algorithm="maxima")
```

```
[Out] a*log((a - b*sin(x)/(cos(x) + 1) + sqrt(a^2 + b^2))/(a - b*sin(x)/(cos(x) +
 1) - sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b) + log(sin(x)/(cos(x) + 1))/b
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(44) = 88.
time = 2.34, size = 142, normalized size = 2.96

$$\frac{\sqrt{a^2+b^2}\,a\log\left(\frac{2\,ab\cos(x)\sin(x)-(a^2-b^2)\cos(x)^2-a^2-2\,b^2-2\,\sqrt{a^2+b^2}\,(a\cos(x)-b\sin(x))}{2\,ab\cos(x)\sin(x)-(a^2-b^2)\cos(x)^2+a^2}\right)-(a^2+b^2)\log\left(\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)+(a^2+b^2)\log\left(-\frac{1}{2}\,\cos\left(x\right)+\frac{1}{2}\right)}{2\,(a^2b+b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(b*cos(x)+a*sin(x)),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(a^2 + b^2)*a*log((2*a*b*cos(x)*sin(x) - (a^2 - b^2)*cos(x)^2 - a^
2 - 2*b^2 - 2*sqrt(a^2 + b^2)*(a*cos(x) - b*sin(x)))/(2*a*b*cos(x)*sin(x) -
 (a^2 - b^2)*cos(x)^2 + a^2)) - (a^2 + b^2)*log(1/2*cos(x) + 1/2) + (a^2 +
b^2)*log(-1/2*cos(x) + 1/2))/(a^2*b + b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot\left(x\right)}{a\sin\left(x\right)+b\cos\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(b*cos(x)+a*sin(x)),x)
```

```
[Out] Integral(cot(x)/(a*sin(x) + b*cos(x)), x)
```

Giac [A]
time = 0.45, size = 75, normalized size = 1.56

$$\frac{a\log\left(\frac{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,a-2\,\sqrt{a^2+b^2}\right|}{\left|2\,b\tan\left(\frac{1}{2}\,x\right)-2\,a+2\,\sqrt{a^2+b^2}\right|}\right)}{\sqrt{a^2+b^2}\,b}+\frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(b*cos(x)+a*sin(x)),x, algorithm="giac")
```

```
[Out] a*log(abs(2*b*tan(1/2*x) - 2*a - 2*sqrt(a^2 + b^2))/abs(2*b*tan(1/2*x) - 2*
a + 2*sqrt(a^2 + b^2)))/(sqrt(a^2 + b^2)*b) + log(abs(tan(1/2*x)))/b
```

Mupad [B]
time = 0.98, size = 123, normalized size = 2.56

$$\frac{\ln\left(\frac{\sin\left(\frac{x}{2}\right)}{\cos\left(\frac{x}{2}\right)}\right)}{b}-\frac{2\,a\,\mathrm{atanh}\left(\frac{\sqrt{a^2+b^2}\,\left(4\mathrm{i}\,\sin\left(\frac{x}{2}\right)\,a^2+2\mathrm{i}\,\cos\left(\frac{x}{2}\right)\,a\,b+1\mathrm{i}\,\sin\left(\frac{x}{2}\right)\,b^2\right)}{a^3\,\sin\left(\frac{x}{2}\right)\,4\mathrm{i}+a^2\,b\,\cos\left(\frac{x}{2}\right)\,1\mathrm{i}+a\,b^2\,\sin\left(\frac{x}{2}\right)\,3\mathrm{i}+b\,\cos\left(\frac{x}{2}\right)\,\left(a^2+b^2\right)\,1\mathrm{i}}\right)}{b\,\sqrt{a^2+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(b*cos(x) + a*sin(x)),x)
```

```
[Out] log(sin(x/2)/cos(x/2))/b - (2*a*atanh(((a^2 + b^2)^(1/2)*(a^2*sin(x/2)*4i +
 b^2*sin(x/2)*1i + a*b*cos(x/2)*2i))/(a^3*sin(x/2)*4i + a^2*b*cos(x/2)*1i +
 a*b^2*sin(x/2)*3i + b*cos(x/2)*(a^2 + b^2)*1i)))/(b*(a^2 + b^2)^(1/2))
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:    #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```